## San Francisco, January 3-6, 2024 <br> Abstracts of the $1192 n$ Meeting

## 00 General

## 1192-00-25396

Suzanne L Weekes*, SIAM. Mathematics in (and for) the Real World.
"Work to prepare for complex futures is timely, given the recent, global examples of extreme weather, clean energy and its role in energy security, the global pandemic, and supply chain challenges. There is a need to ensure that students in all majors eventually develop an increased awareness of these problems through interdisciplinary courses and project work so they understand their role as a citizen and professional, and have the capacity and skills to develop and contribute to solutions." This statement is from the report of the SIAM Convening on Climate Science, Sustainability, and Clean Energy which is available at https://siam.org/publications/reports. In this talk, we consider opportunities to engage and prepare mathematical sciences students to solve real-world problems and real problems facing the world.
(Received June 05, 2023)

1192-00-25403
Terence Tao*, UCLA. Machine Assisted Proof.
For centuries, mathematicians have relied on computers to perform calculations, to suggest conjectures, and as components of mathematical proofs. In the light of more modern tools such as interactive theorem provers, machine learning algorithms, and generative AI, we are beginning to see machines used in more creative and substantive ways in our work. In this talk we survey some historical and recent developments, and speculate on the future roles of machine assistance in mathematics. (Received June 05, 2023)

Suzanne Marie Lenhart*, University of Tennessee, Knoxville. Natural System Management: A Mathematician's Perspective. Mathematical modeling can represent the population dynamics of a variety of natural systems. Specific management features can be included directly into the model framework, and model outcomes can suggest the need for external actions further affecting the environment. Models with discrete and continuous time steps, and some spatial features will be presented. In one example, optimal control techniques to design time varying harvest rates of anchovy stock will be illustrated in a food chain system of differential equations for the Turkish coast of the Black Sea, using landing data. In another example, we find the relationship between air temperature and emergence success of hatchlings across multiple nesting seasons to better understand the potential impact of climate change on Loggerhead sea turtle populations. We demonstrate the effect of changing hatchling emergence success on the juvenile and adult populations using a combination of a statistical model and a discrete time model (with two time scales). (Received June 05, 2023)

## 1192-00-25408

Katherine Ensor*, Rice University. Celebrating Statistical Foundations Driving 21 st -Century Innovation.
Statistical foundations are without question at the core of modern innovation. In today's economy, a common phrase is "data is the new gold". Certainly, we live in an age where data is large, ubiquitous, and comes in many forms. The contributions from the statistical sciences go beyond "data". We are emerging from a pandemic where statisticians around the globe saved lives by contributing critical understanding to vaccines, treatments, pandemic policies, and management. The contributions are universal - from self-driving cars to Mars rovers, to sustainable and improved infrastructure, to clean energy and environmental stewardship, to financial markets and investing, to advances in medicine and medical practices, and even toward a better understanding of the communities in which we live, work, learn and play. This talk will highlight these important contributions and the innovations they made possible and will look to innovations on the horizon.
(Received June 05, 2023)

## 1192-00-31764

Eric Babson*, University. Combinatorial Homotopy Theory. Preliminary report.
This is a placeholder.
(Received September 12, 2023)

1192-00-32432
Feryal Alayont, Grand Valley State University, Ghanshyam Bhatt*, Tennessee State University, Tavish Dunn, University of Texas at Austin, Luella Jill Fu, San Francisco State University, Victor H Moll, Tulane University. The Method of Brackets: A heuristic method of calculating definite integrals. Preliminary report.
The method of brackets is a new integration method, based on a few rules. Some of these have been rigorously established. The current talk will describe this method and present examples containing Bessel functions."
(Received September 12, 2023)

1192-00-33438
Max Lieblich*, University of Washington. Cohomological universality for projective space.
Various results and conjectures, such as the norm residue isomorphism theorem and the cyclicity conjecture, point to some kind of cohomological universality for projective space with respect to cohomology. I will explain the connection and describe recent work in this direction. In particular, I will discuss new techniques for simultaneously generating large subgroups of the cohomology of a variety from a single correspondence with a small projective space. The techniques I use ultimately rely on an
analysis of the Picard groups of certain stacky curves.
(Received September 16, 2023)

## 1192-00-33441

Daniel Bragg*, University of Utah. A Stacky Murphy's Law for the Stack of Curves.
We show that every Deligne-Mumford gerbe over a field occurs as the residual gerbe of a point of the moduli stack of curves. Informally, this means that the moduli space of curves fails to be a fine moduli space in every possible way. We also show the same result for a list of other natural moduli problems. This is joint work with Max Lieblich
(Received September 16, 2023)

## 1192-00-33845

Juliet Eyituoyo Ekoko*, Howard University Student. Making The Precise Definition Of The Logarithmic \& Exponential
Functions More Intuitive.
Before the reform of calculus instruction, textbooks defined the natural logarithm function prior to introducing the exponential function as its inverse. Although this approach was a precise and efficient introduction for Calculus learners, it was tedious because it was hard to motivate. Therefore, with the evolution of calculus education, there has been a shift towards a different strategy. Post-reform, textbooks have embraced a method that employs numerical and geometric evidence to motivate the introduction of the exponential function first, aiming to enhance conceptual understanding and motivation. Recognizing the importance of balancing precision and intuition, the central goal of this research is to make the traditional definition of natural logarithm as a definite integral more intuitive, by first defining the exponential function in a way that is more intuitive and precise. Thereby bridging the gap between precision and intuition, this study will show how to present exponential and logarithmic functions in an introductory analysis course.
(Received September 26, 2023)

1192-00-33874
Chavier Franklin-Alexander McDaniel*, Morehouse student. Optimization tool for beam alignment: The power of Tao in SLAC's cu_\{s\}xr accelerator.
In this study, I present the development of a novel orbit optimization tool, implemented in Tao, a modern interactive software designed for charged particle optics applications. The primary objective of this tool is to fine-tune corrector magnets in the LTU and BSY sections of the Cu SXR accelerator. To achieve this, we investigate the effectiveness of three distinct optimization algorithms: Levenberg-Marquardt, Differential Evolution, and Singular Value Decomposition (SVD). The overarching goal of this research is to facilitate precise beamline alignment, ultimately enhancing the performance and accuracy required for the successful commissioning of LCLS 2. The proposed tool is designed to be practical and user-friendly, empowering accelerator operators to optimize the Cu SXR line performance efficiently. Tao, chosen as the software framework for this development, offers advanced features tailored for solving optimization problems. By minimizing corrector magnet strengths, our tool allows for safe and flexible performance adjustments while significantly improving tuning speed for rapid beam alignment. This balance between enhanced performance and adherence to operational safety standards is a key feature of our approach. The workflow of the tool involves data acquisition through a dedicated script, integration with Tao for data input, running the selected optimization algorithm to find the optimum settings, evaluating the most suitable algorithm based on historical data, applying the chosen algorithm to various data sets, and storing the optimized settings to EPICS for practical implementation. Notably, the integration of Python and Tao enables comprehensive data range exploration and utilization of corrector magnet spectra, showcasing promising potential for enhanced orbit flattening. This development signifies significant progress in the Cu SXR accelerator alignment, with ongoing exploration and further optimization opportunities. (Received September 27, 2023)

## 01 History and biography

## 1192-01-25743

Aditya Kolachana*, Indian Institute of Technology Madras. Mādhava's sophisiticated spherical trigonometry in verse. Mādhava of Sañgamagrāma (c. 14th century CE) is a towering figure in the history of mathematics and astronomy, credited with a number of important results, including infinite series for pi, the arctangent, and the sine and cosine functions, in addition to a number of astronomical innovations. Celebrated as the golavid, or the 'knower of the celestial sphere', Mādhava also made important contributions to the advancement of spherical trigonometry. In his Lagnaprakaraṇa, an astronomical text dedicated to the determination of the ascendant, Mādhava gives sophisticated results in spherical trigonometry, employing imaginative projections and principles of planar geometry. More remarkably, these results are stated in the form of eloquent verses, in a variety of poetic metres, without the loss of mathematical precision or clarity. In this paper, we discuss some of the important results given by Mādhava, along with the likely techniques employed by him in arriving at them. We also highlight the significance of the poetic pedagogy employed by Mādhava in mathematics education.
(Received July 07, 2023)

1192-01-25948
Dana N Mackenzie*, Freelance Writer. Mathematical Royalty: The Curious Case of Princess Elisabeth of Bohemia.
Mathematical Royalty: The Curious Case of Princess Elisabeth of Bohemia As a teenager, Princess Elisabeth Simmern von Pallandt of Bohemia and the Palatinate became acquainted with René Descartes, while they were both living in exile in The Hague. From 1643 until Descartes' premature death from pneumonia in 1650, they carried on a voluminous correspondence. Though most of their discussion was about philosophy, Descartes also instructed her in mathematics, and (as I will explain) he discovered what is now known as Descartes' Circle Theorem as a result of a homework assignment gone awry. I will argue that Princess Elisabeth should be considered a major contributor to this discovery and, indeed, should be recognized as the first known female mathematician after Hypatia.
(Received July 16, 2023)

## 1192-01-26379

Lawrence Arthur D'Antonio*, Ramapo College. Richard Towneley, astronomer of Towneley Hall. Preliminary report. In this talk we look at the career and times of Richard Towneley (1629-1707), an important, but often overlooked, scientist of the 17 th century. For example, Robert Boyle attributed the discovery of what we call Boyle's Law to Towneley. For centuries, the Towneleys were an influential family in Lancashire (Turner painted Towneley Hall). The Lancashire astronomers, Jeremiah Horrocks, William Crabtree, and William Gascoigne were close friends of Richard's uncle Christopher Towneley. But the Towneleys were recusants (remaining Catholic and refusing to recognize the Church of England). This led to Richard Towneley being isolated from the major scientific circles of England. He could not attend Cambridge or Oxford. He was not a member of the Royal Society. In light Towneley's many accomplishments as a Catholic scientist, we will look at the Merton thesis that the Puritan ethos was essential to the development of science in England in the 17th century.
(Received July 26, 2023)

## 1192-01-26754

Daniel J O'Leary*, Independent. A Gear in a Turing Machine. Preliminary report.
Alan Turing had a strong interest in electromechanical computers and designed two of significance - one related to the Riemann zeta function and one related to breaking German Enigma ciphers. In 1935 and 1936, Titchmarsh determined the number of zeros of the Riemann zeta function using extensive hand calculations (which included adding machines). He showed that, up to $t=1468$, there are 1,041 zeros all on the critical line. Turing had long been interested in the Riemann zeta function and believed the Riemann Hypothesis to be false. He proposed to design and build, with the help of Donald MacPhail, an electromechanical computer to extend Titchmarsh's calculations. (Titchmarsh said that he was tired of the calculations and did not want to go further.) Turing's inspiration was Lord Kelvin's tide predicting machine at Liverpool with its extensive system of pulleys and strings. In 1939, Turing and MacPhail decided to use meshing gears driven by an electric motor to calculate and sum the individual terms produced by the Riemann-Seigel formula. McPhail drew the blueprint and they started cutting gears. WWII intervened, and they didn't complete the machine. Turing had been recruited by Max Newmann for the UK Government Code and Cypher School, GC\&CS, and had attended courses on cryptoanalysis before WWII. (Later Newmann and Bertrand Russell sponsored Turing for an FRS.) When WWII was imminent, Turing went to Bletchley Park to work on breaking Enigma the German military mechanical encryption machine. The Polish Cipher Bureau, starting in 1933, had broken the Enigma with great success. Their initial methods used sheets of paper with punched holes, Zygalski sheets. To manage the traffic volume the Poles built an electromechanical computer, the Bomba. After changes to the Enigma and the German key system, the Polish approach stopped working. Turing developed another method and designed an electromechanical computer, the Bombe, to help implement it. This paper examines the common thread of Turing and the electromechanical computer. The first part looks at the Titchmarsh calculations and Turing's computer. The second part looks at the Enigma and the Bombe. (Received August 04, 2023)

## 1192-01-26813

Steven M. Deckelman*, University of Wisconsin-Stout. Physical Mathematics and the Dirichlet Principle. Preliminary report The Dirichlet Principle is the moniker originated by Riemann in his 1857 paper on Abelian functions to describe P.G.L. Dirichlet's method for solving the boundary value problem for the Laplacian (the Dirichlet problem) by means of an energy integral. Possibly an instance of the Boyer-Stigler law of mathematical eponymy, it has been suggested by Felix Klein that Riemann coined this moniker owing more to the veneration he held for his teacher Dirichlet, than as being the source of the original idea, which can be found also in the writings of Green, Gauss, and Willam Thompson. The word energy is suggestive of a physical interpretation and part of the folklore of mathematics is that Dirichlet's original chain of thought was guided by physical considerations involving electrostatic fields. We use the term folklore because the argument doesn't appear to be written down in many places, and where it is, it is often in a redacted form that omit either mathematical details or physics details. Nonetheless we argue that this historical example of reasoning by analogy remains of interest because it illustrates the way physics influenced the creative work of mathematicians of the past. In this talk we will give an overview of the physical motivation for the Dirichlet principle by combining some of the disparate accounts appearing in the literature. Only elementary calculus and simple ideas about electricity and electrostatic fields are used.
(Received August 06, 2023)

## 1192-01-27245

Maritza M. Branker*, Niagara University. Cauchy's Persuasive Appeal.
In 1821, Augustin Louis Cauchy published his Cours d'analyse de L'Ecole royale polytechnique as required by the Conseil of the L'Ecole. It is arguably one of the most influential texts ever published despite the fact it was never included in the list of recommended textbooks for the students. We discuss the impact of this single book on the mathematical landscape, both in Cauchy's time and ours.
(Received August 12, 2023)

1192-01-27403
Jacqueline M. Dewar, Loyola Marymount University, Sarah J. Greenwald*, Appalachian State University. 17 th century mathematics in Kircher's Arithmologia and Schott's Cursus Mathematicus.
As teacher and pupil, mentor and mentee, and fellow Jesuits, Athanasius Kircher and Gaspar Schott provide insight into 17th century mathematics. The two individually authored a total of more than 50 books on a wide variety of subjects, ranging from China, Egypt, and theology to mathematics, mechanics, and physics. Although Kircher's reputation faltered at times, he has been acknowledged as a major source of inspiration for mathematicians in the 17th and 18th centuries (Findlen, 2004). Schott is said to have functioned as a German Mersenne (Alexanderson and Klosinski, 2015) and he greatly improved on the existing mathematical encyclopedias in a number of ways (Knobloch, 2011). In this talk, we will first discuss their lives and education and some of the books they published. Then we will explore their motivations for writing two of their mathematical works, Kircher's Arithmologia sive De abditis Numerorum mysteriis and Schott's Cursus Mathematicus sive absoluta omnium mathematicarum disciplinarum encyclopædia, and showcase some of the illustrations they contain. Finally, we will discuss some of the connections and discoveries we made by exploring their lives and works.
(Received August 14, 2023)

## 1192-01-28177

Brenda Davison*, SFU. Stieltjes and Asymptotic Expansions. Preliminary report.
Thomas Jan Stieltjes' (1856-1894) 1886 PhD thesis titled Recherches sur quelques séries semi-convergentes was a contribution to the theory of asymptotic expansions. In his thesis, Stieltjes found asymptotic expansions for several different functions which he was then able to approximate with an error bound. In this talk, I will situate his work in the context of the analysis of continued fractions for which Stieltjes is well-known, explain the method and discuss how it compared to the contemporaneous work of Poincaré on the same topic.
(Received August 27, 2023)

## 1192-01-28295

Rosie Lev Halutz*, Tel-Aviv University. Euclid's Victorian Rivals and the 'dangerous' Direction-Theory. Preliminary report. The story of direction-theory in Victorian Britain is not a success story. The theory, which attempted to rival Euclid's standard treatment of straight and parallel lines by using the idea of direction, encountered a fierce opposition from the local mathematical community and was eventually abandoned. In 1888, the British mathematician Charles L. Dodgson (1832-1898), also known as the writer Lewis Carrol, called the idea of direction 'dangerous'. Why did Dodgson refer to direction-theory as 'dangerous'? Which mathematicians of the nineteenth-century British mathematical community shared Dodgson's view and which mathematicians advocated the theory? What motivated each side and how did the controversies shape the faith of the theory - from its reception, through its development, to the final abandonment? And lastly, what can we learn from this episode about the Victorian mathematical community? To answer these questions, I analyze the epistemic values that lie at the basis of direction-theory and the wide-ranging meanings that these values had in the Victorian setting. In addition, I examine how committed were the mathematicians that advocated the idea of direction to these values and to the idea of direction itself, and whether they operated independently or viewed themselves as a nonconformist group that develops a controversial theory. (Received August 29, 2023)

## 1192-01-28324

Albert C. Lewis, Independent historian, Karen H Parshall*, University of Virginia. Mathematicians Confront Political Tests: The American Mathematical Society and the Red Scare in 1954.
The Red Scare of the 1950s was an era of great disquiet in the United States. When four mathematicians-one in New York, two in Michigan, and one in Tennessee-lost their teaching jobs in 1954 for failing to answer questions about their political beliefs before different investigative committees, their plight came before an American Mathematical Society unsure of how far it could or should go in asserting itself in the political, as opposed to the mathematical, arena. This talk will explore these four cases and examine the AMS's response to them in the broader context of the Red Scare.
(Received August 29, 2023)

## 1192-01-28490

Ken Saito*, Osaka Prefecture University (emeritus). It all began with the pentagon: polyhedra and irrational lines in Euclid's Elements.
We examine Book XIII of the Elements which treats the five regular polyhedra, and the theory of the (so-called) irrational lines in Book X of the Elements about whose origin the historians are not yet unanimous. Although Book XIII uses a part of the results of Book X for the comparison of the side of polyhedra and the diameter of the circumscribed sphere, many historians feel that this application is not enough to justify the construction of the huge theory of no less than 13 irrational lines in Book X, of which only two appear in Book XIII. We re-examine the propositions of Book XIII concerning polyhedra paying attention to the geometrical proof arguments and techniques of handling quantitative relations, which have not received due attention of modern scholars. These arguments of Book XIII explain the motivation of introducing the concept of rational/irrational lines and new irrational lines in Book X.
(Received August 31, 2023)

## 1192-01-28527

Jemma Lorenat*, Pitzer College. "The science of Mathematics is not crystallized into text-books" : The Bryn Mawr Mathematical Journal Club Notebooks (1896 - 1924).
As mathematics departments in the United States began to shift toward standards of original research at the end of the nineteenth century, many adopted journal clubs as forums to engage with new periodical literature. The Bryn Mawr Mathematical Journal Club, maintained episodically between 1896 and 1924, began as a supplement to the graduate course offerings. Each semester student and professor participants focused on a single disciplinary area or surveyed what had been published lately. The Notebooks containing these reports were stored on the open shelves of the college library. These collectively composed documents record ways in which graduate students transcribed and interpreted contemporary literature. This talk focuses on the entries of Virginia Ragsdale in which she formulated research questions, tested potential strategies, and pursued novel results in topology.
(Received August 31, 2023)

1192-01-29377
Margaret Stawiska-Friedland*, AMS/Mathematical Reviews. The first journal of the Polish Mathematical Society (19211952).

In this talk we present the first journal of the Polish Mathematical Society, Annales de la Société Polonaise de Mathématique, appearing in 1921-1952, with a 6 -year break during World War II. While less known than the specialized journals associated with Warsaw and Lwów Schools of mathematics, it was also important. We discuss in particular its role in fostering international exchange of mathematical ideas and in promoting work of junior scholars, including women mathematicians. We also highlight some lasting achievements of mathematicians beyond Warsaw and Lwów Schools published in the journal. The talk is based on joint work with Marta Kosek (Jagiellonian University).
(Received September 06, 2023)

Janet Heine Barnett*, Colorado State University Pueblo. A discourse view of mathematical communities: Examples from 19th-century analysis. Preliminary report.
Studying the ways in which groups of individuals linked by some common mathematical interest work together as "communities" is now a standard theme among historians of mathematics. In this talk, I adopt a somewhat non-standard view of what it means to be a mathematical community by drawing on Anna Sfard's theory of commognition, a term that combines "cognition" and "communication" to emphasize that these two activities are "but different manifestations of basically the same phenomenon" (Sfard, 2008, p. 83). Within this framework, "doing" mathematics at either the individual or community level is the act of participating in a mathematical discourse, where discourse is an activity regulated by rules at two levels. Objectlevel rules reflect regularities in the behavior of the discursive (e.g., mathematical) objects - the rules involved in "just doing the math." Metadiscursive rules reflect regularities in the discourse itself. For example, it is the metadiscursive rules that govern what constitutes a proper definition or an adequate justification within a particular discourse. Within the commognitive framework, a mathematical community is thus a group of individuals who think and talk about the same mathematical objects in ways that follow the same metadiscursive rules. Following a brief introduction to the relevant aspects of Sfard's theory including the notion of commognitive conflict - I will explore this view of mathematical communities and its relevance to studying the history of mathematics using examples drawn from the discourses on analysis that existed in the 19th century. References Sfard, Anna. 2008. Thinking as communicating: Human development, the growth of discourses, and mathematizing. Leiden: Cambridge University Press.
(Received September 07, 2023)

1192-01-29796
William Thomas Archibald*, Simon Fraser University. Charles Hermite, Analysis, and Mathematics in the Lycée.
A great deal of the mathematics of Charles Hermite addresses analysis, a subject that he taught at the École Polytechnique and the Sorbonne, and also to students at the \{ecole normale supérieure. In this paper we look at some features of those courses as they relate to his own research style and direction. We also consider aspects of the ways in which his values were transmitted via preparatory training in secondary schools. Among other things, this was done via thesis students, and we will consider some examples. (Received September 07, 2023)

## 1192-01-29821

Ethan J Berkove*, Lafayette College. The 15 Puzzle and Ambrose Bierce.
The 15 puzzle is a well-known mathematical recreation which many of us encountered in our youth. The puzzle consists of a $4 \times 4$ grid, with 15 tiles numbered from 1 to 15 . There is one open slot which allows the tiles to slide around the grid. Once the tiles are randomized, the goal is to arrange the tiles so that the numbers 1 to 15 are in order from left to right and top to bottom. It is possible to order the tiles in other configurations too, although not all arrangements are realizable. The 15 puzzle was a viral sensation in around 1880, similar to the Rubik's Cube craze that would occur nearly 100 year later. Interest in the puzzle spread from coast to coast, and the puzzle was the subject of much media attention. One columnist who wrote about the 15 puzzle, Ambrose Bierce, is still well-known today. He put together an entire column on the analysis of the puzzle, which appeared in May 1880 in the Argonaut, a San Francisco journal. In this talk we will provide some historical background about the 15 puzzle and take a look at Bierce's analysis. His was an unusually good amateur attempt at a solution, and there nothing else like it in Bierce's known writings. Although Bierce's argument didn't constitute a proof, he included subtleties that are not usually found in the popular press. We also describe how the puzzle was analyzed by contemporaneous mathematicians who published articles around the same time. Overall, this story is interesting from many points of view. It is an early example of a popular mathematical puzzle and it gives insight into some of what was considered "state of the art" in mathematical research. This is joint work with Lawrence Berkove
(Received September 08, 2023)

1192-01-29951
Suzanne Sumner*, University of Mary Washington. How Poetry informs the History of Mathematics.
The non-Western origins of popular topics in mathematics known as Pascal's triangle and the Fibonacci sequence involve two poets: Acharya Pingala and Omar Khayyam. Interestingly, the ancient Sanskrit scholar Acharya Pingala of India was likely the first to use the Pascal's triangle and Fibonacci sequence numbers in his poetry. Khayyam (1048-1124 or 1129 CE) claimed to have been the first mathematician to publish how to calculate roots of any order in a now-lost treatise titled Problems of Arithmetic. His method would have used the Binomial Theorem. The coefficients of this binomial expansion are frequently arranged in a triangular form known today as "Pascal's Triangle" named after the 17th century mathematician Blaise Pascal. However, the origins of the numbers in Pascal's triangle date much further back to India with Acharya Pingala's (ca. 200 BCE) study of meter and sound combinations for Sanskrit poetry. A 10th century Indian commentator, Halayudha, gave the first appearance of a triangular arrangement of the binomial coefficients for the number of combinations of short and long syllable patterns, but not for finding n-th roots. In addition to Khayyam, the Chinese mathematician Jia Xian (ca. 1050 CE) had discovered the method of finding n-th roots with Pascal's triangle, yet his work is also not extant. Thus, we rely on two lost manuscripts, over approximately the same time period, to determine the Islamic and Chinese origins of Pascal's triangle for finding n-th roots. A related topic is Fibonacci's Liber Abaci, published in Latin in 1202 CE. The history of mathematics cites the rabbit problem in Liber Abaci that generates the Fibonacci sequence as one of the earliest examples of the recursive process. Yet once again Pingala appears to have much earlier studied the Fibonacci numbers as they relate to meter in Sanskrit poetry. Pingala’s Chandaḥśāstra even gave an alternate name for the Fibonacci sequence: mātrāmeru. (Received September 08, 2023)

## 1192-01-30025

Kevin Lambert*, California State University Fullerton. Kites and Letters: How Peter Guthrie Tait and William Rowan Hamilton Remade Quaternions.
In the manuscripts of Trinity College Dublin there is a dispatch book into which a remarkable set of letters have been glued. The Anglo-Irish mathematician William Rowan Hamilton wrote the letters and they were pasted into the book by the Scottish
mathematician and physicist Peter Guthrie Tait．In this paper，I will seek to reconstruct the making of the letter－book to show how，for a brief moment，Hamilton and Tait worked together to achieve an intense correspondence，a＂vivid presence＂ analogous to that achieved by jazz musicians when they improvise together．Mathematics is not music，of course，and so Hamilton and Tait＇s＂vivid presence＂was not in real time but in the virtual space created by their mathematical constructions． This＂tuned in＂correspondence，I will argue，is an example of how the materiality of mathematics allows mathematicians to work closely together even when they are physically distant，and that what Hamilton and Tait achieved through their exchange of letters was crucial for the making of quaternions as a theoretical practice and for the eventual development of vector calculus．
（Received September 08，2023）

## 1192－01－30199

Alicia Zelenitsky Hill＊，Simon Fraser Univesity．Sawaguchi Kazuyuki（沢口一之）and the Kokon Sanpōki（古今算法記）． Preliminary report．
The Kokon Sanpōki（古今算法記）is a seven－volume mathematical text dating from 1671 Japan．Its authorship is commonly attributed to Sawaguchi Kazuyuki（沢口一之）a carpenter based in Osaka and student of Hashimoto Masakazu（橋本正数）．The first three volumes present mathematics common in other texts of the time，mainly an exposition of soroban（abacus） arithmetic and explicit examples of area and volume calculations．The next volumes are believed to be the first works in Japan to depict tianyuan（天元），a method from China for solving equations by representing what we would term polynomials with counting rods．Sawaguchi Kazuyuki applied this technique correctly to 150 questions from an earlier text，the Sanpō Kongenki （算法根源記）．Most modern scholarship relates to the 15 idai or bequeathed problems in the last volume which were later solved by Seki Takakazu（関孝和）．In this brief presentation，I will discuss key aspects of this work． （Received September 09，2023）

## 1192－01－30495

Linda McGuire＊，Muhlenberg College．Honoring the Past：Historical women of the EvenQuads project．
This presentation will focus on historical women honored on the EvenQuads decks of cards．Topics discussed will include methods employed to obtain accurate and verifiable information about figures whose stories often went untold and the amazing contributions these women made to the world of mathematics．From research mathematics to teaching and pedagogy， from writing mathematical texts and treatises to pioneering efforts to bring more women into the field，this presentation will highlight the interleaving ways in which the＂historic women＂of the EvenQuads decks shaped the discipline we know today． Biographies of some honorees will be used as case studies and examples to share interesting information this process made visible about these women and to describe academic conundra peculiar to profiling historic figures．The EvenQuads project strives to be thoughtful，deliberate and intentional when profiling notable women mathematicians across demographics， mathematical specialities，multimodal contributions，and time．The project invited biographical and portrait entries from a wide range of contributing authors and artists（undergraduate students，graduate students，faculty，and other professionals）． As such，questions about how all involved collaborated to navigate fact－finding and writing challenges will also be addressed． （Received September 10，2023）

## 1192－01－30600

E．A．Hunter＊，University of Chicago．A New Look from the Archives：A Revaluation of Archimedes＇Work in the United Kingdom during the 17－18th Centuries．Preliminary report．
A standard history of calculus typically starts nearly two thousand years before the discovery of calculus by Isaac Newton and Gottfried Wilhelm Leibniz，with the third－century BCE Greek mathematician Archimedes．Beginning in the seventeenth and eighteenth centuries，many mathematicians believed，or at least wanted to connect，Archimedes＇work on the properties of tangents to spirals，the method of exhaustion，and the quadrature of the parabola as precursors in both theory and practice to limits，integration，and infinite series．This claim is even repeated today by modern historians and textbooks on calculus． Nevertheless，there has yet to be a detailed analysis of the extent to which Archimedes＇work impacted mathematicians working in the early beginnings of calculus．This paper takes specific examples from various archives in the United Kingdom to evaluate how，or if at all，the ancient mathematician＇s work guided British mathematicians like Isaac Barrow，Colin Maclaurin， Newton，and John Wallis．Internal and external questions related to the history of mathematics are considered，like what defines calculus and its practices．
（Received September 10，2023）

1192－01－31050
Andrew Fiss＊，Michigan Technological University．＂Compute，observe，and mark my word＂：Collaborative Mathematics and Joint Authorship through Harvard＇s Observatory Pinafore，1879．Preliminary report．
This paper draws attention to the archival documents surrounding the Observatory Pinafore，a parodic play from the 1879 Harvard Observatory，a site that has become paradigmatic in studies of computing．Building on recent analyses of algorithms from historians of mathematics and science，it particularly locates the construction of parody in workplace practices that blurred categories of critique／computation，human／machine，and author／worker．The Observatory Pinafore took the people and concerns of Gilbert \＆Sullivan＇s opera HMS Pinafore and mapped them on to the observatory workplace，and in doing so，it made many ambiguities more explicit．While the content of the play commented more on the gender and class negotiations in shifting workplace roles，its creation draws attention to the reliance on collaborative mathematics and joint authorship in a site constructed around the writing and publication of stellar maps and astronomical calculations．Astronomer Winslow Upton generally has been understood as the sole author of the Observatory Pinafore，though astronomer Williamina Fleming was proposed instead in the document＇s rediscovery in 1929．The archival documents suggest a process of revision：ending in a full，clean copy in Williamina Fleming＇s hand but beginning in lines from Winslow Upton with a plethora of marginal comments from at least one other person．The play＇s lyric＂compute，observe，and mark my word＂comes then to illustrate the group＇s understood tasks，not only mathematical and observational but critical，marking words and numbers，retaining them and also literally writing on top of them．This paper，as a whole，then urges historians of mathematics to investigate claims of sole authorship，particularly in mathematical workplaces known for joint publication and collaboration．
（Received September 11，2023）

Aubrey Clayton*, Harvard University, Division of Continuing Education. R.A. Fisher, Eugenics, and the Foundations of Probability.
Ronald Fisher (1890-1962) was an incredibly influential mathematician who, more than any other person, molded the discipline of statistics into what it is today and created the analytical tools that now dominate empirical scientific research. Like his intellectual predecessors Francis Galton and Karl Pearson, he was also a passionate advocate for the eugenics movement. Recent controversies over awards and monuments dedicated in Fisher's name have prompted the question: Just how separable is Fisher's statistical legacy from his involvement in eugenics? In this talk, we will review Fisher's contributions to statistics in the context of eugenics with the goal of partially answering this question. Contrary to the convenient narrative that eugenics was a regrettable side-project with no real bearing on Fisher's other work, we will argue that the goals of the eugenics movement animated Fisher's fundamental approach to statistical inference and the foundational concepts of mathematical probability. Therefore, by continuing to view these problems as Fisher did, we are in a sense furthering the legacy of eugenics.
(Received September 11, 2023)

## 1192-01-31336

Shawn L. McMurran*, CSU San Bernardino, James J. Tattersall, Providence College. Mary Lucy Cartwright: An Extraordinary Life. Preliminary report.
Mary Cartwright was a remarkable 20th century mathematician. She was the first woman to receive an honors degree in mathematics at Oxford and among the first women to be elected to the Royal Society of London. She was mentored by G.H. Hardy and collaborated with J.E. Littlewood. Her research spanned many areas of mathematics including real and complex analysis, differential equations, and topology. As mistress of Girton College, Cambridge, she served as an outstanding role model for her students and supported women in their pursuit of higher education. In this talk we share some key accomplishments in her extraordinary life.
(Received September 11, 2023)

1192-01-31487
Deborah Kent*, University of St. Andrews, Yansong Li, University of St Andrews. "Responsibility for this falls only on us:" Chinese observations of the 1941 total solar eclipse. Preliminary report.
In 1934, the Nanking Zijinshan Observatory announced that a total solar eclipse would cross 4000km through 8 Chinese provinces on 21st September 1941. The China Solar Eclipse Observation Committee spent years making extensive preparations, but wartime conditions in 1941 debilitated every aspect of the planned expedition. The commitment of government resources and the determination of Chinese observers reflect patriotic motivations echoed throughout widespread media coverage of the total solar eclipse.
(Received September 11, 2023)

1192-01-32221
Eun-Joo Ahn*, Yale University. Gendered Calculations: Human Computers at Mount Wilson Observatory in the Early Twentieth Century.
Computing, or mathematical calculations, is an important component of astronomical research. In the nineteenth and early twentieth century, human computers carried out this work that required various mathematical and astronomical knowledge and skills. With the adoption of photographic techniques in astronomical research from the mid-nineteenth century, computing became more prevalent as astronomers analyzed photographic plates to study positional and physical characteristics of celestial objects. Hence, human computers were an integral part of the scientific staff in many leading research observatories. Mount Wilson Observatory (MWO) was one such place. Founded by astrophysicist George Ellery Hale in 1904 with funding from the Carnegie Institution of Washington. Since then, it has become one of the most prominent astronomical observatories during the first half of the twentieth century, whose astronomers contributed to understanding the characteristics of the sun and the structure of our universe. At MWO, and other research observatories in the United States, despite the quantitative nature of the work and the fact that both men and women astronomers computed, the work had gendered implications. To male astronomers, computing was an integral skill that enhanced their capability as an astronomer. To female astronomers, computing often was a routine task with little room for independent research. By following the work carried out by astronomers at MWO during the early decades of the twentieth century, I highlight the gendered perception of computing and how sociocultural norms and geography contributed to the gendered glass ceiling in astronomy. (Received September 12, 2023)

## 1192-01-32418

Jessica Otis*, George Mason University. Death by Numbers: Analyzing Mortality Statistics in 17 th Century London.
The London Bills of Mortality are a series of weekly and annual broadsides containing local mortality statistics that were publicly distributed throughout in the city of London during the seventeenth and eighteenth centuries. The public bills derived from a series of sixteenth-century government documents, which coopted the city's existing religious infrastructure of parish clerks to generate plague and general mortality statistics. This allowed the city mayor, city alderman, and monarch to examine the ratio of plague to non-plague deaths during various outbreaks and make decisions based on quantitative information. These early bills and their public successors abstracted the dead into a series of numbers; the bills contained counts of those who died of plague in each parish, those who died more generally in each parish, and those who died of any particular cause of death city-wide. Unlike other medieval and early modern mortality lists, however, they divorced causes of death from the context of a person's name, age, or occupation, and transformed the dead into anonymous numbers. This paper analyzes the arithmetic of the bills and the wider circulation of their summary statistics, in order to contextualize the arithmetic of the bills in the changing numerical landscape of the late sixteenth and seventeenth century and place the bills in a larger narrative about the increasing quantification of everyday life in early modern Britain.
(Received September 12, 2023)

Julia Tomasson*, Columbia University. Shatranj: Chess and Mathematics in the Islamicate World. Preliminary report. Across the premodern Islamicate world, chess was forbidden and ubiquitous; chess or shatranj was ruled /emphharam or forbidden by many schools of Islamic law but was at the same time a beloved pastime played in the streets as well as the palace. This talk analyzes Arabic chess manuals as an epistemic genre that is well-documented and well-defined but that blurred the boundaries between the mathematical/occult sciences, legality/illegality, theory/practice, and elite/popular culture. Traditional histories of chess look at these handbooks for clues to the origin of modern chess, to the evolution of the physical game, or as an 'exotic' but 'quaint' historical aside before Europeans (literally and otherwise) changed the rules of the game and made chess part of the mystical ideal of European 'Genius.' Moreover, traditional histories also cite these harsh legal rulings against chess as proof that chess-playing and the exercise of reason were stifled in Muslim empires presaging the rise of European chess. But how would this history change if we were to instead take these rulings as proof that some saw chess as a social problem because it was omnipresent? Drawing upon manuscript as well as other textual and material evidence, I take these chess books seriously on their own terms as repositories of knowledge, experience, and ideas about rules (of chess and law), the place of the exercise of individual reason, epistemic virtues and vices, and different knowledge economies in pre/early modern Muslim societies.
(Received September 12, 2023)

Bill Linderman*, King University. Archimedes' Quadrature of the Parabola. Preliminary report.
Archimedes showed that the area of the region bounded by a parabola and a chord is equal to four thirds of the area of the triangle with its base as the segment of the chord and with its height as the distance from the chord to the point on the parabola whose tangent is parallel to the chord. We explore the method of exhaustion Archimedes used to establish this result and then show how examples can easily be verified using a little calculus and linear algebra.
(Received September 12, 2023)

## 1192-01-32771

James Phillip Ascher*, University of Edinburgh. English Periodicals and Marketing Mathematics in Seventeenth-Century Europe. Preliminary report
This paper argues that mathematical knowledge was made into a commodity in the seventeenth century, transforming reading. After the Restoration of Charles II in 1660, a group of English scholars formalized their meetings and acquired special royal permission to write, print, and disseminate knowledge unmolested by censors. This Royal Society used their authority to license and print books contributing new knowledge in all fields, including mathematics. Under this authority the Society licensed well-known books like Robert Hooke's Micrographia (1665) and Isaac Newton's Principia (1687), as well as John Evelyn's Sylva (1662). They also licensed a roughly monthly newsletter, The Philosophical Transactions (1665-), which established the form for periodical publishing, and eventually learned journals, along with encouraging similar newsletters from several other countries. It has been frequently overlooked that the Transactions was marketed as a commercial venture. Recent work from Aileen Fyfe has shown that in later years the journal consistently lost money, but that it was continued because it was seen as effectively disseminating knowledge. While the finances of the earliest years are not well known, the journal does establish a form for the commercial distribution of mathematical knowledge. It establishes the article as a unit and provides an everyday reader with a chance to access what had previously been the private correspondence of the res publica. This modified newsletter becomes a new form of printing and bookselling, which establishes the learned journal, the now standard mechanism for sharing new knowledge. This paper will outline the legal and commercial history around the very beginning of this journal, The Philosophical Transactions, and how the Royal Society used its legal framework to disseminate knowledge within commercial networks. Combining printing history and legal history, it argues that making mathematical knowledge a commodity in the seventeenth century had world-wide results by inspiring competing journals and demonstrating a way of sharing new knowledge sold alongside other kinds of books.
(Received September 12, 2023)

## 1192-01-33185

Charlotte Aten*, University of Denver. Bourbaki's mathematical structures and their legacy.
I will discuss the attempt by the Bourbaki group to rigorously define the notion of a mathematical structure in a way which would suffice for their textbook series, as well as, ambitiously, all of mathematics. This will include the downfall of the idea as category theory subsumed the relevant notions, and the group's unsuccessful attempt to reconcile their notion of structure with that of category. While this talk was motivated by my thesis work, I will also disucss related concepts such as the theory of combinatorial species, model theory, and the advent of Lawvere's algebraic theories.
(Received September 13, 2023)

## 03 Mathematical logic and foundations

## 1192-03-26800

Dhruv Kulshreshtha*, University of Michigan, Aristotelis Panagiotopoulos, Carnegie Mellon University. On the Nonclassifiability of Homeomorphisms of the Sierpiński Carpet. Preliminary report.
Self-homeomorphisms of the interval $[0,1]$ can be classified up to conjugacy by using certain countable structures as invariants. The problem becomes much more complex if one is to replace the interval $[0,1]$ with the square $[0,1]^{2}$. Indeed it is a theorem of Hjorth that there exists no definable way to classify self-homeomorphisms of the square $[0,1]^{2}$ by using countable structures as invariants; that is, the conjugacy equivalence relation on the homeomorphism group of the square reduces a turbulent action. In this talk, we briefly discuss Hjorth's theory of turbulence used to prove the above result. We then take a step towards studying the more general interplay between dimension and turbulence by arguing that even the homeomorphisms of the Sierpiński carpet, the one-dimensional universal plane curve, cannot be classified up to conjugacy in this manner.
(Received August 05, 2023)

## 1192-03-27424

Ilir Ziba*, University of Michigan. Extension of Post's Lattice to Countable-Borel Clones. Preliminary report.
Given a set of cardinals $N$ and an underlying set $X$, an $N$-ary function clone on $X$ is a set of functions $f: X^{n} \backslash \operatorname{ra} X$ for $n \in N$, which contains all projection functions and is closed under composition. In 1941, Emil Post fully characterized all clones of finite functions $f: 2^{n} \backslash \mathrm{ra} 2$ in a lattice famously titled Post's lattice, ordered by inclusion. We seek to extend the view of clones to include countably-infinite Borel functions and characterize this extended notion of Post's lattice in terms of the preservation of countable relations.
(Received August 14, 2023)

## 1192-03-28074

Aristotelis Panagiotopoulos, Carnegie Mellon University, Allison Wang*, Carnegie Mellon University. Every CBER is smooth below the Carlson-Simpson generic partition.
One difficulty that arises in studying the class of countable Borel equivalence relations (CBERs) is that in many cases, the complexity of a CBER lies on a "small" set. For instance, a result of Hjorth-Kechris, Sullivan-Weiss-Wright, and Woodin states that every CBER on a Polish space is hyperfinite when restricted to some comeager set. Another result, due to Mathias, shows that every CBER on the Ellentuck Ramsey space is hyperfinite when restricted to some pure Ellentuck cube. In this talk, we will show that every CBER on the space of all infinite partitions of the natural numbers coincides with equality below a Carlson-Simpson generic element. This is joint work with Aristotelis Panagiotopoulos.
(Received August 25, 2023)

## 1192-03-28195

Jacob Michael Ginesin*, Northeastern University. Language Bounds from Regular Expressions: Extraction and Applications. Regular expressions are pivotal in computing, enabling efficient pattern matching and text manipulation across diverse applications. As such, studying the languages of regular expressions and their language-equivalent automata is of critical importance. We demonstrate the problem of finding the upper and lower bounds of the language accepted by any regular expression, deterministic finite automata, non-deterministic finite automata, or Büchi automata can be reduced to solving either an integer linear programming (ILP) problem or a satisfiability modulo theories (SMT) problem. Solving ILP and SMT problems is generally NP-hard, but in practice solutions are found efficiently by solvers such as Gurobi and z3 with heuristicbased approaches. We show such a problem is worth solving by demonstrating language bounds can overapproximate the language inclusion problem, underapproximate the language intersection emptiness problem, and help efficiently decide membership checking in practice.
(Received August 28, 2023)

## 1192-03-28248

Riley Thornton*, Carnegie Mellon University. Greedy algorithms and differential equations.
The differential equations method is a powerful tool from probabilistic combinatorics for analyzing greedy algorithms. In this talk, I'll present a framework for applying the method in measurable combinatorics.
(Received August 28, 2023)

## 1192-03-28338

Francois Loeser*, Institut Universitaire de France, Sorbonne. Model Theory and Non-Archimedean Geometry. We shall present an overview of applications of Model Theory of valued fields to Non-Archimedean Geometry. We will start with the Bieri-Groves theorem and then focus on more recent results obtained in collaboration with several coauthors (A. Ducros, E. Hrushovski and J. Ye). (Received August 29, 2023)

## 1192-03-28345

Sumun Iyer*, Cornell University, Forte Shinko, UCLA. Generic actions of free groups.
Let $G$ be a countable free group, possibly with infinitely many generators. We consider the Polish space $\operatorname{Act}(G)$ of all continuous $G$-actions on compact Polish spaces. Frisch-Kechris-Shinko-Vidnyanszky proved that the generic element of $\operatorname{Act}(G)$ generates a measure-hyperfinite equivalence relation. The main theorem will be that the generic element of Act $(G)$ generates a hyperfinite equivalence relation. The proof uses a projective Fraisse limit result due to Kwiatkowska as well as the theory of Borel asymptotic dimension due to Conley, Jackson, Marks, Seward, and Tucker-Drob. This is joint work with Forte Shinko.
(Received August 29, 2023)

## 1192-03-28470

Java Villano*, University of Connecticut. Computable categoricity relative to a c.e. degree.
A computable structure $\mathcal{A}$ is said to be computably categorical relative to a degree $\mathbf{d}$ if for all d-computable copies $\mathcal{B}$ of $\mathcal{A}$, there exists a d-computable isomorphism between $\mathcal{A}$ and $\mathcal{B}$. In a 2021 result by Downey, Harrison-Trainor, and Melnikov, it was shown that there exists a computable graph $\mathcal{G}$ such that for an infinite increasing sequence of c.e. degrees
$\mathbf{x}_{0}<_{T} \mathbf{y}_{0}<_{T} \mathbf{x}_{1}<_{T} \mathbf{y}_{1}<_{T} \ldots, \mathcal{G}$ was computably categorical relative to each $\mathbf{x}_{i}$ but not computably categorical relative to each $\mathbf{y}_{i}$. In this talk, we will show how to extend this result for partial orders of c.e. degrees. This work was advised by Reed Solomon.
(Received August 31, 2023)

Robert M Anderson, UC Berkeley, Haosui Duanmu*, Harbin Institute of Technology, Aniruddha Ghosh, University of Queensland, Mohammed Ali Khan, Johns Hopkins University. Existence of Berk-Nash equilibrium with infinite spaces. Model misspecification is a critical issue in many areas of economics. In the context of misspecified Markov Decision Processes, Esponda and Pouzo (2021) defined the notion of Berk-Nash equilibrium and established its existence with finite state and action spaces. However, many substantive applications (including two of the three motivating examples presented by Esponda and Pouzo) involve continuous state or action spaces, and are thus not covered by the Esponda-Pouzo existence theorem. We extend the existence of Berk-Nash equilibrium to compact action spaces and sigma-compact state spaces, with possibly unbounded payoff functions. A complication arises because Berk-Nash equilibrium depends critically on RadonNikodym derivatives, which are bounded in the finite case but typically unbounded in misspecified continuous models. The proofs rely on nonstandard analysis, and draw on novel argumentation traceable to work of the second author on nonstandard representations of Markov processes.
(Received August 31, 2023)

## 1192-03-28792

David Meretzky*, The University of Notre Dame. Recent Results in Differential Field Arithmetic.
Motivated by classical algebraic theorems relating Galois cohomological and field arithmetical properties, I will discuss some results from my upcoming thesis, relating the differential Galois cohomology of Ellis Kolchin to differential field arithmetical properties. The approach in this work is fundamentally informed by mathematical logic, specifically model theory, and I will describe some related results in a general model theoretic environment.
(Received September 03, 2023)

## 1192-03-28869

Douglas Cenzer*, University of Florida. Dense Computability of Closed Sets. Preliminary report.
We develop some notions of densely computable trees and related notions of densely computable closed sets. Another topic is densely computable members of closed sets. This continues recent work of the authors on densely computable structures. (Received September 04, 2023)

## 1192-03-28872

Renling Jin*, College of Charleston. Multilevel infinities and they applications. Preliminary report.
By an iterated ultrapower construction we obtain a nonstandard universe with multilevel of infinities and various versions of transfer principle. The model offers strength and flexibility when dealing with problems in combinatorial number theory. (Received September 04, 2023)

Maryanthe Malliaris*, University of Chicago. Model theory and algorithms. Preliminary report.
The talk will report on recent connections of model theoretic classification theory with algorithmic questions in learning. (Received September 05, 2023)

1192-03-29143
Valentina S Harizanov, George Washington University, Keshav Srinivasan*, The George Washington University. Cohesive ultrapowers of directed graphs. Preliminary report.
We consider a computability-theoretic ultrapower construction for structures. We start with a computable structure, and consider its countable ultrapower over a cohesive set of natural numbers. A cohesive set is an infinite set of natural numbers that is indecomposable with respect to computably enumerable sets. It plays the role of an ultrafilter, and the elements of a cohesive power are the equivalence classes of certain partial computable functions. Thus, unlike many classical ultrapowers, a cohesive power is a countable structure. We focus on cohesive powers of graphs, equivalence structures, and computable structures with a single unary function satisfying various properties, which can also be viewed as directed graphs. For these computable structures, we investigate the isomorphism types of their cohesive powers, as well as the properties of cohesive powers when they are not isomorphic to the original structure.
(Received September 05, 2023)

1192-03-29230
Michael Wolman*, Caltech. Combinatorial expansions on countable Borel equivalence relations. Preliminary report.
Many combinatorial problems, such as graph colouring, can be viewed as problems of expansions on classes of structures. We study such expansion problems in the context of descriptive combinatorics, both in terms of definable expansions of structures on countable Borel equivalence relations, as well as equivariant expansions of structures on groups. We also look at expansions of invariant random structures on groups.
(Received September 06, 2023)

1192-03-29427
Shay Allen Logan*, Kansas State University. Varieties of Variable Sharing or: How I Stopped Worrying and Learned to Love Nonuniform Substitutions.
Relevance and variable sharing have gone hand-in-hand since the very start. The relationship in fact predates the seminal texts of the movement, as one can see by comparing the publication dates of \cite\{Belnap1960\} and \cite\{AndersonBelnap1975\}. But both both parties to this marriage have changed over the decades, and the marriage looks quite different now than it did when it first began. In particular, variable sharing results no longer play at being quite so hard to get. This is the result of a novel way to prove strong variable sharing results. The key bit (introduced in \cite\{Logan2022\}; \cite\{Logan2023\}) is the use of nonuniform substitutions. It turns out that this key bit is key in more than one way: not only does it unlock easy proofs of
strong variable sharing results, it also opens a door behind which hide a plethora of novel and quite unanticipated forms of variable sharing as well. For each of these forms of variable sharing, a proof that is not interestingly different from the proof of the main result in \cite\{Logan2022\} shows that weak-enough logics exhibit that form of variable sharing. Given all this, the goal of this talk is twofold. First, I'll survey the state of the art in order to show you how to use nonuniform substitutions to achieve profit and fame. After that, I'll try to convince you that you shouldn't feel bad about doing so. \scshape Nuel D. Belnap, \itshape Entailment and relevance, \bfseries\itshape Journal of Symbolic Logic, vol. 25 (1960), no. 2, pp. 144-146. \scshape Alan Ross Anderson and Nuel D. Belnap, \itshape Entailment: The Logic of Relevance and Neccessity, Vol. I, \bfseries\itshape Princeton, Princeton University Press (1975)., \scshape Shay Allen Logan, \itshape Depth Relevance and Hyperformalism, \bfseries \itshape Journal of Philosophical Logic, vol. 51 (2022), no. 4, pp. 721-737. \scshape Shay Allen Logan, \itshape Correction to: Depth Relevance and Hyperformalism, \bfseries\itshape Journal of Philosophical Logic, vol. 52 (2023), no. 4, p. 1235.
(Received September 06, 2023)

## 1192-03-29457

Fred Halpern*, Preservation theorems via Smullyan clashing tableau. Preservation theorems via Smullyan clashing tableau. Preliminary report.
\document \abstract We adopt Smullyan's Clashing Tableaux to prove preservation theorems in a unified manner. An Algebraic Description $D$ is a set of structure names and a set of relation names along with a set of "conditions". Description $D$ is simple if it describes two structures ( $A$ and $B$ ) and a relation $\mu$ between them along with possible conditions: $\mu$ is domain-onto, $\mu$ is range-onto, and $\mu$ preserves the formulas $\Phi$. We associate $\exists$ with domain-onto and $\forall$ with range-onto. $D_{Q}$ denotes the quantifiers associated with the onto conditions of $D$. Theorem 1 . The formulas preserved under simple $D$ (relative to $\Sigma$ ) are the closure of $\Phi$ under $D_{Q}, \vee$, and $\wedge$. The method parallels Natural Deduction. Just as we systematically search for a contradictory tableau to show a sentence is unsatisfiable, we search for a clashing tableau system $C$ which shows nonpreservation is unsatisfiable. We recursively compute from $C$ the preserved formula. A key observation is that a tableau for $\Sigma$ yields a weakening sentence $\sigma$ such that $\Sigma \vdash \sigma$. The weakening has Craig interpolation theorem traits. $\Sigma$ is a Cantor-Bernstein theory if $A$ and $A^{\prime}$ are sub-structures of each other imply $A$ and $A^{\prime}$ are isomorphic ( $\Sigma \vdash A, A^{\prime}$ ). Theorem 2 . If $\sum$ is a CantorBernstein theory, then every formula is equivalent to a lattice combination of existential and universal sentences, relative to $\Sigma$. Theorem 2 says that Cantor-Bernstein theory fills the gap between model complete theory and inductive theory. A generalization of Theorem 2 yields the Craig interpolation theorem, Beth definability theorem, and Svenonius' theorem. They can all be interpreted as preservation under two descriptions ( $D, D^{\prime}$ ) results. \endabstract $\backslash e n d d o c u m e n t$ (Received September 06, 2023)

## 1192-03-29475

Antoine Poulin*, McGill University. Borel Complexity of Archimedean Orders.
We present results on the Borel complexity of the action of GL_ $\{2\}(Z)$ on the Archimedean orders of $Z^{\wedge}\{2\}$. This mimics a result of F. Calderoni, A. Shani, D. Marker and L. Motto Ros for $\mathrm{Q}^{\wedge}\{2\}$. We discuss possible generalizations to different groups, including for intermediate rings $\mathrm{Z} \subset \mathrm{R} \subset \mathrm{Q}$ and $\mathrm{Z}^{\wedge}\{\mathrm{n}\}$.
(Received September 06, 2023)

## 1192-03-29607

Julia F. Knight*, University of Notre Dame, emerita. Complexity of well-ordered sets in an ordered Abelian group.
Preliminary report.
This is joint work with Christopher Hall and Karen Lange. We first ask how hard it is to say of a set, known to be well-ordered, that it has type at least $\alpha$. We measure complexity in the Borel and effective Borel hierarchies. The class of well orderings is not Borel, so we use Calvert's notions of complexity and completeness within. Turning to groups, we ask how hard it is to say of well-ordered sets $A, B$ in an ordered Abelian group $G$ that the set $A+B$ has type at least $\alpha$. The question is more interesting in the case where $A, B$ have order type less than $\alpha$. We have good results in the case where $G$ is Archimedean. Similarly, we ask how hard it is to say of a well-ordered set $A$ of non-negative elements in an ordered Abelian group $G$ that the set of finite sums of elements of $A$ has type at least $\alpha$. Again, we have good results in the case where $G$ is Archimedean. (Received September 07, 2023)

1192-03-29664
David Reed Solomon*, University of Connecticut. The tree pigeonhole principle in the Weihrauch degrees. Preliminary report.
The pigeonhole principle says that every finite coloring of the natural numbers has an infinite homogeneous set. The tree pigeonhole principle, $\mathrm{TT}^{1}$, is a natural extension to finite colorings of the full binary branching tree. $\mathrm{TT}^{1}$ was first studied in the context of reverse mathematics by Chubb, Hirst and McNicholl who showed that it implied $B \Sigma_{2}^{0}$. It was shown to sit strictly above $B \Sigma_{2}^{0}$ by Corduan, Groszek and Mileti, and more recently, to sit strictly below $I \Sigma_{2}^{0}$ by Chong, Li, Wang and Yang. In this talk, I will survey recent results with Damir Dzhafarov and Manlio Valenti on the tree pigeonhole principle and its first order part in the Weihrauch degrees.
(Received September 07, 2023)

## 1192-03-29798

David Alex Vogel Gonzalez*, University of California, Berkeley. Scott Sentence Complexities of Linear Orderings.
Martin Davis was an early pioneer in the study of hyperarithmetic sets and the hyperarithmetic hierarchy. The concept of Scott sentence complexity was introduced by Alvir, Greenberg, Harrison-Trainor and Turetsky and gives a way of assigning countable structures to elements of the boldface hyperarithmetic hierarchy. By calculating the Scott sentence complexities occurring in a class of structures we obtain a detailed picture of the descriptive complexity of its isomorphism relation. We study possible Scott sentence complexities of linear orderings using two approaches. First, we investigate the effect of the Friedman-Stanley embedding on Scott sentence complexity and show that it only preserves $\Pi_{\alpha}^{\text {in }}$ complexities. We then take a
more direct approach and exhibit linear orderings of all Scott sentence complexities except $\Sigma_{3}^{\mathrm{in}}$ and $\Sigma_{\lambda+1}^{\mathrm{in}}$ for $\lambda$ a limit ordinal. We show that the former can not be the Scott sentence complexity of a linear ordering. In the process we develop new techniques which appear to be helpful to calculate the Scott sentence complexities of structures in general. This talk is based on joint work with Dino Rossegger.
(Received September 07, 2023)

## 1192-03-29799

Cameron E. Freer*, Massachusetts Institute of Technology. Feedback Computability.
The notion of a feedback query is a natural generalization of choosing for an oracle the set of indices of halting computations. Notice that, in that setting, the computations being run are different from the computations in the oracle: the former can query an oracle, whereas the latter cannot. A feedback computation is one that can query an oracle, which itself contains the halting information about all feedback computations. Although this is self-referential, sense can be made of at least some such computations. We show that feedback Turing machines yield exactly hyperarithmetic computability. Joint work with Nate Ackerman and Bob Lubarsky.
(Received September 07, 2023)

## 1192-03-29800

Nathanael Leedom Ackerman*, Harvard University. On the computability of graph Turing machines.
In this talk we will consider "graph Turing machines", a model of parallel computation on a graph, in which each vertex is only capable of performing one of a finite number of operations. This model of computation is a natural generalization of several well-studied notions of computation, including ordinary Turing machines, cellular automata, and parallel graph dynamical systems. We analyze the power of computations that can take place in this model and show that properties of the underlying graph have significant consequences for the power of computation thereby obtained. In particular, we show that every arithmetically definable set can be computed by a graph Turing machine in constant time, and that every computably enumerable Turing degree can be computed in constant time by a graph Turing machine whose underlying graph has finite degree. This is joint work with Cameron Freer.
(Received September 07, 2023)

1192-03-30320
Bjørn Kjos-Hanssen*, University of Hawaii at Manoa. Diophantine equations exemplify Bennett's logical depth for automatic complexity. Preliminary report.
The solvability of a diophantine equation over the integers is an undecidable problem thanks to the work of Matiyasevich (1970), Robinson, Davis, and Putnam. Martin Davis described the topic as his "lifelong obsession". Bennett's logical depth (1988) of a word $x$ is the computation time required to verify an optimal description of $x$. For automatic complexity (introduced by Shallit and Wang 2001), this verification involves the unique solvability of certain constrained diophantine equations over $\mathbb{N}$. I present my work towards a NP-hardness result giving a precise sense in which logical depth for automatic complexity is simply the number of variables required in such a diophantine equation.
(Received September 09, 2023)

## 1192-03-30336

Dana Bartosova, University of Florida, Lynn Scow*, California State University, San Bernardino. Semi-retractions and reducing one Ramsey problem to another.
In this talk, we will define semi-retractions and discuss their role in transferring the Ramsey property from one class of structures to another. We will compare semi-retractions to other methods of encoding one Ramsey property in another, and discuss the potential for semi-retractions to explain the relative strength of Ramsey theorems.
(Received September 09, 2023)

## 1192-03-30481

Rachel Greenfeld*, Institute for Advanced Study. Undecidability of translational monotilings. Preliminary report.
Translational tiling is a covering of a space (e.g., $\mathbb{Z}^{d}$ ) using translated copies of some building blocks, called the "tiles", without any positive measure overlaps. Given a finite collection of building blocks, is it possible to decide whether they admit a translational tiling? Back in the 60's, Berger famously solved this problem in the negative. However, the decidability of translational monotilings (tilings by translations of a single tile) remained unsolved. In a recent work, joint with Terence Tao, we use a certain hierarchical structure to finally settle this problem in the negative. In the talk, we will survey the history of the problem and discuss some recent developments which eventually led to our result.
(Received September 10, 2023)

1192-03-30680
Dino Rossegger, TU Wien, Theodore Allen Slaman*, University of California, Berkeley, Tomasz Steifer, Polish Academy of Sciences. Learnability of Borel Equivalence Relations.
Let $\sim$ denote a Borel equivalence relation on $2^{\omega}$. We say that $\sim$ is learnable if there is an $A \in 2^{\omega}$ and a computable functional $\Phi$ such that for any $Y \in 2^{\omega}$ and any countable sequence of elements $\left(X_{i}\right)$ from $2^{\omega}$, if there is a $j \in \omega$ such that $Y \sim X_{j}$ then $\lim _{n \rightarrow \infty} \Phi\left(n, A \oplus Y \oplus\left(X_{i}\right)\right)$ is defined and equal to such a $j$. That is to say that $A$ can eventually recognize some element of $Y^{\prime}$ s equivalence class whenever presented with an infinite set of candidates which includes at least one which is equivalent to $Y$. We show that $\sim$ is learnable if and only if it is $\boldsymbol{\Sigma}_{2}^{0}$. Further, whether a given $\boldsymbol{\Pi}_{2}^{0}$-equivalence relation is learnable is a $\boldsymbol{\Pi}_{1}^{1}$ complete property of its defining formula.
(Received September 10, 2023)

Uri Andrews, University of Wisconsin-Madison, Meng-Che Ho*, California State University, Northridge. Word problem of groups as ceers.
Classically, the word problem of a group is the set of words equal to the identity of the group, and we analyze them using Turing reductions. In this talk, we consider the word problem of a group as a computably enumerable equivalence relation (ceer), namely, two words are equivalent if and only if they are equal in the group. We also consider the computable reduction between equivalence relations: $E$ is reducible to $F$ if there is a computable function $f$ so that $i E j$ if and only if $f(i) F f(j)$. We will discuss some recent results and compare them with their classic counterparts. It turns out that the landscape of word problems as ceers are very different from the classical theory.
(Received September 11, 2023)

## 1192-03-30921

Åsa Hirvonen*, University of Helsinki. Games for Measuring Distances Between Metric Structures.
In classical model theory, Ehrenfeucht-Fraïssé games are used to study similarities between structures. This can, e.g., be applied to prove inexpressibility results (finite games) or for building Scott sentences capturing isomorphism of countable structures (infinite or dynamic games). Several authors have considered Ehrenfeucht-Fraïssé games - or their cousin, back-and-forth systems - on metric structures. In a metric setting new phenomena arise, related to various forms of approximation, both within structures and between structures. In a game setting these show up as approximate answers to moves, and approximate preservation of formulae. It turns out that elementary equivalence up to a given quantifier depth can be subdivided by the accuracy and steepness of preserved formulae. These notions reconstruct the ability to prove inexpressibility results, but also enable capturing new phenomena, such as given distances between models with respect to various natural pseudometrics.
(Received September 11, 2023)

## 1192-03-31022

C. Ward Henson*, Univ. of Illinois. Measure algebras of Loeb spaces.

Measure algebras are the basis for some of the currently best understood and most important examples in continuous model theory. Loeb measure spaces provide useful tools in studying the model theory of measure algebras as well as illuminating examples. Most of this talk is based on joint work with Alexander Berenstein that appears in the paper Model theory of probability spaces, which is published in the book Model Theory of Operator Algebras edited by Isaac Goldbring; de Gruyter series in Logic and Applications, vol. 11, 2023, pp. 159-213. If time permits, we will also mention limitations on the Maharam invariants of the measure algebras of Loeb spaces that were found by Jin and Keisler.
(Received September 11, 2023)

## 1192-03-31042

Toby Meadows*, University of California, Irvine. A Modest Foundational Argument for the Generic Multiverse.
The generic multiverse is a system of set theoretic universes that is, roughly speaking, closed under the operations of generic extension and its inverse. The underlying idea can be axiomatized and the resultant theory MV might be thought of as a competitor to ZFC. In this talk, I want to make a modest philosophical argument for the value of MV as a foundation for set theory. I'll start by discussing the way in which ZFC provides a satisfying foundation for almost all of mathematics. I'll then argue ZFC does not provide a similarly satisfying foundation for contemporary set theory. Finally, I'll argue that MV can fulfill this role in a very natural manner.
(Received September 11, 2023)

## 1192-03-31123

Timothy H. McNicholl, Department of Mathematics, Iowa State University, Brian Zilli*, Department of Mathematics, Iowa State University. On the complexity of spectra of bounded analytic functions.
The set of accumulation points of zeros of a bounded analytic function on the disc is known as its spectrum. We consider the closed-set complexity of spectra of computable bounded analytic functions. In particular, all such spectra are $\Sigma_{3}^{0}$-closed and there exists a $\Sigma_{3}^{0}$-complete spectrum, but there also exists a $\Sigma_{2}^{0}$-closed set which is not the spectrum of any such function, while every $\Pi_{2}^{0}$-closed set is. Time permitting, we may discuss an effectivization of a theorem of Matheson concerning uniform Frostman functions.
(Received September 11, 2023)

## 1192-03-31267

Joshua Frisch*, University of California San Diego. Equivalence Relations Classifiable by Abelian Groups.
In this talk I will discuss which Borel equivalence relations can be reduced to orbit equivalence relations of Polish Abelian groups. This is joint work with Forte Shinko.
(Received September 11, 2023)

## 1192-03-31392

Ruiyuan Chen*, University of Michigan, Antoine Poulin, McGill University, Ran Tao, Carnegie Mellon University, Anush Tserunyan, McGill University. Treeing Borel quasi-trees.
A countable Borel equivalence relation is said to be treeable if there is a Borel assignment of a tree on each equivalence class. We prove various results showing that every Borel assignment of "large-scale approximate trees" can be turned into genuine trees, thereby yielding new sufficient criteria for treeability. The proof uses tools from lattice theory and metric geometry, namely median graphs/CAT(0) cube complexes. This is joint work with Antoine Poulin, Ran Tao, and Anush Tserunyan.
(Received September 11, 2023)

Wesley Calvert, Southern Illinois University, Johanna N Y Franklin*, Hofstra University, Daniel Turetsky, Victoria University of Wellington. Reticence in computable structure theory.
We consider the distinction between positive information and complete information in the context of highness for computable structure theory. We say that a degree that is uniformly high for isomorphism "in the reticent sense" will not only compute an isomorphism when one exists but also fail to compute an apparent isomorphism where one does not. While a number of uniform highness notions have been separated, their reticent versions turn out to characterize the same class of degrees. (Received September 12, 2023)

## 1192-03-31935

Mariana Vicaria*, University of California, Los Angeles. Model Theory of Valued Fields.
Model theory is a branch of mathematical logic that studies structures (that is sets equipped with relations, functions and constants) and their definable sets. There is a more general class of subsets that one could study, called the interpretable sets, obtained by taking the quotient of a definable set by a definable equivalence relation. A natural question is: given a structure can one classify the interpretable sets in that structure? A valued field is a field $K$ equipped with a distinguished subset $\mathcal{O}$, a valuation ring. Examples of valued fields are the $p$-adic field $\mathbb{Q}_{p}$ or the Laurent series over the complex numbers $\mathbb{C}((t))$. Given $\mathcal{O}$ a valuation ring of a field and $\mathcal{M}$ its maximal ideal, we commonly refer to the additive quotient $\mathcal{O} / \mathcal{M}$ as the residue field, while the multiplicative quotient $K^{\times} / \mathcal{O}^{\times}$is an ordered abelian group and it is called the value group. One of the most striking results in the model theory of valued fields is the Ax-Kochen/ Ershov theorem which roughly states that the first order theory of an unramified henselian valued field is completely determined by the first order theory of its residue field and its value group. A principle follows from this theorem: the model theory of valued field is controlled by its residue field and its value group. In this talk I will make a brief description of valued fields and their model theory. I'll present how the problem of classifying interpretable sets in henselian valued fields can be approached in an Ax-Kochen style: What obstructions come from the residue field? and from the value group? I will conclude presenting the classification of the interpretable sets in valued fields obtained in joint work with Rideau-Kikuchi. M. Hils and S. Rideau-Kikuchi, Un principe D'Ax-Kochen-Ershov imaginaire , lurl\{Pre-print: https://arxiv.org/pdf/2109.12189.pdf\}. M. Vicaría, Elimination of Imaginaries in $\mathbb{C}((t))$, Journal of the London Mathematical Society, Vol. 108, 2, (2023), 482-544
(Received September 12, 2023)

## 1192-03-32004

Gihanee Senadheera*, Winthrop University. Embedding of well-known degrees to PAC/PACi degrees.. Preliminary report. In 1984 Leslie Valiant introduced the Probably Approximately Correct (PAC) learning model. There is a natural reducibility to PAC learnability and the PACi reducibility is a less restricted version of PAC predictability. The ' $i$ ' in PACi reducibility refers to the PAC reducibility independent of size and computation time. Since PAC and PACi reducibilities resemble the Turing reducibility we can define PAC and PACi degrees. If there exists an embedding from well-known degrees to PAC or PACi degrees, then we can infer all the properties of well-known degrees to PAC or PACi degrees. (Received September 12, 2023)

## 1192-03-32224

David Schrittesser*, Harbin Institute of Technology. Loeb measure and nonstandard measure in statistics and economical theory.
Recently nonstandard methods have been used to give an exact characterization of admissibility in statistical decision theory, in joint work by the speaker with Duanmu and Roy. This has application regarding rationalizability in economics (joint with Khan and Pedersen).
(Received September 12, 2023)

1192-03-32335
Dima Svetosla Sinapova*, Rutgers University. Combinatorial Principles at Successors of Singular Cardinals.
Given a singular cardinal $\kappa$, mutual stationarity asserts that sequences of stationary subsets of regular cardinals with limit $\kappa$ have a "simultaneous witness" for their stationarity. This was first defined by Foreman and Magidor in 2001, who showed it holds when restricted to points of cofinality $\omega$. The case for higher cofinalities remained open until a few years ago Ben Neria showed its consistency from large cardinals. In Ben Neria's model SCH naturally holds at $\aleph_{\omega}$. We show that we can obtain mutual stationarity at $\left\langle\aleph_{n} \mid n<\omega\right\rangle$ for any fixed cofinality together with the failure of SCH at $\aleph_{\omega}$ (joint with Will Adkisson). The we will discuss what this means for various combinatorial principles at $\aleph_{\omega+1}$. (Received September 12, 2023)

1192-03-32344
David Belanger, Nanyang Technological University, Damir D. Dzhafarov*, University of Connecticut, Jun Le Goh, National University of Singapore. Non-injection principles and uniformity.
For positive numbers $n>k$, let $n \hookrightarrow k$ denote be the problem whose instances are functions $f: n \rightarrow k$, with a solution to any such $f$ being a pair $(a, b)$ of numbers smaller than $n$ such that $f(a)=f(b)$. The problems $n \hookrightarrow k$ are trivial in terms of computational properties, but their Weihrauch jumps, $(n \hookrightarrow k)^{\prime}$, where the instances $f$ are given by $\boldsymbol{\Delta}_{2}^{0}$ approximations, are more interesting. We study the uniform content of these problems using Weihrauch-style computable analysis, and show that the relationships between them (for different values of $n$ and $k$ ) as well as between them and other problems from the literature, are quite intricate. Although the proofs of the results are mostly elementary, the underlying combinatorics are surprisingly deep and difficult to fully describe. For example, the question of whether $\lim _{n+1} \leq_{\mathrm{W}}\left(n^{2} \hookrightarrow n\right)^{\prime}$ is equivalent to the existence of an affine plane of order $n$, which is a longstanding open problem in combinatorics. (Received September 12, 2023)

## 1192-03-32471

Noah Schweber*, Proof School. The Harrison order as an ultraproduct. Preliminary report
The Harrison order $\mathcal{H} \cong \omega_{1}^{C K}(1+\eta)$ is an important (counter)example in computable structure theory. We will show how $\mathcal{H}$ can be construed as a kind of "admissible ultraproductr" of computable ordinals. This provides an interesting parallel with the behavior of ultraproducts with respect to countably complete ultrafilters in set theory, and strongly contrasts with a morestudied effective analogue of ultraproduct and ultrapower constructions, the "cohesive product/power." We will examine the basic properties of these ultraproduct-like constructions, and highlight some questions they raise about structures of high Scott rank.
(Received September 12, 2023)

## 1192-03-32509

Matthew Harrison-Trainor*, University of Illinois Chicago. The Complexity of Classifying Topological Spaces.
Given a topological space, how difficult is it to characterize it up to homeomorphism? The unit interval, for example, is the unique metrizable continuum with exactly two non-cut points. We would like to, first of all, measure the complexity of such a characterization, and second of all, prove that it is best possible. I will talk about these problems both for particular examples and as a more general theory. This is the topological analogue of the theory of Scott sentences and Scott complexity for countable structures.
(Received September 12, 2023)

## 1192-03-32859

Vince Guingona, Towson University, Miriam Parnes*, Towson University, Lynn Scow, California State University, San Bernardino. Properties and Products for Classes of Finite Structures
In this talk, we will explore various properties and products for classes of finite structures and the interaction between them. Properties to be discussed include indivisibility and the amalgamation property. We will consider whether these properties are preserved under three distinct notions of products on classes of finite structures: lexicographic product, full product, and free superposition.
(Received September 12, 2023

## 1192-03-33134

Slawomir Solecki*, Cornell University. Descriptive Set Theory and Generic Measure Preserving Transformations.
One of the areas of interest of Descriptive Set Theory is dynamics of Polish groups, that is, groups carrying a group topology that is separable and completely metrizable. Such groups are not, in general, locally compact. Therefore, in studying their dynamics, classical methods relying on Haar measure are not available. These methods can sometimes be replaced by descriptive set theoretic tools. I will describe how the descriptive set theoretic point of view led to a recent answer to an old question in Ergodic Theory. The question lies within a long-established theme, going back to the work of Halmos and Rokhlin, of investigating generic measure preserving transformations. The answer to the question rests on an analysis of unitary representations of a certain non-locally compact Polish group that can be viewed as an infinite dimensional torus. (Received September 13, 2023)

## 1192-03-33875

Jad Damaj*, UC Berkeley. Degree Spectra of Relations of $(\mathbb{N},<)$. Preliminary report.
Given a relation (or function) on a mathematical structure, the computational properties of the relation might depend on how the structure is presented. For example, in some ways of presenting a vector space, independence might be computable, while in other presentations, it might not be. The degree spectrum of the relation measures this. In this talk we will introduce degree spectra and focus on relations on the natural numbers as a linear order. We attempt to answer the "on-a-cone" version of a question posed by Wright about which degree spectra are possible on $(\mathbb{N},<)$ by showing the existence of many previously unknown degree spectra.
(Received September 27, 2023)

## 05 Combinatorics

## 1192-05-25391

Peter M Winkler*, Dartmouth College. Permutons.
Permutons are probability distributions on the unit square with uniform marginals. Like graphons (for graphs), permutons are limit structures for permutations, and with the help of a variational principle, can tell us much about what large random permutations look like. We'll give an overview of what's happening in the permuton world, including the emergence of random permutons, the question of what sets can support a permuton, and some of the many intriguing open problems.
(Received June 05, 2023)

1192-05-25399
Anne Schilling*, University of California, Davis. The Ubiquity of Crystal Bases.
Abstract: Crystal bases are combinatorial skeletons of Lie algebra representations. They appeared in the work of Kashiwara, Lusztig and Littelmann on quantum groups and the geometry of flag varieties. Crystal bases arise in many unexpected places from mathematical physics to probability and number theory. In this talk, I will showcase ten reasons and applications of how crystal theory can be used to solve problems in representation theory, geometry and beyond.
(Received June 05, 2023)

## 1192-05-25400

Maria Chudnovsky*, Princeton University. What Makes a Problem Hard?.
Graph theory is a powerful mathematical tool for modeling real life situations. But once the model is built, how useful is it for finding a solution? The answer depends on how complicated the graph we have constructed is. What makes a graph complicated and when is finding a solution difficult? There are many possible answers to this question; in this talk we will discuss some of them. We will start with a brief overview of the history of the subject. No prior knowledge of graph theory is assumed.
(Received June 05, 2023)

## 1192-05-25450

Jacob Fox, Stanford University, Zoe Himwich, Columbia University, Nitya Mani*, Massachusetts Institute of Technology,
Yunkun Zhou, Stanford University. Sidorenko-type properties in tournaments. Preliminary report.
We study variants of Sidorenko's conjecture in tournaments, where new phenomena arise that do not have clear analogues to the setting of undirected graphs. We first consider oriented graphs that are systematically under-represented in tournaments (i.e. that are tournament anti-Sidorenko). We prove that such oriented graphs must be quite sparse; specifically, we show that the maximum number of edges of a $k$-vertex oriented graph which is tournament anti-Sidorenko is $(1+o(1)) k \log _{2} k$. We also give several novel constructions of oriented graphs that are systematically over-represented in tournaments (tournament Sidorenko); as a representative example, we show that most ways to delete an edge from a transitive tournament yield a tournament Sidorenko oriented graph. As an illustration of our methods, for the special case of stars, we completely understand the tournament Sidorenko and anti-Sidorenko orientations.
(Received June 09, 2023)

## 1192-05-25657

Andrew Vince*, University of Florida. The Average Order of the Connected Induced Subgraphs of a Graph and UnionIntersection Systems.
Because connectivity is such a basic concept in graph theory, extremal problems concerning the average order of the connected induced subgraphs of a graph have been of notable interest. A particularly resistant open problem is whether or not, for a connected graph $G$ of order $n$, all of whose vertices have degree at least 3 , this average is at least $n / 2$. One of our results is that if $G$ is a connected, vertex transitive graph, then the average order of the connected induced subgraphs of $G$ is at least $n / 2$. The exteremal problems above concerning connectivity lead to a broader theory. The concept of a UnionIntersection System wll be introduced in the talk and several open problems posed. (Received June 30, 2023)

1192-05-25745
Adam Knudson*, Brigham Young University. A Nordhaus-Gaddum type problem for the normalized Laplacian spectrum and graph Cheeger constant
For a graph $G$ on $n$ vertices with normalized Laplacian eigenvalues $0=\lambda_{1}(G) \leq \lambda_{2}(G) \leq \cdots \leq \lambda_{n}(G)$ and graph complement $G^{c}$, we prove that

$$
\max \lambda_{2}(G), \lambda_{2}\left(G^{c}\right) \geq \frac{2}{n}^{2}
$$

We do this by way of lower bounding $\max \left\{i(G), i\left(G^{c}\right)\right\}$ and $\max \left\{h(G), h\left(G^{c}\right)\right\}$ where $i(G)$ and $h(G)$ denote the isoperimetric number and Cheeger constant of $G$, respectively. (Received July 07, 2023)

1192-05-25792
Dingding Dong*, Harvard University, Nitya Mani, Massachusetts Institute of Technology, Yufei Zhao, Massachusetts Institute of Technology. On the number of error correcting codes.
We show that for a fixed $q$, the number of $q$-ary $t$-error correcting codes of length $n$ is at most $2^{(1+o(1)) H_{q}(n, t)}$ for all $t \leq\left(1-q^{-1}\right) n-2 \sqrt{n \log n}$, where $H_{q}(n, t)=q^{n} / V_{q}(n, t)$ is the Hamming bound and $V_{q}(n, t)$ is the cardinality of the radius $t$ Hamming ball. This proves a conjecture of Balogh, Treglown, and Wagner, who showed the result for $t=o\left(n^{1 / 3}(\log n)^{-2 / 3}\right)$.
(Received July 11, 2023)

## 1192-05-26276

Mariel Vazquez*, University of California, Davis. Topological Considerations in Genome Biology.
Living organisms and some viruses carry their genetic code in very long, tightly packaged, DNA molecules. Understanding the geometry and topology of DNA is key to understanding the mechanisms of viral infection and the inner workings of a cell. We study the changes in DNA topology mediated by essential processes such as DNA packing and transcription of DNA into RNA. These processes are highly regulated, and even small structural changes can lead to catastrophic effects. We use a variety of techniques from knot theory and low-dimensional topology, aided by discrete methods and computational tools, to analyze molecular biology data produced by us and by our collaborators. In this lecture I discuss DNA packaging in viruses and the formation and entanglement of DNA:RNA hybrids that arise during transcription. The presentation is accessible to students and suitable for a diverse interdisciplinary audience.
(Received July 24, 2023)

Anna Coleman*, Southeastern University, Joshua Harrington, Cedar Crest College, Maggie X. Lai*, Tulane University, Philip D. Thomas*, Kutztown University, Tony Wing Hong Wong, Kutztown University of Pennsylvania. Zero-Sum-Free Graph Labelings.
A $k$-tuple over $\mathbb{Z}_{n}$ is zero-sum-free if every nonempty subset of its components has a nonzero sum. We provide results on zero-sum-free tuples with distinct entries and generalize the concept to zero-sum-free graph labelings. Given a graph $\Gamma$ with vertices $v_{1}, v_{2}, \ldots, v_{k}$, a $\mathbb{Z}_{n}$-labeling on $\Gamma$ is an injective function that maps $v_{i}$ to $a_{i} \in \mathbb{Z}_{n}$. A $\mathbb{Z}_{n}$-labeling on $\Gamma$ is zero-sum-free if $\sum_{i=1}^{j} a_{\sigma_{i}} \neq 0$ for all positive integers $j$ and all path subgraphs $v_{\sigma_{1}} v_{\sigma_{2}} \cdots v_{\sigma_{j}}$ in $\Gamma$. We find results for $\mathbb{Z}_{n}$-zero-sum-free labelings on classes of simple graphs and extend the notion of a $\mathbb{Z}_{n}$-zero-sum-free labeling to nonabelian groups. (Received July 26, 2023)

## 1192-05-26419

Maria Monks Gillespie*, Colorado State University, Sean Griffin, UC Davis. Generalized Springer fibers, battery-powered Young tableaux, and the Delta conjecture.
We present new geometric and combinatorial results relating to the generalized Springer theory developed by Borho and Macpherson. Motivated by the Delta Conjecture in symmetric function theory, we show that the so-called Delta-Springer modules of Griffin, Levinson, and Woo are instances of the generalized Springer fibers from the Borho-Macpherson theory, and use it to present new Schur positive formulas for specializations of the Delta conjecture polynomials. Our formulas involve a new type of Young tableau and the classical cocharge statistic, and we show that the combinatorics of the case we are considering implies special rational smoothness properties of the corresponding generalized Springer fibers.
(Received July 27, 2023)

## 1192-05-26535

George E. Andrews*, Pennsylvania State University. Parity in MacMahon's Partition Anlaysis. Preliminary report. This is a report on joint work with Peter Paule. The object is to illustrate how parity may be used in applications of P.A. MacMahon's Partition Analysis. We begein with a description of how Partition Analysis with a parity component leads us directly to a combinatorial understanding of the $q$-series in the first Goellnitz-Gordon identity. We also show how the addition of parity to Partition Analysis leads to a new interpretation of Hei-Chi Chan's "cubic partitions." We close the talk with an account of some of the more esoteric theorems arising from this new aspect of Partition Analysis. (Received July 31, 2023)

## 1192-05-26660

Karthekeyan Chandrasekaran, University of Illinois, Urbana-Champaign, Sheldon Jacobson, University of Illinois, UrbanaChampaign, Ian Ludden, Rose-Hulman Institute of Technology, Ellen Veomett*, University of San Francisco. Connected Recursive Bijection and Perfect Hierarchical Matchings.
In recent years, the redistricting process been examined and tested by the mathematical and computational communities. One area of study in this arena is that of redistricting protocols enacted as two-player games. The two partisan sides repeatedly take turns in constructing the redistricting map. The purpose and goal of such protocols is to ensure that resulting map does not significantly benefit one party over the other. Here, we consider one such protocol, called Connected Recursive Bisection (CRB). CRB repeatedly splits each part of the current map partition in half, while maintaining part connectivity and terminates when it reaches a specified number of parts. Given a final partition produced by CRB, recovering the sequence of bisections is equivalent to finding a sequence of perfect matchings in the dual graph, in which each vertex represents a part and edges correspond to part adjacencies. We define this sequence of matchings as a perfect hierarchical matching (PHM), which in turn gives rise to a PHM tree. We first characterize PHM trees by an efficient edge labeling scheme and count the number of PHM trees of the complete graph up to isomorphism. We give an efficient algorithm fo recognizing series-parallel graphs with PHMs and generalize it to a slicewise polynomial (XP) algorithm on graphs with constant treewidth. We prove tight max-degree and diameter bounds for PHM trees, as well as a matroid-like edge exchange property. We conclude with hardness results for recognizing specific PHM tree families in general graphs and a few open problems. (Received August 02, 2023)

## 1192-05-26671

Sam Spiro*, Rutgers University. The Random Turán Problem.
Let $G_{n, p}$ denote the random $n$-vertex graph obtained by including each edge independently and with probability $p$. Given a graph $F$, let $\operatorname{ex}\left(G_{n, p}, F\right)$ denote the size of a largest $F$-free subgraph of $G_{n, p}$. When $F$ is non-bipartite, the asymptotic behavior of $\operatorname{ex}\left(G_{n, p}, F\right)$ was determined in breakthrough work done independently by Conlon-Gowers and by Schacht. In this talk we discuss some recent results for bipartite $F$ (where much less is known), as well as for the analogous problem for $r$ partite $r$-graphs.
(Received August 02, 2023)

## 1192-05-26699

Owen Huang*, Georgia Tech. Groups and Graphs of Polynomial Growth. Preliminary report.
If $G$ is a finitely generated group, then one can use its Cayley graph to study its geometric properties. For example, one can use the natural graph metric to define balls and spheres and how fast they may grow. A celebrated theorem of Gromov states that this growth rate is intimately tied not just to the graph, but actually to properties of the group! On the other hand, given the structure of a Cayley graph, one might wonder what group properties we can deduce from the structure. This interplay between groups and graphs is studied in geometric group theory. In addition to a introduction to geometric group theory, we present our recent result stating that for $d=1,2,3$, a group has growth $r^{d}$ if and only if for all finite, symmetric generating sets $S$, its Cayley graph is isomorphic to a subgraph of $\mathbb{Z}_{\infty}^{d} \boxtimes K_{n}$ for some $n \in \mathbb{N}$. In doing so, we introduce a novel property of groups called $d$-embeddability, which roughly characterize how close to $\mathbb{Z}^{d}$, the $d$-dimensional lattice, it looks. This talk is aimed towards any students who have had some exposure to group and graph theory.

Sophia Child*, Colgate University, Samal Omshi*, Colgate University. Exploring Degree, Diameter and Connectedness for Graph Products. Preliminary report.
Large scale computer networks are necessary for simulations, data visualization, and other scientific computations both in industry and in academia. These networks can have tens to hundreds of thousands of processing units - at that scale, it is pertinent that the networks are designed in a way that is efficient and cost-effective. The network can be interpreted as a graph whose vertices represent the compute nodes and edges represent the links between them. Thus it is important to study this problem mathematically. We explore different graph products to examine their viability for network design, analyzing their degree, diameter, and connectedness. We give a sharp diameter bound as well as a necessary and sufficient connectedness condition for the weak modular product. We also examine the zig-zag product, providing counterexamples to previously published diameter bounds, and proving stronger connectedness results than previously known.
(Received August 04, 2023)

## 1192-05-26970

Felice Manganiello, Clemson University, Freeman Slaughter*, Clemson University. The Generic Error Syndrome Decoding Problem.
We introduce the Generic Error Syndrome Decoding Problem, with error sets beyond those defined by a metric, and use the set-theoretic difference operator to characterize when error sets can be detected or corrected by codes. We prove the existence of a general, metric-less form of the Gilbert-Varshamov bound, and show that - like in the Hamming setting - a random code corrects a generic error set with overwhelming probability. We show that the decisional variant of this problem is NP-Complete. Finally, we identify a vulnerability of this problem for a specific family of codes: those defined over large extension fields and without a very high rate.
(Received August 08, 2023)

## 1192-05-26975

Zachary Lubberts*, University of Virginia, Yu Tian, Nordita, Stockholm University, Melanie Weber, Harvard University. Curvature-based Clustering on Graphs.
Unsupervised node clustering (or community detection) is a classical graph learning task. In this paper, we study algorithms, which exploit the geometry of the graph to identify densely connected substructures, which form clusters or communities. Our method implements discrete Ricci curvatures and their associated geometric flows, under which the edge weights of the graph evolve to reveal its community structure. We consider several discrete curvature notions and analyze the utility of the resulting algorithms. In contrast to prior literature, we study not only single-membership community detection, where each node belongs to exactly one community, but also mixed-membership community detection, where communities may overlap. For the latter, we argue that it is beneficial to perform community detection on the line graph, i.e., the graph's dual. We provide both theoretical and empirical evidence for the utility of our curvature-based clustering algorithms. In addition, we give several results on the relationship between the curvature of a graph and that of its dual, which enable the efficient implementation of our proposed mixed-membership community detection approach and which may be of independent interest for curvaturebased network analysis.
(Received August 08, 2023)

## 1192-05-27004

Karen R. Gunderson, University of Manitoba, Jd Nir*, Oakland University. A New Direction: Chromatic Number Thresholds in Random Oriented Graphs.
The Erdős-Rényi random graph $\mathcal{G}(n, d / n)$ and the random $d$-regular graph both model graphs with average degree $d$. In 2013, building on more than sixty years of results, Coja-Oghlan et al. showed that for large values of $d$, the chromatic number of these models is a.a.s. concentrated on a single value. We apply the diverse set of tools developed to answer this question to a new problem. The oriented chromatic number of a oriented graph $\vec{G}$, introduced by Courcelle in 1989, is the size of the smallest tournament to which $\vec{G}$ has a homomorphism. In joint work with Karen Gunderson, we bound the oriented chromatic number of directed analogs of $\mathcal{G}(n, d / n)$ the and random $d$-regular graph. Our results indicate that this problem presents unique and interesting challenges that are worthy of further study.
(Received August 08, 2023)

1192-05-27206
Ethan Borsh*, Allegheny College, Jensen Bailey Bridges*, Oklahoma State University, Millie Jeske*, University of Texas at Tyler. Enumeration of Cyclic Permutations in One-Line and Cycle Notation. Preliminary report
A permutation is said to avoid a given pattern if there is no subsequence of the permutation in the same relative order as that pattern. This notion of pattern avoidance has several applications, including applications to computer science, algebraic combinatorics, and dynamical systems. We investigate cyclic permutations that avoid a pattern $\boldsymbol{\sigma}$ in its one-line notation and another pattern $\boldsymbol{\tau}$ in its cycle notation. In this, we will prove a bijective mapping from previous permutations to those of the desired $n$ by using recursive sequences.
(Received August 12, 2023)

Chirag Kaudan*, San Jose State University, Rachel Taylor, DePaul University. An Inverse Approach to Characterizing All Graphs With Failed Zero Forcing Number of 2
Given a graph $G$, the zero forcing number of $G, Z(G)$, is the smallest cardinality of any set $S$ of vertices on which repeated applications of the forcing rule results in all vertices being in $S$. The forcing rule is: if a vertex $v$ is in $S$, and exactly one
neighbor $u$ of $v$ is not in $S$, then $u$ is added to $S$ in the next iteration. Hence the failed zero forcing number of a graph was defined to be the cardinality of the largest set of vertices which fails to force all vertices in the graph. In 2021 Gomez et al. proved that there are 15 graphs with a failed zero forcing number of 2 , but their proof was complicated requiring the analysis of many graph families. Using a new approach we present a shorter proof which shows all graphs have a failed zero forcing set of two isolated vertices. Furthermore, we characterize the graphs which have failed zero forcing set consisting of two isolated vertices, two adjacent vertices, and which graphs have both. This approach also has greater potential for extending to cases where $F(G)>2$.
(Received August 13, 2023)

1192-05-27260
Van Thanh Son Nguyen*, University of Minnesota, Andrew Sack, University of California, Los Angeles. Face Numbers of Poset Associahedra.
For every poset $P$, its poset associahedron $\mathscr{A}(P)$, introduced by Galashin, is a simple polytope whose facets correspond to proper tubes of $P$ and whose vertices correspond to maximal tubings of $P . \mathscr{A}(P)$ can also be described as a compactification of the configuration space of order-preserving maps $P \rightarrow \mathbb{R}$. Poset associahedra bear resemblance to graph associahedra. However, while the face numbers of graph associahedra have been studied extensively, e.g. by Postnikov-Reiner-Williams, little is known about those of poset associahedra. We show some results concerning the face numbers of poset associahedra. First, we prove that the face numbers of $\mathscr{A}(P)$ only depend on the comparability graph of $P$. Secondly, we give a combinatorial interpretation of the $h$-vectors of some poset associahedra as descents of stack-sorting preimages. We use this connection to prove real-rootedness of their $h$-vectors and nonnegativity of their $\gamma$-vectors. Finally, we study a family of poset associahedra that are not flag and show that their $\gamma$-vectors are also nonnegative.
(Received September 25, 2023)

## 1192-05-27265

AJ Harris, University of Minnesota, Van Thanh Son Nguyen*, University of Minnesota. Growth Diagrams for Bumpless Pipe Dream RSK..
The well-known Robinson-Schensted-Knuth correspondence is a bijection between matrices with non-negative integer entries and pairs of column-strict tableaux. The growth diagram, developed by Fomin, is a tool to break down this complicated correspondence into a simple set of local rules. The growth diagram has also been used to study other operations such as jeu-de-taquin and evacuation. Bumpless pipe dream is an object used to study Schubert polynomial and Schubert constants, whose combinatorial interpretation has been a 30-year open problem. Recently, Huang and Pylyavskyy introduced bumpless pipe dream RSK that recovers the classical RSK as a special case. We will introduce the growth diagram and local rules for this bumpless pipe dream RSK.
(Received August 13, 2023)

1192-05-27294
Ari Isaac Hughes Benveniste*, Pomona College, Angela Cai, University of Pennsylvania. Sparse Graphs That Admit Two Distinct Eigenvalues.
Given a graph $G$, the parameter $q(G)$ gives the minimum number of distinct eigenvalues in the spectrum of a weighted adjacency matrix for $G$ (i.e. the symmetric matrices whose off-diagonal zero pattern exactly matches that of the adjacency matrix of $G$ ). This parameter arises in the Inverse Eigenvalue Problem for Graphs: given a graph $G$, which multisets of real numbers can be obtained as the spectrum of some weighted adjacency matrix for $G$ ? Graphs with $q(G)=2$ are exactly the graphs with an orthogonal weighted adjacency matrix, and their classification is an important problem in the study of zero patterns of orthogonal matrices. It was recently shown by Barrett et al. (Barrett et al. Sparsity of graphs that allow two distinct eigenvalues. Linear Algebra Appl. 674(2023), 377-395) that graphs on $n$ vertices with $q(G)=2$ have $|E(G)| \geq 2 n-4$, and that the graphs meeting this bound are exactly the double-ended candles. They also show that the odd order graphs with $|E(G)|=2 n-3$ and $q(G)=2$ are the single-ended candles. We complete the characterization of graphs with $|E(G)|=2 n-3$ and $q(G)=2$ by treating the even order case. We also resolve two open questions of Barrett et al. by: classifying the condensations of $K_{5}-e$ with $q(G)=2$; and, determining for each double-ended candle $H$, the sets of nonedges $S$ for which $q(H+S)=2$.
(Received August 15, 2023)

## 1192-05-27448

Aaron Williams*, Williams College. Unlocking New Solutions: Puzzle Design using Grey Codes (not just Gray Code!). Preliminary report.
The binary reflected Gray code, or simply Gray code, provides the solution to a wide variety of puzzles. These classic (and obscure) puzzles include the Towers of Hanoi, Chinese Rings, Spin-Out by Binary Arts, and The Brain by Mag-Nif. In these puzzles, the current state can be represented by an $n$-bit binary string. The player interacts with the puzzle by flipping individual bits, and the challenge comes from the fact that some bits are locked based on the values of other bits. In fact, at most two bits are unlocked in a given state, and the state space is simply a path of length $2^{n}$ from $000 \cdots 0$ to $100 \cdots 0$ that follows Gray code order. In other words, the solutions are long, and the solver is always limited to taking one step forward or one step backward. We present new orders of binary strings called grey codes which are generalizations of Gray codes Puzzles using grey codes still have binary state spaces, but they feature more decision points and alternate solutions with shorter lengths. Each generalization can be understood in terms of a key that unlocks bits and allows them to be flipped, with original Gray code puzzles using key $\_10^{n-1}$. In particular, grey code puzzles with key $\_10$ have solutions as short as $n^{2}$ steps (and as long as $2^{n}$ steps). (The term Gray code is in reference to one of its discoverers: Frank Gray. We playfully use the term grey in homage to a frequent misspelling of Gray, and to indicate that the solutions are not so black-and-white!) Joint work with Petr Gregor and Ondrej Mička at Charles University.
(Received August 15, 2023)

Katelyn Buck*, The University of Texas at Austin, Catherine Bess Jacobs*, Wellesley College, Amelia Julian, Plymouth State University. Multi-Dimensional Graphs Modeling Self-Assembling DNA Nanostructures.
Employing tools from graph theory and linear algebra, we model the biological process of the creation of nanostructures from self-assembling DNA complexes. We represent $k$-armed branch junction molecules with tiles which are vertices in a graph with half-edges. The half-edges depict the cohesive-end types of a DNA strand. We aim to determine the minimum number of tiles and cohesive-end types necessary to form the complete complex of a given multi-dimensional graph structure. The problem of modeling DNA self-assembly is particularly challenging when considering graph families which change in multiple dimensions. In this research, we present the minimum number of tiles and cohesive-end types necessary to create the stacked book graphs, the square lattice graphs, and the Mongolian tent graphs, under different laboratory constraints. (Received August 16, 2023)

1192-05-27668
Aryan Bora*, MIT PRIMES, Yunseo Choi, Harvard University, Lucas Tang*, MIT PRIMES. On the Spum and Sum Diameter of Paths.
In a sum graph, the vertices are labeled with distinct positive integers, and two vertices are adjacent if and only if the sum of their labels is equal to the label of some other vertex. The spum of a graph $G$ is defined as the minimum difference between the largest and smallest labels of a sum graph that consists of $G$ and a minimum number of isolated vertices. More recently, Li introduced the sum diameter of a graph $G$, which modifies the definition of spum by removing the requirement that the number of isolated vertices must be minimal. In this paper, we settle conjectures by Singla, Tiwari, and Tripathi and Li by evaluating the spum and the sum diameter of paths. We also improve the bounds of the integral spum of paths and evaluate the integral sum diameter of paths, which are the generalization of spum and sum diameter over sum graphs of distinct integer labels respectively.
(Received August 17, 2023)

## 1192-05-27706

Eric Shen*, Harvard University. Tighter Bounds on the Expected Running Time of Ungarian Markov Chains.
In 2023, Defant and Li defined the Ungarian Markov Chain associated to a lattice $L$. This Markov chain has state space $L$, and from any state $x \in L$ transitions to the meet of $\{x\} \cup T$, where $T$ is a randomly selected subset of the elements of $L$ covered by $x$. They investigated the expected number of steps $\mathcal{E}(L)$ for the maximal element $\hat{1}$ of $L$ to transition to minimal element $\hat{0}$ in the Ungarian Markov Chain, and conjectured that when $L$ is $S_{n}$ or the $n$th Tamari lattice $\operatorname{Tam}_{n}, \mathcal{E}(L)$ is linear in $n$. To this end, they proved a linear lower bound on $\mathcal{E}\left(S_{n}\right)$ and a linear upper bound on $\mathcal{E}\left(\mathrm{Tam}_{n}\right)$. We now prove their conjecture for $\mathcal{E}\left(S_{n}\right)$, and prove an $n^{1-o(1)}$ lower bound on $\mathcal{E}\left(\operatorname{Tam}_{n}\right)$.
(Received August 18, 2023)

## 1192-05-27746

James Anderson, Georgia Institute of Technology, Anton Bernshteyn, Georgia Institute of Technology, Abhishek Dhawan*, Georgia Institute of Technology. Improved bounds on the AKS Conjecture for $K_{1, t, t}$.
Alon, Krivelevich, and Sudakov conjectured in 1999 that for every finite graph $F$, there exists a quantity $c(F)$ such that
$\chi(G) \leq(c(F)+o(1)) \Delta / \log \Delta$ whenever $G$ is an $F$-free graph of maximum degree $\Delta$. The largest class of connected graphs $F$ for which this conjecture has been verified so far, by Alon, Krivelevich, and Sudakov themselves, comprises the almost bipartite graphs (i.e., subgraphs of the complete tripartite graph $K_{1, t, t}$ for some $t \in \mathbb{N}$ ). However, the optimal value for $c(F)$ remains unknown even for such graphs. Bollobás showed, using random regular graphs, that $c(F) \geq 1 / 2$ when $F$ contains a cycle. On the other hand, Davies, Kang, Pirot, and Sereni recently established an upper bound of $c\left(K_{1, t, t}\right) \leq t$. We improve this to a constant, showing $c(F) \leq 4$ for every almost bipartite graph $F$.
(Received August 18, 2023)

## 1192-05-27792

Christopher Bao*, The Davidson Academy, Yunseo Choi, Harvard University, Katelyn Gan*, Sage Hill School, Owen Zhang*, Interlake High School. On a Conjecture by Baril, Cerbai, Khalil, and Vajnovszki on Two Restricted Stacks. Let $s$ be West's stack-sorting map, and let $s_{T}$ be the generalized stack-sorting map, where instead of being required to increase, the stack avoids subpermutations that are order-isomorphic to any permutation in the set $T$. In 2020, Cerbai, Claesson, and Ferrari introduced the $\sigma$-machine $s \circ s_{\sigma}$ as a generalization of West's 2 -stack-sorting-map $s \circ s$. As a further generalization, in 2021, Baril, Cerbai, Khalil, and Vajnovski introduced the $(\sigma, \tau)$-machine $s \circ s_{\sigma, \tau}$ and enumerated
$\left|\operatorname{Sort}_{n}(\sigma, \tau)\right|$-the number of permutations in $S_{n}$ that are mapped to the identity by the $(\sigma, \tau)$-machine-for six pairs of length 3 permutations $(\sigma, \tau)$. In this work, we settle a conjecture by Baril, Cerbai, Khalil, and Vajnovski on the only remaining pair of length 3 patterns $(\sigma, \tau)=(132,321)$ for which $\left|\operatorname{Sort}_{n}(\sigma, \tau)\right|$ appears in the OEIS. In addition, we enumerate
$\left|\operatorname{Sort}_{n}(123,321)\right|$, which does not appear in the OEIS, but has a simple closed form.
(Received August 20, 2023)

1192-05-27816
Sanjana Das*, Massachusetts Institute of Technology. Local properties problem for difference sets. We study the following local properties problem: if a set of real numbers has the 'local property' that every small subset has many distinct differences, must the entire set have many distinct differences? More precisely, we define $g(n, k, \ell)$ to be the minimum value of $|A-A|$ over all $n$-element sets $A \subseteq \mathbb{R}$ such that every $k$-element $A^{\prime} \subseteq A$ satisfies $\left|A^{\prime}-A^{\prime}\right| \geq \ell$. We view $k$ and $\ell$ as fixed, and attempt to understand the asymptotic behavior of $g(n, k, \ell)$ as $n \rightarrow \infty$. One of our main results concerns the quadratic threshold, the minimum value of $\ell$ (as a function of $k$ ) for which $g(n, k, \ell)=\Omega\left(n^{2}\right)$; we determine this value exactly for even $k$, and up to an additive constant for odd $k$. We also prove a more general family of upper and lower bounds;
together, these bounds imply that for all exponents $1<c \leq 2$, the 'threshold' for $\Omega\left(n^{c}\right)$ is quadratic in $k$. (Received August 21, 2023)

## 1192-05-27863

Robert M Sulman*, SUNY Oneonta. The Root Tree of a Three-Cycle Forest. Preliminary report.
The Root-Tree of a Three-Cycle Forest We view an unlabelled directed 3-cycle forest (trees at cycle nodes) as a function F of the graph vertices. Each composite $\mathrm{F} \circ \mathrm{F}$ as well as square-root $g$ (where $\mathrm{g}{ }^{\circ} \mathrm{g}=\mathrm{F}$ ) is also a 3-cycle forest. We construct a new tree with base node F and all square-roots of F (also viewed as tree vertices) connected to F with an edge. If a given squareroot $g$ of $F$ has square-roots, they will be vertices connected to vertex $g$ with an edge. This process must terminate, and the result is called the "Root-Tree of $F$ ". This concept leads to a variety of ideas including: (i) Sequences Fj of forests whose corresponding root-trees follow notable growth patterns (ii) A class of forests that are considered the "same" with respect to a given root-tree (iii) Construction techniques to manipulate root-tree structure (iv) Larger base-cycle (v) Algebraic connections (vi) Computer-assisted explorations
(Received August 21, 2023)

## 1192-05-27908

Lisa Seo Hyun Baek*, Brown University, Ethan Daniel Bove*, Brown University, Michael Cho*, Stanford University, Xingyi Zhang*, Carleton College. Optimal Constructions for DNA Self-Assembly of $k$-Regular Graphs. Within biology, it is of interest to construct DNA complexes of a certain shape. These complexes can be represented abstractly in the language of graph theory, where edges are understood to be strands of DNA joined at junctions, represented by vertices. Because guided construction of such structures is inefficient, design strategies for DNA self-assembly are desirable. Branched DNA molecules are referred to as tiles, each consisting of flexible unpaired cohesive ends with the ability to form bond-edges. We thus consider the minimum number of tiles and bond-edge types to construct a graph $G$ (corresponding to a DNA target structure) without allowing the formation of smaller graphs, or non-isomorphic graphs of the same size. In this paper, we specifically investigate the case where $G$ is $k$-regular. We introduce the concept of (un)swappable graphs, using the property to establish lower bounds on bond-edge and tile types in the unswappable case. We also introduce a method of generating upper bounds using a vertex-cover model. We apply both of these methods to prove new bounds on a number of regular families, including crown graphs, prism graphs, Kneser graphs, Johnson graphs, Antiprism Graphs and Archimedean solid graphs.
(Received August 22, 2023)

## 1192-05-27946

Sinan G Aksoy*, Pacific Northwest National Laboratory, Mark Kempton, Brigham Young University, Stephen J Young, Pacific Northwest National Laboratory. Spectral Threshold for Extremal Cyclic Edge-Connectivity.
The universal cyclic edge-connectivity of a graph is the smallest number of edges whose removal disconnects the graph into components all containing a cycle. This cycle condition is natural in applications, such as network reliability, since the existence of cycles is necessary to guarantee multiple paths between vertices. We prove an upper bound on universal cyclic edge-connectivity for regular graphs of sufficient degree and girth, and provide a spectral condition for when this upper bound is achieved. We discuss these results in the context of network design and spectral graph theory.
(Received August 23, 2023)

## 1192-05-27947

Sinan G Aksoy*, Pacific Northwest National Laboratory, Ilya Amburg, Pacific Northwest National Laboratory, Stephen J Young, Pacific Northwest National Laboratory. Scalable tensor methods for nonuniform hypergraphs.
While multilinear algebra appears natural for studying the multiway interactions modeled by hypergraphs, tensor methods for general hypergraphs face theoretical and practical barriers. A recently proposed adjacency tensor is applicable to nonuniform hypergraphs, but is prohibitively costly to form and analyze in practice. We develop tensor times same vector (TTSV) algorithms for this tensor which improve complexity from $O\left(n^{r}\right)$ to a low-degree polynomial in $r$, where $n$ is the number of vertices and $r$ is the maximum hyperedge size. Our algorithms are implicit, avoiding formation of the order $r$ adjacency tensor. We explain how our methods find application in hypergraph centrality and clustering algorithms, and compare them to graphreduction approaches on data.
(Received August 23, 2023)

1192-05-27958
Kohei Motegi*, Tokyo University of Marine Science and Technology. Some algebraic and geometric formulas for skew Schur/Grothendieck polynomials.
We present some formulas for skew Schur/Grothendieck polynomials using the techniques of integrability. We use the YangBaxter algebra to derive identities between partition functions of lattice models. These identities can be transformed into formulas for Gysin maps in algebraic geometry, and the simplest ones become identities for skew Schur/Grothendieck polynomials generalizing the ones for nonskew Schur/Grothendieck polynomials by Feher-Nemethi-Rimanyi and Guo-Sun. We will also report several other results related to this and other techniques. (Received August 23, 2023)

1192-05-27968
Sean Jay Fiscus*, Co-author, Eric Myzelev*, University of Pennslyvania, Hongyi Zhang*, Haverford College. A New Class of Geometrically Defined Hypergraphs Arising From the Hadwiger-Nelson Problem.
We prove a partial equivalence between a famous problem in geometric graph theory and its hypergraph generalization. The Hadwiger-Nelson problem asks for the chromatic number of the unit distance graph on $\mathbb{R}^{d}$, the least number of colors needed to color $\mathbb{R}^{d}$ such that no two points in $\mathbb{R}^{d}$ Euclidean-distance 1 apart are monochromatic. Its generalization asks for the
chromatic number of a geometrically defined hypergraph on $\mathbb{R}^{d}$, by which we mean a hypergraph with vertex set $\mathbb{R}^{d}$ and edge set $\{e \mid e$ is congruent to $M \in S\}$, for some set of shapes $S$. A variation of a theorem of Erdős and De-Bruijn reduces the problem of coloring infinite hypergraphs to coloring its finite subhypergraphs. Via a construction built on a sequence of these finite hypergraphs, we found a class of geometrically defined hypergraphs of arbitrarily large edge cardinality whose proper colorings coincide with the proper colorings of the unit distance graph on $\mathbb{R}^{d}$. That is, for any positive integer $m>2$, there is a set $S$ of $m$-gons such that colorings which forbid monochromatic copies of these $m$-gons also forbid monochromatic points distance 1 apart. We partially generalize this result to arbitrary real normed vector spaces.
(Received September 01, 2023)

Wayne D. Goddard, Clemson University, Julia Carolyn Vanlandingham*, Clemson University. Weighted Integrity for Generalized Vulnerability of Graphs. Preliminary report.
The integrity of a graph $G$ is defined as the minimum value of $|S|+m(G-S)$ taken over all $S \subseteq V(G)$, where $m(H)$ denotes the maximum cardinality of a component of graph H . In this note, we investigate bounds on the maximum and minimum values of the weighted version of this parameter. We also consider the same question for the related parameter vertex-neighborintegrity.
(Received September 25, 2023)

## 1192-05-28004

Egor Lappo*, Stanford University, Noah A. Rosenberg, Stanford University. Enumeration of rankings for a class of rankable phylogenetic networks.
Phylogenetic networks are mathematical objects often used to represent evolutionary relationships among biological lineages. A ranking of a phylogenetic network is a temporal ordering of its internal vertices, corresponding to the sequence of events in the evolutionary history of the lineages represented by the network. Enumeration of the possible rankings of a network can aid in evaluating the computational complexity of phylogenetic computations and inference algorithms. Exact expressions for the number of rankings for phylogenetic trees are well-known; recently, Bienvenu et al. (2022; Random Structures \& Algorithms 60:653-689) have conjectured that rankings can also be enumerated for a class of phylogenetic networks that can be termed rankable tree-child networks. Here, we solve the problem of Bienvenu et al., providing the enumeration of rankings for rankable tree-child networks. The solution relies on an equivalence between rankings for a phylogenetic tree $T$ and a certain path-counting problem on a certain lattice associated with the partial order described by $T$. The counted paths possess
"roadblocks" specified by the network structure, so that the enumeration of rankings for a network $N$ corresponds to the enumeration of non-roadblocked paths on a lattice associated with a tree displayed by $N$. Our construction introduces a novel algebraic structure into mathematical phylogenetics and provides a conceptual framework for analysis of networks through their displayed trees.
(Received August 24, 2023)

## 1192-05-28008

Saima Parveen*, Government College University, Faisalabad, Pakistan. Multicriteria Decision making for the ranking of the Drugs Used in Autoimmune Disease Vitiligo Treatment.
A well-known depigmenting skin condition called vitiligo is distinguished by an entirely absent population of melanocytes under a microscope, as well as idiopathic, acquired, progressive, circumscribed hypomelanosis of the skin and hair. The terrible skin condition vitiligo has a substantial impact on the patient's quality of life. We present the concept of OR (Operations Research), an area of mathematics related to chemical graph theory. By using quantitative structure-property relationship (QSPR) modeling, chemical graph theory enables us to generate highly resistant research on any structure for the purpose of discovering and creating novel therapeutic molecules. In this work, we visualized additional medication ranking information that the QSPR may offer. The values of the correlation coefficients and the errors produced by the specific QSPR modeling were used to visualize the findings acquired for boiling points and enthalpy of vaporizations by QSPR. The application of VIKOR gives the best ranking for each anti vitiligo medicine when considering the stated qualities, and the findings from this research effort suggest an alternative route for biologists to develop the best combinations while taking the study produced by QSPR into consideration. (Received October 11, 2023)

## 1192-05-28009

Honglin Zhu*, MIT. Evacuating " $O$ "- and " $Y$ "-shaped houses on fire: the connectivity of friends-and-strangers graphs on complete multipartite graphs.
For simple graphs $X$ and $Y$ on $n$ vertices, the friends-and-strangers graph $\mathrm{FS}(X, Y)$ is the graph whose vertex set consists of all bijections $\sigma: V(X) \rightarrow V(Y)$, where two bijections $\sigma$ and $\sigma^{\prime}$ are adjacent if and only if they agree on all but two adjacent vertices $a, b \in V(X)$ such that $\sigma(a), \sigma(b) \in V(Y)$ are adjacent in $Y$. Resolving a conjecture of Wang, Lu, and Chen, we completely characterize the connectedness of $\mathrm{FS}(X, Y)$ when $Y$ is a complete bipartite graph. We further extend this result to when $Y$ is a complete multipartite graph. We also determine when $\operatorname{FS}(X, Y)$ has exactly two connected components where $X$ is bipartite and $Y$ is a complete bipartite graph.
(Received August 24, 2023)

Nathan Albin, Kansas State University, Adriana M Ortiz Aquino*, Kansas State University. Modulus of Edge Covers and Stars. Preliminary report.
Modulus on graphs is a flexible and general tool for measuring the richness of families of objects defined on a graph. It has been shown that the modulus of special families generalizes classical network theoretic quantities such as shortest path, max flow/min cut, and effective resistance. This talk explores the modulus of the family of edge covers on a discrete graph. This modulus is closely related to that of the larger family of fractional edge covers; the modulus of the latter family is guaranteed to approximate the modulus of the former within a multiplicative factor based on the length of the shortest odd cycle in the
graph. The bounds on edge cover modulus can be computed efficiently using a duality result that relates the fractional edge covers to the family of stars.
(Received August 24, 2023)

## 1192-05-28120

Sergio Ricardo Zapata Ceballos*, Lakehead University. Distribution of the p-Torsion of Jacobian Groups of Metric Graphs. We determine the distribution of the $p$-torsion of Jacobian groups of metric graphs. We show that the Jacobian groups with nontrivial $p$-torsion correspond to $\mathbb{F}_{p}$-rational points on configuration hypersurfaces. As a result, we establish a connection between the distribution of their $p$-torsion and the number of $\mathbb{F}_{p}$-rational points on these hypersurfaces. By counting points over finite fields, we prove that this distribution is asymptotically equivalent to $1 / p$.
(Received September 25, 2023)

## 1192-05-28156

Laura Scull*, Fort Lewis College. Path Categories for Graphs
Homotopy for graphs has traditionally been defined by using a product (most commonly, either $\times$ or $\square$ product) with an interval graph $I_{n}$. The resulting homotopy theories, $\times$-homotopy and $A$-homotopy respectively, both seem to resist the usual structure of Quillen model structures, resulting in difficulties in understanding and working with their homotopy categories. In this talk I will discuss an alternate approach to defining homotopy for graphs which may offer some improvement in this area This approach builds off the idea of a path category described in [BG], originally designed to generalize Moore homotopies and applied to topological spaces and simplical sets. I will discuss how to define a path category for graphs (in two flavours, one corresponding to $\times$-homotpy, and one to $A$-homotopy) built out of exponential graphs $X^{I_{n}}$ (where the exponential is defined by the adjoint to the chosen product). I will explain this construction and how it defines notions of homotopy for graphs. The path category offers a lot of structure to the resulting homotopy category automatically, including well-behaved weak factorization systems. This new path category notion of homotopy does not coincide exactly with the original: any maps which are homotopic in the traditional $\times$ - or $A$-homotopy sense are homotopic via the path category, but not necessarily vice versa. I will explain what we know (and what we still don't know) about the relationship between these ideas. This is joint work with R. Cockett (University of Calgary) and R. Hardeman (University of Alberta). [BG] van den Berg, Garner, Topological and simplicial models of identity types
(Received August 27, 2023)

## 1192-05-28205

Neil Makur*, Fremont Christian School. Dr. Amitabh Chaudhary, University of Chicago.. Multi-item Balanced Transportation Problem: Formulation, Solutions and Applications.
The transportation problem, where commodities have to be shipped from a set of sources to a set of sinks while minimizing the total cost, has been studied in various forms. Balanced transportation problem is a transportation problem where the total availability at the origin is the same as the total requirement at the destination, and the goal is to find the subset of edges to use to transport everything to meet all requirements while minimizing a given cost function. In this research, we explore a novel formulation of the balanced transportation problem with two new characteristics - firstly, there are multiple types of items to be transported each with their own demand and supply at various vertices. Also, the edges are fixed cost i.e. the edge cost is not dependent on the number of items, or item types that are transported along it. Having multiple types of items explodes the number of possible edge subsets to be considered for a successful solution. We find an algorithm that can find an optimal plan minimizing the total cost. We also devise and test several algorithms - deterministic and randomized, study their runtime complexity, and compare them for results and runtime in $\mathrm{C}++$. We investigate the immediate application of this solution to efficiently distribute all extra food from food donors to food banks in any local area while fulfilling the banks' needs. (Received August 28, 2023)

## 1192-05-28208

Jacob Ashworth*, Rose-Hulman Institute of Technology, Luca Grossman*, Tufts University, Fausto Navarro*, Johns Hopkins University. Algorithmic Generation of DNA Self-Assembly Structures.
Recent advancements in microbiology have motivated the study of the production of DNA nanostructures. One method to construct these structures is to use self-assembling DNA molecules with branches made up of complementary cohesive ends. Mathematically, we can model the target nanostructure as a graph, with the vertices of the graph representing molecules and the edges representing the bonds between their branches. Thus, determining a collection of molecules which will selfassemble into a given nanostructure is equivalent to a graph theoretical problem. We study the combinatorial and algebraic properties of this problem to develop several algorithms tackling DNA self-assembly, including an integer linear programming approach. Given a target undirected graph, these algorithms output an optimal collection of component building blocks for the corresponding nanostructure
(Received August 28, 2023)

## 1192-05-28239

Thomas Kelly*, Georgia Institute of Technology. Robustness for hypergraph embeddings via spreadness. In this talk, we will discuss robustness results which lie in the intersection of both extremal and probabilistic combinatorics. In joint work with Kang, Kühn, Methuku, and Osthus, we proved the following: If $p \geq C \log ^{2} n / n$ and $L_{i, j} \subseteq[n]$ is a random subset of $[n]$ where each $k \in[n]$ is included in $L_{i, j}$ independently with probability $p$ for each $i, j \in[n]$, then asymptotically almost surely there is an order- $n$ Latin square in which the entry in the $i$ th row and $j$ th column lies in $L_{i, j}$. We prove analogous results for Steiner triple systems and 1-factorizations of complete graphs. These results can be understood as stating that these "design-like" structures exist "robustly". In joint work with Kang, Kühn, Osthus, and Pfenninger, we proved various results stating that if $\mathcal{H}$ is an $n$-vertex $k$-uniform hypergraph satisfying some minimum degree condition and $p=\Omega\left(n^{-k+1} \log n\right)$, then asymptotically almost surely a $p$-random subhypergraph of $\mathcal{H}$ contains a perfect matching. These results can be understood as "robust" versions of hypergraph Dirac-type results as they simultaneously strengthen Johansson,

Kahn, and Vu's seminal solution to Shamir's problem on the threshold for when a binomial random $k$-uniform hypergraph contains a perfect matching. In joint work with Müyesser, and Pokrovskiy, we proved similar results for hypergraph Hamilton cycles. All of these results utilize the recent Park-Pham Theorem or one of its variants. A crucial notion for this is that of the spreadness of a certain type of probability distribution.
(Received August 28, 2023)

## 1192-05-28246

John Urschel*, MIT. Nodal Counts for Symmetric Matrices.
The sign structure of a symmetric matrix and that of its corresponding eigenvectors are closely related. In this talk, we give a brief review of both classical and modern results quantifying this relationship (including Fielder's matrix tree theorem, discrete versions of Courant's nodal domain theorem, Berkolaiko's theorem, and others), and present some new results. (Received August 28, 2023)

## 1192-05-28270

Shuangping Li*, Stanford University. Spectral clustering in the geometric block model.
Gaussian mixture block models are distributions over graphs that strive to model modern networks: to generate a graph from such a model, we associate each vertex with a latent feature vector sampled from a mixture of Gaussians, and we add edge if and only if the feature vectors are sufficiently similar. The different components of the Gaussian mixture represent the fact that there may be different types of nodes with different distributions over features-for example, in a social network each component represents the different attributes of a distinct community. Natural algorithmic tasks associated with these networks are embedding (recovering the latent feature vectors) and clustering (grouping nodes by their mixture component). In this talk, we focus on clustering and embedding graphs sampled from high-dimensional Gaussian mixture block models where the dimension of the latent feature vectors goes to infinity as the size of the network goes to infinity. This highdimensional setting is most appropriate in the context of modern networks, in which we think of the latent feature space as being high-dimensional. We analyze the performance of canonical spectral clustering and embedding algorithms for such graphs in the case of 2 -component spherical Gaussian mixtures and begin to sketch out the information-computation landscape for clustering and embedding in these models. This is based on joint work with Tselil Schramm.
(Received August 29, 2023)

## 1192-05-28294

Jisun Huh, Ajou University, Jang Soo Kim, Sungkyunkwan University, Christian Krattenthaler, Universitat Wien, Soichi Okada*, Nagoya University. Affine bounded Littlewood identities and cylindric standard tableaux. The bounded Littlewood identities are determinant formulas for the sum of Schur functions indexed by partitions with bounded height. These lead to interesting combinatorial consequences concerning standard Young tableaux of bounded height. In this talk, we give affine analogs of the bounded Littlewood identities. These are determinant formulas for sums of cylindric Schur functions. As an application we obtain equinumerous results between cylindric standard Young tableaux and $r$-noncrossing and $s$-nonnesting matchings.
(Received August 29, 2023)

## 1192-05-28299

Emelie J Curl, Hollins University, Jurgen Kritschgau, Portland State University, Carolyn Reinhart*, Swarthmore College, Hein Van der Holst, Georgia State University. The inverse eigenvalue problem for probe graphs. Preliminary report The inverse eigenvalue problem for a graph considers a family of matrices whose zero-nonzero pattern is defined by the graph and asks which spectra are achievable by matrices in this family. A probe graph is a graph whose vertices are partitioned into probe vertices and non-probe vertices, such that the non-probe vertices form an independent set. Any number of edges may be added between non-probe vertices. In this talk, we introduce the study of the inverse eigenvalue problem for probe graphs. We will discuss bounds on the probe graph maximum nullity and characterize probe graphs with extreme maximum nullity. We will also present results on the probe graph zero forcing number. (Received August 29, 2023)

1192-05-28300
Kristin Heysse, Macalester College, Kate J. Lorenzen, Linfield University, Carolyn Reinhart*, Swarthmore College, Xinyu Wu, Carnegie Mellon University. Graphs with non-trivial Jordan blocks for the non-backtracking matrix. Preliminary report. A non-backtracking walk in a graph is any traversal of the vertices such that no edge is immediately repeated. The nonbacktracking matrix $B$ of a graph is indexed by the directed edges of the graph, and encodes if two edges can be traversed in succession. Since this matrix is not symmetric, the question of when the matrix is diagonalizable is of interest to those who study it. Equivalently, such graphs have a non-trivial Jordan block. In this talk, we present a construction for an infinite family of graphs with a non-trivial Jordan block. We also present some numerical results about the number of such graphs on at most 10 vertices.
(Received August 29, 2023)

## 1192-05-28375

Tolson Hallauer Bell*, Carnegie Mellon University, Alan Frieze, Carnegie Mellon University. Giant Rainbow Trees. For any small constant $\epsilon>0$, the sharp threshold for $G(n, p)$ to contain a connected component with $\epsilon n$ vertices is $p=\frac{1}{n}\left(1+\frac{\epsilon}{2} \pm O\left(\epsilon^{2}\right)\right)$. Let $G_{c}(n, p)$ be obtained by assigning each edge in $G(n, p)$ a color in $[c]$ independently and uniformly. Cooley, Do, Erde, and Missethan conjectured that for any fixed $\alpha>0, p=\frac{1}{n}\left(1+\frac{\epsilon}{2} \pm O\left(\epsilon^{2}\right)\right)$ still suffices for $G_{\alpha n}(n, p)$ to contain a rainbow tree (a tree that does not repeat colors) which covers $\epsilon n$ vertices, and proved $p=\frac{1}{n}\left(1+\frac{(\alpha+1) \epsilon}{\alpha} \pm O\left(\epsilon^{2}\right)\right)$ suffices. I will present our proof that their conjecture is correct up to a logarithmic factor in the
error term, as we show that $p=\frac{1}{n}\left(1+\frac{\epsilon}{2} \pm O\left(\epsilon^{2} \log (1 / \epsilon)\right)\right)$ suffices for $G_{\alpha n}(n, p)$ to with high probability contain a rainbow tree which covers $\epsilon n$ vertices. arxiv.org/abs/2308.14141.
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## 1192-05-28377

Nadia Benakli*, New York City College of Technology, CUNY, Novi Herawati Bong, University of Delaware, Shonda
Dueck, The University of Winnipeg, Linda Eroh, University of Wisconsin Oshkosh, Beth A Novick, Clemson University, Ortrud R. Oellermann, The University of Winnipeg. The Threshold Strong Dimension of Trees.
Let $G$ be a graph and $W$ be a set of vertices of $G$. A vertex $w$ in $W$ is said to strongly resolve two vertices $u$ and $v$ in $G$ if there is either a shortest $u-w$ path that contains $v$ or a shortest $v-w$ path that contains $u$. The set $W$ is called a strong resolving set if every pair of vertices in $G$ is strongly resolved by a vertex of $W$. A smallest strong resolving set is called a strong basis and its cardinality, the strong dimension, denoted $\beta_{S}(G)$. When additional edges are added to a graph $G$, the strong dimension $\beta_{S}(G)$ can either increase, decrease, or remain the same. This observation leads to the introduction of a new parameter called the threshold strong dimension of $G$, denoted $\tau_{S}(G)$. It represents the smallest strong dimension among all graphs that contain $G$ as a spanning subgraph. Finding the threshold strong dimension of a tree $T$ is a challenging problem. In this work, we present some results and insights regarding the threshold strong dimension of trees.
(Received September 09, 2023)

## 1192-05-28381

Anthony Bonato*, Toronto Metropolitan University. Why you should care about the localization number.
The localization number is defined by a Cops and Robbers style pursuit-evasion game with an invisible robber on a graph detectable by distance probes. Connections with the metric dimension and the chromatic number make the parameter a natural one to study by graph theorists. We give a sample of recent results on the localization number for various graph families, ranging from planar graphs, hypercubes, Kneser and Moore graphs, locally finite graphs, directed graphs, and graphs arising from designs and Latin squares.
(Received August 30, 2023)

## 1192-05-28405

Quentin Dubroff, Rutgers University, Jacob Fox, Stanford University, Max Wenqiang Xu*, Stanford University. On Erdős distinct subset sums problem.
We present two short proofs giving the best known asymptotic lower bound for the maximum element in a set of $n$ positive integers with distinct subset sums.
(Received August 30, 2023)

1192-05-28413
Swee Hong Chan*, Rutgers University. Random linear extensions of infinite posets.
For a partially ordered set (poset) P , a linear extension of P is a linear ordering (permutation) of elements of P that is consistent with the partial order. In contrast to its finite counterpart, the linear extensions of infinite posets have received less focus in the literature. In this talk we will explore diverse results concerning infinite linear extensions through the lens of algebra, combinatorics, and probability. This talk is based on a joint work with Igor Pak. (Received August 30, 2023)

## 1192-05-28414

Noah A. Rosenberg, Stanford University, Chloe E. Shiff*, Stanford University. Enumeration of Binary Perfect Phylogenies. A perfect phylogeny is a rooted unlabeled tree, each of whose leaves is associated with a positive integer, which can be used to characterize genetic sequences in biological samples. Rooted binary perfect phylogenies provide a generalization of rooted binary unlabeled trees (OEIS A001190), as rooted binary unlabeled trees are rooted binary perfect phylogenies for a sample of entirely distinct sequences, and thus all integer labels are 1 . We enumerate the rooted binary perfect phylogenies with $n$ leaves and sample size $s, s \geq n$ : the rooted binary unlabeled trees with $n$ leaves in which a sample of size $s \geq n$ lineages is distributed across the $n$ leaves. We separately enumerate (1) all rooted binary perfect phylogenies with $n$ leaves and sample size $s \geq n$; (2) the rooted binary perfect phylogenies with sample size $s \geq n$ that are associated with a specific rooted binary unlabeled tree with $n$ leaves; and (3) all rooted binary perfect phylogenies with sample size $s$ and $1 \leq n \leq s$ leaves. For this last quantity, we obtain and analyze a generating function, showing that asymptotically, the number of rooted binary perfect phylogenies with sample size $s$ grows faster than the rooted binary unlabeled trees. The set of rooted binary perfect phylogenies, which contains the set of rooted binary unlabeled trees, can provide a set of structures useful for biological time scales short enough that mutations do not make all lineages in evolutionary trees genetically distinguishable. (Received August 30, 2023)

## 1192-05-28433

Grant Kopitzke*, University of Wisconsin - Stevens Point. The Gini index of an integer partition and GLn-harmonic polynomials.
The Gini index is a function that attempts to measure the amount of inequality in the distribution of a finite resource throughout a population. It is commonly used in economics as a measure of inequality of income or wealth. We will define a discrete Gini index on the set of integer partitions with at most $n$ parts and will show how this function emerges as the top degree in which an irreducible rational representation of $G L_{n}$ appears in the gradation of the $G L_{n}$-harmonic polynomials by homogeneous degree.
(Received September 25, 2023)

Ortrud R. Oellermann*, The University of Winnipeg. Threshold Dimension and Threshold Strong Dimension of a Graph: Characterizations and Irreducible Structures.
Let $u, v, w$ be vertices of a graph $G$. If $u$ and $v$ are distinct distances from $w$, then $w$ resolves $u$ and $v$. If there is a shortest $u-$ $w$ path that contains $v$ or a shortest $v-w$ path that contains $u$, then $w$ strongly resolves $u$ and $v$. The cardinality of a smallest set $W$ of vertices of $G$ such that every two vertices $G$ are resolved by some vertex of $W$ is called the metric (respy. strong) dimension of $G$ and is denoted by $\beta(G)$ (respy. $\beta_{s}(G)$ ). The smallest metric dimension that can be achieved by adding some set of edges to $G$ is the threshold dimension $\tau(G)$ of $G$. The threshold strong dimension $\tau_{s}(G)$ of $G$ is defined analogously. We describe geometric characterizations for the threshold and threshold strong dimensions of $G$. If $\tau(G)=\beta(G)$ (respy.
$\tau_{s}(G)=\beta_{s}(G)$ ) we say that $G$ is $\beta$-irreducible (respy. $\beta_{s}$-irreducible). For a given integer $n>1$ and $b \in\{1,2, \ldots, n-1\}$ we show that there is $\beta$-irreducible graph (and $\beta_{s}$-irreducible graph) of order $n$ and dimension $b$ (resp., strong dimension $b$ ). (Joint work with N. Benakli, N. Bong, S.M. Dueck, L. Eroh, B. Novick)
(Received August 30, 2023)

## 1192-05-28456

Alexander Divoux*, Georgia Institute of Technology, Thomas Kelly, Georgia Institute of Technology, Camille Elisabeth Kennedy, Northwestern University, Jasdeep Sidhu, Stanford University. Subsquares in random Latin squares and rectangles. A $k \times n$ partial Latin rectangle is $C$-sparse if the number of populated entries in each row and column is at most $C$, and each symbol is used at most $C$ times. We prove that, for any $\alpha, \epsilon>0$, there exists some $\beta>0$ such that the probability a uniformly random $k \times n$ Latin rectangle, where $k<(1 / 2-\alpha) n$, contains a $\beta n$-sparse partial Latin rectangle with $\ell$ nonempty entries is $\left(\frac{1 \pm \epsilon}{n}\right)^{\ell}$ for sufficiently large $n$. Using this result, we prove that a uniformly random order- $n$ Latin square asymptotically almost surely has no order- $C \sqrt{n \log n}$ Latin subsquare, making progress on a conjecture of McKay and Wanless. This is a joint work with Tom Kelly, Camille Kennedy, and Jasdeep Sidhu. (Received August 31, 2023)

1192-05-28509
Michael Tait*, Villanova. Even-cycle creating Hamilton paths.
We consider maximizing the number of Hamilton paths such that the union of any of them contains $C_{2 k}$ as a subgraph. This question was originally studied because of its relationship to questions about sets of permutations with given properties. Harcos and Soltész gave a sharp estimate for this quantity when $k=2$ and gave upper and lower bounds for larger $k$. In this talk, we improve their upper bounds for all values of $k>2$. A key element of the proof is to use spectral properties of a not necessarily regular graph to estimate the number of Hamilton cycles the graph contains. This is joint work with John Byrne. (Received August 31, 2023)

1192-05-28511
Michael Tait*, Villanova. Two conjectures on the spread of graphs.
Given a graph $G$ let $\lambda_{1}$ and $\lambda_{n}$ be the maximum and minimum eigenvalues of its adjacency matrix and define the spread of $G$ to be $\lambda_{1}-\lambda_{n}$. In this talk we discuss solutions to a pair of 20 -year-old conjectures of Gregory, Hershkowitz, and Kirkland regarding the spread of graphs. Our results also imply a sharp upper bound on the spread of a nonnegative symmetric matrix with bounded entries. This is joint work with Jane Breen, Alex Riasanovsky, and John Urschel.
(Received August 31, 2023)

## 1192-05-28512

Michael Tait*, Villanova. Graphs with many edges and few short even cycles.
We will discuss the notoriously difficult even-cycle problem. In the talk, we will consider constructions of graphs coming from geometry, combinatorial number theory, and random polynomials which have no or few cycles of length $2 k$. I will discuss joint work with Boris Bukh, Sunny He, and Craig Timmons.
(Received August 31, 2023)

## 1192-05-28564

Anton Bernshteyn, Georgia Institute of Technology, Felix Weilacher*, Carnegie Mellon University. Borel versions of the Local Lemma and LOCAL algorithms for graphs of finite asymptotic separation index.
Asymptotic separation index (asi) is a parameter inspired by Gromov's asymptotic dimension measuring how easily a Borel graph can be approximated by its subgraphs with finite components. We present a Borel version of the Lovász Local Lemma, a powerful general-purpose tool from probabilistic combinatorics, under a finite asi assumption. This is the first result of its kind that applies to a wide class of graphs of exponential growth rate, and it also improves previous results in the measure and category contexts. We discuss an important application relating to randomized distributed algorithms and Borel "local" combinatorial problems.
(Received August 31, 2023)

1192-05-28647
Aditya G. Nair, University of Nevada, Reno, Kunihiko Taira*, University of California, Los Angeles. Turbulence through the Lens of Graph Theory.
Turbulence exhibits highly complex dynamics generated by interactions amongst vortices. These interactions can be elucidated using graph theory, dynamical systems/control theory, and data science. The fusion of graph theory with fluid mechanics facilitates the extraction of key interactions and clusters involving vortical elements, modal structures, and particle trajectories. Techniques like phase-space analysis and time-delay embedding enable a network-centric exploration of timeseries measurements, revealing patterns of visibility, recurrence, and cluster transitions. Armed with insights gained from
these approaches, the graph theory empowers the analysis, modeling, and control of fluid flows, particularly emphasizing interactive dynamics. This presentation provides an overview of advancements in using graph-based methodologies to investigate the interaction dynamics that define turbulent flows. The talk will also offer outlook and challenges on networkbased analysis of fluid flows and their potential for improving energy efficiencies and performance of fluid-based engineering systems.
(Received September 01, 2023)

## 1192-05-28684

Colin Defant*, Massachusetts Institute of Technology, Rupert Li, Massachusetts Institute of Technology, Evita Nestoridi, Stony Brook University. Toggle Markov Chains.
Rowmotion is a certain well-studied bijective operator on the distributive lattice $J(P)$ of order ideals of a finite poset $P$. We introduce the rowmotion Markov chain $\mathbf{M}_{J(P)}$ by assigning a probability $p_{x}$ to each $x \in P$ and using these probabilities to insert randomness into the original definition of rowmotion. More generally, we introduce a very broad family of toggle Markov chains inspired by Striker's notion of generalized toggling. We characterize when toggle Markov chains are irreducible, and we show that each toggle Markov chain has a remarkably simple stationary distribution. We also bound the mixing time of $\mathbf{M}_{J(P)}$ for an arbitrary finite poset $P$. In the special case when $P$ is an antichain, we use spectral methods to obtain much stronger estimates on the mixing time, showing that rowmotion Markov chains of Boolean lattices exhibit the cutoff phenomenon. (Received September 01, 2023)

## 1192-05-28702

Tapendra BC, University of Winnipeg, Shonda Dueck*, The University of Winnipeg. The metric dimension of circulant graphs. Preliminary report.
A pair of vertices $x$ and $y$ in a graph $G$ are said to be resolved by a vertex $w$ if the distance from $x$ to $w$ is not equal to the distance from $y$ to $w$. We say that $G$ is resolved by a subset of its vertices $W$ if every pair of vertices in $G$ is resolved by some vertex in $W$. The minimum cardinality of a resolving set for $G$ is called the metric dimension of $G$. The problem of determining the metric dimension of a graph is known to be NP-hard (Khuller et al 1994). The metric dimension of a graph has applications in network discovery and verification, combinatorial optimization, chemistry, and many other areas, and consequently this graph parameter has received a great deal of attention from researchers recently, the main goal being to determine the metric dimension of certain classes of graphs. In particular, there is great interest in finding classes of graphs whose metric dimension does not grow with the number of vertices. Such graphs are said to have bounded metric dimension. In this talk, we consider the metric dimension of a certain class of circulant graphs that have been shown to have bounded metric dimension (Grigorius et al 2014), namely $C_{n}(1,2, \ldots, t)=C a y\left(\mathbb{Z}_{n}: \pm 1, \pm 2, \ldots, \pm t\right)$. In 2016 , Vetrik improved on the bounds due to Grigorius et al. In 2017, Chau et al showed that, for large enough $n$, the metric dimension of $C_{n}(1,2, \ldots, t)$ is determined by the congruence class of $n$ modulo $2 t$, and they improved on the known bounds on the metric dimension of these graphs for different congruence classes. This year, Gao et al have found better bounds on the metric dimension of these circulants for some congruence classes of $n$ modulo $2 t$. We present some background on the problem and some new results. This is joint work with my undergraduate summer research student Tapendra BC. (Received September 01, 2023)

1192-05-28731
David Conlon, California Institute of Technology, Jacob Fox, Stanford University, Benjamin Gunby-Mann*, Rutgers University, Xiaoyu He, Princeton University, Dhruv Mubayi, University of Illinois at Chicago, Andrew Suk, University of California, San Diego, Jacques Verstraete, University of California San Diego. Off-Diagonal Hypergraph Ramsey Numbers. We discuss off-diagonal 3-uniform Ramsey numbers; that is, we study the behavior of the Ramsey number $R\left(H, K_{n}^{(3)}\right)$ where $H$ is a fixed 3 -uniform hypergraph and $n \rightarrow \infty$. By systematically studying pair-coloring techniques, we show that this Ramsey number is bounded below by $2^{\Omega(n \log n)}$ for a large class of graphs $H$, generalizing work by Fox and He. We also compare Ramsey numbers versus $K_{n}^{(3)}$ to those versus $K_{n, n, n}^{(3)}$. (Received September 02, 2023)

## 1192-05-28751

Yaxin (Casey) Qi*, Columbia University. Graph Reconstruction from Connected Triples.
The problem of graph reconstruction has been studied in its various forms over the years. In particular, the Reconstruction Conjecture, proposed by Ulam and Kelly in 1942, has attracted much research attention and yet remains one of the foremost unsolved problems in graph theory. Recently, Bastide, Cook, Erickson, Groenland, Kreveld, Mannens, and Vermeulen proposed a new model of partial information, where we are given the set of connected triples $T_{3}$ \cite\{BCEGKMV23\}, which is the set of 3 -subsets of the vertex set that induce connected subgraphs. They proved that reconstruction is unique within the class of triangle-free graphs, 2 -connected outerplanar graphs, and maximal planar graphs. They also showed that almost every graph can be uniquely reconstructed from their connected triples. However, little is known about other classes of non-triangle-free graphs within which reconstruction can occur uniquely, nor do we understand what kind of graphs can be uniquely reconstructed from their connected triples without assuming anything about the classes of graphs they belong to. The main result of this paper is a complete characterization of all graphs that can be uniquely reconstructed from their connected triples. We also show that regular planar graphs, 5-connected planar graphs, certain strongly regular graphs, and complete multi-partite graphs are reconstructible whereas $k$-connected planar graphs for $k \leq 4$, Eulerian graphs, and Hamiltonian graphs are not.
(Received September 02, 2023)

1192-05-28827
Alexandr Kostochka, University of Illinois Urbana-Champaign, Ruth Luo, University of South Carolina, Grace McCourt*, University of Illinois at Urbana-Champaign. Dirac-type results for Berge cycles in uniform hypergraphs.

The famous Dirac's Theorem gives an exact bound on the minimum degree of an $n$-vertex graph guaranteeing the existence of a hamiltonian cycle. Dirac also gave a minimum degree bound on the circumference of a graph. Furthermore, he proved that each $n$-vertex 2 -connected graph with minimum degree at least $k$ contains a cycle of length at least $\min \{2 k, n\}$. We consider versions of these results in hypergraphs. A Berge cycle in a hypergraph is an alternating sequence of distinct vertices and edges $v_{1}, e_{2}, v_{2}, \ldots, e_{c}, v_{1}$ such that $\left\{v_{i}, v_{i+1}\right\} \subseteq e_{i}$ for all $i$ (with indices taken modulo $c$ ). We prove bounds on the minimum degree needed to guarantee a Berge cycle of a given length in an $n$-vertex, $r$-uniform hypergraph, and in 2-connected such hypergraphs. The bounds differ depending on the uniformity of the edges and the length of the desired cycle. This work is joint with Alexandr Kostochka and Ruth Luo. (Received September 03, 2023)

1192-05-28898
Guilherme Zeus Dantas E Moura, Haverford College, Daoji Huang, ICERM, Bryan Lu*, Cornell University, Dora Woodruff*, Harvard University. Simplicial Complexes and Jeu de Taquin Theory. Preliminary report.
For every finite, closed interval of Young's lattice, there is an associated simplicial complex. A classical theorem of Bjorner implies that these simplicial complexes are homeomorphic to balls. Our project connects this theory to jeu de taquin. In our setting, there is a simplicial complex associated to every dual equivalence class of skew tableaux. We find that not all of these complexes are homeomorphic to balls, but we classify several cases in which they are: for example, when the tableau indexing the dual equivalence class (via RSK) is rectangular or superstandard. We also show that certain natural tableau operations, such as transpose and evacuation, induce simplicial isomorphisms. Finally, we explore a connection with K-theoretic jdt, a variant of jdt for increasing tableaux.
(Received September 04, 2023)

1192-05-28905
Guilherme Zeus Dantas E Moura, Haverford College, Pavlo Pylyavskyy, University of Minnesota, Ramanuja Charyulu Charyulu Telekicherla Telekicherla Kandalam*, University of Minnesota Twin Cities, Dora Woodruff*, Harvard University. Cluster Monomials in Graph Laurent Phenomenon Algebras. Preliminary report.
LP (Laurent Phenomenon) algebras are a generalization of cluster algebras first defined by Lam and Pylyavskyy. Graph LP algebras are a subclass of LP algebras: to each directed graph, there is an associated LP algebra. Several conjectures and properties of cluster algebras appear to generalize to LP algebras. In our project, we first show that the cluster monomials of the LP algebra $A_{\Gamma}$ associated to the graph $\Gamma$ linearly span $A_{\Gamma}$; an analogous property is known to hold for cluster algebras. We also take steps towards proving that cluster monomials form a positive basis for $A_{\Gamma}$, which is analogous to a well-known conjecture for cluster algebras. In particular, we show that positivity holds whenever $\Gamma$ is a tree or a cycle, and prove other partial results that hold for a general $\Gamma$.
(Received September 04, 2023)

1192-05-28930
Benjamin Brubaker, University of Minnesota - Twin Cities, Michael Hu, UC - Berkeley, Yi Lin, Jilin University, Maria Mihaila*, UC - Davis, Patrick Mullen, University of Michigan, Ethan Stafford, University of Minnesota - Twin Cities. Special Functions from Solvable Lattice Models. Preliminary report.
We present results from the 2023 Polymath Jr. Program on solvable lattice models and their relation to special functions. Solvable lattice models arise naturally from modules for quantum groups. So one may choose such a module, an associated lattice model, and then attempt to characterize the resulting partition functions (i.e., generating functions) of the model. Alternatively, one may select a family of special functions and attempt to provide a solvable lattice model and associated quantum group module. We give one example in each direction. One the one hand, we use a quantum superalgebra to produce special functions. On the other, we provide a model realizing members of Kirillov's family of Hecke-Grothendieck polynomials resulting in special cases of positivity conjectures.
(Received September 04, 2023)

1192-05-28976
sarah-marie belcastro*, Mathematical Staircase, Inc.. Enumerating domino tilings of $2 \times n$ grids on surfaces.
What started as scribbling scratchwork turned into quite a saga. The inductive enumeration of domino tilings of $2 \times n$ rectangles is well known to produce the Fibonacci sequence. Identifying opposite edges-either pair, or both pairs-of a rectangle takes us from the plane to a topological surface (perhaps with boundary). Does the inductive enumeration extend to these cases? What sequences are produced? A few hours of experimentation on an internet-less train produced some tantalizing progress, and on return to civilization more serious investigation commenced. The underlying problems had been solved earlier and in more generality, but the corresponding formulas did not obviously produce integer sequences. In contrast, an inductive enumeration is understandable, if for some surfaces a bit tricky and involving some bijections-and has produced some new sequences for the OEIS. Most interesting may be that one of the sequences is twice another, suggesting a combinatorial proof should be attainable...and therein lies the tale... (Received September 05, 2023)

Qijun He*, University of Virginia. The arithmetic topology of genetic alignments.
We propose a novel mathematical paradigm for the study of genetic variation in sequence alignments. This framework originates from extending the notion of pairwise relations, upon which current analysis is based on, to $k$-ary dissimilarity. This dissimilarity naturally leads to a generalization of simplicial complexes by endowing simplices with weights, compatible with the boundary operator. We introduce the notion of $k$-stances and dissimilarity complex, the former encapsulating arithmetic as well as topological structure expressing these $k$-ary relations. We study basic mathematical properties of dissimilarity complexes and show how this approach captures watershed moments of viral dynamics in the context of SARS-CoV-2 and H1N1 flu genomic data.
(Received September 05, 2023)

## 1192-05-29085

Runze Li*, University of California, Santa Barbara, István Miklós, Rényi Institute, Hungarian Academy of Sciences. Graphic 3 -uniform hypergraph degree sequences. Preliminary report.
A 3-uniform hypergraph is a generalization of simple graphs where each hyperedge has 3 vertices. The degree of a vertex in a hypergraph is the number of hyperedges incident to it. The degree sequence of a hypergraph is the sequence of the degrees of its vertices. The degree sequence problem for 3 -uniform hypergraphs is to decide if a 3 -uniform hypergraph, called a realization, exists with a prescribed degree sequence. Recently, Deza et al. proved that the degree sequence problem for 3uniform hypergraphs is NP-complete. However, some special cases are easy. So far, polynomial algorithms can be used only for some very restricted degree sequences to construct their realizations. Our research mainly shows that if all degrees in a degree sequence $D$ are between $\frac{n^{2}}{27}+O(n)$ and $\frac{2 n^{2}}{27}-O(n)$, further, the number of vertices is at least 38 , and the degree sum is divisible by 3 , then $D$ has a 3-uniform hypergraph realization. Our proof is constructive and it runs in polynomial time. This is the first polynomial algorithm to construct a 3-uniform hypergraph realization of a highly irregular and dense degree sequence.
(Received September 05, 2023)

## 1192-05-29096

Luke Edward Guidry*, Rhodes College, Christopher W. Seaton, Rhodes College. Investigating Spectral Behavior through Digraph Move Sequences.
In a recent paper Farsi, Proctor, and Seaton answered the question of spectral preservation for families of finite digraphs $D$ under the application of six digraph moves that preserve the Morita equivalence class of the associated graph $C^{*}$-algebra. In this presentation, we examine the question of the effect of some of these digraph moves on specific spectra of a finite digraph $D$ and look at properties of these spectra under repeated performance of these digraph moves. We characterize the effects of the sequence of digraph moves $(S S R)$ on the Adjacency Spectrum, Binary Adjacency Spectrum, Symmetric Adjacency Spectrum, and Binary Symmetric Adjacency Spectrum of a finite digraph $D$. We construct two families of digraphs using this move sequence and give explicit formulae for computing the characteristic polynomials of digraphs within these families. We also consider generalizations of this sequence of digraph moves and give further formulae for computing the resulting spectra. (Received September 05, 2023)

## 1192-05-29186

Alexei Borodin*, MIT. Colored interlacing traingles.
We consider probability measures arising from the Cauchy summation identity for the LLT (Lascoux-Leclerc-Thibon) symmetric polynomials of rank $\mathrm{n} \geqslant 1$. We study the asymptotic behaviour of these measures as one of the two sets of polynomials in the Cauchy identity stays fixed, while the other one grows to infinity. At $n=1$, this corresponds to an analogous limit of the Schur process, which is known to be given by the Gaussian Unitary Ensemble (GUE) corners process. Our main result states that, for $\mathrm{n}>1$, our measures asymptotically split into two parts: a continuous one and a discrete one. The continuous part is a product of $n$ GUE corners processes; the discrete part is an explicit finite distribution on interlacing $n$ colourings of $n$ interlacing triangles, which has weights that are rational functions in the LLT parameter $q$. The latter distribution has a number of interesting (partly conjectural) combinatorial properties, such as q-nonnegativity and enumerative phenomena underlying its support. Our main tools are two different representations of the LLT polynomials, one as partition functions of a fermionic lattice model of rank $n$, and the other as finite-dimensional contour integrals, which were recently obtained in arXiv:2012.02376, arXiv:2101.01605. Based on joint work with Amol Aggarwal and Michael Wheeler. (Received September 05, 2023)

## 1192-05-29205

Florian Frick*, Carnegie Mellon University, Andrew Newman, Carnegie Mellon University. A random Borsuk-Ulam theorem. Topological techniques are used across mathematical disciplines to gain insight into global phenomena, while one standard application of probabilistic tools is to understand average-case behavior. I will present a model for random spaces with a free involution, give bounds for their topology and for the dimension $d$ such that any map from such a space to $\mathbb{R}^{d}$ commuting with free involutions must have a zero, thus proving a random Borsuk-Ulam theorem. The Borsuk-Ulam theorem has numerous applications. Its random variant may be used to establish average-case behavior for those applications. (Received September 05, 2023)

## 1192-05-29220

Nadia Benakli, New York City College of Technology, CUNY, Novi Herawati Bong, University of Delaware, Shonda Dueck, The University of Winnipeg, Linda Eroh, University of Wisconsin Oshkosh, Beth A Novick*, Clemson University, Ortrud R. Oellermann, The University of Winnipeg. On the threshold strong dimension of the $n$-cube.
The $n$-cube is the graph whose vertex set consists of all binary vectors of length $n$, with two vertices being adjacent when they differ in precisely one component. While the exact metric dimension of the $n$-cube is not known, it has been shown to be asymptotically equal to $\frac{2 n}{\log _{2}(n)}$. On the other hand, the strong dimension of the $n$-cube is precisely $2^{n-1}$. The threshold strong dimension of a graph $G$ is the smallest strong dimension among all graphs having $G$ as a spanning subgraph. We show that the threshold strong dimension of the $n$-cube is surprisingly smaller than its strong dimension. As a tool, we apply a recent theorem that characterizes the threshold strong dimension of a graph in terms of a specific type of embedding into the strong product of paths. Since there are many applications involving metric dimensions of Cartesian products of graphs, one is motivated to generalize our result, leading to some open questions.
(Received September 05, 2023)

Andrés R. Vindas Meléndez*, UC Berkeley. Stack-sorting simplices: geometry and lattice-point enumeration We study the polytopes that arise from the convex hulls of stack-sorting on particular permutations. We show that they are
simplices and proceed to study their geometry and lattice-point enumeration. First, we prove some enumerative results on $L n 1$ permutations, i.e., permutations of length $n$ whose penultimate and last entries are $n$ and 1, respectively. Additionally, we then focus on a specific permutation, which we call $L^{\prime} n 1$, and show that the convex hull of all its iterations through the stacksorting algorithm share the same lattice-point enumerator as that of the $(n-1)$-dimensional unit cube and lecture-hall simplex. Lastly, we detail some results on the real lattice-point enumerator for variations of the simplices arising from stacksorting $L^{\prime} n 1$ permutations. This then allows us to show that $L^{\prime} n 1$ simplices are Gorenstein of index 2.
(Received September 06, 2023)

1192-05-29276
Andrés R. Vindas-Meléndez*, UC Berkeley. Generalized Parking Function Polytopes.
A classical parking function of length $n$ is a list of positive integers ( $a_{1}, a_{2}, \ldots, a_{n}$ ) whose nondecreasing rearrangement $b_{1} \leq b_{2} \leq \cdots \leq b_{n}$ satisfies $b_{i} \leq i$. The convex hull of all parking functions of length $n$ is an $n$-dimensional polytope in $\mathbb{R}^{n}$, which we refer to as the classical parking function polytope. Its geometric properties have been explored in (Amanbayeva and Wang 2022) in response to a question posed in (Stanley 2020). We generalize this family of polytopes by studying the geometric properties of the convex hull of $\mathbf{x}$-parking functions for $\mathbf{x}=\left(x_{1}, \ldots, x_{n}\right)$, which refer to as $\mathbf{x}$-parking function polytopes. We explore connections between these $\mathbf{x}$-parking function polytopes and other polytopes, such as nestohedra and stellohedra. We also study the graph of the polytope.
(Received September 06, 2023)

1192-05-29288
Anni Hakanen, University of Turku, Ville Junnila, University of Turku, Tero Laihonen*, University of Turku, Ismael Yero, University of Cadiz. Recent Results on Vertices Belonging to All Metric Bases.
A set $S \subseteq V(G)$ is a resolving set in a graph $G=(V(G), E(G))$ if for any pair $u, v \in V(G)$ there exists $s \in S$ such that the distance $d(u, s)$ is different from the distance $d(v, s)$. A metric basis is a resolving set of the smallest possible cardinality. It is known that there are graphs where some vertices must belong to every metric basis. We will discuss recent results regarding such vertices and similar questions concerning other metric dimension variants.
(Received September 06, 2023)

1192-05-29311
Darren A. Narayan*, Rochester Institute of Technology, Brendan Rooney, Rochester Institute of Technology. Removing All Symmetries From Graphs.
A graph $G$ is asymmetric if its automorphism group is trivial. Asymmetric graphs were introduced by Erdős and Rényi (1963). They suggested the problem of starting with an asymmetric graph and removing some number $r$ of edges and/or adding some number $s$ of edges so that the resulting graph is non-asymmetric. Erdős and Rényi defined the degree of asymmetry of a graph to be the minimum value of $r+s$. In this paper, we define another property that measures how close a given non-asymmetric graph is to being asymmetric. We define the asymmetric index of a graph $G$, denoted $a i(G)$, to be the minimum of $r+s$ so that the resulting graph $G$ is asymmetric. We prove that for any non-negative integer $k$, there exists a graph $G$ where $a i(G)=k$. We show that the asymmetric index of a cycle with at least six vertices is two, and provide a complete characterization of all possible pairs of edges that can be added to a cycle to create an asymmetric graph. In addition, we determine the asymmetric index of paths, certain circulant graphs, Cartesian products involving paths and cycles, and bounds for complete graphs, and complete bipartite graphs. In particular we show that the Cartesian Product of K_\{3\} and itself can be made asymmetric by removing one edge and adding another edge, but cannot be made asymmetric by adding two edges or removing two edges.
(Received September 06, 2023)

1192-05-29312
Lon Mitchell*, Eastern Michigan University. On Euclidean Distances and Sphere Representations.
We extend recent results of Abdo Alfakih, who constructed Colin de Verdière matrices for complements of penny graphs from Euclidean distance matrices, by interpreting them using the sphere representations of Kotlov, Lovász, and Vempala. Our results apply to complements of contact graphs of unit spheres in arbitrary dimension.
(Received September 06, 2023)

1192-05-29324
Oscar D Quester*, Bridgewater State University. Van der Waerden Type Numbers for Integer Sequences. Preliminary report. We call a set $D \subseteq \mathbb{N} r$-large if every $r$-coloring of $\mathbb{N}$ admits arbitrarily long monochromatic arithmetic progressions whose gap (or common difference) belongs to $D$. In this language, Van der Waerden's Theorem is translated to the statement that $\mathbb{N}$ is $r$ large for every $r \in \mathbb{N}$. Following along these lines, a set $D \subseteq \mathbb{N}$ is called $r$-accessible if every $r$-coloring of $\mathbb{N}$ admits arbitrarily long monochromatic sequences $\left\{x_{1}, x_{2}, x_{3}, \ldots, x_{k}\right\}$ where $\bar{x}_{i+1}-x_{i} \in D$. We wish to characterize when a set $D \subseteq \mathbb{N}$ is $r$ large/accessible for various values of $r$. A powerful extension of Van der Waerden's Theorem tells us that any set that is generated by a polynomial with rational coefficients is $r$-large (and thus $r$-accessible) for every $r \in \mathbb{N}$. For sets not generated by a polynomial, the question of whether or not the set is $r$-large/accessible is far more open. We are able to show that any set $D \subseteq \mathbb{N}$ whose terms grow sufficiently fast is not 2 -accessible. A consequence of this fact is that the growth rate of the Fibonacci sequence alone is enough to tell us that the set of Fibonacci Numbers is not 4 -accessible. (Received September 06, 2023)

Cristina Ballantine, College of the Holy Cross, Amanda Welch*, Eastern Illinois University. Generalizations of PED and POD Partitions.
Partitions with even (respectively odd) parts distinct and all other parts unrestricted are often referred to as PED (respectively

POD) partitions. In this talk, we generalize these notions and study sets of partitions in which parts with fixed residue(s) modulo $r$ are distinct while all other parts are unrestricted.
(Received September 06, 2023)

## 1192-05-29383

Hannah E. Burson*, University of Minnesota, Hayan Nam, Duksung Women's University, Simone Sisneros-Thiry,
California State University- East Bay. Integer partitions corresponding to numerical semigroups.
Numerical semigroups are cofinite additive submonoids of the natural numbers motivated by the study of linear Diophantine equations. Through a simple injection to Young diagrams, researchers have used known results about numerical semigroups to answer questions about core partitions. In this talk, we will explore this connection between integer partitions and numerical semigroups with a focus on classifying the partitions that appear in the image of the injection from numerical semigroups. This talk is based on joint work with Hayan Nam and Simone Sisneros-Thiry.
(Received September 06, 2023)

## 1192-05-29387

Rhys O'Higgins, Macalester College, Lola Vescovo*, Macalester College. Exact mixing times for random walks on trees of a fixed diameter.
A random walk on a graph is an ordered list of vertices, each chosen randomly from the neighbors of the vertex before it. Using this, we can study the mixing time, which is the expected number of steps to reach a "balanced" distribution from the worst possible starting vertex. We characterize the extremal structures for certain random walks on trees of a fixed diameter and prove that the mixing time is maximized by the double broom.
(Received September 25, 2023)

## 1192-05-29390

Madeline Locus Dawsey*, University of Texas At Tyler. Properties of sequentially congruent partitions.
A sequentially congruent partition, defined by Schneider and Schneider in 2019, is a partition in which each part is congruent to the next part modulo its position in the partition. We introduce a new partition notation specific to sequentially congruent partitions which simplifies known partition bijections, illuminates their corresponding Young diagram transformations, and leads to generalizations of sequentially congruent partitions and related bijections. This is joint work with undergraduate students Ezekiel Cochran, Emma Harrell, and Samuel Saunders from the 2022 REU at the University of Texas at Tyler. (Received September 06, 2023)

## 1192-05-29401

Cong X. Kang, Texas A\&M University, Galveston campus, Eunjeong Yi*, Texas A\&M University, Galveston campus. MakerBreaker Metric Resolving Games on Graphs.
Let $d(x, y)$ denote the length of an $x-y$ geodesic in a graph $G$ with vertex set $V$. For a positive integer $k$, let $d_{k}(x, y)=\min \{d(x, y), k+1\}$ and $R_{k}\{x, y\}=\left\{z \in V: d_{k}(x, z) \neq d_{k}(y, z)\right\}$. A set $S \subseteq V$ is a distance- $k$ resolving set of $G$ if $S \cap R_{k}\{x, y\} \neq \varnothing$ for distinct $x, y \in V$. In this talk, we consider the maker-breaker distance- $k$ resolving game (MB $k$ RG) played on a graph $G$ by two players, Maker and Breaker, who alternately select a vertex of $G$ not yet chosen. Maker wins by selecting vertices which form a distance- $k$ resolving set of $G$, whereas Breaker wins by preventing Maker from winning. We denote by $O_{R, k}(G)$ the outcome of MB $k$ RG. Let $\mathcal{M}, \mathcal{B}$ and $\mathcal{N}$, respectively, denote the outcome for which Maker, Breaker, and the first player has a winning strategy in MB $k$ RG. Given a graph $G$, the parameter $O_{R, k}(G)$ is a non-decreasing function of $k$ with codomain $\{-1=\mathcal{B}, 0=\mathcal{N}, 1=\mathcal{M}\}$. We exhibit pairs $G$ and $k$ such that the ordered pair $\left(O_{R, k}(G), O_{R, k+1}(G)\right)$ realizes each member of the set $\{(\mathcal{B}, \mathcal{N}),(\mathcal{B}, \mathcal{M}),(\mathcal{N}, \mathcal{M})\}$; we provide graphs $G$ such that $O_{R, 1}(G)=\mathcal{B}, O_{R, 2}(G)=\mathcal{N}$ and $O_{R, k}(G)=\mathcal{M}$ for $k \geq 3$.
(Received September 06, 2023)

## 1192-05-29407

Brian Kronenthal*, Kutztown University. Nonisomorphic Maximal Girth Real Algebraically Defined Graphs. In this talk, we will discuss algebraically defined bipartite graphs. Let $\mathbb{F}$ denote a field, and consider the bipartite graph whose partite sets $P$ and $L$ are copies of $\mathbb{F}^{2}$ such that $\left(p_{1}, p_{2}\right) \in P$ and $\left[\ell_{1}, \ell_{2}\right] \in L$ are adjacent if and only if $p_{2}+\ell_{2}=p_{1} \ell_{1}$. This graph has girth six, and of particular interest is the question of whether it is possible to alter this equation to create a nonisomorphic girth six graph. We will explore the case $\mathbb{F}=\mathbb{R}$, where the question may be answered in the affirmative. (Received September 25, 2023)

## 1192-05-29494

Greg Bodwin*, University of Michigan. Girth Problems and their Applications in Theoretical Computer Science.
In extremal combinatorics, the girth problem asks to determine the value of the function $\gamma(n, k)$, which is the maximum possible number of edges that could be packed into an n-node graph that does not have cycles of length k or less. In the early '90s, a seminal work of Althöfer, Das, Dobkin, Joseph and Soares showed that the girth problem is equivalent to the space complexity of "spanner" problems; for example, it controls the amount of memory needed to represent the shortest path distances of an n-node graph within $\cdot(k-1)$ error. We will survey 30 years of research following this fundamental result. We will highlight modern variants of the problem, including the "weighted girth" problem and the "bridge girth" problem, and discuss their analogous applications in network design.
(Received September 07, 2023)

Fan Chung, University of California, San Diego, Nicholas Sieger*, University of California San Diego. A Random Graph Model for Clustering Graphs.
We introduce a random graph model for clustering graphs with a given degree sequence. Unlike previous random graph models, we incorporate clustering effects into the model without any geometric conditions. We show that random clustering graphs can yield graphs with a power-law expected degree sequence, small diameter, and any desired clustering coefficient. Our results follow from a general theorem on subgraph counts which contains an extremal problem which may be of independent interest.
(Received September 07, 2023)

1192-05-29539
Fan Chung, University of California, San Diego, Nicholas Sieger*, University of California San Diego. Random Clustering Graphs.
We introduce a random graph model for clustering graphs with a given degree sequence. Unlike previous random graph models, we incorporate clustering effects into the model without any geometric conditions. We show that random clustering graphs can yield graphs with a power-law expected degree sequence, small diameter, and any desired clustering coefficient. Our results follow from a general theorem on subgraph counts which may be of independent interest, and we pose several questions regarding the relationship between random clustering graphs and more geometric models of small-world networks. (Received September 07, 2023)

## 1192-05-29543

Grace Bielefeldt, St. Olaf College, Iris Horng*, University of Pennsylvania, Holly Luebsen, University of Texas at Austin, Mitchell VonEschen, Lawrence University. Algorithmic Generation of DNA Self-Assembly Graphs. Preliminary report. With recent advancements in the field of nanotechnology, there has been increasing interest in self-assembling nanostructures. These are constructed through the process of branched junction DNA molecules bonding with each other without external guidance. Using a flexible tile-based model, we represent molecules as vertices of a graph and cohesive ends of DNA strands as complementary half-edges allowing the molecules to bond with each other. Due to the unpredictability of DNA self-assembly in a laboratory setting and the risk of undesirable products being incidentally constructed, predicting what structures can be produced from a given list of components, referred to as a "pot of tiles," is useful but has been proven NPhard. This project introduces an algorithm to computationally generate and visualize at least one valid graph and for smaller cases, all non-isomorphic graphs, given a pot of tiles. By adjusting a number of construction parameters, we can produce graphs of various orders and proportions of tiles.
(Received September 07, 2023)

## 1192-05-29550

Tung T. Nguyen*, Western University. On prime Cayley graphs. Preliminary report.
Network theory plays a fundamental role in the study and understanding of collective behaviors of several systems. For example, it has found applications in neuroscience, ecology, neural dynamics, biology, and many physical systems. Typically, we often consider these systems as a single network. However, many real-world systems are multi-layered; namely, the system can have two levels of connections: on the first level, there are connections within each sub-network; on the higher level, there are connections between the sub-networks. In a recent work, we discovered a broadcasting solution mechanism from a reduced network to a larger multilayer network. From this perspective, the quest of decomposing complex networks into smaller, interconnected components is an important problem that could have a wide range of applications. In this talk, we utilize tools from group theory, number theory, and ring theory to study this graph decomposition problem when the network is a Cayley graph. In particular, we answer the following question: Which Cayley graphs are prime? This talk is based on joint work with Maria Chudnovsky, Logan Crew, Jan Mináč, Lyle Muller, Sophie Spirkl, Michal Cizek, and Duy Tan Nguyen. (Received September 07, 2023)

## 1192-05-29586

Jozsef Balog*, University of Illinois at Urbana-Champaign, Haoran Luo, University of Illinois. Turán density of long tight cycle minus one hyperedge.
Denote by $C_{\ell}^{-}$the 3 -uniform hypergraph obtained by removing one hyperedge from the tight cycle on $\ell$ vertices. It is conjectured that the Turán density of $C_{5}^{-}$is $1 / 4$. We make progress toward this conjecture by proving that the Turán density of $C_{\ell}^{-}$is $1 / 4$, for every sufficiently large $\ell$ not divisible by 3 . One of the main ingredients of our proof is a forbiddensubhypergraph characterization of the hypergraphs, for which there exists a tournament on the same vertex set such that every hyperedge is a cyclic triangle in this tournament. A byproduct of our method is a human-checkable proof for the upper bound on the maximum number of almost similar triangles in a planar point set, which recently was proved using flag algebra by Balogh, Clemen, and Lidicky.
(Received September 07, 2023)

1192-05-29591
Bhaswar Bhattacharya, University of Pennsylvania, Yangxinyu Xie*, University of Pennsylvania. Joint Convergence of Monochromatic Edges in Multiplex Hypergraphs.
Given a sequence of $r$-uniform hypergraphs $\left\{G_{n}\right\}_{n>1}$, denote by $T\left(G_{n}\right)$ the number of monochromatic hyperedges in a uniformly random $c_{n}$-coloring of the vertices of $G_{n}$. In this talk we study the joint distribution of monochromatic edges for hypergraphs with multiple layers (multiplex hypergraphs). Specifically, we consider the joint distribution of $\mathbf{T}_{n}:=\left(T\left(G_{n}^{(1)}\right), T\left(G_{n}^{(2)}\right)\right)$, for two sequences of hypergraphs $\left\{G_{n}^{(1)}\right\}_{n \geq 1}$ and $\left\{G_{n}^{(2)}\right\}_{n \geq 1}$ on the same set of vertices. We will show that the joint distribution of $\mathbf{T}_{n}$ converges to (possibly dependent) Poisson distributions whenever the mean vector and the covariance matrix of $\mathbf{T}_{n}$ converge. In other words, the asymptotic joint distribution of $\mathbf{T}_{n}$ is determined only by the convergence of its first two moments. This extends the well-known second moment phenomenon for marginal Poisson convergence to joint distributions and provides a unified framework for establishing the joint Poisson convergence of various
subgraph counting statistics that arise from random vertex and edge coloring of graphs. As an application of our general result, we will derive the asymptotic distribution of $\mathbf{T}_{n}$ for correlated Erdős-Rényi hypergraphs, where interesting phase transitions emerge. Extensions to the joint distribution of more than two hypergraphs will also be discussed.
(Received September 25, 2023)

## 1192-05-29618

Steve Butler*, Iowa State University. The mathematics of discrete periodic patterns....
...or How I learned to stop worrying and love the throw. We give a (brief) survey of the mathematics of juggling and its related variations. We will focus on three common ways to describe patterns: rook placements, cards, and state graphs. For each we will see the techniques that can be used in counting and finding patterns, and give some corresponding results as well. We will also see how we can use the interplay between these different descriptions to gain understanding and prove results (e.g.
Worpitzky's identity).
(Received September 07, 2023)

## 1192-05-29643

Ada Chan, York University, Mark Kempton, Brigham Young University, Sooyeong Kim*, York University, Stephen
Kirkland, University of Manitoba, Adam Knudson, Brigham Young University, Neal Madras, York University. Bounds on Kemeny's constant of a graph and the Nordhaus-Gaddum problem.
Kemeny's constant is an important measure from the theory of Markov chains that has received considerable interest from the graph theory community recently. Our attention focuses on Kemeny's constant within the context of random walks on graphs. Kemeny's constant gives a measure of how quickly a random walker can move around a graph, and thus provides an intuitive measure of the connectivity of a graph. We study Nordhaus-Gaddum problems for Kemeny's constant $\mathcal{K}(G)$ of a connected graph $G$. We prove bounds on $\min \{\mathcal{K}(G), \mathcal{K}(\bar{G})\}$ and the product $\mathcal{K}(G) \mathcal{K}(\bar{G})$ for various families of graphs. In particular, we show that if the maximum degree of a graph $G$ on $n$ vertices is $n-O(1)$ or $n-\Omega(n)$, then $\min \{\mathcal{K}(G), \mathcal{K}(\bar{G})\}$ is at most $O(n)$.
(Received September 07, 2023)

## 1192-05-29657

Aleyah Dawkins*, George Mason University, Rachel Kirsch, George Mason University. Hamiltonicity and related properties in $K_{r+1}$ free graphs. Preliminary report
In this talk, we discuss a new result on best-possible edge density conditions sufficient to imply traceability, Hamiltonicity, chorded pancyclicity, Hamiltonian-connectedness, $k$-path Hamiltonicity, and $k$-Hamiltonicity in $K_{r+1}$-free graphs. The problem of determining the extremal number ex $(n, F)$, the maximum number of edges in an $n$-vertex, $F$-free graph, has been studied extensively since Turán's theorem. Edge density conditions implying these properties also had been found. We bring together these two themes. Equivalently, we introduce variants of the extremal number ex $(n, F)$ in which we require that the graphs not have some Hamiltonian-like property, and we determine their values for $F=K_{r+1}$. We then extend these results to clique density conditions. This talk is based on joint work with Rachel Kirsch.
(Received September 07, 2023)

## 1192-05-29660

Marie Kramer*, Syracuse University. Graph Embeddings and Systole Bounds. Preliminary report
While obstructions to embedding graphs into the plane and the real projective plane are well understood, there is no known complete list for other surfaces such as the torus or the Klein bottle. Work by Kennard, Wiemeler, and Wilking shows that the existence of embeddings of graphs into surfaces implies good bounds on graph systoles, cogirths of regular matroids, and geometric data of special torus representations. For our purposes, we are interested in cubic graphs with small first Betti number. In this talk, we will discuss ongoing work towards a conceptual proof of Chambers' computer assisted results regarding cubic torus obstructions, as well as an analogue for the Klein bottle for small graphs. (Received September 08, 2023)

## 1192-05-29668

Soohyun Park*, Hebrew University of Jerusalem (incoming). Symmetries and intrinsic vs. extrinsic properties of $\overline{\mathcal{M}}_{0, n}$. Our motivation is the following question: How much of the combinatorial structure of the moduli space of stable rational curves with $n$ marked points is "intrinsic" to (the geometry of) the space itself? We view this from the lens of the natural permutation action on the points. In fact, it is known that this action does not extend to other wonderful compactifications of the complement of the $A_{n-2}$ hyperplane arrangement. We determine the differences in intersection patterns of parallel faces of associahedra and permutohedra inducing this rigidity property. As a consequence, we show that this is reflected in most of terms of degree at least 2 in the Chow ring. On the other hand, we consider degree 1 elements from the perspective of $S_{n}-$ invariance (e.g. suitable Lefschetz elements) and how it connects to recent positivity results. In particular, we show that the log concave sequences arising from degree 1 Hodge-Riemann relations using $S_{n}$-invariant elements of the Picard group have a special recursive structure and take the form of (quantum) Littlewood-Richardson coefficients with coefficients involving terms such as partition components, factorials, and multinomial coefficients. The sequences we obtain in these inequalities take the form of (quantum) Littlewood-Richardson coefficients with coefficients involving terms such as partition components, factorials, and multinomial coefficients. Finally, we connect higher degree Hodge-Riemann relations (of other rings) to the geometry of this moduli space via Toeplitz matrices.
(Received September 07, 2023)

Berkeley, Holly Swisher*, Oregon State University. Analogues of Alder-Type Partition Inequalities for Fixed Perimeter Partitions. Preliminary report.
Euler's partition identity, which states that the number of partitions of size $n$ with odd parts equals the number with distinct parts, was shown by Straub in 2016 to also hold for partitions with fixed perimeter rather than size. Fu and Tang then refined and generalized this analogue to $d$-distinct partitions, and obtained a fixed perimeter result related to the first RogersRamanujan identity. Here we use combinatorial techniques to study further generalizations of Alder-type partition functions and inequalities in the fixed perimeter setting.
(Received September 07, 2023)

## 1192-05-29784

Sawyer Jack Robertson*, UC San Diego. Random Walks, Conductance, and Resistance for the Connection Graph Laplacian. We investigate the concept of effective resistance in connection graphs, expanding its traditional application from undirected graphs. We propose a robust definition of effective resistance in connection graphs by focusing on the duality of Dirichlet-type and Poisson-type problems on connection graphs. Additionally, we delve into random walks, taking into account both node transitions and vector rotations. This approach introduces novel concepts of effective conductance and resistance matrices for connection graphs, capturing mean rotation matrices corresponding to random walk transitions. Thereby, it provides new theoretical insights for network analysis and optimization.
(Received September 07, 2023)

1192-05-29793
Novi Herawati Bong, University of Delaware, Mary Flagg*, University of St. Thomas, Mark Hunnell, Winston-Salem State University, John Hutchens, University of San Francisco, Ryan Roger Moruzzi Jr, CSU East Bay, Houston Schuerger, Trinity College, Ben Small, -. Reconfiguration for Skew Zero Forcing Sets.
Reconfiguration studies the ability to transform one feasible solution to a problem into another by a sequence of elementary transformations defined by a reconfiguration rule. Given a graph $G=(V(G), E(G))$ and a problem involving a graph parameter $\mathcal{X}$ with feasible solutions corresponding to subsets $S \subseteq V(G)$, the reconfiguration graph for $\mathcal{X}$ has vertices subsets $S$ with property $\mathcal{X}$ and edges defined by the reconfiguration rules. The rules we study are token exchange and token sliding. In token exchange, two subsets $R$ and $S$ are connected by an edge if one may be obtained from the other by exchanging a single vertex. Token sliding is token exchange with the restriction that the vertices exchanged must be neighbors. We consider token exchange and token sliding for minimum skew zero forcing sets. Skew zero forcing was introduced as a tool bound the maximum nullity of skew symmetric matrices associated to the graph $G$. A set $S \subseteq V(G)$ is a skew zero forcing set if vertices in $S$ are colored blue and the graph is colored blue by repeated applications of the skew forcing rule: if a vertex $v$ has exactly one uncolored neighbor $w$, then $v$ forces $w$ to be colored blue. Skew forcing provides an interesting study for reconfiguration since the empty set may be a skew zero forcing set. We present basic results for common graph families, connections to graph matchings, and a comparison with the reconfiguration graphs for standard zero forcing and positive semidefinite zero forcing for these families.
(Received September 07, 2023)

1192-05-29829
Fadekemi Janet Osaye*, Alabama State University. Wiener index in graphs given girth, minimum, and maximum degrees. Let $G$ be a connected graph of order $n$. The Wiener index $W(G)$ of $G$ is the sum of the distances between all unordered pairs of vertices of $G$. The well-known upper bound $\left(\frac{n}{\delta+1}+2\right)\binom{n}{2}$ on the Wiener index of a graph of order $n$ and minimum degree $\delta$ by Kouider and Winkler was improved significantly by Alochukwu and Dankelmann for graphs containing a vertex of large degree $\Delta$. In this paper, we give upper bounds on the Wiener index of $G$ in terms of order $n$ and girth $g$, where $g$ is a function of both the minimum degree $\delta$ and maximum degree $\Delta$. Our result provides a generalization for these previous bounds for any graph of girth $g$. In addition, we construct graphs to show that, if for given $g$, there exists a Moore graph of minimum degree $\delta$, maximum degree $\Delta$, and girth $g$, then the bounds are asymptotically sharp.
(Received September 25, 2023)

## 1192-05-29841

Wei Gao*, Penn State Abington. Zero-nonzero patterns that allow or require $\mathbb{S}_{n}^{*}$.
A zero-nonzero pattern matrix is a matrix with entries from $\{*, 0\}$, where $*$ is nonzero. Motivated by the possible onset of instability in dynamical systems associated with a zero eigenvalue, the inertia set $S_{n}^{*}$ for $n \geq 2$ is defined to be $\mathbb{S}_{n}^{*}=\{(0, n, 0),(0, n-1,1),(1, n-1,0),(n, 0,0),(n-1,0,1),(n-1,1,0)\}$. In this talk, some known results about zerononzero patterns that allow or require $\mathbb{S}_{n}^{*}$ will be shown.
(Received September 08, 2023)

## 1192-05-29847

Aida Abiad*, Eindhoven University of Technology, Antonina Khramova, Eindhoven University of Technology. A new method to construct cospectral hypergraphs.
Spectral hypergraph theory mainly concerns using hypergraph spectra to obtain structural information about the given hypergraph. This field has attracted a lot of attention over the last years. The spectrum of a hypergraph can be defined in different ways. In this talk, we will focus on the spectrum of two well-known hypergraph representations: adjacency tensors and integer matrices with entries defined by the number of edges that two vertices share. Two hypergraphs are cospectral if they share the same spectrum with respect to a certain representation. The study of cospectral hypergraphs helps in understanding which hypergraph properties cannot be deduced from their spectra. In this talk, a new method for constructing cospectral uniform hypergraphs will be presented.
(Received September 08, 2023)

Chelsea Sato*, Syracuse University. Systole bounds for graphs of small Betti number. Preliminary report.
Recent work of Kennard, Wiemeler, and Wilking observes a link among graph systoles, cogirths of regular matroids, and codimension bounds for fixed-point sets of special torus representation. This talk discusses a new proof of a key graph systole estimate from that paper. One advantage of this proof is that it is independent of a non-trivial result from topological graph theory. Another advantage is that our method provides refined estimates for each Betti number that depend also on the (classical) girth of the underlying graph.
(Received September 10, 2023)

## 1192-05-29898

Jacob Johnston*, Villanova University, Michael Tait, Villanova. Extremal Values for the Spectral Radius of the Normalized Distance Laplacian.
The normalized distance Laplacian of a graph $G$ is defined as $\mathcal{D}^{\mathcal{L}}(G)=T(G)^{-1 / 2}(T(G)-\mathcal{D}(G)) T(G)^{-1 / 2}$ where $\mathcal{D}(G)$ is the matrix with pairwise distances between vertices and $T(G)$ is the diagonal transmission matrix. In this project, we study the minimum and maximum spectral radii associated with this matrix, and the structures of the graphs that achieve these values. In particular, we prove a conjecture of Reinhart that the complete graph is the unique graph with minimum spectral radius, and we give several partial results towards a second conjecture of Reinhart regarding which graph has the maximum spectral radius
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1192-05-29902
Tahda Queer, Hunter College, City University of New York, Cyrus Young*, University of California, Irvine, Wohua Zhou, California State University, East Bay. Colorful Turán Theorems for the Vertex Disjoint Union of Rainbow Triangles. Preliminary report.
Given an edge-colored graph $G$, we denote the number of colors as $c(G)$, and the number of edges as $e(G)$. An edge-colored graph is rainbow if no two edges share the same color. A proper $m K_{3}$ is a vertex disjoint union of $m$ rainbow triangles. Rainbow problems have been studied extensively in the context of anti-Ramsey theory, and more recently, in the context of Turán problems. B. Li. et al. [European J. Combin. 36 (2014)] found that a graph must contain a rainbow triangle if $e(G)+c(G) \geq\binom{ n}{2}+n$. L. Li. and X. Li. [Discrete Applied Mathematics 318 (2022)] conjectured a lower bound on $e(G)+c(G)$ such that $G$ must contain a proper $m K_{3}$. In this poster, we provide a construction that disproves the conjecture. We also introduce a result that guarantees the existence of $m$ copies of rainbow cliques in general graphs, and a sharp result on the existence of proper $m K_{3}$ in complete graphs.
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## 1192-05-29938

Tahda Queer, Hunter College, City University of New York, Cyrus Young, University of California, Irvine, Wohua Zhou*, California State University, East Bay. Colorful Turán Theorems for the Vertex Disjoint Union of Rainbow Cliques. Preliminary report.
labstract Given an edge-colored graph $G$, we denote the number of colors as $c(G)$, and the number of edges as $e(G)$. An edgecolored graph is rainbow if no two edges share the same color. A proper $m K_{3}$ is a vertex disjoint union of $m$ rainbow triangles. Rainbow problems have been studied extensively in the context of anti-Ramsey theory, and more recently, in the context of Turán problems. B. Li. et al. [European J. Combin. 36 (2014)] found that a graph must contain a rainbow triangle if $e(G)+c(G) \geq\binom{ n}{2}+n$. L. Li. and X. Li. [Discrete Applied Mathematics 318 (2022)] conjectured a lower bound on $e(G)+c(G)$ such that $G$ must contain a proper $m K_{3}$. In this presentation, we provide a construction that disproves the conjecture. We also introduce a result that guarantees the existence of $m$ copies of rainbow cliques in general graphs, and a sharp result on the existence of proper $m K_{3}$ in complete graphs. \endabstract (Received September 08, 2023)

## 1192-05-29945

Emily H Dickey*, Stanford University, Noah A. Rosenberg, Stanford University. Labeled Histories for Multifurcating Trees. In mathematical phylogenetics, a labeled history for a rooted labeled bifurcating tree is a temporal sequence of the branching events that give rise to the tree. In other words, given a leaf-labeled tree with $n$ internal nodes, a labeled history is a bijection between those nodes and the set $\{1,2, \ldots, n\}$, such that the label assigned to a given node is strictly greater than the labels assigned to its descendants. Enumerative and probabilistic aspects of labeled histories for bifurcating trees have long been investigated. Here, we generalize the concept of labeled histories to $r$-furcating trees. Consider a rooted labeled tree in which each internal node has exactly $r$ children, $r \geq 2$. We first generalize the equation for the number of labeled histories for a bifurcating tree in the $r=2$ case to enumerate labeled histories for an $r$-furcating tree, with arbitrary $r \geq 2$. We then determine the rooted unlabeled $r$-furcating tree shape on $n$ internal nodes that has the largest number of labeled histories. Finally, we enumerate labeled histories for $r$-furcating trees in a setting that allows for simultaneous branching events. These results advance the study of mathematical phylogenetics by extending computations concerning fundamental features of bifurcating phylogenetic trees to a more general class of multifurcating trees.
(Received October 11, 2023)

## 1192-05-29997

Andrew Hardt*, University of Illinois Urbana-Champaign. Lattice models for motivic Chern classes of Schubert varieties. We describe solvable lattice models whose partition functions are motivic Chern classes of Schubert varieties. Our models generalize previously-considered lattice models for Iwahori Whittaker functions and factorial Schur polynomials. We'll talk about geometric considerations and connections to Kazhdan-Lusztig R-polynomials. This is based on joint work with Ben Brubaker, Daniel Bump, and Hunter Spink.
(Received September 08, 2023)

## 1192-05-30039

Beth Morrison Bjorkman*, Air Force Research Laboratory, Zachary Brennan, Iowa State University, Mary Flagg,
University of St. Thomas, Johnathan Koch, Applied Research Solutions. Power Domination with Random Sensor Failure. Preliminary report.
The power domination problem seeks to find the placement of the mimimum number of sensors called phasor measurement units (PMUs) needed to monitor an electric power network. Vertices with a PMU and vertices adjacent to PMUs are observed
Then, for any observed vertex with exactly one unobserved neighbor, we can use conservation of energy laws to observe the unobserved neighbor. This process continues until no more vertices can be observed. We study the challenges that arise when PMUs are allowed to fail with probability $q$.
(Received September 08, 2023)

## 1192-05-30077

Gabor Lippner*, Northeastern University, Yujia Shi, Northeastern University. Achieving strong quantum state transfer using a bounded potential. Preliminary report.
Quantum state transfer is a phenomenon where the state of a qubit is transferred from one location to another in a network of spin particles. It can be particularly useful in constructing "quantum wires". To that end, one has to ensure that state transfer happens with high fidelity. We study this problem on general graphs, in the presence of magnetic fields. It has been known
that if one applies very strong magnetic fields then the transfer fidelity can get arbitrarily close to 100
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## 1192-05-30081

Zachary Hamaker, University of Florida, Patricia Klein, Texas A\&M University, Anna Weigandt*, University of Minnesota
The Bumpless pipe dream formula for symplectic Schubert polynomials. Preliminary report.
The symplectic group acts on the complete flag variety. Its orbits are indexed by fixed-point-free involutions. Building on work of Brion, Wyser and Yong introduced symplectic Schubert polynomials, which represent cohomology classes of the orbit closures of the symplectic group action. Bumpless pipe dreams are closely connected to the six-vertex model and were introduced by Lam, Lee, and Shimozono to study backstable Schubert calculus. We present a bumpless pipe dream formula for symplectic Schubert polynomials.
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## 1192-05-30093

Brian Hopkins*, Saint Peter's University. Integer Partition Excedances, Antiexcedances, and Generalizations. Preliminary report.
The permutation statistic excedance can be applied to integer partitions: Let $e(n, k)$ be the number of partitions $\lambda=\left(\lambda_{1}, \ldots, \lambda_{s}\right)$ of $n$ with $\lambda_{1} \geq \cdots \geq \lambda_{s}$ such that $k$ parts satisfy $\lambda_{i}>i$. The irregular triangle of $e(n, k)$ values has diagonal sums related to Dyson's crank statistic and the antidiagonal sums match $p(n)+p(n+1)$, sums of consecutive partition numbers. Generalizations of excedances, replacing the condition $\lambda_{i}>i$ with $\lambda_{i}>i+c$ for an integer $c$, help with understanding related integer triangles. There are also patterns in triangles based on antiexcedances $\lambda_{i}<i$. This work relates to a 2022 Ramanujan Journal publication of Blecher and Knopfmacher on fixed points of integer partitions (where $\lambda_{i}=i$ ) and subsequent results of the speaker and James Sellers (arXiv:2305.05096).
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1192-05-30097
Nitya Mani*, Massachusetts Institute of Technology, Edward Yu, Massachusetts Institute of Technology. Turan problems on mixed graphs.
We investigate natural Turán problems for mixed graphs, generalizations of graphs where edges can be either directed or undirected. We study a natural Turán density coefficient that measures how large a fraction of directed edges an $F$-free mixed graph can have; we establish an analogue of the Erdős-Stone-Simonovits theorem and give a variational characterization of the Turán density coefficient of any mixed graph (along with an associated extremal F-free family). This characterization enables us to highlight an important divergence between classical extremal numbers and the Turán density coefficient. We show that Turán density coefficients can be irrational, but are always algebraic; for every positive integer k, we construct a family of mixed graphs whose Turán density coefficient has algebraic degree k.
(Received September 08, 2023)

Seamus Connor, Williams College, Steven Sofos DiSilvio*, Columbia University, Sasha Kononova*, University of California at Los Angeles, Ralph E. Morrison, Williams College, Krish Singal*, Columbia University. On the Size and Complexity of Scrambles.
In the setting of chip-firing games on graphs, the gonality of a graph provides a discrete analog of the gonality of an algebraic curve. Given a graph $G$ on $n$ vertices, the strongest known lower bound on its gonality is its scramble number, denoted $\operatorname{sn}(G)$, which generalizes the notion of bramble number by not requiring subgraphs to touch. A related invariant, disjoint scramble number, denoted $\operatorname{dsn}(G)$, is computed similarly but restricts to disjoint subgraphs. Echavarria et al. showed in 2021 that $\operatorname{sn}(G)$ is NP-hard to compute, but it is not known to be in NP. While a scramble is a natural certificate candidate, its size is potentially exponential in $n$. To study this certificate, we introduce the carton number of a graph, defined as the minimum size of a maximum order scramble. Using $\operatorname{tw}(G)$ as a lower bound on $\operatorname{sn}(G)$ (Harp et al. 2022), we adapt a result on bramble size by Grohe and Marx and prove via a probabilistic argument that bounded degree graphs on $n$ vertices with scramble number $\Omega\left(n^{1 / 2+\epsilon}\right)$ have carton number exponential in $n$. Moreover, we provide an explicit construction of bounded degree expander graphs with scramble number $\Theta(n)$ and carton number exponential in $n$. This invalidates scrambles as NP certificates.

Furthermore, it shows that minor-closed families of graphs with bounded degree have scramble number $O(\sqrt{n})$. We also prove via contradiction that carton number is at least $3 \operatorname{sn}(G)-n$ for graphs of maximum degree less than $\operatorname{sn}(G)$. To conclude our investigation into the computational complexity of computing these invariants, we construct efficient approximation algorithms and consider their fixed-parameter tractability. We apply Courcelle's theorem to prove the fixedparameter tractability of the decision problem $\operatorname{dsn}(G) \geq k$ when parametrized by $\operatorname{tw}(G)$ and $k$. Furthermore, we show that both $\operatorname{sn}(G)$ and gon $(G)$ can be 2 -approximated in polynomial time for graphs of high minimum degree via a known approximation algorithm for the minimum vertex cover problem. Lastly, we remark on the existence of constant factor approximations of these invariants for other graph classes. (Received September 09, 2023)

## 1192-05-30177

Anastasia Halfpap*, University of Montana. Rainbow Turán numbers for paths.
Rainbow Turán problems represent a synthesis of two cornerstones of extremal graph theory: Turán questions and edgecoloring problems. We say that a graph is properly edge-colored if no two incident edges receive the same color, and is rainbow if no two edges receive the same color. Given a forbidden graph $F$, the rainbow Turán number ex ${ }^{*}(n, F)$ is the maximum number of edges in an $n$-vertex graph $G$ such that, under some proper edge-coloring, $G$ does not contain any rainbow copy of $F$. Rainbow Turán problems were introduced by Keevash, Mubayi, Sudakov, and Verstraëte in 2007, and have received substantial attention in the last decade. However, exact values for ex ${ }^{*}(n, F)$ are often elusive, even for small forbidden graphs $F$. In this talk, we focus on rainbow Turán numbers of paths. While "ordinary" Turán numbers for paths are well-understood, the problem of determining $\operatorname{ex}^{*}\left(n, P_{\ell}\right)$ in general seems very difficult. We discuss recent work which determines $\mathrm{ex}^{*}\left(n, P_{5}\right)$, highlighting both some challenges introduced by the rainbow setting and some techniques which made exact determination tractable.
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## 1192-05-30208

Weston Miller*, The University of Texas at Dallas. Rational Catalan Numbers for Complex Reflection Groups. Preliminary report.
The spetsial complex reflection groups are well-generated complex reflection groups that behave as if they were the Weyl group for some connected reductive algebraic group. Analogs of unipotent characters, Harish-Chandra theory, and Lusztig's Fourier transform can be defined combinatorially for these groups, allowing some techniques from the representation theory of finite groups of Lie type to be extended to spetsial complex reflection groups. In a recent paper, Galashin, Lam, Trinh, and Williams introduced a family of rational noncrossing objects for finite Coxeter groups using distinguished subwords. They then gave a type-uniform proof that these objects are counted by rational Coxeter-Catalan numbers by using Hecke algebra traces to compute the point count of braid Richardson varieties. Assuming standard conjectures, I prove that this trace technique extends to irreducible spetsial complex reflection groups. In particular, even though there are not braid Richardson varieties in this context, the trace of a power of a Coxeter element still produces a rational Catalan number. I'll also discuss applications to parking combinatorics.
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## 1192-05-30243

Muralikrishnan Gopalakrishnan Meena*, Oak Ridge National Laboratory. Graph-theoretic analysis and modeling of complex systems in fluid dynamics and mycology.
The interaction-driven evolution of complex systems in both natural and engineering contexts offers a unique opportunity to leverage graph theory for understanding their behavior as well as for modeling and modifying their evolution. This seminar aims to explore the application of graph-theoretic methods in analyzing diverse problems within the realms of fluid dynamics and mycology. The specific applications encompass two main areas: (1) formulating reduced-order models and performing flow modification of complex laminar and turbulent flows and (2) revealing fungal microbe-host interactions to identify pivotal interactions in various fungal communities for biofertilizer development and drug discovery for human pathogenic fungi. The interaction-based framework of graph theory is employed to capture the complex interactions in laminar and turbulent vortical flows. Unsupervised clustering techniques are used to form reduced-order models of such canonical flows. Furthermore, the interactions among the identified vortical structures are used to illustrate the capability of graph-theoretic tools to modify turbulent flows. On the mycology side, graph theory aids in quantifying the interaction of fungal secondary metabolites (SMs) with their source of production. Broadly, two such forms of relationships will be discussed: (1) interactions with the various fungal species that produce SMs and (2) the influence of various chemical treatments that regulate SMs. Insights into commonalities among these complex systems revealed through the interaction-based framework will also be discussed, opening new avenues to analyze these extraordinary scientific problems. (Received September 09, 2023)

## 1192-05-30265

Ayah Almousa*, University of Minnesota - Twin Cities, Shiliang Gao, University of Illinois at Urbana-Champaign, Daoji Huang, ICERM. A Gröbner basis for positroid varieties.
Positroid varieties are subvarieties of the Grassmannian that arise in the study of total positivity. Knutson, Lam, and Speyer described a certain type of Gröbner degeneration called the Hodge degeneration as projections of order complexes of intervals in the Bruhat order, but their description does not give an explicit Gröbner basis nor initial ideal. We give an explicit, combinatorial description of the Gröbner basis and initial ideal corresponding to the Hodge degeneration for an arbitrary positroid variety. As an application, we show that promotion on rectangular-shaped semistandard tableaux gives a bijection between standard monomials of a positroid variety and its cyclic shifts.
(Received September 09, 2023)

Eric G Ramos*, Stevens Institute of Technology. Universal generators for graph configuration spaces, and Robertson's conjecture.
One of the most famous results in graph theory is that of Kuratowski's theorem, which states that a graph $G$ is non-planar if and only if it contains one of $K_{3,3}$ or $K_{5}$ as a topological minor. That is, if some subdivision of either $K_{3,3}$ or $K_{5}$ appears as a subgraph of $G$. In this case we say that the question of planarity is determined by a finite set of forbidden (topological) minors. A conjecture of Robertson, whose proof was recently announced by Liu and Thomas, characterizes the kinds of graph theoretic properties that can be determined by finitely many forbidden minors. In this talk we will present a categorical version of Robertson's conjecture, which we have proven in certain cases. We will then illustrate how this categorification, if proven in all cases, would imply many non-trivial statements in the topology of graph configuration spaces.
(Received September 09, 2023)

## 1192-05-30292

Rachel Kirsch, George Mason University, Jd Nir*, Oakland University. A localized approach to generalized Turán problems. Generalized Turán problems ask for the maximum number of copies of a graph $H$ in an $n$-vertex, $F$-free graph, denoted by ex $(n, H, F)$. We show how to extend the localized approach of Bradač, Malec, and Tompkins to generalized Turán problems. We weight the copies of $H$ (typically taking $H=K_{t}$ ), instead of the edges, based on the size of the largest clique, path, or star containing the vertices of the copy of $H$, and in each case prove a tight upper bound on the sum of the weights. A consequence of our new localized theorems is an asymptotic determination of ex $\left(n, H, K_{1, r}\right)$ for every $H$ having at least one dominating vertex and $\operatorname{mex}\left(m, H, K_{1, r}\right)$ for every $H$ having at least two dominating vertices.
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## 1192-05-30319

Slava Naprienko*, University of North Carolina at Chapel Hill. Free Fermionic Schur Functions.
We use the exactly solvable free fermionic six vertex model to introduce a new family of supersymmetric Schur functions that depend on two sequences of parameters. These free fermionic Schur functions have a hidden symmetry between the two sets of parameters that allows us to generalize and unify factorial, supersymmetric, and dual Schur functions from literature. Using the refined Yang-Baxter equations, we show that these functions satisfy the supersymmetric Cauchy identity, which is independent of the both sequences of parameters.
(Received September 09, 2023)

1192-05-30325
Ville Junnila*, University of Turku, Tero Laihonen, University of Turku, Havu Miikonen, Department of Mathematics and Statistics, University of Turku. On Vertices Forced in All Minimum Locating-Dominating Sets of a graph.
Let $G$ be a finite, simple and undirected graph with the set of vertices $V(G)$ and edges $E(G)$. A nonempty subset $S \subseteq V(G)$ is called locating-dominating in $G$ if for all vertices $u \in V(G) \backslash S$ the intersection of $S$ and the closed neighbourhood $N[u]$ is nonempty and unique. Notice that no vertex $u \in V(G)$ belongs to all locating-dominating sets in $G$ with no isolated vertices since, obviously, $V(G) \backslash\{u\}$ is a locating-dominating set. However, in this talk, we show that there exist vertices belonging to all minimum locating-dominating sets in $G$. In particular, we prove that the path $P_{5 k}(k \geq 1)$ of order $5 k$ contains $2 k$ such vertices; in other words, the ratio of such vertices compared to the number of vertices of the graph is $2 / 5$. Furthermore, among other related topics, we also discuss the maximum of the previous ratio in any graph $G$.
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1192-05-30372
Cristina Ballantine, College of the Holy Cross, Hannah E. Burson, University of Minnesota, William Craig, Universität Köln, Amanda Folsom, Amherst College, Boya Wen*, University of Wisconsin-Madison. Hook length bias in odd versus distinct partitions.
In this talk, I will introduce recent joint work with Ballantine, Burson, Craig, and Folsom, where we show that there are more hooks of length 2 , respectively 3 , in all odd partitions of $n$ than in all distinct partitions of $n$, and make the analogous conjecture for arbitrary hook length $t \geq 2$. We also establish additional bias results on the number of gaps of size 1 , respectively 2 , in all odd versus distinct partitions of $n$. A key component of the proof of our bias result for hooks of length 3 is a linear inequality involving $q(n)$, the number of distinct partitions of $n$, which is of independent interest. I will also survey the methods we use to obtain these results, which are both analytic and combinatorial.
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1192-05-30376
Runze Li*, University of California, Santa Barbara, István Miklós, Rényi Institute, Hungarian Academy of Sciences. 3uniform hypergraphic degree sequences. Preliminary report.
A 3-uniform hypergraph is a generalization of simple graphs where each hyperedge has 3 vertices. The degree of a vertex in a hypergraph is the number of hyperedges incident to it. The degree sequence of a hypergraph is the sequence of the degrees of its vertices. The degree sequence problem for 3 -uniform hypergraphs is to decide if a 3 -uniform hypergraph, called a realization, exists with a prescribed degree sequence. So far, polynomial algorithms can be used only for some very restricted degree sequences to construct their realizations. Our research mainly shows that if all degrees in a degree sequence $D$ are between $\frac{n^{2}}{27}+O(n)$ and $\frac{2 n^{2}}{27}-O(n)$, further, the number of vertices is at least 38 , and the degree sum is divisible by 3 , then $D$ has a 3 -uniform hypergraph realization. Our proof is constructive and it runs in polynomial time. This is the first polynomial algorithm to construct a 3 -uniform hypergraph realization of a highly irregular and dense degree sequence.
(Received September 09, 2023)

Runze Li*, University of California, Santa Barbara, István Miklós, Rényi Institute, Hungarian Academy of Sciences. Construction of graphic 3-uniform hypergraph degree sequences. Preliminary report.
A 3-uniform hypergraph is a generalization of simple graphs where each hyperedge has 3 vertices. The degree of a vertex in a hypergraph is the number of hyperedges incident to it. The degree sequence of a hypergraph is the sequence of the degrees of its vertices. The degree sequence problem for 3 -uniform hypergraphs is to decide if a 3 -uniform hypergraph, called a realization, exists with a prescribed degree sequence. So far, polynomial algorithms can be used only for some very restricted degree sequences to construct their realizations. Our research mainly shows that if all degrees in a degree sequence $D$ are between $\frac{n^{2}}{27}+O(n)$ and $\frac{2 n^{2}}{27}-O(n)$, further, the number of vertices is at least 38 , and the degree sum is divisible by 3 , then $D$ has a 3 -uniform hypergraph realization. Our proof is constructive and it runs in polynomial time. This is the first polynomial algorithm to construct a 3 -uniform hypergraph realization of a highly irregular and dense degree sequence.
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## 1192-05-30502

Ilaria Seidel*, Harvard University. Feasible Regions for Consecutive Pattern Occurrences in Several Pattern-Avoiding Classes.
A length $k$ consecutive pattern occurrence in a permutation $\sigma$ describes the local behavior of $\sigma$ in a length $k$ segment. We consider the probability distribution given by the proportion of consecutive occurrences of each $\pi \in S_{k}$. Borga and Penaguiao studied this distribution in the limit as $\sigma$ becomes arbitrarily long. They showed that the collection of such limit distributions, called the feasible region, forms a polytope. They conjectured that the feasible region is also a polytope when restricted to patterns which avoid a given set of patterns $B$. In this talk, we resolve this conjecture for certain sets of patterns, including all patterns of length 4.
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1192-05-30520
Alexander Divoux*, Georgia Institute of Technology, Thomas Kelly, Georgia Institute of Technology, Camille Elisabeth Kennedy*, Northwestern University, Jasdeep Sidhu*, Stanford University. Random Latin Squares.
A $k \times n$ partial Latin rectangle is $C$-sparse if the number of populated entries in each row and column is less than $C$, and each symbol is used less than $C$ times. We prove that the probability a uniformly random $k \times n$ Latin rectangle, where $k<(1 / 2-\alpha) n$, contains a $\beta n$-sparse partial Latin rectangle with $\ell$ nonempty entries is $\left(\frac{1 \pm \epsilon}{n}\right)^{\ell}$ for sufficiently large $n$. Using this result, we prove that a uniformly random order- $n$ Latin square asymptotically almost surely has no order- $C \sqrt{n \log n}$ Latin subsquare. This research was completed during the 2023 Georgia Tech Mathematics REU. This project was supported by the NSF grants \#1745583, \#1851843, \#2244427 and the GaTech College of Sciences. (Received September 10, 2023)

1192-05-30522
Karel Devriendt*, Max Planck Institute for Mathematics in the Sciences, Andrea Ottolini, University of Washington, Stefan Steinerberger, University of Washington, Seattle. Graphs with nonnegative resistance curvature.
In recent work, Devriendt and Lambiotte proposed a notion of discrete scalar and Ricci curvature on graphs, based on the concept of effective resistance. The (scalar) resistance curvature at a vertex $v$ is defined as $p(v)=1-\frac{1}{2} \mathbb{E}\left(\operatorname{deg}_{T}(v)\right)$ where $\operatorname{deg}_{T}(v)$ is the degree of $v$ in a random spanning tree $T$. The resistance curvature can be approximated efficiently using Laplacian solvers, which makes it accessible to use in large sparse graphs. In this talk, I will survey some results on weighted graphs with nonnegative resistance curvature; here the tree measure is induced by positive edge weights. We retrieve classical geometric results such as bounds on the diameter and Laplacian eigenvalues, and the more graph-theoretic result that graphs with $p>0$ are 1 -tough, i.e. removing $k$ vertices yields at most $k$ new components. Secondly, we extend a question of Fiedler and ask which graphs admit a weighting of the edges such that $p \geq 0$. We show how this question can be translated into a question about whether two polytopes associated with the graph intersect.
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## 1192-05-30523

Ritik Jain*, Fordham University. The number of zeros of a random polynomial over a finite field.
The distribution of the average number of zeros of polynomials over real and complex fields is a classical topic with many wellknown results. In this talk, we present an equivalent finding for multivariable polynomials over finite fields with arbitrary order $q$. First, we obtain the expected number of zeros for such polynomials. Next, we derive the probability generating function of the distribution in the single-variable case. Notably, as $q$ grows without bound, it asymptotically resembles Poisson distribution with parameter 1 . Then, we compute the probability generating function of the number of zeros in the two-variable case. Though it is not explicitly shown, the method of proof extends easily to the general multivariable case via induction, for which the result is stated. Finally, we discuss some related questions. This is a joint work with Han-Bom Moon and Peter Wu. (Received September 10, 2023)

Cong X. Kang*, Texas A\&M University, Galveston campus, Iztok Peterin, University of Maribor, Eunjeong Yi, Texas A\&M University, Galveston campus. Resolving a Graph Family Simultaneously.
A set of vertices $W$ of a graph $G=(V, E)$ is a resolving set of $G$ if, for any two distinct vertices $x, y \in V$, there exists a $w \in W$ such that $d(w, x) \neq d(w, y)$, where $d(u, v)$ denotes the minimum number of edges in a path linking vertices $u$ and $v$ in $G$. A set $S \subseteq V$ is a simultaneous resolving set for a finite collection $\mathcal{C}$ of graphs on a common vertex set $V$ if $S$ is a resolving set for every graph in $\mathcal{C}$; the minimum among the cardinalities of all such $S$ is called the simultaneous metric dimension of $\mathcal{C}$, denoted by $\operatorname{Sd}(\mathcal{C})$. In this talk, we first discuss the simultaneous metric dimension of graphs $G$ and their complements $\bar{G}$. We characterize graphs $G$ satisfying $\operatorname{Sd}(G, \bar{G})=1$ and $\operatorname{Sd}(G, \bar{G})=|V|-1$, respectively. We show that $\{\operatorname{diam}(G), \operatorname{diam}(\bar{G})\} \neq\{3\}$ implies $\operatorname{Sd}(G, \bar{G})=\max \{\operatorname{dim}(G), \operatorname{dim}(\bar{G})\}$. Via a family of self-complementary split graphs of diameter 3 , we show that $\operatorname{Sd}(G, \bar{G})-\max \{\operatorname{dim}(G), \operatorname{dim}(\bar{G})\}$ can be arbitrarily large. We determine $\operatorname{Sd}(G, \bar{G})$ when $G$ is a tree or a unicyclic graph. We will also discuss results on general families obtained from the fractionalization of this notion, should time permit.
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Igor Araujo, University of Illinois at Urbana-Champaign, Jozsef Balogh, University of Illinois at Urbana-Champaign, Robert A. Krueger*, University of Illinois at Urbana-Champaign, Simon Piga, University of Birmingham, Andrew Treglown, University of Birmingham. Oriented cycles in randomly perturbed digraphs.
In 2003, Bohman, Frieze, and Martin proved that for every $\alpha$, there exists $C$ such that every $n$-vertex graph $G$ with minimum degree $\alpha n$ contains a spanning cycle when $C n$ random edges are added, with probability tending to 1 as $n$ tends to infinity. This result fits between two classical theorems: Dirac's theorem, which states that minimum degree $n / 2$ is necessary and sufficient to ensure a spanning cycle, and a series of theorems which state that random graphs with $C n \log n$ edges have spanning cycles with high probability (for more precise choices of $C$ ). Bohman, Frieze, and Martin also prove a digraph analogue of their result, where given a minimum out- and in-degree condition, they find a spanning cycle with all the edges oriented the same way around the cycle. We generalize this digraph result to find cycles of every length and orientation, simultaneously, with high probability. In this talk, I'll sketch our absorbing technique and discuss related problems and obstacles.
(Received September 10, 2023)

## 1192-05-30633

Christino Tamon*, Clarkson University, Weichen Xie, Clarkson University. Is quantum search possible on infinite graphs?. Farhi and Gutmann (1998) proved that a continuous-time analogue of Grover search (also called spatial search) is optimal on the complete graphs. Consider a scenario where we attach an infinite path (or tail) to our finite graph. We view the tail as an external quantum system which has a limited but nontrivial interaction with our finite quantum system. In this talk, we ask if spatial search remains optimal or it is irretrivably lost.
(Received September 10, 2023)

## 1192-05-30638

Deepak Bal, Montclair State University, Patrick Bennett, Western Michigan University, Emily Heath*, Iowa State University, Shira Zerbib, Iowa State University. Generalized Ramsey numbers from hypergraph matchings.
Given graphs $H$ and $G$ and a positive integer $q$, an $(H, q)$-coloring of $G$ is an edge-coloring in which each copy of $H$ receives at least $q$ colors. Erdős and Shelah raised the question of determining the minimum number of colors, $f(G, H, q)$, which are required for an $(H, q)$-coloring of $G$. Determining $f\left(K_{n}, K_{p}, 2\right)$ for all $n, p$ is equivalent to determining the classical multicolor Ramsey numbers. Recently, Mubayi and Joos introduced the use of a new method for proving upper bounds on these generalized Ramsey numbers; by finding a "conflict-free" matching in an appropriate auxiliary hypergraph, they determined the values of $f\left(K_{n, n}, C_{4}, 3\right)$ and $f\left(K_{n}, K_{4}, 5\right)$. In this talk, we will show how to generalize their approach to give bounds on the generalized Ramsey numbers for several families of graphs.
(Received September 10, 2023)

1192-05-30640
Christino Tamon*, Clarkson University. Is chirality helpful in quantum walk on graphs?. Preliminary report.
Given a graph $G$ with adjacency matrix $A$, a continuous-time quantum walk on $G$ has state transfer between vertices $a$ and $b$ at time $t$ if the $(a, b)$-entry of the unitary matrix $\exp (-i t A)$ has near unit magnitude. We explore the following chirality question of state transfer in quantum walks: is it useful at all to allow complex entries in our adjacency matrix? After motivating this question, we describe old and new observations on monogamy violations (at a distance) of state transfer in quantum walks.
(Received September 10, 2023)

1192-05-30658
Alexander N Wilson*, Oberlin College. Super Multiset RSK and a Mixed Multiset Partition Algebra.
Through dualities on representations on tensor powers and symmetric powers respectively, the partition algebra and multiset partition algebra have been used to study long-standing questions in the representation theory of the symmetric group. These algebras enjoy distinguished bases whose product can be described on graph-theoretic diagrams. We extend this story to exterior powers, leading to the introduction of the mixed multiset partition algebra and a generalization of RSK that links the algebra's graph-theoretic basis to a tableau basis for its irreducible representations.
(Received September 10, 2023)

Jonathan Du*, DSM Academy, Jiahao Li*, DSM Academy, Chenkai Zhang Shen*, DSM Academy. A Recursive Construction of Long Cycles in Hypercube Levels.
The Middle Levels problem, a recently resolved conjecture, posits that the subgraph of a $(2 n+1)$-dimensional hypercube formed by the bitstrings with Hamming weight exactly $n$ or $n+1$ (also referred to as levels $n$ and $n+1$ ) is a Hamiltonian graph. In our research, we tackle an extended problem focused on the existence of saturating (optimal) cycles within the subgraph of a general $n$-dimensional hypercube induced by certain sets $S$ of noncontiguous levels. Building upon $P$. Gregor and T. Mütze's 2018 paper, which introduced a method for trimming and gluing Gray codes to demonstrate the existence of saturating cycles within contiguous level ranges in the hypercube, we adapt this methodology to create a novel recursive approach for constructing long cycles within noncontiguous level ranges. Our approach involves breaking down the binaryreflected Gray code (BRGC) on $n$ bits into smaller subunits isomorphic to the BRGC on $m<n$ bits. Each subunit is then independently modified in an inductive manner and reattached to form the desired cycle. Employing this method, we not only establish the existence of a saturating cycle on any set of $n$ levels in the $n$-hypercube, but also show the existence of asymptotically saturating cycles within subgraphs induced by the removal of any contiguous range of levels from the hypercube, among other results. We anticipate that this method will also be applicable to a wide range of further questions concerning cycles within hypercube levels. This work was supervised by a university professor at the DSM research camp. (Received September 12, 2023)

## 1192-05-30761

David Eppstein*, University of California, Irvine. Games on game graphs.
Many important parameters in graph structure theory, including cop-number, treewidth, nowhere density, and flip-width, can be characterized in terms of certain pursuit-evasion games on a given graph. We review these concepts with particular focus on the graphs of game states, and in particular the state graphs of the Tower of Hanoi puzzle.
(Received September 11, 2023)

## 1192-05-30798

Hanmeng (Harmony) Zhan*, Worcester Polytechnic Institute. Strongly cospectral vertices and their phantom mates
Preliminary report.
Strong cospectrality is an equivalence relation motivated by continous quantum walks. In this talk, I will discuss what happens if the size of a class of strongly cospectral vertices is not a power of 2 . This is joint work in progress with Ada Chan and Krystal Guo.
(Received September 11, 2023)

## 1192-05-30899

William Jonathan Keith*, Michigan Technological University. Properties of the reciprocals of false theta functions.
The false theta functions are defined as the $q$-series

$$
\psi(a, b):=\sum_{n=0}^{\infty} a^{\binom{n+1}{2}} b^{\binom{n}{2}}-\sum_{n=-\infty}^{-1} a^{\binom{n+1}{2}} b^{\binom{n}{2}} .
$$

In this talk we consider the reciprocals $1 / \psi\left( \pm q^{a}, \pm q^{b}\right)$ with independent $\pm$ signs, and discuss such properties as parity, congruences, dissections, asymptotics, and relations to the truncated pentagonal number theorem as considered by Andrews and Merca.
(Received September 11, 2023)

1192-05-30917
Michael Coopman*, University of Florida. A new sampler for permutations distributed according to major index. Preliminary report.
For \$0
(Received September 11, 2023)

## 1192-05-31075

Esther Dawn Conrad, NASA Langley Research Center, Aaron Michael Dutle*, NASA Langley Research Center. Formal Verification, Distributed Computing, and Path Planning Algorithms.
The safety- and mission-critical nature of much of the work done at NASA requires algorithms and software to be exceedingly reliable. Formal methods techniques are one way of ensuring this high level of robustness. This talk will discuss the development and formal verification of autonomous aircraft path planning algorithms related to the Bellman-Ford shortest path algorithm, including consideration of distributed computation of the algorithm.
(Received September 11, 2023)

## 1192-05-31077

Yaacov Kopeliovich*, University Of Connecticut. The Solution Of TATA Mumford's problem on Theta Functions. One of the problems in Tata Lectures on Theta Book I of D.Mumford asks whether analogues of Jacobi formulas are available for derivatives of theta functions with rational characteristics. Using the residue theorem I will show that generically such derivatives can be expressed as rational functions of two cubic polynomials. This has an immediate application to
corresponding problem in $q$ series as the substitution $q=e^{2 \pi \imath \tau}$ shows.
(Received September 11, 2023)

Rosa C. Orellana*, Dartmouth College, Nancy Wallace, York University, Mike Zabrocki, York University. Diagram Algebras. Preliminary report.
Diagram algebras are a widely study class of algebraic structures where the computations are done using diagrams. These algebras arise as centralizer algebras of classical groups and have applications to knot theory, integrable models and statistical mechanics, and quantum computing. An important example of a diagram algebra is the Temperley-Lieb algebras which can be defined combinatorially using non-crossing matchings and they occur in the study of two-dimensional models. In the mid 1990s Martin introduced the partition algebra as a generalization of the Temperley-Lieb algebra for higher dimensional models. The planar partition algebra is the subalgebra of the the partition algebra consisting of non-crossing blocks. It can be shown that this planar partition algebra is isomorphic to the Temperely-Lieb algebra. In this talk we will introduce constructions of planar subalgebras of the Temperley-Lieb algebra (or planar partition algebra) and discuss their representation theory. This is joint work with N. Wallace and M. Zabrocki.
(Received September 11, 2023)

## 1192-05-31134

Tien Chih*, Oxford College of Emory University. $\times$-homotopy covers of graphs. Preliminary report.
We develop conditions for a graph cover to be a $\times$-homotopy cover, satisfying a $\times$-homotopy lifting property analogous to the homotopy lifting property of covers of topological spaces. We then show that this condition is equivalent to $\diamond$ equivalence by Tardiff-Wroncha amd 2-homotopy defined by Matsushita. These works define a universal cover within their respective formulations of this setting, and we provide our own formulation and proof in the context of $\times$-homotopy using groupoids. The universal cover leads to a theory of graph covering and quotients, and we show these are exactly the quotients that lift $\times-$ homotopy. We then give some examples and enumerate the $\times$-homotopy lifting covers.
(Received September 11, 2023)

## 1192-05-31143

Mallory Price, Grand Valley State, Nicholas Alexander Simmons*, Grand Valley State, Andrew Kennedy Wilson, Grand Valley State, Sarah Zaske, Grand Valley State. Distinguishing Index of Mycielskian Graphs. Preliminary report. The distinguishing number and distinguishing index tell us in some sense how symmetric a graph is. We define a distinguishing vertex coloring to be a coloring of the vertices of a graph $G$ such that no non-trivial automorphism preserves the vertex coloring. The distinguishing number, $\operatorname{Dist}(G)$, is the smallest number of colors possible for which there is a distinguishing coloring. Similarly, a distinguishing edge coloring is a coloring of the edges of $G$ such that no non-trivial automorphism preserves the edge coloring, and the distinguishing index, $\operatorname{Dist}^{\prime}(G)$, is the smallest number of colors needed for a distinguishing edge coloring. The Mycielskian of a graph $G$, denoted $\mu(G)$, is an extension of $G$ introduced by Mycielski in 1955. In 2022 Boutin, Cockburn, Keough, Loeb, Perry, and Rombach showed that for graphs on at least 3 vertices $\operatorname{Dist}(\mu(G)) \leq \operatorname{Dist}(G)$. We discuss our progress on extending their results to the distinguishing index.
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1192-05-31144
Ryan C. Bunge, Illinois State University, Brian D. Darrow, Jr., Teachers College, Columbia University, Saad I. El-Zanati, Illinois State University, Kimberly P. Hadaway*, Iowa State University, Megan K. Pryor, North Carolina State University, Alexander J. Romer, Millikin University, Alexandra Squires, Lee University, Anna C. Stover, Grand Valley State University. Decomposing Complete 3-Uniform Hypergraphs into Tight 9-Cycle Subgraphs.
On a vertex set $V$ of $v$ vertices, a complete 3 -uniform hypergraph has order $v$ and its edge set consists of all the 3 -element subsets of $V$. If $H$ is a 3 -uniform hypergraph, then a tight 9 -cycle is defined as a 3 -uniform subgraph of $H$ having, for example, vertex set $\left\{v_{1}, v_{2}, v_{3}, v_{4}, v_{5}, v_{6}, v_{7}, v_{8}, v_{9}\right\}$ and corresponding edge set
$\left\{\left\{v_{1}, v_{2}, v_{3}\right\},\left\{v_{2}, v_{3}, v_{4}\right\},\left\{v_{3}, v_{4}, v_{5}\right\},\left\{v_{4}, v_{5}, v_{6}\right\},\left\{v_{5}, v_{6}, v_{7}\right\},\left\{v_{6}, v_{7}, v_{8}\right\},\left\{v_{7}, v_{8}, v_{9}\right\},\left\{v_{8}, v_{9}, v_{1}\right\},\left\{v_{9}, v_{1}, v_{2}\right\}\right\}$. A decomposition of a hypergraph $H$ is a collection of subgraphs whose edge sets partition the edge set of $H$. In this presentation, necessary and sufficient conditions for the existence of tight 9 -cycle decompositions of complete 3 -uniform hypergraphs are given. A brief discussion of how this research engaged undergraduates as well as pre-service and in-service mathematics teachers is also given.
(Received September 11, 2023)

1192-05-31162
Jane Breen*, Ontario Tech University. Kemeny's constant and random walks on threshold graphs. Preliminary report. Kemeny's constant is an interesting and tricky measure of the connectivity of a graph, measuring the expected length of a random trip between vertices in the graph. Techniques to calculate Kemeny's constant come from a range of approaches, including matrix analysis and generalized inverses, spectral graph theory, and combinatorial-style formulas. In this talk we exhibit a range of these techniques and how they were put to work to consider the range of values of Kemeny's constant for threshold graphs.
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## 1192-05-31190

Florian Frick, Carnegie Mellon University, Jacob Lehmann Duke, Williams College, Arianna Meenakshi McNamara, Purdue University, Hannah Park-Kaufmann, Bard College, Steven Raanes, Vassar College, Steven Simon, Bard College, Zoe Wellner*, Carnegie Mellon University. Topological Methods in Zero-Sum Ramsey Theory.
Erdős, Ginzburg, and Ziv showed that any sequence of $2 n-1$ numbers in $\mathbb{Z} / n$ has a subsequence of length $n$ that sums to zero. This result has inspired numerous generalizations and variants, which became collectively known as zero-sum Ramsey theory. Whereas problems of this type have mostly been approached by algebraic and combinatorial techniques, for a fixed prime $p$, we give a topological criterion that guarantees that for any labelling of the vertices of a $p$-uniform hypergraph by integers, there is a hyperedge whose labels sum to zero modulo $p$. Our topological criterion is from a simplicial complex
associated to a $p$-uniform hypergraph, the $\mathbb{Z} / p$ box complex. This same construction appears prominently for topological lower bounds for chromatic numbers of hypergraphs. While we show that there is no direct connection between the chromatic number of a hypergraph and the property of having a zero-sum hyperedge, the appearance of box complexes nevertheless lets us transfer results for chromatic numbers of hypergraphs to results in zero-sum Ramsey theory. This unifies known results and provides a wealth of new zero-sum Ramsey theorems.
(Received September 11, 2023)

## 1192-05-31213

David Keating*, University of Wisconsin, Madison. Coupled Tilings of the Aztec Diamond.
We will study $k$-tilings ( $k$-tuples of domino tilings) of the Aztec diamond. We assign a weight to each $k$-tiling, depending on the number of "interactions" between the dominos of the different tilings. We will compute the generating polynomials of the $k$ tilings by showing that they can be seen as the partition function of an integrable colored vertex model. These partition functions are related to the LLT polynomials of Lascoux, Leclerc, and Thibon.
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## 1192-05-31217

Natalie Robin Dodson*, Middlebury College, Lani Southern, Willamette University. No Three in a $\theta$ : Variations on the No-Three-in-a-Line Problem.
We pose a natural generalization to the well-studied and difficult no-three-in-a-line problem: How many points can be chosen on an $n \times n$ grid such that no three of them form an angle of $\theta$ ? We classify which angles yield nontrivial problems, noting that some angles appear in surprising configurations on the grid. We investigate the case $\theta=135 \backslash$ degree, using geometric properties of the grid to prove upper and lower bounds and expand our techniques to prove a general upper bound for all angles.
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## 1192-05-31222

Jacob Fox*, Stanford University. Ramsey Cayley graphs, random graph, and information theory
A graph is Ramsey if its largest clique or independent set is of size logarithmic in the number of vertices. While almost all graphs are Ramsey, there is still no known explicit construction of Ramsey graphs. Alon conjectured that every finite group has a Ramsey Cayley graph. We discuss the proof that for almost all $n$, all abelian groups of order $n$ satisfy Alon's conjecture. We also verify a conjecture of Alon and Orlitsky motivated by information theory that there are self-complementary Ramsey Cayley graphs. We further prove general results for clique numbers of random graph models. Along the way, we study some fundamental problems in additive combinatorics, and discover that group structure is superfluous for these problems. Based on joint work with David Conlon, Huy Pham, and Liana Yepremyan.
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## 1192-05-31237

Sinan Aksoy, Pacific Northwest National Laboratory, Helen Jenne, Pacific Northwest National Laboratory, Bill Kay, Pacific Northwest National Laboratory, Hyungro Lee, Pacific Northwest National Laboratory, Jenna Pope*, Pacific Northwest National Laboratory, Madelyn Shapiro, Pacific Northwest National Laboratory, Stephen J Young, Pacific Northwest National Laboratory. Enhanced Molecular Graph Embeddings with Inner Product Laplacians. Preliminary report. At their foundation, chemical structures are simply collections of interacting atoms. Graph theory provides a natural tool for capturing atomic interactions in a concrete mathematical fashion. Recently, graph neural networks (GNNs) have become a popular method for modeling molecular graphs for tasks such as property prediction. In typical cases, the GNN treats node attributes (e.g. atomic number) and edge weights (e.g. the Euclidean distance between atoms) separately. For instance, an embedding is first learned from the node attributes and is then updated based on the edge weights. Graph convolutional networks (GCNs), in particular, make use of the normalized Laplacian during the embedding update phase. In this work, we employ an inner product Laplacian in place of the normalized Laplacian, preserving both edge and vertex information during the update phase.
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## 1192-05-31259

Helene Barcelo*, MSRI / Simons Laufer Mathematical Sciences Institute (SLMath). Discrete cubical homotopy groups and real Eilenberg-MacLane spaces..
Discrete cubical homotopy groups and real Eilenberg-MacLane spaces. In this talk we wish to demonstrate how a theory, developed entirely for the purpose of solving problems stemming from search-and-rescue missions, gave rise to one that in turn has applications to fundamental mathematics. Discrete cubical homotopy theory is a discrete analogue of (singular) simplicial homotopy theory, associating a bigraded sequence of groups to a simplicial complex, capturing some of its combinatorial structure. The motivation for this construction came initially from the desire to find invariants for dynamic processes that were encoded using (combinatorial) simplicial complexes. The invariants should be topological in nature, but should also be sensitive to the combinatorics encoded in the complex, in particular to the level of connectivity among simplices. Over the last few years similar notions have arisen from several areas of mathematics (e.g., geometric group theory, coarse geometry, computer science) signaling both the pressing need for such a theory as well as its universal nature. As an illustration, we will provide a real analogue of Brieskorn's result on complex Eilenberg-MacLane spaces associated with Coxeter groups.
(Received September 11, 2023)
science for security and resilience, which quantitatively characterizes the impacts of disruptions. In this presentation, we highlight the importance of algebraic graph theory concepts in the scientific analysis of security and resilience. Specifically, we argue that algebraic graph theory constructs - including the eigenvalues and eigenvectors of graph Laplacian matrices have fundamental relationships with important security and resilience metrics. It is shown that these relationships can be derived via dynamical- systems approaches, and are valuable for designing secure and resilient complex systems. (Received September 11, 2023)

## 1192-05-31328

Michael Robert Doboli*, Stanford University, Alessandra R.P. Maranca, Stanford University, Noah A. Rosenberg, Stanford University. Extremal Colijn-Plazzotta ranks of unlabeled multifurcating rooted trees.
The unlabeled binary rooted trees are enumerated by the Wedderburn-Etherington numbers, 1, 1, 1, 2, 3, 6, 11, 23, 46, 98.. (OEIS A001190). Colijn \& Plazzotta (2018, Systematic Biology 67:113-126) devised an elegant method of bijectively associating the unlabeled rooted binary trees with the positive integers. In this scheme, the number for a tree is obtained from the numbers for its two immediate subtrees. Maranca \& Rosenberg (Adv Appl Math, doi: 10.1016/j.aam.2023.102612, 2024) provided a natural generalization of the Colijn-Plazzotta (CP) bijection to unlabeled rooted multifurcating trees. For a fixed number of leaves $n$, which multifurcating trees with $n$ leaves attain the maximal and minimal CP rank? We answer this question on two spaces of multifurcating trees: strictly $k$-furcating trees, in which each internal node has exactly $k$ immediate descendants, and at-most- $k$-furcating trees, in which internal node has at least 2 and at most $k$ immediate descendants. In both cases, we find that the "maximally balanced" tree attains the minimal CP rank, whereas the "minimally balanced" tree attains the maximal CP rank. We present recursive formulas for the maximal and minimal CP rank on $n$ leaves in both the strictly and at-most- $k$-furcating cases, studying the asymptotic exponential growth of the maximum and minimum. The results generalize results of Rosenberg (2021, Discrete Applied Mathematics 291: 88-98), which considered the case of $k=2$. The study contributes to enhancing a method of characterizing biological trees in scenarios such as pathogen evolution in which the branching of lineages can occur quickly relative to the time scale on which mutations occur.
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1192-05-31349
George Androulakis, University of South Carolina, Alexander Wiedemann*, Randolph-Macon College. Quantum Dynamical Semigroups: Connections to Digraph Theory.
Consider an open quantum system which is weakly interacting with some external reservoir, e.g. some molecules interacting with a thermal electromagnetic field. Under certain limiting conditions, the Schrödinger time-evolution (how states change in time) is governed by a quantum dynamical semigroup, the action of which can be determined by its time-independent generator. In recent years, digraph induced generators have been introduced and studied, particularly in the context of quantum random walks. In this talk, we construct a general class to which these generators belong which allows for additional interaction coefficients while still preserving their main structural properties. Under certain physical assumptions, we discuss how the given the digraph's (Laplacian) eigenvalues can be used to explicitly compute all invariant states and eigenvalues of the semigroup, and when the generated quantum random walk can be restricted to recover a classical random walk. We also consider the converse construction of generator induced digraphs, and show that under certain assumptions the properties of this digraph can be exploited to gain knowledge of both the number and the structure of the invariant states of the corresponding semigroup.
(Received September 11, 2023)

1192-05-31350
Grigoriy Blekherman, Georgia Tech, Papri Dey*, Georgia Institute of Technology. Combinatorial Aspects of Polynomials with Lorentzian Signature, and its applications in Optimization.
Over the past decade, there has been a notable effort to fuse the techniques of algebraic geometry and combinatorics, crafting a comprehensive framework for addressing long-standing conjectures in theoretical computer science and matroid theory pertaining to unimodality and log-concavity. This endeavor includes an exploration of Lorentzian polynomials, also known as completely log-concave or strongly log-concave polynomials, which establishes a bridge between discrete and continuous convexity. We study a natural generalization to the remarkable class of Lorentzian polynomials, referred to as polynomials with Lorentzian signature (PLS) over convex cones. In this talk, I will discuss some key characteristics of this polynomial class, and using its connection with hereditary polynomials associated with simplicial complexes, I shall focus on its applications, particularly in the context of optimization. This talk is based on an ongoing joint work with Greg Blekherman. (Received September 11, 2023)

## 1192-05-31388

Antonio Acuaviva, Universidad Complutense de Madrid, Ada Chan*, York University, Summer Eldridge, University of Toronto, Chris Godsil, University of Waterloo, Matthew How, McMaster University, Christino Tamon, Clarkson University, Emily Wright, Queen's University, Xiaohong Zhang, University of Montreal. One-way perfect state transfer. A continuous-time quantum walk on a graph $X$ is given by the transition operator $e^{-i t H}$, for some Hermitian matrix associated with $X$. Most existig studies use the adjacency matrix or the Laplacian matrix of $X$ as the Hamiltonian $H$. In this case, perfect state transfer from vertex $a$ to vertex $b$ implies periodicity at both vertices. In this talk, we use complex Hamiltonians to generate walks that have perfect state transfer from $a$ to $b$ without periodicity at $a$.
(Received October 18, 2023)

1192-05-31393
Kanchana Madhumali Gamlath Esweda Gamladdalage*, Department of Mathematics, University of Mississippi, Bing Wei, University of Mississippi. The Total Bondage Number of Connected Graphs. Preliminary report. For a finite graph $G$ without isolated vertices, a vertex set $D$ of $G$ is said to be a total dominating set of $G$, if every vertex $v \in V(G)$ has a neighbor in $D$. Define $\gamma_{t}(G)$ to be the minimum cardinality among all total dominating sets of $G$. $B$ is a total bondage edge set if $B \subset E(G)$ such that $G-B$ does not have isolated vertices and $\gamma_{t}(G-B)>\gamma_{t}(G)$. The total bondage
number of $G$ is denoted by $b_{t}(G)$, and $b_{t}(G)=\min \{|B|: B$ is a total bondage edge set of $G\}$ if $G$ has at least one total bondage edge set. Otherwise, we put $b_{t}(G)=\infty$. In this talk, we provide some upper bounds on $b_{t}(G)$ in terms of the degree sum of the vertices of $G$, especially, $b_{t}(G) \leq \Delta-1$ for trees, where $\Delta \geq 3$ denotes the maximum degree of $G$, and we also show some upper bounds on total bondage number for planar graphs. These results improve some previously published upper bounds.
(Received September 11, 2023)

## 1192-05-31407

Mark Kempton*, Brigham Young University. Zero Forcing and Topological Graph Properties.
The zero forcing parameter was introduced to bound the maximum nullity of all symmetric matrices corresponding to a graph. I will present results on how topological properties of a graph - such as planarity, outerplanarity, and connectivity - can give information on how big the zero forcing number of the graph can be.
(Received September 11, 2023)

## 1192-05-31430

Austin Mohr*, Nebraska Wesleyan University. An Extended Deletion-Contraction Recurrence for the Chromatic Polynomial. Preliminary report.
A proper coloring of a graph is an assignment of colors to the vertices in such a way that adjacent vertices receive different colors, and the chromatic polynomial counts the number of such colorings. The well-known deletion-contraction recurrence allows one to determine the chromatic polynomial of a graph by instead determining it for two related, simpler graphs. The combinatorial explosion resulting from applying this recurrence many times can be intractable for large graphs. The author introduces an extension to the deletion-contraction recurrence that allows for the deletion and contraction of multiple edges simultaneously, thereby significantly reducing the depth of recursion. The author also applies the recurrence to develop summation formulas for the chromatic polynomial of certain graph families. (Received September 25, 2023)

1192-05-31433
Zhanar Berikkyzy, Fairfeld University, Joe Miller, Iowa State University, Elizabeth Sprangel, Iowa State University, Shanise Walker*, Clark Atlanta University, Nathan Warnberg, University of Wisconsin-La Crosse. Anti-van der Waerden Numbers of Graph Products with Trees.
Given a graph $G$ an exact $r$-coloring of $G$ is a surjective function $c: V(G) \rightarrow[1, \ldots, r]$. An arithmetic progression in $G$ of length $j$ with common difference $d$ is a set of vertices $\left\{v_{1}, \ldots, v_{j}\right\}$ such that $\backslash \operatorname{dist}\left(v_{i}, v_{i+1}\right)=d$ for $1 \leq i<j$. An arithmetic progression is rainbow if all of the vertices are colored distinctly. The fewest number of colors that guarantees a rainbow arithmetic progression of length 3 is called the anti-van der Waerden number of $G$ and is denoted aw $(G, 3)$. It is known that $3 \leq \operatorname{aw}(G \square H, 3) \leq 4$. Here we determine exact values aw $\left(T \square T^{\prime}, 3\right)$ for some trees $T$ and $T^{\prime}$, determine aw $(G \square T, 3)$ for some trees $T$ an determine $\operatorname{aw}(G \square H, 3)$ for some graphs $G$ and $H$.
(Received September 11, 2023)

## 1192-05-31442

Whitney Lunceford*, Brigham Young University. Improving Reservoir Computer Response Diversity via Thinned Networks. Reservoir computers rely on an internal network to predict the future state(s) of dynamical processes. To understand how a reservoir's accuracy depends on this network we study how varying the network's topology affects the reservoir's ability to predict the chaotic dynamics on the Lorenz and other attractors. We find that thinned networks, in which a large majority of the network's edges have been removed, statistically outperform standard networks typically used in reservoir computing. We show that thinned networks can create more diverse responses from the nodes that make up the reservoir's internal network allowing for a more accurate reconstruction of chaotic attractors. This leads us to propose an answer to a long-standing question in reservoir computing regarding how to choose a reservoir's spectral radius where the optimal spectral radius depends on how thinned the reservoir's internal network is. We also show how the oscillations in a reservoir's response to input is determined by the reservoir's spectral radius for isolated nodes in thinned networks.
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1192-05-31446
Joshua E. Arroyo, University of Florida, Zachary Hamaker, University of Florida, Graham Hawkes, Purdue Global, Jianping Pan*, NCSU. Stable Grothendieck Polynomials and shifted tableaux. Preliminary report.
Grothendieck polynomials are the polynomial representatives of the Schubert classes in the K-theory of flag varieties. We will focus on the stable version of these polynomials in types B and C. When the indexing Coxeter elements are isotropic (resp. Lagrangian) Grassmannian, Ikeda and Naruse show the stable Grothendieck polynomials are the K-theoretic Schur P (resp. Q) functions, which are sum over set-valued shifted tableaux. Kirillov and Naruse gave a combinatorial model for these polynomials in general involving the Yang-Baxter relations. We will talk about expansion formulas of stable Grothendieck polynomials of type B and C in some special cases, and a conjecture for the general case.
(Received September 11, 2023)

## 1192-05-31493

Victor Falgas-Ravry, University of Umea, Amites Sarkar*, Western Washington University. Bootstrap percolation in random geometric graphs.
Random geometric graphs were invented by E.N. Gilbert in 1961 to model communications networks. Bootstrap percolation was invented by Chalupa, Leath and Reich in 1979 to model magnetism. Both models have since been used to study many other things. This is because many spatial networks, such as the brain, can be modeled as random geometric graphs, and many natural processes on networks, such as the activation of neurons in the brain, or the spread of beliefs on social media, can be
modeled by bootstrap percolation. So it was natural when, in 2014 , Bradonjic and Saniee put the two models together, and studied bootstrap percolation in random geometric graphs. In my talk, I'll describe an (almost) complete solution of the Bradonjic-Saniee model. The proofs use variational methods, tiling arguments, and discrete isoperimetric inequalities. (Received September 11, 2023)

1192-05-31495
Robert W. Bell*, Michigan State University. On the weak cop number of certain planar locally finite graphs. Preliminary report.
The game of weak cops and robbers is a two player game on a combinatorial graph in which players take turns moving pawns to adjacent vertices of the graph. The cop player wins by finding a strategy by which cop pawns can prevent a robber pawn from visiting any of the vertices of the graph infinitely often-possibly, but not necessarily, by capturing the robber. If the graph is locally finite, winning is equivalent to finding a strategy that forces the robber to leave every compact set. We show how a strategy of guarding isometric rays leads to a bound on the weak cop number of any locally finite graph that is embedded in the plane without vertex accumulation points. This extends a classical bound on the cop number of planar graphs in the ordinary game of cops and robbers on finite graphs.
(Received September 25, 2023)

## 1192-05-31502

Sinan Aksoy, Pacific Northwest National Laboratory, Alyson Bittner, Pacific Northwest National Laboratory, Bill Kay, Pacific Northwest National Laboratory, Stephen J Young*, Pacific Northwest National Laboratory. Reimagining Spectral Graph Theory. Preliminary report.
While spectral methods provide far-ranging insights on graph structure, there remain significant challenges in their application to real data. Most notably, spectral methods do not incorporate information that may be available beyond adjacency. A common approach to incorporating such additional information is to encode this information in an ad-hoc manner into weights associated with the edges. Not only does this have limited expressivity but it is also restricted by graph structure: if two vertices are not adjacent, then edge weights cannot capture any closeness implied by metadata. We address this issue by introducing the inner product Hodge Laplacian for an arbitrary simplicial complex. Within this framework we prove generalizations of foundational results in spectral graph theory, such as the Cheeger inequality and the expander mixing lemma, and show our framework recovers the usual combinatorial and normalized Laplacians as special cases. Our framework allows for the principled synthesis of combinatorial approaches in network science with more metadata driven approaches by using latent space encodings of the metadata to define an inner product both the vertices and the edges. We demonstrate this synthesized approach on cyber data by using the latent space embedding generated by an autoencoder on the metadata to define the inner product. Joint work with Sinan Aksoy, Alyson Bittner, and Bill Kay
(Received September 11, 2023)

1192-05-31504
Nicholas Stanley Geis, Graduate Student, Kayta Gheorghian*, Boston College, Hannah Sheats, Graduate Student, Caroline Terry, The Ohio State University. The VC-dimension of Semi-algebraic Sets in $\mathbb{R}^{2}$ and $\mathbb{R}^{3}$. Preliminary report. Given a set $X$ and a family of subsets $\mathcal{F}$ of $X$ we define shattering to assess $\mathcal{F}$ 's capacity to represent subsets of $X$. Specifically, for any subset $A \subseteq X$, we say $\mathcal{F}$ shatters $A$ if for each $A^{\prime} \subseteq A$ there exists $F \in \mathcal{F}$ such that $F \cap A=A^{\prime}$. The Vapnik-Chervonenkis (VC) dimension of $X$ with respect to $\mathcal{F}$ is the size of the largest $A \subseteq X$ that is shattered by $\mathcal{F}$. Originally emerging in the context of machine learning, the VC-dimension serves as a metric for evaluation the adaptability of a hypothesis set, $\mathcal{F}$, to different data sets. It has extensive applications in statistical learning theory, particularly in areas dealing with neural networks and decision trees. While much research has been conducted on the VC-dimension from a settheoretic perspective, this paper explores a recent trend which involves considering $X$ as a group and constructing $\mathcal{F}$ using the group operation. We focus on the Heisenberg group, $H(\mathbb{Z})$, defined as the set of matrices of the form

$$
\text { \{\pmatrix 1\&a\&b 0\&1\&c 0\&0\&1 lendpmatrix: a,b,c } \in \backslash m a t h b b\{Z\} ~\}
$$

with matrix multiplication as the group operation. We define a ball centered at the origin as
B_ $_{4}\left\{r_{-}\{1\}, r_{-}\{2\}, r_{-}\{3\}\right\}=\left\{|\operatorname{pmatrix} 1 \& a \& b 0 \& 1 \& c 0 \& 0 \& 1| \operatorname{endpmatrix}:|a| \leq r_{-}\{1\},|b| \leq r \_\{2\},|c| \leq r_{-}\{3\} ; r_{-}\{1\}, r_{-}\{2\}, r_{-}\{3\} \in \in\right.$
and construct the family $\mathcal{F}$ as the collection of all left translates of all such balls. Our primary result reveals that the VCdimension of $H(\mathbb{Z})$ with respect to $\mathcal{F}$ is either 6 or 7 . We establish a direct correlation between balls in the Heisenberg group and axis-parallel parallelograms in $\mathbb{R}^{2}$. Previously, the only known values for the VC-dimension of quadrilaterals in $\mathbb{R}^{2}$ were that of the axis-parallel squares (with VC-dimension 3), axis-parallel rectangles (with VC-dimension 4), and arbitrary quadrilaterals (with VC-dimension 9). We found quadrilateral set systems that have VC-dimension $5,6,7$, and 8 and find that arbitrary trapezoids have the same VC-dimension of arbitrary quadrilaterals, meaning a more restrictive hypothesis class describes the same data.
(Received September 11, 2023)

## 1192-05-31524

Zachary Halberstam*, Harvard University, Carl Schildkraut, Stanford University. Words with Repeated Letters in Grids. Preliminary report.
It is natural to ask about the maximum possible number of contiguous occurrences (allowing diagonals) of a given word in a large $d$-dimensional grid. Alon and Kravitz recently resolved this problem for words with no repeated letters. We establish analogous results for several classes of words with repeated letters; some of our "extremal" configurations look like those in the work of Alon and Kravitz, and others do not. We also briefly discuss some natural variations, such as maximizing the total number of appearances of two different words. Joint work with Carl Schildkraut.
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## 1192-05-31533

Olya Mandelshtam*, University of Waterloo. Formulas for Macdonald polynomials via interacting particle models. We describe some recently discovered connections between one-dimensional interacting particle models and Macdonald polynomials and show the combinatorial objects that make this connection explicit. The first such model is the multispecies asymmetric simple exclusion process (ASEP) on a ring, linked to the symmetric Macdonald polynomial $P_{\lambda}(X ; q, t)$ through its partition function, with multiline queues as the corresponding combinatorial object. The second particle model is the multispecies totally asymmetric zero range process (TAZRP) on a ring, which was recently found to have an analogous connection to the modified Macdonald polynomial $\widetilde{\mathrm{H}}_{\lambda}(X ; q, t)$. The combinatorial object interpolating between probabilities of the TAZRP and the modified Macdonald polynomials turn out to be tableaux with a queue inversion statistic. We explain the plethystic relationship between multiline queues and queue inversion tableaux, and along the way, derive a new formula for $P_{\lambda}(X ; q, t)$ using the queue inversion statistic. This plethystic correspondence is closely related to fusion in the setting of integrable systems.
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## 1192-05-31543

Giuseppe Cotardo*, Virginia Tech, Anina Gruica, Eindhoven University of Technology, Alberto Ravagnani, Eindhoven University of Technology. The Diagonals of Ferrers Diagrams.
In 1986, Garsia and Remmel defined the q-rook polynomials for Ferrers diagrams. They showed that they share many properties with the rook numbers introduced by Riordan and Kaplansky. In 1998, Haglund enstablished connections between q -rook polynomials and matrices over finite fields. In this talk, we reconstruct the theory of q-rook polynomials for Ferrers diagrams by focusing on the properties of their diagonals. We show that the diagonals define an equivalent relation on the set of Ferrers diagrams and we establish connections with the problem of counting matrices of given rank supported on a Ferrers diagram.
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1192-05-31554
Timothy Blanton*, University of California, Davis, Isabelle Hong*, University of California, Los Angeles, Zhan Zhan*, Michigan State University. Parking Functions on Hypergraphs. Preliminary report.
Classical chip-firing is a game played on a graph $G$, where chips are placed on the vertices of $G$ and distributed according to simple rules. The dynamics of chip-firing has found applications in many areas of mathematics and physics, and for instance gives rise to $G$-parking functions and the critical group of $G$. We study an analogue of superstable configurations on hypergraphs, where edges can have multiple vertices. This is motivated by a notion of chip-firing where chips have a choice of where to go when fired. Such configurations can also be seen as the standard monomials of a monomial ideal defined by the cuts in a hypergraph $H$. From $H$, one can consider various directed graphs by choosing orderings of the vertices and show that all superstable configurations of $H$ can be obtained via chip-firing on some such digraph. We use these to define parking functions on $H$ and relate them to vector parking functions in the case of complete hypergraphs. We prove that the set of maximal parking functions of $H$ are in bijection with the set of acyclic orientations of $H$ with unique sink, implying that the degree sequence of $H$-parking functions forms a pure $O$-sequence. Along the way we study burning algorithms and notions of spanning trees in the context of hypergraphs.
(Received September 11, 2023)

1192-05-31567
Dorian Smith*, University of Minnesota Twin Cities. Sandpile Group of Cone over Trees.
Abstract: The sandpile group $K(G)$ of a graph $G$ is a finite abelian group, isomorphic to the cokernel of the reduced graph Laplacian of $G$. We study $K(G)$ when $G=\operatorname{Cone}(T)$. The graph Cone $(T)$ is obtained from a tree $T$ on $n$ vertices by attaching a new cone vertex attached to all other vertices. For two such families of graphs, we will describe $K(G)$ exactly: the fan graphs $\operatorname{Cone}\left(P_{n}\right)$ where $P_{n}$ is a path, and the thagomizer graph $\operatorname{Cone}\left(S_{n}\right)$ where $S_{n}$ is the star-shaped tree. The motivation is that these two families turn out to be extreme cases among $C o n e(T)$ for all trees $T$ on $n$ vertices. (Received September 11, 2023)

## 1192-05-31570

Hermie Monterde*, University of Manitoba. State transfer on joins
A quantum walk on a graph $X$ is determined by the complex symmetric unitary matrix $U(t)=\exp (i t H)$, where $i^{2}=-1$, $t \in \mathbb{R}$ and $H$ is either the adjacency or Laplacian matrix of $X$. We say that vertex $u$ is periodic in $X$ if $\left|U(\tau)_{u, u}\right|=1$ for some time $\tau>0$, and we say that perfect state transfer (PST) occurs between vertices $u$ and $v$ in $X$ if $\left|U(\tau)_{u, v}\right|=1$ for some time $\tau>0$. In this talk, we investigate the existence of periodicity and PST in join graphs. Here, we define the join of $X$ and $Y$ as the graph $X \vee Y$ obtained from $X$ and $Y$ by adding all possible edges between $X$ and $Y$.
(Received September 11, 2023)

1192-05-31579
Kaioke Begay, Central Washington University, Kieran Cook, Central Washington University, Athaliah Mackewicz*, Central Washington University, Aaron Montgomery, Central Washington University. Chip-firing Stability within the Unknown Stability Interval.
The chip-firing game is a well-studied dynamic system with simple rules leading to complex behavior. It is particularly important in studying networks of self-governed objects with no influence from outside agents: neurons, avalanches, circulation of currency, and other entities able to be represented by a graph having vertices weighted by values. In a network given $n$ chips with $e$ number of edges and $v$ number of vertices, stability is guaranteed if $n<e$, and unstable behavior is guaranteed when $n>2 e-v$, leaving an interval with graphs' stability ungoverned by these theorems. In our research, we
examined stability for particular graph configurations within the unknown stability interval to find a relationship between stability and number of chips. A relationship is found for cycles and complete graphs, and our exploration of three-regular graphs has led to generalized theorems for graphs in the unknown stability interval. This work was funded by the CC-REU NSF REU grant (DMS-2050692),
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1192-05-31600
Mark Kempton, Brigham Young University, Kellon G Sandall*, Brigham Young University, Trevor Wai, Brigham Young University. Pretty Good State Transfer in Extended Double Star Graphs.
In quantum computing, it is possible to represent networks of qubits as graphs. Our research deals with a family of graphs which we call extended double stars. These are graphs which consist of two stars connected by a path of arbitrary length. We explore how information can move throughout these graphs. We are interested in particular with what is called Pretty Good State Transfer or PGST. PGST is what occurs when we are able to transmit a quantum state from one node to another arbitrarily close. We present some results which we have gathered over the past few months
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## 1192-05-31646

Adam Clay*, Purdue University, Tibor Jordan, Etovos Lorand University, John Palmer, Pepperdine University. Minimally rigid tensegrities on the line.
We consider rigid tensegrity frameworks on the line, and tensegrity graphs which can be realized as rigid or globally rigid tensegrity frameworks on the line. We prove tight upper bounds on the number of members of such a framework or graph, assuming that they are minimal with respect to the given property. We also characterize the extremal frameworks and graphs. (Received September 11, 2023)

1192-05-31647
Joseph Alameda, United States Naval Academy, Jurgen Kritschgau*, Portland State University. Algorithms for leaky forcing and leaky power-domination.
Zero forcing and power-domination are graph processes related to the control and observability of networks. Leaky forcing, and leaky power-domination by extension, are generalizations of zero forcing and power-domination that model adversarial interruptions in the dynamics of the networks underlying these graph processes. With this in mind, algorithms that can produce leaky forcing and leaky power-dominating sets are a key step in converting the theory of these fields into practice. This talk will focus on using mixed integer programs for finding leaky forcing and leaky power-dominating sets. (Received September 11, 2023)

1192-05-31658
Oliver Andres Alvarado Rodriguez, New Jersey Institute of Technology, Ilya Amburg, Pacific Northwest National Laboratory, Jessalyn Bolkema, California State University, Dominguez Hills, Philip Chodrow, Middlebury College, Thomas Grubb, UC San Diego, Daniel Kaiser, Indiana University, Bill Kay, Pacific Northwest National Laboratory, Jurgen Kritschgau*, Portland State University, Fangfei Lan, University of Utah, Sepideh Maleki, University of Texas. Community Detection in Hypergraphs via Mutual Information Maximization.
The hypergraph community detection problem seeks to identify groups of related nodes in hypergraph data. We propose an information-theoretic hypergraph community detection algorithm which compresses the observed data in terms of community labels and community-edge intersections. This algorithm can also be viewed as maximum-likelihood inference in a degreecorrected microcanonical stochastic blockmodel. We perform the inference/compression step via simulated annealing. Unlike several recent algorithms based on canonical models, our microcanonical algorithm does not require inference of statistical parameters such as node degrees or pairwise group connection rates. Through synthetic experiments, we find that our algorithm succeeds down to recently-conjectured thresholds for sparse random hypergraphs. We also find competitive performance in cluster recovery tasks on several hypergraph data sets.
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1192-05-31730
Jing Yu*, Georgia Tech. Large-scale geometry of graphs of polynomial growth.
In 1995, Levin and Linial, London, and Rabinovich conjectured that every connected graph $G$ of polynomial growth admits an injective homomorphism to the $n$-dimensional grid graph for some $n$. Moreover, they conjected that if every ball of radius $r$ in $G$ contains at most $O\left(r^{\rho}\right)$ vertices, then one can take $n=O(\rho)$. Krauthgamer and Lee confirmed the first part of this conjecture and refuted the second in 2007. By constructing some finite expander graphs, they showed best possible upper bound on $n$ is $O(\rho \log \rho)$. Prompted by these results, Papasoglu asked whether a graph $G$ of polynomial growth admits a coarse embedding into a grid graph. We give an affirmative answer to this question. Moreover, it turns out that the dimension of the grid graph only needs to be linear in the asymptotic growth rate of $G$, which confirms the original Levin-Linial-LondonRabinovich conjecture "on the large scale." Besides, we find an alternative proof of the result of Papasoglu that graphs of polynomial growth rate $\rho<\infty$ have asymptotic dimension at most $\rho$. Furthermore, our proof works in the Borel setting and shows that Borel graphs of polynomial growth rate $\rho<\infty$ have Borel asymptotic dimension at most $\rho$. This is joint work with Anton Bernshteyn.
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## 1192-05-31737

Houston Schuerger*, Trinity College. Zero Forcing, Independent Sets, and Vertex Cuts. Preliminary report.
We consider a recent, but equivalent, reformulation of standard zero forcing referred to as parallel increasing path covers. In the context of parallel increasing path covers, additional focus is put on the set of time-steps during which vertices are active over the course of a zero forcing process. This increased focus identifies a distinct connection between zero forcing processes
and certain collections of vertex cuts of a graph giving rise to a new family of lower bounds on the zero forcing number of graphs. In addition, the natural connection between independent sets and vertex cuts allows for the introduction of further bounds comparing zero forcing numbers and independence numbers of graphs. In particular, we provide a partial answer to a conjecture made by the automated conjecturing program TxGraffiti.
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## 1192-05-31752

Charles Gong, Carnegie Mellon University, Kyle Alexander Kelley*, Kenyon College, Philip D. Thomas*, Kutztown University. Properties of Addsub Configuration Graphs.
Given some $n \in \mathbb{Z}^{+}$, the addsub (configuration) graph $\Gamma_{n}$ is the directed graph whose vertices are $V\left(\Gamma_{n}\right)=\mathbb{Z}_{n} \times \mathbb{Z}_{n}$ and for $\vec{x}, \vec{y} \in V\left(\Gamma_{n}\right)$, where $\vec{x}=\left(x_{1}, x_{2}\right)$ and $\vec{y}=\left(y_{1}, y_{2}\right)$, there exists a directed edge $(\vec{x}, \vec{y}) \in E\left(\Gamma_{n}\right)$ if and only if
$\vec{y}=\left(x_{1}+x_{2}, x_{2}-x_{1}\right)$. We consider the structures of addsub graphs, their symmetries, and their relation to subgroup structures. We also investigate the number of weakly connected components in addsub graphs and their relation to the structure of $\mathbb{Z}_{n}$. Finally, we discuss possible generalizations of configuration graphs to other groups and to higher dimensions. (Received September 12, 2023)

## 1192-05-31758

Kathleen M O'Hara*, Philadelphia, PA, Dennis Stanton, U. of Minnesota. Notes for Neighborly Partitions.
Another proof of the first Rogers Ramanujan partition identity is given using neighborly partitions as defined by Moshen and Mourtada. We re-interpret neighborly partitions in the context of Wilf's sieve-equivalent partition theory. Next we describe a cancelling involution whose null set leads to a nice recursion, the backdrop of the new proof. More needs to be done to provide a truly combinatorial proof of the RR identities.
(Received September 12, 2023)

## 1192-05-31766

Emina Soljanin*, Rutgers University. Service Rates of MDS Codes \& Fractional Matchings in Quasi-uniform Hypergraphs. Preliminary report.
The service rate region of a code is a performance metric of a distributed system that stores data redundantly using the code. It measures the system's ability to simultaneously serve multiple users requesting different data objects. The service rate region of an $[n, k]$ code is a polytope in $\mathbb{R}^{k}$. We first show that this polytope is a linear map image of the fractional matching polytope of a hypergraph defined by the code generator matrix. We then focus on a large class of MDS codes whose associated hypergraphs are quasi-uniform and characterize their service rate regions.
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1192-05-31776
Fan Chung, University of California, San Diego, Michael G Rawson, PNNL, Zhaiming Shen, University of Georgia, Murong Xu*, The University of Scranton. The Ricci Curvatures and Fast Algorithms for Random Clustering Graphs.
Curvature of a graph can be used to examine the local structure of the graph. We consider several notions of Ricci curvature for graphs in the setting of random clustering graphs which can be used to model real-world networks. Unlike most random graph models, a random clustering graph typically has dense neighborhoods with local structures that can be characterized by Ricci curvatures. We compute the expected Ricci curvatures via Lin-Lu-Yau (LLY) and Forman Ricci curvature. In addition, we give a fast approximation algorithm to compute LLY Ricci curvature of graphs. The curvature approximation allows the analysis of real-world networks arising from large datasets. Finally, we compare our theoretical results with empirical results on real-world networks.
(Received September 12, 2023)

## 1192-05-31787

József Balogh, University of Illinois at Urbana-Champaign, Dingding Dong, Harvard University, Bernard Lidicky*, Iowa State University, Nitya Mani, Massachusetts Institute of Technology, Yufei Zhao, Massachusetts Institute of Technology. Almost all $k$-sat functions are unate.
We prove that $1-o(1)$ fraction of all $k$-SAT functions on $n$ Boolean variables are unate (i.e., monotone after first negating some variables), for any fixed positive integer k and as n goes to infinity. This resolves a conjecture by Bollobás, Brightwell, and Leader from 2003. Dong, Mani, and Zhao reduced the conjecture to a Turán problem on partially directed hypergraphs. We solve this Turán problem. (Received September 12, 2023)

1192-05-31832
Lora Bailey, Grand Valley State University, Heather Smith Blake*, Davidson College, Garner Cochran, Berry College, Nathan Harel Fox, Canisius College, Michael Levet, College of Charleston, Reem Mahmoud, Virginia Commonwealth University, Elizabeth Matson, Alfred University, Inne Singgih, University of Cincinnati, Grace Stadnyk, Furman University, Elena Wang, Michigan State University, Alexander Wiedemann, Randolph-Macon College. Combinatorial Models for Genome Rearrangement.
Gene rearrangement is a common mode of molecular evolution which dates back to the pioneering work of Dobzhansky and Sturtevant in 1936. However, even for moderate size inputs, there is a tremendous number of optimal rearrangement scenarios, sequences of mutations that transform one genome into another. In hypothesizing, giving one optimal solution might be misleading and cannot be used for statistical inference. While many models for genome rearrangement have been proposed, there is a need for biologically relevant models which are computationally approachable. With a focus on the Single Cut-and-Join model, we summarize the state-of-the-art in computational complexity and uniform sampling questions for several models surrounding optimal scenarios.

Reha Oguz Selvitopi*, Lawrence Berkeley National Laboratory. Partitioning models for obtaining special sparse matrix structures.
Structured sparse matrices can greatly benefit applications when a special matrix structure is favored and can be exploited by the application for performance reasons. In this talk, we present several combinatorial algorithms based on graph/hypergraph partitioning in order to obtain certain sparse matrix forms that are beneficial for a number of applications. The partitioning models can be used to reorder the sparse matrices in a pre-processing phase into the special structure which the application can take advantage of. Such a pre-processing phase can especially be detrimental for performance if the application repeatedly relies on the same matrix structure or the operations on the matrix are computationally expensive. In this respect, we propose custom partitioning models to obtain the desired form of a sparse matrix in which the relevant objective and constraint in partitioning correlates to the performance metrics in the application. Among the sparse matrix forms we present are singly- or doubly-bordered block-diagonal forms, non-empty off-diagonal block minimization, and sparse matrix tiling. Examples of application areas that benefit from such forms include parallel solutions of linear programs, parallel training of graph neural networks, parallel iterative solvers, and sparse direct solvers.
(Received September 12, 2023)

1192-05-31893
Maya Sankar*, Stanford University. Homotopy and the Homomorphism Threshold of Odd Cycles.
Fix $r \geq 2$ and consider a family $\mathcal{F}$ of $C_{2 r+1}$-free graphs, each having minimum degree linear in its number of vertices. Such a family is known to have bounded chromatic number; equivalently, each graph in $\mathcal{F}$ is homomorphic to a complete graph of bounded size. We disprove the analogous statement for homomorphic images that are themselves $C_{2 r+1}$-free. Specifically, we construct a family of dense $C_{2 r+1}$-free graphs with no $C_{2 r+1}$-free homomorphic image of bounded size. This provides the first nontrivial lower bound on the homomorphism threshold of longer odd cycles and answers a question of Ebsen and Schacht. Our proof introduces a discretized notion of homotopy equivalence for walks in graphs, which has connections to the neighborhood complex.
(Received September 12, 2023)

## 1192-05-31911

Emmy Huang*, Lexington High School, Ray Tang, Massachusetts Institute of Technology. Minimum Decomposition on Maxmin Trees.
Maxmin trees consist of nodes that are either local minimums or maximums. Such trees were first studied by Postnikov. Later Dugan, Glennon, Gunnells, and Steingrimsson introduced the concept of weight to these trees and proved a bijection between maximum weight maxmin trees and permutations. Thus, weight was also defined for permutations. In addition, the $q$-Eulerian polynomial $E_{n}(x, q)$ is defined which relates descents and weights of permutations. This polynomial was later proven to exhibit a stabilization phenomenon. Extracting the formal power series $W_{d}(t)$ from the stabilization of these coefficients, $W_{d}(t)$ was conjectured to partially correspond to OEIS Sequence A256193. In our paper, we introduce a process called minimum decomposition to help us better understand minmax trees. Using minimum decomposition, we present a new way to calculate the weight of different minmax trees and prove the bijection between the coefficients of $W_{d}(t)$ and A256193. (Received September 25, 2023)

## 1192-05-31922

Marchelle Beougher*, Macalester College, Nila Cibu*, University of California, Berkeley, Cassie Ding, Brown University, Steven Sofos DiSilvio, Columbia University, Sasha Kononova, University of California at Los Angeles, Chan Lee, Williams College, Ralph E. Morrison, Williams College, Krish Singal, Columbia University. On the Gonality of Banana Graphs. Baker first developed the notion of graph gonality in 2008 by adapting divisor theory of algebraic curves to finite graphs Computing graph gonality is NP-hard in general, as proven by Gijswijt et al. in 2020. However, by studying families of graphs such as banana graphs, which are path graphs that allow for multi-edges, we can ascertain properties of graph gonality. For example, banana graphs were used to study higher graph gonalities by both Aidun et al. in 2021 and Fessler et al. in 2023. These graphs are structurally simple while still giving rise to rich divisorial properties, making them an ideal testing ground for graph gonality. Expanding on Aidun et al.'s work from 2021, we develop and prove recursive formulas via dynamic programming for gonality and other powerful, related invariants of banana graphs. We also prove that graph gonality can increase or decrease by any integer with the addition or deletion of a single edge through the use of Dhar's burning algorithm and modular arithmetic. Furthermore, we prove that all banana graphs satisfy the Gonality Conjecture which states that a graph G's gonality is upper bounded by $\frac{(g+3)}{2}$ where $g=|E(G)|-|V(G)|+1$. (Received September 12, 2023)

1192-05-31946
Ae Ja Yee*, Pennsylvania State University. Strict partitions and their 2-core/quotient decompositions. Preliminary report. In 2016, Alex Berkovich and Ali Uncu studied partitions with fixed number of odd and even indexed odd parts noticing a connection to BG-rank. Some results from their study played an important role in the recent work of Chern, Li, Stanton, Xue and Yee on the enumeration of Kleshchev biparitions with a fixed partition residue statistic. BG-rank and the residue statistics considered for Kleshchev bipartitions are closely connected to 2-core partitions. In this talk, I will discuss 2-core/quotient decompositions for partitions into distinct parts and present combinatorial proofs of some results of Berkovich and Uncu. This is a preliminary report on a joint project with R. Li, S. Seo and D. Stanton. (Received September 12, 2023)

Mackenzie Bookamer, Tulane University, Sarah Capute, Middlebury College, Natalie Robin Dodson, Middlebury College, Carmen Jackson, Northwestern University, Lani Southern, Willamette University, Liza Ter-Saakov*, Rutgers University -New Brunswick. Classes of Graphs that Admit Sparse Universal Graphs. Preliminary report.
A graph $U$ is universal for a class of graphs $\mathcal{H}$ if it contains each graph in $\mathcal{H}$ as a subgraph. A universal graph $U$ for graphs in $\mathcal{H}$ on $n$ vertices is sparse if there exists some $\epsilon>0$ such that the number of edges in $U$ is less than $O\left(n^{2-\epsilon}\right)$. We investigate classes of graphs that admit sparse universal graphs and techniques for constructing universal graphs. (Received September 12, 2023)

## 1192-05-31979

Oscar Murillo-Espinoza*, California State University Monterey Bay. Arithmetical Structures on Canoe Paddle Graphs. Preliminary report.
Given a connected graph $G$ with $n$ vertices, an arithmetical structure on $G$ is a pair of vectors $(\mathbf{d}, \mathbf{r}) \in \mathbb{Z}_{>0}^{n} \times \mathbb{Z}_{>0}^{n}$ satisfying $(\operatorname{diag}(d)-A) r=0$, where $A$ is the adjacency matrix of $G$. These arithmetical structures originally arose in the work of Lorenzini on degenerations of curves in algebraic geometry. In his work, Lorenzini proved there are finitely many arithmetical structures on any connected graph. This raises the natural question of counting how many arithmetical structures there are on a particular graph $G$. The number of arithmetical structures on paths, cycles, and trees have been counted in previous work. In our work, we investigate methods of obtaining and counting arithmetical structures on canoe paddle graphs, i.e., on graphs containing a cycle of $n$ vertices connecting to the start of a path containing $m$ vertices.
(Received September 12, 2023)

## 1192-05-31993

Will McCreary*, Carleton College. Demonstrating a Novel Excluded Minor for Bimodular Representability. Preliminary report.
One of the driving problems in the research of Matroid Theory is the search for excluded minors of minor-closed classes of matroids. This is largely because attaining a complete list of excluded minors presents a straightforward way to check whether a matroid is contained in such a class. In this paper, we will demonstrate a novel excluded minor for the class of bimodularrepresentable matroids inspired by Oxley and Walsh's paper on bimodular matrices. In doing so we will demonstrate a technique for checking for excluded minors of $\Delta$-modular-representable matroids using SageMath and the Sandwich Factory lattice polytope classification algorithm. In addition, we use this approach to rule out several novel dyadic excluded minors as candidates for bimodular-representable excluded minors.
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## 1192-05-32006

Chris Godsil, University of Waterloo, Wanting Sun, University of Waterloo, Xiaohong Zhang*, University of Montreal. Cospectral graphs.
In this talk, we will use 1-walk regular graphs to construct families of graphs that are pairwise cospectral with respect to the adjacency, Laplacian, unsigned Laplacian, and normalized Laplacian matrices. For example, with this construction, we obtain a family of 250 non-isomorphic graphs on 25 vertices, all cospectral with respect to the above four matrices.
(Received September 12, 2023)

1192-05-32029
Colin Defant, Massachusetts Institute of Technology, Rachana Madhukara*, MIT, Hugh Thomas, UQAM. Permutoric Promotion: Gliding Globs, Sliding Stones, and Colliding Coins.
Defant recently introduced toric promotion, an operator that acts on the labelings of a graph $G$ and serves as a cyclic analogue of Schützenberger's promotion operator. Toric promotion is defined as the composition of certain toggle operators, listed in a natural cyclic order. We consider more general permutoric promotion operators, which are defined as compositions of the same toggles, but in permuted orders. We settle a conjecture of Defant by determining the orders of all permutoric promotion operators when $G$ is a path graph. In fact, we completely characterize the orbit structures of these operators, showing that they satisfy the cyclic sieving phenomenon. The first half of our proof requires us to introduce and analyze new broken promotion operators, which can be interpreted via globs of liquid gliding on a path graph. For the latter half of our proof, we reformulate the dynamics of permutoric promotion via stones sliding along a cycle graph and coins colliding with each other on a path graph.
(Received September 12, 2023)

1192-05-32040
Kenneth S. Berenhaut*, Wake Forest University. Bonded and dissipative edges in the context of communities and cohesion. Preliminary report.
Networks can exhibit varying levels of informative community structure. In this talk, we introduce a novel partitioning method based on the concept of cohesion introduced recently by Berenhaut, Moore and Melvin [Proceedings of the National Academy of Sciences, 119 (4) (2022)]. We define a measure of dissipation of cohesion over edges, as well as an accompanying threshold which distinguishes dissipative from bonded edges. The resulting network of bonded edges provides a partitioning of the graph which does not suffer from resolution issues and is entirely parameter free. Importantly, the method can identify crucial and intermediary independent nodes, which can be informative in social and other settings. Applications to real world data and stochastic block models are considered.
(Received September 12, 2023)

1192-05-32041
Tomasz Slusarczyk*, Massachusetts Institute of Technology. Multilinear forms on random graphs. Preliminary report. For simple graphs $G$ and $H$ with $|V(H)|=k$ let $\operatorname{Hom}(H, G)$ denote the set of homomorphic images $\left(x_{1}, \ldots, x_{k}\right) \in V(G)^{k}$ of
$H$ in $G$. For functions $f_{1}, \ldots, f_{k}: V(G) \rightarrow \mathbb{R}$ Bhowmik, Iosevich, Koh, and Pham (2023) introduced the notion of normalized vertex-weighted homomorphism count

$$
\left.\Lambda_{H}^{G}\left(f_{1}, \ldots, f_{k}\right)=\frac{1}{\mid} \operatorname{Hom}(H, G) \right\rvert\, \sum f_{1}\left(x_{1}\right) \cdots f_{k}\left(x_{k}\right)
$$

where the sum runs over $\left(x_{1}, \ldots, x_{k}\right) \in \operatorname{Hom}(H, G)$. They studied the inequality

$$
\Lambda_{H}^{G}\left(f_{1}, \ldots, f_{k}\right) \leq O_{H}(1)\left\|f_{1}\right\|_{p_{1}} \cdots\left\|f_{k}\right\|_{p_{k}}
$$

with $\|\cdot\|$ being the normalized $L^{p}$ norm. For several small, fixed $H$ and $G$ restricted to a family of "grid-like" graphs on $\mathbb{F}_{q}^{d}$ they try to classify the possible tuples of exponents $\left(p_{1}, \ldots, p_{k}\right)$ such that the above inequality holds for all $f_{i}$, independently of $G$. We extend the analysis to $G$ being an Erdős-Renyi random graph of same edge density as the previous construction. In this new setting we show that all results of Bhowmik, Iosevich, Koh, and Pham hold with high probability. We also simplify existing proofs and introduce new spectral methods. Our techniques can be used to prove similar results for different models of random graphs as long as they satisfy certain quasirandomness properties related to subgraph counts. (Received September 12, 2023)

1192-05-32073
Kaioke Begay*, Central Washington University, Kieran Cook, Central Washington University, Athaliah Mackewicz, Central Washington University. A Variation on Labeled Chip Firing: Firing Chips Simultaneously.
The labeled chip-firing game is a variation of one dimensional chip-firing in which each chip is assigned a numeric value. When a node is selected to fire, two chips are compared with one another and the chip with the greater value is sent to the right while the chip with the lesser value is sent to the left. It has been proven that starting with an even number of chips results in a sorted final configuration. Here a variation of labeled chip-firing in which sets of chips are selected to be compared is introduced. We show that while the chips do not sort in this variation, the largest two and smallest two chips cannot end up more than one away from their final sorted position. We further conjecture that no chip can end up more than one away form its final sorted position, and introduce theorems that may prove helpful in exploring this conjecture.
(Received September 12, 2023)

## 1192-05-32092

Darlison Nyirenda*, University of the Witwatersrand. A generalization of Subbarao's finitization theorem.
Let $m>1, r \geq 0$ be integers and $c_{m, r}(n)$ be the number of partitions of $n$ in which all even multiplicities of the parts are less than $2 m$, and all odd multiplicities are at least $2 r+1$ and at most $2(m+r)-1$. Let $d_{m, r}(n)$ be the number of partitions of $n$ in which parts are either odd but congruent to $2 r+1(\bmod 4 r+2)$ or even but not congruent to $0(\bmod 2 m)$. M. V. Subbarao proved that $c_{m, r}(n)=d_{m, r}(n)$ for all $n \geq 0$. In this talk, we give a generalization of this theorem and discuss some of the combinatorial consequences.
(Received September 12, 2023)

## 1192-05-32099

Marchelle Beougher, Macalester College, Nila Cibu, University of California, Berkeley, Cassie Ding*, Brown University, Steven Sofos DiSilvio, Columbia University, Sasha Kononova, University of California at Los Angeles, Chan Lee*, Williams College, Ralph E. Morrison, Williams College, Krish Singal, Columbia University. Gonality of Chess Graphs and Platonic Solids.
Ever since Baker and Norine formulated a discrete analogue of the Riemann-Roch theorem and linear systems in 2007, divisor theory on graphs has been used to explore properties of algebraic curves. Specifically, the gonality of metric graphs serves as a useful tool for computing the gonality of certain algebraic curves, since it gives a lower bound on that of its associated algebraic curves. To provide more graph-theoretic tools for algebraic geometers, we determine the gonality of chess graphs and Platonic solid graphs. Extending Speeter and Morrison's work on the gonality of rook's and queen's graphs, we compute the gonality of certain families of knight's, king's and bishop's graphs (as well as their toroidal variations) and establish a lower bound on the carton number of rook's graphs. Further, we find the gonality of all five 3-dimensional Platonic solid graphs. Our techniques for determining these gonalities include scramble number computations, tree-cut decompositions, and Dhar's burning algorithm. We also consider a long-standing open question: the gonality of $n$-dimensional hypercubes, conjectured to be $2^{n-1}$ by van Dobben de Bruyn and Gijswijt in 2014. We propose a way to determine the gonality of hypercubes using a new graph invariant, screewidth.
(Received September 12, 2023)

1192-05-32130
Vince Guingona, Towson University, Miriam Parnes, Towson University, Christian A Pippin*, Towson University. Classes Of Graphs With Forbidden Induced Subgraphs.
Many families of graphs can be characterized by a set of graphs which are forbidden from appearing, in some particular sense, in any graph in the family. A famous example is the family of planar graphs. Kuratowski's theorem states that planar graphs are exactly the graphs which forbid the complete graph $K_{5}$ and the complete bipartite graph $K_{3,3}$ as subdivisions. Our research focuses on families of graphs where particular graphs are forbidden from appearing as induced subgraphs. We investigate whether these families have certain properties. The first property we study is indivisibility, which is a special case of the Ramsey property. A family $\mathcal{F}$ of graphs is indivisible if for any graph $A$ in $\mathcal{F}$, there exists a larger graph $B$ in $\mathcal{F}$ such that every 2 -coloring of $B$ yields a monochromatic copy of $A$ as an induced subgraph of $B$. We also consider whether these families have the amalgamation property. A family $\mathcal{F}$ of graphs has the amalgamation property if for all graphs $A$ in $\mathcal{F}$ and all embeddings of $A$ into some $B_{0}$ and into some $B_{1}$ in $\mathcal{F}$, there exists a $C$ in $\mathcal{F}$ where $B_{0}$ and $B_{1}$ have embeddings into $C$ which agree on the images of $A$. We have found that the family Forb $\left(P_{n}\right)$, which forbids a path on $n$ vertices, has the amalgamation
property for $n \leq 3$; however for $n>3$, amalgamation does not hold. Additionally, we know that Forb $\left(P_{n}\right)$ is indivisible for $n \leq 4$ and conjecture that $\operatorname{Forb}\left(P_{n}\right)$ is indivisible for $n>4$. Some of the other families of graphs we are interested in studying are chordal graphs, which forbid cycles of length greater than 3; and perfect graphs, which forbid cyles of odd length greater than or equal to 5 and their complements.
(Received September 12, 2023)

## 1192-05-32148

Bryan L Shader*, University of Wyoming. Musings on Minors.
We will discuss various results that show placing some type of restrictions (e.g., a large number of equal minors, or bounding above the absolute values of all principal minors) on the minors of a matrix induces significant structural constraints on the matrix. In particular, we study the structure of skew-symmetric matrices in which each off-diagonal entry in $\pm 1$, and whose principal minors are at most $k^{2}$.,
(Received September 12, 2023)

1192-05-32154
Adam Lee Schultze*, Lewis University. Towards a Charge Statistic Beyond Type A. Preliminary report.
(Received September 12, 2023)

1192-05-32201
Neal Bushaw*, Virginia Commonwealth University, Craig Larson, Virginia Commonwealth University. Graph Hamiltonicity and Automated Conjecturing.
We present results on new sufficient or necessary conditions for the existence of a Hamilton cycle in a graph. We are especially interested in finding conditions which are not implied by any of a number of well-known theorems in the literature on graph hamiltonicity. We also report a number of unresolved conjectures. This work was undertaken as part of a large-scale collaborative lab with high school students, undergraduates, graduate students, and faculty working together using an opensource automated conjecturing program. We will also discuss this working paradigm, and how it influenced our results; we believe this will be of independent interest to the mixed audience of this session.
(Received September 12, 2023)

## 1192-05-32237

Charles Gong, Carnegie Mellon University, Kyle Alexander Kelley*, Kenyon College, Philip D. Thomas*, Kutztown University. Structural Properties of Move Graphs Generated by Group Actions.
Let $G$ and $A$ be abelian groups. We introduce the new concept of a "move graph" $\Gamma_{G, A}$. Here the nodes of $\Gamma_{G, A}$ are vectors $\mathbf{x}=\left(x_{1}, x_{2}, \ldots, x_{m}\right) \in A^{m}$ and $(\mathbf{a}, \mathbf{b})$ is a directed edge provided $b_{i}=g \cdot a_{i}$ subject to a specified group action on the $i^{t h}$ coordinate, $1 \leq i \leq m$. Our main interest will be the case in which $G=\langle M\rangle$ is the cyclic group generated by a nonsingular integral $m \times m$ matrix $M$, and $A$ is a finite cyclic group. In this context we refer to $M$ as the move matrix, and we write $\Gamma_{M, A}$ in place of $\Gamma_{G, A}$. Our primary objective is to study structural properties of such move graphs, their symmetries, and their relation to subgroup structures. We also investigate the number of weakly connected components in move graphs and their relation to the structure of $\mathbb{Z}_{n}$. Finally, we discuss possible generalizations of move graphs to other groups and to higher dimensions.
(Received September 12, 2023)

1192-05-32253
Steffen Borgwardt, University of Colorado Denver, Calum Buchanan, University of Vermont, Eric D Culver, Brigham Young University, Bryce Frederickson, Emory University, Puck Rombach*, University of Vermont, Youngho Yoo, Texas A\&M University. Path Odd-Covers of Graphs.
We introduce and study "path odd-covers", a weakening of Gallai's path decomposition problem and a strengthening of the linear arboricity problem. The "path odd-cover number" $p_{2}(G)$ of a graph $G$ is the minimum cardinality of a collection of paths whose vertex sets are contained in $V(G)$ and whose symmetric difference of edge sets is $E(G)$. We prove an upper bound on $p_{2}(G)$ in terms of the maximum degree $\Delta$ and the number of odd-degree vertices $v_{\text {odd }}$ of the form $\max \left\{v_{\text {odd }} / 2,2\lceil\Delta / 2\rceil\right\}$. This bound is only a factor of 2 from a rather immediate lower bound of the form $\max \left\{v_{\text {odd }} / 2,\lceil\Delta / 2\rceil\right\}$. We also investigate some natural relaxations of the problem which highlight the connection between the path odd-cover number and other wellknown graph parameters. For example, when allowing for subdivisions of $G$, the previously mentioned lower bound is always tight except in some trivial cases. Further, a relaxation that allows for the addition of isolated vertices to $G$ leads to a match with the linear arboricity when $G$ is Eulerian. (Received September 12, 2023)

1192-05-32281
Van Magnan*, University of Montana. More on Intersecting Problems via Delta-System Methods.
In extremal set theory, we often ask questions about maximizing the size of a family subject to certain intersecting conditions. The celebrated Erdős-Ko-Rado Theorem describes how the maximum size of any intersecting family can be achieved by a 'trivially' intersecting family, in which all members contain a common element. This, and similar problems, can be approached using delta-system methods. These leverage the existence of sunflower-like subfamilies to make structural and quantitative observations about extremal constructions. In this talk, we share recent findings and present novel applications of deltasystem methods to intersecting problems.
(Received September 12, 2023)

Nathan Lemons*, Los Alamos National Laboratory. Some combinatorial problems in quantum computing. Preliminary report. A major scientific challenge of our times is to understand (and realize) the potentials of quantum computing. Current quantum computers are not yet able to support fault-tolerant computation. Quantifying the potential of these impressive, but noisy quantum computers is thus a timely goal. We survey some of the research in this quickly evolving field from a combinatorial perspective and identify open research areas where combinatorial results could be useful. Particular focus will be on quantum optimization paradigms such as the Quantum Approximate Optimization Algorithm (QAOA) and Quantum Annealing. (Received September 12, 2023)

## 1192-05-32289

Anton Michael Dochtermann*, Texas State University. Simple homotopy of flag complexes and contractible transformations on graphs.
A simple homotopy equivalence is a combinatorial strengthening of homotopy involving sequences of collapses and expansions of a simplicial (or CW-) complex. We investigate this notion in the context of flag simplicial complexes, where the operations can be understood in terms of the underlying graph. We relate these constructions to a notion of graph homotopy studied by Ivashchenko (who introduced 'contractible transformations' on graphs in his study of molecular spaces), as well as the sdismanlability of Boulet, Fieux, Jouve (in turn based on the strong homotopy of Barmak and Minian). Restricting to certain contractible transformations recovers other well-studied classes of complexes, as well as potential applications to the computation of persistent homology. This represents joint work with J.F. Espinoza, M.E. Frias-Armenta, H.A. Hernandez, and T. Matsushita.
(Received September 12, 2023)

1192-05-32303
Vishal Gupta*, University of Delaware. A lower bound on the smallest eigenvalue of a graph and an application to the associahedron graph. Preliminary report
In this talk, we will discuss a lower bound for the smallest eigenvalue of a regular graph containing many copies of a smaller fixed subgraph. This generalizes a result of Aharoni, Alon, and Berger in which the subgraph is a triangle. We apply our results to obtain a lower bound on the smallest eigenvalue of the associahedron graph, and we prove that this bound gives the correct order of magnitude of this eigenvalue. If time allows, we will also discuss what is known regarding the second-largest eigenvalue of the associahedron graph.
(Received September 12, 2023)

1192-05-32331
Sinan Aksoy, Pacific Northwest National Laboratory, Juan Andres Escobedo Contreras, Pacific Northwest National Laboratory, Jesun Firoz, Pacific Northwest National Laboratory, Roberto Gioiosa, Pacific Northwest National Laboratory, Tobias Hagge, Pacific Northwest National Laboratory, Mark Kempton, Brigham Young University, Mark Raugas, Pacific Northwest National Laboratory, Stephen J Young*, Pacific Northwest National Laboratory. SpectralFly: Ramanujan Graphs as Flexible and Efficient Interconnection Networks.
In recent years, graph theoretic considerations have become increasingly important in the design of HPC interconnection topologies. One approach is to seek optimal or near-optimal families of graphs with respect to a particular graph theoretic property, such as diameter. For example, the SlimFly topology is based on a construction of McKay, Miller, and Širáň which produces a diameter two graph on a number of nodes approaching the Moore bound, i.e. the largest possible diameter two graph with a fixed radix. Motivated by recent work of Aksoy, Bruillard, Young, and Raugas, we consider topologies which optimize the spectral gap rather than the diameter. In particular, we introduce a novel HPC topology, SpectralFly, designed around the Ramanujan graph construction of Lubotzky, Phillips, and Sarnak (LPS). In this work, we show that the combinatorial properties, such as diameter, bisection bandwidth, average path length, and resilience to link failure, of SpectralFly topologies are better than, or comparable to, similarly constrained DragonFly, SlimFly, and BundleFly topologies. Additionally, we simulate the performance of SpectralFly topologies on a representative sample of physics-inspired HPC workloads using the Structure Simulation Toolkit Macroscale Element Library simulator and demonstrate considerable benefit to using LPS construction as the basis of the SpectralFly topology.
(Received September 12, 2023)

1192-05-32365
Casey M. Pinckney*, University of Maine. Independence Complexes of Finite Groups.
Understanding generating sets for finite groups has been explored previously via the generating graph of a group, where vertices are group elements and edges are given by pairs of group elements that generate the group. We generalize this idea by considering minimal generating sets for subgroups of finite groups. These form a simplicial complex, which we call the independence complex, whose vertices are nonidentity group elements and whose faces of size k correspond to minimal generating sets of size k . We provide constructive algorithms and enumerative results for the independence complexes of cyclic groups whose order is a squarefree product of primes, finite abelian groups whose order is a product of powers of distinct primes, and a particular nonabelian class of semidirect products. We introduce a new tool called a combinatorial diagram to construct the independence complexes for this last class of groups, and we include visualizations created using GAP and Polymake. (Received September 12, 2023)

1192-05-32372
Calum Buchanan*, University of Vermont, Puck Rombach, University of Vermont. A lower bound on the saturation number of a triangle-free graph.
For a graph $H$ and an integer $n$ at least the order of $H$, the saturation number $\operatorname{sat}(n, H)$ is the minimum size of an $n$-vertex graph which does not contain a subgraph isomorphic to $H$, but to which the addition of any edge creates such a subgraph. Erdös, Hajnal, and Moon first studied the saturation number of the complete graph, and the first nontrivial general lower
bound on $\operatorname{sat}(n, H)$ was obtained by Cameron and Puleo. We build upon this general lower bound, presenting a bound with a marked improvement for triangle-free graphs. We also discuss certain classes of trees for which this new lower bound is tight, addressing a question of Faudree, Faudree, Gould, and Jacobson. This talk is based on joint work with Puck Rombach. (Received September 12, 2023)

1192-05-32384
Peter Oden Kagey*, Harvey Mudd College, Bill Keehn, Prison Mathematics Project. Counting arbitrary tilings of the $n \times m$ square grid, cylinder, and torus..
We count all tilings of the $n \times m$ rectangular grid, cylinder, and torus with arbitrary tile sets up to arbitrary subsets of the dihedral group of the square $D_{8}$. These results build on work by Ethier and Lee counting tilings of the torus by tiles of two colors. This is joint work with Bill Keehn via the Prison Mathematics Project (PMP).
(Received September 25, 2023)

## 1192-05-32399

Rhys O'Higgins, Macalester College, Lola Vescovo*, Macalester College. Exact mixing times for random walks on trees of a fixed diameter.
A random walk on a graph is an ordered list of vertices, each chosen randomly from the neighbors of the vertex before it. Using this, we can study the mixing time, which is the expected number of steps to reach a "balanced" distribution from the worst possible starting vertex. We characterize the extremal structures for certain random walks on trees of a fixed diameter and prove that the mixing time is maximized by the double broom.
(Received September 12, 2023)

1192-05-32403
Audrey Baumheckel*, California State University, Fresno, Oscar Vega, California State University, Fresno. Realizable groups of graphs in the the $\Delta-y$ families of $K_{n}$. Preliminary report.
We will present results on positively realizable automorphism groups of graphs in the $\Delta-Y$ families of $K_{n}$, for $n \leq 8$. (Received September 12, 2023)

## 1192-05-32414

Jinting Liang*, Michigan State University. Enriched toric $[\vec{D}]$-partitions.
In this talk I will discuss the theory of enriched toric $[\vec{D}]$-partitions. Whereas Stembridge's enriched $P$-partitions give rises to the peak algebra which is a subring of the ring of quasi-symmetric functions, our enriched toric $[\vec{D}]$-partitions will generate the cyclic peak algebra which is a subring of cyclic quasi-symmetric functions. In the same manner as the peak set of linear permutations appears when considering enriched $P$-partitions, the cyclic peak set of cyclic permutations plays an important role in our theory.
(Received September 12, 2023)

## 1192-05-32416

Sarah Capute, Middlebury College, Medha Durisheti, Virginia Tech, Grant Shirley, East Tennessee State University, Liza Ter-Saakov*, Rutgers University -- New Brunswick. An overview of new uses of DeBruijn cycles for combinatorial objects such as ranked permutations, integer compositions, and injective functions.
In this talk we will describe several generalizations of DeBruijn cycles to objects such as ranked permutations, integer compositions, and injective functions. We use a combination of universal cycles, graph universal cycles, and hypergraph universal cycles for these constructions.
(Received September 12, 2023)

1192-05-32430
Alex Morales, University of Puerto Rico at Mayagüez, Grant Shirley*, East Tennessee State University, Steven Vasquez, University of Puerto Rico at Mayagüez. Rose Conjecture: Total Acquisition Number in Diameter Two Graphs.
In 2015, Rose McCarty led an REU team that showed, in an unpublished work, that the acquisition number of a diameter 2 graph is less than or equal to 4 given that the Rose conjecture is true. The Rose Conjecture supposes that any $n \times n$ matrix with row and column sums equal to $n$ can be decomposed into a pair of matrices where one has distinct row and column sums equal to $\{1,2, \ldots, n\}$, and the other has distinct row and column sums equal to $\{0,1, \ldots, n-1\}$. In this talk, we will summarize attempts to prove this conjecture that took place over a different REU in 2023. (Received September 12, 2023 )

1192-05-32433
Mackenzie Bookamer, Tulane University, Natalie Robin Dodson, Middlebury College, Medha Durisheti*, Virginia Tech, Carmen Jackson, Northwestern University. Quantum Expressiveness. Preliminary report.
Given a binary non-zero matrix $A$, and two random binary matrices $X$ and $Y$, we discuss the expressiveness of $A$. In this context, expressiveness is characterized by computing the probability that for each column $a$ of $A$, the sum of the binary dot products of $a$ with each column of $X$ equals that of $Y$. By looking at different compositions of $A$, we determine general bounds on expressiveness.
(Received September 12, 2023)

Adam Lee Schultze*, Lewis University. Towards a Combinatorial Model for $q$-weight Multiplicities of Simple Lie Algebras. Preliminary report.
Kostka-Foulkes polynomials are Lusztig's $q$-analogues of weight multiplicities for irreducible representations of semisimple Lie algebras. It has long been known that these polynomials have non-negative coefficients. A statistic on semistandard Young tableaux with partition content, called charge, was used to give a combinatorial formula exhibiting this fact in type $A$. Defining a charge statistic beyond type $A$ has been a long-standing problem. Here, we take a completely new approach based on the definition of Kostka-Foulkes polynomials as an alternating sum over Kostant partitions, which can be thought of as formal sums of positive roots. We use a sign-reversing involution to obtain a positive expansion, in which the relevant statistic is simply the number of parts in the Kostant partitions. The hope is that the simplicity of this new crystal-like model will naturally extend to other classical types. We will conclude with a discussion of progress made in types $B, C$ and $D$ during summer research with undergraduate students.
(Received September 12, 2023)

## 1192-05-32459

Mallory Price*, Grand Valley State. Properties of Edge Cover Sequences and Edge Cover Polynomials of Certain Graph Families. Preliminary report.
An edge cover of a simple graph is a subset of the edges chosen so that each vertex is incident with at least one vertex. The edge covers of certain graph families provide new combinatorial interpretations of known sequences, such as the Fibonacci and Lucas numbers being the number of edge covers of path and cycle graphs, respectively. An edge cover polynomial is the generating function of the number of edge covers of a graph. It is known that the edge cover polynomials of path and cycle graphs have real roots, and hence these polynomials have log-concave and unimodal coefficients. In this talk, we present similar results on edge cover sequences and edge cover polynomials of graph families obtained by modifying path and cycle graphs.
(Received September 12, 2023)

1192-05-32460
Mallory Price*, Grand Valley State, Andrew Kennedy Wilson*, Grand Valley State. Edge Covers of Modified Path and Cycle Graphs. Preliminary report.
A graph is a mathematical structure consisting of vertices (dots) and edges (lines) that connect pairs of vertices. Graphs are used in modeling relationships between discrete objects. When a specific graph structure can be extended in a consistent pattern, we get graph families such as path and cycle graphs. An edge cover of a graph is a subset of the graph's edges chosen so that each vertex is an endpoint of at least one edge in this subset. In this project, we studied the sequences formed by counting the total number of edge covers in a graph family. It is known that the edge cover totals of certain graph families, such as the path and cycle graphs, give rise to known sequences, the Fibonacci and Lucas numbers, respectively. This allows us to obtain new combinatorial interpretations of known sequences or to generate new sequences from edge cover totals. In this presentation, we will report on our results on the edge cover sequences for graphs obtained by attaching 3 -vertex path and cycle graphs. (Received September 12, 2023)

## 1192-05-32466

Pieter Ghysels*, Lawrence Berkeley National Laboratory. Physics Informed Graph Neural Networks for Power Grid DC Blocker Placement. Preliminary report.
From a grid resilience perspective, the most consequential and hardest to predict risks are called high-impact low-frequency events, which are a great challenge for protecting the bulk electric system (BES). For instance geomagnetic disturbances (GMDs) can disrupt the Earth's magnetic field and drive geomagnetically induced currents (GICs) in the conductive infrastructure. The research community has developed several approaches to help plan for and mitigate the potential impacts of GMDs on the BES. One promising approach is the addition of transformer neutral blocking devices. Transformer neutral blocker placement - GIC dc-current blocking devices - consists of injecting a shunt capacitor in series with the grounding point of transformer neutrals. Given the high cost of these devices, sparse placement is necessary as it is an unrealistic expectation to install such devices at all transformer neutrals. We developed a heterogeneous graph neural network (HGNN) to predict optimal blocker placement. Labels for training are obtained using PowerModelsGMD.jl, which formulates the blocking placement problem as a mixed integer nonlinear problem, which is then tackled using heuristic solvers. These traditional optimization solvers are computationally very expensive and not scalable to large "real-world" electrical grids. The machine learning models can be used to generate solutions for these problems. We use a novel physics inspired heterogeneous graph neural network (PIHGNN), which combines the strengths of HGNNs with the physics-based modeling of the power grid. The benefits of the PIHGNN approach are improved accuracy, reliability and flexibility. The model parameters are optimized using a Bayesian optimization search algorithm from the DeepHyper framework, which is also use for network architecture optimization.
(Received September 12, 2023)

1192-05-32487
Bowen Li*, Carleton College, István Miklós, Rényi Institute, Hungarian Academy of Sciences, Carter Rockhold Teplica, Columbia University. The Connectedness of the Solution Space in the Black-and-White Graph Pressing Game. The pressing game on black-and-white graphs is defined as follows: Given a graph $G=(V, E)$ where its vertices are colored either black or white, any black vertex $v$ can be "pressed." Pressing $v$ results in the following changes: (a) all neighbors of $v$ switch colors-black neighbors become white and vice versa, (b) all pairs of neighbors of $v$ toggle connectivity-unconnected pairs become connected, and connected pairs become unconnected, and (c) $v$ itself becomes an isolated white vertex. The aim of the game is to transform $G$ into an all-white, empty graph. It is known that an all-white empty graph is reachable through the pressing game if each component of $G$ contains at least one black vertex. Moreover, for a fixed graph, any successful transformation results in the same number of pressed vertices. In this talk, we will discuss the connectedness of the solution space. We define a graph $\mathfrak{S}$ whose vertices represent successful pressing paths. Two paths are considered neighbors if they can be rendered identical by deleting at most 4 terms from each. The 4 -reversal conjecture states that $\mathfrak{S}$ is connected. We will
present a proof with an emphasis on its underlying linear algebraic ideas. Additionally, we will discuss the game's connections to genome rearrangement, sorting signed permutations through reversals, and infinite site models.
(Received September 12, 2023)

1192-05-32504
Boyoon Lee, University of South Florida, Theodore Molla*, University of South Florida, Brendan Nagle, University of South Florida. Two-coloring bipartite uniform hypergraphs.
Extending the notion of a bipartite graph, we define a hypergraph to be bipartite if there exists a bipartition of the vertex set such that every hyperedge intersects both parts. For $k \geq 3$, Lovász proved that it is NP-complete to decide whether a $k$ uniform hypergraph is bipartite. In this talk, we will discuss an algorithm that, when given a bipartite $k$-uniform hypergraph, constructs a bipartition in average running time $O\left(n^{k}\right)$ over all $n$-vertex $k$-uniform bipartite hypergraphs. We will also cover some related results.
(Received September 12, 2023)

## 1192-05-32521

George Brooks*, University of South Carolina, Fadekemi Janet Osaye, Alabama State University, Anna Schenfisch,
Eindhoven University of Technology, Zhiyu Wang, Louisiana State University, Jing Yu, Georgia Institute of Technology Outerplanar graphs with positive Lin-Lu-Yau curvature. Preliminary report.
Various notions of graph curvature have been of particular interest in recent years. In this talk, we consider the Lin-Lu-Yau curvature of outerplanar graphs. More precisely, we prove that $\Delta(G) \leq 9$ for all positively curved outerplanar graphs with $\delta(G) \geq 2$. Using this result, we investigate an upper bound on $|V(G)|$ when further conditions are imposed. (Received September 12, 2023)

## 1192-05-32538

Jeffrey Chen, Massachusetts Institute of Technology, Jesse Selover*, UMass Amherst. Positivity properties of $q$-hit numbers in the finite general linear group \par.
We consider the problem of counting matrices over a finite field with fixed rank and support contained in a fixed set. The count of such matrices gives a $q$-analogue of the classical rook number, although it is not-polynomial in general. Lewis and Morales defined a corresponding $q$-analogue of hit numbers, and conjectured several positivity properties for them. We find explicit formulas for the residues of $q$-hit numbers in low degrees, and prove these residues are non-negative, partially answering one of their conjectures. \par
(Received September 12, 2023)

1192-05-32546
Wayne D. Goddard*, Clemson University. Coloring Graphs to Produce Walks Without Forbidden Repeats.
We consider the problem of coloring the edges of a graph such that every pair of vertices are joined by an $\ell$-rainbow walk, that is, a walk where for every sub-walk of length at most $\ell+1$ its edges receive different colors. We show that the minimum number of colors needed is at most $2 \ell+1$ if the graph has a cycle of length at least $\ell$. We also provide a sharp bound for $\ell=2$ in bridgeless graphs, and general bounds for sufficiently large bridgeless graphs, and show that if the graph contains two sufficiently long edge-disjoint closed trails then the number of colors needed is at most $\ell+2$. Joint work with Deirdre LaVey (Received September 12, 2023)

## 1192-05-32550

Jeffrey Charles Venable*, California State University, Stanislaus. Combo Stats.
Let $[n]$ denote the set $\{1, \ldots, n\}$. We uniformly select $x$ observations from $[n]$ and order the observations from least to greatest. We were able to identify alternative proofs for the Expected Values and Variances for any order statistic of the selected observations. Under a geometric distribution, the distribution of the first order statistic is well known. For future goals, we seek to identify the distribution of the largest observation and that of any order statistic.
(Received September 12, 2023)

## 1192-05-32583

William T. Dugan*, University of Massachusetts Amherst. On the f-vector of flow polytopes for complete graphs-preliminary report. Preliminary report.
The Chan-Robbins-Yuen polytope $\left(C R Y_{n}\right)$ of order $n$ is a face of the Birkhoff polytope of doubly stochastic matrices that is also a flow polytope of the directed complete graph $K_{n+1}$ with netflow $(1,0,0, \ldots, 0,-1)$. The volume and lattice points of this polytope have been actively studied, however its face structure has not. We give generating functions and recurrences to compute the $f$-vector by using Hille's (2007) result bijecting faces of a flow polytope to certain graphs, as well as AndresenKjeldsen's (1976) result that enumerates certain subgraphs of the directed complete graph. We extend our results to flow polytopes over the complete graph having other (non-negative) netflow vectors and study the face lattice of $C R Y_{n}$. (Received September 25, 2023)

1192-05-32586
Jordan Broussard*, Whitworth University. Template Arrays and Two-Dimensional Recurrence Relations. Preliminary report. In a two-dimensional recurrence relation, there is an underlying structure composed of the two-dimensional sequences (arrays) in which the set of indices is extended from $\mathbb{N}$ to $\mathbb{Z}$. We will look at a set of initial conditions sufficient to build a uniquely-determined array from a given recurrence and initial conditions, as well as look at how to construct a Schauder basis using elementary arrays for the set of arrays.

## 1192-05-32591

Linda Eroh*, University of Wisconsin Oshkosh, Cong X. Kang, Texas A\&M University, Galveston campus, Eunjeong Yi, Texas A\&M University, Galveston campus. The Connected Metric Dimension of a Graph.
A set $S$ of vertices in a connected graph $G$ is a resolving set if, for every pair of distinct vertices $x$ and $y$ in $G$, there is some $w \in S$ such that $d(x, w) \neq d(y, w)$. A resolving set $S$ is connected if $S$ induces a connected subgraph in $G$. For a vertex $v \in V(G), \operatorname{cim}_{G}(v)$ is the minimum cardinality of a connected resolving set containing $v$. The connected metric dimension of $G, \operatorname{dim}(G)$, is the minimum cardinality of a connected resolving set in $G$. We characterize graphs $G$ and vertices $v$ such that $\operatorname{cdim}_{G}(v)=1$ and such that $\operatorname{cdim}_{G}(v)=|V(G)|-1$. We show that $\operatorname{cdim}(G)=2 \operatorname{implies} G$ is planar, although it is known that there is a non-planar graph $H$ with $\operatorname{dim}(H)=2$. We show that $\operatorname{cdim}(G)-\operatorname{dim}(G)$ can be arbitrarily large, and characterize trees and unicyclic graphs such that $\operatorname{cdim}(G)=\operatorname{dim}(G)$.
(Received September 12, 2023)

## 1192-05-32602

Mackenzie Bookamer*, Tulane University. $K_{d+2, d+2}$ is fully reconstructible in $\mathbb{R}^{d}$. Preliminary report.
A bar and joint framework in $\mathbb{R}^{d}$ is a pair $(G, \mathbf{p})$ consisting of a graph $G$ on vertex set $V$ and a function $\mathbf{p}: V \rightarrow \mathbb{R}^{d}$. We call $\mathbf{p}$ the realization of the joints. Intuitively, one can think of a bar and joint framework as a physical construction of $G$, using rigid bars as edges and placing the vertices according to $\mathbf{p}$. Rigidity theorists are interested in questions such as whether a given framework can be deformed without changing the lengths of any of the edges. The question for my talk is:

$$
\text { Isthecompletebipartitegraph } K_{d+2, d+2} \text { fullyreconstructiblein } \mathbb{R}^{d} \text { foranyarbitraryd } \in \mathbb{N}, d>3 \text { ? }
$$

(Received September 12, 2023)

## 1192-05-32603

Emily J Olson*, Millikin University. Working with Undergraduates on the Game of Cycles. Preliminary report.
The Game of Cycles is a game that is played by two players on a simple, undirected, connected planar graph, called a board. Each player takes turns directing edges on a board with an arrow without allowing a source or sink to occur. The game continues until a player creates a directed cycle cell or plays the last legal move. In this talk, we present results related to the Game of Cycles and lessons learned while working with undergraduates at a primarily undergraduate institution on mathematical research. We will also discuss the benefits of applying for a grant to compensate the undergraduates, purchase supplies, and fund travel.
(Received September 12, 2023)

## 1192-05-32617

Carmen Jackson*, Northwestern University, Olivia LeBlanc, Colorado State University. The Expected Number of Distinct Non-Consecutive Patterns in a Random Permutation. Preliminary report.
A permutation $\pi$ on $[n]$ is said to contain a particular pattern $\mu$ of length $k$ if there exist $k$ terms in $\pi$ whose relative order is the same as that of the terms in $\mu$. We will discuss the expected number of distinct patterns in a given permutation on $[n]$ and use subadditivity to conclude that this quantity is asymptotic to $c^{n}$ for some $c$ in $(1,2)$. (Received September 12, 2023)

## 1192-05-32628

James Clayton Kerce*, Georgia Tech Research Institute. Directed Graph Augmentation for Improved Performance in Message Passing Graph Neural Networks.
We leverage traditional approaches for interacting particle systems on graphs to propose a new approach to message passing graph neural networks that can significantly improve performance with directed graphs. We demonstrate the method using the state-of-the-art Graph Convolutional Network via Initial Residual and Identity mapping (GCNII), which employs a diffusionbased model that mitigates over-smoothing during training. Although successful in geometric deep-learning, such a diffusion based approach cannot faithfully represent data flow through inherently directed graphs and performance can still degrade as layers are added. In this paper, we leverage concepts of advection-diffusion from fluid-flow physics to adapt GCNII to directed graphs. We present a directed propa gator version, DPGCNII, which demonstrates uniform reductions in prediction error by 33 (Received September 12, 2023)

## 1192-05-32631

Kristin Heysse*, Macalester College, Kate J. Lorenzen, Linfield University, Carolyn Reinhart, Swarthmore College, Xinyu Wu, Carnegie Mellon University. Graphs with nontrivial Jordan blocks for the nonbacktracking matrix.
The non-backtracking (NB) matrix of a graph is the transition matrix of a random walk on the graph which cannot traverse the same edge in succession. This NB matrix and its eigen-information have garnered interest of late, particularly in application to network analysis. However, the lack of symmetry in the NB matrix allows for the existence of graphs without a full basis of NB eigenvectors. In this talk, we will construct an infinite family of graphs with nontrivial Jordan blocks and utilize a decomposition to make searching for such graph families easier.
(Received September 12, 2023)

Neal Bushaw, Virginia Commonwealth University, Daniel P Johnston*, Trinity College, Puck Rombach, University of Vermont. Rainbow Saturation.
A graph $G$ is rainbow $H$-saturated if there is some proper edge coloring of $G$ which is rainbow $H$-free (that is, it has no copy of $H$ whose edges are all colored distinctly), but where the addition of any edge makes such a rainbow H -free coloring impossible. Taking the maximum number of edges in a rainbow $H$-saturated graph recovers the rainbow Turán numbers whose systematic study was begun by Keevash, Mubayi, Sudakov, and Verstraëte. In this talk, we introduce and examine the corresponding rainbow saturation number - the minimum number of edges among all rainbow $H$-free graphs.
(Received September 12, 2023)

## 1192-05-32691

Cash Bortner, California State University, Stanislaus, Paola Viviana Campos*, California State University, Stanislaus, Jessica De Silva, California State University, Stanislaus, Jeffrey Charles Venable*, California State University, Stanislaus. Maximum Stars in Pendant Ladder Graphs. Preliminary report.
For a graph $G$, a family of independent $r$-sets is an $r$-star centered at $v \in V(G)$ if $v$ is in every $r$-set of the family. Since the intersection of any two sets in an $r$-star contains the center, an $r$-star is an example of a pairwise intersecting subfamily of the independent $r$-sets of $G$. Building upon the Erdős-Ko-Rado (EKR) theorem for sets, a graph is $r$-EKR if the size of every pairwise intersecting subfamily of independent $r$-sets is at most the size of an $r$-star. Holroyd, Talbot, and Spencer conjecture that $G$ is $r$-EKR if $1 \leq r \leq \frac{\mu(G)}{2}$, where $\mu(G)$ is the minimum size of a maximal independent set of $G$. Talbot suggests if a counterexample to their conjecture exists, it may be found in the class of well-covered graphs. We consider a subclass of wellcovered graphs, namely pendant graphs denoted $G^{*}$, obtained by appending one pendant edge to each vertex in any base graph $G$. In our first result, we prove that the $k$-star centered at a pendant vertex in $G^{*}$ is of maximum size if, for all $1 \leq r \leq k$, the $r$-star in the base graph $G$ centered at the corresponding base vertex is of minimum size. In our second result, we determine the vertex in the pendant ladder graph which yields the $k$-star of maximum size by applying our first result in proving the $r$-star of the corresponding base vertex in the ladder graph has the minimum size for all $1 \leq r \leq k$. (Received September 12, 2023)

1192-05-32692
Janabel Xia*, Massachusetts Institute of Technology. Deterministic Stack-Sorting for Sock Orderings. Preliminary report. A sock ordering is a sequence of letters, representing socks of various colors. A sock ordering is sorted if all socks of the same color appear consecutively in the sequence. The notion of stack-sorting sock orderings was originally introduced by Defant and Kravtiz. In this talk, we define a new deterministic stack-sorting map $\phi_{\sigma}$ that uses a $\sigma$-avoiding stack, where pattern containment need not be consecutive. Given a stack-sorting map, we say that a sock ordering is $k$-stack-sortable if it can be sorted by at most $k$ iterations of $\phi_{\sigma}$. When $\sigma=a b a$, we find that any sock ordering on $n$ colors is $n$-stack-sortable, and furthermore this bound is tight for $n \geq 3$. We obtain a fine-grained enumeration of the number of 1 -stack-sortable sock orderings of length $n$ and $r$ distinct colors, and also obtain asymptotics for the number of 1 -stack-sortable sock orderings of length $n$.
(Received September 12, 2023)

## 1192-05-32701

Alexandr Kostochka, University of Illinois Urbana-Champaign, Ruth Luo, University of South Carolina, Grace McCourt*, University of Illinois at Urbana-Champaign. Minimum degree ensuring that a hypergraph is hamiltonian-connected. A hypergraph $H$ is hamiltonian-connected if for any distinct vertices $x$ and $y, H$ contains a hamiltonian Berge path from $x$ to $y$ . We find for all \$3\leq r
(Received September 12, 2023)

1192-05-32758
Trent G. Marbach*, Toronto Metropolitan University. The localization number and metric dimension of graphs of diameter 2. We consider the localization number and metric dimension of certain graphs of diameter 2, focusing on families of Kneser graphs and graphs without 4-cycles. For the Kneser graphs with diameter 2, we find upper and lower bounds for the localization number and metric dimension, and in many cases these parameters differ only by an additive constant. Our results on the metric dimension of Kneser graphs improve on earlier ones, yielding exact values in infinitely many cases. We determine bounds on the localization number and metric dimension of Moore graphs of diameter 2 and polarity graphs. (Received September 12, 2023)

## 1192-05-32760

Rida Ait El Manssour*, Université Paris Cité, CNRS, IRIF, Gleb Pogudin, Ecole polytechnique. Multiplicity and inverse system of arc spaces. Preliminary report.
Consider the arc space of the fat point given by $x^{\wedge}\{\mathrm{m}\}=0$. This is defined by the quotient of $\mathrm{k}\left[\mathrm{x}, \mathrm{x}^{\prime}, \mathrm{x}^{\prime \prime}, \ldots\right]$ over the differential ideal generated by $x^{\wedge}\{m\}$. This algebra admits a natural filtration by finite dimensional algebras corresponding to the truncation of arcs. We show that the dimensions of these algebras are given by the sequence $m \wedge\{h+1\}$, where $h$ is the order of truncation. The proof of this result is based on determining the initial ideal of the differential ideal generated by $x^{\wedge}\{m\}$ with respect to lexicographical ordering. The description of the initial ideal was previously conjectured by Afsharijoo, which serves in finding new partition identities generalizing Roger-Ramanujan identities Furthermore, we provide a full description of the inverse system of the truncated ideals corresponding to those algebras in the case of double points. We link the elements of the inverse system to differentially homogeneous polynomials. This is a joint work with Gleb Pogudin. (Received September 12, 2023)

Radio Frequency (RF) signals are ubiquitous in communication media. Radios, transmitters, and receivers all produce some kind of RF signals. Information theory is the study of the level of uncertainty (entropy) inherent in communication systems. In this talk, we discuss at a high level the role entropy can play in RF analysis.
(Received September 12, 2023)

Kassie Archer, United States Naval Academy, Alexander Diaz-Lopez, Villanova University, Darren Glass, Gettysburg College, Joel Louwsma*, Niagara University. Critical groups of arithmetical structures on star graphs and complete graphs. An arithmetical structure on a finite, connected graph without loops is an assignment of positive integers to the vertices that satisfies certain conditions. Associated to each of these is a finite abelian group known as its critical group. We show how to determine the critical group of an arithmetical structure on a star graph or complete graph in terms of the entries of the arithmetical structure. We use this to investigate which finite abelian groups can occur as critical groups of arithmetical structures on these graphs.
(Received September 12, 2023)

Francesca Gandini*, St. Olaf College, Russ Woodroofe, University of Primorska. Posets of Group Action Arrangements. Preliminary report.
Let $G$ be a finite group acting linearly on a vector space $V$. For each group element $g \in G$, the graph of the linear action of $g$ is a subspace $V_{g}=\{(v, v \cdot g)\}$ in $V \oplus V$. The collection of these subspaces is an example of a subspace arrangement. Ideals associated to subspace arrangements are generated by linear forms and have been shown to have nice homological properties (e.g., their regularity is bounded above by the cardinality of the arrangement). On the other hand, one can study the intersection poset of a subspace arrangement. It turns out that the intersection poset arising from a group action arrangement is deeply connected to another group theoretic poset: the coset poset. In collaboration with Russ Woodroofe, we have seen that for the regular action, the two posets are isomorphic and we are studying the consequences of this result for permutation actions more generally.
(Received September 12, 2023)

## 1192-05-32775

Charles Matthew Farmer*, UNC Greensboro (UNCG). The Noncrossing Bond Poset.
The partition lattice and noncrossing partition lattice are well studied objects in combinatorics. Given a graph $G$ on vertex set $\{1,2, \ldots, n\}$, its bond lattice, $L_{G}$, is the subposet of the partition lattice formed by restricting to the partitions whose blocks induce connected subgraphs of $G$. In this article, we introduce a natural noncrossing analogue of the bond lattice, the noncrossing bond poset, $N C_{G}$, obtained by restricting to the noncrossing partitions of $L_{G}$. Both the noncrossing partition lattice and the bond lattice have many nice combinatorial properties. We show that, for several families of graphs, the noncrossing bond poset also exhibits these properties. We present simple necessary and sufficient conditions on the graph to ensure the noncrossing bond poset is a lattice. Additionally, for several families of graphs, we give combinatorial descriptions of the Möbius function and characteristic polynomial of the noncrossing bond poset. These descriptions are in terms of a noncrossing analogue of non-broken circuit (NBC) sets of the graphs and can be thought of as a noncrossing version of Whitney's NBC theorem for the chromatic polynomial. We also consider the shellability and supersolvability of the noncrossing bond poset, providing sufficient conditions for both. We end with some open problems.
(Received September 25, 2023)

## 1192-05-32783

Vaidyanathan Sivaraman*, Mississippi State University, Rebecca Whitman, University of California, Berkeley. Hereditary Nordhaus-Gaddum graphs. Preliminary report.
We study the class of graphs, all of whose induced subgraphs almost satisfy the Nordhaus-Gaddum inequality. This class is characterized by exactly 52 forbidden induced subgraphs. Relationship with perfect graphs and algorithmic problems will be discussed.
(Received September 12, 2023)

1192-05-32866
Cliff Joslyn*, Pacific Northwest National Laboratory, Audun Myers, Pacific Northwest National Laboratory, Emilie Purvine, Pacific Northwest National Laboratory. Temporal Hypergraphs: Analysis and Dynamics. Preliminary report.
So-called "high order networks" and simplicial structures, basically combinatorial network structures like hypergraphs operating at higher levels of abstraction from graphs, are rapidly emerging as effective methods for representing complex systems information. Real-world applications then require additional attendance to the data being "carried on" or recorded within these structures, for example measured quantities, typing categories, identifying labels, or uncertainty information. Specifically temporal information is perhaps the most important adjunct to high order networks. Here we report on two recent approaches to the analysis of temporal hypergraphs represented as a time-filtered trajectory over the space of hypergraphs of a given set of vertices: zigzag homology to identify topological features persistent across multiple time slices, and multiple styles of edit cost or edit distance among the time slices. Where our current analysis is based on measured data presenting as hypergraphs, our ultimate goal is a dynamical model over the space of hypergraphs, against which such trajectories would be sampled.
(Received September 12, 2023)
$|G|+1$. The Nordhaus-Gaddum graphs are the class of graphs satisfying this inequality with equality, and are wellunderstood. In this talk we introduce a hereditary generalization of Nordhaus-Gaddum graphs (and implicitly of threshold graphs): graphs $G$ for which all induced subgraphs $H$ of $G$ satisfy $\chi(H)+\chi(\bar{H}) \leq|H|-a$, for some constant $a$. We discuss the forbidden induced subgraph characterizations of these classes, intersection with a number of common classes, and optimization results.
(Received September 13, 2023)

## 1192-05-32888

Abdullah Al Rafi Mahmud*, Villanova University, Michael Tait, Villanova. Improved Lower Bound on the Size of Maximum $(d, d+2)-G e n e r a l$ Sets in $\mathbb{F}_{q}^{t}$.
A set of points in $\mathbb{F}_{q}^{t}$ is called $(d, s)$-general set if no $s$ points are in a $d$-dimensional affine space. In this project, we study the maximum size of $(\stackrel{q}{d}, d+2)$-general sets. The best-known lower bound is by using a greedy or probabilistic argument and is of order $\left(q^{t}\right)^{\frac{1}{d+1}}$. We use results from coding theory to give an improved lower bound of order $\left(q^{t}\right)^{\frac{m+1}{m^{2}+m+1}}$ for sufficiently large t . (Received September 13, 2023)

## 1192-05-32893

Hari Prasad Sitaula*, Montana Technological University. Algorithm to Compute Discrete Residues of a Rational Function. The classical notion of residue, for a rational function with complex coefficients, is a powerful and ubiquitous tool, having applications in many different areas. For example: Complex Analysis, Physics, Number Theory, Differential Equations, and Combinatorics, to name a few. In the last decade several new notions of discrete residues have been developed by different researchers, all of which have in common the following obstruction-theoretic feature: a given rational function $f(x)$ is "special" (e.g., rationally integrable, or rationally summable, or rationally $q$-summable) if and only if all of its corresponding residues are zero. All of these notions of residue (both the classical one and also its discrete variants) are originally defined in terms of a complete partial fraction decomposition of the given rational function $f(x)$, which is too expensive to carry out in practice due to the high computational cost of finding the complete factorization of the denominator. The main contribution of this talk is the development of an efficient factorization-free algorithm to compute the discrete residues of a rational function. (Received September 12, 2023)

## 1192-05-32894

Jacob Fox*, Stanford University. Triangle Ramsey numbers.
If every two-coloring of the edges of a graph $G$ has a monochromatic clique on $n$ vertices, how few edges can $G$ have? In joint work with Jonathan Tidor and Shengtong Zhang, we answer this question of Sam Spiro for $n$ sufficiently large.
(Received September 12, 2023)

1192-05-32908
Alex Somto Arinze Alochukwu*, Department of Mathematics, Computer Science and Physics, Albany State University, Albany-GA,USA. On Uniquely Colourable Trees.
Consider a scenario in which we wish to assign broadcast frequencies to radio stations so that when two radio stations are assigned the same frequency, they are located sufficiently far apart so that neither broadcast interferes with the reception of the other. Now assume this problem is modelled by a graph: given an undirected graph $G$, we seek for a partition,
$X=\left\{X_{1}, X_{2}, \ldots, X_{k}\right\}$, of the vertex set of $G$ into disjoint color classes such that any two vertices belonging to the color class, $X_{i}$, are at distance at least $i+1$. In graph theory terminology, $X$ is a packing colouring of $G$ and each colour class $X_{i}$ is an $i$-packing. The minimum order $k$ of a packing colouring is called the packing chromatic number of $G$, denoted by $\chi_{\rho}(G)$. In this talk, we present some known results on the packing chromatic number of a graph and investigate the existence of trees $T$, a special class of graph, for which there is only one packing colouring using $\chi_{\rho}(T)$ colours. For the case $\chi_{\rho}(T)=3$, we completely characterise all such trees and, in addition, discuss the monotocity of the packing colouring. (Received September 25, 2023)

1192-05-32950
Heather M. Russell*, University of Richmond - Richmond, VA, Julianna Tymoczko, Smith College. Stranding $\mathfrak{s l}_{n}$ webs. Preliminary report.
A web is a graph that can be interpreted as a special kind of linear transformation which preserves certain permutations. More formally, an $\mathfrak{s l}_{n}$ web is a morphism in a diagrammatic category that encodes the representation theory of the quantum group $U_{q}\left(\mathfrak{s l}_{n}\right)$. Webs are used by topologists to compute quantum knot invariants and by geometers to study certain flag varieties called Springer varieties. They are also interesting objects of study in their own right. In this talk, we introduce webs and describe strandings of webs. We also talk about some of the constructions from the literature - like Khovanov-Kuperberg flows for $\mathfrak{s l}_{3}$ webs - that strandings generalize. This work was done while Heather M. Russell (University of Richmond) and Julianna Tymoczko (Smith College) were Directors' Mathematicians in Residence at BSM.
(Received September 13, 2023)

1192-05-32951
Alexandr Kostochka, University of Illinois Urbana-Champaign, Mina Nahvi*, University of Illinois Urbana-Champaign, Douglas B. West, University of Illinois Urbana-Champaign and Zhejiang Normal University, China, Dara Zirlin, University of Illinois Urbana-Champaign. Trees with at least $6 \ell+11$ vertices are $\ell$-reconstrictible
The $(n-\ell)$-deck of an $n$-vertex graph is the multiset of (unlabeled) subgraphs obtained from it by deleting $\ell$ vertices. An $n$ vertex graph is $\ell$-reconstructible if it is determined by its $(n-\ell)$-deck, meaning that no other graph has the same deck. In this talk, I will share the history of different variations of the Reconstruction Problem with a focus on trees. Furthermore, recent
findings on reconstruction of trees will be presented, including our result which proves that every tree with at least $6 \ell+11$ vertices is $\ell$-reconstructible
(Received September 13, 2023)

## 1192-05-32962

John Bright, University of Florida, Kevin G Milans*, West Virginia University, Jackson Porter, West Virginia University. Ordered Turán Numbers.
An ordered hypergraph $G$ is a hypergraph whose vertices are linearly ordered. The ordered Turan Number of an $r$-uniform hypergraph $G$, denoted $\mathrm{ex}_{<}(n, G)$, is the maximum number of edges in an $n$-vertex $r$-uniform ordered graph not containing $G$ as an ordered subgraph. When $s \leq 2 r-1$, we obtain the ordered Turan Numbers of the tight $r$-uniform $s$-vertex path $P_{s}^{(r)}$ exactly, implying that $\mathrm{ex}_{<}\left(n, P_{s}^{(r)}\right)=\left(1-\frac{1}{2^{s-r}}+o(1)\right)\binom{n}{r}$. We also discuss the case of longer tight paths when $r=3$. This is joint work with John Bright and Jackson Porter.
(Received September 13, 2023)

## 1192-05-32986

Felicia Elizabeth Flores*, Smith College, Eleanor Gallay, Smith College, Emily Hafken, Smith College, Malia Hanes, Smith College, Kerry Elizabeth Seekamp, Smith College, Orit Tashman, Smith College. Stranding webs and Springer fibers.
Webs are a kind of graph that encodes linear transformations between representations of a quantum group, namely vector spaces with a very special kind of group action. Graph-theoretic operations on webs correspond to linear algebra operations making it possible to solve familiar linear algebra problems - like identifying bases or matrices - just from the combinatorics. Webs arise naturally in fields like knot theory and topology, too. We describe strandings, which are special ways to color webs. We then show how strandings can be used to identify the points in an algebraic variety called the Springer fiber. Finally, we talk about combinatorial operations on strandings like flips and rotations, and relate these to some of the classical algebraic, combinatorial, and geometric properties of the underlying objects.
(Received September 13, 2023)

## 1192-05-33031

Jeremy Quail*, University of Vermont, Puck Rombach, University of Vermont. Constructing Graphic Matroids from Decorated Permutations.
Postnikov developed a combinatorial decomposition of the Grassmannian using a special type of matroid, called a positroid, into positroid cells. Postnikov further showed that these positroids are in bijection with decorated permutations. We focus on graphic matroids, and show that the map provided by this bijection is onto: i.e. there exists a graphic matroid for every decorated permutation. Therefore, every positroid cell contains a graphic matroid.
(Received September 13, 2023)

Aleyah Dawkins, George Mason University, Vishal Gupta, University of Delaware, Mark Kempton, Brigham Young University, William Linz, University of South Carolina, Jeremy Quail, University of Vermont, David Harry Richman*, University of Washington, Zachary Stier, UC Berkeley. Ricci flow on graphs from effective resistance. Preliminary report An edge-weighted graph can be viewed as an electrical network of resistors, and in this way there is an effective resistance between any two vertices. We define a Ricci curvature on edges of a graph based on this effective resistance, inspired by previous work of Devriendt and Lambiotte. From this Ricci curvature, we consider the resulting Ricci flow on weighted graphs where the edge weights (or, edge resistances) change over time. We will discuss some examples and results concerning this Ricci flow coming from preliminary investigations, including some visual illustrations. In particular, we explain why the set of graphs with nonnegative Ricci curvatures is stable under this Ricci flow.
(Received September 13, 2023)

1192-05-33152
Van Magnan, University of Montana, Cory Palmer, University of Montana, Ryan Wood*, University of Montana, Missoula, MT. A generalization of diversity for intersecting families.
Let $\mathcal{F} \subseteq\binom{[n]}{r}$ be an intersecting family of sets and let $\Delta(\mathcal{F})$ be the maximum degree in $\mathcal{F}$, i.e., maximum number of edges of $\mathcal{F}$ containing a fixed vertex. The diversity of $\mathcal{F}$ is defined as $d(\mathcal{F}):=|\mathcal{F}|-\Delta(\mathcal{F})$ and can be viewed as a measure of distance from the 'trivial' maximal intersecting family from the Erdős-Ko-Rado Theorem. Indeed, the diversity of this family is 0 . Moreover, the diversity of the largest non-trivial intersecting family à la Hilton-Milner is 1 . It is known that the maximum possible diversity of an intersecting family $\mathcal{F} \subseteq\binom{[n]}{r}$ is $\binom{n-3}{r-2}$ so long as $n$ is large enough. In this talk, we consider a recent generalization of diversity in which we weight the maximum degree in $\mathcal{F}$. We define the $C$-weighted diversity as $d_{C}(\mathcal{F}):=|\mathcal{F}|-C \cdot \Delta(\mathcal{F})$. We discuss the maximum value of $d_{C}(\mathcal{F})$ over intersecting families $\mathcal{F} \subseteq\binom{[n]}{r}$ for general $C$ and give a complete characterization for $C \in[0,7 / 3)$ which leads to a asymptotic resolution of a conjecture of Frankl and Wang on a related 'diversity-like' measure.
(Received September 13, 2023)

1192-05-33168
József Balogh, University of Illinois at Urbana-Champaign, Ce Chen*, University of Illinois Urbana-Champaign, Grace
McCourt, University of Illinois at Urbana-Champaign, Cassie Murley, University of Illinois Urbana-Champaign. RamseyTurán problems with small independence numbers.
Given a graph $H$ and a function $f(n)$, the Ramsey-Turán number $R T(n, H, f(n))$ is the maximum number of edges in an $n$ -
vertex $H$-free graph with independence number at most $f(n)$. For $H$ being a small clique, many results about $R T(n, H, f(n))$ are known and we focus our attention on $H=K_{s}$ for $s \leq 13$. By applying Szemerédi's Regularity Lemma, the dependent random choice method and some weighted Turán-type results, we prove that these cliques have the so-called phase transitions when $f(n)$ is around the inverse function of the off-diagonal Ramsey number of $K_{r}$ versus a large clique $K_{n}$ for some $r \leq s$. (Received September 13, 2023)

1192-05-33190
Dheer Noal Desai*, University of Memphis. Bipartite Spectral Turán problems.
Turán numbers are a cornerstone of extremal graph theory. Their asymptotics are completely known when forbidden graphs have chromatic number at least three, however they remain unknown for several basic bipartite graphs. Nikiforov introduced a spectral analogue to Turán problems referred to as spectral Turán problems. Here our objective is to maximize the spectral radius of the adjacency matrices of graphs not containing some subgraphs. Such a study may give strong upper bounds for the associated Turán problems. While the asymptotics of spectral Turán numbers are known for graphs with chromatic number at least three, several families of bipartite graphs remain open in this scenario too. In this talk we will share recent progress on the spectral Turán numbers for some families of bipartite graphs. In some cases, this will strengthen the previous upper bounds for the associated Turán numbers. We will also discuss the associated minor-free Spectral Turán results that follow from our result. This extends past ideas developed for the spectral even cycle problem and a spectral Erdős-Sós theorem. (Received September 13, 2023)

## 1192-05-33224

Harley Meade*, Colorado State University. A Combinatorial Approach to Classifying Spark of Equiangular Tight Frames. Preliminary report.
Full spark frames are maximally robust to erasures and thus are important to many fields. Using combinatorial properties such as the matroidal structure of frames and group theoretic properties such as subsets being uniformly distributed over the subgroups of the overarching group, we work towards generalizations of results known for submatrices of discrete Fourier transform matrices. This is work towards fully characterizing the spark of such frames.
(Received September 13, 2023)

1192-05-33240
Jinyoung Park*, Courant Institute of Mathematical Sciences, NYU, Michail Sarantis, Carnegie Mellon University, Prasad Tetali, Carnegie Mellon University. Antichains in generalizations of the Boolean lattice.
The Dedekind number asks the number of antichains, $a(n)$, in the n-dimensional Boolean lattice [2]^ \{n\}. While the exact formula for the Dedekind number is still unknown, its asymptotic formula has been well-studied. Since any subsets of a middle layer of the Boolean lattice is an antichain, the logarithm of $a(n)$ is trivially bounded below by the size of a middle layer. In 1960's, Kleitman proved that this trivial lower bound is optimal in the logarithmic scale, and the actual asymptotics was also proved by Korshunov in 1980's. The number of antichains in the generalized Boolean lattice, [ t$] \wedge\{\mathrm{n}\}$ (where t and n both grow), has also received a fair amount of attention, in particular with a connection to a certain Ramsey problem. In this talk, we will discuss a recent development on this subject and open questions.
(Received September 13, 2023)

1192-05-33330
Bing Wei, University of Mississippi, Lei Zhong*, Department of Mathematics, university of Mississippi. Saturated Double Stars.
A graph $G$ is $H$-saturated if $G$ contains no copy of the graph $H$, but for any missing edge $e$ of $G$, there exists a copy of $H$ in $G+e$. The saturation number of the graph $H$, denoted by $\operatorname{sat}(n, H)$, is the minimal number of edges among all $H$-saturated graphs with $n$ vertices. In this talk, we focus on the saturation number $\operatorname{sat}\left(n, S_{t+1, t+1}\right)$, where $S_{t+1, t+1}$ is called a balanced double star obtained by adding an edge between the centers of two stars $S_{t+1}$. We firstly prove the new upper bounds $\operatorname{sat}\left(n, S_{t+1, t+1}\right) \leq \frac{t n}{2}+\frac{t+1}{2}$ and establish the graph achieving the upper bound. Specifically, we will determine the saturation number for $S_{t, t}$ for sufficiently large $n$ and small $t$.
(Received September 13, 2023)

1192-05-33352
Kenneth S. Berenhaut, Wake Forest University, Chi Mayson Zhang*, Colorado State University. Disparity-persistence and the multistep friendship paradox.
In this talk, we consider the friendship paradox in the context of random walks and paths. Among our results, we give an equality connecting long-range degree correlation, degree variability, and the degree-wise effect of additional steps for a random walk on a graph. Random paths are also considered, as well as applications to acquaintance sampling in the context of core-periphery structure.
(Received September 25, 2023)

1192-05-33361
Zeyu Zheng*, Carnegie Mellon University. Common kings of a chain of cycles in a strong tournament.
It is known that every strong tournament has directed cycles of any length, and thereby strong subtournaments of any size. In this note, we prove that they also can share a common vertex which is a king in all of them. This common vertex can be any king in the whole tournament. Further, the Hamiltonian cycles in them can be recursively constructed by inserting an additional vertex to one arc.
(Received September 14, 2023)

Carson R Mitchell*, UC Berkeley. The Geometry and Lattice-Point Enumeration of Stack-Sorting Simplices.
We study the polytopes that arise from the convex hulls of stack-sorting on particular permutations. We show that they are simplices and proceed to study their geometry and lattice-point enumeration. First, we prove some enumerative results on Ln1 permutations, i.e., permutations of length $n$ whose penultimate and last entries are $n$ and 1, respectively. Additionally, we then focus on a specific permutation, which we call L'n1, and show that the convex hull of all its iterations through the stack-sorting algorithm share the same lattice-point enumerator as that of the ( $\mathrm{n}-1$ )-dimensional unit cube and lecture-hall simplex. Lastly, we detail some results on the real lattice-point enumerator for variations of the simplices arising from stack-sorting L'n1 permutations. This then allows us to show that L'n1 simplices are Gorenstein of index 2.
(Received September 14, 2023)

## 1192-05-33417

Vince Guingona, Towson University, Felix Nusbaum*, Williams College, Zain Luis Garrett Padamsee*, Towson University, Miriam Parnes, Towson University, Christian A Pippin, Towson University, Ava Zinman*, Harvard University. Model Theoretic Properties for Classes of Graphs. Preliminary report.
A graph is considered "sparse" if its edge density is at most a certain upper bound. In particular, a graph $G$ is called " $\alpha$ sparse" if, for every subgraph of $G$, the ratio of vertices to edges is at least $\alpha$. In this study, we consider graphs as structures in a language whose only element is a binary edge relation. With this framework, we investigate model theoretic properties of the classes $\mathbb{K}_{\alpha}$, each of which consists of all $\alpha$-sparse graphs. We seek to characterize $\mathbb{K}_{\alpha}$ both directly and using the concept of boundary: a unique, minimal set made up of substructures of all graphs not in $\mathbb{K}_{\alpha}$. Other properties of interest include indivisibility, a coloring property related to the Ramsey property, and the amalgamation property, which addresses closure of classes under appropriate gluings. We fully describe $\mathbb{K}_{\alpha}$ and construct its boundary for $\alpha \geq 1$. For $\alpha<1$, we establish partial descriptions of the contents of and relationships between classes of $\alpha$-sparse graphs. Furthermore, we find that $\mathbb{K}_{\alpha}$ is indivisible if and only if $\alpha>2$, and has the amalgamation property if and only if $\alpha>\frac{3}{2}$. Our model theoretic approach to sparse graph classes may allow us to generalize our results to many classes of structures in relational languages. In addition, graph sparsity has applications in computer science to efficient data storage and access.
(Received September 15, 2023)

1192-05-33420
Devavrat Dabke*, Princeton University, Olga Dorabiala, University of Washington. A Novel Method for Vertex Clustering in Dynamic Networks.
We will introduce and describe spatiotemporal graph k-means (STGkM), a novel, unsupervised method to cluster vertices within a dynamic network. Drawing inspiration from traditional k-means, STGkM finds both short-term dynamic clusters and a "long-lived" partitioning of vertices within a network whose topology is evolving over time. We provide an exposition of the algorithm, illuminate its operation on synthetic data, and apply it to detect political parties from a dynamic network of voting data in the United States House of Representatives. One of the main advantages of STGkM is that it has only one required parameter, namely k; we therefore suggest the range of this parameter and guidance on selecting its optimal value. We also give certain theoretical guarantees about the correctness of our algorithm.
(Received September 15, 2023)

## 1192-05-33473

Josiah Hudock*, Washington \& Jefferson College. Counting connected subsets of an $n \times 2$ rectangular grid. Preliminary report.
We consider dividing an $n \times 2$ rectangular grid into $k$ connected pieces. We count the number of ways to carry out this division and observe the inherent recurrence relations. We also produce closed-form expressions for these formulas for small values of $k$.
(Received September 18, 2023)

1192-05-33482
Rhianna Herlihy*, Worcester Polytechnic Institute. On the number of independent sets in various classes of bipartite graphs. Given a graph $G$, a subset $S \subset V(G)$ is independent if no two vertices in $S$ are adjacent in $G$. We let $i(G)$ denote the number of independent sets of any size in a graph, often referred to as the independent set index or the Fibonacci number of the graph. This talk focuses on bipartite graphs, giving exact values of $i(G)$ for some specific trees and provides a sharp bound for general bipartite graphs in terms of their girths. We additionally explore the inverse problem: given a natural number $n$, can we construct a graph $G$ such that $i(G)=n$ ? For this question, we call $n$ constructible if such a graph does exist. It turns out that not every natural number is constructible when we restrict ourselves to the class of trees. We present several results related to constructibility on trees and conjecture that there are infinitely many non-constructible numbers.
(Received September 18, 2023)

## 1192-05-33512

Will Perkins*, Georgia Tech. Searching for (Sharp) Thresholds in Random Structures: Where are We Now?.
Threshold phenomena in random structures are fundamental in several areas of mathematics and computer science. These phenomena include, among many others, percolation in mathematical physics, emergence of structural properties in random graphs in probabilistic combinatorics, and satisfiability of random constraint satisfaction problems in computer science. For several decades, researchers have studied thresholds in random structures both in specific cases and in generality, searching for universal behavior, pinpointing specific thresholds, and even describing scaling windows around certain thresholds. On the occasion of the recent proof of the Kahn-Kalai conjecture by Park and Pham, we survey the current landscape of our understanding of threshold phenomena, describing some recent developments in both the general theory and in specific models.
(Received September 19, 2023)

## 1192-05-33513

Erin Meger*, Queen's University. Exploring Generalized Complex Networks.
Complex Networks appear in applications all around us, from the internet to social networks. In this talk, we will define the Iterated Independent Model for Complex Networks. This model focuses on the relationships between node adjacencies and non-adjacencies. At each time-step, every node is either cloned or anti-cloned in any combination, where a clone preserves adjacencies, and an anti-clone adds adjacencies to all non-adjacent vertices. This represents the dynamic and ever changing nature of social connections. In this talk, we will discuss the diameter and domination of the graph, as well as show that every finite graph is a subset of the eventual model.
(Received September 19, 2023)

1192-05-33614
Ling Chen*, Occidental College, Zain Shields*, University of California, Berkeley. Analogues of Alder-Type Partition Inequalities for Partitions with Fixed Perimeters
In a 2016 paper, Straub proved an analogue to Euler's celebrated partition identity for partitions with a fixed perimeter. Later, Fu and Tang provided both a refinement and generalization of Straub's analogue to $d$-distinct partitions. They also prove a related result to the first Rogers-Ramanujan identity by defining two new functions, $h_{d}(n)$ and $f_{d}(n)$ for a fixed perimeter $n$, that resemble the preexisitng $q_{d}$ and $Q_{d}$ functions. Motivated by generalizations of Alder's ex-conjecture, we further generalize the work done by Fu and Tang by introducing a new parameter $a$, similar to the work of Kang and Park. We observe the prevalence of binomial coefficients in our study of fixed perimeter partitions and use this to develop a more direct analogue to $Q_{d}$. Using combinatorial techniques, we find Alder-type partition inequalities in a fixed perimeter setting, specifically a reverse Alder-type inequality
(Received September 24, 2023)

## 1192-05-33672

Toby Anderson*, Harvey Mudd College, Olivia Greinke, Transylvania University, Iskandar Nazhar*, University of Massachusetts Amherst, Luis Santori*, University of Central Florida. Construction Strategies for Modeling Lattice and Prism Graphs with Self Assembling DNA.
Motivated by recent advancements in nanotechnology and the discovery of new laboratory techniques using properties of complementary DNA strands, graph theory has recently become useful in the study of self-assembling DNA complexes. Branched molecules of DNA bond to each other with complementary strands of base pairs to form a target nanostructure. Using a flexible tile model, these structures can be modeled mathematically as graphs that are constructed from vertices with extending half-edges. These vertices are referred to as tiles, and their extending half-edges are labeled with bond-edge types that represent different extending sequences of DNA base pairs. Collection of unique tile types are referred to as a pot. In addition to finding optimal constructions for various graphs, our research focuses on searching for underlying relationships between the minimum number of bond-edge types and tile types needed to construct a graph across three scenarios of increasing restriction. In particular, one open question asks if and in what cases we can find optimal constructions for graphs which simultaneously minimizes the use of both tile and bond-edge types in the same pot. When tackling the question, our method involved exploring various lattice and extended-prism graphs including hexagonal lattices, stacked prisms, and web graphs. This poster presents results for constructing these families of graphs in addition to providing a potential answer to this open question.
(Received September 22, 2023)

## 1192-05-33673

Toby Anderson*, Harvey Mudd College, Olivia Greinke, Transylvania University, Iskandar Nazhar*, University of Massachusetts Amherst, Luis Santori*, University of Central Florida. Optimal Constructions for Low-Order Graphs With SelfAssembling DNA.
DNA self-assembly is an emerging area of study within the field of biology and nanotechnology with applications in targeted drug delivery, biomolecular computing, and biosensing. Branched molecules of DNA bond to each other with complementary strands of base pairs to form a target nanostructure. Using a flexible-tile model, these structures can be modeled mathematically as graphs that are constructed from vertices with extending half-edges. These vertices are referred to as tiles, and their extending half-edges are labeled with bond-edge types that represent different extending sequences of DNA base pairs. Collection of unique tile types are referred to as a pot. Most mathematical research using the flexible-tile model seeks to find optimal constructions for various graphs and graph families in an effort to maximize laboratory efficiency within three scenarios of increasing restriction. One open question in this field is whether we can find a construction for a given graph that optimizes bond-edge and tile types simultaneously. In exploring this question, we highlight potential relationships between the constructions of various low-order graphs. In particular, we provide complete results for all graphs of order five and below, and conclude that they all have a construction that optimizes tile and bond-edge types simultaneously in all scenarios. (Received September 22, 2023)

1192-05-33711
Jiah Jin*, The Cooper Union, Sajid Bin Mahamud*, Reed College, Angela Yuan*, University of Texas at Austin. Coalescing results for the distance Matrix. Preliminary report.
Given a graph $G$, the distance matrix $\backslash \mathrm{cD}(G)$ has rows and columns indexed by the vertices of $G$ with the $(u, v)$ entry being $\backslash \operatorname{dist}(u, v)$, the distance between vertices $u$ and $v$. Let $G \stackrel{B}{\circ} H$ be the graph resulting from gluing a copy of $H$ onto each vertex of $B \subseteq V(G)$ in $G$. We say $\left(G_{1}, B_{1}\right)$ and $\left(G_{2}, B_{2}\right)$ are gluing cospectral if $G_{1}{ }_{\circ}^{B_{1}} H$ and $G_{2}{ }^{B_{2}} \mathrm{O} H$ are distance cospectral (have the same eigenvalues for the distance matrix) for all graphs $H$. We give a sufficient condition for $\left(G_{1}, B_{1}\right)$ and $\left(G_{2}, B_{2}\right)$ to be gluing cospectral in terms of a block similarity matrix that commutes with the all ones matrix. This sufficient condition explains all gluing cospectral pairs up through 9 vertices. Moreover, when this condition holds, if $\left(G_{1}, B_{1}\right)$ and $\left(G_{2}, B_{2}\right)$ are gluing cospectral, then $\left(G_{1}, V\left(G_{1}\right) \backslash B_{1}\right)$ and $\left(G_{2}, V\left(G_{2}\right) \backslash B_{2}\right)$ are also gluing cospectral.

Hannah Graff*, Creighton University, Taylor Luck, Xavier University, Noah Owen, University of Kentucky. On The Generalized Distance Matrix.
Given a graph $G$ and a function $f$, the generalized distance matrix $\mathcal{D}(G, f)$ has rows and columns indexed by the vertices of $G$ with the $(u, v)$ entry being $f(\operatorname{dist}(u, v))$ where $\operatorname{dist}(u, v)$ is the distance between vertices $u$ and $v$. Two graphs $G$ and $H$ are said to be strongly distance cospectral if $D(G, f)$ and $D(H, f)$ have the same characteristic polynomial for all $f$. We give a sufficient condition for two graphs to be strongly distance cospectral in terms of simultaneous similarity of related 0-1 matrices. Moreover, we give a sufficient condition in terms of a block similarity matrix for two strongly distance cospectral graphs to remain strongly distance cospectral after gluing arbitrary graphs on to subsets of the vertices (i.e. coalescing). (Received September 25, 2023)

## 1192-05-33744

Kengbo Lu*, Cooper Union. The union-closed sets conjecture and Reimer's average set size theorem.
The Union-Closed Sets Conjecture, often attributed to Peter Frankl in 1979, remains an open problem in discrete mathematics. It posits that for any finite family of sets $\mathcal{S} \neq\{\varnothing\}$, if the union of any two sets in the family is also in the family, then there must exist an element that belongs to at least half of the member sets. We will refer to the italicized portion as the abundance condition. In 2001, David Reimer proved that the average set size of a union-closed family $\mathcal{S}$ must be at least $\frac{1}{2} \log _{2}|\mathcal{S}|$. When proving this result, he showed that a family being union-closed implies that the family satisfies certain conditions, which we will refer to as the Reimer's conditions. Therefore, as seen in the context of Tim Gowers' polymath project on the Union-Closed Sets Conjecture, it is natural to ask if all families that satisfy Reimer's conditions meet the abundance condition. A minimal counterexample to this question was offered by Raz in 2017. In this poster, we will discuss work from this past summer providing a general method to construct infinitely many such counterexamples with any fixed lower bound on the size of the member sets. Furthermore, we will discuss some properties related to these counterexamples, especially those focusing on how far these counterexamples are from being union-closed.
(Received September 25, 2023)

## 1192-05-33750

Kyle Patrick Mulkins*, Marist College. Tracking Train Tracks: "Can you get there from here?". Preliminary report. Constructing train track configurations can be a lot of fun, but can also be very frustrating. A lot of time may be spent setting up tracks to build an elaborate network containing multiple switches only to find that once a train starts moving around the configuration, the train is stuck on a portion of the track. In order to figure out why this happens and how to avoid this annoying situation, we use graph theory to model train track configurations and then analyze their traits to determine which of these configurations would lead to trains getting stuck.
(Received September 25, 2023)

## 1192-05-33755

Faun C C Doherty, Washington \& Jefferson College, Josiah Hudock, Washington \& Jefferson College, Madison A. Ramsey*, Washington \& Jefferson College. Plane Trees and Foldable Words. Preliminary report.
Plane trees have been used as a modeling tool in molecular biology, as RNA secondary structures can be abstracted by wrapping a sequence $w$, called a word, around a plane tree. Forming a general model of how complementary nucleotides bond to each other, words can be "folded" around plane trees such that complementary letters are matched as half edge labels. If a word $w$ can be folded around a plane tree, we say the plane tree is " $w$-valid". We investigate properties of words that have a specific number of $w$-valid plane trees focusing on those that are 3 -foldable.
(Received September 25, 2023)

## 1192-05-33761

Celia Kerr*, College of William \& Mary, Nicholas Mayers, North Carolina State University, Nicholas Russoniello, College of William \& Mary. Shellability of Kohnert posets. Preliminary report.
In his dissertation, A . Kohnert defined a set of operations on diagrams, i.e., arrays of finitely many cells in $\mathbb{N} \times \mathbb{N}$, called Kohnert moves. Given a diagram $D$, the set $K D(D)$ is the set of all diagrams that can be obtained via applications of Kohnert moves to $D$. Such sets are studied by those in representation theory, and as noted by S. Assaf, each $K D(D)$ can naturally be assigned a poset structure defined in terms of Kohnert moves. In this talk, we present recent results regarding the "ELshellability" - a condition satisfied by posets whose Hasse diagrams admit certain edge-labelings - of such posets. In particular, we provide necessary conditions for the EL-shellability of a general Kohnert poset $(K D(D), \leq)$ in terms of subconfigurations of cells in $D$. Moreover, in the case of Rothe diagrams, we give a full classification of "pure" EL-shellable Kohnert posets in terms of permutation patterns.
(Received September 25, 2023)

1192-05-33777
Kenneth S. Berenhaut, Wake Forest University, Liangdongsheng Lyu, Northwestern University, Yuxiao Zhou*, Wake Forest University. Bonded and Dissipative Edges in Networks. Preliminary report.
Networks can exhibit varying levels of informative community structure. In this poster, we introduce a novel partitioning method based on the concept of cohesion introduced recently by Berenhaut, Moore and Melvin [Proceedings of the National Academy of Sciences, 119 (4) (2022)]. We define a measure of dissipation of cohesion over edges, as well as an accompanying threshold which distinguishes dissipative from bonded edges. The resulting network of bonded edges provides a partitioning of the graph which does not suffer from resolution issues and is entirely parameter free. Importantly, the method can identify crucial and intermediary independent nodes, which can be informative in social and other settings. Applications to real world

## 1192-05-33779

Ameer Muhialdeen*, Brown University, Eric Myzelev*, University of Pennslyvania, Hongyi Zhang*, Haverford College. New Bounds on The Independence and Chromatic Numbers of Cyclic Van der Waerden Hypergraphs.
The cyclic Van der Waerden hypergraph is a hypergraph whose vertex set is $\mathbb{Z} / n \mathbb{Z}$, and whose edges are all $k$-term arithmetic progressions in $\mathbb{Z} / n \mathbb{Z}$ for positive integers $n, k$. The independence number of such a hypergraph $b(n, k)$ represents the cardinality of the largest subset of $\mathbb{Z} / n \mathbb{Z}$ that does not contain $k$-term arithmetic progressions, and we have the trivial bound $r-1 \leq b(n, r)<n$. No non-trivial general bounds have been given for $b(n, k)$. In this paper, we give an explicit non-trivial lower and upper bound on $b(n, k)$ for composite $n$ by decomposing $Z / n Z$ into prime cycles $Z / p Z$ 's via the Chinese remainder theorem. In particular, we reduce the problem of upper bounding $b(n, k)$ into a combinatorial problem about subsets of cartesian products of finite sets, to which we provide partial results. A related quantity is the Cyclic Van der Waerden Number $W_{c}(k, r)$, the smallest $N$ such that for all $n \geq N$, every $r$-coloring of the hypergraph has a monochromatic edge. We use a similar technique to find new lower bounds on $W_{c}(k, r)$, which improves previous results when $r \leq 7$. (Received September 26, 2023)

## 1192-05-33783

Kaioke Begay*, Central Washington University, Athaliah Mackewicz, Central Washington University. A Variation on Labeled Chip Firing: Simultaneous Chip Firing.
The labeled chip-firing game is a variation of one dimensional chip-firing in which each chip is assigned a numeric value. When a node is selected to fire, two chips are compared with one another and the chip with the greater value is sent to the right while the chip with the lesser value is sent to the left. It has been proven that starting with an even number of chips results in a sorted final configuration. Here a variation of labeled chip-firing in which sets of chips are selected to be compared is introduced. We show that while the chips do not sort in this variation, the largest two and smallest two chips cannot end up more than one away from their final sorted position. We further conjecture that no chip can end up more than one away form its final sorted position, and introduce theorems that may prove helpful in exploring this conjecture.
(Received September 25, 2023)

1192-05-33802
Ilha Hwang*, Undergraduate Researcher, Dallin Seyfried*, Undergraduate Researcher, John Warnock*, Undergraduate Researcher. Graph Modifications to Bound the Fiedler Value.
We explore a process of proving a new lower bound on algebraic connectivity of a graph by modifying a family of diameter three graphs. We also explore an analogous process for complement graphs of this family. We further prove properties of this complement family in relation to the original family and discuss conjectures to investigate in the future.
(Received September 26, 2023)

1192-05-33807
Matthew Taylor Cowen*, University of the Pacific. Induced Forests and Pseudoforests in Planar Graphs as an Avenue to the Four Color Theorem. Preliminary report.
The Four Color Theorem states that no more than four colors are needed to color the regions of a map such that no two regions that are adjacent share the same color. Despite its fame, this theorem is incredibly difficult to prove, and is the first major theorem in mathematics proven using a computer, and as such its proof is rejected by many mathematicians, although it has repeatedly been shown to hold true. We consider the following conjecture regarding induced forests in planar graphs, stating that in any planar graph $G$, there exists an induced forest containing at least half of the vertices of $G$. If this conjecture holds true, then it would, in turn, provide a proof of the four color theorem. The conjecture has already been shown to hold true for outerplanar graphs and triangle-free graphs, and we prove that for any planar graph $G$ with no edge adjacent triangles, such a graph may be partitioned into the pseudoforests $P_{1}$ and $P_{2}$. We go on to show that for any planar graph G with triangles $\delta_{1}, \ldots, \delta_{n}$, provided there exists at least one vertex in each $\delta_{i}$ with degree 2 , then $G$ may be partitioned into two induced forests. Additionally, we prove that for planar graphs possessing a bounded number of triangles, there exists an induced forest on at least half of the vertices. Using this result, we will conduct an inductive proof on the number of triangles in $G$, in hopes of finding an alternative proof to the Four Color Theorem.
(Received September 26, 2023)

## 1192-05-33810

Henry Chimal-Dzul, University of Notre Dame, Samuel H Dekleva*, University of Notre Dame, Antonio Dolojan, University of Notre Dame. Permutations and the n-Queens Problem. Preliminary report.
The $n$-queens problem and its toroidal variation concern the total number of ways of placing $n$ queens on an $n \times n$ (toroidal resp.) chessboard such that no two queens attack one another. These problems have intrigued mathematicians for more than 150 years and asymptotic bounds have been recently proven. In this work, solutions to the n-queens problem are identified with permutations on the symmetric group on $n$ elements. Throughout this approach, the cycle decomposition of solutions is studied from which bounds and new integer sequences (not reported in OEIS) are obtained.
(Received September 26, 2023)

Luke Green*, Brigham Young University, Mark Kempton, Brigham Young University, Davi Zacheu*, Brigham Young University. Construction and Properties of Cospectral Non-Backtracking Matrices of Graphs. Preliminary report. We investigate the relation between the generalized characteristic polynomial and the determinant of the non-backtracking matrix of a graph. Our result is a method of generating graphs which have cospectral non-backtracking matrices, by using GM-
switching. We investigate properties of these families of graphs, as well as properties of graphs which have cospectral nonbacktracking matrices and do not belong to these families. We compare the number of cospectral graphs with respect to adjacency, Laplacian, normalized Laplacian, sign-less Laplacian, Ihara, and non-backtracking matrices.
(Received September 27, 2023)

Olivia Elias*, University Of Colorado - Colorado Springs, Ian Farish*, California State Polytechnic University - Pomona, Josh Oti Kyei*, Morehouse College. Positive Semidefinite Leaky Zero Forcing: Preliminary Report. Preliminary report. The concept of zero forcing is a relatively new phenomenon among graph theory topics. As this concept provides a way to look at networks in computer science or physics, or even real-life examples such as resource distribution, there has been work to understand more about the nuances of zero forcing. Through existing research, our group has viewed plenty of different rules given for various graphs, and alternative ways of seeing the process through work such as using forts to graph systems. However, in these operations we have seen there are alternate forms of zero forcing, namely positive semi-definite (PSD) and leaky forcing, which changes a rule in regular forcing, but causes noticeable differences in the way a graph is forced. Our work involves discovering the new rules of PSD leaky forcing and understanding more complex ways to understand networks through these lenses of graph forcing in hopes of furthering the study for future endeavors. (Received September 26, 2023)

1192-05-33830
Marchelle Beougher*, Macalester College, Nila Cibu*, University of California, Berkeley, Cassie Ding*, Brown University, Steven Sofos DiSilvio*, Columbia University, Sasha Kononova*, University of California at Los Angeles, Chan Lee*, Williams College, Ralph E. Morrison, Williams College, Krish Singal*, Columbia University. Bananas, Eggs, and Chips: A Recipe for Graph Gonality.
In 2008, Baker pioneered the study of the gonality of algebraic curves in the context of discrete finite graphs. This concept, known as graph gonality, has since been extensively investigated, especially in relation to chip-firing games on graphs. The significance of this study lies in the fact that the gonality on metric graphs provides a lower bound for the associated algebraic curves, making the computation of graph gonality a valuable tool for examining algebraic curves. In our research, we delve into the gonality of specific families of knight's, king's, and bishop's graphs, including their toroidal variations. Recent studies have demonstrated that graph gonality can vary based on graph operations like subgraphs, minors, and uniform subdivisions (Josse van Dobben de Bruyn et al 2021). Our exploration of banana graphs, which are path graphs with possibly multiple edges, led to a new discovery about the ascertain property of graph gonality. Namely, we estabish that graph gonality can increase or decrease with the addition or deletion of a single edge by any integer through the use of Dhar's burning algorithm and modular arithmetic. Notably, while determining graph gonality is generally NP-hard, as proven by Gijswijt et al. in 2020, we formulated a recursive equation specifically for banana graphs. Beyond the realm of graph gonality, there is a growing interest in the computational complexity of its established lower bounds. Echavarria et al. in 2021 showed that scramble number $\operatorname{sn}(G)$, the strongest known lower bound on gonality, is NP-hard to compute. Despite the scramble being a promising certificate candidate, its size can grow exponentially with $n$. To further understand this certificate, we introduce the carton number of a graph, defined as the minimum size of a maximum order scramble. We prove via contradiction that carton number is at least $3 \operatorname{sn}(G)-n$ for graphs of maximum degree less than $\operatorname{sn}(G)$. Additionally, we show that scramble number cannot serve as polynomial-size NP certificates. This conclusion is based on a probabilistic approach, showing that bounded degree graphs on $n$ vertices with scramble number $\Omega\left(n^{1 / 2+\epsilon}\right)$ have carton number exponential in $n$.
(Received September 26, 2023)

## 1192-05-33869

Christopher Luke Uchizono*, University of the Pacific. Mutations of Quivers with Potential and Dimer Models. Preliminary report.
In this work, we aim to give a brief exposition to a relationship between mutations of quivers with potential and dimer models. Quiver mutation arose naturally from studying the flips of triangulations of regular $n$-gons and are combinatorial in nature. Dimer models can be interpreted as finite bipartite tilings of a compact oriented Riemann surface and arose from statistical physics. The work of Derksen, Weymann, and Zelevinsky provide an approach to lift quiver mutation to an algebraic setting by introducing the notion of a quiver with potential. Every dimer model has a naturally associated quiver with potential known as its dimer quiver. We show that a specific kind of transformation, known as urban renewal, of a dimer model induces mutation of the dimer quiver in a natural way with respect to quiver potential. (Received September 27, 2023)

## 1192-05-33885

Jean-Pierre Appel, Moravian University, Gabrielle Fischberg, Tufts University, Kyle Alexander Kelley, Kenyon College, Nathan Shank, Moravian University, Eliel Sosis*, University of Michigan. $k$-Total Bondage in Graphs. Preliminary report. Let $G=(V, E)$ be a finite undirected graph with no isolated vertices. A set $S \subseteq V$ is said to be a total dominating set of $G$ if every vertex in $V$ is adjacent to some vertex in $S$. The total domination number, $\gamma_{t}(G)$, is the minimum cardinality of a total dominating set in $G$. We define the $k$-total bondage to be the minimum number of edges to remove from $G$ so that the resulting graph has a total dominating number at least $k$ more than $\gamma_{t}(G)$. In this work we establish general properties of $k$ total bondage, exact values for certain graph classes including paths, cycles, and wheels, and obtain upper bounds for complete and complete bipartite graphs.
(Received September 27, 2023)

## 1192-05-33901

Vincent Bonini, California Polytechnic State University, Daniel Chamberlin, California Polytechnic State University, Stephen G Cook*, California Polytechnic State University, Parthiv Seetharaman, California Polytechnic State University, Tri Tran, California Polytechnic State University. Condensed Ricci Curvature on Paley Graphs and Their Generalizations.

Preliminary report.
We study a modified notion of Ollivier's Coarse Ricci Curvature on graphs first introduced by Lin, Lu, and Yau. We derive explicit formulas for the Ricci curvature on certain Paley graphs and their generalizations. Interestingly, our methods rely on the construction of families of complete subgraphs and our Ricci curvature formulas relate to the number of solutions to the equation $x^{k}+y^{k}=z^{k}$ over finite fields.
(Received September 27, 2023)

Aidan Bucko, University of Utah, Selvi Kara, Bryn Mawr College, Josie Blue Marshall, University of Utah, Kayden Perez, University of Utah, Kiyanna Porter, University of Utah, Lia Smith, University of Utah, Nicholas Van Fleet, University of Utah, Amelia Walden*, University of Utah, Deborah Wooton, University of Utah. Higher dimensional chip-firing games. Higher dimensional chip-firing games were first described by Felzenszwalb-Klivans in which the game would be played on a two-dimensional grid with an initial configuration of flows that started on the edges of the grid. The game would be played by rerouting, or "firing" through adjacent squares. The game is then finished when no more flows are able to fire and you are now left with a final configuration. Felzenszwalb-Klivans also introduced the idea of a topological hole where it would either act as a source or a sink for any flows on its edges. While examining conservative pulses around a topological hole, they found that it would always result in the same final configuration no matter what firing choices they made and they named this type of final configuration a Mayan Diamond. We have decided to further the research of Felzenszwalb-Klivans by exploring three main questions. First, how does the final configuration change when dealing with multiple large topological holes? Second, how far away from the topological hole can a conservative pulse be so that it still terminates at the Mayan Diamond? And third, how many different final configurations can a conservative pulse of $n$ units have without a topological hole present? We share some of our results related to these questions and also discuss potential future directions for our research. Our aim is to better understand the behavior of these games and their fascinating patterns. (Received September 29, 2023)

## 06 Order, lattices, ordered algebraic structures

## 1192-06-27844

Michael Robinson*, American University. Fast mixture separation using the Dowker complex.
Indepedent mixture models (IMMs) are widely used to represent data in which observations are drawn from several qualitatively different subpopulations. The probability of an observation is then formed as a sum of conditionally independent distributions called classes, the idea being that the classes and the subpopulations are the same. If one has observations drawn from an IMM, there is a tradeoff between accuracy (representing the probability of the mixture exactly) and explainability (identifying the most salient classes). Most of the literature on recovering IMMs from data attempts to find a decomposition using expectation maximization. The resulting process is called latent class analysis (LCA). Under broad theoretical conditions, LCA yields a decomposition of the empirical distribution into classes. LCA posits that we must focus on accuracy as our primary aim. As such, LCA tends to result in too many low-probability classes, and the resulting mixture is hard to interpret. Explainability degrades with an increasing number of classes. What a data analyst really needs is not necessary captured by the most accurate model, but rather one that is both accurate and explainable. Instead, let us permit coarser decompositions that bound classes. By doing this, we entertain a plurality of inferred IMMs, as the problem becomes substantially underdetermined. We argue explainable models are parsimonious: they contain the minimum number of classes to characterize the data with acceptable error. Indeed, the minimum error/maximum likelihood decomposition is unlikely to be parsimonious if the number of classes is not known. This talk will explain that there is an ordering on inferred IMMs via refinement, and that this is none other than the face partial order on the Dowker complex. Using this topological fact, one can ask for the most parsimonious such inferred IMM. This IMM fits the data as well as LCA does when the data are undersampled, as is often the case. But this most parsimonious IMM gives much better explainability, since there is just the minimal number of classes. Moreover there are topological guarantees about the discovered classes with respect to the actual (latent) classes, namely that the latent ones are a refinement of the discovered ones.
(Received August 21, 2023)

## 1192-06-28311

Amit Patel*, Colorado State University. Möbius Homology.
This paper introduces Möbius homology, a homology theory for representations of finite posets into abelian categories. While the connection between poset topology and Möbius functions is classical, we establish a direct connection between poset topology and Möbius inversions. More precisely, the Möbius homology categorifies the Möbius inversion because its Euler characteristic is equal to the Möbius inversion of the dimension function of the representation. We also introduce a homological version of Rota's Galois Connection Theorem which relates the Möbius homology over two posets connected by a Galois connection. Our main application is to persistent homology over general posets. We show that under one definition, the persistence diagram is an Euler characteristic over a poset of intervals and hence Möbius homology is a categorification of the persistence diagram. This provides a new invariant for persistent homology over general posets. Finally, we use our homological Rota's Galois Connection Theorem to prove several results about the persistence diagram. (Received August 29, 2023)

1192-06-31241
Etienne Phillips*, North Carolina State University. On Kohnert Posets That are Lattices. Preliminary report.
Kohnert polynomials are a relatively new family of polynomials appearing in algebraic combinatorics and related areas. They are indexed by diagrams, which are collections of unit cells, $D \subseteq \mathbb{N} \times \mathbb{N}$. Given a diagram $D$, we obtain other Kohnert diagrams by moving cells down following certain rules, giving an induced poset, $K D(D)$. This poset structure was studied by Sami Assaf who noted in particular that $K D(D)$ were not lattices in general. I investigate this question, namely: when is $K D(D)$ a lattice? First, I find and prove some special cases of "simple" diagram are distributive lattices. Generalizing these results, I obtain that if $D$ contains only one cell per column, $K D(D)$ is a lattice if and only if $D$ doesn't contain a particular sub-diagram. Using this foundation, I give several other families of more complex diagrams that are also lattices, and show
how these general results can apply to the families of diagrams Sami Assaf studied. I also consider other properties, for example I show that for a diagram $D$ containing no cells in rows 1 or 2 , then $K D(D)$ is a modular lattice if and only if it is "simple," and conjecture a total classification of diagrams with modular posets.
(Received September 25, 2023)

1192-06-31583
Karl-Dieter Crisman, Gordon College, Erin McNicholas*, Willamette University, Kathryn Nyman, Willamette University, Michael Orrison, Harvey Mudd College. Voting on Relations, from Kemeny to Borda. Preliminary report. The Kemeny rule has a natural interpretation in terms of minimizing the distance between the winner(s) of an election and the ballot profile. While the Borda count is not defined in terms of distances, it, like Kemeny, is determined by pairs information, and so can be reframed as a minimization rule over a different set of alternatives. In this talk, we study both Borda and Kemeny from a geometric perspective on distance interpretations. These interpretations are more fully realized when we loosen the constraints on the space of voter inputs. By allowing voters to indicate preferences on general relations among underlying alternatives, we are able to embed many common profile spaces as subspaces of a higher dimensional, highly symmetric space. This symmetry leads to elegant and simple eigenspace decompositions of the space of profiles, shedding light on results that seem less natural in restricted situations, such as voting on full rankings or partially ordered sets. This allows us to not only compare Borda and Kemeny, but to use the eigenvalues in reverse as tuning parameters to create and analyze methods which accentuate various features of the ballot, regardless of the set of ballots we allow. (Received September 11, 2023)

1192-06-32375
Gabrielle De Micheli*, UCSD. Reductions from module lattices to free module lattices, and application to dequantizing module-LLL.
In this presentation, we give evidence that free modules (i.e., modules which admit a basis) are no weaker than arbitrary modules, when it comes to solving cryptographic algorithmic problems (and when the rank of the module is at least 2 ). More precisely, we show that for three algorithmic problems used in cryptography, namely the shortest vector problem, the Hermite shortest vector problem and a variant of the closest vector problem, there is a reduction from solving the problem in any module of rank $n \geq 2$ to solving the problem in any free module of the same rank. As an application, we show that this can be used to dequantize the LLL algorithm for module lattices presented by Lee et al. (Asiacrypt 2019).
(Received September 12, 2023)

1192-06-33111
Christian Corbett*, Florida Atlantic University. Exploring Topological Insights: $d$-elements, Hull-Kernel and Inverse Topology in Frames. Preliminary report.
Given an algebraic frame $L$ with the finite intersection property (FIP), we say an element $x \in L$ is a $d$-element if $x=\bigvee\left\{c^{\Perp}: c \in \mathfrak{K} L, c \leq x\right\}$. The collection of all $d$-elements of $L$ is denoted as $d L$, By Zorn's lemma, there exist maximal $d$ elements, and so we may equip the maximal elements of $d L$ with the hull-kernel topology, and we call this topology $M a x(d L)$. In this presentation, we will discuss some of the topological properties of $\operatorname{Max}(d L)$; namely, when $\operatorname{Max}(d L)$ is Hausdorff and when $\operatorname{Max}(d L)$ is zero-dimensional.
(Received September 13, 2023)

1192-06-33312
Tom A Bohman, Carnegie Mellon University, Fei Peng*, National University of Singapore. A construction for Boolean cube Ramsey numbers.
Let $Q_{n}$ be the poset that consists of all subsets of a fixed $n$-element set, ordered by set inclusion. The poset cube Ramsey number $R\left(Q_{n}, Q_{n}\right)$ is defined as the least $m$ such that any 2-coloring of the elements of $Q_{m}$ admits a monochromatic copy of $Q_{n}$. The trivial lower bound $R\left(Q_{n}, Q_{n}\right) \geq 2 n$ was improved by Cox and Stolee, who showed $R\left(Q_{n}, Q_{n}\right) \geq 2 n+1$ for $3 \leq n \leq 8$ and $n \geq 13$ using a probabilistic existence proof. In this paper, we provide an explicit construction that establishes $R\left(Q_{n}, Q_{n}\right) \geq 2 n+1$ for all $n \geq 3$. The best known upper bound, due to Lu and Thompson, is $R\left(Q_{n}, Q_{n}\right) \leq n^{2}-2 n+2$. (Received September 13, 2023)

1192-06-33768
Anna Hugo, Davidson College, Nzingha Joseph*, Carleton College, Anna Pun, CUNY Baruch College. The Stable Tamari Order.
The classical Tamari order is a poset of Catalan objects that is useful across algebraic combinatorics. Recent work in this area has been motivated by Mark Haiman's study of trivariate diagonal harmonics using the Tamari lattice. Early generalizations of the classical Tamari order include the $r$-Tamari order developed by Francois Bergeron, a poset of $r$-Dyck paths represented by their area sequences. Motivated by the construction of the $r$-Tamari order, we present a new poset called the stable Tamari order, a generalization of the classical Tamari order to all non-negative integer sequences. We draw connections between the algebraic and combinatorial properties of the stable Tamari order, showing possible links to graph theory with a preliminary result. We also present discoveries and conjectures about the relative sizes of lower order ideals and maximal chains between sequences in our poset. This poster is based on work conducted at the NYC Discrete Math REU at Baruch College (CUNY), in Summer 2023. This work was mentored by Professor Anna Pun (Baruch College), and conducted in collaboration with Anna Hugo (Davidson College).
(Received September 25, 2023)

## 08 General algebraic systems

Marion Scheepers*, Boise State University. Magic Squares. Preliminary report.
Magic squares have been considered by mathematicians for millenia. In these works magic squares with integer entries, i.e. magic squares over the integers, are studied. We generalize the concept to a broader range of algebraic platforms, and generalize some known results on magic squares over the integers to the broader framework.
(Received September 11, 2023)

1192-08-33089
Danielle C Bowerman*, Missouri University of Science and Technology, Matt Insall, Missouri University of Science and Technology. Loeb Measures and Universal Algebra. Preliminary report.
In this talk, we will discuss the properties of an algebra (A, F) where F is a family of operations on A of either finitary or infinitary type, set A, as well as all of its powers, are endowed with measures, and the operations are compatible with the given measures. We will focus on properties related to their associated Loeb measures. Our natural notion of compatibility of the operations will be that they are measurable functions. In particular, we will focus on some specific examples. (Received September 13, 2023)

## 1192-08-33658

Charles Thomas Fanning*, Lewis University. Bounds for Anti - Associative Magmas.
A magma is a set together with a closed binary operator. Some subclasses of magmas are well studied, such as quasigroups semigroups, and monoids; however, the more general classes of magmas are not well studied. In this paper, we attempt to construct a lower bound for the number of anti - associative magmas of order $n$. We also construct a lower and upper bound for a subclass of the anti - associative magmas of order n by considering a related class of graphs.
(Received September 22, 2023)

## 11 Number theory

## 1192-11-25406

Terence Tao*, UCLA. Correlations of Multiplicative Functions.
The Liouville function assigns a sign to each natural number: +1 if the number is the product of an even number of primes, and -1 if the number is the product of an odd number of primes. A famous conjecture of Chowla asserts consecutive values of the Liouville function are asymptotically uncorrelated to each other; this can be viewed as a simplified analog of the twin prime conjecture. This conjecture is still open today, but significant progress has been made towards it; while we still cannot quite detect cancellations in the Liouville function at small enough scales to imply the Chowla conjecture, we have been able to see such cancellations in slightly larger scales. Nevertheless, there still appears to be a key ingredient missing to complete the story. We survey some of the recent developments in this area.
(Received June 05, 2023)

## 1192-11-25901

Fred B. Holt*, T-Mobile. Patterns among the Primes: a study of Eratosthenes sieve as a discrete dynamic system. We study the gaps between primes by studying Eratosthenes sieve as a discrete dynamic system. At each stage of the sieve there is a cycle of gaps $G\left(p^{\wedge} \#\right)$ of length $\backslash p h i\left(p^{\wedge} \#\right)$ and span $p^{\wedge} \#$. A recursion produces the next cycle $G\left(p \_\{k+1\}^{\wedge} \#\right)$ from $\mathrm{G}\left(\mathrm{p} \mathrm{k}^{\wedge} \#\right)$. We model the recursion across these cycles of gaps in Eratosthenes sieve as a discrete dynamic system. We produce exact population models for those gaps $g<2 p_{k+1}$ among candidate primes. We prove a Polignac result that every gap $g=2 k$ arises in these cycles of gaps and produce asymptotic relative populations for every gap. We show that computer enumerations of gaps among primes will be biased by transient stages in the evolutions of these populations of gaps. Every admissible $k$-tuple (constellation of gaps) arises and persists in the cycles of gaps. This constructive approach provides coordinates for interesting constellations, including those for consecutive primes in arithmetic progression and Engelsma's counterexamples to the convexity conjecture for primes. The prime numbers are precisely those candidates that survive the sieve, and the gaps between primes are the gaps that survive the sieve. So after establishing the results for gaps and constellations in Eratosthenes sieve, we study the mechanisms for survival. The samples of gaps within the horizons of survival agree with the relative populations of the gaps at that stage of the sieve. This study is an extended example in combinatorial number theory. The results within Eratosthenes sieve are exact and deterministic, but our studies for survival introduce statistical estimates. Taken as a whole then, this work establishes strong results for gaps and constellations within Eratosthenes sieve, that provide intuitive support for the corresponding results between primes.
(Received July 14, 2023)

1192-11-25923
Ken Ono*, University of Virginia. Counting finite field matrix points on curves and surfaces.
Here we count finite field "matrix points" on certain elliptic curves and $K 3$ surfaces. For example, for every $n \geq 1$, we consider the matrix elliptic curves

$$
B^{2}=A\left(A-I_{n}\right)\left(A-a I_{n}\right),
$$

where $(A, B)$ are commuting $n \times n$ matrices over a finite field $\mathbb{F}_{q}$ and $a \neq 0,1$ is fixed. Our formulas are assembled from special values of finite field hypergeometric functions and $q$-multinomial coefficients. We use these formulas to prove Sato-Tate distributions for the error terms for matrix point counts for these curves and some families of $K 3$ surfaces. (Received July 15, 2023)

Bogdan Felix Jones*, Walter Payton College Preparatory High School, Chicago, Illinois, USA. Gaussian primes with a pseudoprime coordinate. Preliminary report.
We study a pattern in the sequence of primes, motivated by two letters of Pierre de Fermat from 1640: primes of the form $c^{2}+n^{2}$, where $c, n$ are positive integers and $c$ is a Carmichael number or a pseudoprime. We prove that such primes exist and that there are fewer primes of the form $c^{2}+n^{2}$, where $c$ is a Carmichael number or a pseudoprime, than there are primes of the form $\ell^{2}+n^{2}$, where $\ell$ is another prime. Our findings complement those of Étienne Fouvry and Henryk Iwaniec from 1997 and of Peter Cho-Ho Lam, Damaris Schindler, and Stanley Yao Xiao from 2020, regarding Gaussian primes with a prime coordinate, and echo results of Paul Erdös from 1956 and of Carl Pomerance from 1981 regarding Carmichael numbers and pseudoprimes, respectively.
(Received July 20, 2023)

## 1192-11-26266

Sarah Days-Merrill*, University of Vermont. Generalizing the Polynomial Ring Learning with Errors Problems for NonMonogenic Number Fields. Preliminary report.
Over the last twenty years, lattice-based cryptosystems have gained interest due to their level of security against attacks from quantum computers. The main cryptosystems are based on the hardness of Ring Learning with Errors (RLWE); in this poster, we will present on a variation called Polynomial Ring Learning with Errors (PLWE). Let $f(x) \in \mathbb{Z}[x]$ be a monic irreducible polynomial of degree $n$ and $q$ be a prime such that $f(x)(\bmod q)$ factors completely into linear terms. Consider the polynomial ring $P=\mathbb{Z}[x] /\langle f(x)\rangle$, then $P / q P=\mathbb{F}_{q}[x] /\langle f(x)\rangle$. Given a secret polynomial $s(x) \in P / q P$, PLWE samples are of the form $(a(x), b(x)) \in P / q P \times P / q P$ where $a(x)$ is drawn uniformly at random and $b(x)=a(x) s(x)+e(x)$ for a "small" error term $e(x) \in P / q P$. To draw a small error, we draw the coefficients of $\tilde{e}(x) \in P$ according to a discrete Gaussian distribution over the integers. When a number field $K$ defined by $f(x)$ is monogenic, the polynomial ring $P$ is isomorphic to the ring of integers $\mathcal{O}_{K}$ of $K$. So the process of sampling the error terms amounts to choosing a "small" vector in $\mathcal{O}_{K}$ and then reducing it modulo $q$. The PLWE problem is not typically defined for non-monogenic number fields. However, the DedekindKummer Theorem tells us that while $P$ is not isomorphic to $\mathcal{O}_{K}$ in most cases, $\mathcal{O}_{K} /(q) \cong \mathbb{F}_{q}[x] /\langle f(x)\rangle$ whenever $q$ does not divide the index of $P$ in $\mathcal{O}_{K}$. Our work studies the possibility of sampling "small" error vectors in $\mathbb{F}_{q}[x] /\langle f(x)$ directly to expand the PLWE problems to a wider class of rings of integers. (Received July 23, 2023)

## 1192-11-26475

Sky Pelletier Waterpeace*, Unaffiliated. A Novel Generalization of the Liouville Function $\lambda(n)$ and a Convergence Result for the Associated Dirichlet Series.
We introduce a novel arithmetic function $w(n)$, a generalization of the Liouville function $\lambda(n)$, as the coefficients of a Dirichlet series, and as a special case of a parametrized family of functions $w_{m}(n)$. We prove some useful special properties of these arithmetic functions and then focus on convergence of their Dirichlet series. In particular, we show that each function $w_{m}(n)$ injectively maps $\mathbb{N}$ into a dense subset of the unit circle in $\mathbb{C}$ and that $F_{m}(s)=\sum_{n} \frac{w_{m}(n)}{n^{s}}$ converges for all $s$ with $\mathfrak{R}(s) \in\left(\frac{1}{2}, 1\right)$. Finally, we show that the family of functions $w_{m}(n)$ converges to $\lambda(n)$ and that $F_{m}(s)$ converges uniformly in $m$ to $\sum_{n} \frac{\lambda(n)}{n^{s}}$, implying convergence of that series in the same region and thereby proving a particularly interesting property about a closely related function.
(Received September 15, 2023)

1192-11-26632
Ken Ono*, University of Virginia. Distributions on integer partitions.
In this talk the speaker will discuss statistical distributions on integer partitions. The results include Benford laws, Dyson's ranks, and the distribution of parts and hook lengths. This is joint work with Michael Griffin, Larry Rolen and Wei-Lun Tsai. (Received August 02, 2023)

## 1192-11-26763

Jon Frederick Grantham*, IDA/CCS. No new Goormaghtigh primes up to $10^{500}$. Preliminary report.
The Goormaghtigh conjecture states that the only two numbers which have non-trivial representations as repunits to two different bases are 31 and 8191 . We show that no other primes less than $10^{500}$ satisfy that condition.
(Received August 04, 2023)

1192-11-26783
Nickolas Andersen, Brigham Young University, Clayton Williams*, University of Illinois at Urbana-Champaign. An Infinite Family of Vector Valued Mock Theta Functions.
We exhibit an infinite family of vector-valued mock theta functions indexed by positive integers coprime to 6 . These are built from specializations of Dyson's rank generating function and related functions studied by Watson, Gordon, and McIntosh. The associated completed harmonic Maass forms transform according to the Weil representation attached to a rank one lattice. This strengthens a 2010 result of Bringmann and Ono and a 2019 result of Garvan.
(Received August 04, 2023)
digits.
For any positive integer $n$, let $s_{b}(n)$ be the sum of the digits of $n$ in its base- $b$ representation. We call $n$ a $b$-Niven number if $s_{b}(n)$ divides $n$. In this project, we focus on arithmetic progressions of positive integers. In particular, we study the maximum length of an arithmetic progression where every term is a $b$-Niven number. We also show that every arithmetic progression of infinite length contains infinitely many $b$-Niven numbers.
(Received August 09, 2023)

## 1192-11-27102

Marie-Hélène Tomé*, Duke University. Arithmetic of Hecke L-functions of quadratic extensions of totally real fields. Deep work by Shintani in the 1970's describes Hecke $L$-functions associated to narrow ray class group characters of totally real fields $F$ in terms of what are now known as Shintani zeta functions. However, for $[F: \mathbb{Q}]=n \geq 3$, Shintani's method was ineffective due to its crucial dependence on abstract fundamental domains for the action of totally positive units of $F$ on $\mathbb{R}_{+}^{n}$, so-called Shintani sets. These difficulties were recently resolved in independent work of Charollois, Dasgupta, and Greenberg and Diaz y Diaz and Friedman. For those narrow ray class group characters whose conductor is an inert rational prime in a totally real field $F$ with narrow class number 1, we obtain a natural combinatorial description of these sets, allowing us to obtain a simple description of the associated Hecke $L$-functions. As a consequence, we generalize earlier work of Girstmair, Hirzebruch, and Zagier, that offer combinatorial class number formulas for imaginary quadratic fields, to real and imaginary quadratic extensions of totally real number fields $F$ with narrow class number 1 . For such extensions, our work may be viewed as an effective affirmative answer to Hecke's Conjecture that the relative class number has an elementary arithmetic expression in terms of the relative discriminant. (Received August 10, 2023)

## 1192-11-27129

Samantha Platt*, University of Oregon. Visual Study of Gaussian Periods and Analogues,
Gaussian periods have been studied for centuries in the realms of number theory, field theory, cryptography, and elsewhere. However, it was only within the last decade or so that they began to be studied from a visual perspective. By plotting Gaussian periods in the complex plane, many interesting and insightful patterns can be seen, leading to various conjectures and theorems about their properties. In this talk, we offer a description of Gaussian periods, along with examples of the structure that can occur when plotting them in the complex plane. In addition to this, we offer two ways in which this study can be generalized to other situations-one relating to supercharacter theory, the other relating to class field theory. (Received August 10, 2023)

## 1192-11-27179

Eleanor McSpirit*, University of Virginia. An Infinite Family of Quantum Modular 3-Manifold Invariants.
In 1999, Lawrence and Zagier established a connection between modular forms and the Witten-Reshetikhin-Turaev invariants by constructing $q$-series whose radial limits at roots of unity recover these invariants for particular manifolds. These $q$-series gave rise to some of the first examples of quantum modular forms. Using a 3 -manifold invariant recently developed Akhmechet, Johnson, and Krushkal, one can obtain an infinite family of quantum modular invariants which realize the series of Lawrence and Zagier as a special case. This talk is based on joint work with Louisa Liles. (Received August 11, 2023)

## 1192-11-27240

Olivia Beckwith*, Tulane University, Martin Raum, Chalmers Technical University, Olav Richter, University of North Texas. Imaginary quadratic fields with $\ell$-torsion-free class groups and specified split primes.
We use Zagier's weight 3/2 Eisenstein series to prove results on the classification of Ramanujan-type congruences for Hurwitz class numbers. As an application, we show that for any odd prime $\ell$ and finite set $S$ of odd primes, there exists an imaginary quadratic field which splits at each prime in $S$ and has class number indivisible by $\ell$. This result is in the spirit of results by Bruinier, Bhargava (when $\ell=3$ ) and Wiles, but the methods are completely different.
(Received August 12, 2023)

## 1192-11-27267

Henri Darmon*, McGill University. Fourier Coefficients of Modular Forms.
Modular forms are holomorphic functions of a complex variable that satisfy an inordinate amount of symmetry: in particular, they are periodic under translation and therefore admit fourier expansions. The coefficients in these expansions give rise to a remarkable variety of arithmetically interesting sequences, related, notably, to the combinatorics of the partition function, representation numbers of quadratic forms, elliptic curves, and two dimensional representations of the automorphism group of the field of algebraic numbers. I will survey some of these classical connections and explain how the coefficients of certain first-order deformations of modular forms also carry deep information about algebraic cycles and abelian extensions of number fields.
(Received August 13, 2023)

1192-11-27365
James Leng*, University of California, Los Angeles. The Equidistribution of Nilsequences. Preliminary report. Let $G$ be a nilpotent Lie group, and let $\Gamma$ be a discrete subgroup of $G$ whose topological quotient $G / \Gamma$ is compact. Certain challenges in multiplicative number theory and arithmetic combinatorics involve an equidistribution theory on nilmanifolds. This theory pertains to the analysis and categorization of orbits within the space $G / \Gamma$. In 2012 , Green and Tao established a significant result in this area by proving a quantitative equidistribution theorem for nilmanifolds, presenting a polynomial rate of equidistribution. In this discussion, we will explore a recent advancement in the realm of quantitative equidistribution, which improves the bounds of Green and Tao. Additionally, we will outline various implications of this in both arithmetic
combinatorics and multiplicative number theory.
(Received August 14, 2023)

1192-11-27485
Joshua Harrington*, Cedar Crest College. Exceptional Totient Numbers.
A positive integer $n$ is called an exceptional totient number if the set
$R_{e}(n)=\{x \in \mathbb{Z}: 1 \leq x<n, \operatorname{gcd}(n, x)=\operatorname{gcd}(n, x-1)=1\}$ can be partitioned into two disjoint subsets of equal sum. Take, for example, $R_{e}(7)=\{2,3,4,5,6\}$. This can be partitioned into the subsets $\{2,3,5\}$ and $\{4,6\}$ whose elements each add to 10 . In this talk, we provide a complete classification of exceptional totient numbers.
(Received August 15, 2023)

## 1192-11-27591

Tewodros Amdeberhan, Tulane University, George E. Andrews, Pennsylvania State University, Cristina Ballantine*, College of the Holy Cross. Hook Length and Symplectic Content in Partitions.
The irreducible polynomial representations of the general linear group are indexed by partitions. The dimension of a representation is given by a formula involving the hook lengths and the contents of cells in the Young diagram of the indexing partition. There are also analogous formulas for irreducible polynomial representations of symplectic and orthogonal groups. I will discuss the combinatorial nature of identities involving hooks and the symplectic (and orthogonal) contents. These have been conjectured by Amdeberhan and are inspired by the Nekrasov-Okounkov hook-length formula.
(Received August 16, 2023)

## 1192-11-27592

Cristina Ballantine*, College of the Holy Cross, Mircea Merca, University Politehnica of Bucharest, Cristian-Silviu Radu,
Johannes Kepler University. Congruences for the number of 3 and 6-regular partitions and quadratic forms. Preliminary report. We denote by $b_{k}(n)$ the number of $k$-regular partitions of $n$, i.e., partitions with no parts divisible by $k$. Using quadratic forms, we find infinite families of Ramanujan-type congruences for $b_{3}(n)$ and $b_{6}(n)$. For 6 -regular partitions, the result is obtained via local obstructions to representations by quadratic forms. For 3 -regular partitions, we go beyond the local obstructions and investigate the number of representations of an integer by the relevant quadratic form. The difficulty arises from the need to investigate the form $x^{2}+a y^{2}$, where $a$ is not an Euler convenient number.
(Received August 16, 2023)

## 1192-11-27622

Alex Rice*, Milsaps College. Reciprocal sums and counting functions.
Motivated by the gentle exploration of the distribution of prime numbers typical of an undergraduate number theory course, as well as by a recent breakthrough result in arithmetic combinatorics, we explore connections between the counting function, which counts the number of elements up to a given threshold, and the reciprocal sum function, which adds the reciprocals of the elements up to a given threshold, for subsets of the natural numbers. This talk is based on an American Mathematical Monthly article that received a 2023 Paul R. Halmos-Lester R. Ford award.
(Received August 16, 2023)

1192-11-27675
Mayank Pandey*, Princeton University. Squarefree numbers in short intervals and short arithmetic progressions to smooth moduli. Preliminary report.
We discuss some results on the distribution of squarefree numbers in short intervals and short arithmetic progressions to smooth moduli.
(Received September 01, 2023)

## 1192-11-27702

Sergei Gukov*, California Institute of Technology. Going to the other side, in modularity, algebra, and topology.
I will describe a peculiar bijection between a large class of $q$-series with integer coefficients that is a vast generalization of the pairing between mock modular forms and false theta-functions. It includes pairs of $q$-series associated to arbitrary graphs whose vertices are labeled by integers and characters of logarithmic vertex algebras in positive / negative Kazhdan-Lusztig zones. Both of these families have relation to quantum groups. This aspect will be discussed in a companion talk at the AMS Special Session SS27A on Modular Tensor Categories and TQFTs beyond the Finite and Semisimple. (Received August 18, 2023)

1192-11-27711
Kim Klinger-Logan*, Kansas State University. Applications of differential equations in automorphic forms to string theory. Certain eigenvalue problems involving the invariant Laplacian on moduli spaces have potential applications to scattering problems in physics. Green, Russo, Vanhove, et al., discovered the behavior of gravitons (hypothetical particles of gravity represented by massless string states) is also closely related to eigenvalue problems for the Laplace-Beltrami operator on various moduli spaces. In this talk we will examine applications and results related to solutions $f$ of $(\Delta-\lambda) f=E_{a} E_{b}$ where $E_{s}$ is a non-homorphic $G L(2)$ Eisenstein series and $\Delta=y^{2}\left(\partial_{x}^{2}+\partial_{y}^{2}\right)$ on $S L_{2}(\mathbb{Z}) \backslash S L_{2}(\mathbb{R}) / S O_{2}(\mathbb{R})$. One such interesting finding from this work is a family of identities relating convolution sums of divisor functions to Fourier coefficients on modular forms. This work is in collaboration with Ksenia Fedosova, Stephen D. Miller, Danylo Radchenko, and Don Zagier. (Received August 18, 2023)

Giorgos Kotsovolis, Princeton University, Katharine Woo*, Princeton University. Prime number theorems for polynomials from homogeneous dynamics. Preliminary report.
We establish a new class of examples of the multivariate Bateman-Horn conjecture by using tools from homogeneous dynamics. Let $V$ be a linear space with a $\mathbb{Z}$-structure and $\varrho: G \rightarrow \mathrm{GL}(V)$ be a $\mathbb{Q}$-representation, where $G$ is a connected and simply connected algebraic group such that $G(\mathbb{R})$ has no compact factors. Let $F$ be an invariant polynomial under the group action such that $\{v \in V: F(v)=1\}$ is the union of finitely many orbits $v_{0} G$ for $v_{0} \in V$. Then under certain conditions on $\operatorname{stab}_{v_{0}}(\mathbb{R})$ and local equidistribution properties for the values of $F$, we derive an asymptotic for

$$
\pi_{-}\{V, F\}(T)=\#\{v \in V:|v| \leq T, F(v) \text { is prime }\}
$$

that matches the Bateman-Horn prediction. Examples of such polynomials include the determinant polynomial, the Pfaffian, and the determinant polynomial on the space of symmetric matrices.
(Received August 18, 2023)

## 1192-11-27808

Chengyang Bao*, The University of Chicago. Computation of finite quotients of crystalline deformation rings via Taylor-WilesKisin patching method.
Let $\bar{\rho}: G_{\mathbb{Q}_{p}} \rightarrow \mathrm{GL}_{2}\left(\overline{\mathbb{F}}_{p}\right)$ be an absolutely irreducible representation. Fix an integer $n$, and let $R_{n}$ denote the crystalline deformation ring whose $\overline{\mathbb{Q}}_{p}$-points are crystalline with Hodge-Tate weights $[0, n-1]$ and with fixed determinant. On the one hand, Kisin has showed that $R_{n}$, if nontrivial, has formally smooth generic fibers for all $n$ and he has computed the presentation of $R_{n}$ for $1 \leq n \leq 2 p-1$. As $n$ grows by multiples of $p-1, R_{n}$ can be more complicated, in the sense that the Hilbert-Samuel multiplicity of the special fiber of $R_{n}$ is increasing (not strictly) and is unbounded as predicted by the BreuilMézard conjecture. We present in this talk a way to compute finite quotients of $R_{n}$ by computing certain global Hecke algebras via the Taylor-Wiles-Kisin patching method under the assumption that $\bar{\rho}$ has a nice globalization. With the data of finite quotients in some examples, we can answer the question whether the dimension of the tangent space of the special fiber of $R_{n}$ can be 3 as $n$ grows in those cases. (The dimension is always bounded by 3.)
(Received August 20, 2023)

## 1192-11-27921

Theresa Anderson*, Carnegie Mellon. Some recent developments in arithmetic statistics. Preliminary report.
Arithmetic statistics, or the counting of objects of algebraic interest, has seen a lot of development in the last twenty years. We will take a glimpse into just a few of the advances that have happened, with an emphasis on the wide interplay of tools and techniques that have entered this playing field.
(Received August 22, 2023)

1192-11-27988
Deewang Bhamidipati*, UC Santa Cruz. A p-adic analytic Brauer group \par. Preliminary report.
The Brauer group of a local field plays a central role in class field theory. On one hand, the Brauer group of a field $K$ classifies central simple algebras, up to Morita equivalence. On the other hand, the Brauer group $\operatorname{Br}(K)$ is identified with the Galois cohomology $H^{2}\left(\mathrm{Gal}_{K}, \bar{K}^{\times}\right)$. In this talk, we introduce a variant in which the algebraic closure of a $p$-adic field is replaced by its completion $\mathbb{C}_{p}$. This locates the Brauer group $\operatorname{Br}(K)$ within a larger $p$-adic analytic Brauer group, interacting with $p$-adic
Hodge theory. \par
(Received August 23, 2023)

## 1192-11-28001

Arseniy Sheydvasser*, Bates College. Is Unreasonable Slightness a General Phenomenon?.
When is an arithmetic group generated by its upper and lower triangular matrices? This question was originally considered by algebraists like P.M. Cohn, but has gained some interest in more recent times in connection to model theoretic questions. In this talk, we will consider this question for three families of arithmetic groups: one for $\mathrm{SO}(1,3), \mathrm{SO}(1,4)$, and $\mathrm{SO}(1,5)$, respectively. For each of these families, we'll show that there is a simple, natural, and surprising dividing line having to do with the presence or absence of an analog of the classic Euclidean algorithm on a corresponding maximal discrete ring. For the simplest family, these are discrete subrings of the complex numbers; for the other two, they are discrete subrings of the quaternions!
(Received August 24, 2023)

## 1192-11-28015

Thomas Luckner*, Flagler College. Consecutive primes which are widely digitally delicate and Brier numbers.
Making use of covering systems and a theorem of D. Shiu, M. Filaseta and J. Juillerat showed that for every positive integer $k$ there exist $k$ consecutive widely digitally delicate primes. They also noted that for every positive integer $k$, there exist $k$ consecutive primes which are Brier numbers. We show that for every positive integer $k$, there exist $k$ consecutive primes that are both widely digitally delicate and Brier numbers. This is joint work with M. Filaseta and J. Juillerat. (Received August 24, 2023)

Tejasi Bhatnagar*, University of Wisconsin Madison. Size of isogeny classes of certain abelian varieties over finite fields. In this talk, we will discuss the size of the isogeny classes of certain abelian varieties over a finite field. There is an expected estimate that we get from the theory of "stratification" of the moduli space of abelian varieties. We will see some previous cases, including ordinary abelian varieties where the (lower) bound is confirmed. Moreover, we will talk about some new results in the case of non-ordinary Newton strata.
(Received August 24, 2023)

## 1192-11-28054

Erdenebileg Erdenebat*, Brigham Young University - Hawaii, Ka Lun Wong, Brigham Young University - Hawaii. The error term of the sum of digital sum functions in arbitrary bases.
Let $k$ be a non-negative integer and $q>1$ be a positive integer. Let $s_{q}(k)$ be the sum of digits of $k$ written in base $q$. In 1940 , Bush proved that $A_{q}(x)=\sum_{k \leq x} s_{q}(k)$ is asymptotic to $\frac{(q-1) x \log x}{2 \log q}$. In 1968, Trollope proved an explicit formula for the error term of $A_{2}(n-1)$, labeled by $-E_{2}(n)$, where $n$ is a positive integer. In 1975, Delange extended Trollope's result to an arbitrary base $q$ by another method and labeled the error term $n F_{q}\left(\frac{\log n}{\log q}\right)$. When $q=2$, the two formulas of the error term are supposed to be equal, but they look quite different. We proved directly that those two formulas are equal. More interestingly, Cooper and Kennedy in 1999 applied Trollope's method to extend $-E_{2}(n)$ to $-E_{q}(n)$ with a general base $q$, and we also proved directly that $n F_{q}\left(\frac{\log n}{\log 2}\right)$ and $-E_{q}(n)$ are equal for any $q$.
(Received August 24, 2023)

## 1192-11-28061

Yuya Murakami*, Kyushu University. False theta functions whose radial limits are quantum invariants.
Quantum invariants are important topological objects motivated by physics. They are also important in number theory since they relates to modular forms. In fact, the radial limit conjectures states that quantum invariants appear in radial limits of certain false theta functions. In this talk, I prove this conjectures for some cases with three key ideas: (1) To develop a new asymptotic formula by the Euler-Maclaurin summation formula; (2) To prove that the conjectures is deduced from the holomorphy of a certain rational function; (3) To prove the holomorphy by the induction on pruning of a plumbing graph. (Received August 25, 2023)

1192-11-28112
Toka Diagana*, University of Alabama in Huntsville. Analysis of Some p-adic Dynamical Systems in the Projective Line $\mathbb{P}^{1}\left(\mathbb{Q}_{p}\right)$. Preliminary report.
Let $p \geq 2$ be a prime. The study of $p$-adic dynamical systems arises in Diophantine geometry in the constructions of canonical heights, used for counting rational points on algebraic vertices over a number field. In recent years, p-adic dynamical systems have been proposed to model many practical problems including memory retrieval. Other applications of $p$-adic dynamical systems arise in biology, physiology, computer science, combinatorics, automata theory and formal languages, numerical analysis and simulations, etc. In this talk, we are interested in studying the long-term behavior of p-adic dynamical systems associated with some rational functions in the projective line $\mathbb{P}^{1}\left(\mathbb{Q}_{p}\right)$. The fixed points, maximal Siegel disks, attractors, and periodic trajectories for the above-mentioned p-adic dynamical systems are analyzed. Moreover, the corresponding Julia and Fatou sets are also examined.
(Received August 26, 2023)

## 1192-11-28138

James A. Sellers*, University of Minnesota Duluth. Elementary Proofs of Congruences for POND and PEND Partitions.
Recently, Ballantine and Welch considered various generalizations and refinements of POD and PED partitions. These are integer partitions wherein the odd parts must be distinct (in the case of POD partitions) or the even parts must be distinct (in the case of PED partitions). In the process, they were led to consider two classes of integer partitions which are, in some sense, the "opposite" of POD and PED partitions. They labeled these POND and PEND partitions, which are integer partitions wherein the odd parts cannot be distinct (in the case of POND partitions) or the even parts cannot be distinct (in the case of PEND partitions). In this work, we study these two types of partitions from an arithmetic perspective. Along the way, we are led to prove the following two infinite families of Ramanujan-like congruences: For all $\alpha \geq 1$ and all $n \geq 0$, \align* $\operatorname{pond}\left(3^{\wedge}\{2 \alpha+1\} \mathrm{n}+\backslash \operatorname{frac}\left\{23 \cdot 3^{\wedge}\{2 \alpha\}+1\right\} 8\right) \& \equiv 0(\bmod 3)$, and $\operatorname{pend}\left(3^{\wedge}\{2 \alpha+1\} \mathrm{n}+\backslash \operatorname{frac}\left\{17 \cdot 3^{\wedge}\{2 \alpha\}-1\right\} 8\right) \& \equiv 0(\bmod 3)$ lendalign* where $\operatorname{pond}(n)$ counts the number of POND partitions of weight $n$ and pend $(n)$ counts the number of PEND partitions of weight $n$. All of the proof techniques used herein are elementary, relying on classical $q$-series identities and generating function manipulations, along with mathematical induction.
(Received August 27, 2023)

## 1192-11-28143

Carlos A Rivera*, University of Washington. Reciprocity obstruction for rational points over p-adic function fields.. Preliminary report.
In 2021, Olivier Wittenberg introduced a new obstruction to the local to global principle for varieties over $p$-adic function fields, the reciprocity obstruction, analogue to the Brauer-Manin obstruction for number fields. We propose a geometrization of it using discrete adic spaces. As applications we extend Wittenberg's definition to non-proper varieties, show that the obstruction is the only one for zero dimensional varieties, and exhibit an example of a Châtelet surface showing that individual classes can indeed impose strictly stronger conditions by using the new obstruction than the one imposed by a predecessor of it introduced by Harari, Szamuely and Scheiderer.
(Received August 27, 2023)

Alexey Pozdnyakov*, University of Connecticut. Arithmetic Geometry with a View toward Machine Learning.
From the prime number theorem to the Birch and Swinnerton-Dyer conjecture, major results in number theory are often informed by computations followed by pattern recognition. With modern computing and machine learning techniques, we now have access to both super-human computation and super-human pattern recognition, allowing us to probe for new structure and new relationships within arithmetic geometry. As an example, the recently discovered murmurations phenomenon, a bias in the distribution of Frobenius traces, was first observed through a machine learning investigation of the LMFDB. We will present murmurations and related problems in the theory of $L$-functions through a machine learning approach. (Received August 27, 2023)

## 1192-11-28173

Jonah Klein*, University of South Carolina. Bounding the $j$-th smallest modulus in a covering system with distinct moduli. A covering system is a finite set of arithmetic progressions with the property that each integer belongs to at least one of them. In 2015, Hough resolved the famous minimum modulus problem of Erdős, showing that the smallest modulus in a covering system with distinct moduli is always smaller than $10^{16}$. Building upon his work, in 2022, Balister, Bollobás, Morris, Sahasrabudhe, and Tiba reduced this bound to 616000 with a method they named the distortion method. A natural next question is to ask if the $j$-th smallest modulus of a covering system with distinct moduli is bounded. In 2022, Cummings, Filaseta and Trifonov showed that it is bounded, but give no explicit bounds. Recently, in joint work with Dimitris Koukoulopoulos and Simon Lemieux, we analysed a slight modification of the distortion method, and combined this with a combinatorial argument based on a theorem of Crittenden and Vanden Eynden to show that the $j$-th smallest modulus of a covering system with distinct moduli is smaller than $\exp \left(c j^{2} / \log (j+1)\right)$ for some absolute constant $c$. The goal of this presentation is to discuss this result and its proof in further detail.
(Received August 27, 2023)

## 1192-11-28204

Carolina Rizzi*, Texas A\&M International University, Holly Vanlooy*, Boise State University. Complex Continued Fractions and Inadmissible Sequences.
It is known that Serret's Theorem does not hold for Hurwitz Continued Fractions in the complex plane due to Lakien's counterexample. We expand on this counterexample by applying algorithms that convert inadmissible sequences into their admissible forms. We provide additional counterexamples and prove that there exists an uncountably infinite family of counterexamples.
(Received August 28, 2023)

## 1192-11-28307

Jacob Mayle*, Wake Forest University, Jeremy A. Rouse, Wake Forest University. Odd order reduction for abelian surfaces. Preliminary report.
Let $E / \mathbb{Q}$ be an elliptic curve and $P \in E(\mathbb{Q})$ be a point of infinite order. One can ask: What is the density of primes $p$ such that the order of $P$ in $E\left(\mathbb{F}_{p}\right)$ is odd? While this question remains open in general, in the generic case the answer is $\frac{11}{21}$. In this talk, we will discuss an ongoing joint work with Jeremy Rouse that considers this problem for abelian surfaces in the generic case. (Received August 29, 2023)

Benjamin York*, University of Connecticut. Models of CM elliptic curves with prescribed $\ell$-adic Galois image. In 2018, Lozano-Robledo provided a classification for $\ell$-adic Galois representations attached to elliptic curves with complex multiplication (CM). In this talk, we will discuss a classification of Weierstrass models for CM elliptic curves with specified $\ell$ adic Galois representation, and discuss our methods for proving this classification. This is joint work with Enrique GonzálezJiménez and Álvaro Lozano-Robledo.
(Received August 29, 2023)

1192-11-28320
Shen Chan Huang*, University of Nevada Las Vegas, Anthony G Shannon, University of New South Wales, Peter J-S Shiue, University of Nevada Las Vegas. Notes on Generalized and Extended Leonardo Numbers.
This presentation extends and generalizes properties recently proposed by multiple authors concerning terms of the Leonardo sequence $\left\{L e_{n}\right\}$ within the framework of second-order recursive sequences. This is achieved by exploring both the difference equation characteristics of the homogeneous Fibonacci sequence and the non-homogeneous traits of their counterparts in the Leonardo sequence. As a result, several new identities emerge related to a generalized Leonardo sequence $\left\{\mathcal{L}_{k, n}\right\}$. In addition, we will consider the type of series in the following form: $\sum_{n \geq 0}^{\infty} \frac{\mathcal{L}_{k, n}}{10^{n+1}}$.
(Received August 31, 2023)

## 1192-11-28321

John Rufus Hall*, University of Kentucky. Pairs of quadratic forms over a p-adic field.
Given two quadratic forms $Q_{1}, Q_{2}$ over a $p$-adic field $K$ in $n$ variables, we consider the pencil $\mathcal{P}_{K}\left(Q_{1}, Q_{2}\right)$, which contains all nontrivial $K$-linear combinations of $Q_{1}$ and $Q_{2}$. We define $D$ to be the maximal dimension of a subspace in $K^{n}$ where $Q_{1}$ and $Q_{2}$ both vanish. We define $H$ to be the maximal number of hyperbolic planes that a form in $\mathcal{P}_{K}\left(Q_{1}, Q_{2}\right)$ splits off over $K$. In this talk, we will discuss which values for $(D, H)$ are possible for a nonsingular pair of quadratic forms over a $p$-adic field $K$. (Received August 29, 2023)

Hung-Ping Tsao*, None. A Well-rounded Distribution of Small Prime Numbers as Terminal Values in the Process of Successively Summing up all Prime Factors of a Natural Number. Preliminary report.
We provide here a new platform for exploration of prime numbers. For our purpose, we define the terminal value $t(n)$ for each natural number $n$ by successively summing up the prime factors. We first define $t(1)=1, t(2)=2, t(3)=3, t(5)=5$ and $t(7)=7$. The process of summing up successively the prime factors of 4 would end up with $t(4)=4$, albeit the process continues forever, which would cause a problem in computer programming. Since $6=2 \times 3$ and $2+3=5$, we define $t(6)=5$. Since $8=2 \times 2 \times 2$ and $2+2+2=6$, we can define $t(8)=t(6)=5$, without having to repeat the process of $2+3=5$. Likewise, we can define $t(9)=5$, since $9=3 \times 3$ and $3+3=6$.To facilitate our observations, we define two sets $A=5$ and $B=7,11,13$. We shall call 5 the A-terminal value and 7,11 or 13 a B-terminal value. OBSERVATION. Up to $n=2400$, the percentages for single, double, triple and quadruple terminal values from A and B combined are consistently 16, 4 and 1, respectively. Note that a triple terminal value will be counted as two double terminal values, for example. CONJECTURE. In a long run, there would not be any n-tuple terminal values from A or B for $\mathrm{n}>10$.
(Received September 15, 2023)

## 1192-11-28409

Zafer Selcuk Aygin*, Northwestern Polytechnic, Khoa Nguyen, University of Calgary. $N$-colored generalized Frobenius partitions: generalized Kolitsch identities.
Let $N \geq 1$ be squarefree integer coprime to 6 . Let $c \phi_{N}(n)$ denote the number of $N$-colored generalized Frobenius partitions of $n$ introduced by Andrews in 1984, and $P(n)$ denote the number of partitions of $n$. We prove that

$$
c \phi_{N}(n)=\sum_{d \mid N} N / d \cdot P\left(\frac{N}{d}^{2} n-\frac{N^{2}-d^{2}}{2} 4 d^{2}\right)+b(n)
$$

where $C(z):=(q ; q)_{\infty}^{N} \sum_{n=1}^{\infty} b(n) q^{n}$ is a cusp form in $S_{(N-1) / 2}\left(\Gamma_{0}(N), \chi_{N}\right)$. This extends and strengthens earlier results of Kolitsch and Chan-Wang-Yang which treated the cases when $N$ is a prime. We then use properties of modular forms to show that $b(n)$ is small compared to the linear combinations of partition function in our formula. This establishes an asymptotical formula for $c \phi_{N}(n)$ in terms of the partition function. These results are from a joint work with Professor Khoa Dang Nguyen of the University of Calgary.
(Received August 30, 2023)

1192-11-28416
Alexander Cowan, Harvard University, Sam Frengley, University of Cambridge, Kimball L. Martin*, University of Oklahoma. Rational genus 2 curves with real multiplication.
A genus 2 curve has real multiplication if the endomorphism ring of its Jacobian contains a real quadratic order. Explicit models of such curves are known when the quadratic order has discriminant 5 or 8 . We give models for generic families over $\mathbb{Q}$ for many more discriminants. Our approach is to algorithmically transform the Mestre obstruction on the associated Hilbert modular surface to into a nice minimal form.
(Received August 30, 2023)

1192-11-28419
Andrew Scott Kolesar*, United States Naval Academy. Geometry of Numbers over Real Quadratic Number Fields and Applications to Quadratic Forms. Preliminary report.
In the early 1990s, Conway published the 15 -Theorem, and in the mid 2000s Bhargava and Hanke announced the 290Theorem. Both concern universal quadratic forms, but the latter in particular referenced 6436 universal quaternary forms; moreover, it relied heavily on the theory of modular forms. Sadly, neither the list of forms nor the proof has formally appeared in print. We take 348 of the 6436 escalators, and using more elementary and classical Geometry of Numbers (GoN) methods will provide for many the first publicly-available proofs of their universality (as 303 are not proven universal by the 15 Theorem). Beginning with elementary methods, we use pre-existing GoN-based results regarding representation by ternary subforms-ternary results in fact proven by undergraduates in a 2015 REU and in a 2022 USNA honors thesis-to show the universality of 62 of the 348 . For the remainder, we highlight GoN techniques from the early 2010 s of Clark, Hicks, and Thompson involving a small multiple theorem and reduction; however, whereas Clark etal focused on $\mathbb{Z}$ we will extend these GoN techniques and results to $\mathbb{Z}[\sqrt{2}], \mathbb{Z}[\sqrt{3}]$ and $\mathbb{Z}[(1+\sqrt{5}) / 2]$.
(Received August 30, 2023)

1192-11-28454
Hasan Saad*, University of Virginia. Automorphic Forms and Point Distributions on K3 Surfaces.
Automorphic forms are ubiquitous in mathematics. In this talk we discuss how mock modular forms and holomorphic modular forms make it possible to determine the limiting distributions of point counts of certain families of $K 3$ surfaces over finite fields. Namely, we obtain the $O(3)$ distribution with an explicit error term. We then show how to count "matrix" points on these surfaces and determine the distributions of those counts.
(Received August 30, 2023)

## 1192-11-28460

Olena Kozhushkina, Ursinus College, MaeKayla Minton*, Kentucky Wesleyan College, Dillon Snyder*, University of Connecticut, Justin G. Trulen, Kentucky Wesleyan College, Laura Vaughan*, Vanderbilt University. p-Adic Valuation Trees and Iterative Sequences. Preliminary report.
The $p$-adic valuation, $\nu_{p}(x)$, is the highest power of a prime $p$ that divides an integer $x$, which can be extended to determine the valuation of the terms in an integer sequence and then visually represented with a tree. A $p$-adic valuation tree, as seen
below for $n^{2}+4$, is comprised of nodes which can be terminating, when every term in the node has the same valuation, or otherwise non-terminating. We investigate how nodes split on a variety of trees, such as those formed from a product of linear polynomials with integer coefficients and from iterative mappings of linear functions. We describe the infinite branches of these polynomial trees and the valuations of their terminating nodes. For iterative mappings, we conjecture their behaviors and whether the sequences of valuations are constant, periodic, or unbounded, depending on the coefficients and initial values of the linear function.
(Received August 30, 2023)

## 1192-11-28464

John M. Voight*, Dartmouth. Polarized class sets of quaternion orders.
For a definite quaternion order, the class number formula expresses the number of isomorphism classes of (right) invertible ideals in terms of arithmetic invariants of the order. A related spinor class number formula was recently proven by Jiangwei Xue and Chia-Fu Yu, using elaborate calculations with the Selberg trace formula. We give an alternate proof for this formula using a direct, conceptual argument: we interpret the set counted by this formula as a polarized class set.
(Received August 30, 2023)

1192-11-28472
Summer Haag, University of Colorado, Boulder, Clyde Kertzer*, University of Colorado, Boulder, James Rickards, University of Colorado, Boulder, Katherine E. Stange, University of Colorado, Boulder. The local-global conjecture for Apollonian circle packings is false.
In a primitive integral Apollonian circle packing, the curvatures that appear must fall into one of six or eight residue classes modulo 24 . The local-global conjecture states that every sufficiently large integer in one of these residue classes will appear as a curvature in the packing. We prove that this conjecture is false for many packings, by proving that certain quadratic and quartic families are missed. The new obstructions are a property of the thin Apollonian group (and not its Zariski closure), and are a result of quadratic and quartic reciprocity, reminiscent of a Brauer-Manin obstruction. Based on computational evidence, we formulate a new conjecture.
(Received August 31, 2023)

1192-11-28499
Chad Awtrey, Samford University, Frank Patane*, Samford University. On the Galois Group of a Reciprocal Even Octic Polynomial.
Let $F$ be a field of characteristic 0 , and let $f(x)=x^{8}+a x^{6}+b x^{4}+a x^{2}+1 \in F[x]$ be irreducible with Galois group Gal $(f)$. We extend results in the literature to give a complete and elementary classification of $\operatorname{Gal}(f)$, identified as a transitive subgroup of $S_{8}$ up to conjugacy. The approach involves testing whether a small number of elements in $F$ are perfect squares. As an application, we give one-parameter families of polynomials for each of the six possible Galois groups.
(Received August 31, 2023)

1192-11-28521
Fnu Rakvi*, University of Pennsylvania. On 3 adic Galois images associated to isogeny torsion graphs of non CM elliptic curves defined over $\mathbb{Q}$.
Let $E$ be a non CM elliptic curve defined over $\mathbb{Q}$. There is an isogeny torsion graph associated to $E$ and there is also a Galois representation $\rho_{E}: G a l(\overline{\mathbb{Q}} / \mathbb{Q}) \rightarrow G L_{2}\left(\mathbb{Z}_{\ell}\right)$ associated to $E$ for every prime $\ell$. In this talk, I will discuss a classification of 3 adic Galois images associated to vertices of isogeny torsion graph of $E$.
(Received August 31, 2023)

1192-11-28543
Ognian Trifonov*, University of South Carolina. Extreme Covering Systems.
We prove that if the least modulus of a distinct covering system is 4 , its largest modulus is at least 60 ; also, if the least modulus is 3 , the least common multiple of the moduli is at least 120 ; finally, if the least modulus is 4 , the least common multiple of the moduli is at least 360 . The constants 60,120 , and 360 are best possible-they cannot be replaced by larger constants. We also consider some open problems on covering systems.
(Received August 31, 2023)

## 1192-11-28549

Vladimir Pyotr Sworski*, Colorado State University. Number of 4-Cycles of the Genus 2 Superspecial Isogeny Graph. The genus 2 superspecial degree-2 isogeny graph over $\mathbb{F}_{p^{2}}$ is a network graph whose nodes are constructed from genus 2 superspecial curves and whose edges are the degree 2 isogenies between them. Flynn and Ti discovered 4 -cycles in the graph, which pose problems for applications in cryptography. Florit and Smith constructed an atlas which describes what the neighborhood of each node looks like. The speaker wrote a program in SageMath that can calculate neighborhoods of these graphs for primes in the range of $10^{2}-10^{6}$. Much of this work is motivated by these computations. We examine the prevalence of 4 -cycles in the graph and, motivated by work of Arpin in the genus 1 situation, in the subgraph called the spine. (Received August 31, 2023)
eta function and its applications in topology and combinatorial geometry. We derive generalized Dedekind sums arising from the Eichler-Shimura isomorphism applied to holomorphic weight $k \geq 3$ Eisenstein series attached to primitive non-trivial Dirichlet characters $\chi_{1}, \chi_{2}$. Applying this isomorphism gives us a degree $k-2$ homogenous polynomial in two variables with complex coefficients; by specializing these variables, we recover a finite sum formula due to Stucker, Vennos, and Young. This result gives us a simple tool for understanding holomorphic Eisenstein series attached to characters through the EichlerShimura isomorphism.
(Received August 31, 2023)

## 1192-11-28573

Jeffrey Yelton*, Wesleyan University. Clusters and non-archimedean uniformization of superelliptic curves.
Let $K$ be a field with a discrete valuation; let $S \subset K \cup\{\infty\}$ be an even-cardinality subset; and choose a prime $p$. Under suitable conditions, such a set $S$ may be used to determine a subgroup $\Gamma<\mathrm{PGL}_{2}(K)$ of automorphisms of $\mathbb{P}_{K}^{1}$ whose action induces a quotient that can be algebraized as a superelliptic curve $C: y^{p}=f(x) / K$ and such that the quotient map induces a bijection between $S$ and the set $R$ of roots of $f$. This raises two natural questions: (1) how may one test whether any given even-cardinality subset $S \subset K \cup \infty$ has the right properties such that it induces a superelliptic curve in this way, and (2) in the case that $S$ does satisfy these properties, how does the cluster data (the distances between the pairs of points in $K$ ) of $S$ compare to that of $R$ ? I will address both of these questions by describing such a test for a given set $S$, and in the case of a positive outcome of the test, a procedure to modify $S$ into a subset of $K \cup\{\infty\}$ whose cluster data directly determines that of $R$.
(Received August 31, 2023)

## 1192-11-28583

Hyun Jong Kim*, University of Wisconsin at Madison. Cohen-Lenstra heuristics and vanishing of zeta functions for cyclic covers of projective lines over finite fields. Preliminary report
Ellenberg-Venkatesh-Westerland proved a Cohen-Lenstra result for imaginary quadratic function fields over finite fields by asymptotically counting points on Hurwitz schemes, which parameterize tamely ramified G-covers of the projective line. Moreover, Ellenberg-Li-Shusterman used the methods of Ellenberg-Venkatesh-Westerland to prove that a fixed complex number vanishes on almost no Zeta functions of hyperelliptic curves over finite fields, with respect to a limit taking the genera of the curves to infinity and then the sizes of the base fields to infinity. I will talk about my thesis work on extending their results to higher degree function fields and curves, possibly with some emperical computational observations.
(Received August 31, 2023)

1192-11-28607
Santiago Arango*, Emory University. $q$-Weil Galois groups in low dimension. Preliminary report.
Given an abelian variety $A$ over a finite field $\mathbb{F}_{q}$, one obtains a Galois group $\operatorname{Gal}(A)$ corresponding to the characteristic polynomial of the Frobenius endomorphism of $A$. Given an abstract Galois group $G$, when can we find an abelian variety $A / \mathbb{F}_{q}$ such that $\operatorname{Gal}(A) \cong G$ ? In this talk, I will report on some progress towards this question in the cases of surfaces and threefolds.
(Received September 01, 2023)

1192-11-28641
Aihua Li*, Montclair State University. Beautiful Mathematical Elements in Architectural Design and Their Roles in Teaching and Research. Preliminary report.
In this talk, the author will demonstrate the beauty of mathematics around our daily lives through a collection of photos and art works that reflect mathematical elements in geometry, number theory, and algebra. A subset of the work is from the beautiful geometric figures appearing in the architectural design of Barcelona buildings. Some of them carry stories in depth and mathematical questions which attracted researchers in mathematics and general audience. The author will also share her experience in integrating the art works, the mathematics ideas behind them, and the research components into her classroom teaching and student research projects.
(Received September 01, 2023)

## 1192-11-28664

Garen Chiloyan*, none. A classification of images of the 2-adic Galois representation attached to rational isogeny-torsion graphs.
Let $\mathcal{E}$ be a $\mathbb{Q}$-isogeny class of elliptic curves defined over $\mathbb{Q}$. The isogeny graph associated to $\mathcal{E}$ is a graph which has a vertex for each elliptic curve in $\mathcal{E}$ and an edge for each $\mathbb{Q}$-isogeny of prime degree that maps one elliptic curve in $\mathcal{E}$ to another elliptic curve in $\mathcal{E}$, with the degree recorded as a label of the edge. An isogeny-torsion graph is an isogeny graph where, in addition, we label each vertex with the abstract group structure of the torsion subgroup over $\mathbb{Q}$ of the corresponding elliptic curve. In this talk, we present the classification of the image of the 2 -adic Galois representation that occurs at each vertex of all isogenytorsion graphs consisting of elliptic curves defined over $\mathbb{Q}$.
(Received September 01, 2023)

1192-11-28694
Timmy Ma*, Xavier University of Louisiana. Generalization of the 2-Fibonacci Sequences and their Binet formula. Preliminary report.
We will explore the generalization of the four different 2-Fibonacci sequences defined by Atnassov. In particular, we will define the recurrence relation of the 2 -Fibonacci sequences, discuss the generating function of each of these sequences, and provide the necessary conditions to obtain the Binet formula for each of these sequences.
(Received September 01, 2023)

Catherine Maria Hsu*, Swarthmore College, Preston Wake, Michigan State University, Carl Wang-Erickson, University of Pittsburgh. Explicit non-Gorenstein $R=\mathbb{T}$ via rank bounds.
As famously illustrated by the Herbrand-Ribet theorem, the intricate relationship between modular forms and Galois representations can be a powerful tool for studying arithmetic properties of number fields. In this talk, I will discuss recent work, joint with Preston Wake and Carl Wang-Erickson, that explores the connection between a Galois deformation problem arising from Eisenstein congruences and the arithmetic of certain number fields with twisted-Heisenberg Galois group. We will focus on the project's computational methods, which utilize the correspondence between $n$-fold Massey vanishing and the existence of Galois extensions with Galois groups of nilpotency class n-1.
(Received September 02, 2023)

## 1192-11-28800

George E. Andrews*, Pennsylvania State University. Partition Analysis and Partitions with n copies of n.
This is a report on joint work with Peter Paule. We begin the talk with an account of how "partitions with $n$ copies of $n$ " arise naturally in physics. We then show how the study of partitions with $n$ copies of $n$ can be easily incorporated in MacMahon's Partition Analysis. We close the talk with a look at how Schmidt-type partitions may arise in this context.
(Received September 03, 2023)

1192-11-28812
Jerson Caro Reyes*, Boston University. Some advanced in a conjecture of Watkins and an analogue over function fields. In 2002, M. Watkins proposed a conjecture stating that for every elliptic curve defined over $\mathbb{Q}$, its Mordell-Weil rank is bounded by the 2 -adic valuation of its modular degree. This talk explores advancements related to Watkins's conjecture, presenting results organized into two main parts. The first part delves into results associated with Watkins's conjecture and explores various approaches to ensure its satisfaction for elliptic curves. In the second part, we extend this conjecture to elliptic curves defined over function fields of positive characteristics. Additionally, we provide concrete examples where Watkins's conjecture holds true; for instance, there exists a family of elliptic curves with unbounded rank that satisfies this version of Watkins's conjecture.
(Received September 03, 2023)

1192-11-28833
Alexandra Florea, University of California Irvine, Edna Jones, Duke University, Matilde N. Lalin*, Université de Montréal. Moments of Artin-Schreier L-functions.
We compute moments of $L$-functions associated to the polynomial family of Artin-Schreier covers over $\mathbb{F}_{q}$, where $q$ is a power of a prime $p$, when the size of the finite field is fixed and the genus of the family goes to infinity. We compute the $k^{\text {th }}$ moment for a large range of values of $k$, depending on the sizes of $p$ and $q$. We also compute the second moment in absolute value, obtaining an exact formula with a lower order term, and confirming the unitary symmetry type of the family.
(Received September 03, 2023)

## 1192-11-28864

Muhammet Boran, Yıldız Technical University, John Byun, Carleton College, Zhangze Li*, University of Michigan, Steven Joel Miller, Williams College, Stephanie Reyes, Claremont Graduate University. Power Sums of Primes in Arithmetic Progression.
Let $\pi(x)=\sum_{p \leq x} 1$ be the prime counting function and for $k \in \mathbb{R}$ define

$$
\pi_{k}(x):=\sum_{p \leq x} p^{k}
$$

Gerard and Washington proved that $\pi_{k}(x)$ is asymptotic to $\pi\left(x^{k+1}\right)$, and they quantified the difference $\pi_{k}(x)-\pi\left(x^{k+1}\right)$ for $k>1$ or $-1<k<0$. We extend these results to primes in arithmetic progressions, exploring

$$
\pi(x ; m, n)=\sum_{p \leq x p \equiv n \$(\bmod \$ m)} 1 a n d \pi_{k}(x ; m, n)=\sum_{p \leq x p \equiv n \$(\bmod \$ m)} p^{k}
$$

We prove that $\pi_{k}(x ; m, n)$ remains asymptotic to $\pi\left(x^{k+1} ; m, n\right)$ where $\operatorname{gcd}(m, n)=1$. We examine this approximation for primes of the form $p \equiv 1(\bmod 4), p \equiv 3(\bmod 4), p \equiv 1(\bmod 5)$, and $p \equiv 3(\bmod 5)$. With above modular cases, we select 9 positive integers between $10^{4}$ and $10^{8}$, and calculate the relative error

$$
\frac{\pi(x ; m, n)-\pi_{k}\left(x^{1 /(k+1)} ; m, n\right)}{\pi}(x ; m, n)
$$

for $k=-1 / 10,-1 / 12,1 / 2,1$. Building on our findings, we further prove from the observation that $\pi_{k}(x ; m, n)-\pi\left(x^{k+1} ; m, n\right)$ tends to be negative when $k>0$ and generally positive when $\$-1$ (Received September 04, 2023)

The method of quadratic Chabauty was a groundbreaking development on the problem of determining explicitly the set of rational points on a curve. A crucial requirement of this method is the computation of values of local height functions at primes of bad reduction. In this talk, I will discuss recent progress in the algorithmic computation of these local height functions for hyperelliptic curves, along with practical examples. This is joint work with Alexander Betts, Juanita Duque-Rosero, and Pim Spelier.
(Received September 04, 2023)

## 1192-11-28883

Daniel Everett Martin*, Clemson University. Maximal divisors and Markoff mod p graphs.
We introduce the notion of maximal divisors of an integer, and we present a method for counting them that is modelled on Ramanujan's theory of "superior highly composite numbers." Our bound on the number of maximal divisors is then put to use in proving connectivity of Markoff mod $p$ graphs.
(Received September 04, 2023)

## 1192-11-28918

Ethan C Smith*, Grove City College. Extremal primes for modular forms
Given a fixed elliptic curve $E / \mathbb{Q}$ and a prime $p$ of good reduction, we say that $p$ is extremal for $E$ if the absolute value of the trace of Frobenius is as large as allowed by the Hasse bound, i.e., if $\left|a_{p}(E)\right|=\lfloor 2 \sqrt{p}\rfloor$. James and Pollack proved asymptotic formula for the number of extremal primes up to $x$ for a fixed CM elliptic curve. We will discuss some of the difficulties that arise when extending the concept to higher weight modular forms. This is joint work with Kevin James and David Penniston. (Received September 04, 2023)

## 1192-11-28927

Joshua Harrington, Cedar Crest College, Yewen Sun*, The Ohio State University, Tony Wing Hong Wong, Kutztown University of Pennsylvania. Covering systems with odd moduli.
The concept of a covering system was first introduced by Erdos in 1950. Since their introduction, a lot of the research regarding covering systems has focused on the existence of covering systems with certain restrictions on the moduli. Arguably, the most famous open question regarding covering systems is the odd covering problem. In this paper, we explore a variation of the odd covering problem, allowing a single odd prime to appear as a modulus in the covering more than once, while all other moduli are distinct, odd, and greater than 1 . We also consider this variation while further requiring the moduli of the covering system to be square-free
(Received September 04, 2023)

1192-11-28937
Joshua D Belden, Clovis North High School, Maria S Nogin*, California State University, Fresno. The Four Numbers Game The classical four numbers game starts with choosing four counting numbers and placing them at the corners of a square. At the midpoint of each edge, we write the difference of the two adjacent numbers, subtracting the smaller one from the larger one. This produces a new list of four numbers, written on a smaller square. We repeat this process again and again until something interesting occurs. This is where the game stops but questions start. Does this phenomenon always occur? Can it occur after any given number of steps? Are there four-tuples of numbers that cannot be obtained in the process? Are there any patterns in the four-tuples obtained? Do the answers to the above questions change if we extend the set of allowed numbers to the set of integers, rational, or all real numbers? More advanced students can even extend to other rings. What if we play with either three or more than four numbers? One of the authors did his Masters project on this game a few years ago. There is a lot of literature on the topic, and there are still many questions that students could investigate.
(Received September 04, 2023)

1192-11-28954
Colette LaPointe*, CUNY Graduate Center. Smoothness and irreducibility of dynatomic modular curves in positive characteristic.
labstract In arithmetic dynamics, the smoothness and irreducibility of the dynatomic modular curves $Y_{1}(n)$ and $Y_{0}(n)$ have frequently been studied for the polynomial family $f_{c}(x)=x^{d}+c$ in both char 0 and positive char $p$, but less is known about the dynamical behavior of other families. I am studying the smoothness and irreducibility of $Y_{1}(n)$ and $Y_{0}(n)$ for families of the form $f_{c}(x)=g^{p}+b x+c$ defined over a field $k$ of positive char $p$, with $g \in k[x, c]$. For example, it has been shown that for such a dynamical system, $Y_{1}(1)$ is smooth, and for $n \geq 2$, we have the following: (i) when $b \neq 1, Y_{1}(n)$ is smooth if and only if $b^{n} \neq 1$, and (ii), when $b=1, Y_{1}(n)$ is smooth if and only if $p \nmid n$, except in the case $p=d=2$ where $Y_{1}(2)$ is smooth, where $d=\operatorname{deg}\left(f_{c}\right)$. More generally, I am interested in asking what dynamical properties of this family can be attributed to them all having a unique critical point at infinity. \endabstract
(Received September 15, 2023)

1192-11-29043
Michael Cerchia*, Emory University, Jesse Franklin, University of Vermont, Evan O'Dorney, Carnegie Mellon University. Section Rings of $\mathbb{Q}$-Divisors on Elliptic Curves.
For any $\mathbb{Q}$-divisor $D$ on a curve $X$, the graded section ring is $R(X, D):=\bigoplus_{d \geq 0} H^{0}(X,\lfloor d D\rfloor)$. In the case that $D=K_{X}$, where $K_{X}$ is the canonical divisor, the section ring is called the "canonical ring" and is a classical object of study. Under the assumption that $X$ has genus 1 , we bound the generators and relations of section rings associated to arbitrary $\mathbb{Q}$-divisors. When $D$ is supported by at most two points, we give a complete description of the canonical ring.
(Received September 05, 2023)

## 1192-11-29045

Matthew de Courcy-Ireland*, Stockholm University. Markoff graphs mod p: non-planarity and short cycles We discuss a family of 3-regular graphs whose vertices are triples ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) modulo a prime number, solving a Markoff-type cubic equation. The three neighbours of ( $x, y, z$ ) are given by changing any one coordinate to the other solution of a quadratic equation. We sketch a proof that these graphs are not planar, except for the primes 2,3 , and 7 . Non-planarity follows from Euler's formula and the fact that there are few short cycles. Non-planarity would also follow from the separator theorem of Lipton and Tarjan, assuming a spectral gap for this family of graphs. Testing the presence of a gap was our original motivation. (Received September 05, 2023)

Marco Baldi, Polytechnic university of Marche, Alessandro Barenghi, Polytechnic University of Milan, Sebastian Bitzer, Technical University of Munich, Patrick Karl, Technical University of Munich, Felice Manganiello*, Clemson University, Alessio Pavoni, Technical University of Munich, Gerardo Pelosi, Polytechnic University of Milan, Paolo Santini, Polytechnic University of Marche, Jonas Schupp, Technical University of Munich, Freeman Slaughter, Clemson University, Antonia Wachter-Zeh, Technical University of Munich, Violetta Weger, Technical University of Munich. CROSS - Codes and Restricted Objects Signature Scheme.
We present CROSS, an official round 1 NIST candidate in the 2023 additional call for signature schemes.CROSS is based on the so-called Restricted Syndrome Decoding Problem (R-SDP), a variant of the classical Syndrome Decoding Problem (SDP), thus, it depends on an NP-complete problem. CROSS is a signature scheme obtained by applying the Fiat-Shamir transform to an interactive Zero Knowledge (ZK) Identification protocol.
(Received September 05, 2023)

## 1192-11-29066

Rodrigo Angelo, Stanford University, Adam Harper, University of Warwick, K Soundararajan, Stanford University, Max Wenqiang Xu*, Stanford University. Recent progress on random multiplicative functions and applications. We highlight some recent progress of random multiplicative functions: probabilistic models for multiplicative functions. We will also mention its inspiration to a deterministic question: counting the number of quadratic characters with the positive definite property.
(Received September 05, 2023)

1192-11-29082
Matthew David Kroesche*, Texas A\&M University. Low-Lying Zeros of a Thin Family of Automorphic L-Functions in the Level Aspect.
We calculate the one-level density of thin subfamilies of a family of Hecke cuspforms formed by twisting the forms in a smaller family by a character. The result gives support up to 1 , conditional on GRH, and we also find several of the lower-order main terms. In addition, we find an unconditional result that has only slightly lower support. A crucial step in doing so is the establishment of an on-average version of the Weil bound that applies to twisted Kloosterman sums. Moreover, we average over these thin subfamilies by running over the characters in a coset, and observe that any amount of averaging at all is enough to allow us to get support greater than 1 and thus distinguish between the SO(even) and SO(odd) symmetry types. Finally, we also apply our results to nonvanishing problems for the families studied.
(Received September 15, 2023)

Evan M. O'Dorney*, Carnegie Mellon University. Diophantine Approximation on Conics.
Given a conic $\mathcal{C}$ over $\mathbb{Q}$, it is natural to ask what real points on $\mathcal{C}$ are most difficult to approximate by rational points of low height. For the analogous problem on the real line (for which the least approximable number is the golden ratio $\phi$, by Hurwitz's theorem), the approximabilities comprise the classically studied Lagrange and Markoff spectra, but work by ChaKim and Cha-Chapman-Gelb-Weiss shows that the spectra of conics can vary. We provide notions of approximability, Lagrange spectrum, and Markoff spectrum valid for a general $\mathcal{C}$ and prove that their behavior is exhausted by the special family of conics $\mathcal{C}_{n}: X Z=n Y^{2}$, which has symmetry by the modular group $\Gamma_{0}(n)$ and whose Markoff spectrum was studied in a different guise by A. Schmidt and Vulakh. The proof proceeds by using the Gross-Lucianovic bijection to relate a conic to a quaternionic subring of $\operatorname{Mat}^{2 \times 2}(\mathbb{Z})$ and classifying invariant lattices in its 2-dimensional representation. (Received September 05, 2023)

1192-11-29126
Bruce C. Berndt*, University of Illinois, Ors Rebak, UiT The Artic University of Norway. Values of Ramanujan's theta function $\varphi(q)$.
In his notebooks, Ramanujan determined some 'new' values for $\varphi(q)$, for example, the value of $\varphi\left(e^{-3 \pi}\right)$. In his lost notebook, Ramanujan provided an 'incomplete value' for $\varphi\left(e^{-7 \pi \sqrt{7}}\right)$, which was recently completely evaluated by Örs Rebák. In joint work with Rebák, we develop general cubic and quintic analogues of Ramanujan's now completed septic formula. As corollaries, we are able to determine several 'new' values of $\varphi\left(e^{-\pi \sqrt{n}}\right)$. (Received September 05, 2023)

Jingbo Liu*, Texas A\&M University-San Antonio. g-invariant on unary Hermitian lattices over imaginary quadratic fields. Let $E=\mathbb{Q}(\sqrt{-d})$ be an imaginary quadratic field for a square-free positive integer $d$, and let $\mathcal{O}$ be its ring of integers. For
each positive integer $m$, let $I_{m}$ be the free Hermitian lattice of rank $m$ over $\mathcal{O}$ with an orthonormal basis, let $\mathfrak{S}_{d}(1)$ be the set consisting of all positive definite integral unary Hermitian lattices over $\mathcal{O}$ that can be represented by some $I_{m}$, and let $g_{d}(1)$ be the smallest positive integer such that all Hermitian lattices in $\mathfrak{S}_{d}(1)$ can be uniformly represented by $I_{g_{d}(1)}$. In this talk, we will determine the explicit form of $\mathfrak{S}_{d}(1)$ and the exact value of $g_{d}(1)$ for all imaginary quadratic fields $E$ with class number 2 or 3 , generalizing naturally the Lagrange's four-square theorem.
(Received September 15, 2023)

## 1192-11-29172

Tyler Genao, The Ohio State University, Tristan Phillips*, Dartmouth College, Frederick V Saia, University of Illinois Chicago, Tim Santens, KU Leuven, John Yin, University of Wisconsin-Madison. Counting Abelian Surfaces. Preliminary report.
How many abelian varieties have a given property? In the case of elliptic curves (i.e., abelian varieties of dimension 1) there have been many recent results which count the number of elliptic curves with a given torsion subgroup, level-structure, or which satisfy a set of local conditions. In this talk we will discuss an approach towards counting abelian surfaces (i.e., abelian varieties of dimension 2) with a given property. More precisely, we will consider the problem of counting the number of twist classes of abelian surfaces with quaternionic multiplication by the indefinite rational quaternion algebra of discriminant 6 whose Igusa invariants (viewed as points in a weighted projective stack) are of bounded height. (Received September 05, 2023)

## 1192-11-29185

Joshua Harrington, Cedar Crest College, Lenny Jones, Shippensburg University, Tristan Phillips*, Dartmouth College. Primitive Covering Numbers.
A covering number is a positive integer $N$ for which a covering system of the integers can be constructed with distinct moduli that are divisors $d>1$ of $N$. If none of the proper divisors of a covering number $N$ are themselves a covering number, then $N$ is called a primitive covering number. Let $a$ and $b$ be positive integers and let $p$ and $q$ be distinct odd primes. In this talk we will discuss necessary and sufficient conditions for a number of the form $2^{a} p^{b} q$ to be a primitive covering number.
(Received September 05, 2023)

1192-11-29231
Samuel Crew, Max Planck Institute - Security and Privacy, Germany, Ankush Goswami*, University of Texas Rio Grande Valley, Texas, US, Robert B. Osburn, University College Dublin, Ireland. Strange identities, the Habiro ring and resurgence. In 1997, Kontsevich introduced a curious expression which does not converge on any open subset of the complex numbers, but is well-defined at a root of unity. Moreover, it is an element of the Habiro ring which satisfies a "strange identity" due to Zagier. In this talk, we discuss resurgence properties for the Borel transform of elements in the Habiro ring which satisfy a general type of strange identity. As an application, we provide evidence for (and against) conjectures in quantum topology due to Costin and Garoufalidis. This is joint work with Samuel Crew and Robert Osburn.
(Received September 06, 2023)

1192-11-29261
Ae Ja Yee*, Pennsylvania State University. The Rogers-Ramanujan Identities and Ariki-Koike Algebras.
In 2000, Ariki and Mathas showed that the simple modules of the Ariki-Koike algebras $\mathcal{H}_{\mathbb{C}, v ; Q_{1}, \ldots, Q_{m}}(G(m, 1, n))$ (when the parameters are roots of unity and $v \neq 1$ ) are labeled by the so-called Kleshchev multipartitions. This together with Ariki's categorification theorem enabled Ariki and Mathas to obtain the generating function for the number of Kleshchev multipartitions by making use of the Weyl-Kac character formula. In this talk, I will revisit this generating function for $v=Q_{1}=\cdots=Q_{a}=-1, Q_{a+1}=\cdots=Q_{m}=1$. This case is particularly interesting, for the corresponding Kleshchev multipartitions have a very close connection to the Rogers-Ramanujan identities. I will discuss an analytic proof of this generating function. The second objective of this talk is to count the number of simple modules of the Ariki-Koike algebra in a fixed block. It is known that the simple modules of the Ariki-Koike algebras in a fixed block are labeled by the Kleshchev multiparitions with a fixed partition residue statistic. This partition statistic was also studied in the works of Berkovich, Garvan, and Uncu. Employing their results, we can get two bivariate generating function identities for $m=2$. I will also discuss these identities. This talk is based on joint work with S. Chern, Z. Li, D. Stanton, and T. Xue. (Received September 06, 2023)

## 1192-11-29323

John Layne, University of Virginia, Samuel Whiting Marshall*, University of Central Florida, Christopher Sadowski, Ursinus College, Emily Shambaugh, Dickinson College. A motivated proof of the Bressoud-Göllnitz-Gordon identities. We present what we call a "motivated proof" of the Bressoud-Göllnitz-Gordon partition identities. Similar "motivated proofs" have been given for the Rogers-Ramanujan, Gordon's, Andrews-Bressoud, and the Göllnitz-Gordon-Andrews partition identities. In our proof, we introduce "ghost series" similar to those introduced in the motivated proof of the AndrewsBressoud identities and use recursions similar to those in the motivated proof of the Göllnitz-Gordon-Andrews identities. We anticipate that this motivated proof of the Bressoud-Göllnitz-Gordon identities will illuminate certain twisted vertex-algebraic constructions. This work was done as part of the Summer 2023 Ursinus College NSF REU, grant number 1851948. (Received September 06, 2023)

1192-11-29397
Jim L. Brown, Occidental College, Felice Manganiello*, Clemson University. The impact of Kevin James beyond Number Theory.
This segment is dedicated to exploring the influence Kevin James has had on mathematics, as evidenced through recorded videos with individuals who have crossed paths with him during his career.

Frank Kschischang, University of Toronto, Felice Manganiello*, Clemson University, Alberto Ravagnani, Eindhoven
University of Technology, Kristen Savary, Clemson University. External Codes for Multiple Unicast Networks via Interference Alignment.
In this seminar, we introduce a formal framework to study the multiple unicast problem for a coded network in which the network code is linear over a finite field and fixed. We show that the problem corresponds to an interference alignment problem over a finite field. In this context, we establish an outer bound for the achievable rate region and provide examples of networks where the bound is sharp. We finally give evidence of the crucial role played by the field characteristic in the problem
(Received September 06, 2023)

1192-11-29442
Sung Min Lee*, University of Illinois, Chicago. The distribution in arithmetic progressions of primes of cyclic reduction for an elliptic curve.
Given an elliptic curve $E / \mathbb{Q}$ and a prime $p$, let $E_{p}$ be the reduction of $E$ modulo $p$. It is known that $E_{p}\left(\mathbb{F}_{p}\right)$ forms an abelian group of rank at most 2 . We call $p$ a prime of cyclic reduction for $E$ if $E_{p}\left(\mathbb{F}_{p}\right)$ is a cyclic group. Recently, Akbal and Güloǧlu considered the question of cyclicity of $E_{p}\left(\mathbb{F}_{p}\right)$ under the restriction that $p$ lies in an arithmetic progression. In this talk, we present a concrete example where there are no primes $p \equiv k(\bmod n)$ for which $E_{p}\left(\mathbb{F}_{p}\right)$ is cyclic, answering a question of Akbal and Güloğlu in the negative. Furthermore, we demonstrate that, statistically, the distribution of primes of cyclic reduction modulo $n$ is always biased, unless $n$ is a power of two. The first part of this talk is a joint work with Nathan Jones. (Received September 15, 2023)

## 1192-11-29449

Maxwell Forst, University of Minnesota Duluth, Lenny Fukshansky*, Claremont McKenna College. On a new absolute version of Siegel's lemma.
We establish a new version of Siegel's lemma over a number field $k$, providing a bound on the maximum of heights of basis vectors of a subspace of $k^{N}, N \geq 2$. In addition to the small-height property, the basis vectors we obtain satisfy certain sparsity condition. Further, we produce a nontrivial bound on the heights of all the possible subspaces generated by subcollections of these basis vectors. Our bounds are absolute in the sense that they do not depend on the field of definition. The main novelty of our method is that it uses only linear algebra and does not rely on the geometry of numbers or the Dirichlet box principle employed in the previous works on this subject.
(Received September 06, 2023)

1192-11-29459
Jim L. Brown*, Occidental College, Felice Manganiello, Clemson University. An introduction to the mathematics of Kevin James.
In this talk we will highlight the mathematical contributions of Kevin James. This talk will highlight a video tribute to Kevin James by his doctoral advisor Andrew Granville.
(Received September 06, 2023)

## 1192-11-29508

Victoria Cantoral-Farfan, Mathematical Institute, Goettingen, Germany, Wanlin Li, Washington University in St. Louis, Elena Mantovan, California Institute of Technology, Rachel J. Pries, Colorado State University, Yunqing Tang*, University of California Berkeley. Reductions of abelian varieties.
For an elliptic curve over $\mathbb{Q}$, it is known that it has a density one of ordinary reduction (after a finite extension of the field) by Serre and it has infinitely many supersingular reductions by Elkies. In this talk, we will discuss some generalization of Serre's theorem to certain higher dimensional abelian varieties parametrized by unitary Shimura varieties of signatures
$(1, n-1),(0, n), \ldots,(0, n)$ and a generalization of Elkies's theorem to certain abelian fourfold parametrized by a certain unitary Shimura curve of genus 0 . This is joint work (in progress) with Victoria Cantoral Farfan, Wanlin Li, Elena Mantovan, and Rachel Pries.
(Received September 07, 2023)

## 1192-11-29535

Wei-Kai Lai*, University of South Carolina Salkehatchie. Digital Root of Power Tower.
The power tower of an integer is defined as the iterated exponentiation of the integer, and is also called a tetration. If it is iterated for $n$ times, it is called a power tower of order $n$. Using the technique of congruence, we find the pattern of the digital root, along with the last digit, of a power tower when its order increases.
(Received September 07, 2023)

1192-11-29549
Sarah Peluse*, University of Michigan, K Soundararajan, Stanford University. Divisibility of character values of the symmetric group. Preliminary report.
In 2017, Miller computed the character tables of $S_{n}$ for all $n$ up to 38 and looked at various statistical properties of the entries. Characters of symmetric groups take only integer values, and, based on his computations, Miller conjectured that almost all entries of the character table of $S_{n}$ are divisible by any fixed prime power as $n$ tends to infinity. In this talk, I will
discuss joint work with K. Soundararajan that resolves this conjecture, and mention some related open problems.
(Received September 07, 2023)

## 1192-11-29616

John Layne*, University of Virginia, Samuel Whiting Marshall*, University of Central Florida, Christopher Sadowski, Ursinus College, Emily Shambaugh*, Dickinson College. Ghost series and a motivated proof of the Bressoud-Göllnitz-Gordon identities.
We present what we call a "motivated proof" of the Bressoud-Göllnitz-Gordon partition identities. Similar "motivated proofs" have been given for the Rogers-Ramanujan, Gordon's, Andrews-Bressoud, and the Göllnitz-Gordon-Andrews partition identities. In our proof, we introduce "ghost series" similar to those introduced in the motivated proof of the AndrewsBressoud identities and use recursions similar to those in the motivated proof of the Göllnitz-Gordon-Andrews identities. We anticipate that this motivated proof of the Bressoud-Göllnitz-Gordon identities will illuminate certain twisted vertex-algebraic constructions. This work was done as part of the Summer 2023 Ursinus College NSF REU, grant number 1851948.
(Received September 09, 2023)

## 1192-11-29667

Breeanne Baker Swart, The Citadel, Susan Crook, Loras College, Helen G Grundman*, Bryn Mawr College, Laura L
Hall-Seelig, Merrimack College. Gaussian B-Happy Numbers.
In this talk, I will review the concept of a $B$-happy number and define its extension from the positive integers to the Gaussian integers. After discussing the fixed points and cycles of the relevant functions, I will provide conditions for the existence and non-existence of arbitrarily long arithmetic sequences of Gaussian $B$-happy numbers. Finally, I will describe an alternative definition of Gaussian happy numbers, one which may seem more natural, yet does not agree with the standard definition when restricted to the positive integers.
(Received September 07, 2023)

## 1192-11-29676

David Hovey, South Texas ISD Science Academy, Abigail Martinez, University of Texas Rio Grande Valley, Emily Payne*, University of Texas Rio Grande Valley. An Extension Of Euler's Partition Identity and Part-Frequency Tables.
We extend Euler's "odd=distinct" partition identity to a two-variable identity which considers the number of times parts divisible by $m$ may appear. While an analytic proof is evident, we employ the part-frequency tables of William Keith et al. to combinatorially prove and further generalize this extension.
(Received September 07, 2023)

1192-11-29700
Maarten Derickx, No affiliation, Anastassia Etropolski, Foursquare, Jackson S. Morrow, University of North Texas, Mark van Hoeij, Florida State University, David Michael Zureick-Brown*, Emory University. Sporadic cubic torsion. I'll survey various recent results about "sporadic" (or "unexpected") points on modular curves, and then focus on recent joint work with Derickx, Etropolski, van Hoeij, and Morrow about torsion on elliptic curves over cubic number fields. (Received September 07, 2023)

Shane Chern*, Dalhousie University, Lin Jiu, Duke Kunshan University. Hankel determinants and Jacobi continued fractions for $q$-Euler numbers.
The $q$-Euler numbers

$$
\left.\varepsilon_{n}:=\frac{1}{( } 1-q\right)^{n} \sum_{k=0}^{n}(-1)^{k}\binom{n}{k} \frac{1+q}{1}+q^{k+1}
$$

were introduced by Carlitz in 1948. In this talk, I will present our recent results on the Hankel determinant evaluations for $\epsilon_{n}$, that is,


Such determinant evaluations are built upon the construction of the associated orthogonal polynomials for $q$-Euler numbers, which are given by a specialization of the big $q$-Jacobi polynomials, thereby yielding the corresponding Jacobi continued fraction expressions. Our results are parallel to those of Chapoton and Zeng on $q$-Bernoulli numbers. This is joint work with Lin Jiu.
(Received September 07, 2023)

1192-11-29703
Angelica Babei, McMaster University, Manami Roy, Lafayette College, Holly Swisher*, Oregon State University, Bella Tobin, Agnes Scott College, Fang-Ting Tu, Louisiana State University. Supercongruences arising from Ramanujan-Sato Series. Preliminary report.
In 2013, Chisholm, Deines, Long, Nebe, and the third author proved a class of Ramanujan type supercongruences arising as truncations of Ramanujan series for $1 / \pi$ evaluated at singular values corresponding to specific elliptic curves with complex multiplication. Recently, the authors together with Beneish established a theorem which generates Ramanujan-Sato series for $1 / \pi$ and provided 11 explicit examples of such series related to modular forms on non-compact arithmetic triangle groups. Here, we utilize the methods and work of Chisholm et al. to prove supercongruences arising as truncations of these 11

Ramanujan-Sato series.
(Received September 07, 2023)

## 1192-11-29714

Florian Pop*, University of Pennsylvania. Definability of Valuations over arithmetically significant Fields. Preliminary report. I plan to report on old and new developments concerning (uniform) first-order definability of valuations of function fields over arithmetically significant base fields, e.g. global and/or local fields. I will also comment on how this type of result can be used to make progress on the (strong) elementary equivalence vs isomorphism problem (EEIP), which is a first-order effectiveness problem in birational arithmetic geometry over the base fields under discussion.
(Received September 07, 2023)

1192-11-29728
Terrence Richard Blackman*, Medgar Evers Community College, CUNY, Zachary Stier, UC Berkeley. Fast Navigation With Icosahedral Golden Gates.
We discuss recent work on the design of universal single-qubit gate sets for quantum computing. Using quaternionicallyderived "super golden gates," we connect the problem of efficient approximate synthesis of given gates to arbitrary precision in quantum hardware design to "icosahedral gates" constructed using the symmetries of the icosahedron, which enjoy a form of optimality. This is joint work with Zachary Stier.
(Received September 07, 2023)

1192-11-29755
Brandon Alberts, Eastern Michigan University, Anh Trong Nam Hoang*, University of Minnesota, Séverin Philip, RIMS, Kyoto University, Allechar Serrano Lopez, Harvard University, Sameera Vemulapalli, Princeton University, David Michael Zureick-Brown, Emory University. Heights on stacks: a comparison.
It is well-known that two popular conjectures in number theory, Malle's conjecture for counting number fields with bounded discriminant and the Batyrev-Manin conjecture for counting rational points of bounded height on Fano varieties, share a very similar formula. In recent works that unify these two conjectures, Ellenberg-Satriano-Zureick-Brown and Darda-Yasuda independently developed different constructions of height for rational points on stacks and gave asymptotic predictions for the number of rational points of bounded height. In this talk, we will compare their frameworks and attempt to match their predictions with known computations of the point counts on modular curves. This is joint with Brandon Alberts, Séverin Philip, Allechar Serrano López, Sameera Vemulapalli, and David Zureick-Brown.
(Received September 07, 2023)

1192-11-29781
Leah Sturman, Bowdoin College, Holly Swisher*, Oregon State University. An asymptotic approach to a conjecture of Kang and Park. Preliminary report.
The Alder-Andrews Theorem, a partition inequality generalizing Euler's partition identity, the first Rogers-Ramanujan identity, and a theorem of Schur to $d$-distinct partitions of $n$, was proved successively by Andrews in 1971, Yee in 2008, and Alfes, Jameson, and Lemke Oliver in 2010. While Andrews and Yee utilized $q$-series and combinatorial methods, Alfes et al. proved the finite number of remaining cases using asymptotics originating with Meinardus and high-performance computing. In 2020, Kang and Park conjectured a "level 2" Alder-Andrews type partition inequality which relates to the second Rogers-Ramanujan identity, and Duncan, Khunger, the second author, and Tamura proved this for all but finitely many cases. Here, we generalize the methods of Alfes et al. to resolve nearly all of the remaining cases of Kang and Park's conjecture. (Received September 07, 2023)

## 1192-11-30003

Sarah Arpin, University of Colorado Boulder, Mingjie Chen, School of Computer Science, University of Birmingham, University Road West, Kristin Lauter, Facebook, Renate Scheidler, University of Calgary, Katherine E. Stange, University of Colorado, Boulder, Ha Tran*, Concordia University of Edmonton. Path-finding algorithms using one endomorphism. The security of isogeny-based cryptosystems depends on several hard problems, one of them is the path-finding problem (to find a path between two specified elliptic curves in a supersingular $\ell$-isogeny graph). In this talk, given an initial curve with a known endomorphism, we use the volcano structure of the oriented supersingular isogeny graph to take ascending/descending/horizontal steps on the graph and deduce path-finding algorithms to the initial curve. (Received September 08, 2023)

## 1192-11-30012

Harold Jimenez Polo*, University of California, Irvine. Atomicity and the Goldbach conjecture for polynomials.
We prove an analogue of the weak Goldbach conjecture for additively reduced and atomic Laurent polynomial semidomains. This is joint work with Nathan Kaplan.
(Received September 08, 2023)
with high probability, and present an implementation of the BGS algorithm. This is joint work with Elisa Bellah, Elena Fuchs and Lynnelle Ye.
(Received September 08, 2023)

## 1192-11-30060

Lea Beneish*, University of North Texas, Jennifer Berg, Bucknell University, Eva G. Goedhart, Franklin \& Marshall College, Hussain Kadhem, University of California, Berkeley, Allechar Serrano Lopez, Harvard University, Stephanie Treneer, Western Washington University. Replicable functions arising from code-lattice VOAs fixed by automorphisms. We ascertain properties of the algebraic structures in towers of codes, lattices, and vertex operator algebras (VOAs) by studying the associated subobjects fixed by lifts of code automorphisms. In the case of sublattices fixed by subgroups of code automorphisms, we identify replicable functions that occur as quotients of the associated theta functions by suitable eta products. We show that these lattice theta quotients can produce replicable functions not associated to any individual automorphisms. Moreover, we show that the structure of the fixed subcode can induce certain replicable lattice theta quotients and we provide a general code theoretic characterization of order doubling for lifts of code automorphisms to the lattice-VOA. Finally, we prove results on the decompositions of characters of fixed subVOAs. This talk is based on joint work with Jennifer Berg, Eva Goedhart, Hussain M. Kadhem, Allechar Serrano López, and Stephanie Treneer. (Received September 08, 2023)

1192-11-30091
Navvye Anand, Sanskriti School, Amit Basistha, Indian Statistical Institute, Bangalore, Alexander Gong*, Columbia University, Steven Joel Miller, Williams College, Stephanie Reyes, Claremont Graduate University, Alexander Zhu, Carleton College. Sum of Consecutive Terms of Pell and Related Sequences.
Linear recurrences, which are discrete analogue of differential equations, arise in many areas of mathematics and science, and thus it is important to understand the properties of the terms of these sequences. Frequently there are fascinating relations among the elements, which reflect interesting behaviors; perhaps the most famous is the ratio of adjacent terms of the Fibonacci sequence ( $F_{n}=F_{n-1}+F_{n-2}$ ) tends to the golden mean, which is ubiquitous throughout nature. Here we study new identities related to the sums of adjacent terms in the Pell sequence, which is defined by $P_{n}=2 P_{n-1}+P_{n-2}$ for $n \geq 2$ and $P_{0}=0, P_{1}=1$ (though many of our results hold for more general recurrences). We prove that the sum of $N$ consecutive Pell numbers is an integer multiple of another Pell number if and only if $4 \mid N$ We also consider the Generalized Pell $(k, i)$ numbers defined by $p(n)=2 p(n-1)+p(n-k-1) \quad n \geq k+1$, with $p(0)=p(1)=\cdots=p(i)=0$ and
$p(i+1)=\cdots p(k)=1$ for $0 \leq i \leq k-1$, and prove that the sum of $N=2 k+2$ consecutive terms is an integer multiple of another term in the sequence, and we prove that for the Generalized Pell $(k, k-1)$-numbers such an integer multiple relation does not exist when $N$ and $k$ are odd.We also give analogous results for the Fibonacci and other related second order recursive sequences.
(Received September 08, 2023)

1192-11-30122
Harold Jimenez Polo*, University of California, Irvine. A Goldbach theorem for Laurent polynomials with positive integer coefficients.
We prove an analogue of the Goldbach conjecture for Laurent polynomials with positive integer coefficients. This is joint work with Sophia Liao.
(Received September 08, 2023)

## 1192-11-30125

Haiyang Wang*, University of Georgia. On the Kodaira Types of Elliptic Curves with Potentially Good Supersingular Reduction. Preliminary report.
Let $O_{K}$ be a Henselian discrete valuation domain with field of fractions K . Assume that $O_{K}$ has algebraically closed residue field $k$. Let $E / K$ be an elliptic curve with additive reduction. The semi-stable reduction theorem asserts that there exists a minimal extension $L / K$ such that the base change $E_{L} / L$ has semi-stable reduction. It is natural to wonder whether specific properties of the semi-stable reduction and of the extension $L / K$ impose restrictions on what types of Kodaira type the special fiber of $\mathrm{E} / \mathrm{K}$ may have. In this talk we will discuss the restrictions imposed on the reduction type when the extension $\mathrm{L} / \mathrm{K}$ is wildly ramified of degree 2 , and the curve $E / K$ has potentially good supersingular reduction. We will also discuss the possible reduction types of two isogenous elliptic curves with these properties. (Received September 08, 2023)

## 1192-11-30126

Joselyne Aniceto*, University of Texas - Rio Grande Valley. Congruences properties of consecutive coefficients in arithmetic progression of Gaussian polynomials..
A 2007 theorem of Dr. Brandt Kronholm on infinite families of consecutive partition congruences in arithmetic progression for the function $p(n, m)$ enumerating partitions of $n$ into at most $m$ parts, is at the center of this research. A second theorem describes another infinite families of closely related partition congruences. This theorem captures a narrow infinite family of partition congruences from the function $p(n, m, N)$ enumerating partitions of $n$ into at most $m$ parts, no part larger than $N$. However, empirically, there appears to be a larger infinite family of congruences awaiting general description. The proposed research project will consist of conjecturing what this larger infinite family is and to then prove that conjecture.
(Received September 08, 2023)

Siddarth Menon, University of California, Berkeley, William Verreault, University of Toronto, Asif Zaman, University of Toronto. Natural Moments of Random Multiplicative Functions over Function Fields.
Little is known about the distribution of the partial sums of random multiplicative functions defined over integers, but the order of magnitude for all moments have been recently determined by Harper in a series of highly sophisticated papers. Building on recent work extending multiplicative and probabalistic number theory to the function field setting, we study the even natural moments of partial sums of Steinhaus and Rademacher random multiplicative functions defined over function fields. Using analytic arguments that parallel the work of Harper, Nikeghbali, and Radziwiłł and Heap and Lindqvist over the integers, as well as combinatorial arguments peculiar to the function field setting, we obtain an exact expression for the 4th moment and an asymptotic expression for the higher natural moments in the large $q$ and large $n$ limits which matches with the results over integers.
(Received September 10, 2023)

## 1192-11-30151

Hung Viet Chu, TEXAS A\&M UNIVERSITY, COLLEGE STATION, Sujith Uthsara Kalansuriya Arachchi, UNIVERSITY OF COLOMBO, Jiasen Liu, University of Southern California, Qitong Luan*, University of California, Los Angeles, Rukshan Marasinghe, UNIVERSITY OF COLOMBO, Steven Joel Miller, Williams College. On a Pair of Diophantine Equations. For relatively prime natural numbers $a$ and $b$, we study the two equations $a x+b y=(a-1)(b-1) / 2$ and $a x+b y+1=(a-1)(b-1) / 2$, which arise from the study of cyclotomic polynomials. Previous work showed that exactly one equation has a nonnegative integer solution, and the solution is unique. Our first result gives criteria to determine which equation is used for a given pair $(a, b)$. We then use the criteria to study the sequence of equations used by the pair $\left(a_{n} / \operatorname{gcd}\left(a_{n}, a_{n+1}\right), a_{n+1} / \operatorname{gcd}\left(a_{n}, a_{n+1}\right)\right)$ from several special sequences $\left(a_{n}\right)_{n \geq 1}$. We also discover that for any $k \in \mathbb{N}$, the sequence of equations used by $\left(\left\lceil 2^{n+k-1} /\left(2^{k}+1\right)\right\rceil\right)_{n \geq 1}$ has each equation appear in groups of $k$ alternatively. Finally, fixing $k \in \mathbb{N}$, we investigate the periodicity of the sequence of equations used by the pair $(k / \operatorname{gcd}(k, n), n / \operatorname{gcd}(k, n))$ as $n$ increases.
(Received September 08, 2023)

1192-11-30166
Sarah Arpin, University of Colorado Boulder, Tyler Raven Billingsley, St. Olaf College of Northfield, MN, Daniel Rayor Hast, Boston University, Jun Bo Lau*, Boston University, Ray Perlner, NIST, Angela Robinson, NIST. On the Decoding Failure Rate of BIKE. Preliminary report.
We study the decoding failure (DFR) of BIKE, a fourth round candidate in the NIST Post-Quantum Cryptography Standardization process. After selecting suitable parameters, we compute the average DFR at the 20-bit security level and identify the waterfall and error floor regions of the DFR curve. We use graph-theoretic methods from low density parity check codes to identify certain trapping sets with near-codewords defined by Vasseur in 2021. This leads to an ongoing project to design probabilistic models and investigate the effect of near-codewords on the bit-flipping decoder. This is joint work with Sarah Arpin, Tyler Billingsley, Daniel Hast, Ray Perlner and Angela Robinson.
(Received September 09, 2023)

## 1192-11-30174

Jiajun Ma, School of Mathematical Sciences, Xiamen University;, Congling Qiu, MIT, Zhiwei Yun, MIT, Jialiang Zou*, University of Michigan, Ann Arbor. Theta correspondence and Springer correspondence.
In this talk, we consider the finite field theta correspondence between principle series. Joint with Jiajun Ma and Congling Qiu, we explicitly describe this correspondence by analyzing the relevant Hecke algebra bimodules and applying a deformation argument. Joint with Zhiwei Yun, we geometrized the whole picture. Consequently, we obtained a relation between the Springer correspondence and theta correspondence.
(Received September 09, 2023)

1192-11-30181
Jordan S Ellenberg, University of Wisconsin-Madison, Boya Wen*, University of Wisconsin-Madison. Exceptional points on Atkin-Lehner quotients of modular curves. Preliminary report.
The curve $X_{0}^{+}(N)$ is the quotient of the modular curve $X_{0}(N)$ by the Atkin-Lehner involution $w_{N}$. A point in $X_{0}^{+}(N)(\mathbb{Q})$ that is neither a cusp nor a CM point is called an exceptional point. Balakrishnan, Best, Bianchi, Dogra, Lawrence, Müller, Triantafillou, Tuitman, and Vonk (in publications by subsets of these authors) developed computational tools to carry out quadratic Chabauty explicitly, and showed that among the curves $X_{0}^{+}(N)$ of prime level $N$ and genus 2 and 3 , the only curves with exceptional rational points are those with level $N=73,103,191$ (all genus 2 ). In joint work with Adžaga, Arul, Beneish, Chen, Chidambaram and Keller, we showed that among the curves $X_{0}^{+}(N)$ of prime level $N$ and genus 4, 5, and 6, the only curves with exceptional rational points are those with level $N=137,311$ (both genus 4). The exceptional points on $X_{0}^{+}(N)$ for $N=73,103,191,137,311$ were known to Galbraith, who also found that the exceptional points on $X_{0}^{+}(N)$ for $N=73,103,191$ arise from the hyperelliptic involution. Little was known as to why exceptional points arise on nonhyperelliptic curves $X_{0}^{+}(137)$ and $X_{0}^{+}(311)$. In this talk, I will report joint work in progress with Jordan Ellenberg concerning these exceptional points, inspired by data obtained from the quadratic Chabauty algorithm.
(Received September 09, 2023)

1192-11-30244
Noah Kravitz*, Princeton University. Recent work on Lonely Runner spectra.
Dirichlet's Theorem says that for every natural number $n$ and every real number $t$, there is some $v \in\{1,2, \ldots, n\}$ such that $t v$ lies within $1 /(n+1)$ of an integer. The Lonely Runner Conjecture of Wills and Cusick asserts that the constant $1 /(n+1)$ cannot be improved by replacing $\{1,2, \ldots, n\}$ with a different set of $n$ nonzero real numbers. In this short talk I will survey recent progress on the "Lonely Runner spectra" that arise naturally in the context of this problem.

Alexander Kalogirou*, University of South Carolina. Minimum modulus problems. Preliminary report.
We describe applications of the moments/distortion method developed by Balister, Bollobás, Morris, Sahasrabudhe, and Tiba. Elaborating on their approach, we are able to obtain answers to a variety of questions on the minimum modulus in a covering system. For example, if $m_{0}$ is the minimum modulus in a covering system with distinct moduli, then is the sum of the reciprocals of the moduli $>1.001$ ? For $m_{0}=4$, this is not necessarily the case. We explore what happens for larger $m_{0}$. This is work in progress with Michael Filaseta, Robert Groth, Jonah Klein and Ognian Trifonov.
(Received September 10, 2023)

1192-11-30418
Maximilian Carl Eric Hofmann, Goethe-University, Frankfurt am Main, Annemily Gammie Hoganson, Carleton College, Siddarth Menon*, University of California, Berkeley, William Verreault, University of Toronto, Asif Zaman, University of Toronto. Integer Moments of Random Multiplicative Functions Over Function Fields.
Little is known about the distribution of the partial sums of random multiplicative functions defined over integers, however, order of magnitude estimates for all moments have been recently determined by Harper in a series of highly sophisticated papers. Building on recent work extending multiplicative and probabalistic number theory to the function field setting, we study the even natural moments of partial sums of Steinhaus and general natural moments of partial sums Rademacher random multiplicative functions defined over function fields. Using analytic arguments that parallel the work of Harper, Nikeghbali, and Radziwill and Heap and Lindqvist over the integers, as well as combinatorial arguments unique to the function field setting, we obtain an exact expression for the 4th moment and an asymptotic expression for the higher natural moments in the large $q$ and large $n$ limits which corroborates the analogous results over integers
(Received September 10, 2023)

## 1192-11-30482

Aaron J Pollack*, University of California, San Diego. Arithmeticity of modular forms on G_\{2\}
Quaternionic modular forms on the exceptional group G_\{2\} were defined by Gan-Gross-Savin, following work of GrossWallach. They are remarkable automorphic functions, in that they admit a refined notion of Fourier expansion and Fourier coefficients, similar to the holomorphic modular forms on Shimura varieties. We will explain the proof that the cuspidal quaternionic modular forms of even weight at least 6 admit an algebraic structure, defined in terms of these Fourier coefficients. The proof is theta functions all the way: one produces numerous such cusp forms with algebraic Fourier coefficients by theta-lifting from anistropic F_\{4\} (using the minimal representation on E_\{8\}), and one shows that these lifts produce enough cusp forms by proving and using a Siegel-Weil theorem for a dual pair D_\{4\} x D_\{4\} in E_\{8\}.
(Received September 10, 2023)

1192-11-30490
Niven Achenjang, MIT, Deewang Bhamidipati, UC Santa Cruz, Aashraya Jha*, Boston University, Caleb Ji, Columbia University, Rose Lopez, UC Berkeley. The Brauer group of stacky $Y_{0}(2)$. Preliminary report. The Brauer group of a DM stack $X$ is a fundamental invariant that describes algebras over $X$ which are étale locally isomorphic to matrix algebras. The important case of $\mathscr{M}_{1,1}$, the moduli stack of elliptic curves was done by Antieau and Meier and Shin. We try and adapt these techniques to find the the Brauer group of $\mathscr{Y}_{0}(2)$ over bases of arithmetic interest. Antieau and Meier compute the cohomology of $\mathscr{Y}(2)$ by realizing it as the classifying stack of $C_{2}$ over the base $X=\mathbb{A}^{1} \backslash\{0,1\}$ (the Legendre family), and then use the Hochschild-Serre spectral sequence associated to the $S_{3}$-Galois cover $\mathscr{Y}(2) \rightarrow \mathscr{M}_{1,1}$. We modify this technique by considering the Hoschild-Serre sequence for $C_{2}$-Galois cover $\mathscr{Y}(2) \rightarrow \mathscr{Y}_{0}(2)$. (Received September 10, 2023)

## 1192-11-30492

Jennifer Balakrishnan, Boston University, Daniel Rayor Hast, Boston University, Aashraya Jha*, Boston University. Quadratic Chabauty over number fields. Preliminary report.
Let $K$ be a number field, and $C / K$ a curve. By theorems of Siegel and Faltings we know the number of integral points on a curve for genus $g \geq 1$ and rational points for a curve $g \geq 2$ are finite. Unfortunately these proofs don't compute the finite list of points on the curves. Quadratic Chabauty has been used extensively to find $\mathbb{Q}$-rational points on curves $C / \mathbb{Q}$. For curves $C / \mathbb{Q}$ there has also been some progress made to compute the set $C(K)$ where $K$ is a quadratic field. In this talk, we discuss the progress made for finding rational and integral points of curves $C / K$ which are not base changes from $\mathbb{Q}$. This is joint work with Daniel Hast and Jennifer Balakrishnan. (Received September 10, 2023)

Antun Milas*, SUNY at Albany. Z-hat invariants and higher depth quantum modular forms. Preliminary report.
A few years ago, Gukov, Pei, Putrov and Vafa introduced certain remarkable q-series, the radial limits of which are expected to yield WRT invariants of plumbed 3-manifolds. Gukov proposed the conjecture that these series are quantum modular forms. In a recent work with Bringmann and Mahlburg, we demonstrated that, for specific integral homology spheres, the Z-hat invariants correspond indeed to quantum modular forms of higher depth. In this presentation, we will provide an overview of our approach towards achieving (higher depth) quantum modularity in a broader context.
(Received September 10, 2023)

Esme Rosen*, Lousiana State University. Computing with Arithmetic Triangle Groups. Preliminary report.
Arithmetic groups have with properties similar to the classical modular curve defined using $S L_{2}(\mathbb{Z})$. On the other hand, triangle groups arise as the monodromy groups of certain hypergeometric differential equations. Particularly special are arithmetic triangle groups, of which there are finitely many, classified by Takuechi. In this talk, we consider questions about the geometry and arithmetic of the curve arising from the arithmetic triangle group $(3,3,5)$ defined over $\mathbb{Q}(\sqrt{5})$. We also present a framework for how these results are related to hypergeometric functions.
(Received September 10, 2023)

1192-11-30518
Abbey Marie Bourdon*, Wake Forest University, Sachi Hashimoto, Brown University, Timo Keller, University of Groningen, Zev Klagsbrun, Center for Communications Research, David Lowry-Duda, ICERM \& Brown University, Travis Morrison, Virginia Tech, Filip Najman, University of Zagreb, Himanshu Shukla, University of Bayreuth. An Algorithm for Isolated $j$-invariants.
In this talk, I will discuss a new algorithm which can be used to determine whether an elliptic curve with rational $j$-invariant gives rise to an isolated point on some modular curve $X_{1}(n)$. Our results are most compelling in the case of $\mathbb{P}^{1}$-isolated points, which are those not induced by a rational map to the projective line of the same degree. Running the algorithm on all elliptic curves presently in the LMFDB gives evidence for the conjecture that $j \in \mathbb{Q}$ is the image of a $\mathbb{P}^{1}$-isolated point if and only if $j$ corresponds to an elliptic curve with complex multiplication or $j=-140625 / 8,-9317,351 / 4$, or -162677523113838677 . (Received September 10, 2023)

1192-11-30521
Abdellatif Anas Chentouf, MIT, Catherine Hazel Cossaboom, University of Virginia, Samuel Goldberg*, University of Virginia, Jack B Miller, Yale University. Patterns of primes in joint Sato-Tate distributions. In 2013, James Maynard demonstrated that there are infinitely many pairs of consecutive primes which differ by 600 or less; the underlying ideas were also developed by Terence Tao independently. Here, we build on the Maynard-Tao machinery to show that there exist bounded gaps in distinguished positive density subsets of the primes, defined as follows. Let $\mathcal{E}_{1}$ and $\mathcal{E}_{2}$ be elliptic curves. For each prime $p$ and $j=1,2$, let $a_{j}(p)$ be the normalized error for the number of solutions mod $p$, i.e.
$a_{j}(p):=\frac{p+1-\# \mathcal{E}_{j}\left(\mathcal{F}_{p}\right)}{2 \sqrt{p}}$. The now-proven Sato-Tate conjecture states that the normalized error terms $a_{j}(p)$ equidistribute with respect to a semicircular distribution on $[-1,1]$. We show that if $\mathcal{E}_{1}$ and $\mathcal{E}_{2}$ are twist-inequivalent, then for all subintervals $I_{1}, I_{2} \subset[-1,1]$, there exist infinitely many bounded gaps between the primes $p$ satisfying both $a_{1}(p) \in I_{1}$ and $a_{2}(p) \in I_{2}$. We additionally prove a common generalization of a Green-Tao style theorem, showing that in our set of primes, there exist arbitrarily long arithmetic progressions infinitely often which meet bounded gaps conditions.
(Received September 10, 2023)

1192-11-30525
Shi Bai*, Florida Atlantic University. Concrete Security of Lattice-Based Cryptography.
One of the most promising candidates for post-quantum cryptography is the lattice-based cryptography. It enjoys strong security proof based on the so-called worst-case to average-case reduction, efficient implementations and algorithmic simplicity. The security of lattice-based cryptography relies on the assumed hardness of solving certain geometric problems on high-dimensional lattices. In this talk, we will discuss several fundamental algorithms for solving such lattice problems and estimating the concrete security of lattice-based cryptography. We will also discuss some recent progress in lattice reduction algorithms in the classic and quantum setting.
(Received September 10, 2023)

## 1192-11-30564

Esther Banaian*, Aarhus University, Archan Sen, University of California Berkeley. Orbifold Markov Numbers.
It is known that Markov numbers can be viewed as a specialization of the cluster variables in the cluster algebra from a oncepunctured torus. We consider "orbifold Markov numbers" which are the result of specializing generalized cluster variables in the generalized cluster algebra (in the sense of Chekhov-Shapiro) from a once-punctured sphere with three orbifold points of order three. It is known by Gyoda that these specializations provide all solutions to a certain generalization of the Markov equation. We provide a direct method for computing orbifold Markov numbers, via snake graphs from orbifolds, and discuss some patterns amongst certain sequences of these numbers. This is based on joint work with Archan Sen. (Received September 10, 2023)

## 1192-11-30565

Qiao He*, Columbia University. Local arithmetic Siegel-Weil formula at ramified primes II.
In this talk I will focus on the analytic side of my joint work with Chao Li, Yousheng Shi and Tonghai Yang on the proof of Kudla-Rapoport conjecture (a.k.a local arithmetic Siegel-Weil formula) for Kramer model. I will discuss what is needed for the analytic side to finish the proof. In particular, we need a surprisingly simple formula for a derived primitive local density, which is one of the crucial ingredients and the major technical innovation of our work. If time permitted, I would also discuss the corresponding case with maximal parahoric level which generalizes the Kramer case.
(Received September 10, 2023)

Timothy J. Huber, University of Texas Rio Grande Valley, James G. Mc Laughlin*, West Chester University, Dongxi Ye, Sun Yat-sen University. Dissections of lacunary eta quotients and identically vanishing coefficients.

For any function $A(q)=\sum_{n=0}^{\infty} a_{n} q^{n}$ define

$$
A_{(0)}:=n \in \mathbb{N}: a_{n}=0
$$

Now suppose $C(q)$ and $D(q)$ are two functions whose $m$-dissections are given by

$$
\begin{aligned}
& C(q)=c_{0} G_{0}\left(q^{m}\right)+c_{1} q G_{1}\left(q^{m}\right)+\ldots+c_{m-1} q^{m-1} G_{m-1}\left(q^{m}\right) \\
& D(q)=d_{0} G_{0}\left(q^{m}\right)+d_{1} q G_{1}\left(q^{m}\right)+\ldots+d_{m-1} q^{m-1} G_{m-1}\left(q^{m}\right)
\end{aligned}
$$

If $c_{i}=0 \Longleftrightarrow d_{i}=0, i=0,1, \ldots, m-1$, then we say that $C(q)$ and $D(q)$ have similar $m$-dissections, and then it is also clear that $C_{(0)}=D_{(0)}$, in which case we say that $C(q)$ and $D(q)$ have identically vanishing coefficients. On the other hand, if $c_{j} \neq 0$ and $d_{j}=0$ for one or more $j \in\{0,1, \ldots, m-1\}$ and otherwise $c_{i}=0 \Longleftrightarrow d_{i}=0$, then $C_{(0)} \varsubsetneqq D_{(0)}$. Previously described experiments indicated, for example, the existence of at least 71 other eta quotients with coefficients that vanish identically with $f_{1}^{4}$ (here $f_{i}=\prod_{n=1}^{\infty}\left(1-q^{i n}\right)$ ). The methods described in the present talk were developed to prove some the experimental results in that previous paper. Some new 4 -dissections of particular eta quotients are developed. These are used in conjunction with known 2 - and 3-dissections to prove many results on the identical vanishing of coefficients in sets of 4-, 6and 8- lacunary eta quotients, results which found experimentally and partially proved in another paper by the present authors. Similar arguments allow many results of the form $C_{(0)} \varsubsetneqq D_{(0)}$ to be proved for many pairs of lacunary eta quotients $C(q)$ and $D(q)$. One sample result we prove using the methods alluded to above is the following. Let $F(q)$ and $G(q)$ be any two eta quotients in the following list:


Then

$$
F_{(0)}=G_{(0)}
$$

(It was shown in a previous paper by the authors that if $A(q)=f_{1}^{4}$ and $F(q)$ is any eta quotient in the list above, then $\left.A_{(0)} \varsubsetneqq F_{(0)}.\right)$
(Received September 10, 2023)

## 1192-11-30580

Yuliann Sepulveda*, University of Texas, Rio Grande Valley. Congruences for coefficients in expansions of certain $q$ hypergeometric series. Preliminary report.
In 2001, Zagier studied various properties of the series $F(q)$, originally introduced by Kontsevich in 1997. In particular, Zagier showed that $F(q)$ satisfies a "strange identity" which leads to its quantum modular transformation properties. In fact, Andrews and Sellers (2016) considered the coefficients in the $(1-q)$-expansion of $F(q)$, also known as the Fishburn numbers, and obtained several congruences. Motivated by this, there has been substantial interest in generalizing these congruences for similar expansions of $q$-hypergeometric series in the Habiro ring that satisfy a "strange identity." In this work, we expand the previous work to two non-Habiro-ring elements considered by Andrews, Jiménez-Urroz and Ono (2001) that also satisfy strange identities. We show that the coefficients (appropriately normalized) in the $(1-q)$-expansions of these series give rise to similar congruences.
(Received September 11, 2023)

## 1192-11-30584

Arseniy Sheydvasser*, Bates College. Ulam Sequences: Chaos and Order and Connections Between the Two.
Back in 1964, Stanisław Ulam defined a funny sequence. Start with 1 and 2, and then every subsequent term will be the next smallest number that can be written as a sum of two distinct prior terms in exactly one way. So, it starts with: $1,2,3,4,6,8$, $11,13,16,18,26,28,36,38,47,48,53,57,62,69,72,77,82,87,97,99,102, \ldots$ and so on. It is very easy to get lots of data and to make conjectures about what should be true of sequences like this. Actually proving results has been much harder, but a significant portion of what is known was established by undergraduate students. One recurring theme, which has come up again and again, is that Ulam-type sequences are very ordered in terms of their larger-scale structure, but seemingly completely chaotic on smaller scales. We'll talk a little about what is known (with additional references for those that want to know more) and much more about what is *not* known and might be fun to explore with students.
(Received September 10, 2023)

## 1192-11-30598

Brian Freidin, Auburn University, Alejandro Tomas Lopez*, Rice University, Bella Villarreal*, Grinnell College, Ren Watson, University of Texas At Austin, Jaedon Whyte, MIT. Cornering The True Proportion of Transverse-Free Projective Plane Curves.
We say that a curve $C \subset \mathbb{P}^{2}$ defined over $\mathbb{F}_{q}$ is transverse-free if every line defined over $\mathbb{F}_{q}$ intersects $C$ at some point with multiplicity at least 2 . Originally raised by Charles Favre, we consider the problem of determining an exact measure on how often these curves occur. Using the framework established by Poonen for computing the density, $\mu$, of curves that satisfy certain local properties, we improve upon Asgarli and Freidin's joint work in the area, and find that for the set of transversefree curves, $\mu \sim q^{-3 q-1}$ as $q \rightarrow \infty$. In particular, we show that there is a special class of curves, which we name trivial blocking curves, that are always transverse-free, use them to derive lower and upper bounds on $\mu$, and then show that nearly all transverse-free curves are trivial blocking curves for sufficiently large $q$ in order to find the asymptotic. Along the way, we
also specialize the computation tools introduced by the aforementioned authors.
(Received September 10, 2023)

## 1192-11-30634

Alejandro Tomas Lopez, Rice University, Bella Villarreal, Grinnell College, Ren Watson, University of Texas At Austin Jaedon Whyte*, MIT. On the Proportion of Transverse-Free Plane Curves.
We say that a curve $C \subset \mathbb{P}^{2}$ defined over $\mathbb{F}_{q}$ is transverse-free if every line defined over $\mathbb{F}_{q}$ intersects $C$ at some point with multiplicity at least 2 . Bounds on the asymptotic proportion of transverse-free curves were previously derived by Asgarli and Freidin in the smooth case. We tackle the general case and show that there is a special class of curves, which we name trivial blocking curves, that are always transverse-free. In particular, we find a lower bound on the size of this class of curves, and show that this bound is asymptotic to the true proportion of transverse-free curves as $q \rightarrow \infty$, that is, nearly all transversefree curves are trivial blocking curves for sufficiently large $q$.
(Received September 10, 2023)

1192-11-30654
Xuancheng Shao*, University of Kentucky. Popular differences in certain polynomial progressions.
A nonlinear version of Roth's theorem states that dense sets of integers contain configurations of the form $x, x+d, x+d^{2}$. We discuss an effective "popular" version of this result, showing that every dense set has some nonzero $d$ such that the number of configurations with difference parameter $d$ is almost optimal. Perhaps surprisingly, the quantitative dependence in this result is exponential, compared to the tower-type bounds encountered in the popular linear Roth theorem. We also discuss generalizations to a multidimensional configuration, configurations with more than three terms, and configurations with some restrictions on $d$. The talk includes joint works with Sarah Peluse, Sean Prendiville, and Mengdi Wang. (Received September 10, 2023)

## 1192-11-30673

Caner Nazaroglu*, University of Cologne. False-Indefinite Theta Functions and Applications. Preliminary report. Indefinite theta functions have an integral place as building blocks of mock modular forms, which then appear in a myriad of physical applications. In recent studies of M5-brane compactifications to three dimensions and its applications to threedimensional topology, it has become clear that false theta functions, which are closely related to indefinite theta functions, also make they appearance and a wider world of holomorphic quantum modularity is expected to emerge. A class of functions beyond these two is the so-called 'false-indefinite theta functions', which in certain special cases can be studied with mock Maass theta functions developed by Zwegers. It is then natural to ask to what extent these new modularity properties can be used to replicate the successes of ordinary modular forms. In this talk, we will investigate this question for algebraic and combinatorial applications.
(Received September 10, 2023)

## 1192-11-30682

Philip Tosteson*, University of North Carolina, Chapel Hilll. Factorization statistics of polynomials via configuration spaces. Given a random polynomial $f$ of degree $d$ over field, what is the expected number of irreducible factors of $f$ ? This question asks about a factorization statistic (the number of irreducible factors of $f$ ). I will talk about connections between the cohomology of configuration spaces and factorization statistics of polynomials over finite and $p$-adic fields. (Received September 10, 2023)

## 1192-11-30697

Robert S Groth*, University of South Carolina. Generalized Sierpiński Number.
A Sierpiński number is a positive odd integer $k$ such that $k \cdot 2^{n}+1$ is composite for all $n \in \mathbb{Z}^{+}$. Fix an integer $A$ with $2 \leq A$. We show there exists a positive odd integer $k$ such that $k \cdot a^{n}+1$ is composite for all integers $a \in[2, A]$ and all $n \in \mathbb{Z}^{+}$. This is joint work with Michael Filaseta and Thomas Luckner.
(Received September 10, 2023)

1192-11-30728
Yuxin Lin, California Institute of Technology, Deepesh Singhal*, University of California, Irvine. Primes in denominators of Algebraic numbers.
Denote the set of algebraic numbers as $\overline{\mathbb{Q}}$ and the set of algebraic integers as $\overline{\mathbb{Z}}$. For $\gamma \in \overline{\mathbb{Q}}$, consider its irreducible polynomial in $\mathbb{Z}[x], F_{\gamma}(x)=a_{n} x^{n}+\cdots+a_{0}$. Denote $e(\gamma)=\operatorname{gcd}\left(a_{n}, a_{n-1}, \ldots, a_{1}\right)$. Drungilas, Dubickas and Jankauskas show in a recent paper that $\mathbb{Z}[\gamma] \cap \mathbb{Q}=\left\{\alpha \in \mathbb{Q} \mid\left\{p \mid v_{p}(\alpha)<0\right\} \subseteq\{p|p| e(\gamma)\}\right\}$. Given a number field $K$ and $\gamma \in \overline{\mathbb{Q}}$, we show that there is a subset $X(K, \gamma) \subseteq \operatorname{Spec}\left(\mathcal{O}_{K}\right)$, for which $\mathcal{O}_{K}[\gamma] \cap K=\left\{\alpha \in K \mid\left\{\mathfrak{p} \mid v_{\mathfrak{p}}(\alpha)<0\right\} \subseteq X(K, \gamma)\right\}$. We prove that $\mathcal{O}_{K}[\gamma] \cap K$ is a principal ideal domain if and only if the primes in $X(K, \gamma)$ generate the class group of $\mathcal{O}_{K}$. More generally, we show that the class group of $\mathcal{O}_{K}[\gamma] \cap K$ is obtained from the class group of $\mathcal{O}_{K}$ by taking the quotient by the subgroup generated by primes in $X(K, \gamma)$. We then fix a number field $K$ and study statistical properties of the ring $\mathcal{O}_{K}[\gamma] \cap K$ as $\gamma$ varies over algebraic numbers of a fixed degree $n \geq 2$. Given $k \geq 1$, we explicitly compute the density of $\gamma$ for which $\mathcal{O}_{K}[\gamma] \cap K=\mathcal{O}_{K}[1 / k]$ and show that this does not depend on the number field $K$. In particular, we show that the density of $\gamma$ for which $\mathcal{O}_{K}[\gamma] \cap K=\mathcal{O}_{K}$ is $\frac{\zeta(n+1)}{\zeta(n)}$.
(Received September 10, 2023)

Alexandra Shlapentokh, East Carolina University, Caleb J Springer*, Heilbronn Institute for Mathematical Research. Definability and decidability for integral functions in infinite algebraic extensions of function fields.
Julia Robinson established that, given any number field $F$, there is a first-order formula defining the ring of integers $\mathcal{O}_{F}$ in $F$. Over the years, this result has been generalized and extended in several ways, including an extension to global fields by Rumely. Intuitively speaking, it appears that each algebraic extension of $\mathbb{Q}$ which is "close to" $\mathbb{Q}$ admits a first-order definition for its ring of integers, while those "close to" the algebraic closure $\overline{\mathbb{Q}}$ do not. For example, a theorem of Shlapentokh generalizing work of Fukuzaki and Videla, proves that every so-called $q$-bounded algebraic extension of $\mathbb{Q}$ has a definable ring of integers. In this talk, we turn our attention to positive characteristic and revisit the notion of $q$-boundedness for algebraic extensions $L$ of $\mathbb{F}_{p}(t)$. Under the $q$-boundedness condition, we establish a first-order definition for $\mathcal{S}$-integers by exploiting the Hasse Norm Principle, and deduce undecidability when the field of constants is infinite.
(Received September 10, 2023)

## 1192-11-30765

Edward Charles Keppelmann*, University of Nevada Reno. Very Triangular Numbers and their Extensions to Pentagonal and other VERY VERY type classifications.. Preliminary report.
Very Triangular numbers are those numbers whose binary representation has a triangular number of ones. The existence of these numbers was investigated by undergraduate Austin Mcewan for his honors thesis under the direction of Edward Keppelmann in 2016. Austin's research raised many related questions about similar notions for square, pentagonal and other kinds of numbers. There are also notions about the distribution and number of non-leading zeroes in these binary representations as well which lead to definitions for the classes of VERY VERY such numbers. By presenting the definitions, some very simple examples, and some basic coding modules in Magma (a mathematical programming language out of the University of Sydney but Mathematica and Maple should also work), math circle participants can explore these notions while also learning to do some mathematical programming. Doing computer searches and looking for patterns are an exciting way to gain an understanding of complex but tractable structures like those presented here. And of course when they are ready, participants can think about writing proofs regarding these objects.
(Received September 11, 2023)

1192-11-30779
Ryan Blau*, The College of Idaho, Joshua Harrington, Cedar Crest College, Sarah Lohrey, Bryn Mawr College, Eliel Sosis, University of Michigan, Tony Wing Hong Wong, Kutztown University of Pennsylvania. Arithmetic Progressions of Integers that are Relatively Prime to their Digital Sums. Preliminary report.
labstract An integer $n \geq 2$ is called $b$-anti-Niven if it is relatively prime to the digit sum of its base-b representation. We explore the maximum lengths of arithmetic progressions (APs) of $b$-anti-Niven numbers. We give bounds for lengths of ( $d$ )-APs of $b$-anti-Niven numbers based on the prime factors of $d$ and $b$, and we show attainability when possible. We also prove that in any base $b$ we can find arbitrarily many consecutive $b$-anti-Niven numbers. \endabstract
(Received September 11, 2023)

1192-11-30802
Abdellatif Anas Chentouf, MIT, Catherine Hazel Cossaboom*, University of Virginia, Samuel Goldberg, University of Virginia, Jack B Miller, Yale University. Patterns of Primes in Joint Sato-Tate Distributions.
In 2013, James Maynard demonstrated that there are infinitely many pairs of consecutive primes which differ by 600 or less; the underlying ideas were also developed by Terence Tao independently. Here, we build on the Maynard-Tao machinery to show that there exist bounded gaps in distinguished positive density subsets of the primes, defined as follows. Let $\mathcal{E}_{1}$ and $\mathcal{E}_{2}$ be elliptic curves. For each prime $p$ and $j=1,2$, let $a_{j}(p)$ be the normalized error for the number of solutions mod $p$, i.e. $a_{j}(p):=\frac{p+1-\# \mathcal{E}_{j}\left(\mathcal{F}_{p}\right)}{2 \sqrt{p}}$. The now-proven Sato-Tate conjecture states that the normalized error terms $a_{j}(p)$ equidistribute with respect to a semicircular distribution on $[-1,1]$. We show that if $\mathcal{E}_{1}$ and $\mathcal{E}_{2}$ are twist-inequivalent, then for all subintervals $I_{1}, I_{2} \subset[-1,1]$, there exist infinitely many bounded gaps between the primes $p$ satisfying both $a_{1}(p) \in I_{1}$ and $a_{2}(p) \in I_{2}$. We additionally prove a common generalization of a Green-Tao style theorem, showing that in our set of primes, there exist arbitrarily long arithmetic progressions infinitely often which meet bounded gaps conditions.
(Received September 11, 2023)

1192-11-30819
Jim L. Brown, Occidental College, Juan Serratos, University of Southern California, Uma Tikekar, University of Maryland, College Park, Jonathan Webb*, University of Idaho. Lattices and their associated theta series for linear codes over $\mathbb{F}_{8}$. Preliminary report.
Let $f=x^{3}+a x+b$ be irreducible over $\mathbb{Q}$ with $a, b \equiv 1(\bmod 2)$ and let $\alpha$ be a root of $f$ in its splitting field. We may adjoin $\alpha$ to $\mathbb{Q}$ to get a number field $K$ of degree three. We specify conditions on $f$ under which $K$ is monogenic and Galois. Our assumption $a, b \equiv 1(\bmod 2)$ guarantees that 2 is inert in $K$, so $\mathcal{O}_{K} / 2 \mathcal{O}_{K} \cong \mathbb{F}_{8}$ where $\mathcal{O}_{K}$ is the ring of integers of $K$. Given a linear code $C \subseteq \mathbb{F}_{8}^{n}$ and the canonical surjection $\pi: \mathcal{O}_{K}^{n} \rightarrow\left(\mathcal{O}_{K} / 2 \mathcal{O}_{K}\right)^{n}$, we associate $C$ with the lattice $\Lambda(C)=\pi^{-1}(C)$. This lattice is integral with respect to the trace form $\operatorname{Tr}_{K / \mathbb{Q}}$. The lattice and trace form together generate a theta series $\Theta_{\Lambda(C)}(q)$. We compute examples of this theta series with $\Lambda(C)=\mathcal{O}_{K}$ for various monogenic and Galois $K$. (Received September 11, 2023)
groups is by examining how they interact with a vector space via a representation. We will study representations of the general linear groups $\mathrm{GL}_{1}\left(\mathbb{F}_{\mathrm{q}}\right)$ and $\mathrm{GL}_{2}\left(\mathbb{F}_{\mathrm{q}}\right)$ using an invariant called the " $\gamma$-factor." This $\gamma$-factor is a finite-field analogue of a $\gamma$-factor appearing in the local Langlands correspondence, and it has historically been defined for certain representations or pairs of representations using the Rankin-Selberg method. In this talk, we review what is known, and we adapt "local" arguments due to Paul Garrett to define and study $\gamma$-factors attached to triples of representations.
(Received September 11, 2023)

## 1192-11-30880

Marcus Appleby, University of Sydney, Steven T Flammia, AWS Center for Quantum Computing, Gene S. Kopp*, Louisiana State University. From class field theory to quantum designs via modular cocycles. Preliminary report.
This talk will connect three seemingly unrelated topics: explicit class field theory, the asymptotic behavior of certain nonmodular $q$-series near points on the real line, and optimal sets of measurements for quantum state tomography. More specifically, the presenter will discuss his work showing that a version of the Stark conjectures over real quadratic fields is equivalent to a statement about the asymptotic behavior of the $q$-Pochhammer symbol $\left(e^{2 \pi i\left(r_{2} \tau-r_{1}\right)}, e^{2 \pi i \tau}\right)_{\infty}$ (with $r_{1}, r_{2} \in \mathbb{Q}$ ) as $\tau$ approaches a real quadratic value along any modular geodesic. That statement may also be phrased in terms of "real multiplication values" of modular cocycles. The presenter will also discuss joint work with Marcus Appleby and Steven Flammia on constructing symmetric informationally complete positive operator-valued measures (SIC-POVMs) from those same special cocycle values.
(Received September 11, 2023)

1192-11-30905
Michael H. Mertens*, RWTH Aachen University. Explicit construction of mock modular forms.
In this text we give a survey of various ways to construct and compute explicit Fourier expansions of mock modular forms, including Poincaré series, holomorphic projection, as well as elliptic and Abelian functions.
(Received September 11, 2023)

## 1192-11-30923

John F. R. Duncan*, Academia Sinica. Congruent Numbers and Umbral Moonshine.
Umbral moonshine interprets the mock theta functions of Ramanujan as distinguished functions arising from infinitedimensional representations of finite groups. In this talk we will explain a connection between umbral moonshine and the congruent number problem of antiquity.
(Received September 11, 2023)

## 1192-11-30934

Jake Chinis*, University of Bristol. On the Liouville Function in Short Intervals.
Assuming the Riemann Hypothesis, we show that the Liouville function exhibits squareroot cancellation in almost all short intervals, provided that the length of the interval is neither too small, nor too large. The proof uses a simple variant of the methods developed by Matomäki and Radziwiłł in their seminal work on multiplicative functions in short intervals, as well as some standard results concerning smooth numbers.
(Received September 11, 2023)

1192-11-30969
Leah Sturman*, Bowdoin College, Holly Swisher, Oregon State University. Resolving Kang and Park's generalization of the Alder-Andrews theorem.
The Alder-Andrews Theorem, a partition inequality generalizing Euler's partition identity, the first Rogers-Ramanujan identity, and a theorem of Schur to $d$-distinct partitions of $n$, was proved in three parts by Andrews in 1971, Yee in 2008, and Alfes, Jameson, Lemke Oliver in 2010. The progress of Andrews and Yee, which deftly utilized $q$-series and combinatorial methods, proved all but finitely many cases of $d$. In contrast, to address the remaining cases Alfes et al. used intricate asymptotics originating with Meinardus to obtain explicit bounds with which to deduce the inequality for all but finitely many $n$. They then used high-performance computing to computationally check the finite number of remaining cases of $d$ and $n$. In 2020, Kang and Park conjectured a "level 2" Alder-Andrews type partition inequality which encompasses the second Rogers-Ramanujan Identity. Duncan, Khunger, the second author, and Tamura proved Kang and Park's conjecture for all but finitely many cases using a combinatorial shift identity. Here, we generalize the methods of Alfes et al. to resolve Kang and Park's conjecture except $d=3,4,5,7$.
(Received September 11, 2023)

## 1192-11-30976

Yousheng Shi*, Zhejiang University. Local arithmetic Siegel Weil formula at ramified primes I.
In this talk I will first give a brief introduction to the (global) arithmetic Siegel-Weil formula and arithmetic theta lift. Then I will introduce the local arithmetic Siegel-Weil formula, also known as the Kudla-Rapoport conjecture, which relates the intersection numbers of special divisors on unitary Rapoport-Zink (RZ) space with central derivatives of local density polynomials. I will focus on my joint work with Qiao He, Chao Li and Tonghai Yang which proves the conjecture for the socalled Kramer model of RZ space. If time permits, I will also talk about some recent progress on the conjecture for RZ spaces of general maximal parahoric level structure over a ramified prime, of which the Kramer model is a special case.
(Received September 11, 2023)

The canonical ring of an algebraic variety is given by $R(X)=\bigoplus_{d \geq 0} H^{0}(X, d D)$. Extending the work of Igusa, we present progress on determining explicit generators and relations for the canonical ring of $\mathcal{A}_{g}$ for $g \leq 6$. This will equivalently provide explicit generators and relations for certain spaces of Siegel modular forms.
(Received September 11, 2023)

## 1192-11-31101

Suphawan Janphaisaeng, Department of Mathematics, Naresuan University, Benchawan Sookcharoenpinyo, Department of Mathematics, Naresuan University, Shayathorn Wanasawat*, Department of Mathematics and Statistics, Thammasat University. On the Sum of Reciprocal of Generalized Fibonacci Sequences.
In this paper, we investigate many types of the summations of the reciprocal of generalized Fibonacci numbers. It can be show that their floor functions becomes the difference of generalized Fibonacci numbers. We also present the exact way to round down such summations by applying some identities to estimate and find their boundaries.
(Received September 11, 2023)

1192-11-31126
Jean-François Biasse, University of South Florida, Claus Fieker, University of Kaiserslautern, Tommy Hofmann, University of Siegen, William Youmans*, Florida Atlantic University. An algorithm for solving the principal ideal problem with subfields. The principal ideal problem (PIP) is the problem of deciding whether a given ideal of a number field is principal and, if it is, of finding a generator. Solving the PIP applies to solving major computational tasks in number theory. It is also connected to the search for approximate short vectors in so-called ideal lattices, which is a crucial problem in cryptography. We present a novel application of norm relations to utilize information from subfields to solve the PIP in fields of degree up to 1800 (Received September 11, 2023)

1192-11-31129
Jonah Gaster*, University of Wisconsin, Milwaukee. Boundary slopes for the Markov ordering of the rationals. A rational number $p / q$ determines a simple closed curve on a once-punctured torus. When the torus is endowed with a complete hyperbolic metric, each rational gets a well-defined length. If the metric is chosen so that the torus is "equianharmonic" (that is, when its holonomy group is the commutator subgroup of $\operatorname{PSL}(2, \mathbb{Z})$ ), the lengths of the curves have special arithmetic significance, with connections to Diophantine approximation and number theory. Taking inspiration from McShane's elegant proof of Aigner's conjectures, concerning the (partial) ordering of the rationals induced by hyperbolic length on the equianharmonic torus, I will describe how hyperbolic geometry can be used to characterize monotonicity of the order so obtained along lines of varying slope in the $(q, p)$-plane. (Received September 11, 2023)

1192-11-31145
Matthew Krauel*, California State University, Sacramento. From decompositions of Jacobi forms to mock modular forms. The connection between mock modular forms and decompositions of Jacobi forms into characters of superconformal algebras has been central to Mathieu and Umbral moonshine. After a brief historical review, we discuss recent results in this area. In particular, we introduce intrinsic vector-valued mock modular forms and a formula, from which, the mock modular form associated with the coefficients of superconformal characters can be readily obtained. Finally, we highlight an application of these results in the theory of vertex operator superalgebras.
(Received September 11, 2023)

## 1192-11-31214

Justin Trias*, Imperial College London. Theta correspondence in families for type II dual pairs.
The classical local theta correspondence for p-adic reductive dual pairs defines a bijection between prescribed subsets of irreducible smooth complex representations coming from two groups ( $\mathrm{H}, \mathrm{H}^{\prime}$ ), forming a dual pair in a symplectic group. Motivated by new perspectives in the local Langlands correspondence for modular representations, Alberto Minguez extended the theta correspondence for type II dual pairs (i.e. when (H, H') is made of general linear groups) to the setting of representations with coefficients in algebraically closed fields of characteristic $l$ as long as the characteristic $l$ does not divide the pro-orders of H and $\mathrm{H}^{\prime}$. More recently, the work of Emerton and Helm extended the local Langlands correspondence to families of representations, that is over coefficient rings, with compatibility to both classical and modular local Langlands for general linear groups. We explain how to build a theta correspondence in families, i.e. with coefficients in rings like $\mathrm{Z}[1 / \mathrm{p}]$, for type II dual pairs that is compatible with reduction to residue fields of the base coefficient ring, where central to this approach is the integral Bernstein centre. We translate some weaker properties of the classical correspondence, such as compatibility with supercuspidal support, as a ring morphism between the integral Bernstein centres of H and $\mathrm{H}^{\prime}$ and interpret it for the Weil representation. This ring morphism between the Bernstein centres brings a richer structure than a simple compatibility of supercuspidal supports and allows to ask new geometric questions for the theta correspondence: we prove that this map is surjective i.e. it is a closed immersion between the associated affine schemes. In particular our result implies a theta correspondence between characters of the Bernstein centre over any coefficient field of characteristic not p. This is joint work with Gil Moss.
(Received September 11, 2023)

1192-11-31234
Jillian Eddy, UC Davis, Elena Fuchs, UC Davis, Matthew Litman*, UC Davis, Daniel Everett Martin, Clemson University, Nico Tripeny, Haverford College. Connectivity of Markoff mod-p Graphs and Maximal Divisors.
Baragar conjectured the Markoff mod- $p$ graphs are connected for all primes $p$. This conjecture was verified by de CourcyIreland and Lee for all primes up to 3,000. In this talk, we will discuss our recent work confirming this conjecture for all primes $p>3.448 \times 10^{392}$ by employing results of Chen and Bourgain, Gamburd, and Sarnak. We introduce the notion of
maximal divisors as well as prove sharp asymptotic and explicit upper bounds on the number of them. This improves the Markoff graph $p$-bound by roughly 140 orders of magnitude as compared with an approach using all divisors.
(Received September 11, 2023)

## 1192-11-31240

Benjamin Rainear, United States Naval Academy, Katherine Thompson*, United States Naval Academy. Elementary Proofs of Representation by Ternary Quadratic Forms. Preliminary report.
Mordell in 1958 gave a new proof of the three squares theorem. Those techniques were generalized by Blackwell, et al., in 2016 to characterize the integers represented by the remaining six "Ramanujan Dickson ternaries." We continue the generalization of these techniques to four additional forms.
(Received September 11, 2023)

## 1192-11-31302

Robert Dicks*, University of Illinois at Urbana-Champaign. Scarce congruences for r-colored partitions.
The study of congruence properties for the partition function, $p(n)$, has a long history. Ramanujan proved congruences of the form $p(\ell n+\beta)$ for the primes $\ell=5,7$, and 11. Many years later, Ahlgren and Boylan showed that there are no similar congruences for primes $\ell \geq 13$. More recently, Ahlgren, Beckwith, and Raum showed that, for primes $Q \geq 5$, congruences of the form $p(\ell Q n+\beta) \equiv 0(\bmod \ell)$ and $p\left(\ell Q^{2} n+\beta\right) \equiv 0(\bmod \ell)$ are scarce in a precise sense. For a positive integer $r$, we study similar congruences for the $r$-colored partition function $p_{r}(n)$. We make crucial use of work of Radu which computes the expansion of half-integral weight modular forms capturing our relevant partition values at a certain cusp of $X(\ell Q)$. We also use modular Galois representations and the arithmetic large sieve.
(Received September 11, 2023)

1192-11-31307
Claire Frechette*, Boston College, Mathilde Gerbelli-Gauthier, Institute for Advanced Study, Alia Hamieh, University of Nothern British Columbia, Naomi Tanabe, Bowdoin College. Large Sums of Fourier Coefficients of Cusp Forms. One common approach to studying the asymptotic behaviour of coefficients of modular forms has been to study the behaviour of their averages. In 1927, Hecke proved that for sufficiently large $x$,

$$
S(x, f):=\sum_{n<x} \lambda_{f}(n) \ll_{f} x^{\frac{1}{2}}
$$

This bound has been gradually improved, and it is conjectured that for a primitive cusp form $f \in S_{k}(N)$, we have $S(x, f)=o(x \log x)$ in the range $x \geq k^{\epsilon}$. Lamzouri proved in arXiv:1703.10582 [math.NT] that this is true under the assumption of the Generalized Riemann Hypothesis (GRH) for $L(s, f)$. In this talk, we will discuss recent results from our paper (arXiv:2308.06311 [math.NT]), which prove that this conjecture holds under a weaker assumption than GRH. In particular, our results show that if the associated L-function $L(s, f)$ has no more than $\epsilon^{2} \log k / 5000$ zeros in a specified region, then for $1 \leq T \leq(\log k)^{\frac{1}{200}}$, we have

$$
S(x, f) \ll \frac{x \log x}{T}
$$

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1192-11-31332
Neil Lyall*, University of Georgia. Graham's Conjecture in Geometric Ramsey Theory. Preliminary report.
During the 1970's Erdos, Graham et al. initiated a study of geometric point configurations which cannot be destroyed by finite partitions of high dimensional Euclidean spaces. They have shown that such configurations must be spherical and in 1994 Graham conjectured that this condition is also sufficient. While these problems were initially studied by purely combinatorial means a surprising density analogue has been shown for simplexes by Bourgain, using Fourier analysis. We will present some recent results toward this conjecture both in the Euclidean setting and in vector spaces over finite fields, and discuss an approach utilizing some modern tools of additive combinatorics, also referred to as "higher order Fourier analysis".
(Received September 11, 2023)

1192-11-31347
Christelle Vincent*, University of Vermont. What can theta functions tell us about abelian threefolds?. Preliminary report. In this talk we will summarize our work of the last six years on the construction of CM Jacobians of dimension 3 over $\mathbb{C}$, done jointly with Sorina Ionica, Pinar Kılıçer, Kristin Lauter, Elisa Lorenzo Garcìa, Maike Massierer, and Adelina Mânzățeanu. We begin with a period matrix for a complex abelian threefold which we have constructed to have complex multiplication by a sextic field, to be simple, and to admit a principal polarization. From these objects we can compute the theta constants associated to the threefold, which are the values taken by some theta functions when evaluated at a period matrix of the threefold. Over the years we have succeeded in wresting a lot of very cool information from these constants!
(Received September 11, 2023)

Kelly O'Connor*, Colorado State University. Relative Oriented Class Groups Of Quadratic Extensions. Preliminary report. In 2019 Zemková defined relative oriented class groups associated to quadratic extensions of number fields $L / K$, extending work of Bhargava concerning composition laws for binary quadratic forms over number fields of higher degree. Indeed, this
work generalized the classical correspondence between the ideal class group of a quadratic number field and classes of binary quadratic forms to any base number field of narrow class number one. Zemková explicitly computed these relative oriented class groups for totally real quadratic extensions of the rationals. We extend Zemková's result for quadratic extensions $L / K$ of a totally real field of narrow class number 1, under certain conditions on the absolute Galois group of $L / \mathbb{Q}$. (Received September 11, 2023)

1192-11-31400
Beaullah Mugwangwavari*, University of the Witwatersrand, Darlison Nyirenda, University of the Witwatersrand. Extensions and variations of Andrews-Merca identities.
G. E. Andrews and M. Merca gave a new combinatorial interpretation for the total number of even parts in all partitions of $n$ into distinct parts. We generalise this result and establish more variations.
(Received September 11, 2023)

## 1192-11-31455

Travis Morrison*, Virginia Tech. Diophantine parametrization of varieties with no rational points.
Let $K$ be a global field. A subset of $K^{n}$ is diophantine over $K$ if it is cut out by a polynomial equations whose coefficients are in $K$, or equivalently, if the subset is defined by a positive-existential formula. The diophantine definability of subsets of K is related to the decidability of $K$ : for example, Hilbert's tenth problem over $\mathbb{Q}$ would be undecidable if $\mathbb{Z}$ were diophantine over $\mathbb{Q}$. In this talk, I will discuss the diophantine definabiliity of some subsets of $K^{n}$ related to equations over $K$ with no $K$ rational solutions. For example, Poonen showed that the non-squares of $K$ are diophantine over $K$, and Colliot-Thélène and Van Geel generalize this: the non-nth powers of $K$ are diophantine over $K$. Koenigsmann showed that conics over $\mathbb{Q}$ are parametrized by a subset of $\mathbb{Q}$ diophantine over $\mathbb{Q}$. I will discuss joint work with Eisenträger in which we generalized Koenigsmann's result to number fields and global function fields of odd characteristic, and related results in higher dimensions.
(Received September 11, 2023)

## 1192-11-31474

Greg McShane*, Institut Fourier, Université Grenoble Alpes. Convexity and uniqueness for Markoff numbers. The Markov numbers have several different interpretations: - abstractly as integers appearing in solutions to the Markoff cubic - geometrically as functions of simple geodesics on the modular torus - combinatorially as the number of perfect matchings for snake graphs. as "specializations" in cluster algebras. In 1913 Frobenius conjectured that the largest Markoff number in a triple determines the two other numbers. This conjecture can be reformulated geometrically as a question of multiplicity for the lengths of simple geodesics on the modular torus. Many people have obtained partial results by imposing additional (divisibility) conditions on the largest number in a triple. In particular Martin Aigner extended results of Button and Baragar. Admitting how difficult the general case is he propose some weaker conjectures concerning the behavior of Markoff numbers in relation to a natural labelling by the extended rational numbers. Two quite different solutions to these conjectures were recently obtained by - C Lagisquet and E. Pelantová and S. Tavenas and L. Vuillon - K. Lee, L. Li, M. Rabideau, R. Schiffler. We present a unified solution to Aigner's conjectures based on basic principles of differential geometry and the relation between the numbers and the lengths of closed hyperbolic geodesics. We will explain why any purely geometric approach to Frobenius' uniqueness should fail.
(Received September 11, 2023)

## 1192-11-31525

Niven Achenjang*, MIT. An Upper Bound for the Average Rank of Elliptic Curves over Global Function Fields, via 2-Selmer Groups.
Let $K$ be any global field (for example, $K=\mathbb{Q}$ or $K=\mathbb{F}_{2}(t)$ ). Given an elliptic curve $E$ over $K$, its set $E(K)$ of $K$-rational points forms a finitely generated abelian group, whose rank has been the subject of much study. In particular, these days, there are vast conjectures (due to many authors) predicting the distribution of $\operatorname{rank}_{\mathbb{Z}} E(K)$ as $E$ ranges over the family of all elliptic curves over $K$. These predictions come from analyzing heuristics for the behavior of $E$ 's so-called " $n$-Selmer groups", one for each $n \geq 1$, and so also predict the distribution of these auxiliary groups. Of note, the conjectures predict that the average value of $\operatorname{rank}_{\mathbb{Z}} E(K)$ should be $\frac{1}{2}$ and that the average size of $n$-Selmer should be $\sum_{d \mid n} d$. For example, the average size of 2-Selmer is predicted to be $3=1+2$. In this talk, I will present on my recent work verifying new cases of this predicted average size, "up to small error term". Specifically, I prove that for a global function field $K$ with constant field $\mathbb{F}_{q}$, the average size of 2 -Selmer groups of elliptic curves over $K$ is bounded above by an explicit expression of the form $3+o(1)$, where the $o(1)$ term vanishes as $q \rightarrow \infty$. From this, one can deduce that the average value of $\operatorname{rank}_{\mathbb{Z}} E(K)$ is at most $\frac{3}{2}+o(1)$ as $q \rightarrow \infty$. Results of this form were known already for 2-Selmer groups when char $K \geq 5$ or when $K$ is a number field (in which case, the predicted average size is known to be exactly correct). The novelty of my recent work is that it works equally well for any global function field, even those in "bad" characteristic.
(Received September 11, 2023)

## 1192-11-31599

Beth Malmskog*, Colorado College, Na'ama Nevo, Northeastern University. Lower Rate Bounds for Hermitian-Lifted Codes for Odd Prime Characteristic.
Locally recoverable codes are error correcting codes with the additional property that every symbol of any codeword can be recovered from a small set of other symbols. This property is particularly desirable in cloud storage applications. A locally recoverable code is said to have availability $t$ if each position has $t$ disjoint recovery sets. Hermitian-lifted codes are locally recoverable codes with high availability first described by Lopez, Malmskog, Matthews, Piñero-Gonzales, and Wootters. The codes are based on the well-known Hermitian curve and incorporate the novel technique of lifting to increase the rate of the code. Lopez et al. lower bounded the rate of the codes defined over fields with characteristic 2 . In this talk, I will discuss recent work with then-undergraduate student Na'ama Nevo, generalizing earlier work to show that the rate of Hermitian-lifted
codes is bounded below by a positive constant depending on $p$ when $q=p^{l}$ for any odd prime $p$.
(Received September 11, 2023)

1192-11-31601
Colby Austin Brown*, University of California, Davis. Computational methods for investigating the Markoff graph modulo p. The solutions to the Markoff equation modulo $p, x^{2}+y^{2}+z^{2}-3 x y z \equiv 0 \bmod p$, form the Markoff graph, where solutions are connected via Vieta involutions. The algorithm given by Bourgain, Gamburd, and Sarnak for testing the connectivity of the graph depends on the orbits of triples under repeat Vieta involutions and is best suited for primes $p$ where $p \pm 1$ has few prime factors. Fortunately, values of $p \pm 1$ with many prime factors are well suited to work with computers. However, since the size of the graph grows as $O\left(p^{2}\right)$, probing the structure of the Markoff graph modulo $p$ computationally presents challenges for very large $p$. In this talk, we will discuss computational methods for searching the Markoff graph, generating orbits of triples under Vieta involutions, and computationally establishing the connectivity of Markoff graphs modulo $p$. (Received September 11, 2023)

192-11-31617
aBa Mbirika*, University of Wisconsin-Eau Claire, Jürgen Spilker, University of Freiburg, Mckenzie West, University of Wisconsin-Eau Claire. The Narayana sequence and a tanatalizing connection with primes of the form $x^{2}+31 y^{2}$. Preliminary report.
While there is a wealth of research on sequences given by second-order linear recurrences such as the Fibonacci and other well-known Lucas sequences, there is far less on the third-order recurrence known as the Narayana sequence. The Narayana sequence $\left(N_{n}\right)_{n \geq 0}$ arose from a combinatorial problem proposed by the $14^{\text {th }}$ century Indian mathematician named Nārāyaṇa Paṇdita in his epic Sanskrit text, Ganita Kaumudi. Set $\mathcal{N}(k)$ to be the greatest common divisor of all sums of $k$ consecutive Narayana numbers. We give a closed form for the value $\mathcal{N}(k)$ and explore a tantalizing connection to primes of the form $x^{2}+31 y^{2}$. Using tools from algebraic number theory, we study this connection. Lastly, we give compelling evidence of a unique relationship between primes of the form $p=x^{2}+31 y^{2}$ and the Pisano period $\pi_{N}(p)$ of the Narayana sequence modulo $p$.
(Received September 11, 2023)

## 1192-11-31645

Garo Sarajian*, United States Military Academy. Densely-divisible numbers and the quadratic sieve. Preliminary report. Several factorization methods use smooth numbers to search for components within Fermat's factorization method. By relaxing the smoothness condition, additional candidates are considered within the search for divisors, coming at the cost of a more intractable numerical linear algebra problem. In this talk, we discuss the empirical results of loosening the smoothness criteria within the quadratic sieve and the computational complexities given the modified sieve.
(Received September 11, 2023)

## 1192-11-31648

Ankit Bisain*, Massachusetts Institute of Technology, Andrei Mandelshtam*, Stanford University, Noah Walsh*, Massachusetts Institute of Technology, Xun Wang*, University of Michigan. Subconvexity Implies Effective Quantum Unique Ergodicity For Hecke Maaß Cusp Forms On $\mathrm{SL}_{2}(\mathbb{Z}) \backslash \mathbb{H}$.
On assuming subconvexity of L functions associated to hecke Maaß cusp forms and holomorhic cusp forms, we establish effective quantum unique ergodicity for hecke Maaß cusp forms over $\mathrm{SL}_{2}(\mathbb{Z}) \backslash \mathbb{H}$. The proof uses spectral decomposition on square-integrable functions in the upper half plane along with careful applications of the Watson-Ichino formula. (Received September 11, 2023)

## 1192-11-31654

Matthew G. Boylan*, University of South Carolina. Hecke operators on modulars forms with eta-multiplier. Preliminary report.
We discuss how Hecke operators permute subspaces of integer and half-integer weight holomorphic modular forms arising as multiples of eta-quotients. Generating functions for certain kinds of partition functions modulo powers of a prime $\ell$ reside uniformly in such spaces. The topic of this talk, modular forms, was studied and enjoyed by our colleague Kevin James. I will spend some time talking about Kevin's impact broadly on number theory in South Carolina and the Southeast. (Received September 11, 2023)

## 1192-11-31660

Jennifer Berg*, Bucknell University. Brauer-Manin obstructions requiring arbitrarily many Brauer classes.
If a variety $X$ over a number field $k$ has points in each completion of $k$, then we can ask whether the set of $k$-rational points is nonempty. To start, we can impose conditions on the collection of local points to narrow down the subset that contains the rational points, should any exist. One fruitful approach uses the Brauer group of X , which defines an obstruction set known as the Brauer-Manin set. For geometrically rational surfaces, if X everywhere locally soluble while $\mathrm{X}(\mathrm{k})$ is empty, then conjecturally a Brauer-Manin obstruction is the reason. In general, when such an obstruction occurs, it arises from a finite number of classes in the Brauer group. One might wonder whether properties of this finite subset can be determined in advance, i.e., without computing the Brauer-Manin set. In the case of cubic surfaces, for example, it is known that just one Brauer class is needed to detect an obstruction. In this talk, we'll discuss work that shows we cannot always hope to give such quantitative bounds; for any integer $\mathrm{N}>0$, we construct conic bundles over the projective line for which the Brauer group modulo constants is generated by N classes and all N generators are required to witness an obstruction. (This is joint work with Pagano, Poonen, Stoll, Triantafillou, Viray, Vogt.)
(Received September 11, 2023)

Georgia Soo Frances Corbett*, Bucknell University. Large Values of Newform Dedekind Sums. Preliminary report. Our project focuses on a generalization of Dedekind sums, called newform Dedekind sums, involving a finite sum of products of primitive Dirichlet characters and Bernoulli functions introduced by Stucker, Vennos and Young in 2020. We denote these sums $S_{\chi_{1}, \chi_{2}}(a, c)$ where $\chi_{i}$ is a nontrivial primitive Dirichlet character modulo $q_{i}$ and $(a, c)$ is the first column of a matrix in a congruence subgroup of $S L_{2}(\mathbb{Z})$. We show that the newform Dedekind sum is rarely substantially larger than log ${ }^{3} c$ by relating the size of the Dedekind sum to continued fractions and applying a result of Hensley. We complement this result by producing examples of large values of the newform Dedekind sum.
(Received September 11, 2023)

## 1192-11-31703

Poornima Belvotagi*, University of California San Diego, Nir Gadish, University of Michigan, Jonathan Richard Love, CRM Montreal, Brett Nasserden, Western University, Libby Taylor, Stanford University. Gluing quotient stacks. Preliminary report.
Given a variety with tame quotient singularities, does it arise globally as the quotient of a smooth variety by a finite group? Geraschenko-Satriano used stacks to give a characterization in the abelian case. This motivates the question: Given a stack with a local quotient presentation, what are the obstructions to realizing this stack as a global quotient? In this talk, we'll report on progress toward this question, which joint with N. Gadish, J. Love, B. Nasserden, and L. Taylor.
(Received September 11, 2023)

## 1192-11-31719

Molly FitzGibbons*, Williams College, Amanda Verga*, Trinity College. Dynamics of the Order of Appearance Map in the Fibonacci Sequence.
The order of appearance $z(n)$ of a positive integer $n$ in the Fibonacci sequence is defined as the smallest positive integer $j$ such that $n$ divides the $j$-th Fibonacci number. A fixed point arises when, for a positive integer $n$, we have that the $n^{t h}$ Fibonacci number is the smallest Fibonacci that $n$ divides. In other words, $z(n)=n$. In 2012, Marques proved that fixed points occur only when $n$ is of the form $5^{i}$ or $12 \cdot 5^{i}$ for non-negative integers $i$. It immediately follows that there are infinitely many fixed points in the Fibonacci sequence. We prove that there are infinitely many integers that iterate to a fixed point in exactly $k$ steps. In addition, we construct infinite families of integers that go to each fixed point of the form $12 \cdot 5^{k}$. We conclude by providing an alternate proof that all positive integers $n$ reach a fixed point after a finite number of iterations of $z$ on $n$.
(Received September 12, 2023)

## 1192-11-31731

Jordan S Ellenberg*, University of Wisconsin-Madison. Topology of Hurwitz stacks and arithmetic statistics. Preliminary report.
We will discuss recent developments in the research program in which homological stability theorems for Hurwitz stacks and related covers of configuration space are used to derive results in arithmetic statistics concerning the variation of arithmetic objects over families of function fields
(Received September 12, 2023)

1192-11-31732
A.C. Cojocaru, University of Illinois Chicago, Auden Hinz*, University of Illinois Chicago, Tian Wang, Max Planck Institute for Mathematics. Quantitative upper bounds related to an isogeny criterion for elliptic curves.
For $E_{1}$ and $E_{2}$ elliptic curves defined over a number field $K$, without complex multiplication, we consider the function $\mathcal{F}_{E_{1}, E_{2}}(x)$ counting non-zero prime ideals $\mathfrak{p}$ of the ring of integers of $K$, of good reduction for $E_{1}$ and $E_{2}$, of norm at most $x$, and for which the Frobenius fields $\mathbb{Q}\left(\pi_{\mathfrak{p}}\left(E_{1}\right)\right)$ and $\mathbb{Q}\left(\pi_{\mathfrak{p}}\left(E_{2}\right)\right)$ are equal. Motivated by an isogeny criterion of Kulkarni,
Patankar, and Rajan, which states that $E_{1}$ and $E_{2}$ are not potentially isogenous if and only if $\mathcal{F}_{E_{1}, E_{2}}(x)=\mathrm{o}\left(\frac{x}{\log x}\right)$, we investigate the growth in $x$ of $\mathcal{F}_{E_{1}, E_{2}}(x)$. We prove that if $E_{1}$ and $E_{2}$ are not potentially isogenous, then there exist positive constants $\kappa\left(E_{1}, E_{2}, K\right), \kappa^{\prime}\left(E_{1}, E_{2}, K\right)$, and $\kappa \prime \prime\left(E_{1}, E_{2}, K\right)$ such that the following bounds hold: (i)
$\mathcal{F}_{E_{1}, E_{2}}(x)<\kappa\left(E_{1}, E_{2}, K\right) \frac{x(\log \log x)^{\frac{1}{9}}}{(\log x)^{\frac{19}{18}}}$; (ii) $\mathcal{F}_{E_{1}, E_{2}}(x)<\kappa^{\prime}\left(E_{1}, E_{2}, K\right) \frac{x^{\frac{6}{7}}}{(\log x)^{\frac{5}{7}}}$ under the Generalized Riemann Hypothesis for Dedekind zeta functions (GRH); (iii) $\mathcal{F}_{E_{1}, E_{2}}(x)<\kappa \prime \prime\left(E_{1}, E_{2}, K\right) x^{\frac{2}{3}}(\log x)^{\frac{1}{3}}$ under GRH, Artin's Holomorphy Conjecture for the Artin $L$-functions of number field extensions, and a Pair Correlation Conjecture for the zeros of the Artin $L$-functions of number field extensions.
(Received September 12, 2023)

1192-11-31824
Ricardo Conceicao*, Gettysburg College, Rachael Kelly, Gettysburg College, Samuel VanFossen, Gettysburg College. On the Markoff equation over polynomial rings.
As part of his studies in diophantine approximation, Markoff proved that all integral solutions of the equation $x^{2}+y^{2}+z^{2}=3 x y z$ can be generated by the action of certain automorphisms of this hyper-surface on the solution $(1,1,1)$. Ever since, several authors have extended Markoff's work to the study of solutions of this equation over finite fields and number fields. Our goal is to discuss some progress made in understanding the solutions of this equation over the polynomial ring $k[t]$, where $k$ is a field with characteristic not equal to 2 .
(Received September 12, 2023)

Molly FitzGibbons, Williams College, Amanda Verga*, Trinity College. Dynamics of the Fibonacci Order of Appearance Map. The order of appearance $z(n)$ of a positive integer $n$ in the Fibonacci sequence is defined as the smallest positive integer $j$ such that $n$ divides the $j$-th Fibonacci number. A fixed point arises when, for a positive integer $n$, we have that the $n^{t h}$ Fibonacci number is the smallest Fibonacci that $n$ divides. In other words, $z(n)=n$. In 2012, Marques proved that fixed points occur only when $n$ is of the form $5^{k}$ or $12 \cdot 5^{k}$ for non-negative integers $k$. In our work, we study repeated iterations of $z$ on $n$. We prove that infinitely many integers iterate to a fixed point in exactly $k$ iterations. In addition, we construct infinite families of integers that iterate to each fixed point of the form $12 \cdot 5^{k}$ and provide a new proof that all positive integers reach a fixed point within a finite number of iterations. We conclude with some open questions and areas of research that we are still exploring.
(Received September 12, 2023)

## 1192-11-31903

Mojtaba Moniri*, Normandale Community College. How Long Can Successions of These Ramsey-Collatz Rare Concurrencies Be?.
For certain rather tough-to-generate sequences of 16 -tuples of numbers below $2^{363}$ where there are clear but unreachable length upper bounds, we achieve some lower bounds for the maximum possible lengths, and we do this in six 'apartness' levels. On one hand this involves a Max-Min notion regarding binary subtrees of edge-labeled ternary trees initiated by Downey-Greenberg-Jockusch-Milans in Combinatorica in 2011 where in depth 5 there are eight possible 'weights'-some of which are very rare. It also involves the Collatz scaled total stopping times for numbers below $2^{363}$, apartness of such values in the 'low' and 'high' halves of each tuple as well as between the corresponding components of terms of the sequence, and reaching high and low outliers about 11 times the standard deviation apart. The paper reports the construction of sizable LoH-i sequences of apartness $>10^{-i}$ for $i=1, \cdots, 6$ that are presented as Mathematical NFTs (opensea.io/collection/lowhigh), illustrates some of those, and provides tools for verification (a task more routine than the performed mining). (Received September 25, 2023)

1192-11-32059
Sarah Arpin*, University of Colorado Boulder, James Clements, University of Bristol, Pierrick Dartois, Centre Inria de l'Universite de Bordeau, Jonathan Eriksen, Norwegian University of Science and Technology, Peter Kutas, Eotvos Lorand University, Benjamin Wesolowski, ENS de Lyon, CNRS, UMPA. Finding orientations of supersingular elliptic curves and quaternion orders.
Orientations of supersingular elliptic curves encode the information of an endomorphism of the curve. Computing the full endomorphism ring is a known hard problem, so one might consider how hard it is to find one such orientation. We prove that access to an oracle which tells if an elliptic curve is $\mathfrak{O}$-orientable for a fixed imaginary quadratic order $\mathfrak{O}$ provides non-trivial information towards computing an endomorphism corresponding to the $\mathfrak{O}$-orientation. We provide explicit algorithms and indepth complexity analysis. We also consider the question in terms of quaternion algebras. We provide algorithms which compute an embedding of a fixed imaginary quadratic order into a maximal order of the quaternion algebra ramified at $p$ and $\infty$. We provide code implementations in Sagemath which is efficient for finding embeddings of imaginary quadratic orders of discriminants up to $O(p)$, even for cryptographically sized $p$.
(Received September 12, 2023)

## 1192-11-32106

Walter Bridges*, University of Cologne. Limit shapes for Andrews-Gordon partitions.
The famous Rogers-Ramanujan and Andrews-Gordon identities equate certain q-hypergeometric series with simple, modular infinite products. Here, the "sum sides" may be interpreted as generating functions for partitions with certain difference conditions between parts, and an algorithm giving this correspondance was the subject of Kurşungöz's Ph.D. thesis. In the ongoing study of Rogers-Ramanujan-type identities, such algorithms are often the only known approach linking the combinatorics with the generating functions. We use Kurşungöz's algorithm to study the long-term statistical behavior of these partitions in a particular case. We prove that, as the size of these partitions tends to infinity, the Young diagrams are almost all near a fixed curve. That is, there is a single limit shape. This is joint work with Kathrin Bringmann. (Received September 12, 2023)

## 1192-11-32124

Hirotaka Kakuhama*, Hokkaido University. Local theta correspondences for quaternionic dual pairs and Langlands parameters. Preliminary report.
An important property of the local theta correspondence is its relation with the local Langlands correspondence. In this talk, we will formulate the conjecture describing the local theta correspondence of irreducible tempered representations of the quaternionic dual pairs of almost equal ranks in terms of Langlands parameters. Here, we use the formulation of local Langlands correspondence for rigid inner twists to define the Langlands parameters. The key ingredient for constructing the correspondence of the parameter side is to link the rigid inner twists of two quaternionic unitary groups consisting of a dual pair.
(Received September 12, 2023)

1192-11-32125
James A. Sellers, University of Minnesota Duluth, Nicolas Allen Smoot*, Research Institute for Symbolic Computation, JKU Linz. Frobenius Partitions and New Congruence Families. Preliminary report.
We will announce the proof of a new congruence family modulo powers of 5 for a certain generalized Frobenius partition function studied by Drake in 2009, and by Jiang, Rolen, and Woodbury in 2022. The family is associated with the classical level

20 modular curve-a Riemann surface of cusp count 6 and genus 1 . This ranks the family within a class of more difficult congruences to prove. Moreover, this family is closely related to the Andrews-Sellers congruences for the generalized 2 -color Frobenius partition function discovered by Sellers and proved by Paule and Radu. We believe that our methods may lead to greater insights about why such congruence families occur.
(Received September 12, 2023)

1192-11-32233
Sarah M Harrison*, Northeastern University. New BPS algebras from superstring compactifications
Borcherds Kac-Moody (BKM) algebras are a generalization of familiar Kac-Moody algebras with imaginary simple roots. On the one hand, they were invented by Borcherds in his proof of the monstrous moonshine conjectures and have many interesting connections to new moonshines, number theory and the theory of automorphic forms. On the other hand, there is an old conjecture of Harvey and Moore that BPS states in string theory form an algebra that is in some cases a BKM algebra. I will briefly review the construction of new BKM superalgebras arising from self-dual vertex operator algebras of central charge 12, and then discuss recent work showing how they arise as algebras of BPS states in physical string theories in 2 dimensions, as well as their connection with automorphic forms. Based on work with N. Paquette, D. Persson, and R. Volpato. (Received September 12, 2023)

1192-11-32311
Wen-Ching Winnie Li*, Pennsylvania State University. Hypergeometric functions, Galois representations, and modular forms.
Historically, the hypergeometric functions were studied through the lens of differential equations. When the parameters are rational numbers, Katz introduced $\ell$-adic Galois representations and developed a theory parallel to that in the classical setting. These Katz representations can be realized on cohomological groups of algebraic varieties, and hence they are expected to be automorphic according to Langlands' philosophy. In this talk we use recent results in this area to showcase the interconnections among hypergeometric functions, Galois representations, and modular forms
(Received September 12, 2023)

1192-11-32323
Alina Carmen Cojocaru*, University of Illinois at Chicago, Auden Hinz, University of Illinois Chicago, Tian Wang, Max Planck Institute for Mathematics. Effective versions of an isogeny criterion for elliptic curves.
For $E_{1}$ and $E_{2}$ elliptic curves defined over a number field $K$ and without complex multiplication, we consider the function $\mathcal{F}_{E_{1}, E_{2}}(x)$ counting non-zero prime ideals $\mathfrak{p}$ of the ring of integers of $K$, of good reduction for $E_{1}$ and $E_{2}$, of norm at most $x$, and for which the Frobenius fields of $E_{1}$ and $E_{2}$ coincide. Motivated by an isogeny criterion of Kulkarni, Patankar, and Rajan about the existence of an isogeny between $E_{1}$ and $E_{2}$, we prove upper bounds for $\mathcal{F}_{E_{1}, E_{2}}(x)$. This is joint work with Auden Hinz and Tian Wang.
(Received September 12, 2023 )

1192-11-32330
Chantal David*, Concordia University. James primes for elliptic curves without complex multiplication.
Let $E$ be an elliptic curve $E$ over $\mathbb{Q}$. A James prime for $E$ is a prime $p$ of good reduction such that the number of rational points on $E$ modulo $p$ is maximal (or minimal), i.e. $a_{p}(E)= \pm[2 \sqrt{p}]$ by the Hasse bound. Those primes were first consider by James, Tran, Trinh, Wertheimer and Zantout in 2016, who conjectured that for any non-CM curve $E$,

## $\#\left\{p \leq x: a \_\{p\}(E)=[2 \sqrt{ } p]\right\} \sim \backslash \operatorname{frac}\{8\} 3 \pi \backslash \operatorname{frac}\left\{x^{\wedge}\{1 / 4\}\right\} \log x$.

This conjecture can then be thought as an intermediate situation between two famous prime counting conjectures associated to elliptic curves over $\mathbb{Q}$, the Sato-Tate conjecture and the Lang-Trotter conjecture. We present in this talk some results giving evidence for this conjecture, namely some some upper bounds for the number of James primes when $E$ is a non-CM curve, assuming that all the symmetric power $L$-functions associated to $E$ are automorphic and satisfy GRH, obtained in joint work with A. Gafni, A. Malik, N. Prabhu and C. Turnage-Butternaugh (2000). We exploit the fact that the James primes are at the end of the Sato-Tate interval, where the measure is small. An unconditional upper bound was also obtained by Gafni, Thorner and Wong (2002).
(Received September 12, 2023)

## 1192-11-32360

Nathan Jones*, University of Illinois Chicago, Francesco Pappalardi, Università Roma Tre, Peter Stevenhagen, Leiden University. Locally imprimitive points on elliptic curves.
Under the Generalized Riemann Hypothesis, any element in the multiplicative group of a number field $K$ that is globally primitive (i.e., not a perfect power in $K^{*}$ ) is a primitive root modulo a set of primes of $K$ of positive density. For elliptic curves $E / K$ that are known to have infinitely many primes $\mathfrak{p}$ of cyclic reduction, possibly under GRH, a globally primitive point $P \in E(K)$ may fail to generate any of the point groups $E\left(k_{\mathfrak{p}}\right)$. We describe this phenomenon in terms of an associated Galois representation $\rho_{E / K, P}: G_{K} \rightarrow G L_{3}(\hat{\mathbb{Z}})$, and use it to construct non-trivial examples of global points on elliptic curves that are locally imprimitive.
(Received September 12, 2023)

Daniel J. Katz, California State University, Northridge, Allison E. Wong*, University of California, Davis. Rationality of FourValued Families of Weil Sums of Binomials.

Given a prime number $p$, a finite field $K$ of characteristic $p$, a positive integer $s$ that is relatively prime to $|K|-1$, and an element $u$ of the unit group of $K$, we have a Weil sum of a binomial, $\sum_{x \in K} \psi\left(x^{s}-u x\right)$, where $\psi$ is the canonical additive character of $K$. The multiset of these Weil sums as $u$ varies over the unit group of $K$ is called the Weil spectrum for the field $K$ and the exponent $s$. Taken individually, Weil sums come up in arithmetic geometry as they count points in algebraic sets over finite fields. When taken together, as Weil spectra, these sums come up in applications such as cryptography (in which we wish to determine which power permutations of finite fields can resist linear cryptanalysis), digital sequence design (in which we wish to find families of sequences that are not strongly correlated with any shifted versions of the others), and the theory of error-correcting codes. In this talk, we will discuss how to use Galois theory and algebraic number theory to probe the structure of Weil spectra with few distinct values. In particular, although values in the Weil spectrum are in general algebraic integers in the $p$ th cyclotomic extension of $\mathbb{Q}$, we can show that they must be rational integers when the Weil spectrum contains four or fewer distinct values, with the exception of one special case.
(Received September 13, 2023)

1192-11-32519
Avalon Ann Blaser*, University of Utah, Molly Bradley*, University of Pennsylvania, Daniel Vargas*, Harvey Mudd College, Kathy Xing*, Amherst College. Sato-Tate Type Distributions for Matrix Points on Elliptic Curves and Some K3 Surfaces.
Generalizing the problem of counting rational points on curves and surfaces over finite fields, we consider the setting of $n \times n$ matrix points with finite field entries. We obtain exact formulas for matrix point counts on elliptic curves and certain $K 3$ surfaces for "supersingular" primes. These exact formulas, which involve partitions of integers up to $n$, essentially coincide with the expected value for the number of such points. Therefore, in analogy with the Sato-Tate conjecture, it is natural to study the distribution of the deviation from the expected values for all primes. We determine the limiting distributions for elliptic curves and a family of $K 3$ surfaces. For non-CM elliptic curves with square-free conductor, our results are explicit. (Received September 12, 2023)

## 1192-11-32539

Hanson Smith*, California State University San Marcos. On the Monogenicity of Iterated Polynomials. Preliminary report. By adjoining a root of an irreducible polynomial $f(x) \in \mathbb{Z}[x]$ to the rationals, we obtain a larger field that can be thought of as a generalization of $\mathbb{Q}$. Such extensions are called number fields. Just as $\mathbb{Z} \subset \mathbb{Q}$, each number field $K$ has an analogue of the integers which we call the ring of integers of $K$ or simply a number ring. Studying the properties of number rings is a key aspect of algebraic number theory. A number field is monogenic over $\mathbb{Q}$ if the ring of integers admits a power integral basis, i.e., a $\mathbb{Z}$-basis of the form $\left\{1, \alpha, \ldots, \alpha^{n-1}\right\}$. Here we call $\alpha$ a monogenerator. If $f(x)$ is the minimal polynomial of $\alpha$, then we say that $f(x)$ is monogenic. This talk will survey current work and open questions regarding the monogenicity of iterates of polynomials in $\mathbb{Z}[x]$.
(Received September 12, 2023)

## 1192-11-32549

Kristin E. Lauter*, Meta AI Research (FAIR). SALSA, PICANTE y VERDE: Machine Learning attacks on LWE with small sparse secrets.
Learning with Errors (LWE) is a hard math problem underpinning many proposed Post-Quantum Cryptosystems (PQC). The only PQC key exchange standardized by NIST is based on module LWE, and current publicly available PQ Homomorphic Encryption (HE) libraries are based on ring LWE. The security of LWE-based PQ cryptosystems is critical, but certain implementation choices could weaken them. One such choice is sparse binary secrets, desirable for PQ HE schemes for efficiency reasons. This talk presents novel machine learning-based attacks against LWE schemes with sparse binary secrets. Our initial work, SALSA, demonstrated a proof of concept machine learning-based attack on LWE with sparse binary secrets in small dimensions ( $\mathrm{n}<=128$ ) and small Hamming weights ( $\mathrm{h}<5$ ). Our more recent work, PICANTE and VERDE, recovers secrets in much larger dimensions (up to $n=512$ ) and with larger Hamming weights (roughly $n / 10$, and up to $h=60$ for $n=350, h=63$ for $\mathrm{n}=512$ ). We achieve this dramatic improvement via a novel preprocessing step, which allows us to generate training data from a linear number of eavesdropped LWE samples $(4 \mathrm{n})$ and changes the distribution of the data to improve transformer training. We also improve the secret recovery methods of SALSA and introduce a novel cross-attention recovery mechanism allowing us to read off the secret directly from the trained models. In VERDE, we extend the attack to apply to sparse ternary and Gaussian secrets. While PICANTE does not threaten NIST's proposed LWE standards, it demonstrates significant improvement over SALSA and could scale further, highlighting the need for future investigation.
(Received September 12, 2023)

1192-11-32620
Sogol Cyrusian, UC Santa Barbara, Alex Domat, Trinity College, Eric Ren*, Arizona State University, Mayla Ward, Western Washington University. Generalized Delta sets of Numerical Semigroups. Preliminary report.
A Numerical Semigroup is a cofinite submonoid of $\left(\mathbb{N}_{0},+\right)$. These semigroups admit non-unique factorization into irreducibles; the factorization set of a single element is conventionally denoted as a set of vectors. Extensive existing literature considers the set of 1 -norms of said vectors (the "length set") and the gaps between consecutive lengths (the "delta set"). We consider a more general family of length sets by considering the $t$ norm for arbitrary $t \in(1, \infty)$ (under the usual $l_{p}$ space definition), and identify properties of the associated Delta sets. In particular, for $t=0$, the $\Delta_{0}$-sets of all semigroups up to three generators, as well as maximal embedding dimension semigroups, semigroups with generators in generalized arithmetic progression, and semigroups with generators in a compound sequence, are explicitly given. For $t=\infty$, the $\Delta_{\infty}$-sets of semigroups with two generators are explicitly given, and the contents of other generalized families, including semigroups with generators in generalized arithmetic progression, are analyzed. The periodicity of the $\Delta_{0}$ and $\Delta_{\infty}$ sets of individual semigroup elements is also proven, along with general results for $t$-lengths between 1 and $\infty$. We also relate semigroup trade structure, $t$-catenary degree, and $\Delta_{t}$ sets.
(Received September 12, 2023)
M. A. K. Ahmad, Universiti Teknologi Malaysia, Murat Alp, College of Engineering and Technology, American University of the Middle East, Kuwait, Olena Kozhushkina, Ursinus College, Jane Holsapple Long*, Stephen F. Austin State University, Mansur Saburov, College of Engineering and Technology, American University of the Middle East, Kuwait, Justin G. Trulen, Kentucky Wesleyan College. Describing the 2-adic Valuation Trees Generated by Depressed Cubic Polynomials. Preliminary report
For $p$ prime, the $p$-adic valuation $\left\{\nu_{p}(x)\right\}$ of an integer $x$ is the highest power of $p$ which divides $x$. Consider a sequence of 2 adic valuations $\left\{\nu_{2}(f(n))\right\}$, generated by a depressed cubic polynomial $f(n)=n^{3}-a n+b$ with 2 -adic coefficients. We can represent this sequence using a binary tree diagram that could be infinite, finite, or even a single dot. We provide the criterion for solvability of a depressed cubic equation and the number of solutions over some domains of $\mathbb{Q}_{2}$, which we utilize to describe the properties of 2 -adic valuation trees associated to depressed cubic functions. (This is a joint work with M. A. K. Ahmad, M. Alp, M. Saburov, and J. Trulen.)
(Received September 12, 2023)

1192-11-32757
Connor F Morrow*, University of Florida. Congruences mod powers of 5 for Andrews's even parts below odd parts partition function.
We prove congruences mod powers of 5 for George Andrews's partition function, $\overline{\mathcal{E O}}(n)$, the number of partitions of $n$ in which every even part is less than each odd part and the largest even part occurs an odd number of times. This function is related to Ramanujan's third order mock theta function $\nu(q)$. The result is equivalent to Shane Chern's result for 1-shell totally symmetric plane partitions. Using Atkin-Lehner involutions we show equivalence with Dohoon Choi, Soon-Yi Kang, and Jeremy Lovejoy's congruences for the crank parity function. (Received September 12, 2023)

## 1192-11-32800

Sadman Rahman*, Undergraduate Student, Department of Mathematics, North Central College, David J Schmitz, Professor, Department of Math \& Actuarial Science, North Central College. On Some Properties of Higher-Order Rectangular Numbers. A pronic number is a product of two consecutive integers, that is, a number of the form $n(n+1)$. The study of pronic numbers dates back to Aristotle. They are also called rectangular numbers. In our study, we explore some interesting properties of higher-order rectangular numbers. The $n$-th rectangular number of order $h$, written $R_{h}(n)$, is a number of the form $n(n+h)$. In this terminology, the pronic numbers are exactly the rectangular numbers of order $h=1$. Motivated by the properties of the first-order rectangular numbers, we investigate if similar properties hold for rectangular numbers of an arbitrary order $h$. Although the cases $h=1$ and $h>1$ share many properties, we discovered that several properties of the first-order numbers are significantly different than their higher-order cousins. Among the main results of our study, we state a necessary and sufficient condition for $R_{h}(n)$ to be a perfect square, and explore a sufficient condition for $R_{h}(n)$ to be a perfect cube. We also explore several generalizations of properties satisfied by the first-order rectangular numbers to those of higher-orders. Finally, we state a conjecture about the 'necessity' of our sufficient condition for being a perfect cube.
(Received September 12, 2023)

1192-11-32827
Morgan Fiebig*, University of Wisconsin-Eau Claire, aBa Mbirika, University of Wisconsin-Eau Claire, Jürgen Spilker, University of Freiburg. Patterns in the Pisano period and entry points of linear recurrence sequences modulo m. Preliminary report.
The Pisano periods $\pi_{F}(m)$ and entry points $e_{F}(m)$ in the Fibonacci sequence has been a well-studied subject since the seminal paper by D. D. Wall in 1960. In this talk, we extend this study to other well-known sequences. In particular, we explore the following second-order linear recurrence sequences modulo $m$ : Lucas, Pell, associated Pell, balancing, Lucas-balancing, cobalancing, and Lucas-cobalancing. Through data collected through programming in Mathematica, we observe a myriad of intriguing results (and conjectures) relating the Pisano periods and entry points of these sequences. In some sequences, we observe a palindromic behavior in the actual terms of the fundamental period. (Received September 12, 2023)

## 1192-11-32865

Joni Teräväinen*, University of Turku. Uniformity of the Möbius Function in Short Intervals.
In this talk, I will discuss the short interval distribution of the Möbius function. The Möbius function is expected to behave randomly in short intervals, and one way to formalize this is to assert that its Gowers uniformity norms are small over such intervals. I will explain what this notion means and what are some of its implications. I will then discuss joint work with Kaisa Matomäki, Maksym Radziwiłł, Terence Tao and Tamar Ziegler where we showed that the Möbius function is indeed Gowers uniform in almost all short intervals of polynomial length. I will also mention some applications related to Chowla's conjecture and some further progress made on this topic.
(Received September 12, 2023)

## 1192-11-33047

Cathy Barrish*, Muhlenberg College, Charles S Ohanian, Muhlenberg College, Amanda Sodl, Muhlenberg College. Average Class Number of Almost Eisenstein Triples.
For a fixed integer $D$, an Almost Eisenstein Triple is a set of 3 integers, $a, b, c$, that satisfies $a^{2}+a b+b^{2}=c^{2}+D$. Inspired by Gauss' work with average class numbers, we set out to develop a formula for the average class number of Almost Eisenstein Triples (AET's), specifically when $D<0$. Using a double summation method, we found that the average class number of AET's with $\$ 0<-\mathrm{D}$
(Received September 13, 2023)

Jingbo Liu, Texas A\&M University-San Antonio, Matthew Treviño*, Texas A\&M University - San Antonio, Liliana B Villanueva*, Texas A\&M University - San Antonio. A study on the LLL-algorithm and the shortest vector problem.
Let $L=\mathbb{Z} \overrightarrow{v_{1}}+\cdots+\mathbb{Z} \overrightarrow{v_{n}}$ be a lattice with $\overrightarrow{v_{1}}, \ldots, \overrightarrow{v_{n}}$ are linearly independent vectors in $\mathbb{R}^{n}$. One interesting and challenging question in lattice theory is to find or approximate a shortest nonzero vector of $L$. The Lenstra-Lenstra-Lovász (LLL)-algorithm is one of the most useful tools in the algorithmic study of lattices. It provides a partial answer to the shortest vector problem within polynomial time and approximates a shortest nonzero vector of an $n$-dimensional lattice up to a factor of $(4 / 3)^{(n-1) / 2}$. For binary lattices, LLL-algorithm always produces a reduced basis containing a shortest nonzero vector when the parameter $\alpha \geq 1 / 3$. However, a counterexample shows up for ternary lattices. In this project, we will focus on higher dimensional lattices $(n \geq 3)$ and study the probability that the LLL-algorithm produces a shortest nonzero vector. This work is supported by the MAA's National REU Program.
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## 1192-11-33125

Hua Lin*, Northwestern University. Connections between Dirichlet L-functions and random matrix theory.
We compute the one-level density of zeros of order $\ell$ Dirichlet $L$-functions over function fields $\mathbb{F}_{q}[t]$ for $\ell=3,4$ in the Kummer setting $(q \equiv 1 \bmod \ell)$ and for $\ell=3,4,6$ in the non-Kummer setting $(q \not \equiv 1 \bmod \ell)$. In each case, we obtain a main term predicted by Random Matrix Theory (RMT) and lower order terms not predicted by RMT. We also confirm the symmetry type of the families is unitary, supporting Katz and Sarnak's philosophy.
(Received September 13, 2023)

## 1192-11-33193

Carlos A. Agrinsoni*, Department of Mathematics, Purdue University, Moises R. Delgado, University of Puerto Rico, Cayey, Heeralal Janwa, University of Puerto Rico, Rio Piedras. Some New Techniques for the Resolution of the Exceptional Almost Perfect Nonlinear Conjecture.
An almost perfect non-linear (APN) function is a function $f: \mathbb{F}_{2^{n}} \rightarrow \mathbb{F}_{2^{n}}$ satisfying the property that its directional derivative at every nonzero point is two to one. APN functions arise in many areas of mathematics. For example, in cryptography, the APN function appears as the function that minimizes the probability of success of the differential cryptanalysis. An APN function is said to be an exceptional APN if it is an APN over infinitely many extensions of $\mathbb{F}_{2^{n}}$. Aubry McGuire and Rodier conjecture that up to CCZ equivalence, the only exceptional APN functions are the Gold and Kasami-welch monomials. In this presentation, we will provide some new criteria for the absolute irreducibility of polynomials over finite fields. Consequently, we will settle some open cases of the conjecture by using some new techniques based on combinatorial arguments. (Received September 13, 2023)

1192-11-33248
Samantha Duckworth, Smith College, Daniel Friend, Smith College, Manasi Gore*, Smith College. Continued fractions, minimum excluded algorithm, and complementary sequences.
Two sequences of positive integers are complementary if their union is the positive integers and their intersection is empty. A naive approach to determining whether two sequences are complementary is to use a "sieving" algorithm, whose more refined version is called the minimum excluded algorithm. A continued fraction is an iteration of the process "add a fraction to an existing fraction's denominator." While elementary enough to study in high school, continued fractions have a deep and surprising relationship to diverse areas, including Euclid's algorithm for real numbers. In this talk we will present some interesting connections between complementary sequences, continued fractions and the minimum excluded algorithm. (Received September 13, 2023)

## 1192-11-33257

Jesse C Elliott*, California State University, Channel Islands. Analytic number theory and algebraic asymptotic analysis. In this talk, I will elucidate and extend several theorems and conjectures in analytic number theory via two new analyticalgebraic notions, that of the degree and the logexponential degree of a real function. The Riemann hypothesis, for example, is equivalent to the statement that the degree of the function $\pi(x)-\operatorname{li}(x)$ is $1 / 2$, where $\pi(x)$ is the prime counting function and $\operatorname{li}(x)$ is the logarithmic integral function. This talk is a summary of a forthcoming research monograph entitled Analytic Number Theory and Algebraic Asymptotic Analysis, to be published in World Scientific's Monographs in Number Theory series.
(Received September 13, 2023)

1192-11-33302
Angel Kumchev, Towson University, Nathan Gregory McNew*, Towson University, Ariana Park, University of Minnesota Twin Cities. Short Interval Results For Powerfree Polynomials Over Finite Fields.
Let $k \geq 2$ be an integer and $\mathbb{F}_{q}$ be a finite field with $q$ elements. We prove several results on the distribution in short intervals of polynomials in $\mathbb{F}_{q}[x]$ that are not divisible by the $k$ th power of any non-constant polynomial. Our main result generalizes a recent theorem by Carmon and Entin on the distribution of squarefree polynomials to all $k \geq 2$. We also develop polynomial versions of the classical techniques used to study gaps between $k$-free integers in $\mathbb{Z}$. We apply these techniques to obtain analogues in $\mathbb{F}_{q}[x]$ of some classical theorems on the distribution of $k$-free integers. The latter results complement the main theorem in the case when the degrees of the polynomials are of moderate size.
(Received September 13, 2023)

Parker Hayden*, Washington College. Magic Squares Through a p-adic Lens. Preliminary report.
For thousands of years, humans have been enchanted by a mathematical object called a Magic Square. These squares contain a grid of numbers such that when you add along a row, column, or diagonal, the sum is equal. Although magic squares have been studied for centuries, there is not much known about viewing magic squares through a $p$-adic lens. For example, this work investigates given a random square, called $A$, for what primes is $A$ a magic square when the entries are all reduced modulo $p$. Additionally, we investigate what can be said if we view $A$ as having entries in $\mathbb{Z}_{p}$. This poster presentation will include our findings as well as the code we used to generate our results.
(Received September 15, 2023)

1192-11-33430
Sophia Liao*, MIT-PRIMES, Harold Polo, University of California, Irvine. The Goldbach Theorem for Laurent Polynomials with Positive Integer Coefficients.
The Goldbach conjecture posits that every even natural number can be written as the sum of two primes. While it has neither been proven true or false, variations of this problem have been solved: In particular, in polynomial extensions of certain rings and fields, it has been shown that elements can be written as the sum of two irreducible polynomials. In this paper, we establish an analogue of the Goldbach conjecture for Laurent polynomials with positive integer coefficients. (Received September 21, 2023)

## 1192-11-33498

Elizabeth Athaide*, Massachusetts Institute of Technology, Emma Cardwell, Harvard University, Christina Thompson, Stanford University. Class Number Formulas for Certain Biquadratic Fields.
We consider the class numbers of imaginary quadratic extensions $F(\sqrt{-p})$, for certain primes $p$, of totally real quadratic fields $F$ which have class number one. Using seminal work of Shintani, we obtain two elementary class number formulas for many such fields. The first expresses the class number as an alternating sum of terms that we generate from the coefficients of the power series expansions of two simple rational functions that depend on the arithmetic of $F$ and $p$. The second makes use of expansions of $1 / p$, where $p$ is a prime such that $p \equiv 3(\bmod 4)$ and $p$ remains inert in $F$. More precisely, for a generator $\backslash \mathrm{ep}_{F}$ of the totally positive unit group of $\backslash \mathrm{cao}_{F}$, the base- $\backslash \mathrm{ep}_{F}$ expansion of $1 / p$ has period length $\ell_{F, p}$, and our second class number formula expresses the class number as a finite sum over disjoint cosets of size $\ell_{F, p}$.
(Received September 19, 2023)

## 1192-11-33598

Guy Moshkovitz, Department of Mathematics, City University of New York (Baruch College)", Daniel Zhu*, Princeton University. Quasi-linear relation between partition and analytic rank.
An important conjecture in additive combinatorics posits that the partition rank and analytic rank of tensors are equal up to a constant, over any finite field. We prove the conjecture up to logarithmic factors. Our proof is largely independent of previous work, utilizing recursively constructed polynomial identities and random walks on zero sets of polynomials. We also introduce a new, vector-valued notion of tensor rank ("local rank"), which serves as a bridge between partition and analytic rank and which may be of independent interest.
(Received September 21, 2023)

1192-11-33674
Archer Clayton*, Brigham Young University, Paul Jenkins, Brigham Young University. The Effect of The Trace Operator on the Duality of Modular Grids in Genus Zero.
Griffin, Jenkins, and Molnar studied coefficient duality for canonical bases for a broad range of weakly holomorphic modular forms, showing that the Fourier coefficients of canonical basis elements appear as negatives of Fourier coefficients for elements of a canonical basis of another space of forms. We investigate the effect of the trace operator on this duality in spaces of genus zero and show exactly when duality still holds after applying the trace operator. (Received September 22, 2023)

## 1192-11-33736

Connor Lane*, Rose-Hulman Institute of Technology. Asymptotic distribution of Residue Classes in Pascal's Triangle mod p. Let $p$ be a fixed prime and consider the entries of Pascal's triangle mod $p$. Let $A_{n}(r)$ be the number of entries congruent to $r$ $\bmod p$ in the first $n$ rows of Pascal's triangle. In 1947, Nathan Fine began the study of $A_{n}(r)$ by showing that $A_{n}(0)$ grows faster than $A_{n}(r)$ for nonzero $r$. In 2001, Guy Barat and Peter J. Grabner showed that for nonzero $r, A_{n}(r)$ are all asymptotically equal. We use a new method to obtain Barat and Grabner's results, along with obtaining some new effective bounds.
(Received September 26, 2023)

1192-11-33740
Brian Li*, Mission San Jose High School. Whittaker Vectors For Subregular W-algebra and Nonstandard r-matrices. Preliminary report.
Let $\mathfrak{g}=\mathfrak{g l}_{n}$ be the Lie algebra of $n \times n$ matrices. To any nilpotent element in $\mathfrak{g}$, we can associate a certain associative algebra called the finite W -algebra. In this paper, we consider three types of nilpotent elements: the regular one, the subregular one, and the rectangular one. We study the decomposition of a certain class of modules of these W -algebras into the sum of irreducibles. Namely, we construct the generating set of vectors using the explicitly defined elements of W -algebras first introduced by Brundan-Kleshchev. Then, we show that the natural tensor product structure on them quantizes the nonstandard solution of the classical Yang-Baxter equation constructed from the trace pairing with the same nilpotent element on a certain quasi-parabolic subalgebra of $\mathfrak{g}$. Keywords: Capelli elements, Nilpotent subalgebra, Pyramids, Quantization,

Brianna Evans*, University of Wisconsin-Eau Claire. Identities of 3rd Order Linear Recurrence Sequences. Preliminary report.
Recent advances have been made in the study of 3rd order linear recurrence sequences. Results have been found regarding divisibility of the Tribonacci numbers by Evink and Helminck and of the Narayana numbers by aBa, Spilker, and West. In our research, we explore sequences similar to the Tribonacci and Narayana sequences and replicate various identities. For these 3rd order linear recurrence sequences, we find and prove a variety of identities about the sums and products of consecutive terms, using algebraic and combinatorial arguments.
(Received September 25, 2023)

## 1192-11-33787

Elena O'Grady*, Reed College, Melinda Yang*, Pomona College. Adinkras as Origami. Preliminary report.
Around 20 years ago, physicists Michael Faux and Jim Gates invented Adinkras as a way to better understand Supersymmetry.
These are bipartite graphs whose vertices represent bosons and fermions, and whose edges represent operators which relate the particles. Recently, Doran et al. determined that Adinkras are a type of Dessin d'Enfant by explicitly exhibiting a Belyĭ map as a composition $\beta: S \rightarrow \mathbb{P}^{1}(\mathbb{C}) \rightarrow \mathbb{P}^{1}(\mathbb{C})$. We are interested in exhibiting the same Belyĭ map as a different composition $\beta: S \rightarrow E(\mathbb{C}) \rightarrow \mathbb{P}^{1}(\mathbb{C})$.
(Received September 26, 2023)

## 1192-11-33788

Leanna Breland*, Mississippi State University, Kevin Le*, Texas A\&M University, Jingchen Ni*, Emory University, Laura O'Brien*, University of California - Santa Barbara, Hui Xue, Clemson University, Daozhou Zhu, Clemson University. INTERLACING OF ZEROS OF PERIOD POLYNOMIALS.
It is known that the zeros of the period polynomial for a newform $f \in S_{k}\left(\Gamma_{0}(N)\right)$ all lie on the circle $|z|=1 / \sqrt{N}$. In this paper we show that these zeros satisfy various interlacing properties for fixed $N$ and varying $k$ when either $k$ or $N$ is large. We also present a complete result when $N=1$. Lastly, we establish the interlacing properties when $k$ is fixed and $N$ varies. (Received September 26, 2023)

1192-11-33793
Paul Arthur Fili, Oklahoma State University, Vyn C Hubbard*, Oklahoma State University. p-adic equidistribution. Preliminary report.
Classical equidistribution (uniform distribution) looks at how a sequence of points distribute on the unit interval. It has been shown that any sequence that is integer multiples of an irrational number are equidistributed on the unit interval. Equidistribution can be extended beyond just the unit interval however. The Sato-Tate conjecture shows how the number of points on elliptic curves over rational numbers modulo different primes are equidistributed. Bilu has also proved that the roots of irreducible polynomials of order $n$ with small coefficients are also equidistributed on the unit circle as $n$ approaches infinity. However, there are interesting things to be said about these classical equidistribution results if we look at them in the context of p-adic numbers. Bombieri and Zannier have shown a p-adic analog of Bilu's equidistribution theorem with polynomials that have p-adic coefficients. Our goal is to look at p-adic analogs of the Sato-Tate conjecture and the equidistribution of Kloosterman sums.
(Received September 26, 2023)

1192-11-33821
Jewel Aho*, University of St. Thomas, Louis Burns*, Pomona College, Thea Nicholson*, Xavier University. Minimal discriminants and additive reduction of elliptic curves with a 4-isogeny. Preliminary report.
Elliptic curves over $\mathbb{Q}$ that admit a cyclic isogeny of degree $n$ are parameterizable. In this talk, we consider the family of parameterized elliptic curves corresponding to an isogeny class degree of 4 . We classify their minimal discriminants and give necessary and sufficient conditions for determining the primes at which additive reduction occurs.
(Received September 26, 2023)

1192-11-33852
Elena De Leon, Texas Woman's University, Wade McCormick*, UC Berkeley. Image of newform Dedekind sums attached to Quadratic Characters. Preliminary report.
The classical Dedekind sum, first introduced to describe how the Dedekind eta function transforms, has seen application within and outside of number theory in fields as diverse as combinatorics and mathematical physics. A generalization put forward by Stucker, Vennos, and Young, the newform Dedekind sum is denoted $S_{\chi_{1}, \chi_{2}}(\gamma): \Gamma_{0}\left(q_{1} q_{2}\right) \rightarrow \mathbb{C}$, where the $\chi_{i}$ are primitive Dirichlet characters modulo $q_{i}>1$ such that $\chi_{1} \chi_{2}(-1)=1$. Among other intriguing properties, these newform Dedekind sums provide a non-trivial homomorphism from $\Gamma_{1}\left(q_{1} q_{2}\right)$ to $\mathbb{C}$ and, much like classical Dedekind sums, satisfy a family of reciprocity laws. A recent paper by Majure classified the structure of the image of these sums, showing that $S_{\chi_{1}, \chi_{2}}\left(\Gamma_{1}\left(q_{1} q_{2}\right)\right)$ forms a full rank lattice in the number field generated by the values of the $\chi_{i}$. We investigate the generators of these lattices in the case in which $\chi_{1}$ and $\chi_{2}$ are quadratic. Numerical evidence strongly indicates that these lattices are always generated by an even integer, and we put forward a number of partial results in this direction. Furthermore, utilizing finite Fourier analysis, we develop a closed-form formula for $S_{\chi_{1}, \chi_{2}}(\gamma)$ in certain special cases. Complementing the work of Corbett, who showed that values of $S_{\chi_{1}, \chi_{2}}(\gamma)$ are generally very small, we produce infinite families of points at which large values are attained.

Felix Filozov*, Brooklyn College, City University of New York, Joseph Vandehey, University of Texas at Tyler. Sequences of admissible digits in three-dimensional continued fractions. Preliminary report
Continued fractions are a way of representing a number as a sequence of inversions and additions by integers. While classically studied over the reals, several multi-dimensional variants have attracted recent interest. Here we examine threedimensional continued fractions over a cubical lattice. We show which sequences of digits are admissible by considering the inversion of ten particular subsets of the unit cube. We also provide some calculations for estimating the invariant measure and the dual space.
(Received September 26, 2023)

## 1192-11-33855

Evelina Dubovski*, Staten Island Technical High School. Dense Properties of Power Divisor Functions.
We consider power divisor functions $\sigma_{s}(n)=\sum_{d \mid n} d^{s}$ and investigate the density problem of $f_{s}(n)=\frac{\sigma_{s}(n)}{n^{s}}$. It is known that for $s=1$ (the standard divisor function) the range of $f_{1}$ is dense in $[1, \infty)$. Moreover, it has already been proved that for any $a>1,\left|f_{1}(n)-a\right|<\frac{1}{n^{0.4-\varepsilon}}$ for any small $\varepsilon>0$ and some natural number $n$. We prove that there exists a threshold $t$, $1<t<2$, such that the values of $f_{s}$ are dense in $[1, \infty)$ for $\$ 0<$ st. Fors=0 functionf_0
takesonlynaturalvaluesand, thus, losesthedensitypropertyatthelowerthresholds $=0$. Forthestandardfunctionf 1 wealsoshowthattheabovedensityestimatecanbestrengthenedto|f_1(n) -a|<\frac \{1\}\{n^\{1-|varepsilon \}\} andoutlinethepathtowardssimilarestimates fors\neq 1
.Thetheoretical findingsaresupportedbycomputationalexamplesdemonstratingthedensepropertiesfor $0<$ st $\$$. (Received September 26, 2023)

1192-11-33856
Nickolas Andersen, Brigham Young University, Gradin Anderson*, Brigham Young University, Amy Woodall, University of Illinois at Urbana-Champaign. The Weil Bound for Generalized Kloosterman Sums of Half-Integral Weight.
Let $L$ be an even lattice of odd rank with discriminant group $L^{\prime} / L$, and let $\alpha, \beta \in L^{\prime} / L$. We prove the Weil bound for the
Kloosterman sums $S_{\alpha, \beta}(m, n, c)$ of half-integral weight for the Weil Representation attached to $L$. We obtain this bound by proving an identity that relates a divisor sum of Kloosterman sums to a sparse exponential sum. This identity generalizes Kohnen's identity for plus space Kloosterman sums with the theta multiplier system.
(Received September 26, 2023)

## 12 Field theory and polynomials

## 1192-12-29380

Kathryn Hechtel*, University of Kentucky. Getting a grip on the degree of a skew-polynomial. Preliminary report.
A skew-polynomial ring is a polynomial ring over a field where one must apply an automorphism to commute coefficients with x. It was first introduced by Ore in 1933 and since the 1980s has been used to study skew-cyclic codes. The dimension of a skew-cyclic code depends on the degree of its generating skew-polynomial. However, due to the skew multiplication rule, the degree of a skew-polynomial can be smaller than its number of roots and hence tricky to predict. I will discuss how we can use tools offered by Leroy in 2012 to connect the degree of a skew-polynomial to linear independence of field elements related to the roots.
(Received September 06, 2023)

## 13 Commutative rings and algebras

## 1192-13-26218

Pat Lank*, University of South Carolina. Strong generation for module categories. Preliminary report.
This talk is concerned with strong generation in the module category of a commutative noetherian ring, which is linked to the rouquier dimension of the corresponding bounded derived category. A sufficiency criterion is establish for such rings to admit strong generators in their module category, and as a consequence, it answers affirmatively to a question of Iyengar and Takahashi. In particular, it is shown that any noetherian quasi-excellent ring of finite krull dimension admits strong generators in their module category, and explicit examples are established.
(Received September 25, 2023)

1192-13-26352
Andreas Reinhart*, University of Graz. Orders in quadratic number fields with unusual delta set.
Let $\mathcal{O}$ be an order in a quadratic number field. The delta set $\Delta \mathcal{O})$ of $\mathcal{O}$ is the set of positive integers $n$, for which there exist positive integers $r, s$ with $n=r-s$ and a nonzero nonunit $x \in \mathcal{O}$ such that for each positive integer $m$ with $s \leq m \leq r, x$ is a product of $m$ atoms of $\mathcal{O}$ if and only if $m \in\{s, r\}$. It is well known that $\mathcal{O}$ is half-factorial if and only if $\Delta(\mathcal{O})=\varnothing$. If $\mathcal{O}$ is half-factorial, then we set $\min \Delta(\mathcal{O})=0$. Our main goal is to characterize the orders $\mathcal{O}$ for which $\min \Delta(\mathcal{O})>1$. Besides that, we classify the quadratic number fields that possess an order $\mathcal{O}$ with $\min \Delta(\mathcal{O})>1$. As a supplement, we discuss some open problems and we provide various examples of orders $\mathcal{O}$ with $\min \Delta(\mathcal{O})>1$.
(Received July 26, 2023)

Sarah Poiani*, University of New Mexico. The Duality of Pair Operations. Preliminary report.
Expanding on the work of Kemp, Ratliff and Shah, for any closure cl defined on a class of modules modules over an Noetherian ring, we develop the theory of cl-prereductions of submodules. For any interior i on a class of $R$-modules, we also develop the theory of i-postexpansions. Using the duality of Epstein, R.G. and Vassilev, we show that if i is the interior dual to cl, then these notions are in fact dual to each other. We further the thematic notion of duality and seek to understand how it arises in the context of properties pair operations (a generalization of closure and interior operations) can be endowed with. In particular, we will see that the notions of order reversing and involutive are self dual, and that independence is dual to spanning. Because involutive and idempotent are strong requirements, we also extend our analysis to the weaker notions of pre- and postinvolutive and pre- and post-idempotent $\left(L \subseteq p^{2}(L, M), p^{2}(L, M) \subseteq L, p(L, M) \subseteq p^{2}(L, M)\right.$, and $p^{2}(L, M) \subseteq p(L, M)$, respectively).
(Received August 03, 2023)

## 1192-13-27779

Victor Fadinger, University of Graz, Sophie Frisch*, TU Graz, Sarah Nakato, Kabale University, Uganda, Daniel Smertnig, University of Graz, Austria, Daniel Windisch, Graz University of Technology. Non-unique factorization in rings of integer-valued polynomials.
The ring of integer-valued polynomials on a domain $D$ with q.f. $(D)=K$,

$$
\operatorname{Int}(D)=f \in K[x] \mid f(D) \subseteq D
$$

provides examples of wild non-unique factorization of elements into irreducibles, in contrast to the comparatively tame nonunique factorization in Krull rings (where unique factorization of divisorial ideals is always lurking in the background). When $D$ is a Dedekind domain with finite residue fields, $\operatorname{Int}(D)$ is Prüfer, but not Krull, and its multiplicative monoid is not transferKrull. It is, however, monadically Krull, meaning that for each element the monoid consisting of its divisors is a Krull monoid. We give an overview of some recent results regarding - ize
- sets of lengths
- absolutely and non-absolutely irreducible elements
- non-prime absolutely irreducible elements
- lengths of factorizations of powers of irreducible elements \enditemize V. Fadinger, S. Frisch, S. Nakato, R. Rissner, D. Smertnig, D. Windisch, Primes and irreducible elements in atomic domains, to appear. S. Frisch, Relative polynomial closure and monadically Krull monoids of integer-valued polynomials, in S. Chapman, M. Fontana, A. Geroldinger, B. Olberding (eds.) "Multiplicative ideal theory and factorization theory", Springer 2016, 145-157. S. Frisch, S. Nakato, A graph-theoretic criterion for absolute irreducibility of integer-valued polynomials with square-free denominator, Comm. Algebra 48 (2020) 3716-3723. S. Frisch, S. Nakato, R. Rissner, Split absolutely irreducible integer-valued polynomials over discrete valuation domains, J. Algebra 602 (2022) 247-277. S. Frisch, S. Nakato, R. Rissner, Sets of lengths of factorizations of integer-valued polynomials on Dedekind domains with finite residue fields, J. Algebra 528 (2019) 231-249. R. Rissner, D. Windisch, Absolute irreducibility of the binomial polynomials, J. Algebra 578 (2021) 92-114. \enddocument
(Received August 19, 2023)


## 1192-13-28079

Igor Klep, University of Ljubljana, Victor Magron, LAAS CNRS, Jurij Volčič*, Drexel University. Sums of squares certificates for polynomial moment inequalities.
Moment polynomial inequalities and optimization are ubiquitous in probability, statistics, operator theory, economics, partial differential equations, and most recently, quantum information theory. This talk presents results on positivity and optimization of moment polynomials, which are polynomial expressions in variables and their formal mixed moments. A positive solution to Hilbert's 17 th problem for pseudo-moments is given. On the other hand, moment polynomials positive on actual measures are shown to be sums of squares and formal moments of squares up to arbitrarily small perturbation of their coefficients. When only measures supported on a bounded semialgebraic set are considered, a stronger positivity certificate is given; as an application, certain nonlinear Bell inequalities from quantum physics are settled.
(Received August 25, 2023)

## 1192-13-28249

Kevin Steine Harris*, Tacoma Community College. The Clebsch-Gordan Problem for Truncated Polynomial Rings in One Variable.
We investigate how modules decompose over principal subalgebras of certain truncated polynomial rings. In particular, we will investigate how a module decomposition may (or may not) change when we decompose over different principal subalgebras. Varying decompositions are related to the notion of rank varieties. Finally, we will examine how one might extend the notion of rank varieties to more general truncated polynomial rings and investigate the Clebsch-Gordan problem for truncated polynomial rings in one variable.
(Received August 28, 2023)

## 1192-13-28316

Eugene Gorsky*, University of California, Davis. Jet spaces in link homology. Preliminary report.
I will give an overview of some jet schemes and arc schemes appearing in link homology, the combinatorics of their Hilbert series and the associated Koszul homology. In particular, I will highlight some open problems in Khovanov homology of torus knots.
(Received August 29, 2023)
of Noetherian UFDs and Quasi-Excellent Domains.
We show that every finite poset is isomorphic to a saturated subset of the spectrum of a Noetherian unique factorization domain. In addition, we show that every finite poset is isomorphic to a saturated subset of the spectrum of a quasi-excellent domain. As a consequence, the spectra of Noetherian UFDs and quasi-excellent domains can contain finite parts that are very far from being catenary
(Received August 29, 2023)

## 1192-13-28618

Yuelin Li*, Carnegie Mellon University. Strength and Symmetry of Polynomials. Preliminary report.
In this presentation we work with polynomials involving multiple variables. The first part of the talk deals with the strength of specific classes of polynomials. The strength of a given polynomial is the least number of reducible polynomials that it can be written as a sum of. We will present a lower bound for the strengths of elementary symmetric polynomials, as well as propose a possible upper bound. The second part of the talk deals with products, powers, and Hilbert functions of symmetric ideals. We will present some results involving polynomial rings with two variables. We will also present some upper bounds for the numbers of minimal generators of the powers of symmetric ideals involving any number of variables. These results are based on joint work of the 2023 Polymath Jr. group in commutative algebra.
(Received September 01, 2023)

1192-13-28733
Gyu Whan Chang*, Department of Mathematics Education, Incheon National University. PvMDs, Prüfer domains and their class groups.
Let $D$ be a P $v \mathrm{MD},\left\{X_{\alpha}\right\}$ be an infinite set of indeterminates over $D, S$ be the saturated multiplicative subset of $D\left[\left\{X_{\alpha}\right\}\right]$ generated by all nonconstant prime polynomials, and $R=D\left[\left\{X_{\alpha}\right\}\right]_{S}$. In this talk, we show that $R$ is a Prüfer domain with $C l(R)=C l(D)$.
(Received September 02, 2023)

1192-13-28737
Alessandra Costantini*, Oklahoma State University, Edward F. Price, Colorado College, Matthew James Weaver, University of Notre Dame. Rees algebras of linearly presented ideals. Preliminary report.
The Rees algebra of an ideal $I$ is an invaluable tool in the study of the algebraic properties of I , as it encodes information on the asymptotic growth of the powers of I. Moreover, as $\operatorname{Proj}(\mathcal{R}(I))$ is the blowup of an affine scheme along $V(I)$, Rees algebras represent an essential tool in the study of singularities. As the blowup construction describes $\operatorname{Proj}(\mathcal{R}(I))$ via parametric equations, a fundamental problem is to find the implicit equations of blowups. This is a difficult problem in general, as a priori one would need to determine all possible relations among the generators of all powers of I. In this talk, I will restrict to the case when I is a codimension-two perfect ideal in a polynomial ring $k\left[x_{1}, \ldots, x_{d}\right]$, admitting a presentation matrix consisting of linear entries. Most of the existing literature in this setting assumes the so-called $G_{d}$ condition that the Fitting ideals $\operatorname{Fitt}_{i}(I)$ have codimension at least $i+1$ for $i=1, \ldots, d-1$. Moving away from this assumption, we determine the defining ideal of the Rees algebra of $I$ by requiring only that this codimension constraint is satisfied for $i=1, \ldots, d-2$. This is part of joint work with E. Price and M. Weaver, available on arXiv, arxiv:2308.16010.
(Received September 02, 2023)

## 1192-13-28747

Matyas Domokos*, Alfréd Rényi Institute of Mathematics, Budapest, Hungary. On a quantity associated with finitely generated Krull monoids.
Questions in multiplicative ideal theory led to the study of zero-sum sequences over abelian groups, and in particular to the introduction of the Davenport constant of a finite abelian group. The Davenport constant of a finite abelian group equals the Noether number of the same group. This observation highlights a close connection between zero-sum theory and invariant theory, that was developed further recently by introducing the so-called separating Davenport constant, which mirrors a change of focus from systems of generators of rings of polynomial invariants to systems of separating invariants. In the talk we shall overview results about the separating Davenport constant, its comparison with the Davenport constant, and some applications of concepts of actions of algebraic groups in its investigation.
(Received September 02, 2023)

## 1192-13-28778

Xianglong Ni*, UC Berkeley. Weyman's generic ring for free resolutions of length three.
One approach to studying the structure of finite free resolutions is to construct and analyze universal examples. Unfortunately, Bruns proved that these examples typically do not exist-but they do if one weakens the standard notion of universality to allow for non-unique specialization. Weyman explicitly constructed such "generic" examples for length three resolutions, with a careful handling of this non-uniqueness. Moreover, this apparent defect of the construction actually endows the generic example with additional symmetry, from which a surprising connection to the ADE classification arises.
(Received September 03, 2023)

Hwankoo Kim*, Hoseo University. Almost Gorenstein Dedekind Domains.
An integral domain $R$ is said to be locally G-Dedekind if the Gorenstein global dimension of $R_{\mathfrak{m}}$ is at most one for each maximal ideal $\mathfrak{m}$. In this talk we show that an integral domain $R$ is not necessarily G-Prüfer even if R is a locally G-Dedekind domain, which gives a negative answer to the question raised by S. Xing. It follows that the localization of the G-Prüfer domain is different from the classical case of the Prüfer domain. We also study coherent local G-Dedekind domains, called almost GDedekind domains. The almost G-Dedekind domains need not be integrally closed and fill the gap between the G-Dedekind
domain and the G-Prüfer domain. Several examples are given to illustrate the new concept. This is a joint work with S. Xing and L. Qiao
(Received September 04, 2023)

## 1192-13-28867

Roswitha Rissner*, University of Klagenfurt. Powers of irreducibles in rings of integer-valued polynomials.
Non-unique factorization of elements into irreducibles has been observed in the ring of integer-valued polynomials and its generalizations. It is known that every (multi-)set consisting of integers greater than 1 can be realized as the (multi-)set of lengths of an integer-valued polynomial over a Dedekind domain $D$ with infinitely many maximal ideals whose residue fields are finite. Moreover, under the same assumptions on $D, \operatorname{Int}(D)$ is not transfer Krull. The proofs of these two statements are build on the constructions of integer-valued polynomial whose factorization behaviour can be fully controlled. For this, it is crucial to avoid the situation of a factorization in which an irreducible factor occurs more than once. This is because in nonunique factorization domains there is in general no saying how the powers of an irreducible element factor. From a factorization-theoretic point of view, one wants therefore identify those elements among the irreducibles whose powers factor uniquely. We call such elements absolutely irreducible. This talk provides an overview on recent results characterizing absolutely irreducible elements in rings of integer-valued polynomials. The results presented in this talk are joint work with S. Frisch, M. Hiebler, S. Nakato, and D. Windisch.
(Received September 04, 2023)

1192-13-28919
Sarasij Maitra*, University of Utah, Vivek Mukundan, Indian Institute of Technology, Delhi. Reduced type of one dimensional complete local domains.
Recently in an attempt to solve a long standing problem (popularly known as Berger's Conjecture) regarding the torsion of the module of differentials of a one dimensional complete local domain over a field, the numerical invariant "reduced type" was introduced by C. Huneke et. al. In this talk, we will discuss the properties of this invariant and focus on the extremal values (i.e., max/min values). Specifically, we will talk about how studying this invariant translates to studying the same for the associated numerical semigroup ring coming from the valuations. We also will briefly talk about some finiteness results of the category of reflexive modules for rings of extremal reduced type. This is based on joint work with Vivek Mukundan. (Received September 04, 2023)

## 1192-13-28922

Matthew Mastroeni*, Iowa State University, Jason McCullough, Iowa State University, Irena Peeva, Cornell University. Koszul Graded Möbius Algebras and Strongly Chordal Graphs.
The graded Möbius algebra of a matroid is a graded commutative algebra that encodes the combinatorics of the lattice of flats of the matroid. As a special subalgebra of the augmented Chow ring of the matroid, these rings played an important role in the recent proof of the Dowling-Wilson Top Heavy Conjecture, which interpolates between the algebraic properties of the augmented Chow ring and the combinatorics of the graded Möbius algebra. Recently, Mastroeni and McCullough proved that the Chow ring and augmented Chow ring of a matroid are Koszul. We study when graded Möbius algebras are Koszul. Our results suggest a new characterization of strongly chordal graphs via edge orderings.
(Received September 04, 2023)

Matthew Robert Ballard, University of South Carolina, Srikanth Iyengar, University of Utah, Patrick Lank, University of South Carolina, Alapan Mukhopadhyay, University of Michigan, Josh Pollitz*, Syracuse University. Frobenius pushforwards generate the bounded derived category
By now it is quite classical that one can understand singularities in prime characteristic commutative algebra through properties of the Frobenius endomorphism. The foundational result illustrating this is a celebrated theorem of Kunz characterizing the regularity of a noetherian ring (in prime characteristic) in terms of whether a Frobenius pushforward is flat. In this talk, I'll discuss a structural explanation of the theorem of Kunz, that also recovers it, and other theorems of this ilk. Namely, I'll discuss recent joint work with Ballard, Iyengar, Lank, and Mukhopadhyay where we show that over an F-finite noetherian ring of prime characteristic high enough Frobenius pushforwards generate the bounded derived category. (Received September 05, 2023)

## 1192-13-29088

Jim Coykendall, Clemson University, Grant Moles*, Clemson University. Orders in a Number Field with $\bar{R}=R \cdot U(\bar{R})$. A subring of a number field is called an order if it is a free $\mathbb{Z}$-submodule of the associated number ring (the ring of algebraic integers) of rank equal to the degree of the number field. If an order $R$ in a number field has the additional property that $\bar{R}=R \cdot U(\bar{R})$, then we can say a great deal more about the multiplicative structure of $R$ and related orders. This talk will explore the properties of such orders. (Received September 05, 2023)

Andras Cristian Lorincz, University of Oklahoma, Michael Perlman*, University of Minnesota. Equivariant D-modules on spaces with finitely many orbits.
We consider a smooth complex variety endowed with the action of a linear algebraic group, such as a toric variety, space of matrices, or flag variety. Given an orbit, the local cohomology modules with support in its closure encode a great deal of information about its singularities and topology. We will discuss how techniques from representation theory, D-modules, and quivers may be used to compute these local cohomology modules and their related invariants. Our focus will be the space of 2x2xn hypermatrices and its orbit closures.
(Received September 06, 2023)

1192-13-29420
Felix Gotti*, Massachusetts Institute of Technology. A Generalization of the Finite Factorization Property.
An atomic monoid is called a finite factorization monoid (FFM) if every non-invertible element has finitely many factorizations, while an atomic domain is called a finite factorization domain (FFD) if its multiplicative monoid is an FFM. The finite factorization property was introduced back in 1990 by D. D. Anderson, D. F. Anderson, and M. Zafrullah for integral domains, and it was later generalized by F. Halter-Koch to commutative monoids. More recently, A. Geroldinger and Q. Zhong introduced the following natural generalization of the finite factorization property: an atomic monoid $M$ is called a lengthfinite factorization monoid (LFFM) if for any positive integer $\ell$ every non-invertible element of $M$ has only finitely many factorizations with length $\ell$, while an atomic domain is called a length-finite factorization domain (LFFD) if its multiplicative monoid is an LFFM. We obtain from the definitions that every FFD is an LFFD. In this talk, we will discuss recent progress on the length-finite factorization property, putting special emphasis on the setting of integral domains.
(Received September 06, 2023)

## 1192-13-29507

Benjamin Briggs, University of Copenhagen, James Cameron, University of Utah, Janina C Letz, Bielefeld University, Josh Pollitz*, Syracuse University. Koszul homomorphisms and universal resolutions in local algebra. The phenomenon of Koszul duality has been observed in many forms across algebra, geometry and topology. In this talk, I'll discuss an instance in local commutative algebra. In particular, I will introduce a class of local homomorphisms where one can transfer free resolutions over the source to ones over the target via $\mathrm{A}_{\infty}$-structures. This generalizes constructions of Shamash and Eisenbud for complete intersection rings, a construction of Burke for Golod rings, and provides many new examples where one can transfer resolutions. This is all joint work with Ben Briggs, James Cameron and Janina Letz. (Received September 07, 2023)

1192-13-29552
Christopher O'Neill*, San Diego State University. Classifying numerical semigroups using polyhedral geometry. A numerical semigroup is a subset of the natural numbers that is closed under addition. There is a family of polyhedral cones $C_{m}$, called Kunz cones, for which each numerical semigroup with smallest positive element m cooresponds to an integer point in $C_{m}$. Recent work has demonstrated that if two numerical semigroups correspond to points in the same face of $C_{m}$, they share many important properties, such as the number of minimal generators and the Betti numbers of their defining toric ideals. In this way, the faces of the Kunz cones naturally partition the set of all numerical semigroups into "cells" within which any two numerical semigroups have similar algebraic structure. In this talk, we survey what is known about the face structure of Kunz cones, and how studying Kunz cones can inform the classification of numerical semigroups. No familiarity with numerical semigroups or polyhedral geometry will be assumed for this talk. (Received September 07, 2023)

1192-13-29617
Ilya Dumanski*, MIT. Reduced structure of arc spaces. Preliminary report.
The arc space of a scheme may be non-reduced even if the scheme itself is reduced. The problem of finding its reduced structure is deeply connected to algebraic geometry, representation theory, and differential algebra. I will report on combinatorial, algebraic, and representation-theoretic methods towards this problem. Based on joint works with Evgeny Feigin, Ievgen Makedonskyi, and Igor Makhlin.
(Received September 07, 2023)

1192-13-29626
Adam Lee Boocher*, University of San Diego, Milena Hering, The University of Edinburgh. Betti Numbers Large and Small. The Betti numbers of graded module over a polynomial ring encode, in some sense, its algebraic complexity. In this talk I'll discuss how recent work has clarified some information about how large these Betti numbers can be in the aggregate, and also how in special cases (including embeddings of projective space) we can study the question of when a particular Betti number is zero.
(Received September 07, 2023)

## 1192-13-29638

Bhargav Bhatt, Princeton University / IAS, Linquan Ma, Purdue University, Zsolt Patakfalvi, EPFL, Karl Schwede*, University of Utah, Kevin Tucker, University of Illinois At Chicago, Joe Waldron, Michigan State University, Jakub
Witaszek, Princeton University. Perfectoid pure singularities. Preliminary report.
We will present a some results of perfectoid pure singularities, a mixed characteristic analog of $\log$ canonical and $F$-pure singularities from characteristic zero and characteristic $p>0$ respectively.
(Received September 07, 2023)

Maxine Elena Calle*, University of Pennsylvania, Sam Ginnett, Reed College. Ideals in the Burnside Tambara functor on a cyclic group.
We give a concrete description of the prime ideals in the Burnside Tambara functor on a cyclic group, extending results of Dress and Lewis. Based on joint work with S. Ginnett.
(Received September 07, 2023)

1192-13-29815
Jim Coykendall*, Clemson University. The strong finite type property in ring extensions.
The SFT (for strong finite type) property for a commutative ring with 1 is a near-Noetherian property that was first explored by J. Arnold in 1973 as a central object in the study of Krull dimension behavior of power series rings. Since that time there has been much interest in the SFT property and some of its variants. In this talk we will explore the SFT property: its history, some known results and variants of the property, and some new results concerning the question as to whether the SFT property is preserved in polynomial extensions.
(Received September 08, 2023)

## 1192-13-29932

Austin Wei*, Ohio State University. almost Prüfer domains. Preliminary report.
A domain $D$ is an almost Prüfer domain if there exists $z \in D$ so that for each finitely generated ideal $I$ of $D$, the ideal $(I, z)$ is invertible. Equivalently, a ring $D$ is almost Prüfer if there exists $z \in D$ so that for any other $x \in D$, and maximal ideal $\mathfrak{m}$ of $D$, either $\frac{x}{z}$ or $\frac{z}{x} \in D_{\mathfrak{m}}$, and such an element $z$ is locally comparable. We will discuss various analagous properties of almost Prüfer domains to the Prüfer case, with a focus on the $\operatorname{ring} \operatorname{Int}\left(\mathbb{Z}^{n}\right)$ of integer-valued polynomials in several variables.
(Received September 08, 2023)

## 1192-13-29939

Pete L. Clark*, University of Georgia. Claborn's Theorem on Class Groups.
In 1966, Luther Elic Claborn solved the Inverse Class Group Problem by proving that any commutative group whatsoever can serve as the ideal class group of a Dedekind domain. Subsequent proofs were given by Leedham-Green and the speaker, the latter involving elliptic curves. In this talk we will discuss the latter proof and then survey subsequent work in this area, focusing on the question of what else can be said or imposed on Dedekind domains with a given commutative group as its ideal class group.
(Received September 08, 2023)

## 1192-13-29983

Juliette Emmy Bruce*, University of California, Berkeley. Finite Generation of Multigraded Regularity.
If $X$ is a smooth projective toric variety with $\mathbb{Z}^{r}$-multigraded total coordinate ring $S$ then the multigraded regularity of a $S$ module $M$ is a subset $\operatorname{reg}(M)$ of $\mathbb{Z}^{r}$. I will discus when the multigraded regularity of a finitely generated $S$-module $M$ is finitely generated in the sense that $\operatorname{reg}(M)$ has finitely many "corners". As an application of this work we will characterize the asymptotic behavior of the multigraded regularity of powers of ideals.
(Received September 08, 2023)

## 1192-13-30268

Justin Lyle*, Self. Endomorphism Algebras Over Commutative Rings and Torsion in Tensor Products. Preliminary report. Let $R$ be a commutative Noetherian local ring and $M$ a finitely generated $R$-module. We analyze certain tensor products involving $M$ through the natural left module structure of $\operatorname{End}_{R}(M)$. In particular, we study a wide generalization of the classical trace map, and use it to provide an extension of a result of Lindo that calculates the center of End ${ }_{R}(M)$ under mild conditions. As an application of our methods, we provide partial results on a open question of Sega on the vanishing of $\operatorname{Tor}_{i}^{R}(M, M)$.
(Received September 09, 2023)

1192-13-30279
Clay Adams*, Harvey Mudd College, Francesca Cantor*, Swarthmore College, Anese Gashi*, Williams College, Semir Mujevic*, Brown University, Sejin Park*, Brown University, Austyn Simpson, University of Michigan, Jenna Zomback, University of Maryland, College Park. Uniform Arithmetic in Local Rings via Ultraproducts. Preliminary report.
We reinterpret many properties of noetherian local rings via the existence of some $n$-ary numerical function satisfying certain uniform bounds. We provide such characterizations for seminormality, weak normality, generalized Cohen-Macaulayness, and $F$-purity, among others. Our proofs that such numerical functions exist are nonconstructive and rely on the transference of the property in question from a local ring to its ultrapower or catapower.
(Received September 09, 2023)

The syzygies of projective curves are fairly well understood - for instance, an influential theorem of Gruson, Lazarsfeld, and Peskine gives optimal bounds on their regularity. There has been a flurry of activity in the past two decades centered around generalizing work on syzygies to the toric setting. I will discuss work that lifts machinery for understanding syzygies of
projective curves to obtain a bound on the multigraded regularity of a curve in a product of projective spaces. (Received September 09, 2023)

1192-13-30351
Moses Samuelson-Lynn*, University of Utah. Seed complex efficiency for small cluster algebras of finite type, and optimal quivers.
Cluster algebras were introduced in 2002 and have since found use in numerous fields of math and science. Understanding the structure of cluster algebras requires knowledge of its elements, known as seeds, which grow unwieldy quite quickly if a good initial seed is not chosen. In this project, the efficiency of initial seed complexes for simply-laced finite cluster algebras is assessed via various graph centrality algorithms, as well as through explicit measures of Laurent polynomial degree and magnitude to provide a firm handle on the size and complexity of the variables in each seed pattern. This allows for efficient explicit representation of cluster algebras in terms of a finite set of variables with well-defined algebraic relations, giving a concrete framework for these classes of objects. In many cases, these align with established alternatives to the familiar Cartan-Killing forms, although there are some surprising exceptions whose structure may merit further investigation. These results suggest a potential alternative set of interpretations of cluster algebras, most notably in the $D_{n}$ case, where the apparent optimal case shows an unexpected pattern with embedded triangles, contrary to the intuitive layout of a simple stellated polygon. \par
(Received September 09, 2023)

1192-13-30363
Isidora Dare Bailly-Hall, Grinnell College, Christine Berkesch, University of Minnesota, Karina Dovgodko, Columbia University, Sean Guan, University of California, Berkeley, Saisudharshan Sivakumar, University of Florida, Jishi Sun*, University of Michigan. On virtual resolutions of points in a product of projective spaces.
Free resolutions, or syzygies, with a graded structure are algebraic objects that encode many geometric properties. This correspondence lies at the heart of classical projective algebraic geometry. Virtual resolutions were recently introduced by Berkesch, Erman, and Smith to produce a similar correspondence for smooth toric varieties. We will describe two methods for producing nice virtual resolutions for a finite sets of points in $\mathbb{P}^{n} \times \mathbb{P}^{m}$. We first extend to $\mathbb{P}^{n} \times \mathbb{P}^{m}$ a result of Harada, Nowroozi, and Van Tuyl for $\mathbb{P}^{1} \times \mathbb{P}^{1}$ by intersecting with a sufficiently high power of one set of variables. Then, we describe an explicit virtual resolution for a set $X$ in sufficiently general position in $\mathbb{P}^{1} \times \mathbb{P}^{2}$; this is a subcomplex of the free resolution for $X$. Along the way, we provide an explicit relationship between Betti numbers and higher difference matrices of Hilbert functions.
(Received September 09, 2023)

## 1192-13-30365

Isidora Dare Bailly-Hall*, Grinnell College, Karina Dovgodko*, Columbia University, Saisudharshan Sivakumar*, University of Florida, Jishi Sun*, University of Michigan. Virtual resolutions of points in a product of projective spaces. Free resolutions, or syzygies, with a graded structure are algebraic objects that encode many geometric properties. This correspondence lies at the heart of classical projective algebraic geometry. Virtual resolutions were recently introduced by Berkesch, Erman, and Smith to produce a similar correspondence for smooth toric varieties. We will describe two methods for producing nice virtual resolutions for a finite sets of points in $\mathbb{P}^{n} \times \mathbb{P}^{m}$. We first extend to $\mathbb{P}^{n} \times \mathbb{P}^{m}$ a result of Harada, Nowroozi, and Van Tuyl for $\mathbb{P}^{1} \times \mathbb{P}^{1}$ by intersecting with a sufficiently high power of one set of variables. Then, we describe an explicit virtual resolution for a set $X$ of sufficiently general position in $\mathbb{P}^{1} \times \mathbb{P}^{2}$; this is a subcomplex of the free resolution for $X$. Along the way, we provide an explicit relationship between Betti numbers and higher difference matrices of Hilbert functions.
(Received September 09, 2023)

1192-13-30474
Mounir Hajli*, Shanghai Jiao Tong University, Hussein Mourtada, Université Paris Cité. A geometric interpretation of plane partitions. Preliminary report.
We consider the study of plane partitions of integers by representing them as global sections of an infinite dimensional toric variety $V\left(\mathcal{P}_{2}\right)$. We see $V\left(\mathcal{P}_{2}\right)$ as the projective limit of a sequence of finite dimensional normal toric varieties; we study the geometry of these latter varieties in relation with integer partitions. This is joint work in progress with Hussein Mourtada. (Received September 11, 2023)

1192-13-30494
Antun Milas*, SUNY at Albany. Principal Subspaces and Jet Algebras. Preliminary report.
Nearly two decades ago, Capparelli, Lepowsky, and the author initiated a study of characters of principal subspaces of vertex algebras, employing the use of bosonic vertex operators. In our framework, a pivotal role was played by certain "shift" maps known as simple current operators and q-recursions. As a result of this analysis, we were able to compute the Hilbert series of the arc algebra of the fat point, expressed as famous Andrews-Gordon series. Arc algebras/spaces have recently acquired increased interest within the field of vertex algebra, primarily due to their significance in the context of $4 \mathrm{~d} / 2 \mathrm{~d}$ dualities in physics. In my talk, we will discuss Hilbert series of certain n-jet algebras and arc algebras in connection to a particular class of principal subspaces studied by Penn and myself. We employ various analytic tools to study growth of coefficients of these series. We also discuss how methods based on vertex operators naturally lead to questions about "classical freeness" of vertex algebra modules, which we will elaborate on. This talk is partially based on a joint work with $\mathrm{H} . \mathrm{Li}$. (Received September 10, 2023)

Minerals, Bruce Olberding*, New Mexico State University. The core of an ideal in a Prüfer domain.
An ideal $J \subseteq I$ of a commutative ring $R$ is a reduction of $I$ if $J I^{n}=I^{n+1}$ for some positive integer $n$. The core of $I$ is the intersection of all reductions of $I$, and it is a well-studied and useful feature of ideals in Noetherian rings. We investigate properties of cores of ideals in the very different context of Prüfer domains and show how the existence of desirable properties of the core coincides with the presence of important structural properties of Prüfer domains.
(Received September 10, 2023)

Anna Brosowsky*, University of Michigan. Cartier algebras through the lens of p-families.
A Cartier subalgebra of a prime characteristic commutative ring $R$ is an associated non-commutative ring of operators on $R$ that play nicely with the Frobenius map. When $R$ is regular, its Cartier subalgebras correspond exactly with sequences of ideals called $F$-graded systems. One special subclass of $F$-graded system is called a $p$-family; these appear in numerical applications such as the Hilbert-Kunz multiplicity and the $F$-signature. In this talk, I will discuss how to characterize some properties of a Cartier subalgebra in terms of its $F$-graded system. I will further present a way to construct, for an arbitrary $F$ graded system, a closely related $p$-family with especially nice properties.
(Received September 10, 2023)

## 1192-13-30590

Bjorn Cattell-Ravdal*, Metropolitan State University of Denver, Erin Delargy*, Binghamton University, Akash Ganguly*, Carleton College, Sean Guan, University of California, Berkeley, Trevor Karn, University of Minnesota, Saisudharshan Sivakumar*, University of Florida. On ideals preserved by linear changes of coordinates in positive characteristic. Preliminary report.
Let $k$ be an infinite field of positive characteristic. The general linear group $\mathrm{GL}_{n}(k)$ acts on the polynomial ring
$S=k\left[x_{1}, \ldots, x_{n}\right]$ by linear substitution of variables. We study the $\mathrm{GL}_{n}(k)$-stable ideals of $S$, specifically their $\mathrm{GL}_{n}(k)$ submodule structure and syzygies. We prove a decomposition theorem for $\mathrm{GL}_{n}(k)$-stable ideals of $S$, and find that the depth of $\mathrm{GL}_{n}(k)$-stable ideals is 0 . Furthermore, we provide a method for obtaining minimal generating sets and minimal free resolutions for a wide class of $\mathrm{GL}_{2}(k)$-stable ideals generated in a single degree.
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## 1192-13-30594

Bjorn Cattell-Ravdal, Metropolitan State University of Denver, Erin Delargy, Binghamton University, Akash Ganguly, Carleton College, Sean Guan, University of California, Berkeley, Trevor Karn, University of Minnesota, Saisudharshan Sivakumar*, University of Florida. Ideals preserved by linear changes of coordinates in positive characteristic. Preliminary report.
Let $k$ be an infinite field of positive characteristic. The general linear group $\mathrm{GL}_{n}(k)$ acts on the polynomial ring $S=k\left[x_{1}, \ldots, x_{n}\right]$ by linear substitution of variables. We study the $\mathrm{GL}_{n}(k)$-stable ideals of $S$, specifically their $\mathrm{GL}_{n}(k)$ submodule structure and syzygies. We prove a decomposition theorem for $\mathrm{GL}_{n}(k)$-stable ideals of $S$, and find that the depth of $\mathrm{GL}_{n}(k)$-stable ideals is 0 . Furthermore, we provide a method for obtaining minimal generating sets and minimal free resolutions for a wide class of $\mathrm{GL}_{2}(k)$-stable ideals generated in a single degree.
(Received September 10, 2023)

1192-13-30614
Jack Jeffries*, University of Nebraska, David Lieberman, University of Nebraska-Lincoln. Sandwich Bernstein-Sato polynomials. Preliminary report.
The Bernstein-Sato polynomial is a well-studied invariant of elements of a polynomial ring over a field of characteristic zero; this invariant is based on the action of the ring of differential operators $D_{R}$ on $R$. Its existence is a consequence of Bernstein's inequality. Recent results have extended the existence of Bernstein-Sato polynomials and Bernstein's inequality to various classes of (mildly) singular rings. Motivated by extending Bernstein's inequality, we introduce a two-sided analogue of Bernstein-Sato polynomials (dubbed sandwich Bernstein-Sato polynomials); these invariants are based considering elements of $R$ as elements of $D_{R}$, instead of as a $D_{R}$-module. Existence of these sandwich Bernstein-Sato polynomials has a close relationship to Bernstein's inequality; in particular, we apply these to establish new cases of Bernstein's inequality. We also give a number of examples. This is based on joint work with David Lieberman.
(Received September 10, 2023)

1192-13-30683
Hailong Dao, University of Kansas, Souvik Dey, Charles University, Prague, Monalisa Dutta*, University of Kansas, USA. Ulrich Split Rings.
We introduce the notion of Ulrich split rings. A local Cohen-Macaulay ring is called Ulrich split if any short exact sequence of Ulrich modules is split exact. We characterize such rings from a category theoretic viewpoint, and also via the annihilator of Ext. We explicitly characterize Ulrich split rings among the complete local Cohen-Macaulay rings of dimension two with minimal multiplicity and complex residue field. This also allows us to characterize surface cyclic quotient singularities for which the test ideal associated to the family of maximal Cohen-Macaulay modules (in the sense of F. Pérez and R.R.G.) is equal to the maximal ideal. For hypersurfaces of minimal multiplicity, the Ulrich split property stays preserved under taking the double branched cover. For 1-dimensional local Cohen-Macaulay rings, the Ulrich split property behaves well when taking the fiber product.
(Received September 25, 2023)

Teresa Yu*, University of Michigan. Moment varieties and determinantal ideals.
Determinantal ideals are a class of ideals defined via minors of matrices whose entries are ring elements. In this talk, we will give examples of determinantal ideals and describe some of their properties. We will then see how to apply these results to the field of algebraic statistics.
(Received September 10, 2023)

## 1192-13-30729

Swaraj Pande*, University of Michigan. A Frobenius version of Tian's alpha-invariant..
In this talk, we will discuss certain asymptotic invariants of section rings of projective varieties in positive characteristic. Inspired by the alpha-invariant of complex varieties, we introduce the "Frobenius-alpha" invariant of N-graded section rings using Frobenius splittings. This invariant is closely related to (and shares many properties with) the F-signature, another asymptotic invariant of singularities. We will discuss these relations, and some special properties of the Frobenius-alpha invariant when the section ring comes from a Fano variety. Time permitting, we will also discuss its semicontinuity properties. (Received September 10, 2023)

## 1192-13-30902

Cheng Meng*, Purdue University, Alapan Mukhopadhyay, University of Michigan. h-function of local rings of characteristic $p$.

For a Noetherian local ring R of characteristic p , we will study a multiplicity-like object called h-function. It is a function of a real variable s that estimates the asymptotic behavior of the sum of ordinary power and Frobenius power. The h-function of a local ring can be viewed as a mixture of the Hilbert-Samuel multiplicity and the Hilbert-Kunz multiplicity. In this talk, we will prove the existence of $h$-function and the properties of h-function, including differentiability, additivity, and behavior under ring maps. We will express Taylor's s-multiplicity, Trivedi's Hilbert-Kunz density function, and Mukhopadhyay's FrobeniusPoincaré function as functionals of h-function and show that they exist in more general settings. We will also show how hfunction recovers other invariants in characteristic p, including the Hilbert-Kunz multiplicity, the F-signature, and the Fthreshold.
(Received September 11, 2023)

1192-13-31034
Graham J. Leuschke*, Syracuse University, Tim Tribone, University of Utah. Branched covers and matrix factorizations. A theorem due to Knörrer states that there are finitely many isomorphism classes of maximal Cohen-Macaulay modules over a hypersurface ring $R=S /(f)$ if and only if the same is true for the double branched cover of $R$, that is, the hypersurface ring defined by $f+z^{2}$ in $S[[z]]$. We consider an analogue of this statement in the case of the hypersurface ring defined instead by $f+z^{d}$ for $d \geq 2$. In particular, we show that this hypersurface, which we refer to as the $d$-fold branched cover of R, has finite Cohen-Macaulay representation type if and only if, up to isomorphism, there are only finitely many indecomposable matrix factorizations of $f$ with $d$ factors. As a result, we give a complete list of polynomials $f$ with this property in characteristic zero. (Received September 11, 2023)

## 1192-13-31052

Devlin Mallory*, University of Utah. Finite F-representation type for homogeneous coordinate rings.
Finite $F$-representation type is an important notion in characteristic- commutative algebra, and is closely connected to the behavior of differential operators. Despite this, explicit examples of varieties with or without this property are few. We prove that a large class of homogeneous coordinate rings (essentially, those of Calabi-Yau or general-type varieties) will fail to have finite $F$-representation type, via an analysis of their rings of differential operators. This also provides instructive examples of the structure of the ring of differential operators for non- $F$-pure varieties, which to this point have largely been unexplored. Time permitting, we may also discuss conjectural evidence of the behavior of FFRT in the Fano case. (Received September 11, 2023)

## 1192-13-31068

John A Christian, Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, Harm Derksen*,
Northeastern University. Invariant Theory for Spacecraft Navigation.
In spacecraft navigation, vision-based sensors can be used to get information about the pose of a spacecraft. The observed data can relate to star patterns, or features of a terrain such as craters for example. To compare the sensor-data to known databases of geometric patterns, one would like to extract features that are invariant under symmetry transformations. How many invariant features one can extract is not always a straightforward problem. I will discuss an invariant theory approach to analyse this problem.
(Received September 11, 2023)

1192-13-31224
David Eisenbud*, MSRI. Observations and conjectures on infinite free resolutions. Preliminary report.
Most of the work on infinite free resolutions over commutative local rings has centered on the ranks of the modules in the resolutions - the Poincare series. But there is much more to a resolution. Recent observations by the presenter and Hai Long Dao, and work with Dao, Michael K Brown and Prashanth Sridhar have exposed surprising new properties of the matrices in (many) such resolutions, and led to a number of new results and conjectures. I will explain some of these, as an invitation to further work in this direction.
(Received September 11, 2023)

Trevor Arrigoni*, University of Kansas. F-Invariants of Simple Algebroid Plane Branches.
We present algorithms for computing certain positive characteristic invariants, namely Frobenius thresholds and test ideals, associated to a natural family of irreducible power series in two variables. These algorithms build on the integer programs recently defined by Hernández and Witt to compute roots of the Bernstein-Sato polynomial for this family of curves.
(Received September 11, 2023)

1192-13-31423
Michael Morrow*, University of Kentucky, Uwe Nagel, University of Kentucky. Syzygy Computations in OI-Modules. Given a sequence of related modules $M_{n}$ defined over a sequence of related polynomial rings, one may ask how to simultaneously compute the syzygy module of each $M_{n}$. Working in the setting of OI-modules over Noetherian polynomial OIalgebras, we present an OI-analogue of Schreyer's theorem for computing syzygies. Here, OI denotes the category of totally ordered finite sets with order-preserving injective maps.
(Received September 11, 2023)

1192-13-31432
Le Tran*, New Mexico State University. Initially regular sequences on cycles and unicyclic graphs. Preliminary report.
Let $I$ be a homogeneous ideal in a polynomial ring $R$. Initially regular sequences on $R / I$ are a type of sequence that behaves like regular sequences and whose length provides a lower bound for the depth of $R / I$. We discuss the notion of initially regular sequences on $R / I$ and give an explicit description of an initially regular sequence of length equal to the depth of $R / I$, where $I$ is the edge ideal of any cycle $C_{n}$, for $n \geq 3$. The approach involves examining the associated primes of ideals of the form in $_{>}(I, f)$ for arbitrary monomial ideals $I$ and $f$ linear sums. We give a description of the minimal and the embedded associated primes of these ideals in terms of the minimal primes of $I$. Moreover, we compute accurately the depth of certain unicyclic graphs.
(Received September 11, 2023)

## 1192-13-31576

Fulvio Gesmundo, University of Saarland, Hang Huang*, Texas A\&M University, Henry K. Schenck, Auburn University, Jerzy Weyman, Instytut Matematyki, Jagiellonian University. Resolving the $2 \times 2$ permanents of a $2 \times n$ matrix. Preliminary report.
I will discuss how to describe the minimal free resolution for the $2 \times 2$ permanents of a generic matrix $M$. In contrast to the case of $2 \times 2$ determinants, the $2 \times 2$ permanents define an ideal $I$ which is neither prime nor Cohen-Macaulay. The starting point combines works of Laubenbacher-Swanson on the Groebner basis of an ideal of $2 \times 2$ permanents of a generic matrix with the previous work connecting the initial ideal of $2 \times 2$ permanents to a simplicial complex. The main technical tool is a spectral sequence arising from the Bernstein-Gelfand-Gelfand correspondence.
(Received September 11, 2023)

## 1192-13-31640

Claudiu Raicu, University of Notre Dame, Keller VandeBogert*, University of Notre Dame. Projective Dimension of Weyl Modules over the Schur Algebra.
In 1988, Akin-Buchsbaum posed the problem of computing the "best" resolution of a Weyl module by tensor products of divided powers in arbitrary characteristic. Outside of special cases, these resolutions have remained elusive even though such resolutions have recently become desirable for the purposes of computing stable sheaf cohomology on flag varieties. In this talk, I will discuss new progress that approaches this conjecture through the lens of sheaf cohomology. More precisely, we are able to construct the (conjecturally) shortest possible resolution of a Weyl module via a categorified row-insertion procedure that takes advantage of cohomological properties of certain classes of vector bundles on projective space.
(Received September 12, 2023)

## 1192-13-31761

Ethan Reed*, University of Notre Dame. An Isomorphism Theorem of Certain Arithmetic Complexes.
We consider generalizations of certain arithmetic complexes appearing in work of Raicu and VandeBogert in connection with the study of stable sheaf cohomology on flag varieties. Defined over the ring of integer valued polynomials, we prove an isomorphism of these complexes as conjectured by Gao, Raicu, and VandeBogert. In particular, this gives a more conceptual proof of an identification between the stable sheaf cohomology of hook and two column partition Schur functors applied to the cotangent sheaf of projective space. This talk is based on joint work with Luca Fiorindo, Shahriyar Roshan-Zamir, and Hongmiao Yu.
(Received September 12, 2023)

1192-13-31763
Alison E. Becker, D.O.D., Tom G. Stojsavljevic*, Beloit College. n-Absorbing Ideals and Graded Commutative Rings. Preliminary report.
Let $G$ be a group with identity $e$. We say that a $G$-graded ring is a ring $R$ together with a decomposition $R=\bigoplus_{g \in G} R_{g}$ as a $\mathbb{Z}$ -module such that $R_{g} R_{h} \subseteq R_{g+h}$ for all $g, h \in G$. An ideal $I$ of a graded ring $R$ is said to be a graded ideal (or homogeneous ideal) whenever $I=\bigoplus_{g \in G}\left(I \cap R_{g}\right)=\bigoplus_{g \in G} I_{g}$. In this talk, we will explore a special type of graded ideal defined as a 2absorbing ideal. We extend this notion to consider what it means to consider what it means for a graded ideal to be $n$ absorbing and weakly $n$-absorbing in this space.
(Received September 12, 2023)

Alexandra Pevzner*, University of Minnesota - Twin Cities. Symmetric group fixed quotients of polynomial rings. Let the symmetric group act on the polynomial ring $S=k\left[x_{1}, \ldots, x_{n}\right]$ via variable permutation. We consider the quotient module $M$ which sets a monomial equal to all of its images under the action. This is a module over the ring of invariants, with relatively little known about its structure. When using integer coefficients, we can embed $M$ as an ideal inside the ring of symmetric polynomials. Doing so gives rise to a family of ideals - one for each $n$. Localizing the coefficient ring of $S$ at a prime $p$ reveals striking behavior in these ideals, which stay stable (in a sense) as $n$ grows, but jump in complexity each time $n$ equals a multiple of $p$. In this talk, we will discuss the construction of this family of ideals, as well as some results and conjectures on its structure.
(Received September 12, 2023)

## 1192-13-31916

Karthik Ganapathy*, University of Michigan. Prime ideals in equivariant rings. Preliminary report.
Prime ideals in a noetherian ring R play a crucial role in understanding it's module theory. For example, every finitely generated R-module admits a finite filtration whose graded pieces have prime annihilators. In equivariant commutative algebra, our goal is to uncover noetherian-like phenomena in non-noetherian rings with a large group of symmetry G , like the set of infinite invertible matrices GL. In this talk, I will define the notion of a "G-prime" ideal in this setting, and explain why these ideals take the role of prime ideal. Over a field of positive characteristic, I will prove that the Frobenius powers of certain determinantal ideals are GL-prime using Grobner theory, and state some foundational open questions regarding GLprime ideals in positive charateristic.
(Received September 12, 2023)

## 1192-13-31928

Karthik Ganapathy*, University of Michigan. Equivariant commutative algebra in positive characteristic. Preliminary report. In the presence of a large group action, even non-noetherian rings sometimes behave like noetherian rings. For example, Cohen proved that every symmetric ideal in the infinite variable polynomial ring $k\left[x_{1}, x_{2}, \ldots, x_{n}, \ldots\right]$ is generated by the orbit of finitely many polynomials. In this talk, I will give a brief introduction to equivariant commutative algebra where we systematically study such noetherian phenomena in infinite variable polynomial rings, and explain my work over fields of positive characteristic.
(Received September 12, 2023)

## 1192-13-32039

Mahrud Sayrafi*, University of Minnesota, Twin Cities. Splitting of vector bundles on toric varieties.
In 1964, Horrocks proved that a vector bundle on a projective space splits as a sum of line bundles if and only if it has no intermediate cohomology. Generalizations of this criterion, under additional hypotheses, have been proven for other toric varieties, for instance by Eisenbud-Erman-Schreyer for products of projective spaces, and in joint work with Michael Brown for smooth projective toric varieties of Picard rank 2. This talk is about a splitting criterion for arbitrary smooth projective toric varieties.
(Received September 12, 2023)

## 1192-13-32044

Devlin Mallory, University of Utah, Mahrud Sayrafi*, University of Minnesota, Twin Cities. Computing summands of the Frobenius pushforward over Cox rings. Preliminary report.
It is known that the Frobenius pushforward of the structure sheaf on toric varieties splits as a sum of line bundles, and high enough pushforwards of it generate the derived category. Working over the multigraded Cox ring of a Mori Dream Space, we discuss algorithms for determining the summands of the Frobenius pushforward of the structure sheaf.
(Received September 12, 2023)

1192-13-32226
Daniel Erman*, University of Hawaii. From Hilbert to Mirror Symmetry.
In 1890, Hilbert transformed algebra with a collection of finiteness results. I will discuss what Hilbert's ideas say about matrices of polynomials, as well as how those ideas have evolved over the decades, through interactions with modern algebraic geometry and, quite recently, through a connection with mirror symmetry.
(Received September 12, 2023)

1192-13-32318
Adam L. Van Tuyl, McMaster University, Jay Yang*, Washington University in St. Louis. Techniques for Virtual Resolutions of Monomial Ideals.
I will discuss a couple techniques for constructing virtual resolutions of monomial ideals. The focus will be on recent work with Adam Van Tuyl where we provide a powerful new technique for finding virtually Cohen-Macaulay monomial ideals. (Received September 12, 2023)

## 1192-13-32428

Megumi Harada, McMaster University, Alexandra Seceleanu, University of Nebraska-Lincoln, Liana M. Sega*, University of Missouri Kansas City. The minimal free resolution of a general principal symmetric ideal.
We introduce the class of principal symmetric ideals, which are ideals generated by the orbit of a single polynomial under the action of the symmetric group. Fixing the degree of the generating polynomial, this class of ideals is parametrized by points in a suitable projective space. We show that the minimal free resolution of a principal symmetric ideal is constant on a nonempty

Zariski open subset of this projective space and we determine this resolution explicitly.
(Received September 12, 2023)

1192-13-32582
Lauren Cranton Heller*, University of California - Berkeley. Explicit constructions of short virtual resolutions. Preliminary report.
In 2022 several groups of researchers independently proved the existence of short virtual resolutions on a simplicial projective toric variety X , meaning that for each multigraded module there exists a free complex of length at most dim X which gives a resolution of the corresponding sheaf. I will discuss constructions of such resolutions using commutative algebra.
(Received September 12, 2023)

1192-13-32753
Rankeya Datta, University of Missouri, Neil Epstein, George Mason University, Takumi Murayama, Purdue University, Karl Schwede, University of Utah, Kevin Tucker*, University of Illinois At Chicago. Test Ideals in some non- $F$-finite rings with Phantom F-trace. Preliminary report.
Test ideals were first introduced by Mel Hochster and Craig Huneke in their celebrated theory of tight closure, and are closely tied to the theory of Frobenius splittings. Test ideals have also found subsequent application far beyond their original scope in algebraic, analytic, and arithmetic geometry. Classically, test ideals were developed in the $F$-finite setting, where Frobenius is a finite morphism and key tools such as duality give rise to a robust theory with many important properties. For excellent non-$F$-finite rings, however, the situation is far less transparent and a number of open questions remain. In this talk, I will discuss recent work in progress on the properties of test ideals in some non- $F$-finite rings - those which are quotients of regular rings with "sufficiently many" Frobenius splittings in a precise sense. In particular, this applies to affinoid algebras over spherically complete fields.
(Received September 12, 2023)

1192-13-32763
Christine Berkesch*, University of Minnesota, C-Y. Jean Chan, Central Michigan University, Patricia Klein, Texas A\&M
University, Laura Felicia Matusevich, Texas A\&M University, Janet Page, North Dakota State University, Janet Cowden Vassilev, University of New Mexico. Differential operators of toric face rings.
Toric face rings, introduced by Stanley, are simultaneous generalizations of Stanley-Reisner rings and affine semigroup rings, among others. We use the combinatorics of the fan underlying these rings to inductively compute their rings of differential operators. Along the way, we discover a new differential characterization of the Gorenstein property for affine semigroup rings.
(Received September 12, 2023)

1192-13-33092
Maya Banks*, University of Wisconsin - Madison, Keller VandeBogert, University of Notre Dame. Differential Modules and Deformations of Free Complexes.
A differential module is a module equipped with a square-zero endomorphism. By developing a theory for deforming an arbitrary free complex into a differential module, we classify (up to quasi-isomorphism) the free differential modules whose homology is equal to a given module M . We use an iterative approach to parameterize the deformations and obstructions in terms of certain Ext groups, and apply this theory to study certain rigidity properties of free resolutions and related rank conjectures.
(Received September 13, 2023)

## 1192-13-33108

Maya Banks*, University of Wisconsin - Madison. Syzygies of Varieties in Weighted Projective Space.
A fundamental idea in algebraic geometry is that information about a variety in $\mathbb{P}^{n}$ is encoded in the syzygies of it's defining ideal in the graded polynomial ring $S=k\left[x_{0}, \ldots, x_{n}\right]$. Examples of this phenomenon include Castelnuovo-Mumford regularity, the linear strand of a free resolution, Green's $N_{p}$ conditions, and Mumford's famous 'quadric generation' result. For varieties embedded into weighted projective space we can find similar connections between the geometry of the embedding and the algebra of the defining ideal in a nonstandard graded polynomial ring. In this talk, I'll discuss weighted analogs of some of the classical ideas in the geometry of syzygies such as those mentioned above.
(Received September 13, 2023)

## 1192-13-33303

David Baron*, Williams College, Ammar Eltigani, Williams College, Anamaria Perez, Harvard College, Mayah Teplitskiy, Union College. Characterizations of local quasi-excellent integral domains..
Complete local rings are well understood due to a powerful theorem of Cohen's. If $M$ is the maximal ideal of a local ring, then its completion can be defined using the $M$-adic metric. Characterizing the relationship between a local ring and its completion allows us to deduce results about the original ring. In particular, we characterize the relationship between quasi-excellent integral domains and their completions. We prove that $T$, a complete local ring of characteristic 0 with maximal ideal $M$, is the completion of a local quasi-excellent integral domain if and only if no nonzero integer of $T$ is a zero-divisor and $T$ is reduced. We also provide necessary and sufficient conditions for countable local quasi-excellent domains. One notable application of our results is that there is no bound on how non-catenary a quasi-excellent integral domain can be, a result that was previously unknown.
(Received September 13, 2023)

Henry Jiang, MIT PRIMES-USA, Shihan Kanungo*, MIT PRIMES-USA, Hwisoo Kim*, Phillips Academy. A weaker notion of the finite factorization property.
An (additive) commutative monoid is called atomic if every given non-invertible element can be written as a sum of atoms (i.e. irreducible elements), in which case, such a sum is called a factorization of the given element. The number of atoms (counting repetitions) in the corresponding sum is called the length of the factorization. Following Geroldinger and Zhong, we say that an atomic monoid $M$ is a length-finite factorization monoid if each $b \in M$ has only finitely many factorizations of any prescribed length. An additive submonoid of $\backslash \mathrm{rr}_{\geq 0}$ is called a positive monoid. Factorizations in positive monoids have been actively studied in recent years. The main purpose of this paper is to give a better understanding of the non-unique factorization phenomenon in positive monoids through the lens of the length-finite factorization property. To do so, we identify a large class of positive monoids which satisfy the length-finite factorization property. Then we compare the length-finite factorization property to the bounded and the finite factorization properties, which are two properties that have been systematically investigated for more than thirty years.
(Received September 14, 2023)

## 1192-13-33522

David Baron*, Williams College, Ammar Eltigani*, Williams College, Anamaria Perez*, Harvard College, Mayah Teplitskiy*, Union College. Controlling Formal Fibers of Countably Many Principal Prime Ideals. Although Cohen's Structure Theorem fully characterizes complete local rings, the structure of an arbitrary local ring that is not complete remains elusive. A standard technique used to understand the prime spectrum of a local ring is to study the formal fiber rings with respect to its completion. There are many results in the literature that begin to take control of these formal fibers. Nonetheless, we are far from having full control over them. In this project, we extend some of these efforts by characterizing completions of local integral domains where we can control formal fibers of countably many principally generated ideals. We also provide necessary and sufficient conditions for completions of countable local domains, quasiexcellent local domains and excellent local domains with identical properties. (Received September 20, 2023)

## 1192-13-33734

Ethan Yiheng Liu*, The Harker School. Canonical Forms of Vanishing Polynomials over Finite Commutative Rings with Identity.
A vanishing polynomial over a commutative ring $R$ is a polynomial in $R[X]$ such that all elements of $R$ evaluate to zero. We note that all finite rings can be uniquely decomposed into rings of prime power order. Thus, in determining the vanishing polynomials over finite rings, it suffices to determine the vanishing polynomials over rings of orders $p, p^{2}, p^{3}, \ldots$ for prime $p$. In this work, we derive a canonical form for vanishing polynomials over rings of order $p^{3}$ and $p^{2}$. We prove that in each ring, each vanishing polynomial can be uniquely written in the canonical form, and each polynomial in the form vanishes. We integrate these results with existing canonical forms to obtain a complete classification consisting of canonical forms of vanishing polynomials over rings of order $p^{k}$ for prime $p$ and positive integer $k \leq 3$. We present a new approach that uses the vanishing properties of formal derivatives. We also provide a separate solution for the special case $p=2$. These approaches are applicable and valuable for future endeavors to rings of higher prime power order. We derive the number of distinct polynomial functions over each ring of order $p, p^{2}$, and $p^{3}$ using the canonical forms obtained. We also demonstrate the computational advantage of the canonical form in vanishing checks. In addition, we provide a canonical form of vanishing polynomials with vanishing derivatives over finite fields, and we establish an upper bound on the number of polynomial functions over any general finite ring.
(Received September 24, 2023)

1192-13-33829
Akilah Goldson*, Sarah Lawrence College, Erik M. Imathiu-Jones*, California Institute of Technology, Matilda LaFortune*, Scripps College, Eli Pregerson*, Harvey Mudd College. Trace Ideals Over Numerical Semigroup Rings. Numerical semigroups have been studied formally starting in the early 1900s by mathematician Ferdinand Georg Frobenius (1849-1917). Every numerical semigroup can be enriched to have a ring structure called a numerical semigroup ring. Numerical semigroup rings are a more recent object of study and have applications in algebraic geometry as quotients of toric ideals and appear frequently in combinatorial algebra. In this project we study semigroup rings using trace ideals. Trace ideals are a generalization of the trace property and provide a rich theory to detect properties of rings and modules. Our research studies the structure of trace ideals of numerical semigroup rings with few generators. We proved three propositions concerning trace ideals of monomial ideals of numerical semigroup rings.
(Received September 26, 2023)

## 1192-13-33870

Andrew Lin*, MIT PRIMES, Henrick Rabinovitz*, MIT PRIMES, Qiao Zhang*, MIT PRIMES. The Furstenberg property in Puiseux monoids.
Let $M$ be a commutative monoid. The monoid $M$ is called atomic if every non-invertible element of $M$ factors into atoms (i.e. irreducible elements), while $M$ is called a Furstenberg monoid if every non-invertible element of $M$ is divisible by an atom. Additive submonoids of $\mathbb{Q}$ consisting of nonnegative rationals are called Puiseux monoids, and their atomic structure has been actively studied during the past few years. The primary purpose of this paper is to investigate the property of being Furstenberg in the context of Puiseux monoids. In this direction, we consider some properties weaker than being Furstenberg, and then we connect these properties with some atomic results which have been already established for Puiseux monoids. (Received September 27, 2023)

A central problem in commutative algebra concerns understanding the structure of an ideal in a polynomial ring. Abstractly, the structure of an ideal can be represented by an object called a minimal resolution. There is no explicit method to obtain a minimal resolution in general, even for more fundamental classes of ideals. In this poster, we focus on resolutions of monomial ideals, one such simpler class. In particular, we investigate minimal free resolutions of matroid ideals using a recent construction called Barile-Macchia resolutions. Our main goal is to identify when Barile-Macchia resolutions provide minimal free resolutions of matroid ideals, such as those generated by lattice path matroids.
(Received September 29, 2023)

## 14 Algebraic geometry

## 1192-14-26642

Max Lieblich, University of Washington, Juan Salinas*, University of Washington. Moduli of Desargues Configurations. Preliminary report.
We study configurations of lines in projective space $\mathbb{P}^{2}$ over a field and determine properties of their stack-theoretic moduli space. The primary example will be that of a Desargues configuration: let $U_{D} \subset H i l b_{\mathbb{P}^{2}}$ denote the locus in the Hilbert scheme of (nondegenerate) Desargues configurations, then the quotient stack $\left[U_{D} / P G L_{3}\right]$ is a separated, reduced, regular, Noetherian Deligne-Mumford stack of dimension 3 with affine diagonal. We then generalize to planar configurations that arise as scheme-theoretic images of pinhole cameras $\mathbb{P}^{3} \rightarrow \mathbb{P}^{2}$ of a union of lines $X$ in $\mathbb{P}^{3}$, and determine properties of the a similar quotient stack $\left[U_{X} / P G L_{3}\right]$.
(Received August 02, 2023)

## 1192-14-27061

Johnny Dahl*, Lawrence University, Minjun Lee*, Lawrence University, Fadila Louleid*, Lawrence University, Julie Rana, Lawrence University, Liu Scott*, Lawrence University. Curve Your Enthusiasm: Coding Genus 2 Fibrations. Preliminary report.
Classification problems play a central role in algebraic geometry. A natural first step in classifying algebraic objects is to generate a plethora of examples. In this talk, we discuss code that we modified and developed in order to generate examples of objects known as KSBA-stable algebraic surfaces, particularly those that arise from genus 2 fibrations. Our results were obtained during a summer research project at Lawrence University, funded by an NSF-LEAPS MPS grant. This talk will be accessible to anyone who has seen some multivariable calculus. (Received August 09, 2023)

## 1192-14-27075

Erin Connelly, University of Washington, Timothy Duff, University of Washington, Jessie Loucks*, University of Washington. Algebra and Geometry of Camera Resectioning. Preliminary report. We study certain algebraic varieties associated with the camera resectioning problem. We characterize these resectioning varieties' multigraded vanishing ideals using Gröbner basis techniques. As an application, we derive and re-interpret celebrated results in computer vision due to Carlsson, Weinshall, and others related to camera-point duality. We also clarify some relationships between the classical problems of optimal resectioning and triangulation, state a conjectural formula for the Euclidean distance degree of a resectioning variety, and discuss how this conjecture relates to the recently-resolved multiview conjecture.
(Received August 09, 2023)

1192-14-27667
Danai Deligeorgaki*, KTH Royal Institute of Technology, Alex Markham, KTH Royal Institute of Technology, Pratik Misra KTH Royal Institute of Technology, Liam Solus, KTH Royal Institute of Technology. On the marginal independence structure of $D A G$ models.
We consider the problem of estimating the marginal independence structure of a DAG model from observational data. In order to so, we divide the space of directed acyclic graphs (DAGs) into certain equivalence classes, where each class can be represented by a unique undirected graph called the unconditional dependence graph. The unconditional dependence graphs satisfy certain graphical properties, namely having equal intersection and independence number. Using this observation, we can construct a Grobner basis for an associated toric ideal and define additional binomial relations to connect the space of unconditional dependence graphs. With these moves, we can implement a search algorithm, GrUES (Grobner-based Unconditional Equivalence Search), that estimates the conditional independence structure of the graphical model. The implementation shows that GrUES recovers the true marginal independence structure via a BIC-optimal or MAP estimate at a higher rate than simple independence tests while also yielding an estimate of the posterior. This is based on our preprint with Alex Markham, Pratik Misra and Liam Solus: arXiv:2210.00822.
(Received September 09, 2023)

## 1192-14-27744

Karl Christ*, University of Texas at Austin. Irreducibility of Severi varieties on toric surfaces.
Severi varieties parametrize curves of fixed geometric genus in a given linear system on a surface. In this talk, I will explain how tropical methods can be used to show the irreducibility of such spaces. This implies the irreducibility of other moduli spaces of curves, and relies on an explicit study of the tropicalization of families of curves. It is ongoing joint work with Ilya Tyomkin and Xiang He.
(Received August 18, 2023)

Given a tropical cycle X, one can talk about a notion of "tropical" vector bundles on X having tropical fibers. By restricting our attention to bounded rational sections of these bundles, one can develop a good notion of characteristic classes that behave as expected classically. We present further results on these characteristic classes and use these properties to prove a Porteous' formula for these bundles, which gives a determinantal expression of the fundamental class of degeneracy loci of a (tropical) bundle morphism in terms of their Chern classes.
(Received August 22, 2023)

## 1192-14-27991

Tafari Clarke-James*, The University of Washington. Homotopy Theory in Real Closed Spaces. Preliminary report. Let $A$ be a commutative ring with unity. Just as we form the geometric space spec $(A)$, whose points are prime ideals of $A$, we can form a space sper $(A)$, whose points are in some sense orderings of the ring $A$. We can glue these \underline\{real spectra\} to form \underline\{real closed spaces\}, which generalize semialgebraic sets as schemes generalize algebraic varieties. We review model-theoretic notions to work with (pro)constructible subsets of real closed spaces, and to develop a structure sheaf on a given real closed space. In sorting out the details for singular (co)homology real closed spaces, we noticed that homotopies are poorly behaved in this category and cannot lead to a meaningful singular (co)homology theory. My current work focuses on creating meaningful homotopies in the category of real closed spaces and fixing this deficiency. (Received August 23, 2023)

## 1192-14-27994

Tafari Clarke-James*, The University of Washington. No Homotopies in Real Closed Spaces. Preliminary report. Let $A$ be a commutative ring with unity. Just as we form the geometric space $\operatorname{spec}(A)$, whose points are prime ideals of $A$, we can form a space $\operatorname{sper}(A)$, whose points are in some sense orderings of the ring $A$. We can glue these \underline\{real spectra\} to form \underline\{real closed spaces\}, which generalize semialgebraic sets as schemes generalize algebraic varieties. We review model-theoretic notions to work with (pro)constructible subsets of real closed spaces, and to develop a structure sheaf on a given real closed space. In sorting out the details for singular (co)homology of real closed spaces, we noticed that homotopies are poorly behaved in this category and cannot lead to a meaningful singular (co)homology theory. My current work focuses on creating meaningful homotopies in the category of real closed spaces and fixing this deficiency. Joint work with Sándor Kovács.
(Received August 24, 2023)

1192-14-28616
Santiago Arango, Emory University, Changho Han, University of Waterloo, Oana Padurariu, Max-Planck-Institut für Mathematik, Sun Woo Park*, University of Wisconsin-Madison. Counting points on $x^{2}+y^{2}=z^{4}$ and 5-isogenies of elliptic curves over $\mathbb{Q}$. Preliminary report.
We count the number of $\mathbb{Q}$-rational points on the generalized Fermat stacky curves of signature $(2,2, r)$ of bounded height. As an application, we relate this counting problem to the enumeration, by naïve height, of the number of elliptic curves over $\mathbb{Q}$ admitting a 5 -isogeny. This is based on joint work with Santiago Arango-Piñeros, Changho Han, and Oana Padurariu. (Received September 01, 2023)

1192-14-28665
Cash Bortner, California State University, Stanislaus, Seth Sullivant*, North Carolina State University. Structural Identifiability of Series-Parallel LCR Systems.
We consider the identifiability problem for the parameters of series-parallel LCR circuit networks. We generalize results of Madhi-Meshkat-Sullivant from viscoelastic models used in biological modeling. We prove that for networks with only two classes of components (inductor-capacitor (LC), inductor-resistor (LR), and capacitor-resistor (RC)), the parameters are identifiable if and only if the number of non-monic coefficients of the constitutive equations equals the number of parameters. The notion of the "type" of the constitutive equations plays a key role in the identifiability of LC, LR, and RC networks. We also investigate the general series-parallel LCR circuits (with all three classes of components), and classify the types of constitutive equations that can arise, showing that there are 22 different types. However, we produce an example that shows that the basic notion of type that works to classify identifiability of two class networks is not sufficient to classify the identifiability of general series-parallel LCR circuits.
(Received September 01, 2023

## 1192-14-28714

Patricio Gallardo, UC Riverside, Jose Gonzalez, University of California, Riverside, Javier Gonzalez Anaya*, Harvey Mudd College, Evangelos Routis, University of Warwick (formerly). Higher-dimensional Losev-Manin spaces and their geometry. The classical Losev-Manin space can be interpreted as a toric compactification of the moduli space of $n$ points in the affine line modulo translation and scaling. Motivated by this, we study its higher-dimensional toric counterparts, which compactify the moduli space of $n$ distinct labeled points in affine space modulo translation and scaling. We show that these moduli spaces are a fibration over a product of projective spaces - with fibers isomorphic to the Losev-Manin space - and that they are isomorphic to the normalization of a Chow quotient. Moreover, we present a criterion to decide whether the blow-up of a toric variety along the closure of a subtorus is a Mori dream space. As an application, we demonstrate that a related generalization of the moduli space of pointed rational curves constructed by Chen, Gibney, and Krashen is not a Mori dream space when the number of points is at least nine, regardless of the dimension. This is a report on joint work with Patricio Gallardo, Javier Gonzalez Anaya and Evangelos Routis
(Received September 01, 2023)

Li, Brown University. Kapranov degrees.
The moduli space of stable rational curves with marked points has two distinguished families of maps: the forgetful maps, given by forgetting some of the markings, and the Kapranov maps, given by complete linear series of psi-classes. The collection of all these maps embeds the moduli space into a product of projective spaces. Kapranov degrees are the multidegrees of this embedding. Using a result from the theory of error correcting codes, we characterize the positivity of Kapranov degrees, which gives a new proof of Laman's theorem characterizing generically rigid graphs in the plane. (Received September 02, 2023)

## 1192-14-28916

Michael Cerchia, Emory University, Alexis Newton*, Emory University. Quartic torsion and quartic points on some rank 0 modular curves. Preliminary report.
We will present some new progress on the classification of the finite groups which appear as the torsion subgroup of $E(K)$ as $K$ ranges over quartic number fields. In particular, we concentrate on determining the quartic points on certain modular curves $X_{1}(N)$ for which the rank of $J_{0}(N)$ is zero.
(Received September 04, 2023)

1192-14-28968
Eran Assaf*, Dartmouth College, Shiva Chidambaram, Massachusetts Institute of Technology, Sam Frengley, University of Cambridge, Samuel Schiavone, MIT, Rachel Webb, UC Berkeley. Canonical Rings of Stacky Surfaces. Preliminary report. Generalizing the methods of Voight and Zureick-Brown for stacky curves, and works of Neves and Reid on algebraic surfaces, we describe generators and relations for canonical rings of some stacky surfaces, including explicit Gröbner bases, working in an algebro-geometric context. Sample applications include Hilbert modular surfaces.
(Received September 05, 2023)

Michael Montoro, SUNY at Buffalo, James Austin Myer*, CUNY Graduate Center, Raymond van Bommel, MIT. (Toward) an algorithm for resolution of singularities (for curves) in positive characteristic. Preliminary report.
Hironaka proved in 1964 that we can resolve singularities in characteristic 0, and there have since been substantial efforts to simplify the algorithm, e.g. by Kollar. The state of the art is an algorithm of Abramovich, Temkin, and Wlodarczyk, and independently, McQuillan, wherein the category of schemes is enlarged to that of stacks. To this day, no one knows whether we can resolve the singularities of an algebraic variety of any dimension defined over a field of positive characteristic. So, curiosity led the three of us to investigate what exactly goes awry in the case of positive characteristic, and we aimed to adjust the work of Abramovich et al. to handle the case of positive characteristic. For simplicity, we only studied curves. We hope this method will blaze a trail toward varieties of higher dimension, but this requires a closer look at the invariant in this case of positive characteristic. Even though this algorithm cannot handle any new cases, there is still a good chance that the algorithm will be more efficient than existing methods to resolve singularities. (Received September 05, 2023)

## 1192-14-29009

Andres Fernandez Herrero*, Columbia University. Relative etale slices and cohomology of moduli spaces.
If two varieties X and Y belong to a smooth projective family over a connected base, then X and Y share many geometric properties. For example, they are homeomorphic and their Hodge numbers agree. In moduli theory, it is often the case that moduli spaces are singular even though they come from GIT quotients of a smooth variety (this is usually the case when there are strictly semistable objects). In this talk, I will explain how to use local structure theorems for stacks to relate two such moduli spaces X and Y that fit into a proper family over a base, even when neither X nor Y are smooth. This is based on joint work with Mark Andrea de Cataldo and Andres Ibañez Nuñez.
(Received September 05, 2023)

1192-14-29011
Sarah Frei*, Dartmouth College, Katrina Honigs, Simon Fraser University, John M. Voight, Dartmouth. Abelian varieties whose torsion is not self-dual.
Given an abelian variety over a number field $K$, one can ask how the n-torsion subgroup compares to the n-torsion subgroup of the dual abelian variety. In general, these finite group schemes need not be isomorphic. In joint work with K. Honigs and J. Voight, we produce examples of abelian surfaces over Q where the p-torsion subgroup schemes are not isomorphic for small primes $p$. We do this by studying the action of the Galois group on the p-adic Tate module and its reduction modulo $p$. In this talk, I will discuss our construction as well as various applications, extensions, and computational questions. (Received September 05, 2023)

Juanita Duque-Rosero, Boston University, Christopher Keyes, King's College London, Andrew Kobin, Emory University, Manami Roy, Lafayette College, Soumya Sankar, Utrecht University, Yidi Wang*, University of Pennsylvania. Local-global principles for integral points on stacky curves. Preliminary report.
The primitive solutions of a generalized Fermat equation, i.e., Diophantine equations of the form $A x^{p}+B y^{q}=C z^{r}$, can be studied as integral points on certain stacky curves. In a paper by Bhargava and Poonen, an explicit example of such a curve of genus $1 / 2$ violating local-global principle for integral points was given. However, a general description of stacky curves failing the local-global principle is unknown. In this talk, we report on our progress on studying the local-global principle for certain classes of examples.
(Received September 05, 2023)

Alexandre Zotine*, Queen's University. Computing Higher Direct Images of Toric Morphisms.
Higher direct images are relative cohomological invariant generalizing sheaf cohomology. In practice, it is difficult to compute sheaf cohomology, let alone a relative generalization. By specializing to toric varieties, we obtain a combinatorial framework for computing these higher invariants. Given a surjective toric morphism $\pi: X \rightarrow Y$ between smooth projective toric varieties, we will discuss an algorithm implemented in Macaulay2 for computing the higher direct images of any line bundle over $X$. This is joint work with Mike Roth and Greg Smith.
(Received September 05, 2023)

1192-14-29221
Jason Lo*, California State University, Northridge. Bridgeland stability for line bundles, deformed Hermitian-Yang-Mills equations, and Catalan numbers.
In the 1980s, Donaldson and Uhlenbeck-Yau established a correspondence between the existence of solutions to the Hermitian-Yang-Mills equation associated to a vector bundle on a compact Kahler manifold, and the Mumford-Takemoto stability of the vector bundle. Motivated by the recent developments of deformed Hermitian-Yang-Mills (dHYM) equations and Bridgeland stability conditions, Collins-Yau asked if a similar relation holds between dHYM equations and the Bridgeland stability of line bundles. In this talk, we will present a partial result on elliptic surfaces, and explain how Catalan numbers arise in the process of solving an equation of Bridgeland stability conditions. This talk is based on joint work with Tristan Collins, Yun Shi, and Shing-Tung Yau, as well as joint work with Rimma Hamalainen and Edward Morales.
(Received September 06, 2023)

## 1192-14-29251

Hannah K. Larson, Harvard University and UC Berkeley, Ravi D Vakil*, Stanford University. Bott periodicity, algebrogeometrically. Preliminary report.
I will report on joint work with Hannah Larson, and hopefully joint work in progress with Jim Bryan, in which we try to make sense of Bott periodicity from a naively algebro-geometric point of view. (Received September 06, 2023)

## 1192-14-29305

Samir Canning*, ETH Zurich. Relationships between cycles on moduli spaces of curves and abelian varieties.
I will explain the existence of a non-tautological class on the moduli space of principally polarized abelian sixfolds. The proof uses the pullback to the moduli space of genus 6 curves of compact type via the Torelli morphism. In a surprising twist, the proof sheds light on Pixton's conjecture for the structure of the tautological ring of moduli spaces of curves. This is joint work with Dragos Oprea and Rahul Pandharipande. (Received September 06, 2023)

1192-14-29353
Michael Perlman*, University of Minnesota, Gregory G Smith, Queen's University. Cohomology of toric vector bundles. Preliminary report.
A toric vector bundle on a toric variety is a vector bundle endowed with a torus action compatible with the one on the underlying space. In contrast to the case of line bundles, these are not purely combinatorial objects, as they also depend on linear algebraic information known as the Klyachko data. We will discuss a new family of free complexes that may be used to approximate the cohomology of these bundles and all their twists by line bundles. This leads to computational efficiency and a new approach to vanishing theorems.
(Received September 06, 2023)

Nathan Pflueger, Amherst College, Noah Solomon*, Georgia Institute of Technology. Twice-Marked Banana Graphs \& BrillNoether Generality. Preliminary report.
Finite graphs model many of the properties enjoyed by algebraic curves, allowing the development, spearheaded by Baker and Norine, of a Brill-Noether theory for such graphs. A central question of this theory is: which graphs are Brill-Noether general?
In this talk we discuss a family of graphs known as banana graphs, with two marked vertices, through the lens of Hurwitz-BrillNoether theory. This talk will describe how properties of these graphs can be used to construct explicit new examples of finite graphs which are Brill-Noether general. These are the first such examples since the analysis of chains of loops by Cools, Draisma, Payne and Robeva. We also highlight that almost all banana graphs of genus at least 3 cannot be used for this purpose, due either to failure of a submodularity condition or to the presence of far too many inversions in certain permutations associated to divisors called transmission permutations. This is joint work with Nathan Pflueger. (Received September 06, 2023)

1192-14-29415
Ryan M. Shifler*, Salisbury University. Quantum cohomology determined with negative structure constants present. Let $I G:=I G(2,2 n+1)$ denote the odd symplectic Grassmannian of lines which is a horospherical variety of Picard rank 1. The quantum cohomology ring $Q H^{*}(I G)$ has negative structure constants. For $n \geq 3$, we give a positivity condition that implies the quantum cohomology ring $Q H^{*}(I G)$ is the only quantum deformation of the cohomology ring $\mathrm{H}^{*}(I G)$ up to the scaling of the quantum parameter. This is a modification of a conjecture by Fulton.
(Received September 06, 2023)

Isabel Vogt*, Brown University. Stability of Pushforwards.
Given a finite cover of curves $\alpha: X \rightarrow Y$, it is natural to consider the vector bundles on $Y$ that arise as pushforwards of vector bundles from $X$. In this talk we will consider the stability properties of the vector bundles $\alpha_{*} V$ that arise in this way. This talk is based on joint works with Izzet Coskun and Eric Larson.
(Received September 06, 2023)

1192-14-29517
Emily Clader*, San Francisco State University, Chiara Damiolini, University of Texas at Austin, Christopher Eur, Harvard University, Daoji Huang, ICERM, Shiyue Li, Brown University, Rohini Ramadas, University of Warwick. Permutohedral complexes, multimatroids, and curves with cyclic action.
Although the moduli space of genus-zero curves is not toric, it shares an intriguing amount of the combinatorial structure that a toric variety would enjoy. In fact, by adjusting the moduli problem slightly, one finds a moduli space that is indeed toric, known as Losev-Manin space. The associated polytope is the permutohedron, which has a wealth of other connections: notably, to the structure of the symmetric group and to the combinatorics of matroids. Baytrev and Blume generalized this story by constructing a type-B version of Losev-Manin space, whose associated polytope is a signed permutohedron that relates to the group of signed permutations as well as (via the work of Eur-Fink-Larson-Spink) to the combinatorics of "delta matroids." In joint work with C. Damiolini, D. Huang, S. Li, and R. Ramadas, we carry out the next stage of generalization, defining a family of moduli spaces of rational curves with $\mathbb{Z}_{r}$-action that can be encoded by a "permutohedral complex" for a more general complex reflection group, which specializes when $r=2$ to the type-B case. In ongoing work with the above authors and C. Eur, we also begin a study of how this permutohedral complex relates to the combinatorics of multimatroids.
(Received September 07, 2023)

1192-14-29545
Peter McDonald*, University of Utah. Multiplier ideals and klt singularities via (derived) splittings.
Thanks to the Direct Summand Theorem, splinter conditions have emerged as a way of studying singularities in commutative algebra and algebraic geometry. In characteristic zero, work of Kovács (2000) and Bhatt (2012) characterizes rational singularities as derived splinters. In this talk, I will present an analogous characterization of klt singularities by imposing additional conditions on the derived splinter property. This follows from a new characterization of the multiplier ideal, an object that measures the severity of the singularities of a variety, viewing it as a sum of trace ideals. This perspective also gives rise analogous description of the test ideal in characteristic as a corollary to a result of Epstein-Schwede (2014) and has applications to the study of closure operations in characteristic zero. (Received September 07, 2023)

1192-14-29559
Austen James, Flexport, Anthony Varilly-Alvarado*, Rice University. Probabilistic approaches to rational points on algebraic surfaces. Preliminary report.
The Brauer group of a del Pezzo over a number field is thought to govern the existence of rational points. A large piece of this group is determined by the Galois-module structure on the geometric Picard group of a surface. I will present work in progress that, give an equation for a cubic surface over the rationals, determines its algebraic Brauer group with a high degree of confidence. I will also indicate certificates for the probabilistic results. Technology permitting, I will show a live demo. This is joint work with Austen James.
(Received September 07, 2023)

## 1192-14-29592

Vance Blankers, Northeastern University, Sebastian Bozlee*, Tufts University. On moduli spaces of curves with colliding points.
The moduli space $\mathcal{M}_{g, n}$ of smooth algebraic curves of genus $g$ with $n$ distinct marked points is not compact. However, it admits many compactifications that are themselves moduli spaces, and it remains an outstanding problem in algebraic geometry to classify these modular compactifications. An important family of examples is that of the moduli spaces of "weighted pointed" curves constructed by Hassett, in which a vector of real numbers determines which of the $n$ marked points are permitted to come together. In this talk, I will present modular compactifications of $\mathcal{M}_{g, n}$ using a simplicial complex rather than vector of weights as an input. Not only do the resulting "simplicial" moduli spaces generalize Hassett's, but they also classify the modular compactifications coming from colliding markings. If time permits, we will also discuss how this idea can be combined with an earlier classification result to produce a classification of modular compactifications of $\mathcal{M}_{1, n}$ by Gorenstein curves admitting collisions.
(Received September 07, 2023)

1192-14-29596
Asher Auel, Dartmouth College, Richard Haburcak*, Dartmouth College, Hannah K. Larson, Harvard University and UC Berkeley. The Maximum Gonality in a Brill-Noether Locus.
Brill-Noether theory studies the question of when a general curve of genus $g$ admits a $g_{d^{\prime}}^{r}$ namely, a linear system of degree $d$ and rank $r$. A refined Brill-Noether theory aims to answer the question of when a curve with a $g_{d}^{r}$ admits another $g_{e}^{s}$. That is, we are interested in how Brill-Noether loci stratify the moduli space of curves $\mathcal{M}_{g}$. We study the restriction of Brill-Noether loci to the gonality stratification of the moduli space of curves of fixed genus. As an application, we give new proofs that BrillNoether loci with $\rho=-1$ and $\rho=-2$ have distinct support, and give lower bounds on when roughly half of the noncontainments of the Maximal Brill-Noether Loci Conjecture hold. This is joint work with Asher Auel and Hannah Larson. (Received September 07, 2023)

Anton Leykin*, Georgia Institute of Technology. Learning to find one good solution to a nonlinear problem. Preliminary report.
Suppose $f_{p}(x)$ is a parametric family of polynomial systems with real coefficients that has finitely many solutions for a generic value of the parameters $p$. Suppose $g=g(p)$ is a function defined on a (large) finite set $D$ of the parameter space such that $f_{p}(g(p))=0$. One may interpret this as a data set prescribing a unique "good" solution for values of parameters in a large (but finite) set $D$. We design a learning strategy for selecting a starting problem $p_{0} \in D$ such that $g\left(p_{0}\right)$ can be numerically continued to a solution of an arbitrary given problem of interest $p_{1}$ with high probability of obtaining the good solution $g\left(p_{1}\right)$ if $p_{1} \in D$. For the applications that we consider it is crucial that the resulting algorithm that finds $p_{0}$ and tracks a polynomial homotopy continuation path originating from $g\left(p_{0}\right)$ is fast, while the process of training of the neural network that is a part of this algorithm is allowed to be slow. This strategy was successfully applied to minimal problems for recovery of relative pose for two and three calibrated cameras (with Duff, Hruby, and Pajdla) and is a part of ongoing work on state estimation of a moving frequency source using Doppler effect (with Christian, Huang, and Manchini).
(Received September 07, 2023)

## 1192-14-29820

Holly Paige Chaos*, University of Vermont, Christelle Vincent, University of Vermont. Weierstrass points on Shimura curves. Preliminary report.
In this talk we introduce Weierstrass points on curves, which we can think of as the zeros of a certain Wronskian on the curve We then recall results of Rohrlich and Ahlgren-Ono that show that on the curve $X_{0}(p)$, for $p$ a prime, the reduction of this Wronskian modulo $p$ is a power of the Hasse invariant. Finally, we discuss the generalization of these results to the setting of Shimura curves parametrizing abelian surfaces with quaternionic multiplication.
(Received September 08, 2023)

1192-14-29840
Jae Hwang Lee*, Colorado State University. A Quantum $H^{*}(G)$-module via Quasimap Invariants. Preliminary report. For $X$ a smooth variety or Deligne-Mumford stack, the quantum cohomology ring $Q H^{*}(X)$ is a deformation of the usual cohomology ring $H^{*}(X)$, where the product structure is modified to incorporate quantum corrections. These correction terms are defined using Gromov-Witten invariants. For a GIT quotient $V / / G$, the cohomology ring $H^{*}(V / / G)$ also has the structure of a $H^{*}(G)$-module. In this work, we use quasimap invariants with light points and a modified version of the WDVV equation to define a quantum deformation of this $H^{*}(G)$-module structure. Using localization, we explicitly compute this structure for the Hirzebruch surface of type 2 . We conjecture that this new quantum module structure is isomorphic to the Batyrev ring when the target is a semipositive toric variety. (Received September 08, 2023)

## 1192-14-30014

Pieter Belmans, University of Luxembourg, Jishnu Bose, University of Southern California, Sarah Frei*, Dartmouth
College, Ben Gould, University of Michigan, James Hotchkiss, University of Michigan, Alicia Mae Lamarche, University of Utah, Jack Petok, Dartmouth College, Cristian Rodriguez Avila, University of Massachusetts, Amherst, Saket Shah University of Michigan. Fano schemes of $k$-planes on the intersection of two quadrics. Preliminary report.
The derived category of a smooth complete intersection $X$ of two quadrics in $\mathbb{P}^{2 g+1}$ is classically understood: it is built out of certain line bundles along with the derived category of a hyperelliptic curve $C$ which is naturally associated to $X$. Since much of the geometry of quadrics is controlled by linear subspaces, it is natural to also consider $F_{k}(X)$, the Fano scheme parametrizing projective $k$-planes on $X$. For $k=g-1, g-2, F_{k}(X)$ and its derived category again depend on $C$. In this talk I will report on joint work in progress with Belmans, Bose, Gould, Hotchkiss, Lamarche, Petok, Rodríguez Avila, and Shah, in which we work to understand the derived category of $F_{k}(X)$ for $\$ 1 \backslash l e q \mathrm{k}$ (Received September 08, 2023)

1192-14-30019
Anastasia Nathanson, University of Minnesota, Lauren Nowak, University of Washington, Patrick Vincent O'Melveny, University of Washington, Dustin Ross*, San Francisco State University. New perspectives on tropical intersection theory. Motivated by moduli spaces of rational pointed curves and their tropical counterparts, recent developments have led to new perspectives on tropical intersection theory that allow us to import tools from toric and convex geometry into the picture. In this talk, I'll discuss some of these recent developments and applications.
(Received September 08, 2023)

1192-14-30082
Karina Elle Cho*, Stony Brook University. Approaching the Defining Ideal through Cones. Preliminary report.
Given a smooth projective variety, how can we geometrically construct generators of its ideal? One way is to consider cones over the variety, and looking at the ideal generated by these cones. This cone ideal may not capture all of the polynomials through the variety, but in sufficiently high degree, the cone ideal and the defining ideal agree. I will describe my work in studying the degree in which cones generate the defining ideal.
(Received September 08, 2023)
have dimension $n$; and their multiplicities are given as a polynomial of the parts of the corresponding partitions.
(Received September 09, 2023)

1192-14-30216
Giovanni Inchiostro*, University of Washington. Compactifications of moduli of plane curves via birational geometry. One way to compactify moduli of curves is via fixing an embedding of a curve into a surface, and study how the pair consisting of the surface and the curve degenerates. I will discuss some new compactifications of the moduli space of pairs $\left(\mathbb{P}^{2}, C\right)$ where $C$ is a smooth planar curve. Our compactification interpolates between the more classical KSBA and K-stability compactifications. This is joint work with K. Ascher, D. Bejleri, H. Blum, K. DeVleming, Y. Liu, X. Wang (Received September 09, 2023)

1192-14-30259
Hans Schoutens*, CUNY. Geometric tools for the decidability of the existential theory of $F_{p}[[t]]$..
I will give a brief survey how tools from algebraic geometry can be used in finding solutions to Diophantine equations over $F_{p}[[t]]$ and similar rings. These tools include arc spaces, motives and resolution of singularities. This approach yields the definability of the existential theory of $F_{p}[[t]]$ (in the ring language with a constant for $t$ ) contingent upon the validity of resolution of singularities (Denef-Schoutens). Anscombe-Fehm proved a weaker result using model-theoretic tools and together with Dittmann, they gave a proof assuming only the weaker 'local uniformization conjecture'
(Received September 09, 2023)

1192-14-30285
Silviana Amethyst, University of Wisconsin - Eau Claire, Morgan Fiebig, University of Wisconsin-Eau Claire, Caden Danger Joergens*, University of Wisconsin-Eau Claire, Briar Weston*, University of Wisconsin-Eau Claire. Improvements to Snap-Together Algebraic Surfaces. Preliminary report.
This ongoing mathematical art project aims toward assemblable and 3d printable algebraic surfaces using a plug and socket modality. Technical and theoretical barriers include internal geometry due to solidification at singularities, computation of direction for alignment of plugs and sockets, and quality of the triangulation. This poster explores an alternative to offsetting or solidification by using methods that preserve the structure of singularities. Plugs and sockets are improved by computing the direction using bottlenecks and the medial axis. We improve the smoothness of the triangulation using a remeshing algorithm.
(Received September 09, 2023)

1192-14-30288
Yuxin Lin*, California Institute of Technology, Elena Mantovan, California Institute of Technology, Deepesh Singhal, University of California, Irvine. Abelian covers of $\mathbb{P}^{1}$ of p-ordinary Ekedahl-Oort type.
Given a family of abelian covers of $\mathbb{P}^{1}$ and a prime $p$ of good reduction, by considering the associated Deligne-Mostow Shimura variety, we obtain lower bounds for the Ekedahl-Oort type, and the Newton polygon, at $p$ of the curves in the family. In this paper, we investigate whether such lower bounds are sharp. In particular, we prove sharpeness when the number of branching points is at most five and $p$ sufficiently large. Our result is a generalization under stricter assumptions of [1, Theorem 6.1] by Bouw, which proves the analogous statement for the $p$-rank, and it relies on the notion of Hasse-Witt triple introduced by Moonen in [2].
(Received September 09, 2023)

1192-14-30324
Tyler Campos, Yale, Makenna Greenwalt*, University of Oregon, Lisa Liu, Stanford University. A Canonical Coordinate System On Curves Defined Over $\overline{\mathbb{Q}}$.
Previous work by Penkava and Mulase developed a canonical coordinate system for curves defined over $\overline{\mathbb{Q}}$ using the Strebel differential of a Riemann surface $C$. In this talk, we incorporate the result that every curve defined over $\mathbb{Q}$ admits a facebipartite triangulation into an alternative construction of the canonical coordinate system. In these new "galaxy coordinates," there is a simple local expression for a meromorphic 1 -form which extends to a globally defined meromorphic 1 -form on $C$. In the following presentation, this 1 -form will be used to construct connections on curves defined over $\overline{\mathbb{Q}}$.
(Received September 11, 2023)

1192-14-30434
Shiyue Li*, Brown University. Multimatroids and rational curves with cyclic action. Preliminary report.
I will describe moduli spaces of rational stable curves with finite cyclic action. The intersection theory of these moduli spaces is governed by the combinatorics of multimatroids. These multimatroids, introduced by Bouchet, generalize matroids and delta-matroids. The geometry of these moduli spaces in turn informs us about multimatroids. Based on past and present joint works with Emily Clader, Chiara Damiolini, Christopher Eur, Daoji Huang, and Rohini Ramadas. (Received September 10, 2023)

1192-14-30457
Courtney George*, University of California, Riverside, Christopher Manon, University of Kentucky. Combinatorial Confirmation of Mori Dream Spaces.
A projective, normal variety, X , is called a Mori dream space (MDS) when its Cox ring, Cox(X), is finitely generated. While Mori dream spaces exhibit nice behavior, no complete classification of them is known. Due to their combinatorial nature, one
natural class of candidates for Mori dream spaces is projectivized toric vector bundles. In 2012, Jose Gonzalez proved that all rank-2 projectivized toric vector bundles are MDS but complete conditions on rank-r bundles being MDS are still unknown. Kaveh and Manon (2019) gave a combinatorial description of toric vector bundles that we use to describe multiple families of rank-r toric vector bundles that are MDS. We conclude with computational examples and directions for future research, including an algorithmic implementation.
(Received September 10, 2023)

## 1192-14-30631

Sumayya Mohsin, Lahore University of Management Sciences, Shaheen Nazir, Lahore University of Management Sciences, Lahore, Muhammad Imran Qureshi*, King Fahd University of Petroleum and Minerals, Saudi Arabia. Constructions and Deformations of Calabi-Yau 3-folds in codimension 4.
Over the last 4 decades, the study of Calabi-Yau 3-folds has been an important area of research in algebraic geometry and string theory, paving the way for con- nections between the two areas. Over the last 4 decades, the study of Calabi-Yau 3-folds has been an important area of research in algebraic geometry and string theory, paving the way for connections between the two areas. We present families of Calabi-Yau 3-folds with isolated orbifold points in codimension 4 that can be described as $2 \times 2$ minors of generic rank 3 matrix. We show how to use Tom and Jerry, two Pfaffian matrix formats, to study the existence of further deformation families of the CY 3-folds lying in the same Hilbert scheme. We compute the Hodge numbers of these Calabi-Yau 3 -folds and the corresponding manifolds obtained from their crepant resolution. We find a Calabi-Yau manifold with a pair of Hodge numbers that does not appear in the famously known list of 30108 distinct Hodge pairs of Kruzer-Skarke, in the list of 7890 distinct Hodge pairs corresponding to complete intersections in the product of projective spaces and in Hodge paris obtained from Calabi-Yau 3-folds having low codimension embeddings in weighted projective spaces, giving it a distinctive position in the CY landscape.
(Received September 25, 2023)

## 1192-14-30632

Martin Helmer*, North Carolina State University. Effective Whitney Stratification of Real Algebraic Varieties.
We describe an algorithm to compute Whitney stratifications of real algebraic varieties. The basic idea is to first stratify the complexified version of the given real variety using conormal techniques, and then to show that the resulting stratifications admit a description using only real polynomials. This method also extends to stratification problems involving certain basic semialgebraic sets as well as certain algebraic maps. One of the map stratification algorithms described also yields a new method for solving the real root classification problem. This is joint work with Vidit Nanda (University of Oxford). (Received September 10, 2023)

## 1192-14-30756

Stephen Pietromonaco*, University of Michigan. Curve counting and arithmetic for banana nano-manifolds. In this talk I will introduce four new Calabi-Yau threefolds which we call "banana nano-manifolds." These are rigid threefolds fibered by Abelian surfaces, and whose singular fibers contain a "banana configuration" of rational curves. In the context of counting curves, we compute the genus $g \geq 2$ Gromov-Witten potentials in three of the four independent curve classes, and show that the resulting series is the Fourier expansion of a Siegel modular form of weight $2 g-2$ for a certain discrete subgroup of $\mathrm{Sp}_{4}(\mathbb{R})$. This is consistent with general expectations from mirror symmetry. The banana nano-manifolds are rigid Calabi-Yau threefolds, and consequently also have interesting arithmetic structure. In each case, we compute the weight four cusp modular form whose Fourier coefficients effectively encode the point counts over finite fields. This is joint work with Jim Bryan and Mike Roth.
(Received September 11, 2023)

## 1192-14-30942

Arthur Baragar*, University of Nevada, Las Vegas. Circle packings and the Markoff equation.
There have been a number of results concerning the Apollonian circle packing (and its variations) that are strikingly similar to results concerning the Markoff equation and its variations. In this talk, we explore some of the connections between these seemingly disparate topics.
(Received September 11, 2023)

## 1192-14-31037

Dante Luber*, TU Berlin. Singular Initial degenerations of Grassmannian via matroid subdivisions of hypersimplices We discuss matroid representations and initial degenerations of the Grassmannian. We show that for $n<12$, all $\mathbb{C}$ representation spaces of $(3, n)$-matroids are smooth, and that singular representation spaces exist for $n \geq 12$. As an application of these results, we show that all initial degenerations of the $(3,8)$-Grassmannian are smooth, and that singular initial degenerations exist for $(3, n)$-Grassmannian where $n \geq 12$. Explanations will be provided as to why our results do not cover the cases where $n \in\{9,10,11\}$. We exploit the containment of the tropical Grassmannian in the Dressian to study initial degenerations. That is, a nonempty initial degeneration of $\operatorname{Gr}(d, n)$ is induced by a point $w$ in the tropical Grassmannian $\operatorname{TGr}(d, n)$. On the combinatorial side, this point $w$ induces a regular matroidal subdivision of the ( $d, n$ )-hypersimplex. Hence, we use properties of such a subdivision to study the corresponding initial degeneration. Our techniques employ tropical, algebraic, and polyhedral geometry, as well as commutative algebra and computation using OSCAR and Polymake. This is joint work with Daniel Corey. (Received September 11, 2023)

Integer partitions are ubiquitous in mathematics. I will report on a link between an algebro-geometric invariant of singularities of algebraic varieties and the theory of integer partitions. This link provides algebro-geometric interpretations of some identities in the theory of partitions that were imagined by Ramanujan and allows one to guess and prove new partition identities.
(Received September 11, 2023)

## 1192-14-31189

Aaron Bertram, University of Utah, Alicia Mae Lamarche*, University of Utah. Root systems, moduli interpretations, and their derived categories. Preliminary report.
Based on ongoing work with Aaron Bertram, we will explore properties of toric varieties constructed from root systems, their moduli space interpretations due to Losev-Manin and Batyrev-Blume, and decompositions of their bounded derived categories of coherent sheaves.
(Received September 11, 2023)

## 1192-14-31281

Aaron Pixton*, University of Michigan. Piecewise polynomial formulas for tautological classes.
The tautological ring of the moduli space of stable curves is a subring of the Chow ring consisting of the cycles produced by forgetful and gluing morphisms. Many geometrically natural classes admit formulas in terms of these tautological classes. I will describe a recent perspective on some of these tautological formulas, which is that they can often be interpreted nicely in terms of piecewise polynomials on the dual cone complex to the moduli space. I will survey some of these piecewise polynomial formulas
(Received September 11, 2023)

1192-14-31289
Erin Connelly, University of Washington, Timothy Duff, University of Washington, Jessie Loucks*, University of Washington. Camera Resectioning and Carlsson-Weinshall Duality. Preliminary report.
We study certain algebraic varieties associated with the camera resectioning problem. We characterize these resectioning varieties' multigraded vanishing ideals using Gröbner basis techniques. As an application, we derive and re-interpret celebrated results in computer vision due to Carlsson, Weinshall, and others related to camera-point duality. (Received September 11, 2023)

1192-14-31295
Patricio Gallardo*, UC Riverside, Luca Schaffler, Università Roma Tre. Almost Toric Compactifications of the Moduli Space of Lines in the Plane.
Thanks to the work of V. Alexeev, we know there exists a projective toric compactification of the moduli space of lines in the plane, with the associated polytope being a fiber polytope. Moreover, this compactification can be interpreted as a moduli of weighted stable pairs. This talk will discuss our joint work with L. Schaffler regarding a non-toric compactification obtained by performing a blow-up of such a toric variety. As a direct application, we will demonstrate that any $\mathbb{Q}$-factorialization of this blown-up space is not a Mori dream space when considering a sufficient number of lines.
(Received September 11, 2023)

1192-14-31304
Erin Connelly, University of Washington, Timothy Duff, University of Washington, Jessie Loucks-Tavitas*, University of Washington. Algebraic Vision: A Gentle Introduction. Preliminary report.
Algebraic vision, lying in the intersection of computer vision and projective geometry, is the study of three-dimensional objects being photographed by multiple pinhole cameras. Two natural questions arise: (1) Given a 3-D object or scene and multiple images of it, can we determine the (relative) positions of the cameras in the world? And, (2) given multiple images as well as (relative) camera locations, can we reconstruct the scene or object being photographed? We will discuss certain algebraic varieties, called the multiview and resectioning varieties, that arise from these questions. We will summarize recent research (joint with Erin Connelly and Tim Duff) regarding the resectioning variety. Time-permitting, we will also explore potential future directions for all levels of mathematical maturity. (Received September 11, 2023)

## 1192-14-31315

Sameer Agarwal, Google, Erin Connelly*, University of Washington, Alperen Ergur, University of Texas at San Antonio, Rekha Rachel Thomas, University of Washington, Cynthia Vinzant, University of Washington. The Geometry of Rank Drop in Two-View Image Reconstruction.
Given $2 \leq k \leq 9$ points $\left(x_{i}, y_{i}\right) \in \mathbb{P}^{2} \times \mathbb{P}^{2}$ we characterize rank deficiency of the $k \times 9$ matrix $Z$ with rows $x_{i}^{\top} \otimes y_{i}^{\top}$ in terms of the geometry of the point sets $\left\{x_{i}\right\}$ and $\left\{y_{i}\right\}$. For $k=6$ the rank drop locus is captured by the classical theory of cubic surfaces; for $k=7$ and $k=8$ the rank drop locus is described via the interplay of quadric surfaces, cubic curves, and Cremona transformations.
(Received September 11, 2023)

## 1192-14-31507

Juliette Emmy Bruce*, University of California, Berkeley. Matroids and the Moduli Space of Abelian Varieties. Inspired by recent work calculating the top weight cohomology of the moduli space $\mathcal{A}_{g}$ of principally polarized abelian varieties of dimension $g$ for small values of $g$ I will discuss a connection between matroids and compactifications of $\mathcal{A}_{g}$ that is
analogous to the connection between graphs and compactifications of the moduli space of curves. Given time I will also discuss recent work computing the homology of various matroid complexes.
(Received September 11, 2023)

## 1192-14-31529

Saber Ahmed*, Hamilton College, Mboyo Esole, Northeastern University. Hyperplane Arrangement and Flop Transitions of $E_{6}$-models. Preliminary report.
Elliptic fibrations are fascinating geometries that exist at the intersection of algebraic geometry, number theory, representation theory, and geometry of string theory. Recently, there have been huge advancements in understanding the geometry of crepant resolutions of Weierstrass models using ideas from the physics community. This talk will discuss the geometry of elliptic fibrations of type $\mathrm{E}_{6}$ characterized by a generic fiber of Kodaira type IV* related to Step 8 of Tate's algorithm. Their Weierstrass models generically have 14 possible distinct crepant resolutions that can flop to each other following a graph coinciding with the incidence graph of the chambers of the hyperplane arrangement $\mathrm{I}\left(\mathrm{E}_{6}, \mathbf{2 7}\right)$ defined by the kernels of the weights of one of the minuscule representations of $\mathrm{E}_{6}$ restricted to the open dual fundamental Weyl chamber. We will also explore the fiber geometry of such crepant resolutions, providing examples of new non-Kodaira fibers. (Received September 11, 2023)

## 1192-14-31538

Chiang-Heng Chien*, Brown University, Benjamin Kimia, Brown University. When Homotopy Continuation Meets Computer Vision: Making Multiview Geometry Tasks Practical using GPU-HC.
Systems of polynomial equations arise frequently in computer vision tasks. Especially in multiview geometry of structure from motion (SfM) pipelines, efficiency is critical as a polynomial system is typically solved repeatedly under a RANSAC scheme for robust estimation. For small scale problems, Gröbner basis or elimination template methods are widely used as they are efficient solvers. Large scale, high complexity problems, on the other hand, have evaded practical solutions from these methods due to large number of variables and roots. Such limitations have encouraged a mindset to eliminate variables, reducing the problem scale into a solvable range. This is not always possible, and even when it is, stability issue arise in highdegree simplified systems. Homotopy continuation (HC) is able to solve very complex problems while avoiding stability issue; however, it is known to have high computational cost, discouraging many to adopt it as a practical solver. In this talk, we introduce GPU-HC: a GPU design and implementation of an HC solver shifting the paradigm from that of eliminating variables to arrive at a reduced systems of much higher degree with stability risks, to the mindset of directly solving the original large, low-degree system efficiently. GPU-HC can be generically applied to a range of computer vision problems. It has also been successfully applied to old problems that were previously unsolvable because of the large problem scale. Notable examples such as trifocal relative points from points at lines and three view relative pose of a generalized camera are showcases to present the power of GPU-HC. It opens a door for easy problem formulation and solutions of a range of computer vision tasks, making them practical in various real-life applications.
(Received September 12, 2023)

1192-14-31575
Javier Gonzalez Anaya, Harvey Mudd College, Brett Nasserden*, Western University, Alexandre Zotine, Queen's University. Surjective endomorphisms of projectivized toric bundles.. Preliminary report. The study of the automorphism group of a projective variety provides insight into the symmetries of that variety. However, when we broaden our scope to all surjective endomorphisms $f: X \rightarrow X$ we uncover an interesting relationship: the existence of a surjective endomorphism not being an automorphism often implies special geometric characteristics for $X$. For example, we obtain a classification of smooth projective curves by their surjective endomorphisms: genus zero curves have ramified surjective endomorphisms of a degree higher than one, while genus one curves possess unramified surjective endomorphisms of higher than one. Finally, curves of genus at least 2 lack any surjective endomorphism exceeding a degree of one. Nakayama (2002) proved that the smooth rational projective surfaces with a non-isomorphic surjective endomorphism are precisely the smooth toric surfaces. However, in higher dimensions, the situation becomes more complex. This leads us to explore the projectivizations of toric vector bundles. Fix a smooth toric variety $X_{\Sigma}$ and a toric vector bundle $E$ over $X_{\Sigma}$. The projective bundle $\mathbb{P}(E)$ is not, in general, a toric variety, but admits a torus action that makes the bundle projection $\pi: \mathbb{P}(E) \rightarrow X_{\Sigma}$ equivariant. In this talk, we will study equivariant surjective morphisms $f: \mathbb{P}(E) \rightarrow \mathbb{P}(E)$ such that $\pi \circ f=g \circ \pi$ for some surjective morphism $g: X_{\Sigma} \rightarrow X_{\Sigma}$. As an application of our results, we will prove the Kawaguchi-Silverman conjecture for equivariant surjective endomorphisms of projective toric bundles. Using our insight from the equivariant situation, we will then generalize our method to certain projective bundles on (not necessarily toric) smooth rationally connected varieties. (Received September 11, 2023)

## 1192-14-31580

Thomas Yahl*, University of Wisconsin - Madison. The Unbalanced Procrustes Problem and Algebraic Optimization. Given a set of vectors, the unbalanced Procrustes problem concerns finding a set of orthonormal vectors from a lower dimensional space that best approximates the given vectors. We explore the algebraic properties of the unbalanced Procrustes problem by studying the critical points of this optimization problem. In simple cases, the unbalanced Procrustes problem reduces to a closest point problem to the Stiefel manifold where the critical points and solutions may be computed explicitly via singular value decomposition. More generally, numerical methods are used to compute solutions. We provide a new numerical algorithm for computing solutions to the unbalanced Procrustes problem based off our analysis of the problem and compare it to existing algorithms.
(Received September 11, 2023)

1192-14-31593
Claudiu Raicu, University of Notre Dame, Keller VandeBogert*, University of Notre Dame. Cohomological Stability for Polynomial Functors on Projective Space.
Schur functors are particularly well-behaved polynomial functors on the category of vector spaces, and any Schur functor
applied to the cotangent bundle on projective space may be realized as the pushforward of a line bundle on a flag variety. However, one can ask the question: is there a notion of stability for arbitrary polynomial functors applied to the cotangent bundle, similar to that witnessed for the cohomology of line bundles on flag varieties? The answer to this question is yes, and in this talk I will make precise this stability statement and some of its consequences. This stable cohomology satisfies many desirable properties such as a Künneth formula and invariance under Frobenius, and in many cases can be computed explicitly through the use of well-chosen resolutions. This is joint work with Claudiu Raicu.
(Received September 11, 2023)

## 1192-14-31596

Adam French*, Stony Brook University, Raasikh Shahid*, Hunter College, Kevin Tang*, Tufts University. Combinatorics of Extremal Assignments in Genus 1. Preliminary report.
A large class of modular compactifications of the moduli space $\mathcal{M}_{g, n}$ of smooth $n$-pointed algebraic curves of genus $g$ are indexed by "extremal assignments," large collections of combinatorial data identified by Smyth. We significantly reduce the combinatorial data involved in the genus one case, identifying a bijection of extremal assignments with certain families of set partitions on $n+2$ labels. A key insight is the reduction to earlier results of Moon et al in genus zero by means of a gluing functor $\mathcal{F}: \overline{\mathcal{M}}_{0, n+2} \rightarrow \overline{\mathcal{M}}_{1, n}$. In the future, we hope to extend this technique to arbitrary genus, and investigate the compactifications generated by certain classes of extremal assignments, along with possible algorithms to calculate these assignments.
(Received September 11, 2023)

1192-14-31605
Jordy Lopez Garcia*, Texas A\&M University. Operators, Polytopes and Toric Geometry. Preliminary report.
Bloch varieties arise in the spectral theory of Schrödinger operators on periodic graphs. For the square lattice, Bättig constructed a toric compactification of the Bloch variety and classified its singularities. Additionally, he constructed an infinitedimensional vector bundle over this compactified variety and rewrote the original spectral problem on different coordinate patches. In this talk, we give recent developments of this bundle construction that apply to more general periodic graphs. This is joint work with Matt Faust and Frank Sottile.
(Received September 11, 2023)

1192-14-31655
Eric Larson*, Brown University. The Minimal Resolution Conjecture for points on general Brill-Noether curves.
Let $X$ be a projective variety, with Hilbert polynomial $P_{X}(t)$, and let $\Gamma$ be a general subset of at least $P_{X}(\operatorname{reg}(X))$ points on $X$ . One of the most fundamental invariants of any projective variety is the shape of its minimal free resolution, i.e., its Betti table; the natural conjecture, known as the Minimal Resolution Conjecture, predicts the Betti table of Gamma in terms of the Betti table of $X$. When $X$ is a general Brill-Noether curve of degree $d$ and genus $g$ in $\mathbb{P}^{r}$, many counterexamples are known, including asymptotically $100 \%$ of the cases when $d<2 r$. In this talk, I will describe recent joint work with G. Farkas, in which we prove this conjecture when $d \geq 2 r$.
(Received September 11, 2023)

## 1192-14-31739

Andrew R Stout*, Borough of Manhattan Community College, CUNY. The Jet Operator: From Local to Global Deformations. We consider jet schemes of flat deformations over $k[t] /\left(t^{n+1}\right)$. It is shown that, for locally complete intersection varieties, these spaces can be viewed as global flat deformations over $\mathbb{A}_{k}^{n}$ of the classical jet scheme of order $n$. This has natural implications for defining a motivic volume and developing an analogue of the motivic Milnor fiber. These results were obtained by further developing previous work done by the author on Auto Arc Spaces (a generalized jet scheme), and it is directly related to questions concerning the non-reduced structure of jets of $\operatorname{Spec}\left(k[t] /\left(t^{n+1}\right)\right)$.
(Received September 12, 2023)

## 1192-14-31872

Claudiu Raicu*, University of Notre Dame, Keller VandeBogert, University of Notre Dame. Stable sheaf cohomology on flag varieties.
A fundamental problem at the confluence of algebraic geometry, commutative algebra and representation theory is to understand the structure and vanishing behavior of the cohomology of line bundles on flag varieties. Over fields of characteristic zero, this is the content of the Borel-Weil-Bott theorem and is well-understood, but in positive characteristic it remains wide open, despite important progress over the years. By embedding smaller flag varieties as Schubert subvarieties in larger ones, one can compare cohomology groups on different spaces and study their eventual asymptotic behavior. In this context I will describe a sharp stabilization result, and discuss some consequences and illustrative examples. Joint work with Keller VandeBogert.
(Received September 12, 2023)

1192-14-31963
Jose Israel Rodriguez*, University of Wisconsin Madison. Algebraic degrees in optimization: from triangulation with multiview varieties to Procrustes problems. Preliminary report.
Algebraic degrees in optimization: from triangulation with multiview varieties to Procrustes problems The algebraic degree of an optimization problem is a well studied topic in applied algebraic geometry. It appears in statistics, semidefinite programming, computer vision and physics. Geometrically, these degrees are the degree of a projection map and can be computed by determining the fiber over a general point. It counts the number of solutions to a particular system of polynomial equations. With my collaborators, we have studied the degree of maximum likelihood estimation (ML degree), nearest point problems (ED degree) and linear optimization on varieties (LO degrees) in terms of Euler characteristics and Chern-Mather
classes with results for multiview varieties. In this talk, we recall these techniques in the context of the unbalanced Procrustes problem (UPP) to introduce the algebraic degree of UPPs (joint work with Thomas Yahl). - ize J. I. Rodriguez and B. Wang. The maximum likelihood degree of mixtures of independence models. SIAM J. Appl. Algebra Geom., 1(1):484-506, 2017. L. G. Maxim, J. I. Rodriguez, and B. Wang, "Euclidean distance degree of the multiview variety," SIAM J. Appl. Algebra Geom., vol. 4, no. 1, pp. 28-48, 2020. L. G. Maxim, J.I. Rodriguez, B. Wang, L. Wu, "Linear optimization on varieties and Chern-Mather classes," arXiv:2208.09073, 2022. \enditemize
(Received September 12, 2023)


## 1192-14-31980

William Balderrama, University of Virginia, Kyle M Ormsby*, Reed College, J.D. Quigley, University of Virginia. A motivic analogue of the $K(1)$-local sphere spectrum.
We identify the motivic $K G L / 2$-local sphere as the fiber of $\psi^{3}-1$ on $(2, \eta)$-completed Hermitian $K$-theory, over any base scheme containing $1 / 2$. This is a motivic analogue of the classical $K(1)$-local sphere, and extends to a description of the $K G L / 2$-localization of any cellular motivic spectrum. Our proof relies on a novel conservativity argument that should be of broad utility in stable motivic homotopy theory.
(Received September 12, 2023)

## 1192-14-32018

Mario Morán Cañon*, University of Oklahoma, Julien Sebag, Universite de Rennes. Nilpotency in the arc scheme. The arc and jet schemes of an integral variety defined over a field of characteristic zero are not necessarily reduced. Various results, e.g., those by M. Mustaţă or J. Sebag, emphasize that the existence of nontrivial nilpotent functions of the arc or jet scheme is related to the singularities of the base variety, and can be read on the topology of the jet schemes associated to the same variety. Furthermore, nilpotency on arc scheme is also connected to other areas, such as differential operators or vertex algebras. After providing a brief overview of the main results concerning nilpotency in the arc scheme, we will present a description of the nilpotent functions in terms of a smooth birational model of the base variety, such as a resolution of singularities. We will also explain how this description provides effective algorithms for computing nilpotent functions of the arc scheme that live in the jet scheme of a given level.
(Received September 12, 2023)

## 1192-14-32032

Michael Brown, Auburn University, Mahrud Sayrafi*, University of Minnesota, Twin Cities. Short resolutions of the diagonal and a Horrocks-type splitting criterion for toric varieties of Picard rank 2.
In 1964, Horrocks proved that a vector bundle on a projective space splits as a sum of line bundles if and only if it has no intermediate cohomology. Then in 2015, Eisenbud-Erman-Schreyer used the BGG correspondence to prove a version of this criterion under an additional hypothesis for products of projective spaces. This talk is about the key ingredient for proving a Horrocks-type splitting criterion for vector bundles over a smooth projective toric variety $X$ of Picard rank 2: a short resolution of the diagonal sheaf on $X \times X$ consisting of finite direct sums of line bundles. I'll discuss the construction via a variant of Weyman's "geometric technique," as well as additional properties and applications. (Received September 12, 2023)

1192-14-32042
Jane Ivy Coons, University of Oxford, St. Johns College, Mark Curiel*, University of Hawai'i at Manoa, Elizabeth Gross, University of Hawai`i at Manoa. Mixed volumes of networks with binomial steady-states.. Preliminary report.
Mass-action kinetics on a chemical reaction network gives rise to a polynomial dynamical system. The number of complex solutions to this system is called the steady-state degree - it is a measure of the algebraic complexity of solving the system. The steady-state degree is difficult to compute in general, however the mixed volume of the system can provide a decent upper bound. We exploit the geometry of partitionable binomial networks to give a method for computing the mixed volume via a matrix determinant.
(Received September 12, 2023)

## 1192-14-32191

Peter Haine*, UC Berkeley. Spectral weight filtrations.
This talk is a report on joint work with Piotr Pstrągowski. Pstrągowski defined a left adjoint $\mathrm{SH}(\mathbb{C})^{\text {cell }} \rightarrow$ Syn ${ }^{\text {ev }}$ from cellular $\mathbb{C}$-motivic spectra to even (MU-based) synthetic spectra. This functor refines the Betti realization of a cellular motivic spectrum, and for any prime $p$, restricts to an equivalence on $p$-complete objects. We'll explain how to further refine the Betti realization functor $\mathrm{SH}(\mathbb{C}) \rightarrow \mathrm{Sp}$ to a left adjoint $\mathrm{SH}(\mathbb{C}) \rightarrow \mathrm{Syn}$ to all synthetic spectra. To do this, we'll give a description of motivic spectra as sheaves on a subcategory of compact pure motives. This description also lets us give a new construction of (and refine) the Gillet-Soulé weight filtration on the compactly supported integral Betti cohomology of a complex variety. (Received September 12, 2023)

1192-14-32327
Lisa Liu*, Stanford University. Belyi Maps for Curves Defined over $\overline{\mathbb{Q}}$. Preliminary report.
Given a Riemann surface $C$, Belyi's Theorem states that $C$ is defined over $\overline{\mathbb{Q}}$ if and only if there exists a map $\beta: C \rightarrow \mathbb{P}^{1}$ such that all critical values of $\beta$ are in the set $\{0,1, \infty\}$. Such a map is called a Belyi map, and if it is "clean," which means that the profile of 1 is $(2,2, \ldots, 2)$, then the inverse image $\beta^{-1}[(0,1)]$ can be realized as a graph on the original surface $C$. By composing arbitrary Belyi maps with constructed maps from $\mathbb{P}^{1}$ to $\mathbb{P}^{1}$, we prove that there always exists a clean Belyi map from a surface $C$ defined over $\overline{\mathbb{Q}}$ that can be realized as a face-bipartite triangulation of $C$.

Owen Biesel*, Southern Connecticut State University. A Norm Functor for Quadratic Algebras.
Given a finite branched cover of schemes $Y \rightarrow X$, we can ask for a "trace" or "norm" homomorphism taking algebraic data over $Y$ to algebraic data over $X$. We construct a norm functor for sheaves of quadratic algebras, assigning to each branched double-cover of $Y$ a branched double-cover of $X$ in a way that is compatible with other trace and norm operations. We also conjecture a relationship between discriminant algebras and this new norm functor.
(Received September 25, 2023)

## 1192-14-32383

Pierrick Bousseau*, University of Georgia. Quivers and curves in higher dimension.
Quiver Donaldson-Thomas invariants are integers determined by the geometry of moduli spaces of quiver representations.
They play an important role in the description of BPS states of supersymmetric quantum field theories. I will describe a correspondence between quiver Donaldson-Thomas invariants and Gromov-Witten counts of rational curves in toric and cluster varieties. This is joint work with Hulya Arguz (arXiv:2302.02068 and arXiv:arXiv:2308.07270).
(Received September 12, 2023)

1192-14-32444
Pat Lank, University of South Carolina, Peter McDonald, University of Utah, Sridhar Venkatesh*, University of Michigan. Singularities via generation in derived categories. Preliminary report.
We characterize well known classes of singularities of complex projective varieties in terms of statements about thick subcategories of the derived category of coherent sheaves. This includes classical cohomological singularities such as rational singularities and Du Bois singularities. This is part of joint work with Pat Lank and Peter McDonald.
(Received September 12, 2023)

1192-14-32454
Ritvik Ramkumar*, Cornell University, Alessio Sammartano, Politecnico di Milano. Hilbert schemes of points on threefolds. I will discuss some of the recent developments on the geometry of the Hilbert schemes of threefolds. In particular, I will focus on certain parity conjectures and how they relate to the reducednes of these Hilbert schemes. Some of what I will discuss is joint work with Alessio Sammartano.
(Received September 12, 2023)

## 1192-14-32498

Luca Battistella, Università di Bologna, Francesca Carocci, University of Geneva, Jonathan Wise*, University of Colorado Boulder. Logarithmic linear series. Preliminary report.
I will describe progress towards constructing complete moduli of linear series for variable semistable curves, including curves not of compact type. The definition makes use of the logarithmic Picard group and an interpretation of tropical curves as algebraic stacks (known as Artin fans). The moduli space admits a virtual fundamental class, clutching constructions, and a regeneration theorem, but it also presents new subtleties in the spirit of matroid realization. (Received September 12, 2023)

## 1192-14-32507

Lucas Mioranci*, University of Illinois at Chicago. Algebraic hyperbolicity of very general hypersurfaces in homogeneous varieties. Preliminary report.
A complex projective variety $X$ is (Brody) hyperbolic when it admits no nonconstant holomorphic map $\mathbb{C} \rightarrow X$, that is, when it contains no entire curves. In dimension one, hyperbolic curves are those with genus greater or equal to 2 . In higher dimensions, it is a difficult and important problem to characterize hyperbolic varieties. It motivates celebrated conjectures such as the Lang Conjectures, Green-Griffiths Conjecture, and Manin's Conjecture. Algebraic hyperbolicity has been introduced as an algebraic analogue for hyperbolicity: we say $X$ is algebraically hyperbolic if there exists an ample divisor $H$ and a real number $\epsilon>0$ such that the geometric genus $g(C)$ and the degree of any integral curve $C \subset X$ satisfy the inequality

$$
2 g(C)-2 \geq \varepsilon \operatorname{deg}_{H}(C) .
$$

In particular, algebraically hyperbolic varieties do not contain any rational or elliptic curves. Every hyperbolic variety is algebraically hyperbolic, and Demailly conjectured that the converse holds. The algebraic hyperbolicity of very general hypersurfaces in projective space is almost completely classified by the results of Clemens, Ein, Voisin, Pacienza, Coskun and Riedl, and Yeong. By building on their techniques, I extended the classification to the much more general case of homogeneous varieties, thus obtaining explicit bounds for the hyperbolicity in plenty of open cases, including Grassmannians, flag varieties, and their products.
(Received September 25, 2023)

1192-14-32536
Roi Docampo*, University of Oklahoma. Invariants of singularities and the geometry of arc spaces.
Since their introduction into algebraic geometry by J. Nash, arc spaces and jet schemes have been used to analyze and control invariants of singularities of algebraic varieties. Traditionally this is done by studying the topological structure of arc spaces. For example, understanding containments between closed subsets of arc spaces allows us to control classical invariants like
discrepancies and log canonical thresholds. But recent developments suggest that the geometric structure of arc spaces (their singularities, their non-reduced structure) should also play an important role. For instance, embedding dimensions and codimensions in arc spaces are directly related to Mather and Mather-Jacobian discrepancies. In collaboration with C. Chiu and T. de Fernex, we have been developing a toolbox for the study of the schematic structure of arc spaces and jet schemes. In this talk I will give an overview of the latest developments in this area.
(Received September 12, 2023)

1192-14-32587
Shari Hoch, Indiana University of Pennsylvania, Anna Marti, Georgia College and State University, Ethan Speiser Soloway*, University of Pennsylvania. Transverse-Free Affine Curves. Preliminary report.
In the affine plane over a finite field, an algebraic curve $C$ is said to be transverse-free if every line is either tangent to $C$ or incident with a singularity of $C$. We present lower and upper bounds on the frequency of transverse-free affine curves in three cases, extending work of Asgarli and Freidin for projective curves. These bounds are distinguished by the number of singularities: we consider smooth curves, curves with one singularity, and curves with any number of singularities. (Received September 12, 2023)

## 1192-14-32670

James Hotchkiss*, Columbia University. The period-index conjecture for abelian threefolds.
The period-index conjecture is a longstanding conjecture in the theory of Brauer groups. I will give an introduction to the conjecture, and describe joint work with Alex Perry which proves the conjecture for Brauer groups of abelian threefolds. The proof relies on a combination of tools from derived categories and enumerative geometry.
(Received September 12, 2023)

## 1192-14-32673

Logan Hambric*, Lehigh University, Andrew Harder, Lehigh University. Homotopy Equivalence Between Algebraic
Hypersurfaces and Coamoebae. Preliminary report.
A common structure in the study of algebraic hypersurfaces is the Pair of Pants, $P^{n-1}=V\left(z_{1}+\ldots+z_{n}+1\right) \subset\left(\mathbf{C}^{*}\right)^{n}$. This structure can be generalized to the simplicial algebraic hypersurface, which is given by
$\tilde{P}^{n-1}=V\left(z_{1}^{p_{11}} \cdots z_{n}^{p_{1 n}}+\ldots+z_{1}^{p_{n 1}} \cdots z_{n}^{p_{n n}}+1\right)$ such that the convex hull of the integral points $\left(p_{i 1}, \ldots, p_{i n}\right)$ along with
$(0, \ldots, 0)$ forms a non-degenerate $n$-simplex. In a 2021 paper, C. Arnal states a conjecture that there exists a homotopy equivalence between a simplicial algebraic hypersurface and its coamoeba which preserves the action of complex conjugation on homology. In this work we prove the conjecture stated by Arnal, using some constructions given by G. Kerr and I. Zharkov in their 2016 paper.
(Received September 12, 2023)

## 1192-14-32709

J. Maurice Rojas*, Texas A\&M University. Computing Isotopy Type for Real Circuit Sums.

Consider an $n$-variate polynomial $f$ with degree $d$, Newton polytope of dimension $n$, exactly $n+k$ monomial terms, and all coefficients in $\{ \pm 1, \ldots, \pm H\}$. We consider the complexity of explicitly computing the isotopy type for the positive zero set of $f$. While earlier work has yielded algorithms with arithmetic complexity $d^{\Omega(n)}$, we present a deterministic algorithm for the case $k \leq 2$ with bit-complexity $(\log (d H))^{O(n)}$. While the case we consider is highly sparse, the underlying tools are important for the case $k=3$ and beyond. In particular, we see how further speed-ups are possible for a large set of inputs. This is joint work with Frederic Bihan, Erika Cory, Weixun Deng, Kaitlyn Phillipson, and Robert J. Rennie.
(Received September 12, 2023)

1192-14-32716
Ritvik Ramkumar*, Cornell University, Alessio Sammartano, Politecnico di Milano. Cartwright-Sturmfels ideals and their moduli.
Cartwright-Sturmfels ideals, CS-ideals for short, are multigraded ideals whose generic initial ideals are radical. Some examples include the ideals of maximal minors of a matrix of linear forms, binomial edge ideals, and multiview ideals. I will discuss the geometry of CS-ideals inside the multigraded Hilbert scheme, with a particular focus on bigraded CS-ideals. This is joint work with Alessio Sammartano.
(Received September 12, 2023)

1192-14-32765
Hernan Iriarte*, University of Texas at Austin. Weak continuity on the variation of Newton-Okounkov bodies. Preliminary report.
We study the space of higher rank quasi-monomial valuations on the function field of an algebraic variety. This gives us a space of higher rank valuations that we endow with a weak "tropical" topology. In this setting, we show the joint continuity of the Newton-Okounkov body with respect to its defining divisor and valuation. We explain how this result fits in the literature and how it gives us a restriction in the existence of mutations of Newton-Okounkov bodies. Joint work with Omid Amini. (Received September 12, 2023)
bundles. Bayer-Popescu-Sturmfels generalized Beilinson's result to a cellular resolution of the diagonal for what they called "unimodular" toric varieties (a more restrictive condition than being smooth), which can also be extended to smooth toric varieties and global quotient toric DM stacks of a smooth toric variety by a finite abelian group, if we allow our resolution to have cokernel which is supported only along the vanishing of the irrelevant ideal. Here we show implications for exceptional collections of line bundles and a positive example for the modified King's conjecture by giving a strong, full exceptional collection of line bundles on a smooth, non-unimodular nef-Fano complete toric surface.
(Received September 12, 2023)

## 1192-14-32934

Jack Jeffries, University of Nebraska, David Lieberman*, University of Nebraska-Lincoln. From Bernstein to Sandwiches: Proving Bernstein's Inequality in Singular Rings.
In the realm of differential operators over a polynomial ring in characteristic zero, an exceptionally useful result is Bernstein's Inequality. This inequality puts a lower bound on the dimension of $D$-modules (where $D$ is the ring of differential operators). Bernstein's inequality can be used to show the existence of Bernstein-Sato polynomials and some finiteness conditions for local cohomology modules. In this talk, we show that Bernstein's inequality holds in some singular rings. In particular we demonstrate new methods that show Veronese rings (a new proof of a known result) and Segre product rings (a new result) satisfy Bernstein's inequality. We also present a new two-sided version of Bernstein-Sato polynomials called Sandwich Bernstein-Sato Polynomials. These are motivated in form by Bernstein-Sato Polynomials, but are equalities that live in $D$ (rather than while viewing $R$ and a $D$-module). Existence of such sandwich equations also has close ties to Bernstein's inequality, and can be used to show new realms where the inequality holds. This is joint work with Jack Jeffries. (Received September 13, 2023)

1192-14-32953
Alana Campbell*, Fordham University, Flora Dedvukaj, Fordham University, Donald McCormick III, Fordham University, Han-Bom Moon, Fordham University, Joshua Morales, Fordham University. On Algebraic Space Filling Curves. Poonen and Gabber independently showed that any smooth geometrically irreducible projective scheme over a finite field has a smooth space filling curve, that is, a smooth curve defined over the field and passes through all points over the field. However, except the case of projective plane, no concrete example was found in literature. In this note, we construct explicit examples of algebraic space filling curves in three dimensional projective space, in particular the ones with minimum degree. (Received September 13, 2023)

1192-14-33001
Tyler Campos*, Yale, Makenna Greenwalt, University of Oregon, Lisa Liu, Stanford University. Studying Holomorphic Connections on Curves Defined over $\overline{\mathbb{Q}}$ Using Meromorphic Connections Arising from the Assignment of Lie Group Elements to Half-Edges of Ribbon Graphs.
We show that assigning Lie group elements to half-edges of a ribbon graph embedded in a curve $C$ defined over $\overline{\mathbb{Q}}$ naturally produces a vector bundle over $C$. The meromorphic 1 -form defined in the previous section extends to a meromorphic connection on $C$. The monodromy representations of $\pi_{1}(C)$ can then be understood as the subset of the monodromy representations of $\pi_{1}\left(C \backslash\left\{p_{1}, \ldots, p_{n}\right\}\right)$, where $p_{i} \in C$ are the singularities of the aforementioned meromorphic 1 -form. This method provides possible first steps to understanding the holomorphic connections and the Riemann-Hilbert Correspondence in the special case of curves defined over $\overline{\mathbb{Q}}$ in a concrete sense by providing an explicit way to construct monodromy representations.
(Received September 13, 2023)

1192-14-33010
Lev Borisov, Rutgers, The State University of New Jersey, Mattie Ji*, Brown University, Yanxin Li, Rutgers, The State University of New Jersey, Sargam Mondal, Middlesex County Academy for Science, Mathematics \& Engineering Technologies. On the Geometry of a Fake Projective Plane with 21 Automorphisms.
A fake projective plane is a complex surface with the same Hodge numbers as $\mathbb{C} P^{2}$ but not biholomorphic to it. While all 100 fake projective planes have been classified, the question of constructing explicit polynomial equations for each surface is largely unsolved. In this work, we study the fake projective plane $\mathbb{P}_{\text {fake }}^{2}=\left(a=7, p=2, \varnothing, D_{3} 2_{7}\right)$ in the Cartwright-Steger classification. We exploit the large symmetries given by $\operatorname{Aut}\left(\mathbb{P}_{\text {fake }}^{2}\right) \stackrel{\text { fake }}{=} C_{7} \rtimes C_{3}$ to construct an explicit embedding of $\mathbb{P}_{\text {fake }}^{2}$ into $\mathbb{C} P^{5}$ as a system of 56 sextics with coefficients in $\mathbb{Q}(\sqrt{-7})$, which is an improvement in dimensions from the previous embedding into $\mathbb{C} P^{9}$ by Borisov and Keum. For each torsion line bundle $T \in \operatorname{Pic}\left(\mathbb{P}_{\text {fake }}^{2}\right)$, we also compute and study the linear systems $|n H+T|$ with small $n$, where $H$ is an ample generator of the Néron-Severi group. In particular, while all fake projective planes with known explicit equations could be embedded into $\mathbb{C} P^{9}$ using $6 H$, we disprove a similar conjecture that 5 H is very ample for all fake projective planes.
(Received September 13, 2023)

1192-14-33016
Wei Gu, Department of Physics, Virginia Tech, Leonardo Constantin Mihalcea, Department of Mathematics, Virginia Tech, Eric Sharpe, Department of Physics, Virginia Tech, Weihong Xu*, Department of Mathematics, Virginia Tech, Hao Zhang, Department of Physics, Virginia Tech, Hao Zou, Beijing Institute of Mathematical Sciences and Applications. Quantum K Whitney relations for partial flag varieties.
The quantum K-theory ring of a smooth projective variety is a deformation of its K-theory ring of algebraic vector bundles. The (equivariant) K-theory ring of the partial flag variety $\mathrm{Fl}\left(r_{1}, \ldots, r_{k} ; \mathbb{C}^{n}\right)$ can be constructed from its tautological vector bundles and the associated Whitney relations. Inspired by the physics of Gauged Linear Sigma Models (GLSM), we give a conjectured deformation of these Whitney relations in (equivariant) quantum K-theory, generalizing the Grassmannian case proved by Gu,

Mihalcea, Sharpe, and Zou. We prove this conjecture for the incidence variety $\mathrm{Fl}\left(1, n-1 ; \mathbb{C}^{n}\right)$; for the complete flag variety $\mathrm{Fl}\left(1, \ldots, n-1 ; \mathbb{C}^{n}\right)$, we reduce this conjecture to a conjecture of Buch and Mihalcea on Chevalley-type K-theoretic GromovWitten invariants.
(Received September 13, 2023)

## 1192-14-33025

Rebekah Palmer*, Unaffiliated. Canonical quaternion algebras.
Invariants of hyperbolic manifolds stretch into various mathematical fields. We will focus on three invariants. On the one hand, every hyperbolic 3-manifold has an associated quaternion algebra coming from a geometric representation of its fundamental group into $\mathrm{PSL}_{2}(\mathbb{C})$. On the other hand, every hyperbolic 3-manifold has an $\mathrm{SL}_{2}(\mathbb{C})$ character variety coming from all representations of its fundamental group to $\mathrm{SL}_{2}(\mathbb{C})$. These two invariants can be united by the canonical quaternion algebra. In this talk, we will discuss the construction and interplay of all three objects.
(Received September 13, 2023)

1192-14-33256
Ivona Grzegorczyk, California State University Channel Islands, Ricardo Suarez*, University of Torino, Italy. Spinor Abelian Varieties. Preliminary report.
A spinor Abelian variety is an Abelian variety whose tangent space at the origin, $T_{0}(A)$, is a space of spinors for some Clifford algebra. Hence $\backslash \mathrm{C}_{q}(V) \cong \operatorname{End}\left(T_{0} A\right)$ for some quadratic vector space $(V, q)$. We examine intrinsic properties of such a variety as well as symmetries on the two torsion points. Then we go on find a singular class of curves whose Jacobian is a spinor Abelian variety and examine some their intrinsic properties. We conclude with extensions of Clifford multiplication on the line bundles on the singular class of curves.
(Received September 13, 2023)

1192-14-33368
Shari Hoch, Indiana University of Pennsylvania, Anna Marti, Georgia College and State University, Ethan Speiser
Soloway*, University of Pennsylvania. Transverse-Free Affine Curves (Poster Presentation). Preliminary report.
In the affine plane over a finite field, an algebraic curve $C$ is said to be transverse-free if every line is either tangent to $C$ or incident with a singularity of $C$. We present lower and upper bounds on the frequency of transverse-free affine curves in three cases, extending work of Asgarli and Freidin for projective curves. These bounds are distinguished by the number of singularities: we consider smooth curves, curves with one singularity, and curves with any number of singularities. (Received September 14, 2023)

## 1192-14-33440

Katrina Honigs*, Simon Fraser University. Derived equivalence of generalized Kummers over Q.
An abelian variety and its dual are derived equivalent via the Poincaré bundle. However, it is an open question whether generalized Kummer varieties attached to an abelian surface and its dual are derived equivalent. In recent work joint with Frei and Voight, we produced examples of abelian surfaces defined over Q whose associated Kummer 4-fold is not equivalent over Q to that of its dual. I will describe how this question about derived categories has gone hand-in-hand with the production of examples of abelian surfaces whose torsion has certain properties.
(Received September 16, 2023)

1192-14-33516
Elise Farr*, Boston University, Galileo Fries*, Colorado College, Julian Hutchins*, Morehouse College, Vuong Trieu Nguyen Hoang*, Wingate University. The Geometry of Small Chemical Reaction Networks. Chemical reaction network theory is a field of applied mathematics concerned with the modeling of chemical systems. These models can be used in other contexts such as in systems biology to study cellular signaling pathways and epidemiology to study the effect of human interaction on the spread of disease. For this presentation, we are concerned with understanding a reaction network's equilibrium points through the lens of algebraic geometry. Specifically we are interested in computing the positive part of the steady-state variety defined by polynomial equations arising from the assumption of mass-action kinetics. We provide a systematic classification of all positive steady-state varieties produced by 2 -species, 2 -reaction networks, grounded in combinatorial and algebraic characteristics of the networks. Currently, there is no algorithm to calculate the positive part of the steady-state variety of a chemical reaction network, further necessitating their computation and documentation. Our classification theorems seek to not only expand the fields breadth of understanding but also provide a foundation for future analysis of larger reaction networks.
(Received September 19, 2023)

1192-14-33587
Richard Kyung, CRG-NJ, Zimo Li*, United World College of South East Asia. Study on Geometrical Properties of Networks Using Spectral Analysis in Graphing Theory.
The use of the adjacency matrix and the Laplacian matrix in graph theory is an efficient approach for analyzing complex networks. The information obtained from these matrices and their eigenvalues is valuable to figure out the connectivity and topology of networks. The largest eigenvalue of the adjacency matrix is an important parameter for understanding a network's connectivity. In this paper, the Gershgorin Circle Theorem was used to estimate the spectrum of a matrix, which refers to the set of eigenvalues associated with that matrix. In the process, the shape, connectivity, and overall structure of a network were examined to understand the structural and topological properties of complex networks. This research can extend to study the practical applications and contribute to understanding the interconnected nature of various real-world phenomena.
(Received September 21, 2023)

Holly Krieger*, University of Cambridge. Uniformity When Arithmetic Meets Geometry.
In 1983, Faltings proved Mordell's famous 1922 conjecture relating arithmetic to geometry: that for a polynomial equation $f(x, y)=0$, if the topology of the set of solutions over the complex numbers is sufficiently complicated, then the set of solutions with rational numbers is finite. Once we know a set is finite, natural questions arise: can we compute this finite set? How large can it be? What input data does its size depend on? Recent work of Dimitrov-Gao-Habegger and Kühne have provided a strong and striking answer to this last question, proving what is known as the Uniform Mordell-Lang Conjecture for curves embedded into Jacobians, posed by Mazur in 1986. Here the word 'uniform' roughly means that the size bound depends only on the genus of the curve (a measure of the topological complexity of the complex solutions) as well as a notion of algebraic complexity for the field in which we search for points satisfying the polynomial equation. This talk will introduce the history of Mordell's conjecture and highlight the near-century of progress leading to the breakthrough work of Dimitrov-GaoHabegger and Kühne. Without assuming any background in algebraic geometry or number theory, we will discuss their proofs, some of the applications of their results, and connections with other questions of uniformity in the interactions between arithmetic, algebra, and geometry.
(Received September 22, 2023)

## 1192-14-33742

Ravi D Vakil*, Stanford University. Passing a Curve through N Points - Solution of a 100-Year-Old Problem.
Through two randomly chosen points in the plane, indeed in $n$-space, there is a line. Through five randomly chosen points in the plane, there is a conic. But in higher-dimensional space, through even four randomly chosen points, there can't be a conic, because all conics have to lie on a plane, and four randomly chosen points don't. Through four randomly points in the plane, you can find a cubic $y=a x^{3}+b x^{2}+c x+d$, and if you're not looking just for a graph of an equation, you can find a cubic $a x^{3}+b x^{2}+\cdots+h x+i y=1$ through nine randomly chosen points. For centuries, the "interpolation problem" has arisen in many contexts: is there a curve of some "type" through a bunch of generally chosen points? Even making this precise has led to important definitions and notions and theorems. In this talk, I'll discuss Eric Larson and Isabel Vogt's proof of the interpolation problem (in its modern incarnation) in full generality, bringing on beautiful ideas both old and new. We will start with some elementary observations and intuitions, and gently build up to some of the ideas behind their tour-de-force solution. (Received September 25, 2023)

## 1192-14-33784

Alex Matthew Abrams*, Loyola Marymount University, Tesfa Asmara*, Pomona College, David W. Bonds*, California State University Los Angeles, Aniyah Stephen*, Hartwick College. Abelian Division Fields Over Real Quadratic Fields. Preliminary report.
An $n$ division field of an elliptic curve is an extension field containing all points of $n$ torsion. It is of interest to find when these fields are abelian. Previously, Enrique González-Jiménez and Álvaro Lozano-Robledo showed what $n$ it is possible to have abelian division fields for elliptic curves defined over $\mathbb{Q}$. In this project we investigate when abelian division fields of non-CM elliptic curves arise after a base change from $\mathbb{Q}$ to $\mathbb{Q}(\sqrt{5})$.
(Received September 25, 2023)

## 15 Linear and multilinear algebra; matrix theory

## 1192-15-25445

Yiu Tung Poon*, Iowa State University, Nyle Alexander Sutton, Department of Mathematics, Iowa State University. Generalized matricial ranges and positive definiteness.
Numerical range is the subject of study for over a century. Its extension to matricial range is defined in terms of completely positive maps. A linear map $\phi$ between matrix algebras is completely positive if and only if its Choi matrix $C(\phi)$ is positive semidefinite. In this paper, we extend the study to some joint matricial ranges defined by those $\phi^{\prime}$ 's where $C(\phi)$ is Hermitian with specified spectrum. We prove some convexity theorems which extend previous results on generalized joint numerical ranges and matricial ranges. We also extend Bohnenblust's result on joint positive definiteness of Hermitian matrices and Friedland and Loewy's result on the existence of a nonzero matrix with multiple first eigenvalue in subspaces of Hermitian matrices.
(Received June 09, 2023)

1192-15-25557
Joseph P. Stover*, Gonzaga University. Spectral radius bounds using spectral radius-preserving row sum expansions. We provide a new method to find bounds on the spectral radius of nonnegative matrices using the novel concept of row sum expansions which is an operation to increase the size of a matrix while preserving its spectral radius. The standard minimum and maximum row sum as bounds on the spectral radius is a special case of this new method. Our method involves increasing or decreasing elements of the matrix to create a block matrix with constant row sums in each block. We then reduce the dimension of the modified matrix while preserving its spectral radius. This allows us to calculate the spectral radius of a matrix of smaller dimension which is a bound on that for the original matrix. Spectral radius-preserving row sum expansions is also a concept accessible to undergraduates. It allows one to create arbitrarily large matrices with a specified spectral radius and arbitrary row sums. Published paper: https://journals.uwyo.edu/index.php/ela/article/view/6981
(Received June 20, 2023)

Processing Units), has become an import tool in scientific computing (broadly defined), machine learning, finance, and related fields. Graphics cards are highly optimize to perform a fixed linear algebra operation (particularly translation and rotation, but increasingly any operator application) on a huge number of inputs. This makes them well suited for applications which can be parallelized, particularly if those operations use (or are) numerical linear algebra. This talk will detail the speaker's experiences using Matrix Algebra on GPU and Multi-core Architectures (MAGMA), a software library to ease rewriting numerical linear algebra code that runs on the CPU to utilize the GPU. I will cover what precisely MAGMA does, how it use it, and key issues I encountered in using it. No previous experience in numerical linear algebra will be assumed.
(Received August 07, 2023)

1192-15-26919
Tin-Yau Tam*, University of Nevada, Reno. Asymptotic results on the positive semi-definite part of a square matrix. Preliminary report.
A result of Nayak asserts that $\lim _{m \rightarrow \infty}\left|A^{m}\right|^{1 / m}$ exists for each $n \times n$ complex matrix $A$, where $|A|=\left(A^{*} A\right)^{1 / 2}$, and the limit is
given in terms of the spectral decomposition. We extend the result of Nayak, namely, we prove that the limit of $\lim _{m \rightarrow \infty}\left|B A^{m} C\right|^{1 / m}$ exists for any $n \times n$ complex matrices $A, B$, and $C$, where $B$ and $C$ are nonsingular; the limit is obtained $m \rightarrow \infty$
and is independent of $B$. We then provide generalization in the context of real semisimple Lie groups.
(Received August 07, 2023)

1192-15-27006
Stephan Ramon Garcia*, Pomona College. Fast Food for Thought: What Can Chicken Nuggets Tell Us About Linear Algebra?.
A simple question about chicken nuggets connects everything from analysis and combinatorics to probability theory and computer-aided design. Linear algebra is a recurring theme: determinantal formulas, piecewise-polynomial interpolation, "random norms", trace polynomials, and operator algebras all make an appearance. This talk is aimed at a general mathematical audience: students are invited to attend!
(Received August 08, 2023)

## 1192-15-27391

Edward Poon*, Embry-Riddle Aeronautical University. The simultaneous zero inclusion property and Birkhoff-James orthogonality.
A normed space $\mathcal{X}$ is said to have the Simultaneous Zero Inclusion (SOI) property if, for every invertible bounded linear operator $T$ on $\mathcal{X}, 0$ lies in the spatial numerical range of $T$ if and only if 0 lies in the spatial numerical range of $T^{-1}$. Previously the only known spaces with the SOI property were inner product spaces, corresponding to the classical numerical range. By connecting the SOI property to Birkhoff-James orthogonality, we show that there are non-inner product spaces based on Radon planes that have the SOI property.
(Received August 14, 2023)

1192-15-28010
Mao-Ting Chien, Soochow University, Taiwan, Stephen Kirkland, University of Manitoba, Chi-Kwong Li*, College of William and Mary, Hiroshi Nakazato, Hirosaki University. Numerical ranges of cyclic shift matrices.
We study the numerical range of an $n \times n$ cyclic shift matrix, which can be viewed as the adjacency matrix of a directed cycle with $n$ weighted arcs. In particular, we consider the change in the numerical range if the weights are rearranged or perturbed. In addition to obtaining some general results on the problem, a permutation of the given weights is identified such that the corresponding matrix yields the largest numerical range (in terms of set inclusion), for $n \leq 6$. We conjecture that the maximizing pattern extends to general $n \times n$ cylic shift matrices. For $n \leq 5$, we also determine permutations such that the corresponding cyclic shift matrix yields the smallest numerical range.
(Received August 24, 2023)

1192-15-28013
Mike Michailidis*, The MathWorks, Inc.. Linear Algebra Inspired Machine Learning: a MATLAB Demo.
You've probably heard of machine learning. Artificial intelligence and neural networks are slowly taking over the world. But did you know that linear algebra can be a great introduction to machine learning? It can help uncover the essence of AI, why it works and what happens when it doesn't. In this talk, we will explore a modern application of linear algebra for machine learning. A linear algebra inspired, digit recognition algorithm will be presented, based exclusively on linear algebra concepts studied in a 1st linear algebra class taken by mathematicians and STEM students. The overview of the algorithm will be followed by an interactive demo in MATLAB, that will also allow the user to test its efficacy on their own handwriting. (Received August 24, 2023)

1192-15-28081
Harm Derksen, Northeastern University, Igor Klep, University of Ljubljana, Visu Makam, Radix Trading, Jurij Volčič*, Drexel University. Ranks of linear pencils separate matrix similarity orbits.
Two matrices $A, B$ are called similar if there is an invertible matrix $P$ satisfying $A P=P B$. As is well known, complex matrices are up to similarity uniquely determined by their Jordan canonical form. This talk discusses possible extensions to (joint) similarity of tuples of matrices. Tuples $\left(A_{1}, \ldots, A_{n}\right)$ and $\left(B_{1}, \ldots, B_{n}\right)$ are called similar if there is an invertible matrix $P$ such that $A_{j} P=P B_{j}$ for all $j$. The classification of matrix tuples up to similarity has been deemed a "hopeless problem", but is widely studied due to its importance in operator theory, invariant and representation theory, algebraic geometry and computational complexity. This talk presents a natural collection of separating invariants for matrix tuples, along the way solving a 2003 conjecture of Hadwin and Larson, which itself was an adaptation of a 1985 conjecture of Curto and Herrero.

Sheldon Axler*, San Francisco State University. The Stupendous Singular Value Decomposition.
This talk will discuss the remarkable singular value decomposition of a linear map on a finite-dimensional inner product space. The focus here will be on applications of the singular value decomposition within linear algebra, including the following topics. - ize
- understanding the adjoint, inverse, and pseudoinverse of a linear map
- norms of linear maps
- best approximation of a linear map by linear maps with smaller rank
- polar decomposition
- operators applied to ellipsoids and parallelepipeds
- change of volume via singular values \enditemize
(Received August 26, 2023)


1192-15-28128
Leslie Hogben*, Iowa State University \& American Institute of Mathematics. Uniform and apportionable matrices
There has been extensive study of diagonalization of matrices, or finding the Jordan Canonical Form for a matrix that is not diagonalizable. Diagonalization can be viewed as using a similarity to concentrate the magnitude of all the entries with a small subset of entries. Here we study what can be viewed as reversing this process, spreading out the magnitudes as uniformly as possible. A uniform matrix plays the role of a diagonal matrix in this process. A square complex matrix is uniform if all entries have the same absolute value and a square complex matrix is apportionable if it is similar to a uniform matrix; the problem of apportioning by unitary similarity is also studied. Hadamard matrices and discrete Fourier transforms are important examples of uniform matrices. Matrix apportionment has connections to classical problems of combinatorics, including graceful labeling of graphs, and connections with the new study of instantaneous uniform mixing in quantum walks. Various results and examples are presented. Every rank one matrix is unitarily apportionable and there is a procedure to find a unitary apportioning matrix. A necessary condition for a matrix to be apportioned by unitary matrix is established and this condition is used to construct a set of matrices with nonzero Lebesgue measure that are not apportionable by a unitary matrix. There are examples of matrices $A$ such at there are infinitely many possible magnitudes of entries of the uniform matrices $M A M^{-1}$ and exaples of spectra that are not attainable by any uniform matrix.
(Received August 26, 2023)

1192-15-28176
Samir Mondal, Indian Institute of Technology Madras, Koratti C Sivakumar, Indian Institute of Technology Madras, Michael J. Tsatsomeros*, Washington State University. P-matrix powers. Preliminary report. A P-matrix is a matrix all of whose principal minors are positive. Positive definite matrices, M-matrices and diagonally dominant matrices with positive diagonal matrices are examples of P-matrices. We demonstrate that the fractional powers (roots) of a P-matrix are also P-matrices. This insight allows us to affirmatively address a longstanding conjecture raised by Danny Hershkowitz and Charlie Johnson: If $A^{k}$ is a P-matrix for all positive integers $k$, then the eigenvalues of $A$ are positive. (Received August 27, 2023)

1192-15-28533
Louis A Deaett, Quinnipiac University, Shaun M Fallat, University of Regina, Veronika Furst, Fort Lewis College, John Hutchens*, University of San Francisco, Lon Mitchell, Eastern Michigan University, Yaqi Zhang, Drexel University. Spark and rank of symmetric matrices associated with a graph. Preliminary report. The space of real symmetric matrices whose off-diagonal pattern of zero and nonzero entries correspond to the adjacencies of a graph $G$ is denoted by $\mathcal{S}(G)$. The spark of a matrix is the minimum size of any set of linearly dependent columns, and thus indicates the size of the smallest support of any null vector for the given matrix. A fort of a graph $G$ with respect to standard zero forcing is a set of vertices of $G$ that is the complement of a maximal failed zero forcing set. Forts in $G$ correspond to possible supports of null vectors for symmetric matrices in $\mathcal{S}(G)$. To study the relationship between spark and rank of symmetric matrices we study the connection between forts and connectivity in the graph $G$ with the spark and rank of matrices in $\mathcal{S}(G)$.
(Received August 31, 2023)

1192-15-28542
Shmuel Friedland*, University of Illinois at Chicago. Semidefinite programming characterizations of the numerical radius and its dual norm from complex and quaternion matrices. Preliminary report.
We give a semidefinite programming (SDP) characterization of the dual norm of the numerical radius for matrices over the field of complex numbers $\mathbb{C}$ and the skew-field of quaternions $\mathbb{H}$. This characterization yields the SDP characterization of the numerical radius of these matrices. For complex numbers this characterization is due to T. Ando 1973. We show that the computation of the numerical radius and its dual norm within $\varepsilon$-precision is polynomial in the data and $|\log \varepsilon|$ using the short step, primal interior point method. References: [1] S. Friedland, On semidefinite programming characterizations of the numerical radius and its dual norm for quaternionic matrices, in preparation. [2] S. Friedland and C.-K. Li, On semidefinite programming characterizations of the numerical radius and its dual norm, arXiv:2308.07287 (14 Aug 2023) (Received August 31, 2023)

1192-15-28608
Stephan Ramon Garcia*, Pomona College. A second course in linear algebra: a call for the early introduction of complex numbers.
A second course in linear algebra is increasingly important for students of the mathematical sciences. Although many applications involve only real numbers, complex arithmetic often sheds significant light. Most elementary properties of complex numbers have immediate matrix analogues and many important theorems can be deduced, or at least postulated, from the basics of complex arithmetic alone. Partially supported by National Science Foundation Grant DMS-2054002.

Jan Armendariz-Bones*, Author, Peter Carroll*, Author, Natalie Marie Dando*, Author, Jorge Yahel Montes Guzman*, Author, Dr. Padraic Taylor, Advisor. Applications of Google's PageRank Algorithm: Predicting Tennis Matches and Traffic Analysis.
Google's PageRank is the algorithm developed by Larry Page and Sergey Brin to sort search results by relevance based upon the link structure of the Internet. The mathematical backbone of PageRank is Perron's Theorem, and the Power Method is used to implement the algorithm. While Google uses PageRank to rank its webpages, our project uses the algorithm to rank American men's tennis players and to analyze traffic patterns in Portland, Oregon. Our tennis ranking system can compete with the current systems in predicting tennis match outcomes, even outperforming the other systems at predicting American vs. American matches. Our traffic system can find patterns in populated areas by time of day and days of the week, which can indicate numerous trends such as common residential areas, business areas, and nightlife areas.
(Received September 01, 2023)

Gregory Adams, Bucknell University, Pamela Gorkin*, Bucknell University. Packages of curves in the numerical range of a matrix.
Gau and Wu studied compressions of the shift operator and showed that they have a Poncelet-like property; that is the numerical range of such a curve has the property that it is inscribed in a closed convex polygon that is itself inscribed in the unit circle. At the same time, Boris Mirman considered what he called UB-matrices, or matrices for which the addition of a (certain) column and row dilates it to a unitary matrix. Mirman also showed that the numerical range of such a matrix has the Poncelet property. These two results are the same, but they were developed quite differently. Later, complex function theory began to play an important role when it was shown that the vertices of the polygon can be obtained using Blaschke products. Thinking of the circumscribing polygon as joining lines between consecutive points, Mirman considered what he called "Poncelet packages of curves", or curves that are obtained from (possibly non- convex) polygons that result by drawing lines that skip over points. Our talk will begin by considering this history and major consequences of each viewpoint.
(Received September 03, 2023)

## 1192-15-28958

Veronika Furst*, Fort Lewis College. Sparsity of null vectors of symmetric matrices described by a graph
The inverse eigenvalue problem of a graph (IEP-G) is to determine the possible spectra of real symmetric matrices described by a graph, in the sense that an off-diagonal entry of the matrix is nonzero if and only if the corresponding vertices of the graph are connected by an edge. In this talk, we consider the corresponding eigenvectors; in fact, it suffices to consider the null vectors of these matrices. In particular, we ask how sparse such a null vector may be and extend the definition of the spark of a matrix to the spark of a graph. We find the analog of a classical result of Lovász, Saks, and Schrijver and give a necessary and sufficient condition for the existence of a symmetric (not necessarily positive semidefinite) matrix described by a graph with specified rank and full spark. Joint work with Louis Deaett, Shaun Fallat, John Hutchens, Lon Mitchell, and Yaqi Zhang.
(Received September 04, 2023)

1192-15-28960
Devon N Munger, University of Washington Bothell, Andrew Lewis Nickerson, Western Washington University, Pietro
Paparella*, University of Washington Bothell. Demystifying the Karpelevič theorem.
In this talk, it is shown that there is a continuous function $\mu:[0,1] \longrightarrow \mathbb{C}$ such that $\mathrm{P}^{\prime}(\mu(\alpha))=0, \forall \alpha \in[0,1]$, where $\mathrm{P}^{\prime}$ is a Type I reduced Ito polynomial. In addition, demonstrations are given for basic properties of the Karpelevič region, including a rigorous, but elementary argument showing that points on the boundary of the Karpelevič region are extremal whenever $n>3$.
(Received September 04, 2023)

## 1192-15-29130

Eunkyu Kim*, Department of Mechanical Engineering, The Cooper Union. Camera Calibration Algorithm Refinement Via Iterative Experimental Validation. Preliminary report.
Camera calibration is a fundamental process in computer vision and photogrammetry that aims to accurately determine the intrinsic and extrinsic parameters of a camera. By accurately calibrating a camera, one can accurately measure distances, angles, and positions of objects in the real world using images or video acquired from the camera. This paper introduces a linear algebra-based formulation of camera calibration algorithms, focusing on two methods: the regular method and the linearized method. The regular method is computationally more efficient but with no way of determining or improving calibration accuracy. In contrast, the linearized method can be adopted to an iterative process to enhance calibration accuracy. To demonstrate the effectiveness of these methods, a marker-based motion tracking experiment utilizing real-life data is conducted. The results showcase the regular method's computational efficiency, while the linearized method's ability to remove outliers is demonstrated through reverse coordinate computation. With a 2.00 mm upper limit threshold, reverse coordinate computation achieved accuracy up to a Frobenius norm of 0.18 mm . (Received September 05, 2023)

1192-15-29242
Nikolai Anatolievich Krylov*, SIENA COLLEGE. Periodicity and Circulant Matrices in the Riordan Array of a Polynomial. Consider a polynomial $p(t)$ of degree $d$. Columns of the Riordan array $\left(1 /\left(1-t^{d+1}\right), t p(t)\right)$ are eventually periodic sequences with a period of $d+1$. We use circulant matrices to describe the long term behavior of such periodicity when the column's
index of the Riordan array grows indefinitely. We also discuss some combinatorially interesting sequences that appear through the corresponding A - and Z - sequences of such Riordan arrays.
(Received September 06, 2023)

## 1192-15-29289

Seong Jun Park*, ILASSS5A. Modern Theory of Copositive Matrices: Copositive Range and Copositivity Preservers. Copositive matrices is a subclass of nonnegative matrices that has received a considerable attention in recent years and it is ubiquitous in applications, from optimization to data analysis and machine learning. In this talk, we offer a novel factorization result and recent findings on the copositive range. Additionally, we present some current work on finding copositivity preservers.
(Received September 06, 2023)

1192-15-29613
Shaun M Fallat*, University of Regina. Spectral Applications of a Weighted Vertex-Clique Incidence Matrix.
Using the concept of an edge-clique cover of a graph $G$ we consider a weighted version of a vertex-clique incidence matrix as a tool to study the important spectral parameter $q(G)$, known as the minimum number of distinct eigenvalues of the graph $G$. We will discuss some recent work on threshold graphs and present some applications to classes of graphs that allow two distinct eigenvalues. This is joint work with Dr. S.A. Mojallal.
(Received September 07, 2023)

## 1192-15-29614

Shaun M Fallat*, University of Regina. Some Combinatorics behind Certain Totally Nonnegative Determinantal Inequalities. Determinantal inequalities associated with totally nonnegative matrices represent a cornerstone topic for this class of matrices. Using the combinatorial essence of the bidiagonal factorization known for totally nonnegative matrices, we introduce a set of index-row/column set operations that act on such determinantal expressions to produce refinements of known inequalities and enrich the class of inequalities comparing products of minors for totally nonnegative matrices. This work is joint with Dr. P.K. Vishwakarma.
(Received September 07, 2023)

## 1192-15-29802

Jonathan Webb*, University of Idaho. The determinant of the frame operator of an $(\ell, K)$-frame over $\mathbb{F}_{2}$. Preliminary report. A family of vectors $\left\{f_{i}\right\}_{i=1}^{k} \subseteq \mathbb{F}_{2}^{n}$ is called a frame if it spans $\mathbb{F}_{2}^{n}$. The $n \times k$ matrix with $f_{i}$ as its $i$ th column is called the synthesis operator $F$ of $\left\{f_{i}\right\}_{i=1}^{k}$. Let $F$ have rows $\left\{r_{i}\right\}_{i=1}^{n}$. The $n \times n$ matrix $\left[\left(r_{i}, r_{j}\right)\right]_{1 \leq i, j \leq n}$, where $(\cdot, \cdot)$ denotes the $\mathbb{F}_{2^{-}}$ valued dot product, is called the frame operator $A$ of $\left\{f_{i}\right\}_{i=1}^{k}$. If $\left(r_{i}, r_{i}\right)=1$ for $\ell$ of the $r_{i}$, and $\left(r_{i}, r_{j}\right)=1$ for $K$ pairs of distinct $r_{i}$ and $r_{j}$, then $\left\{f_{i}\right\}_{i=1}^{k}$ is called an $(\ell, K)$-frame. We give a general expression for $\operatorname{det}(A)$ and an expression for the expected value of $\operatorname{det}(A)$ when $\left\{f_{i}\right\}_{i=1}^{k}$ is an $(\ell, K)$-frame. (Received September 07, 2023)

1192-15-30074
Maxime Bros*, NIST. A Brief History of MinRank.
The MinRank problem can be described easily: given a set of matrices, find a linear combination of them with rank smaller than a threshold; despite its simplicity, this problem is ubiquitous in public-key cryptography. In fact, since it was first used in 1999 by Kipnis and Shamir in an attack against the HFE cryptosystem, the MinRank problem appeared in a lot of other places, mainly in multivariate-based cryptography. Thus, understanding the complexity of the MinRank problem is very important in cryptography, and this is an active area of research. In this talk, I will go through the first attempts to attack MinRank all the way through the most recent attacks, namely the SupportMinors attack and its improvements. Throughout this MinRank journey, I will speak both about combinatorial and algebraic attacks.
(Received September 08, 2023)

## 1192-15-30189

Larissa Renshaw*, University of Tennessee at Martin. Techniques for Solving Linear Programs including the Simplex Method and Extensions to Quadratically Constrained Quadratic Programs.
We explore the known methods of solving Linear Programming problems, including geometric, algebraic, and the Simplex Method. We then explore the methodology as it pertains to higher orders. Along with this, we hope to expand and create our own method using computer programming as well as potentially extend to a quadratically convex quadratic program. (Received September 09, 2023)

## 1192-15-30201

Xiangyi Zhu*, University of California, Irvine, Yizhe Zhu, University of California Irvine. Non-backtracking eigenvector delocalization for random regular graphs.
The non-backtracking operator of a graph is a powerful tool in the study of spectral graph theory and random matrix theory. Most of the existing results for the non-backtracking operator of a random graph are only about eigenvalues. In this paper, we make a first step in analyzing its eigenvector behaviors. We show that for the non-backtracking operator $B$ of a uniformly chosen $d$-regular random graph with $n$ vertices, its eigenvectors corresponding to nontrivial eigenvalues are completely delocalized, namely, the $\ell_{\infty}$-norm of each non-trivial eigenvectors are bounded by $\frac{\log ^{C}(n)}{\sqrt{n d}}$ with high probability. We introduce a
$2 n \times 2 n$ matrix $\tilde{B}$ based on the Ihara-Bass formula, which has a more direct relationship with the adjacency matrix and shares the same eigenvalues of the non-backtracking operator except for the trivial ones. Using $\tilde{B}$, We expressed the eigenvalues and eigenvectors of $B$ in terms of the corresponding eigenvalues and eigenvectors of the adjacency matrix. From the eigenvector delocalization result for the adjacency matrix shown in Huang and Yau (2021), we prove that non-backtracking eigenvectors are delocalized
(Received September 09, 2023)

1192-15-30284
Tin-Yau Tam*, University of Nevada, Reno. Generalized numerical ranges associated with Lie algebras. Preliminary report. Let $G$ be a complex semisimple Lie group with Lie algebra $\mathfrak{g}, K$ connected subgroup of $G$ with Lie algebra $\mathfrak{k}, B_{\theta}(\cdot, \cdot)$ the inner product on $\mathfrak{g}$ induced by the Killing form $B(\cdot, \cdot)$, where $\theta$ is the Cartan involution of $\mathfrak{g}$ associated with $\mathfrak{k}$. For $X, C \in \mathfrak{g}$, the $C$ numerical range of $X$ is

$$
W_{C}(X):=B_{\theta}(C, \operatorname{Ad}(k) X): k \in K, C, X \in \mathfrak{g} .
$$

We will discuss some geometric properties of $W_{C}(X)$ and related results.
(Received September 09, 2023)

## 1192-15-30317

Sheldon Axler*, San Francisco State University. The Marvelous Minimal Polynomial.
Suppose $V$ is a finite-dimensional vector space over some field and $T$ is a linear map from $V$ to $V$. There is a unique monic polynomial $p$ of smallest degree such that $p(T)=0$ (here monic means that the coefficient of the highest order term equals 1 ). This polynomial $p$ is called the minimal polynomial of $T$. This talk will discuss several aspects and applications of the minimal polynomial that should be better known, including the following. - ize
- The minimal polynomial of \(T\) is easily computable (usually very quickly, even when \(V\) has high dimension).
- The zeros of the minimal polynomial of \(T\) are exactly the eigenvalues of \(T\).
- The minimal polynomial of \(T\) tells us whether or not \(T\) is diagonalizable.
- The minimal polynomial of \(T\) tells us whether or not there exists a basis of \(V\) with respect to which \(T\) has an upper-triangular matrix.
- The minimal polynomial of \(T\) tells us whether or not there exists a basis of \(V\) consisting of generalized eigenvectors of \(T\).
- If \(V\) is a real or complex inner product space, then the minimal polynomial leads to an easy proof of the finite-dimensional spectral theorem. \enditemize
(Received September 09, 2023)


## 1192-15-30415

Samir Raouafi*, Auburn University. Operators with Minimal Pseudospectra and Applications to Numerical Ranges. Preliminary report.
It is well-known that normal operators have minimal pseudospectra and the closure of their numerical ranges is the convex hull of their spectra. However, the converse is not valid in general. In this talk, we will discuss recent characterizations of the normality of some operators based on the spectrum and a single pseudospectrum. Additionally, we will examine the class of non-normal operators with minimal pseudospectra and explore their implications for the numerical range. (Received September 10, 2023)

## 1192-15-30422

Javier Santillan*, UC Berkeley. Implementing a Totally Unimodular Matrix Recognition Algorithm into SageMath. Preliminary report.
A totally unimodular (TU) matrix is a matrix such that all of its square submatrices have their determinants in $0, \pm 1$. Matroids arising from TU matrices are called regular matroids, and their study led to the discovery of Seymour's Decomposition for totally unimodular matrices. We get a polynomial time algorithm for recognizing whether a given matrix is totally unimodular from this decomposition. We'll provide this algorithm, highlighting how Seymour's Decomposition is crucial in allowing for a polynomial time algorithmic reduction to simpler cases. We now have a working implementation of this TU recognition algorithm in SageMath. We'll show how one would then be able to use our implementation in SageMath to test whether a given matrix is TU, together with other relevant and useful functionalities it has to extract information about the matrix decomposition.
(Received September 10, 2023)

## 1192-15-30470

Andres Marcos Encinas, Polytechnic University of Catalunya, Barcelona, Spain, Samir Mondal, Indian Institute of Technology Madras, Koratti C Sivakumar*, Indian Institute of Technology Madras. On an Analogue of a Property of Singular $M$-matrices, for the Lyapunov and the Stein Operators.
For real square matrices, it is known that, if $A$ is a singular irreducible $M$-matrix, then the only nonnegative vector that belongs to the range space of $A$ is the zero vector. In this talk, we present an analogue of this result for the Lyapunov and the Stein operators.
(Received September 10, 2023)

1192-15-30724
Marina Arav, Georgia State University, Hein Van Der Holst*, Georgia State University. A characterization of directed treewidth 1 digraphs using matrices.
For a digraph $D=(V, A)$ with vertex-set $V=\{1, \ldots, n\}$ and $\operatorname{arc}$-set $A$, let $M(D)$ be the set of all $n \times n$ matrices $A=\left[a_{i, j}\right]$ with $a_{i, j} \neq 0$ if $i j \in A, a_{i, i} \neq 0$ for $i \in V$, and $a_{i, j}=0$ if $i \neq j$ and $i j \notin A$. By $N(D)$ we denote the space of all $n \times n$
matrices $X=\left[x_{i, j}\right]$ with $x_{i, j}=0$ if $i=j$ or $i j \in A$. We say that a matrix $A \in M(D)$ satisfies the ASAP if for all $X \in N(D)$, $A X^{T}=0$ and $X^{T} A=0$ implies $X=0$. The stable maximum nullity $M s(D)$ of a digraph $D$ is the maximum nullity of any $A \in M(D)$ satisfying the ASAP. A digraph $D$ has $M s(D)=0$ if and only if $D$ is acylic. In this talk, we show that digraphs $D$ with $M s(D) \leq 1$ are exactly the digraphs with directed treewidth 1 .
(Received September 10, 2023)

## 1192-15-30737

Marina Arav*, Georgia State University, Hein Van Der Holst, Georgia State University. The strong maximum nullity of a bipartite graph.
For a bipartite graph $G$ with bipartition $U, W$ and $|U|=|W|$, in which we allow parallel edges, we denote by $Q(G)$, the set of all real $U \times W$ matrices $M=\left[m_{i, j}\right]$ with $m_{i, j} \neq 0$ if $i$ and $j$ are connected by a single edge, $m_{i, j} \in \mathbb{R}$ if $i$ and $j$ are connected by parallel edges, and $m_{i, j}=0$ if $i$ and $j$ are not adjacent, and we denote by $N(G)$ the set of all real matrices $X=\left[x_{i, j}\right]$ with $x_{i, j}=0$ if $i$ and $j$ are adjacent. A matrix $M$ has the ASAP if there is no nonzero matrix $X \in N(G)$ such that $X^{T} M=0$ and $M X^{T}=0$. There exists a matrix satisfying the ASAP if and only if $G$ has a perfect matching. For a bipartite graph $G$ having a perfect matching, denote by $M_{S}(G)$ the maximum nullity of any $M \in Q(G)$ satisfying the ASAP. In this talk, we discuss the class of bipartite graphs $G$ with $M_{S}(G)=0$ and the class of bipartite graph $G$ with $M_{S}(G) \leq 1$. We do this in terms of forbidden substructures (minors).
(Received September 10, 2023)

## 1192-15-30757

Anastasiia Minenkova*, University of Hartford, Gamal Mograby, University of Maryland. Linear Algebra in Quantum Computing. Preliminary report.
In this talk we discuss the relation between linear algebra and problems in quantum computing: from the perfect quantum state transfer to quantum walks on graphs. Using this relation we give a quick exposition of the mathematical aspects of these concepts.
(Received September 11, 2023)

## 1192-15-30943

Muyan Jiang, University of California, Berkeley, Ilya M Spitkovsky*, New York University Abu Dhabi. On Kippenhahn curves of some structured matrices. Preliminary report.
We consider the so-called reciprocal matrices, i.e., tridiagonal matrices $A$ with a constant main diagonal and mutually inverse pairs $\left\{a_{i, i+1}, a_{i+1, i}\right\}$ of the off-diagonal entries, $i=1, \ldots, n-1$. Conditions are established for their Kippenhahn (a.k.a. numerical range generating) curves to (i) contain at least one ellipse, or (ii) consist entirely of ellipses. Note that these ellipses, if any, have real foci and are either centered at the origin or come in pairs symmetric about the $y$-axis. These results are then used to describe the numerical ranges (and, more generally, rank- $k$ numerical ranges) of some reciprocal matrices. This is a continuation of our earlier work (Special Matrices 10 (2022), 117-130).
(Received September 11, 2023)

## 1192-15-31061

Kennett Dela Rosa*, University of the Philippines Diliman. Zero-dilation indices and numerical ranges. Preliminary report. The zero-dilation index $d(T)$ of a matrix $T$ is the largest integer $k$ for which 0 is in the rank- $k$ numerical range of $T$. We consider the zero-dilation index of $m n$-by- $m n$ matrices of the following form $T=\left[\begin{array}{cccc}0 & & A & \\ B_{0} & B_{1} & \cdots & B_{m-1}\end{array}\right]$ where $A \in M_{m-1}\left(M_{n}(\mathbb{C})\right)$ and $B_{0}, B_{1}, \ldots, B_{m-1} \in M_{n}(\mathbb{C})$. This family of matrices includes block companion matrices. We show that if $A$ is block diagonal nonsingular, then $d(T)$ satisfies the following: if $m \geq 3$ is odd, then $\frac{(m-1) n}{2} \leq d(T) \leq \frac{(m+1) n}{2}$ (and this inequality is sharp) and if $m \geq 2$ is even, then $d(T)=\frac{m n}{2}$. Other related results will be presented. (Received September 11, 2023)

## 1192-15-31203

H. Tracy Hall*, Hall Labs, LLC. Hollow sign patterns with arbitrary traceless spectrum. Preliminary report.

We consider a generalization of spectrally arbitrary sign patterns and zero/nonzero patterns that is adapted to the category of simple digraphs. An $n \times n$ sign pattern or zero/nonzero pattern is called spectrally arbitrary when the characteristic polynomials of real matrices matching the pattern include all monic real polynomials of degree $n$. The $2 n$ Conjecture would imply that at least $2 n$ non-zero entries are required for a spectrally arbitrary pattern, rather than the critical dimension of $2 n-1$ nonzero entries implied by dimension counting under diagonal similarity. Sign patterns and zero/nonzero patterns are associated to digraphs that have no multiple edges but that do allow loops at the vertices. Restricting to simple digraphs is equivalent to considering hollow patterns, for which every diagonal entry is zero. Although a hollow pattern cannot be spectrally arbitrary, every spectrum is equivalent up to translation to a unique traceless spectrum, and a hollow pattern might have arbitrary traceless spectrum. In this case the critical dimension is $2 n-2$ nonzero entries, while the hollow equivalent of the $2 n$ Conjecture would require at least $2 n-1$ nonzero entries. We also discuss recent results, calculations, and questions related to the $2 n$ Conjecture and present an invariant, called the directed cycle fingerprint graph of a sign pattern or zero/nonzero pattern, that takes the form of a larger simple undirected graph and that recognizes many cases of nonisomorphic digraphs whose characteristic polynomials are equivalent.
(Received September 11, 2023)

Alan Krinik, California State Polytechnic University, Pomona, Hubertus Von Bremen*, California State Polytechnic University, Pomona. Diagonalization, Powers and the Exponential Matrix of some special banded matrices. Preliminary report. We consider special matrices that have three nonzero diagonals, one along the main diagonal and the other two placed symmetrically about the main diagonal. For these matrices we provide the eigenvalues and eigenvectors in closed form. We use these to diagonalize the matrices and generate closed form formulas for their powers and exponential matrix. We present examples illustrating the results.
(Received September 11, 2023)

## 1192-15-31429

Mark Hunnell*, Winston-Salem State University. Reconfiguration for Positive Semidefinite Zero Forcing.
Reconfiguration examines the relationship among solutions to a problem. The vertices of the reconfiguration graph are the feasible solutions to the problem, and there exists an edge between two vertices on the reconfiguration graph if the corresponding solutions can be transformed into one another by exactly one application of the reconfiguration rule. We study reconfiguration graphs of minimal positive semidefinite zero forcing sets under a variety of reconfiguration rules. We address questions related to the structure of these reconfiguration graphs, questions of realizability of graph families as reconfiguration graphs, and compare results across different reconfiguration rules. In addition, we develop an approach to address reconfiguration questions when the vertices of the reconfiguration graph are optimal sets of vertices with respect to a graph parameter.
(Received September 11, 2023)

## 1192-15-31472

Stephen Kirkland*, University of Manitoba. Completion Problems for Kemeny's Constant. Preliminary report. Given an irreducible stochastic matrix, Kemeny's constant measures the expected duration of a trip between randomly chosen states in the associated Markov chain. For this reason there is interest in identifying stochastic matrices for which the corresponding value of Kemeny's constant is low. In this talk we consider the problem of completing a partially specified stochastic matrix so as to minimize Kemeny's constant. We show that for any partially specified stochastic matrix for which the problem is well-defined, there is a minimizing completion that is as sparse as possible. We also discuss the completion problem in the case that all specified entries lie on the diagonal.
(Received September 11, 2023)

## 1192-15-31641

Rikhav Shah*, UC Berkeley. Subspace Iteration for Nonnormal Eigenvalue Problems.
Rayleigh-Ritz methods are a class of algorithms for computing some, usually the largest, eigenvalues of a matrix $A$. They do so by solving the eigenvalue problem for the much smaller matrix $Q^{*} A Q$ (called a compression of $A$ ) for some appropriately chosen rectangular $Q$. This talk discusses the stability of the eigenvalues of $Q^{*} A Q$, their relationship to those of $A$, and applications to the convergence of such algorithms.
(Received September 11, 2023)

## 1192-15-31691

L. Guth, MIT, Dominique Maldague*, MIT. A sharp square function estimate for the moment curve in $\mathbb{R}^{n}$. Preliminary report.
I will present recent work which proves sharp $L^{p}$ square function estimates for the moment curve $\left(t, t^{2}, \ldots, t^{n}\right)$, in the critical range $2 \leq p \leq \frac{n(n+1)}{2}+1$. We use a delicate inductive structure and multi-scale analysis inspired by decoupling. To prove the $n$-dimensional result, our argument requires sharp square function estimates for lower dimensional moment curves as well as sharp square function estimates for new shapes that we called $m$ th order Taylor cones. Our work recovers the $L^{4}$ square function estimate for the cone in $\mathbb{R}^{3}$ of Guth, Wang, and Zhang, as well as the $L^{7}$ square function estimate for the 3dimensional moment curve of Maldague. I will explain the differences of working in higher dimensions and with larger exponents $p$. This is joint work in progress with Larry Guth.
(Received September 12, 2023)

## 1192-15-31710

Giuseppe Cotardo*, Virginia Tech, Alain Couvreur, Inria Saclay-Île-de-France Research Centre. Products of MRD Codes. Over the past decade, there has been a renewed interest in studying the component-wise product of codes. This has established new connections between coding theory and additive combinatorics and has found applications in several areas, including secret sharing, multiparty computation, algebraic complexity theory, cryptanalysis, and code-based encryption schemes. While this approach has been widely considered in the context of Hamming-metric codes, it remains relatively unexplored for rank-metric codes. Inspired by the results of Mirandola and Zemor, in this talk we present a lower bound on the dimension of the product of MRD matrix rank-metric codes and we show that Gabidulin codes achieve this bound with equality. In the second part, we discuss the applications of our results in decoding algorithms and public key cryptography. (Received September 11, 2023)

1192-15-31722
Hermie Monterde*, University of Manitoba. Quantum walks on nonnegative matrices.
Let $X$ be a weighted graph. A Hamiltonian of a quantum system represented by $X$ is a Hermitian matrix $H$ that respects the adjacencies in $X$, i.e., $H_{u, v}=0$ if and only if there is no edge between vertices $u$ and $v$. Using $H$, one can then define a quantum walk on $X$ using the complex symmetric unitary matrix $U(t)=\exp (i t H)$, where $i^{2}=-1$ and $t \in \mathbb{R}$. We say that vertex $u$ is periodic in $X$ if $\left|U(\tau)_{u, u}\right|=1$ for some time $\tau>0$ and perfect state transfer (PST) occurs between vertices $u$ and $v$ in $X$ if $\left|U(\tau)_{u, v}\right|=1$ for some time $\tau>0$. In this talk, we discuss the implications on the quantum walk on $X$ whenever $H$ is
taken to be a symmetric nonnegative matrix. In particular, we investigate the role of the nonnegativity of $H$ in the characterizations of periodicity and PST in $X$.
(Received September 12, 2023)

1192-15-31736
L. Guth, MIT, Dominique Maldague*, MIT, John Urschel, MIT. Improved approximations for matrix 2 to $q$ norms in the hypercontractive regime ( $q>2$ ). Preliminary report.
Consider the general set-up of $\ell^{p}\left(\mathbb{R}^{m}\right)$ to $\ell^{q}\left(\mathbb{R}^{n}\right)$ boundedness of an $n$ by $m$ matrix. I will discuss the special case that $p=2$ and $q>2$, where it is known that approximating the matrix 2 to $q$ norm within an arbitrary factor is NP hard. I will explain how we approached this problem with an analyst's perspective and produced a new (polynomial time) algorithm which implies an overall improvement in the worst-case approximation factor for this problem. This is joint work in progress with Larry Guth and John Urschel.
(Received September 12, 2023)

## 1192-15-31774

Shaun M Fallat, University of Regina, H. Tracy Hall, Hall Labs, LLC, Rupert Levene, University College Dublin, Seth A
Meyer*, St. Norbert's College, Shahla Nasserasr, Rochester Institute of Technology, Polona Oblak, University of Ljubljana,
Helena Smigoc, University College Dublin. An example of a graph with a spectrum whose realization is unique.
Consider a simple graph $G$ and the class of symmetric matrices whose off diagonal zero-nonzero pattern agrees with the adjacency matrix of $G$, but whose diagonal is unrestricted. The inverse eigenvalue problem for graphs asks what multisets of eigenvalues can occur for matrices in this class. This talk will discuss a recent development, the invertible subtrees lemma, which takes combinatorial information about the graph and gives structural results on matrices achieving particular eigenvalue multiplicities. This is one of the necessary ingredients to prove that there exist trees that have realizable unordered multiplicity lists that can only be achieved by a unique list of eigenvalues (up to shifting and scaling).
(Received September 12, 2023)

## 1192-15-31782

Jonathan Niño-Cortes, University of Washington, Cynthia Vinzant*, University of Washington, Seattle. The convex algebraic geometry of higher-rank numerical ranges.
The higher-rank numerical range is a convex compact set generalizing the classical numerical range of square complex matrix, first appearing in the study of quantum error correction. In this talk, I will discuss some of the real algebraic and convex geometry of these sets, including a generalization of Kippenhahn's theorem, and describe an algorithm to explicitly calculate the higher-rank numerical range of a given matrix.
(Received September 12, 2023)

## 1192-15-31801

Kevin Vander Meulen*, Redeemer University. Inverse eigenvalue problems for sign patterns.
Inverse eigenvalue problems for sign patterns explore the nature of the possible eigenvalues of a real matrix based on the combinatorial sign structure of the matrix. An analytic technique using Jacobian matrices, originally developed to demonstrate a pattern is spectrally arbitrary, has been adjusted to deal with other classes of patterns, such as inertially arbitrary patterns. Variations on the technique have also provided information about the possible number of distinct eigenvalues of a pattern. Combinatorial techniques have also been developed, including the use of digraphs and matrix centralizers. We will review some of these techniques, some main results, as well as some recent developments, with a special focus on the number of distinct eigenvalues allowed by a pattern.
(Received September 12, 2023)

1192-15-31867
Zhaojun Bai, University of California, Davis, Ding Lu*, University of Kentucky. Joint numerical ranges in optimization and nonlinear eigenvalue problems. Preliminary report.
We discuss intrinsic connection between optimizations over joint numerical ranges and a class of eigenvector-dependent nonlinear eigenvalue problems (NEPv). By exploiting geometrical properties of joint numerical ranges, the NEPv is revealed as the first-order optimality condition for the optimization over joint numerical ranges. The connection allows a geometric interpretation of essential self-consistent-field (SCF) iteration for solving the NEPv and the development of a geometric algorithm to find the global optimizer. Numerical experiments will be used to demonstrate the effectiveness of our new approaches.
(Received September 12, 2023)

1192-15-31892
Adam H Berliner, St. Olaf College, Minerva Catral*, Xavier University, Dale D Olesky, University of Victoria, Pauline van den Driessche, University of Victoria,. Spectral properties of a structured matrix related to a system of second order ODEs. We consider real matrices of the form $C=\left[\begin{array}{cc}A & B \\ I & O\end{array}\right]$ where $A, B$ are square matrices and $I, O$ are the identity matrix and zero matrix, respectively. Such matrices arise from dynamical systems of second-order ordinary differential equations $\ddot{\mathbf{x}}=A \dot{\mathbf{x}}+B \mathbf{x}$ where $A$ and $B$ are real matrices of order $n$. Eigenvalue properties are studied for the sign pattern $\mathcal{C}=\left[\begin{array}{ll}\mathcal{A} & \mathcal{B} \\ \mathcal{D} & O\end{array}\right]$ of order $2 n$, where $\mathcal{A}, \mathcal{B}$ are the sign patterns of $A, B$ respectively, and $\mathcal{D}$ is a positive diagonal sign pattern.
This talk gives an overview of results from joint works with Adam Berliner, D.D. Olesky and P. van den Driessche. (Received September 12, 2023)

Victor Bailey, Georgia Institute of Technology, Yubin Gao, North University of China, Frank Hall, Georgia State University, Zhongshan Li*, Georgia State University. $4 \times 4$ Irreducible sign pattern matrices that require four distinct eigenvalues. A sign pattern matrix is a matrix whose entries are from the set $\{+,-, 0\}$. For a sign pattern matrix $A$, the qualitative class of $A$, denoted $Q(A)$, is the set of all real matrices whose entries have signs given by the corresponding entries of $A$. An $n \times n$ sign pattern matrix $A$ requires all distinct eigenvalues if every real matrix in $Q(A)$ has $n$ distinct eigenvalues. In the article "Sign patterns that require all distinct eigenvalues", JP J. Algebra Number Theory Appl., 2:2 (2002), 161-179, Li and Harris characterized the $2 \times 2$ and $3 \times 3$ irreducible sign pattern matrices that require all distinct eigenvalues, and established some useful general results on $n \times n$ sign patterns that require all distinct eigenvalues. In this paper, we characterize $4 \times 4$ irreducible sign patterns that require four distinct eigenvalues. This is done by characterizing $4 \times 4$ irreducible sign patterns that require four distinct real eigenvalues, that require four distinct nonreal real eigenvalues, or that require two distinct real eigenvalues and a pair of conjugate nonreal eigenvalues. The last case turns out to be much more involved. Some interesting open problems are presented. Three important tools that are used in the paper are the following: the discriminant of a polynomial; the fact that if a square sign pattern matrix $A$ requires all distinct eigenvalues then $A$ requires a fixed number of real eigenvalues; and the known result that if $A$ is a " $k$-cycle" sign pattern then for each $B \in Q(A)$, the $k$ nonzero eigenvalues of $B$ are evenly distributed on a circle in the complex plane centered at the origin. (Received September 12, 2023)

## 1192-15-31945

Marina Arav, Georgia State University, Frank Hall, Georgia State University, Zhongshan Li*, Georgia State University, Aram Mathivanan, Georgia State University, Jiamin Pan, Georgia State University, Hein Van der Holst, Georgia State University, Hanfei Xu, Georgia State University, Zheng Yang, Georgia State University. Advances on similarity via transversal intersection of manifolds.
Let $A$ be an $n \times n$ real matrix. As shown in the recent paper "The bifurcation lemma for strong properties in the inverse eigenvalue problem of a graph", Linear Algebra Appl. 648 (2022), 70-87, by S.M. Fallat, H.T. Hall, J.C.-H. Lin, and B.L. Shader, if the manifolds $\mathcal{M}_{A}=\left\{G^{-1} A G \mid G \in \mathrm{GL}(n, \mathbb{R})\right\}$ and $Q(\operatorname{sgn}(A))$ (consisting of all real matrices having the same sign pattern as $A$ ), both considered as embedded submanifolds of $\mathbb{R}^{n \times n}$, intersect transversally at $A$, then every superpattern of $\operatorname{sgn}(A)$ also allows a matrix similar to $A$. Those authors introduced a condition on $A$ (in terms of certain linear matrix equations) equivalent to the above transversality, called the nonsymmetric strong spectral property (nSSP). In this paper, this transversality property of $A$ is characterized using an alternative, more direct and convenient condition, called the similaritytransversality property (STP). Let $X=\left[x_{i j}\right]$ be a generic matrix of order $n$ whose entries are independent variables. The STP of $A$ is defined as the full row rank property of the Jacobian matrix of the entries of $A X-X A$ at the zero entry positions of $A$ with respect to the nondiagonal entries of $X$. This new approach makes it possible to take better advantage of the combinatorial structure of the matrix $A$, and provides theoretical foundation for constructing matrices similar to a given matrix while the entries have certain desired signs. In particular, several important classes of zero-nonzero patterns and sign patterns that require or allow this transversality property are identified. Examples illustrating many possible applications (such as diagonalizability, number of distinct eigenvalues, nilpotence, idempotence, semi-stability, eigenvalues and their algebraic and geometric multiplicities, Jordan canonical form, minimal polynomial, and rank) are provided. (Received September 12, 2023)

## 1192-15-31955

Angel Chavez*, Regis University. Norms on Complex Matrices Induced by Probability Distributions. The complete homogeneous symmetric polynomials of even degree define norms on the space of Hermitian matrices. These norms are peculiar in that they depend on the eigenvalues of a matrix and not its singular values. Interestingly, these norms are actually an instance of a more general family of norms on Hermitian matrices which are induced by probability distributions. We give an overview of these probabilistic norms and explore how they can be extended to all square complex matrices.
(Received September 12, 2023)

## 1192-15-31995

Xiangyi Zhu*, University of California, Irvine, Yizhe Zhu, University of California Irvine. Majority Dynamics and Projected Power Iterations on Stochastic Block Models.
Community detection in the stochastic block model is one of the central topics in network data science, and many methods have been studied to achieve information-theoretical thresholds in different sparsity regime. We investigate the performance of two very efficient iterative methods: majority dynamics and projected power iterations, for community detection in a 2 -block SBM $G(n, p, q)$ where each group has size $n$. Our simulation reveals that majority dynamics with a Gauss-Seidel or Jacobi update rule converge fast but lack stability (known as the phenomenon of "the power of few" in Erdős-Rényi random graphs), whereas projected power iterations are more stable but with a slower convergence rate. To overcome these limitations, we apply these two methods alternatively in each round with majority dynamics first for speeding up and projected iterations next to eliminate the instability. Our results demonstrate that the combined method achieves remarkable performance in very few rounds for the task of detection and exact recovery in different sparsity regimes. Compared to classical spectral method with running time $O\left(n^{2}\right)$ we conjecture our method achieves detection and exact recovery above the information-theoretical thresholds with time $O(n \log n / \log \log n)$.
(Received September 12, 2023)

1192-15-32009
Joseph Drapeau, Brigham Young University, Joseph Henderson, Brigham Young University, Peter Seely, Brigham Young University, Dallas Smith*, Utah Valley University, Benjamin Webb, Brigham Young University. Complete Equitable Decompostions.

Abstract: A classical result in spectral graph theory states that if a graph $G$ has an equitable partition $\pi$ then the eigenvalues of the associated divisor graph $G_{\pi}$ are a subset of the graph's eigenvalues. A natural question is whether it is possible to recover the remaining eigenvalues of $G$ using this method of creating divisor graphs. Here we show that any weighted undirected graph $G$ can be decomposed into a number of subgraphs each with a nontrivial equitable partition whose collective spectra contain the remaining eigenvalues. Using this decomposition we introduce an algorithm for finding the eigenvalues of a graph (symmetric matrix) with a nontrivial equitable partition. Under mild assumptions on the equitable partition we show that we can find eigenvalues of such a graph faster using this method when compared to the standard method. This is potentially useful as many real-world data sets have a nontrivial equitable partition.
(Received October 30, 2023)

## 1192-15-32179

Colin Garnett*, Black Hills State University. The Nonsymmetric Strong Multiplicity Property. Preliminary report.
Strong Properties for sign patterns have been of interest in studying sign patterns. The Strong Multiplicity Property was introduced in a paper by Shaun Fallat, Tracy Hall, Jephian Lin, and Bryan Shader. The property and other results in this paper relate to symmetric matrices. We will extend the Strong Multiplicity Property to a nonsymmetric Strong Multiplicity Property (nSMP) on nonsymmetric matrices and look at some of the implications of the nSMP.
(Received September 12, 2023)

## 1192-15-32270

Stephan Ramon Garcia, Pomona College, Gizem Karaali, Pomona College, Daniel J. Katz*, California State University, Northridge. Nonvanishing minors and uncertainty principles for Fourier analysis over finite fields.
Chebotarëv's theorem on roots of unity says that every minor of a discrete Fourier transform matrix of prime order is nonzero. We present a generalization of this result that includes analogues for discrete cosine and discrete sine transform matrices as special cases. This leads to a generalization of the Biró-Meshulam-Tao uncertainty principle to functions with symmetries that arise from certain group actions, with some of the simplest examples being even and odd functions. This new uncertainty principle gives a bound that is sharp and, for some classes of functions, stronger than that of Biró-Meshulam-Tao. (Received September 12, 2023)

1192-15-32296
Pan-Shun Lau, University of Nevada, Reno, Chi-Kwong Li, College of William and Mary, Raymond Nung-Sing Sze*, The Hong Kong Polytechnic University. Product of diagonal elements of Normal matrices. Preliminary report.
For positive integer $1 \leq k \leq n$, let

$$
W_{k}(A)=\prod_{i=1}^{k} x_{i}^{*} A x_{i}: x_{1}, \ldots, x_{k} \in \mathbf{C}^{n} \text { areorthonormal. }
$$

We showed that $W_{2}(A)$ is star-shaped when $A$ is a $3 \times 3$ normal matrix. This finding confirms a conjecture of Huang and Nakazato. The same result holds for any $n \times n$ essentially Hermitian matrix $A$.
(Received September 12, 2023)

1192-15-32396
Benjamin Clark*, Washington State University, Pietro Paparella, University of Washington Bothell. Polynomials that preserve nonnegative matrices.
In further pursuit of a solution to the celebrated nonnegative inverse eigenvalue problem, Loewy and London [Linear and Multilinear Algebra 6 (1978/79), no. 1, 83-90] posed the problem of characterizing all polynomials that preserve all nonnegative matrices of a fixed order. If $\mathscr{P}_{n}$ denotes the set of all polynomials that preserve all $n$-by- $n$ nonnegative matrices, then it is clear that polynomials with nonnegative coefficients belong to $\mathscr{P}_{n}$. However, it is known that $\mathscr{P}_{n}$ contains polynomials with negative entries. In this presentation, results for $\mathscr{P}_{n}$ with respect to the coefficients of the polynomials belonging to $\mathscr{P}_{n}$. This talk concludes with a characterization of $\mathscr{P}_{2}$ and places to purse further research.
(Received September 12, 2023)

## 1192-15-32700

Bryan Curtis*, Iowa State University. State of the Inverse Eigenvalue Community.
We will share the progress that has been made in the inverse eigenvalue problem and results on zero forcing during the last 3 years.
(Received September 12, 2023)

1192-15-32961
Kristin A Camenga, Juniata College, Douglas Knowles, Cornell University, Patrick X. Rault, Idaho State University, Rebekah B Johnson Yates*, Houghton University. The Dot and the Line: A Romance in Finite Field Numerical Ranges. Preliminary report.
Numerical ranges over finite fields were first introduced in 2016 in Coons, et al., which classified the numerical ranges of particular types of matrices over finite fields with certain prime numbers of elements; this classification was quickly generalized to all finite fields $\mathbb{F}_{q^{2}}$ where $q$ is a prime power (Ballico 2017,2018 ). The finite field setting introduces several complexities for numerical ranges, including vectors that are orthogonal to themselves. We present preliminary results in classifying the numerical ranges of specific families of matrices over finite fields and examining the geometry thereof, including some results for higher dimension matrices.
(Received September 13, 2023)

Jeffrey Stuart*, Pacific Lutheran University. k-potence of block triangular matrices. Preliminary report.
We investigate the structure of off-diagonal blocks of a block triangular k-potent matrix in Frobenius normal form. We develop necessary and sufficient conditions for a matrix in Frobenius normal form to be k-potent.
(Received September 13, 2023)

1192-15-33120
Tianjian Huang, UWCSEA, Haoxin Liu, asheville school, Ziqi WANG*, UWCSEA. Multiband Wavelet Based Parallel-CNN Image Classification Algorithms. Preliminary report.
Convolutional Neural Network (CNN) as a state of art tool has been widely used in the field of computer vision recently. There are two crucial steps in CNN: (1) Convolution operation being used to extract local features through convolution kernels and (2) the back propagation algorithm being used to train CNN and capture local features more efficiently. The cascade of multilayer CNN helps to concentrate global features of images and classify the images. As a state of art tool for both mathematics and its applications, wavelet can decompose signals into multiple components with different frequencies or scales. The components ideally decompose the variability of the data into physically meaningful and interpretable parts. In this research we propose a Discrete M-Band wavelet Transform (DMWT) based parallel CNN algorithm for image classification.
Decomposition. In our experiments, we applied 4-Band DWT to every $\mathrm{N} x \mathrm{~N}$ image matrix in our data set to decompose it into 16 N/4 x N/4 sub-images with one low-pass sub image and 15 high-pass sub images. Each image reflects different frequency and directional features of the image. Unlike CNN convolutional kernels, which pays more attention to local features. Every wavelet sub image has global feature or sematic information inherited from the original picture. We can then use CNN local features and Multiband wavelet multiresolution global features together to build a new type of Multiband Wavelet parallelCNN and improve the current CNN neural network and achieve better image classification results. Keywords: M-band wavelet transform, Multiview, CNN, Parallel computing.
(Received September 13, 2023)

1192-15-33122
Louis A Deaett*, Quinnipiac University. Matroid theory, zero forcing, and the minimum rank of patterns. Preliminary report. The zero forcing number gives a lower bound on the minimum rank associated with a graph. We highlight how this bound is analogous to the triangle number lower bound on the minimum rank of a matrix pattern. By placing the triangle number bound in the context of matroid theory, we show how it can be improved upon by exploiting the combinatorics of matroids. In particular, a purely combinatorial approach to lower-bounding the rank of a matrix with a specific zero-nonzero pattern is not limited to what the triangle number alone can provide. Returning to our original analogy, this raises the question of whether it is possible to use matroid theory to provide a combinatorial bound on the minimum rank of a graph that goes beyond that provided by the zero forcing number.
(Received September 13, 2023)

1192-15-33207
Gregory E Coxson*, ECE Department, U.S. Naval Academy, Walter Morris, Mathematics Department, George Mason University. Progress on Two Questions Involving Spectra of P-matrices. Preliminary report.
An $N \times N$ real-valued matrix is called a P-matrix if all of its principal minors are positives. One of the subclasses of P-matrices are the Mimes, known at one time as Hidden Minkowski matrices. An $N \times N$ matrix is a Mime if there exist Z-matrices $B$ and $C$ in $R^{N \times N}$ and a nonnegative vector $n \in R^{N}$ such that $A C=B, B u>0$ and $C u>0$. First, we prove that an $N$-degree polynomial

$$
f(x)=a_{0}+a_{1} x+\ldots+a_{N} x^{N}
$$

where the coefficients $a_{k}$ are all real is the characteristic polynomial of an $N \times N$ Mime if and only if $a_{k}(-1)^{k}>0$ for $k=0,1, \ldots, N$. Second, we consider a 1987 paper by Hershkowitz and Johnson in which conditions were developed for the existence of a diagonalizable P-matrix with spectrum $\left\{1,1, \ldots, 1, \omega, \omega^{2}\right\}$, with $\omega$ a complex root of $1+\lambda+\lambda^{2}$. Special attention was paid to the $5 \times 5$ case. We show that determining the existence of a solution for the $5 \times 5$ case reduces to finding ten vectors in the two-dimensional plane whose magnitudes and phase angles satisfy a set of fifteen inequality constraints and a set of four equality constraints.
(Received September 13, 2023)

## 1192-15-33304

Hans Matthew Riess*, Duke University. Algebraic foundations for planning in multi-agent systems. Preliminary report. In the intricate landscape of multi-agent systems, decentralized task coordination emerges as a complex mathematical challenge. Central to our investigation is tropical, i.e. max-plus, algebra, a structured mathematical framework that provides deep insights into the dynamics and behaviors of discrete event systems. Through rigorous mathematical formulations, beginning with cellular sheaf theory, we elucidate how max-plus algebra offers innovative solutions to the inherent challenges of multi-agent systems. Join us on a journey that showcases the intersection of mathematics with practical applications, highlighting the pivotal role of algebraic methodologies in multi-agent system planning.
(Received September 13, 2023)

1192-15-33486
Minh-Tam Quang Trinh, Massachusetts Institute of Technology, Michael Yang*, MIT PRIMES-USA. Rigidity and Rank of Group-Circulant Matrices.
Given a finite group $G$, a ring $\Lambda$, and a function $f: G \rightarrow \Lambda$, a $G$-circulant matrix of $f$ is a $|G| \times|G|$ matrix $M$ with rows and columns indexed by the elements of $G$ for which $M_{x y}=f(x y)$ for all $x, y \in G$. We study two different, fundamental properties of $G$-circulants when $\Lambda$ is an algebraically closed field with good characteristic. We begin by proving new results
about the matrix rigidity of $G$-circulants for nonabelian $G$, which are the first of its kind. We show that for any sequence of finite groups $G_{i}$ whose abelian normal subgroups have sufficiently small index, the family of $G_{i}$-circulants is not Valiant-rigid. Furthermore, we show that this result applies for families of groups $\left\{G_{i}\right\}_{i}$ whose representations are bounded above in degree. Next, we exhibit a formula for the rank of any $G$-circulant in terms of the decomposition of its corresponding function $f: G \rightarrow \Lambda$ into the matrix coefficients of the irreducible representations of $G$. While this was known to Diaconis, we present a more elementary proof that avoids the full strength of Schur Orthogonality. We then apply this formula to the case of $G$ circulants for cyclic $G$. Through this, we generalize a theorem of Zhangchi Chen, providing a necessary and sufficient criterion for when zero-one circulants are always nonsingular. Additionally, we answer an open problem about singular circulant digraphs posed by Lal-Reddy and give a probabilistic estimate for the regularity of zero-one singular circulant matrices. (Received September 24, 2023)

## 1192-15-33535

Joshua Ducey, James Madison University, Lauren Engelthaler, University of Dallas, Jacob Gathje*, College of Saint Benedict \& Saint John's, Brant Jones, James Madison University, Isabel Pfaff*, Oberlin College, Jenna Plute*, Texas A\&M University. The Sandpile Group of Subset Intersection Graphs. Preliminary report.
The sandpile group of a graph is a finite abelian group related to the graph's structure, whose order is equal to the number of spanning trees of the graph. We consider the Laplacian matrix of various subset intersection graphs, such as Kneser and Johnson graphs. Building on the work of Wilson and Bier, we construct matrices that conjugate this Laplacian matrix to a simplified upper triangular form that we can then fully diagonalize. This reveals the corresponding sandpile group.
(Received September 20, 2023)

1192-15-33691
Rohan Das*, BASIS Independent Silicon Valley, Christopher Qiu*, Bridgewater Raritan High School, Shiqiao Zhang*, MIT PRIMES. The distribution of the cokernel of a random integral symmetric matrix modulo a prime power. Preliminary report. Given a prime $p$ and positive integers $n$ and $k$, consider the ring $\mathrm{M}_{n}\left(\mathbb{Z} / p^{k} \mathbb{Z}\right)$ of $n \times n$ matrices over $\mathbb{Z} / p^{k} \mathbb{Z}$. Friedman and Washington computed the number of matrices in $\mathrm{M}_{n}\left(\mathbb{Z} / p^{k} \mathbb{Z}\right)$ with a given residue modulo $p$ and a given cokernel $G$ subject to the condition $p^{k-1} G=0$. Cheong, Liang, and Strand generalized this result by removing the condition $p^{k-1} G=0$, completing the description of the distribution of the cokernel of a random matrix uniformly selected from $\mathrm{M}_{n}\left(\mathbb{Z} / p^{k} \mathbb{Z}\right)$. In this paper, we investigate the distribution of the cokernel of a random symmetric matrix uniformly selected from $\mathrm{M}_{n}\left(\mathbb{Z} / p^{k} \mathbb{Z}\right)$. We prove a symmetric analogue of the result of Cheong, Liang, and Strand by adapting their methods. Our result leads to a refined version of a result of Clancy, Kaplan, Leake, Payne, and Wood. \par
(Received September 23, 2023)

## 1192-15-33797

Mika Campbell*, Spelman College. Understanding Neural Responses to our Moving World using Hypothesis - Free Voxel Decomposition.
Name: Mika Campbell Undergraduate Institution, Grad Year: Spelman College, 2024 Undergrad Major: Mathematics MIT Faculty Advisor: Dr. Nancy Kanwisher MIT Department: Department of Brain and Cognitive Sciences Project Title: Understanding Neural Responses to our Moving World using Hypothesis - Free Voxel Decomposition Mika Campbell ${ }^{1}$, Meenakshi Khosla ${ }^{2}$ and Nancy Kanwisher ${ }^{3}$ Department of Mathematics, Spelman College ${ }^{1}$ Department of Brain and Cognitive Sciences, Massachusetts Institute of Technology ${ }^{2,3}$ Previous work has already identified brain selectivities to static images of varying categories. In this project we ask, what high-level selectivities exist within the brain that allow us to respond to our natural, moving world? To answer this question, we used data-driven component modelling based on voxel-decomposition techniques like Bayesian and Sparse code nonnegative matrix factorization (NMF) algorithms separately, to analyze ventral stream fMRI responses to the dynamic naturalistic stimuli of the Algonauts 2021 data set (Cichy et al., 2021). Selectivities to social interaction, people and scenes were seen with this methodology, however the relatively low inter-subject consistency (Pearson's $\mathrm{R}=0.2-0.5$ ) noted within derived components suggests high inter-subject variability in the organizational dimensions; better inter-subject alignment technique or novel methods may be needed to evaluate the data set in the future. Strong correlations between the components returned by the Sparse and Bayesian NMF algorithms were observed, suggesting that the Bayesian model implicitly reflects a sparse representational structure, despite no explicit constraints. However, this conclusion is premature and would need to be compounded upon by further evidence. (Received September 26, 2023)

1192-15-33853
Chris L Camaño*, San Francisco State University. Randomized Algorithms for Efficient Tensor Network Contraction. Preliminary report.
The contraction of tensor networks is a crucial computational issue in various fields of quantum physics, including quantum information theory and condensed matter physics. As network complexity increases, many iterative algorithms become inefficient and prohibitively expensive. In response to this challenge, we introduce a novel randomized algorithm for contracting high-rank MPO and MPS tensor networks. Our algorithm uses a randomized sketch, represented as a random tensor embedding, to reduce computational overhead in complex networks while maintaining accuracy consistent with traditional non-randomized methods. By leveraging random sampling and decomposition, our approach avoids exhaustive computations and provides efficient tensor contraction approximations that are competitive with current methods. We demonstrate the utility of our algorithm through numerical experiments, specifically focusing on power iteration for ground state identification and Hamiltonian time evolution.
(Received September 26, 2023)
succession of five factorizations, ending with the Singular Value Decomposition (the greatest of all). We will prove the SVD in a new geometric way for 2 by 2 matrices - and derive it from the eigenvalues and eigenvectors of $\mathrm{A}^{\wedge}\{\mathrm{T}\} \mathrm{A}$ and $\mathrm{A} \mathrm{A}^{\wedge}\{\mathrm{T}\}$ for all matrices.
(Received November 03, 2023)

## 16 Associative rings and algebras

## 1192-16-26272

Fuxiang Yang*, University of California, San Diego. Invariant Theory and Group Coaction on Artin-Shelter Regular Algebra. In this paper, we studied group coactions on Artin-Schelter regular algebras which is a noncommutative generalization of the polynomial ring. More specifically, we studied the dual reflection groups of Artin-Schelter regular algebras. We classified the dual reflection groups of global dimension 2 Artin-Schelter regular domains. We showed that a dual reflection group of an Artin-Schelter regular algebra can be extended to a dual reflection group of an Ore extension using semidirect product. This allows us to produce more examples of dual reflection groups.
(Received July 24, 2023)

## 1192-16-27654

Joel A Shelton*, Tusculum University. Where are the Trinions: the search for 3-dimensional $\mathbb{R}$-algebras.
I will discuss the elusive idea of three-dimensional real division algebras and their history, which will be called "Trinions" for the purpose of this presentation. This will also include an introduction to algebras and their mathematical significance, while acknowledging the conclusion drawn from Frobenius' theorem - the non-existence of Trinions within the study of algebra. By unraveling the structure behind real division algebras and delving into Frobenius' theorem, this presentation seeks to provide a brief and introductory understanding of the constraints imposed on the existence of three-dimensional real division algebras. (Received August 17, 2023)

## 1192-16-28014

Kenneth Chan, University of Washington, Jason Gaddis*, Miami University, Robert Won, George Washington University,
James J. Zhang, University of Washington. Ozone groups of PI Artin-Schelter regular algebras.
The ozone group of an algebra $A$ is defined as the group of automorphisms of $A$ which fix every element of its center. In this talk, I will discuss the ozone group in the context of PI Artin-Schelter regular algebras and its applications to characterizing skew polynomial rings and their centers, and to the Zariski Cancellation Problem. Several examples will be given, including a classification of quadratic PI Artin-Schelter regular algebras in global dimension three with trivial ozone group. This is joint work with Kenneth Chan, Robert Won, and James J. Zhang.
(Received August 24, 2023)

1192-16-28080
Jurij Volčič*, Drexel University. Free Bertini's theorem and isospectrality of noncommutative polynomials.
The simplest version of Bertini's irreducibility theorem states that the generic fiber of a non-composite polynomial function is an irreducible hypersurface. This talk presents the analog of Bertini's theorem in a free associative algebra. One of its consequences pertains to evaluations of noncommutative polynomials on tuples of matrices. Two noncommutative polynomials are isospectral if at each matrix tuple, their evaluations have the same spectrum. An algebraic criterion for isospectrality is given, which together with factorization theory for free algebra then leads to a better understanding of isospectral pairs of noncommutative polynomials.
(Received August 25, 2023)

## 1192-16-28226

Eric Swartz, William \& Mary, Nicholas J. Werner*, SUNY at Old Westbury. The Covering Numbers of Rings.
A cover of a ring $R$ is a collection of proper subrings of $R$ whose union is all of $R$, and the covering number $\sigma(R)$ of $R$ is the cardinality of a minimal cover (if one exists). In this talk, we will survey the history of covering number problems for rings and other algebraic structures such as groups, and will discuss how to determine $\sigma(R)$ for any ring $R$ with a finite covering number. Key to our work is the notion of a $\sigma$-elementary ring, which is a ring $R$ such that $\sigma(R)<\sigma(R / I)$ for all nonzero twosided ideals $I$ of $R$. Evidently, if $R$ admits a finite cover, then $\sigma(R)$ is equal to $\sigma(R / I)$ for some $\sigma$-elementary residue ring $R / I$ of $R$. Our main result is a full classification of all $\sigma$-elementary rings with a finite covering number. Using these rings, we can determine $\sigma(R)$ for any ring $R$ admitting a finite cover. Moreover, we prove that almost all positive integers are not the covering number of a ring.
(Received August 28, 2023)

1192-16-28229
Yariana Diaz*, Macalester College. Total stability and Auslander-Reiten theory for Dynkin quivers.
This paper concerns stability functions for Dynkin quivers, in the generality introduced by Rudakov. We show that relatively few inequalities need to be satisfied for a stability function to be totally stable (i.e. to make every indecomposable stable). Namely, a stability function $\mu$ is totally stable if and only if $\mu(\tau V)<\mu(V)$ for every almost split sequences $0 \rightarrow \tau V \rightarrow E \rightarrow V \rightarrow 0$ where $E$ is indecomposable. These can be visualized as those sequences around the "border" of the Auslander-Reiten quiver.
(Received August 29, 2023)

Ruiqi Lin*, Wake Forest University. Invariants of Hopf actions on skew polynomial algebras. Preliminary report.
Let $\mathbb{k}$ be a field of characteristic $p>0$, and let $U$ be the restricted enveloping algebra of the 2 -dimensional nonabelian solvable Lie algebra; $U$ is a non-semisimple Hopf algebra of finite representation type with no-nontrivial grouplike elements H.-X. Chen, D.-G. Wang, and J.J. Zhang have classified all 2- and 3-dimensional quadratic skew polynomial algebras $T$ on which $U$ acts non-trivially. We compute the associated invariant subrings $T^{U}$ and explore their properties, including whether $T^{U}$ is again a skew polynomial ring, as well as bounds on the degrees of a minimal set of generators of $T^{U}$.
(Received August 29, 2023)

## 1192-16-28304

Ellen E Kirkman, Wake Forest University, W Frank Moore, Wake Forest University, Tolulope Oke*, Wake Forest University. Inner-faithful actions of the Drinfeld double of a finite group. Preliminary report. Group actions are very popular in mathematics. Many mathematical objects are best understood by their actions or the kind of objects that act on them. For example, it is of interest to study the actions of finite dimensional semisimple Hopf algebras, and in particular investigate whether they act on algebras with "nice" homological properties. In this talk, I will give a brief description and an example of the inner-faithful action of the Drinfeld double of a finite group on an Artin-Schelter regular algebra.
(Received September 05, 2023)

1192-16-28610
Amrita Acharyya*, University of Toledo. Galois cohomology to analyze crossed modules, quaternion division algebra. \document Group cohomology is intimately connected with algebraic structures. For example, if $G$ is a finite group, then $H^{2}\left(G, K^{*}\right)$, where $K^{*}=K \backslash 0$ classifies "twisted" forms of the group ring $K[G]$. This is a very useful construction in Galois theory: if $G=G a l(K / k)$, then the twisted rings are central simple $k$-algebras representing elements of the relative Brauer group $\operatorname{Br}(K / k)$. For degree three group cohomology, a close analog of twisted group rings is provided by linear Grcategories. If $G$ is, say, a finite group, then $C(G, \omega, k)$ is a category of $G$-graded $k$-vector spaces, but where the usual (graded) tensor product is twisted by correcting the usual canonical associativity of three factors by a scalar linear map $\omega$. To satisfy the axioms, $\omega$ must represent a class in $H^{3}\left(G, k^{*}\right)$, and it is important in applications that $\omega$ be explicitly known. Of interest are the algebras in linear Gr-categories, and in particular the division algebras. We analyze a crossed module structure induced by a central division algebra. In this work we use an explicit example of a non-crossed product division algebra that provides a non commutative structure which is an iterated twisted function field over a quaternion division algebra D that is defined over the number field $Q(\sqrt{3}, \sqrt{-7})$. - ize
- joint work with Ettore Aldrovandi \enditemize \enddocument (Received September 01, 2023)


1192-16-28824
Aleksandr Kleyn*, AMS. Linear Map of Module over Non-commutative Algebra. Preliminary report. Let $A$ be associative $D$-algebra. Let $\overline{\bar{e}}_{1}=\left(\begin{array}{lll}e_{11} & \ldots & e_{1 n}\end{array}\right)$ be a quasibasis of $A$-module $V_{1}$ of columns. Let $\overline{\bar{e}}_{2}=\left(\begin{array}{lll}e_{21} & \ldots & e_{2 m}\end{array}\right)$ be a quasibasis of $A$-module $V_{2}$ of columns. Linear map of $A$-module $V_{1}$ into $A$-module $V_{2}$ has form

$$
\left(\begin{array}{c}
w^{1} \\
\ldots \\
w^{m}
\end{array}\right)=\left(\begin{array}{ccc}
a_{1}^{1} & \ldots & a_{n}^{1} \\
\ldots & \ldots & \ldots \\
a_{1}^{m} & \ldots & a_{n}^{m}
\end{array}\right) \circ\left(\begin{array}{c}
v^{1} \\
\ldots \\
v^{n}
\end{array}\right) \quad a_{j}^{i} \in \mathcal{L}(D ; A \rightarrow A)
$$

with respect to quasibases $\overline{\bar{e}}_{1}, \overline{\bar{e}}_{2}$. Entries of matrix $a$ are unique iff $\overline{\bar{e}}_{1}, \overline{\bar{e}}_{2}$ are bases. \par (Received September 03, 2023)

1192-16-29476
K. R. Goodearl*, UCSB, James J. Zhang, University of Washington. Poisson fields. Preliminary report.

We will discuss fields with Poisson structures. Even the smallest fields that can support nonzero Poisson structures, namely rational function fields in two variables, have enormously many non-isomorphic such structures. Existence, invariants, and some classifications will be discussed.
(Received September 06, 2023)

## 1192-16-29486

Andres Barei, UCSB, Birge Huisgen-Zimmermann*, University of California at Santa Barbara, Ashwin Trisal, UCSB. Finite dimensional representations of quivers with oriented cycles. Preliminary report.
We present a slice of our work targeting the category of finite dimensional $K Q$-modules, where $K$ is a field and $Q$ a quiver with oriented cycles. We tackle this category by exploring the representations of Loewy length $\leq L$, i.e., the modules over the truncation $\Lambda_{L}=K Q /\langle$ the paths of length $L\rangle$ for $L \gg 0$. Our main results concern invariants of the family $\left(\Lambda_{L}\right)_{L \geq 2}$ and the satellite family of strongly tilted algebras $\left(\widetilde{\Lambda_{L}}\right)_{L \geq 2}$.
(Received September 06, 2023)
inner functional structure of biologic systems. These patterns correspond to the basis elements of a Lie algebra sl(2,R) of a special linear group and basis elements of the imaginary part of coquaternion. Coquaternion and Lie algebra operations suggest that the base patterns of a biologic system including an element corresponding to environment can serve as base units for the space of regulatory elements of biologic systems. Closed algebraic structure of sl(2,R) and coquaternion suggests closed relatively autonomous functional properties of biologic systems regulated by the base patterns and their combinations. Some functional (physiologic) interactions of base PNR patterns organized in a sequence can be demonstrated in regulations of the system's behavior. As a dissipative structure a biologic system has open links with the environment. It determines the existence of "unstable" divergent flow trajectories showing how the system evolves in time. Thus, non- equilibrium properties of the system's dynamics are reflected in spiral trajectories propagating along coordinate axes and lying on the surfaces related to the quadratic form of coquaternion. Trajectories follow hyperboloids of two sheets then one sheet hyperboloid and move further to a hyperboloid of two sheets and so on. Trajectories may form loops. Each time changing the surfaces of hyperboloids the trajectories are crossing the surface of a double cone. (Received September 07, 2023)

## 1192-16-29770

Jacob Laubacher*, St. Norbert College. Secondary Hochschild homology and differentials.
In this talk, we present a generalization of Kähler differentials which ultimately correspond to the secondary Hochschild homology associated to a commutative triple. As our main goal, we establish computations in low dimension. (Received September 07, 2023)

## 1192-16-30080

Manuel L. Reyes*, UC Irvine, Daniel S Rogalski, UCSD. When are Koszul algebras domains?. Preliminary report. Based on classical results in commutative algebra, it is natural to expect that a connected graded algebra or local ring with good homological properties should have no zero-divisors. However, such theorems are often difficult to prove for noncomutative rings unless one assumes very strong hypotheses. In this talk we will discuss a homological condition, stated in terms of the Ext algebra, that is necessary and sufficient for a Koszul algebra to be a domain. Similar methods yield a sufficient homological condition for a noncommutative local ring to be a domain and for a Koszul ring to be prime.
(Received September 08, 2023)

## 1192-16-30110

Zain Qudsi*, Clemson University. Solving the $n$-color ice model.
Lattice models arose in statistical mechanics and today enjoy a wide variety of applications across mathematics. Solvable lattice models are those which have a solution to the Yang-Baxter equation, which guarantees certain symmetries on associated "partition" functions. Here we explore $n$-color ice-type models, which in the $n=2$ case reduce to statistical mechanical models of 2D ice sheets. Our models are related to lattice models studied by Perk and Schultz. We give a necessary and sufficient criterion for the existence of solutions to the Yang-Baxter equation for any $n$, extending the work of Baxter and of Brubaker, Bump, and Friedberg. A subset of these solutions has connections to quantum groups like $U_{q}\left(\widehat{\mathfrak{s l}}_{n}\right)$. We also demonstrate invariance of these solutions under some natural transformations, including Drinfeld twists. This work was done as part of the Polymath Jr. 2022 program.
(Received September 08, 2023)

1192-16-30230
Toshitaka Aoki, Kobe University, Emerson G. Escolar, Kobe University, Shunsuke Tada*, Kobe University. Posets whose persistence modules are always interval decomposable and homological invariants.
In one-parameter persistent homology analysis, it is a fundamental fact that every persistence module can be decomposed into interval modules due to Gabriel's theorem, where each interval module is of the form $\mathrm{I}[b, d]$ with $b \leq d$. When dealing with multi-parameter settings, however, there are some difficulties with adapting the same techniques. Recently, there has been growing interest in the use of relative homological algebra to develop invariants using interval covers and interval resolutions (i.e., right minimal approximations and resolutions relative to interval-decomposable modules) for multi-parameter persistence modules. In this talk, we study persistence modules over finite posets. Firstly, we provide another family of posets $C_{m, n}$ (which we call commutative cycles) indexed by two positive integers $m, n$ satisfying the condition that every persistence module over it can be decomposed into interval modules. Conversely, we show that every poset satisfying that condition is either type $A$ quiver or our commutative cycle. Similar to the one-parameter setting, one can consider the notion of persistence diagrams for the commutative cycle. Secondly, we show that over finite posets, the restriction of interval covers of persistence modules to each indecomposable direct summand is injective. This result suggests a way to simplify the computation of interval covers. This talk is based on joint work with Toshitaka Aoki and Emerson G. Escolar.
(Received September 09, 2023)

1192-16-30239
Emily Cliff, University of Sherbrooke, Colin Ingalls*, Carleton University, Charles Paquette, Royal Military Coloege. Quasiuniversal representations and generic bricks. Preliminary report.
This is joint work with Emily Cliff and Charles Paquette. Given a quiver, a dimension vector and a stability condition, Alistair King shows that, if the dimension vector is unimodular, there is a moduli space of stable representations with a universal representation. We extend this result to show that even when the dimension vector is divisible, the moduli space constructed by King has a quasi-universal representation. We are able to use the quasi-universal representation to construct generic bricks in several cases.
(Received September 09, 2023)

Be'eri Greenfeld*, University of Washington. Algebraicity, torsion, growth and freeness.
The Kurosh Problem asks whether a finitely generated algebraic algebra can be infinite-dimensional. Its group-theoretic counterpart, the Burnside Problem, asks whether a finitely generated torsion group can be infinite. We examine the Kurosh Problem under growth restrictions, proving a strong quantitative version of it. Namely, every growth function of an algebra occurs as the growth of an algebraic algebra, up to a polynomial error factor. Applications to questions on nil rings, GKdimension and Lie algebras are derived. In many algebraic contexts, the largest objects are those containing free sub-objects; we discuss to what extent an "almost algebraic algebra" or an "almost torsion group" can contain free substructures from the perspectives of random walks, probabilistic identities and graded rings.
(Received September 10, 2023)

1192-16-30655
Van C Nguyen*, U.s.Naval Academy, Oana Veliche, Northeastern University. Iterated Mapping Cones on the Koszul Complex.
In this talk, we construct iterated mapping cones built on the Koszul complex over a complete intersection ring $R$. We establish exact sequences involving direct sums of the components of the Koszul homology, and express the images of the maps of these sequences as homologies of such iterated mapping cones. As an application of this iterated mapping cone construction, we recover a minimal free resolution of the residue field over $R$, independent from the well-known resolution constructed by Tate by adjoining variables and killing cycles. Through our construction, the differential maps can be expressed explicitly as blocks of matrices, arranged in some combinatorial patterns.
(Received September 10, 2023)

1192-16-30660
Emre Sen, University of Iowa, Gordana Glisa Todorov*, Northeastern University, Shijie Zhu, Nantong University. On Higher Auslander Algebras. Preliminary report.
One of the well-known theorems of Maurice Auslander about artin algebras describes the correspondence between algebras A of finite representation type and algebras B, with gl.dim. $\mathrm{B} \leq 2 \leq \operatorname{dom} . \operatorname{dim}$. B which are now called Auslander algebras. Higher Auslander algebras were introduced by O. Iyama as algebras C, with gl.dim. $\mathrm{C} \leq \mathrm{k} \leq$ dom.dim.C and it was also shown that there is correspondence between higher representation finite algebras and higher Auslander algebras (with certain equivalences - there is more precise statement). I will talk about recent work of Emre Sen who had created several approaches for constructing new families of higher Auslander algebras and as a consequence he obtained new higher representation finite algebras. In addition to that Emre, Shijie Zhu and I, have a joint paper in which we have another method of constructing higher Auslander algebras in the family of Nakayama algebras. Higher Auslander algebras, higher representation finite algebras and higher cluster tilting modules are still not well understood, even in the families of well known classes of algebras, so creating families of such will be useful.
(Received September 10, 2023)

## 1192-16-30676

Peter Goetz*, Humboldt State University. Frobenius Extensions in Noncommutative Invariant Theory. Preliminary report. Let $k$ be a field and let $B \subset A$ be an extension of $k$-algebras. If $A$ is a free $B$-module of finite rank and $A$ is isomorphic to $\operatorname{Hom}_{B}(A, B)$ (as $(A, B)$-bimodules), then $A$ is a Frobenius extension of $B$. If a finite group $G$ grades (in a particular way) an Artin-Schelter regular algebra $A$, and $A_{e}$ is the homogenous component associated to the identity element $e \in G$, then $A_{e} \subset A$ is a Frobenius extension. I will sketch a proof of this fact. Then I will talk about the extent to which this result holds for other types of Hopf algebra actions on regular algebras. \endabstract \enddocument (Received September 10, 2023)

## 1192-16-30721

Keir Lockridge, Gettysburg College, Jacob Elijah Terkel*, Gettysburg College. Fuch's Problem For Linear Groups. Which groups can occur as the group of units in a ring? Such groups are called realizable. Though the realizable members of several classes of groups have been determined (e.g., cyclic, odd order, alternating, symmetric, finite simple, indecomposable abelian, and dihedral), the question remains open. The general linear groups are realizable by definition: they are the units in the corresponding matrix rings. In this paper, we study the realizability of the special linear and affine general linear groups. In particular, we determine which special linear groups of degree 2 over a finite field are realizable by a finite ring, and we determine which affine general linear groups of degree 1 over a cyclic group are realizable by a finite ring. We also give partial results for certain linear groups of other degrees and for rings of characteristic zero. (Received September 10, 2023)

1192-16-30959
Harshit Yadav*, Rice University. Near-group fusion categories. Preliminary report.
Near-group fusion categories are certain fusion categories whose fusion rings are a small generalization of group rings. The existence and construction of such categories is an ongoing endeavor. In this talk, we will discuss some ongoing work with Terry Gannon in this direction.
(Received September 11, 2023)

1192-16-31115
Zhongkai Mi, Louisiana State University, Quanshui Wu, Fudan University, China, Milen Tchernev Yakimov*, Northeastern University. The lowest discriminant ideal of Cayley-Hamilton Hopf algebras.
Discriminant ideals of noncommutative algebras carry important information about their representation theory, automorphisms and isomorphisms. Despite a substantial amount of work on discriminants, very little is known about general discriminant ideals. We will describe a general study of the lowest discriminant ideals of Cayley-Hamilton Hopf algebras in the sense of De Concini, Reshetikhin, Rosso and Procesi, whose identity fiber algebras are basic. The results will be illustrated
with applications to group algebras of central extensions of abelian groups, big quantum Borel subalgebras at roots of unity and quantum coordinate rings at roots of unity.
(Received September 11, 2023)

## 1192-16-31116

W Frank Moore*, Wake Forest University, Kenta Ueyama, Shinshu University. Quantum Determinants and Invariants of Group Actions on Skew Polynomial Algebras. Preliminary report.
We provide a generalization of the notion of determinant which allows one to easily compute the homological determinant of an automorphism $\varphi$ of a skew polynomial ring. In addition, we use this notion to provide a generalization of the classical formula that expresses the trace series of $\varphi$ as the reciprocal of a polynomial related to the characteristic polynomial of $\varphi$. We then use this generalization to give classifications of classical and mystic reflections and bireflections of skew polynomial rings.
(Received September 11, 2023)

## 1192-16-31195

Wenjun Niu*, Perimeter Institute for Theoretical Physics. Non-semisimple braided tensor categories from twisted SUSY gauge theories. Preliminary report.
Supersymmetric (SUSY) twisting is a prominent way of obtaining topological quantum field theories (QFT). In this session, I will report on constructions of non-semisimple braided tensor categories that are motivated by topological twists of $3 \mathrm{~d} N=4$ SUSY gauge theories. This construction uses representation theory of vertex operator superalgebras. Time permit, I will report on the Kazhdan-Lusztig correspondence for these vertex operator superalgebras.
(Received September 11, 2023)

1192-16-31226
Sam Gunningham, Montana State University, David Jordan, University of Edinburgh, Monica Vazirani*, UC Davis. Skeins on Tori. Preliminary report.
We study skeins on the 2-torus and 3-torus via the representation theory of the double affine Hecke algebra. As an application we can compute the dimension of the generic $S L_{N^{-}}$and $G L_{N}$-skein module of the 3 -torus for arbitrary N .
(Received September 11, 2023)

Ilani Axelrod-Freed, Massachusetts Institute of Technology, Claire Frechette*, Boston College, Veronica Lang, Smith College. Measuring the Space of Whittaker Functions using General Metaplectic Ice.
Local Whittaker functions for principal series representations of reductive groups play an integral role in number theory and representation theory, and many of their applications extend to the metaplectic case, where reductive groups are replaced by their metaplectic covering groups. We will examine metaplectic Whittaker functions for covers of $G L_{r}(F)$ through the lens of a solvable lattice model, which provides a quantum group backbone for the behaviour of these analytic objects. In 2010, Brubaker-Bump-Chinta-Friedberg-Gunnells created such a model for one particularly nice cover of $G L_{r}(F)$ for a local field $F$, which was proved solvable in 2018 by Brubaker-Bump-Buciumas and thus provided an isomorphism between the space of Whittaker functions of this cover and a particular quantum group module. Generalizing their work, we prove that such a solvable lattice model exists for all metaplectic covers of $G L_{r}(F)$ and identify the associated quantum group module. Our generalization incorporates additional complexity arising from the center of the group, which was hidden in the particular case, with the added result that the map to the quantum group module is no longer necessarily an isomorphism. However, using combinatorial linear algebra, we may construct two precise formula for the dimension of the space of metaplectic Whittaker functions for an arbitrary cover of $G L_{r}(F)$ and thereby determine when this map remains an isomorphism. (Received September 11, 2023)

## 1192-16-31590

Jason P Bell*, University of Waterloo. Filtered deformations of commutative domains of Krull dimension two.
Let $k$ be a field of positive characteristic and let $A$ be a filtered deformation of a commutative finitely generated $k$-algebra $B$ that is an integral domain. Etingof has asked whether $A$ satisfies a polynomial identity and we show that this is the case when $B$ has Krull dimension at most two.
(Received September 11, 2023)

## 1192-16-31714

Padmini Veerapen*, Tennessee Tech University. Twisted Homogeneous Coordinate Rings of Quadrics.
\document Twisted Homogeneous Coordinate Rings of Quadrics Padmini Veerapen We classify Artin-Schelter regular algebras of dimension four with Hilbert series the same as that of the polynomial ring on four variables that map onto twisted homogeneous coordinate rings of a rank-two quadrics. In doing so, we extend work of Shelton, Vancliff, and Van Rompay on twisted homogeneous coordinate rings of rank-three and rank-four quadrics. This is joint work with R. Chandler, H. Tran and X. Wang. \enddocument
(Received September 12, 2023)

## 1192-16-31716

Daniel S Rogalski*, UCSD, Robert Won, George Washington University, James J. Zhang, University of Washington. Homological Integrals for Weak Hopf Algebras. Preliminary report.
The integral is an important structure in a finite-dimensional Hopf algebra. Lu, Wu, and Zhang generalized this to define a homological integral for any Artin-Schelter Gorenstein Hopf algebra. This homological integral has many applications in the
study of Hopf algebras of small GK-dimension. A weak Hopf algebra is a generalization of a Hopf algebra in which the comultiplication does not necessarily preserve the unit. Weak Hopf algebras arise naturally in the study of tensor categories. We report on work in progress that defines a homological integral for an AS Gorenstein weak Hopf algebra.
(Received September 12, 2023)

1192-16-31741
Niklas Kelly Garner*, University of Washington, Seattle. Vertex Algebras and Non-semisimple Chern-Simons TQFTs.
Preliminary report.
I will describe aspects of certain non-semisimple 3-dimensional topological quantum field theories by way of suitable boundary vertex operator algebras, focusing on simple examples arising from supersymmetric quantum field theories. Time permitting, I will relate these boundary vertex algebras to super Hopf algebras for a family of abelian gauge theories. (Received September 12, 2023)

192-16-31748
Hongdi Huang, Rice University, Van C. Nguyen, U.S. Naval Academy, Charlotte Ure, Illinois State University, Kent B. Vashaw*, Massachusetts Institute of Technology, Padmini Veerapen, Tennessee Tech University, Xingting Wang, Louisiana State University. Twisting Manin's universal quantum groups and comodule algebras.
We consider 2-cocycle twists (and more generally, Morita-Takeuchi equivalences between) Manin's universal quantum groups and their comodule algebras. We show when Zhang twists of connected graded algebras can be realized as cocycle twists, thus concretely connected the (graded) representation theory of an algebra A to the corepresentation theory of its universal quantum group. We also prove that fundamental properties of noncommutative associative algebras, such as Artin-Schelter regularity and Koszuality are preserved under 2-cocycle twist. This is joint work with Hongdi Huang, Van Nguyen, Charlotte Ure, Padmini Veerapen, and Xingting Wang. (Received September 12, 2023)

## 1192-16-32048

Cody Gilbert*, Saint Louis University. Moduli of Representations of Clannish Algebras.
We prove irreducible components of moduli spaces of semistable representations of skewed-gentle algebras, and more generally, clannish algebras, are isomorphic to products of projective spaces. This is achieved by showing irreducible components of varieties of representations of clannish algebras can be viewed as irreducible components of skewed-gentle algebras, which we show are always normal. The main theorem generalizes an analogous result for moduli of representations of special biserial algebras proven by Carroll-Chindris-Kinser-Weyman.
(Received September 12, 2023)

1192-16-32448
Andrew B Conner*, Saint Mary's College of California, Peter Goetz, Humboldt State University. Algebras Associated to Sequences of Truncated Point Schemes. Preliminary report.
Let $\mathbb{k}$ be an algebraically closed field. Let $\mathscr{Z}=\left\{Z_{d} \subset\left(\mathbb{P}^{n}\right)^{\times d}\right\}$ be a sequence of subschemes having the property that $\pi_{1}\left(Z_{d}\right), \pi_{2}\left(Z_{d}\right) \subset Z_{d-1}$, where $\pi_{1}, \pi_{2}$ project onto the first and last $d-1$ factors of $\left(\mathbb{P}^{n}\right)^{\times d}$, respectively. Artin-Tate-van den Bergh described a canonical way to associate a graded algebra $B$ to the sequence $\mathscr{Z}$. Of principal interest in noncommutative projective geometry is the following special case: Let $T$ be a free associative $\mathbb{k}$-algebra, and $I \subset T$ a homogeneous ideal. Let $Z_{d}$ be the subscheme of zeros of elements of $I_{d}$, viewed as multilinear forms on $\left(T_{1}^{*}\right)^{\times d}$. If $A$ is a quadratic, three-dimensional Artin-Schelter (AS) regular algebra, then $Z_{d} \cong Z_{2}$ for all $d \geq 2$, and $Z_{2}$ is the graph of an automorphism $\sigma$ of $E=\pi_{1}\left(Z_{2}\right)$. In that case, $B$ is isomorphic to the twisted homogeneous coordinate ring of $(E, \sigma)$, and $A$ can be recovered from $B$. When $A$ is a non-degenerate 4-dimensional Sklyanin algebra, also an AS-regular algebra, $A$ cannot be recovered from its twisted homogeneous coordinate ring; however, $A$ can be recovered from $B$. In this talk we will discuss the ring structure of $B$, and describe its relationship to $A=T / I$ and to the twisted homogeneous coordinate ring construction.
(Received September 12, 2023)

1192-16-32575
Erin Delargy*, Binghamton University, Rylie Harris*, Wesleyan University, Jiachen Kang*, University of Michigan, Bryan Lu*, Cornell University, Ramanuja Charyulu Charyulu Telekicherla Telekicherla Kandalam*, University of Minnesota Twin Cities. $\mathfrak{S}_{n}$-equivariant Koszul algebras from the Boolean lattice. Preliminary report.
Koszul algebras are a class of quadratic algebras with particularly rich structure. To any Koszul algebra we can associate to it a (generally noncommutative) Koszul dual algebra which inherits any group action on the original algebra. We study two $\mathfrak{S}_{n^{-}}$ equivariant Koszul algebras arising from the Boolean lattice: the Chow ring of the Boolean matroid and the "colorful ring" associated to the barycentric subdivision of a simplex. The dimensions of the graded pieces of both of these algebras correspond to the Eulerian numbers, but the bases and representations for the graded pieces of the two algebras are vastly different. Here we compare and contrast the bases and representations for each of these algebras and their Koszul duals. Along the way, we investigate (sometimes noncommutative) Gröbner bases for these algebras and their Koszul duals, and we prove branching rules for representations of the symmetric group which categorify a recursion on the Eulerian numbers. (Received September 12, 2023)

1192-16-33149
Dalia Artenstein, Universidad de la República de Uruguay, Janina C Letz, Bielefeld University, Amrei Oswald*, University of Iowa, Sibylle Schroll, University of Cologne, Andrea Solotar, Universidad de Buenos Aires. Hochschild cohomology of monomial path algebras. Preliminary report.
We investigate the Gerstenhaber algebra structure of the Hochschild cohomology of triangular monomial algebras. First, we
determine a diagonal map on Bardzell's resolution for monomial path algebras. Then, we restrict our attention to triangular monomial path algebras and describe the cup product on their Hochschild Cohomology. This is joint work with Dalia
Artenstein, Janina Letz, Sibylle Schroll, and Andrea Solotar.
(Received September 13, 2023)

Yvan Grinspan*, Amherst College, Alex Kupersmith*, Amherst College, Seth Yoo*, Amherst College. Classification and Ideal Lattices of Leavitt Path Algebras. Preliminary report.
A Leavitt path algebra is a universal algebra associated to a directed graph, with the generators being the various paths on the graph. In this project, we study the connections between the algebraic properties of Leavitt path algebras and the graphtheoretic properties of their associated directed graphs. Our focus was how the traits of a given directed graph relate to the algebraic properties of its Leavitt path algebra, with an emphasis on the structure of ideal lattices and the distinction between graded and non-graded ideals. In conducting this research, we wrote a new proof for an existing theorem, produced a program that receives directed graphs as input and creates diagrams of their ideal lattice structures, and proved various results about the Leavitt path algebras of directed graphs with two vertices.
(Received September 20, 2023)

1192-16-33724
Ethan Yiheng Liu*, The Harker School. On the structure and generators of the nth-order chromatic algebra. Preliminary report.
The chromatic algebra is a method to study the close connections of chromatic polynomials to physics applications, such as the Potts model and topological quantum field theory. Existing studies focus on homomorphisms between the chromatic algebra and other algebras. This work investigates the intrinsic properties of the chromatic algebra. We establish a bijection between the $n$-th order chromatic algebra basis diagrams and the noncrossing planar partitions without singletons of $2 n$ points. We also determine a canonical representation of the chromatic algebra as strings of length $2 n$, which has applications in implementing the chromatic algebra through code. We then demonstrate that the dimension of the $n$-th order chromatic algebra is the $2 n$-th Riordan number, which exhibits exponential growth. Moreover, we establish a generating set of size $\frac{1}{2} n^{2}-\frac{1}{2} n+1$, and we provide a procedure to construct the basis from the generating set. Additionally, we examine the chromatic trace mapping, an important evaluation function which relates chromatic algebra diagrams to chromatic polynomials. We prove results regarding the kernel of the trace and the traces of chromatic diagrams across rotations. (Received September 24, 2023)

## 17 Nonassociative rings and algebras

1192-17-27585
Katherine Baker, Ursinus College, Shashank Kanade*, University of Denver, Matthew C Russell, University of Illinois Urbana-Champaign, Christopher Sadowski, Ursinus College. Principal subspaces and new multi-sum identities.
I will explain some new multi-sum identities proved in a joint work with Baker, Sadowski and Russell. The first set of identities involves new quadruple sums for Nandi's combinatorial identities. The second set of identities is related to principal characters of level 4 standard modules for the affine Lie algebras $D_{4}^{(3)}$. All of these identities were first conjectured by Baker and Sadowski based on explorations of principal subspaces for basic modules for twisted affine Lie algebras.
(Received August 16, 2023)

## 1192-17-27962

Hope Emily Peck*, William Jewell College. Straightening Identities in the Onsager Algebra of $\mathfrak{s l}_{4}$.
In this work, we proved straightening identities in the Onsager Algebra for $\mathfrak{s l}_{4}$. Lars Onsager was a Nobel Prize-winning physical chemist and theoretical physicist who introduced the Onsager Algebra. Broadly, the Onsager algebra is a Lie algebra, which is a vector space with a Lie bracket. We define the Onsager Algebra for $\mathfrak{s l}_{4}$ to be the subalgebra of the loop algebra of $\mathfrak{s l}_{4}$, which is fixed by the Chevalley involution. The commutative property is not applicable to this Lie algebra. Straightening identities are reordering formulas within the Lie algebra. These identities can be used to formulate linear combinations of the products of the basis elements into a preferred order. Doing so offers a tool for understanding the representation theory of this algebra in characteristic $p>0$. We used double induction to prove these straightening identities. We found a general straightening identity and our Onsager algebra straightening identities were applications of our general formula. (Received August 24, 2023)

1192-17-28219
Hongdi Huang, Rice University, Xin Tang, Math \& Computer Science, Fayetteville State University, Xingting Wang*, Howard University, James J. Zhang, University of Washington. Valuation methods for Poisson algebras. Preliminary report. We study Poisson valuations and provide their applications in solving problems related to rigidity, automorphisms, Dixmier property, isomorphisms, and embeddings of Poisson algebras and fields. This is joint work with Hongdi Huang, Xin Tang, and James Zhang.
(Received August 28, 2023)

1192-17-28609
Shashank Kanade*, University of Denver. Characters of VOAs and invariants of links.
I will explain the relationship of the bosonic forms of the characters of various rational and non-rational VOAs in type A to quantum invariants of torus links. These VOAs include principal W-algebras, ( $1, p$ ) triplet, and ( $1, p$ ) singlet VOAs. In general, it is an important open question to find fermionic formulas for the characters of these VOAs (knot-theoretically or otherwise).

Tevian Dray*, Oregon State University, Corinne A. Manogue, Oregon State University (physics), Robert A. Wilson, Queen Mary University of London. A Division Algebra Description of the Magic Square, including $E_{8}$.
The Freudenthal-Tits magic square of Lie algebras provides an abstract parameterization of a family of Lie algebras in terms of two division algebras, with the exceptional cases all involving the octonions. Here, we provide an explicit interpretation of the magic square in terms of $3 \times 3$ generalized matrices over the appropriate division algebras.
(Received September 05, 2023)

## 1192-17-29000

Tevian Dray, Oregon State University, Corinne A. Manogue*, Oregon State University (physics), Robert A. Wilson, Queen Mary University of London. $E_{8}$ and the Standard Mode.
Using the explicit parameterization in terms of octonions presented in the talk by Tevian Dray, we interpret the elements of the Lie algebra $\mathfrak{e}_{8}$ as objects in the Standard Model of particle physics. We obtain lepton and quark spinors with the usual properties, the Standard Model Lie algebra $\mathfrak{s u}(3)+\mathfrak{s u}(2)+\mathfrak{u}(1)$, and the Lorentz Lie algebra $\mathfrak{s o}(3,1)$.
(Received September 05, 2023)

## 1192-17-29656

Andrew R. Linshaw*, University of Denver, Bailin Song, University of Science and Technology of China. Arc spaces and the chiral de Rham complex.
It is known by work of Bruschek, Mourtada, and Schepers that the Hilbert-Poincaré series of the coordinate rings of certain arc spaces are closely related to the Rogers-Ramanujan identities. Arc spaces have important applications to vertex algebras because in many cases, a vertex algebra can be viewed as a quantization of the arc space of its associated variety. In this talk, I will explain how the invariant theory of arc spaces can be used to give a complete description of the vertex algebra of global sections of the chiral de Rham complex of a compact Ricci-flat Kähler manifold. This is a joint work with Bailin Song.
(Received September 07, 2023)

1192-17-30131
Hongdi Huang*, Rice University, Xin Tang, Math \& Computer Science, Fayetteville State University, Xingting Wang, Louisiana State University, James J. Zhang, University of Washington. Elliptic Poisson algebras in dimension three. A family of quadratic Poisson structures of particular interest is the elliptic Poisson algebra that is defined by Feigin and Odesskii, and later independently, by Polishchuk, which can be regarded as semi-classical limits of the elliptic Sklyanin algebras. In this talk, we will focus on the elliptic Poisson algebra in dimension three that is determined by a (weighted) homogeneous polynomial with isolated singularity. Moreover, we will describe their Poisson valutions and automorphism groups.
(Received September 08, 2023)

## 1192-17-30254

Matvey Borodin*, Brookline High School. The Action of the Cactus Group on Arc Diagrams. Preliminary report.
We study an action of the cactus group $J_{n}$ on the set of arc diagrams. This is a pictorial method for exploring the structure of crystals of tensor products of irreducible $U_{q}\left(\mathfrak{s l}_{2}\right)$-modules. We fully describe the orbits of the action of $J_{n}$ on certain types of arc diagrams. We also describe some invariants that are always preserved by actions of $J_{n}$ on this set. Furthermore, we describe some additional relations that this action places on the generators of $J_{n}$.
(Received September 13, 2023)

## 1192-17-30674

Marco Aldi, Virginia Commonwealth University, Samuel Jameson Bevins*, Virginia Commonwealth University. $L_{\infty^{-}}$ cohomology of completely prunable hypergraphs.
We exploit the procedure to attach a nilpotent strong homotopy Lie algebra to a simple completely prunable hypergraph and prove that the corresponding cohomology groups are generated by taking iterated products and triple Massey products in degree 1. We conclude with a description of the cohomology of the Lie algebras associated with path graphs as a graded commutative algebra.
(Received September 11, 2023)

## 1192-17-30906

Cinzia Bisi*, Ferrara University, Italy. Invariants and Automorphisms for Slice Regular Functions.
Let A be one of the following algebras : $\mathbb{R}_{2} \cong \mathbb{H}$, (the real algebra of quaternions) or the Clifford algebra $\mathbb{R} \cong \mathbb{H} \oplus \mathbb{H}$. For the algebra A, the automorphism group Aut(A) and its invariants are well known. In this seminar we will describe the invariants of the automorphism group of the algebra of slice regular functions over A . Time permitting also the case $A=\mathbb{O}$ the algebra of octonions, will be revised.
(Received September 11, 2023)
except for the antisymmetry of its product. In 1965, Bloh first considered Leibniz algebras, though he called them D-algebras. They were popularized by Loday, who adopted the name Leibniz algebras, in the 1990s. All Lie algebras are Leibniz algebras. Many important results in Lie algebras have been shown to have Leibniz analogs such as Lie's Theorem, Engel's Theorem, and Cartan's criterion. A subalgebra $B$ of Leibniz Algebra $A$ is said to be subinvariant if there exists a chain of ideals $B=B_{0} \triangleleft B_{1} \triangleleft \cdots \triangleleft B_{n}=A$. We give the Leibniz analog of several results in Lie algebras on subinvariance and ascendancy. For example, if a subalgebra $B$ of Leibniz algebra $A$ is subinvariant in two subalgebras $H$ and $K$ of $A$, we show that for certain classes of Leibniz Algebras, $B$ is subinvariant in the subalgebra generated by $H$ and $K$. (Received September 25, 2023)

## 1192-17-32922

Elijah Bodish, MIT, Haihan Wu*, UC Davis. Webs for the Quantum Orthogonal Group
The Jones polynomial can be defined with the Temperley-Lieb category, whose Karoubian completion is equivalent to the representation category of quantum SL(2). In order to generalize the equivalence between a graphical category and the representation category of a quantum group, G. Kuperberg introduced web categories for rank 2 Lie algebras, where trivalent graphs are used in addition to planar matchings. Web categories have been widely studied since then. I will talk about how to define the web category for the quantum orthogonal group, based on joint work with E. Bodish.
(Received September 13, 2023)

## 18 Category theory, homological algebra

## 1192-18-27954

Andrea Stine*, UC Riverside, David Eric Weisbart, University of California Riverside, Adam M Yassine, Bowdoin College. A Rigidity Condition for Composition in an Augmented Span 2-Category.
Augmented generalized span categories provide a compositional framework for mathematical models of simple open systems in classical mechanics. The method of augmenting an object with an element of a vector space extends the earlier idea of an augmentation to a framework that includes Riemannian metrics and symplectic 2 -forms. The procedure for constructing augmentations on composite spans was previously ad-hoc, but a rigidity condition in the extended setting forces a unique compositional rule for augmentations on pullbacks. The procedure involves defining appropriate functors to $k$-Vect and constructing action categories from these functors. The framework we explore should allow the compositional principle to be applied more broadly and with more intrinsic rigidity.
(Received September 05, 2023)

1192-18-28297
Isaac Xiaoran Ren*, KTH. Computing relative Betti diagrams of multipersistence modules using Koszul complexes. In topological data analysis, multiparameter persistence modules do not have simple interval decompositions as in the oneparameter case. To approximate these decompositions, we use tools from relative homological algebra. In this talk, we focus on relative resolutions, which are exact sequences of persistence modules that are projective, in some sense, relative to a given family of "simple" modules. Under certain realizable conditions on this family, these resolutions consist of direct sums of simple modules, whose multiplicities we can then collect in the so-called relative Betti diagrams. We then compute these relative Betti diagrams using Koszul complexes, which is simpler than directly computing the full relative resolutions. This is a joint work with Wojciech Chachólski, Andrea Guidolin, Martina Scolamiero, and Francesca Tombari. (Received August 29, 2023)

## 1192-18-28463

Sofia Rose Rose Martinez Alberga*, Purdue University. Coalgebraic Models for $G$-spaces. Preliminary report
Given a commutative ring $R$, a $\pi_{1}-R$-equivalence is defined to be a continuous map of spaces inducing an isomorphism on fundamental groups and an $R$-homology equivalence between universal covers. If $R$ is the ring of integers then this notion coincides with that of a weak homotopy equivalence. When $R$ is an algebraically closed field, Raptis and Rivera described a full and faithful (co)algebraic model for the homotopy theory of spaces up to $\pi_{1}-R$-equivalence by means of simplicial coalgebras considered up to a notion of weak equivalence created by the cobar functor. Their work extends previous algebraic models for spaces considered up to $R$-homology (Kriz, Goerss, Mandell) by including the information of the fundamental group in complete generality. In this talk, I will describe G-equivariant analogs of this statement obtained through generalizations of a celebrated theorem of Elmendorf.
(Received August 30, 2023)

1192-18-29128
Agustina Czenky*, University of Oregon. Unoriented 2-dimensional TQFTs and the category $\operatorname{Rep}\left(S_{t} \ell \mathbb{Z}_{2}\right)$.
Let $\mathbf{k}$ be an algebraically closed field of characteristic zero. Oriented 2-dimensional cobordisms can be understood in purely algebraic terms using the Deligne category $\operatorname{Rep}\left(S_{t}\right)$, which interpolates the category of finite-dimensional representations of the symmetric group $S_{n}$ from $n \in \mathbb{N}$ to any parameter $t \in \mathbf{k}$. We show an analogous story happens in the unoriented case: it turns out that unoriented 2-dimensional cobordisms recover the generalized Deligne category $\operatorname{Rep}\left(S_{t}\right.$ 亿 $\left.\mathbb{Z}_{2}\right)$. (Received September 05, 2023)
rules. This project investigates braidings on a generalization of these fusion rules to a setting where simple objects are no longer required to be split. In particular, we classified braidings on fusion categories over the reals using techniques from the recent paper by Plavnik, Sanford and Sconce that classifies associators on these non-split categories, considering the three possible cases where objects are real, complex or quaternionic. In the poster, we will introduce some key techniques used in our project that allow us to perform graphical computations with string diagrams, and we will demonstrate some examples of these computations as well as current results.
(Received September 06, 2023)

## 1192-18-29451

Petter Andreas Bergh, NTNU, Julia Yael Plavnik, Indiana University, Sarah J. Witherspoon*, Texas A\&M University. Support varieties without the tensor product property.
Support varieties are important tools in representation theory, starting with representations of finite groups and finite group schemes. Much of the theory has been successfully adapted to many other settings, for example finite tensor categories, while much is still unknown. In this talk, we will explore support varieties for some categories arising as crossed products with finite groups. The variety of a tensor product is not always the intersection of the varieties, as is the case in more traditional settings.
(Received September 06, 2023)

## 1192-18-29478

Beren Sanders*, University of California, Santa Cruz. Descent and the Balmer spectrum.
We'll introduce the Balmer spectrum and the subject of tensor triangular geometry, which regards stable homotopy theories as geometric objects (in much the same way that classical algebraic geometry regards commutative rings as geometric objects). Our ultimate goal is to describe a new theorem - from joint work with Tobias Barthel, Natàlia Castellana, Drew Heard, Niko Naumann and Luca Pol - which describes how the Balmer spectrum behaves under descent. This theorem significantly generalizes (and is inspired by) earlier work of Paul Balmer on separable descent, which afforded tensor triangular geometry with an analog of the étale topology.
(Received September 06, 2023)

1192-18-29620
Qing Zhang*, UC Santa Barbara. Low-rank classification on non-semisimple modular categories.
I will talk about our recent progress on the classification of non-semisimple modular categories by rank. This is based on the joint work with Zhenghan Wang.
(Received September 07, 2023)

## 1192-18-30062

Victor Ostrik*, University of Oregon. Growth in tensor powers.
This talk is based on joint work with K. Coulembier, P. Etingof, D. Tubbenhauer. Let $\Gamma$ be any group and let $V$ be a finite dimensional representation of $\Gamma$ over arbitrary field. We consider tensor powers $V^{\otimes n}$ of V and their decompositions into indecomposable summands. Let $b_{n}(V)$ be the total number of indecomposable summands in $V^{\otimes n}$. We prove that

$$
\lim _{n \rightarrow \infty} \sqrt{ }[n] b_{n}(V)=\operatorname{dim}(V)
$$

Similarly let $d_{n}(V)$ be the number of indecomposable summands of $V^{\otimes n}$ with dimension not divisible by the characteristic of the field. Then we define

$$
\delta(V):=\lim _{n \rightarrow \infty} \sqrt{ }[n] d_{n}(V) .
$$

The real number $\delta(V)$ is an interesting invariant of the representation $V$. Using theory of tensor categories we show that this invariant is additive (under direct sums), multiplicative and takes values in algebraic integers. (Received September 08, 2023)

## 1192-18-30282

Zhenghan Wang*, Microsoft Station Q, UC Santa Barbara. TQFTs with structures. Preliminary report.
Unitary TQFTs, while interesting in physics, are not directly useful for applications to classical topological problems. On the other hand, non-unitary TQFTs spawned from super-symmetric Yang-Mills are powerful tools for studying smooth 4dimensional topology. I will report some preliminary progress in this direction. This is based on joint work with L. Fagan, Q. Kolt, Y. Qiu, and Q. Zhang.
(Received September 09, 2023)

## 1192-18-30354

Morgan Peck Opie*, UCLA. A cofibration category structure on the category of directed graphs.
I will discuss a cofibration category structure on a category of directed graphs with path homology equivalences as the weak equivalences. As time allows, I will discuss possible applications of such a structure to defining $K$-theories of categories of graphs, with an eye towards what problems such a construction might address. This talk is based on join work with Daniel Carranza, Brandon Doherty, Chris Kapulkin, Maru Sarazola, and Liang Ze Wong.
(Received September 09, 2023)

Agnès Beaudry, University of Colorado Boulder, Luisa Boateng*, Stanford University, Michael Hermele, University of Colorado Boulder, Stephanie Oh, University of Colorado Boulder, Marvin Qi, University of Colorado Boulder, Evan Wickenden, University of Colorado Boulder. Explorations in Unitary Modular Tensor Categories for Topological Quantum Computing. Preliminary report.
The field of topological quantum computation is a rapidly emerging and exciting prospect, with its capabilities for building fault-tolerant quantum computers. This is made possible by using quasiparticles called anyons which encode information nonlocally, making systems insensitive to errors caused by local perturbations from their environment. As these systems have certain structural properties, we can model them using the mathematics of category theory. In this talk, we will explore the basics of category theory and the structures that allow us to study topological quantum computing, specifically looking at different constructions of unitary modular tensor categories, which encapsulate the physical rules of braiding and fusion of non-abelian anyons.
(Received September 11, 2023)

## 1192-18-30925

Alapan Mukhopadhyay*, University of Michigan. Generators of bounded derived categories in prime characteristics. Since Bondal- van den Bergh's work on the representability of functors, proving existence of strong generators of the bounded derived category of a scheme has been a central problem. While for a quasi-excellent, separated scheme the existence of strong generators is established, explicit examples of such generators are not common. In this talk, in prime characteristics, we produce explicit generators using the Frobenius pushforward functor. Our description of generators, in particular, recovers Kunz's characterization of regularity in terms of flatness of the Frobenius. Part of the talk is based on a joint work with Matthew Ballard, Srikanth Iyengar, Patrick Lank and Josh Pollitz.
(Received September 11, 2023)

## 1192-18-30972

Travis Wheeler*, University of Kentucky. Bicategorical Character Theory.
In 2007, Nora Ganter and Mikhail Kapranov published a paper that defined the categorical trace. They used the categorical trace to define a character theory for 2-representations. In 2008, Kate Ponto published her thesis, defining a shadow functor for bicategories. With the shadow functor, Dr. Ponto defined the bicategorical trace, which is a generalization of the symmetric monoidal trace for bicategories. Are these two notions of trace related to one another? We look to character theory for 2representations for the answer.
(Received September 11, 2023)

## 1192-18-31656

Jason Schuchardt, UCLA, Ben Spitz*, University of California Los Angeles. Localizations and Spectra of Tambara Functors. Preliminary report.
Tambara functors are usually seen as the natural equivariant generalization of the notion of a commutative ring. As such, we expect there to exist a good theories of commutative algebra and algebraic geometry for Tambara functors. This talk will discuss recent attempts to realize such theories.
(Received September 11, 2023)

1192-18-32129
Pablo S. Ocal*, UCLA. Representations of algebras via universal supports. Preliminary report.
In this talk I will present progress towards understanding the representation theory of noncommutative algebras. The ideas are inspired by the Balmer spectrum of a symmetric tensor triangulated category, a topological tool analogous to the usual spectrum of a commutative ring. I will describe a universal support theory for triangulated categories, and illustrate it with the bounded derived category of path algebras of type A quivers. I will conclude with surprising connections with Verma modules of Khovanov's arc algebra and combinatorics.
(Received September 12, 2023)

1192-18-32208
Jan-Luca Spellmann*, Utah State University. Hirzebruch-Riemann-Roch for twisted equivariant matrix factorizations. Preliminary report.
A categorical Hirzebruch-Riemann-Roch theorem was proved by Shklyarov in [Shk13], counting the dimension of morphism spaces in specific categories in terms of bilinear forms and Chern characters. This theorem is known in the world of 2d TQFTs as the Cardy condition. Getting a concrete understanding of the Chern characters and the bilinear form proves to be difficult in examples. For the dg categories of matrix factorizations and their equivariant counterparts, this has been done by Polishchuk and Vaintrob [PV12] using the computation of Hochschild homology of matrix factorization categories by Dyckerhoff [Dyc11]. In this talk I will present my work understanding Shklyarovs theorem in the context of twisted equivariant matrix factorizations.
(Received September 12, 2023)

Fernando Chegjua Liu Lopez*, Rice University, Chelsea Walton, Rice University. Twists of graded algebras in monoidal categories.
We generalize the notion of a Zhang twist to the setting of closed-monoidal categories, and prove necessary and sufficient conditions for twisted algebras to have equivalent categories of graded modules, with degree preserving morphisms. Along the way, we prove several results about closed-monoidal categories and their categories of modules, particularly when these are viewed as categories enriched over themselves. This is joint work in preparation with Chelsea Walton.
(Received September 12, 2023)

## 1192-18-32581

Robert J Rennie*, Colorado College. Higher Categorical Galois Theories.
We discuss recent results (R., 2022) adapting a 1-categorification of Galois Theorems (Borceaux and Janelidze, 2001) to a higher categorical refinement. The process of doing so reveals two things of note: 1) that the result uniting Galois-style approaches to different areas of Mathematics is mostly formal in its proof, and 2) that the formality suggests a deeply homotopy-theoretic nature to such Galois Theories.
(Received September 12, 2023)

## 1192-18-32601

Matthew Harper*, University of California, Riverside, Peter Samuelson, University of California, Riverside. A Diagrammatic Calculus for Induction and Restriction on Temperley-Lieb Modules.
Building on work of Quinn, we define a diagrammatic monoidal category generated by one object ( $\uparrow$ ) and its dual ( $\downarrow$ ); these objects act by induction and restriction on the sum over $n$ of the categories of Temperley-Lieb modules. These generators satisfy a categorified Weyl relation $\downarrow \uparrow \cong \uparrow \downarrow \oplus P_{0}$; in the action on Temperley-Lieb modules, $P_{0}$ is the projection onto the trivial isotypic component. In analogy with the Heisenberg category of Khovanov, Licata, and Savage, the Grothendeick group and Hochschild homology of our category are closely related to the Weyl algebra and $\mathfrak{s l}_{2}$ double affine Hecke algebra, respectively. (Received September 12, 2023)

1192-18-32958
Akshaya Chakravarthy*, MIT PRIMES-USA, Agustina Czenky, University of Oregon, Julia Yael Plavnik, Indiana University. On Modular Categories With Frobenius-Perron Dimension Congruent to 2 Modulo 4.
A modular (tensor) category is a fusion category with additional structure that has applications in other fields of mathematics as well as quantum computing. We contribute to the classification of modular categories $\mathcal{C}$ with Frobenius-Perron dimension congruent to 2 modulo 4 . We show that such categories have their group of invertibles of even order and that they factorize as $\mathcal{C} \cong \widetilde{\mathcal{C}} \boxtimes \mathcal{P}$, where $\widetilde{\mathcal{C}}$ is an odd-dimensional modular category and $\mathcal{P}$ is the rank 2 pointed modular category. This reduces the classification of these categories to those of odd-dimensional modular categories. It follows that modular categories with Frobenius-Perron dimension congruent to 2 modulo 4 and rank up to 46 are pointed. More generally, we prove that if $\mathcal{C}$ is a weakly-integral modular category and $p$ is an odd prime dividing the order of the group of invertibles that has multiplicity one in the Frobenius-Perron dimension of $\mathcal{C}$, then we have the factorization $\mathcal{C} \cong \tilde{\mathcal{C}} \boxtimes \mathcal{P}$, where $\tilde{\mathcal{C}}$ is a modular category with Frobenius-Perron dimension not divisible by $p$ and $\mathcal{P}$ is a pointed modular category with Frobenius-Perron dimension $p$. Lastly, we show the existence of pointed modular subcategories in pseudo-unitary modular categories. (Received September 13, 2023)

1192-18-33124
Constantin Teleman*, UC Berkeley. Crossed braided tensor categories from Coulomb branches. Preliminary report. I will discuss the appearance of non-semisimple crossed braided tensor categories in 3-dimensional gauge theory for a disconnected compact Lie group, based on twisted sectors of the so-called Chiral Rings (sometimes called Coulomb branches). This is based on a project with Colleen Delaney and John Nolan.
(Received September 13, 2023)

1192-18-33455
Agustina Czenky, University of Oregon, William Gvozdjak*, MIT PRIMES-USA, Julia Yael Plavnik, Indiana University. Classification of Low-Rank Odd-Dimensional Modular Categories.
Modular categories are important algebraic structures that appear in a diverse quantity of applications, including topological quantum field theory, representations of braid groups, quantum groups, von Neumann algebras, conformal field theory, and vertex operator algebras. Motivated by their applications in the study of topological phases of matter and topological quantum computation, it is interesting to classify modular categories. We prove that any odd-dimensional modular category of rank at most 23 is pointed. We also show that an odd-dimensional modular category of rank 25 is either pointed, perfect, or equivalent to $\operatorname{Rep}\left(D^{\omega}\left(\mathbb{Z}_{7} \rtimes \mathbb{Z}_{3}\right)\right)$. Finally, we give partial classification results for modular categories of rank up to 73 . (Received September 17, 2023)

1192-18-33847
Sam Johnson*, Colorado College, Isak Larson*, Colorado College, Brendan McCune*, Colorado College. Formalizing Sylow Theorems in Homotopy Type Theory using Agda. Preliminary report.
Computer formalization allows for a guarantee of correctness in mathematical findings by type checking proofs for consistency with provided mathematical axioms. This eliminates the need for labor intensive and fallible expert confirmations of proposed proofs. Recent work [Wu 2022] has focused on applying large language models toward formalizations of proofs in ML type theory. However, a refinement of this type theory (Homotopy Type Theory) allows for a set of axioms better suited for proofs involving spaces, groupoids, and other mathematical structures with homotopical structure. In addition, the results proven in the latter type theory yield more general results interpretable in other mathematical contexts [Shulman et. al '22]. For example, a pre-existing higher-group generalization of the Sylow Theorems provides promise that a Homotopy Type Theoretic proof of the Sylow Theorems is possible, which would thus yield a general Sylow Theory to interpret in other higher toposes. In this project we use the goal of formalizing Sylow Theorems in HoTT toward a more explicit understanding of how human-level HoTT-fluency might be automated via machine learning.
(Received September 26, 2023)

Jennifer Guerrero*, University of California, Santa Cruz. Describing Units of the A-fibered Burnside Ring. Preliminary report. Assuming that G is a finite group, the Burnside Ring, the Grothendieck group of the category of finite G-sets, is frequently studied. Now, given a finite group G and an Abelian (written multiplicatively) group A we will define the A-fibered Burnside Ring, $B^{A}(G)$, a generalization of the Burnside ring. We will consider the orthogonal unit group, $O\left(B^{A}(G)\right)$, of $B^{A}(G)$ which happens to be the collection of units of $B^{A}(G)$ of finite order. We will look at examples to motivate our description of $O\left(B^{A}(G)\right)$ for certain finite groups G.
(Received August 14, 2023)

## 1192-19-27999

Jonathan Campbell, Center for Communications Research, Josefien Kuijper, Stockholm University, Mona Merling University of Pennsylvania, Inna Zakharevich*, Cornell. Algebra with Geometry.
Algebraic $K$-theory generally works with algebraic objects, such as modules over a ring. Because of this algebraic structure there are powerful techniques for analyzing the structure of algebraic $K$-theory, such as Grothendieck's dévissage and localization theorems. However, the basic idea of algebraic $K$-theory is actually very combinatorial: there are objects and a rule for "adding" or "unifying" them, and we wish to analyze their invariants under this notion of addition. For example, scissors congruence of polytops, piecewise isomorphisms of varieties, and SK-decompositions of manifolds are all examples of invariants which do not possess an overall algebraic structure, but which still exhibit an interesting notion of "addition." In this talk we will discuss "squares $K$-theory" which gives a minimalistic construction of algebraic $K$-theory depending only on combinatorial information, and present several examples of $K$-theories that can be constructed using this definition.
(Received August 24, 2023)

## 1192-19-28000

Jonathan Campbell, Center for Communications Research, Kate Ponto, University of Kentucky, Inna Zakharevich*, Cornell. Another talk on the Dennis trace. Preliminary report.
The Dennis trace is fundamentally simple: it takes the class of a module over a ring $R$ (living in $K_{0}(R)$ ) to the trace of the identity on $R$ (living inside $H H_{0}(R)$ ). Since direct sums of modules lead to sums in the trace, this is a well-defined homomorphism. Amazingly enough, this simple description gives rise to a map on higher $K$-groups that boasts a wealth of information about the higher $K$-groups. In this talk we describe a new perspective on the trace using endomorphism $K$-theory and an interesting splitting of the $K$-theory of automorphisms. (Received August 24, 2023)

## 1192-19-28754

Mona Merling*, University of Pennsylvania. Parametrized $K$-theory of manifolds. Preliminary report.
In this talk we will present a topological version of the cut-and-paste K-theory of manifolds, which we can view as mediating between the MTSO spectrum and classical algebraic K-theory of spaces. This is joint work with George Raptis and Julia
Semikina.
(Received September 12, 2023)

## 1192-19-29642

Maxine Elena Calle*, University of Pennsylvania, David Chan, Michigan State University, Andres Mejia, University of Pennsylvania. A linearization map for equivariant A-theory.
The Waldhausen $A$-theory of a space $X$ encodes important geometric data, but accessing this data is difficult. The linearization map relates the A-theory of a space $X$ to $K\left(\mathbb{Z}\left[\pi_{1} X\right]\right)$ and plays an important role in computations. When $X$ has an action by a finite group $G$, Malkiewich-Merling have constructed a genuine equivariant $A$-theory spectrum for $X, A_{G}(X)$. This talk will discuss how to construct an equivariant linearization map out of $A_{G}(X)$ which lands in the algebraic $K$-theory of a coefficient ring analogous to $K\left(\mathbb{Z}\left[\pi_{1} X\right]\right)$. Based on joint work with D. Chan and A. Mejia.
(Received September 07, 2023)

## 1192-19-31250

Benjamin Garcia*, CCM, UNAM campus Morelia. Groups with isomorphic fibered Burnside rings.
The isomorphism problem of a representation ring associated to finite groups deals with the question of whether two groups having isomorphic representation rings must be isomorphic; the answer to this question is known to be negative for some rings such as the Burnside ring and the complex character ring, yet the study of isomorphisms between representation rings reveals common properties and invariants between the groups. Given a finite group $G$ and an abelian group $A$, the $A$-fibered Burnside ring $B^{A}(G)$ is the Grothendieck ring of the category of $A$-fibered $G$-sets, it was introduced by Dress as a generalization of the Burnside ring and the monomial representation ring. The purpose of this talk is to present some advances on the isomorphism problem of $B^{A}(G)$ : concretely, we give conditions on two groups $G$ and $H$ for the existence of an isomorphism between their $A$-fibered Burnside rings preserving some standard bases, and we show the existence of non-isomorphic groups having isomorphic fibered Burnside rings for a wide family of fiber groups $A$.
(Received September 11, 2023)

Anna Marie Bohmann*, Vanderbilt University, Teena Gerhardt, Michigan State University, Cary Malkiewich, Binghamton University, Mona Merling, University of Pennsylvania, Inna Zakharevich, Cornell. Scissors congruence $K$-theory and group homology.
Scissors congruence, the subject of Hilbert's Third Problem, asks for invariants of polytopes under cutting and pasting operations. One such invariant is obvious: two polytopes that are scissors congruent must have the same volume, but Dehn
showed in 1901 that volume is not a complete invariant. Trying to understand these invariants leads to the notion of the scissors congruence group of polytopes, first defined the 1970s. Elegant recent work of Zakharevich allows us to view this as the zeroth level of a series of higher scissors congruence groups. In this talk, I'll explain how a new approach allows us to build computationally approachable trace maps from these higher scissors congruence groups to group homology
(Received September 11, 2023)

1192-19-31519
Andres Mejia*, University of Pennsylvania. Spherical Group Ring Models for Equivariant A-theory. Preliminary report. Classically, the Algebraic K-theory of spaces ( $A$-theory) was used to study manifold topology from a homotopical perspective. In the equivariant setting, Malkiewich and Merling constructed a genuine $G$-spectrum $A_{G}(X)$ together with an assembly map $A_{G}(*) \wedge \Sigma_{G}^{\infty} M \rightarrow A_{G}(M)$ whose cofiber deloops to the equivariant "stable H-cobordism space" for a smooth $G$-manifold $M$. Non-equivariantly, Waldhausen's original vision for $A$-theory was an interpretation that initiated work in "brave new algebra" that happens on the level of spectra. Moreover, he gave an interpretation of $A$ theory analogous to the $K$ theory of rings where we take $K\left(\Sigma^{\infty} \Omega X\right)$ where we think of $\Sigma^{\infty} \Omega X$ as a "spherical group ring" in analogy with $\mathbb{Z}\left[\pi_{1} X\right]$. A natural question is whether or not there is a similar story for $A_{G}(M)$ and we propose a model that gives a positive answer to this question. As an application, this provides the equivariant linearization map $A_{G}(X) \rightarrow K_{G}\left(\mathbb{Z}\left[\pi_{1} X\right]\right)$ with a purely algebraic interpretation. We will also discuss some future directions in connection with the relationship to $G$-manifolds established by Malkeiwich and Merling. All of this work is joint with Maxine Calle, David Chan, and Anish Chedalavada. (Received September 11, 2023)

John McHugh*, University of California Santa Cruz. On the image of the trivial source ring in the ring of virtual characters of a finite group.
We examine the cokernel of the canonical homomorphism from the trivial source ring of a finite group to the ring of $p$-rational complex characters. We use Boltje and Coşkun's theory of fibered biset functors to determine the structure of the cokernel. An essential tool in the determination of this structure is Bouc's theory of rational p-biset functors. As a result of this work, a "detection theorem" is found that gives a necessary and sufficient condition for a virtual $p$-rational character of a finite group to "come from" a (virtual) trivial source module.
(Received September 12, 2023)

1192-19-32060
Shanna Dobson*, University of California, Riverside. K-Theoretic Time Crystals. Preliminary report.
We construct a K-theoretic equivalent of a time crystal using Morava K-theory. We then discuss the concept of a K-theoretic Everettian phone, and reframe properties of quantum computing using the K-theory time crystals. Recall, a time crystal is, informally, a quantum system of particles that self-organizes in temporal periodicity. Furthermore, the motion of a time crystal is a crypto-equillibrium "motion without energy", as it can continuously cycle between states while simultaneously not requiring or exhausting fuel. It is, simply put, an incredible new phase of non-equilibrium matter.
(Received September 12, 2023)

1192-19-32215
Lucy Yang*, Columbia University. An invitation to genuine equivariant homotopy theory via real K-theory.
Grothendieck-Witt theory and real K-theory are homology theories of schemes refining algebraic K-theory in the presence of duality data. We will see that $C_{2}$-genuine equivariant homotopy theory provides a natural setting for these theories. Similarly to ordinary K-theory, real K-theory admits homological approximations, known as real trace theories. In this talk, we will examine one such instance of a real trace theory, real Hochschild homology (HH). We will compare real HH to ordinary Hochschild homology and show that it is similarly 'nice.'
(Received September 12, 2023)

1192-19-32374
Diego Manco Berrio*, University of Oregon. Pseudo symmetric multifunctors and applications to K-theory..
Multicategories have been used by Mandell and Elmendorf to encode multiplicative structures in the multicategory of permutative categories. They defined $K$-theory as a symmetric multifunctor from permutative categories to connective spectra and showed that it preserves the encoded multiplicative structures. Yau has shown that Mandell's inverse $K$-theory multifunctor from $\Gamma$-categories to permutative categories preserves the action of the symmetric group by permuting factors only up to coherent isomorphisms, i.e., it is pseudo symmetric rather than symmetric, so it is not immediate that it preserves symmetric multiplicative structures. In this talk, we introduce pseudo symmetric multifunctors, including a new definition equivalent to the original one given by Yau, and show some examples and applications that imply that pseudo symmetric multifunctors, and in particular Mandell's inverse $K$-theory multifunctor, preserve certain symmetric multiplicative structures, like $E_{n}$-algebras.
(Received September 12, 2023)

## 20 Group theory and generalizations

## 1192-20-25402

Ruth Charney*, Brandeis University. From Braid Groups to Artin Groups.
Braid groups are a fascinating class of groups with applications to topology, algebra, geometric group theory, and combinatorics. Braid groups belong to a much larger, but less understood class of groups known as Artin groups. Beginning with Braid groups, I will discuss various algebraic and geometric characterizations of Artin groups. I will then survey what is
known and not known about these groups and highlight a few of the new approaches that have been introduced to address some of the more challenging open questions.
(Received June 05, 2023)

## 1192-20-26263

Yulan Qing*, Fudan University SCMS. Quasi-redirecting boundaries of finitely generated groups. Preliminary report.
In this talk I will introduce a compactification of the finitely generated group that is an analogue of the Gromov boundary. This compactification is called quasi-redirecting boundary; it is QI-invariant often compact. It contains Sublinearly Morse boundaries as subspaces and is compact for many classes of groups. If time permits we will discuss its applications and connection with other boundaries. This is joint work with Kasra Rafi.
(Received July 23, 2023)

## 1192-20-26843

Sahana H. Balasubramanya*, Lafayette College. Non recognizing spaces for stable subgroups.
We say an action of a group $G$ on a space $X$ recognizes all stable subgroups if every stable subgroup of $G$ is quasiisometrically embedded in the action on $X$. The problem of constructing or identifying recognizing spaces has been extensively studied for many groups, including mapping class groups and right angled Artin groups- these are well known examples of acylindrically hyperbolic groups. In these cases, the recognizing spaces are the largest acylindrical actions for the group. One can therefore ask the question if a largest acylindrical action of an acylindrically hyperbolic group (if it exists) is a recognizing space for stable subgroups in general. We answer this question in the negative by producing an example of a relatively hyperbolic group whose largest acylindrical action fails to recognize all stable subgroups. This is joint work with Marissa Chesser, Alice Kerr, Johanna Mangahas and Marie Trin.
(Received August 07, 2023)

## 1192-20-26844

Sahana H. Balasubramanya*, Lafayette College. Actions of solvable groups on hyperbolic spaces.
Actions on hyperbolic metric spaces are an important tool for studying groups, and so it is natural, but difficult, to attempt to classify all such actions of a fixed group. Recent work of Carolyn Abbott, Sahana Balasubramanya and Alex Rasmussen build strong connections between hyperbolic geometry and commutative algebra in order to classify the cobounded hyperbolic actions of numerous metabelian groups (up to a coarse equivalence). In particular, we turn this classification problem into the problems of classifying ideals in the completions of certain rings and calculating invariant subspaces of matrices. We use this framework to classify the cobounded hyperbolic actions of many abelian-by-cyclic groups. Our investigations incorporate tools from commutative algebra, such as number systems, factorization in formal power series rings, completions, and valuations. (Received August 07, 2023)

## 1192-20-27017

Zachary Mesyan*, University of Colorado, Colorado Springs. Conjugacy and Least Commutative Congruences in Semigroups. Given a semigroup $S$ and $s, t \in S$, write $s \sim{ }_{s}^{1} t$ if $s=p_{1} \cdots p_{n}$ and $t=p_{f(1)} \cdots p_{f(n)}$, for some $p_{1}, \ldots, p_{n} \in S \cup\{1\}$ and permutation $f$ of $\{1, \ldots, n\}$. This is a natural generalization of primary conjugacy (relating $s$ and $t$ whenever $s=p r$ and $t=r p$, for some $p, r \in S \cup\{1\}$ ), which has been used extensively across many fields of algebra. It turns out that the transitive closure $\sim_{s}$ of $\sim_{s}^{1}$ is the congruence generated by the primary conjugacy relation, and is moreover the least commutative congruence, on any semigroup. We explore general properties of $\sim_{s}$, discuss them in the context of groups and rings, compare them to other semigroup conjugacy relations, and describe their equivalence classes in various types of semigroups, including free, Rees matrix, graph inverse, and transformation semigroups.
(Received August 08, 2023)

## 1192-20-27182

Eleanor McSpirit*, University of Virginia. Zeros in the character tables of symmetric Groups with an $\ell$-core index.
The representation theory of symmetric groups is intimately tied to the theory of partitions. An open question in representation theory is the limiting proportion of zeros in the character tables of symmetric groups. In this talk, I will discuss joint work with Ken Ono, in which we leverage the theory of $\ell$-core partitions to obtain vanishing conditions for certain entries in these character tables as well as asymptotic formulas for zero entries with indexed by $\ell$-core partitions.
(Received August 11, 2023)

1192-20-27625
Sam Kim Miller*, University of California, Santa Cruz. Endotrivial complexes and the trivial source ring.
Let $k$ be a field of prime characteristic and $G$ be a finite group. In this talk, we will introduce the notion of an endotrivial complex, a bounded chain complex of $k G$-modules $C$ for which $E n d_{k}(C) \cong C^{*} \otimes_{k} C \simeq k[0]$, where $\simeq$ denotes homotopy equivalence. Such a complex can be viewed as analogous to endotrivial modules, which are well-studied. We additionally require that endotrivial complexes contain only $p$-permutation modules, since in this case, endotrivial complexes will induce splendid Rickard autoequivalences of the group algebra. We investigate the group of endotrivial complexes of $k G$-modules modulo homotopy, $\mathcal{E}_{k}(G)$. The Brauer quotient gives us an important equivalent definition of endotriviality, and for any endotrivial complex, produces integral invariants for each $p$-subgroup of $G$ which completely determine the homotopy class of the endotrivial complex up to a twist by a Brauer character of degree 1. As a result, $\mathcal{E}_{k}(G)$ is finitely generated with rank bounded by the number of conjugacy classes of $p$-subgroups of $G$. We will discuss these structural results and a few applications, including the complete structure of $\mathcal{E}_{k}(G)$ for some classes of groups, and partially determining the image of $\mathcal{E}_{k}(G)$ in the orthogonal unit group of the trivial source ring via Lefschetz invariants. (Received August 16, 2023)

## 1192-20-27666

Kevin Woods*, Oberlin College. Quasi-polynomial Behavior in Factorizations via Presburger Arithmetic.
A quasi-polynomial is a function $f(t)$ such that there exist a period $m$ and polynomials $f_{0}, \ldots, f_{m-1}$ such that
$f(t)=f_{t \bmod m}(t)$. Many counting functions related to factorizations in rings and semigroups are quasi-polynomials, for sufficiently large $t$. For example, for a fixed numerical semigroup, the following are eventually quasi-polynomial: the number of factorizations of $t$, the number of elements that have factorizations of length at most $t$, and the number of distinct lengths of factorizations of $t$. These can all be generalized to affine semigroups, as well. Similarly, quasi-polynomials also appear when considering affine semigroups whose generators are polynomial functions of $t$ : for example, the Betti numbers and (for numerical semigroups) the genus and Frobenius number. All of these results quickly follow by using tools from Presburger arithmetic, which is the first order logic over the integers whose basic statements are linear inequalities. We describe these tools and give examples of how they can be used in counting problems related to semigroups.
(Received August 17, 2023)

## 1192-20-28069

Spencer Chapman, Trinity University, Eli B. Dugan*, Williams College, Shadi Gaskari, San Diego State University, Ron Lycan II, San Diego State University, Sarah Mendoza De La Cruz, University of Texas at Austin, Christopher O'Neill, San Diego State University, Vadim Ponomarenko, San Diego State University. Generalized Factorization Lengths in Atomic Monoids. Preliminary report.
We generalize the concept of factorization length within atomic semigroups, and present a study of the factorization lengths of iterated powers of elements. We do so through the investigation of several families of semigroups, including numerical semigroups, block monoids, and arithmetical congruence monoids. Our results range from asymptotic behavior to complete, quasi-polynomial characterizations of factorization length.
(Received August 25, 2023)

## 1192-20-28236

Ahmad Mohammed Alghamdi*, Umm Al-Qura University at Makkah, Saudi Arabia. Around Anchors of Irreducible Characters.
Block theory is a modern approach to study representation of a finite group $G$. The concept of p-block relative of a fix prime number $p$ is due to Richard Brauer. He initiated and originated this approach in a pioneering work, research papers and monographs for more that 40 yeas. The pillars of block theory are Brauer first, second and third main theorems. Defect group of a p-block of the finite group $G$ is a p-subgroup of the group G. It is kind of a generalization of Sylow theory. Furthermore, in the algebra setting defect group measures the semi-simplicity of a p-block. Then J. Green extended Brauers' work and opened new approach by invoking indecomposable G-modules. He initiated the so-called vertex theory. His approach unifies both defect theory and vertex theory in one place which is called G-algebras. Green correspondence, Green indecomposable Theorems and relative trace map are the main tools for this approach. The most well understood work in this regards is theory of block with cyclic defect group. Dade's did lots of theory in this direction. The importance of block with cyclic defect group is are exploited in many applications, conjectures and advanced block theory. The concept of anchor of an irreducible character appeared first by Radha Kessar, Marcus Linkelmann and Bukhard Kulshammer. We have the fact the the vertex is inside the anchor and the anchor lies inside the defect. There are lots of interesting examples for these relationship. Our work is around the paper by Radha Kessar, Marcus Linkelmann and Bukhard Kulshammer. . In particular, we are trying to initiate the subject theory of irreducible characters with cyclic anchor groups. Our main equations and study are the following: - When does an irreducible character have a cyclic anchor? - Does the situation in the above question imply that the defect group of the block in which the character sits is Cyclic? - Can one say about the relationship with finite representation type in this setting? - What is the relationship between Navarro vertex and Anchor of an irreducible character?
(Received August 28, 2023)

1192-20-28310
Chad Awtrey*, Samford University, Frank Patane, Samford University. On Solvable Transitive Permutation Groups of Degree $2 p$.
Let $p>2$ be prime and $S_{2 p}$ the symmetric group of degree $2 p$. We give a classification of the solvable transitive subgroups of $S_{2 p}$ that possess blocks of size 2 and size $p$. The classification realizes each such subgroup as a semidirect product of two cyclic groups; a generator for each cyclic group is given explicitly. Further, we describe several applications related to computing Galois groups of irreducible polynomials of degree $2 p$.
(Received August 29, 2023)

## 1192-20-28398

Laura Cossu*, University of Graz (Austria). Factorization in power monoids.
Let $M$ be a multiplicative monoid. When endowed with the operation of setwise multiplication induced by $M$, the collection of all finite subsets of $M$ containing the identity $1_{M}$ is itself a monoid, called the reduced power monoid of $M$. This is a (highly) non-cancellative monoid in which every set is a product of irreducibles. The goal of the talk is to discuss arithmetical properties of such factorizations.
(Received August 30, 2023)

1192-20-28444
Gil Goffer*, University of California at San Diego, Be'eri Greenfeld, University of Washington. Probabilistic laws on infinite groups.
In various cases, a law (that is, a quantifiers free formula) that holds in a group with high probability, must actually hold for all elements. For instance, a finite group in which the commutator law $[x, y]=1$ holds with probability at least $5 / 8$, must be abelian. For infinite groups, one needs to work a bit harder to define the probability that a given law holds. One natural way is by
sampling a random element uniformly from the r-ball in the Cayley graph and taking r to infinity; another way is by sampling elements using random walks. It was asked by Amir, Blachar, Gerasimova, and Kozma whether a law that holds with probability 1 , must actually hold globally, for all elements. In a recent joint work with Be'eri Greenfeld, we give a negative answer to their question. In this talk I will give an introduction to probabilistic group laws and present a finitely generated group that satisfies the law $x^{\wedge}\{p\}=1$ with probability 1 (with respect to all reasonable random works), but yet admits no group law that holds for all elements. In fact, this group contains a free subgroup.
(Received August 30, 2023)

1192-20-28448
Gil Goffer*, University of California at San Diego, Be'eri Greenfeld, University of Washington. Small cancellation methods in probabilistic group laws.
In this short talk I will give an overview of probabilistic laws in countable groups, and illustrate how small cancellation methods can reveal unexpected phenomena. For example, in a joint work with Be'eri Greenfeld, we construct a group in which the law $x^{\wedge}\{p\}=1$ holds with probability 1 , but does not satisfy this law, or any other law, globally. This answers questions of Amir, Blachar, Gerasimova, and Kozma.
(Received August 30, 2023)

## 1192-20-28529

Rylee Alanza Lyman*, Rutgers University-Newark. Outer Automorphism groups of "Plain" Groups
A "plain" group is a finitely generated free product of finite groups and infinite cyclic groups. Among virtually free groups, the plain groups have perhaps the richest collection of outer automorphisms. I will report on recent results about the outer automorphism group of a plain group.
(Received August 31, 2023)

## 1192-20-28611

Jingyin Huang*, Ohio State University. Combinatorial non-positive curvature and the $K(\pi, 1)$-conjecture for reflection arrangement complements.
The $K(\pi, 1)$-conjecture for reflection arrangement complements, due to Arnold, Brieskorn, Pham, and Thom, predicts that certain complexified hyperplane complements associated to infinite reflection groups are Eilenberg MacLane spaces. We establish a close connection between simple properties in metric graph theory and the $K(\pi, 1)$-conjecture, via elements of non-positively curvature geometry. We also propose a new approach for studying the -conjecture. As a consequence, we deduce a large number of new cases of Artin groups which satisfies the $K(\pi, 1)$-conjecture.
(Received September 01, 2023)

## 1192-20-28633

Luise-Charlotte Kappe*, Binghamton University, Arturo Magidin, University of Louisiana at Lafayettee. Generalizing the Chermak-Delgado Lattice I.
The Chermak-Delgado measure of a subgroup $H$ of a finite group $G$ is defined as the product of the order of $H$ with the order of the centralizer of $H$ in $G$, i.e. $m_{G}(H)=|H|\left|C_{G}(H)\right|$, and the set of all subgroups with maximal Chermak-Delgado measure forms a lattice in $G$. Let $w(x, y)$ be a word in the alphabet $\left\{x, y, x^{-1}, y^{-1}\right\}$ and $H$ a subgroup of a group $G$. The following sets are subgroups of $G$ :
\align* $\wedge\{*\} w_{-}\{1\}(H) \&=\{x \in G \mid w(x g, h)=w(g, h)$ for all $g \in G, h \in H\} w_{-}\{1\}^{\wedge}\{*\}(H) \&=\{x \in G \mid w(g x, h)=w(g, h)$ for all $g \in G, h \in H\}$

For instance, if $w$ is the commutator word $w(x, y)=[x, y]$, we have

$$
{ }^{*} w_{1}(H)={ }^{*} w_{2}(H)=C_{G}(H) a n d w_{1}^{*}\left(H_{1}\right)=w_{2}^{*}(H)=C_{G}\left(H^{G}\right)
$$

For $i=1,2$, we can modify the Chermak-Delgado measure by replacing the centralizer with the subgroups ${ }^{*} w_{i}(H)$ and $w_{i}^{*}(H)$, to get $m_{G}(H)=\left.|H|\right|^{*} w_{i}(H) \mid$ and $m_{G}(H)=\left|H \| w_{i}^{*}(H)\right|$. The question arises for which words $w(x, y)$ the subgroups having maximal measure form a lattice. We discuss obstacles that the generalization encounters and some cases in which they can be surmounted.
(Received September 01, 2023)

## 1192-20-28634

Luise-Charlotte Kappe, Binghamton University, Arturo Magidin*, University of Louisiana at Lafayettee. Generalizing the Chermak-Delgado Lattice II.
The Chermak-Delgado lattice of a finite group $G$ consists of all subgroup $H$ for which $m_{G}(H)=|H| C_{G}(H) \mid$ attains its largest possible value. The lattice and the subgroups in it have a number of interesting properties; for example, if $H$ is in the lattice then so is $C_{G}(H)$; if $H$ and $K$ are in the lattice then $\langle H, K\rangle=H K$; and the least element $A$ of the lattice is a characteristic abelian subgroup of $G$ such that for all abelian subgroups $B$ of $G,[G: A] \leq[G: B]^{2}$. We explore the construction that arises when we replace $C_{G}(H)$ with other subgroups; if $H \mapsto M(H)$ is a function that associates subgroups of $G$ to subgroups of $G$, when is the collection of $H$ for which $|H||M(H)|$ attains its largest possible value a lattice? Which other results from the Chermak-Delgado lattice will then follow, and which ones require additional conditions? In particular, we look at the situation with the word $w(x, y)=\left[x^{n}, y\right]$ and $M(H)={ }^{*} w_{1}(H)=\{x \in G \mid w(x g, h)=w(g, h)$ for all $g \in G, h \in H\}$. (Received September 01, 2023)

Shakuan Frankson*, Howard University. The Algebraic Structure of Double and k-Riordan Arrays.
The (ordinary) Riordan group consists of infinite lower-triangular matrices with column entries determined by two ordinary generating functions, $g$ and $f$, where the $h$ th column is given by $g f^{h}$. We refer to $f$ as the multiplier function. These arrays can be row constructed using two sequences $A=\left(a_{n}\right)$ and $Z=\left(z_{n}\right)$, which are weights applied to entries on one row to find the next. Some questions that we plan to explore focus on the algebraic structure of the Riordan group and if they could be extended to the $k$-Riordan group, $k>1$, where the elements are defined by $g$ and $k$ multiplier functions $f_{k}$. When $k=2$, we will refer to the set of matrices as the double Riordan group, DR. Much is still unknown about this group, such as which arrays are pseudo-involutions, what is the commutator subgroup, and how to define another set of matrices called A-matrices that can be used to construct DR arrays. Note that, for ordinary generating functions, $g=\sum g_{n} z^{n}=\sum g_{n} \frac{z_{n}}{r_{n}}$, where $\left(r_{n}\right) \equiv 1$. When we change the denominator sequence from $r_{n}=1$ to $r_{n}=n$ !, the matrices we get are called exponential Riordan arrays and will have an $h$ th column generating function $\frac{g f^{h}}{h!}$. The row construction method for these arrays becomes more difficult as the sequences needed will come from differential equations, denoted by $A_{e}$ and $Z_{e}$. It is known that ordinary double (and $k$-) Riordan arrays can be split into $k$ ordinary Riordan arrays, and this allows us to study the ordinary $k$-Riordan array by looking at ordinary Riordan arrays. There is no known process of doing this for the double exponential arrays. The row construction for exponential $k$-Riordan arrays is even more unclear as there may exist many combinations of differential equations $A_{e}$ and $Z_{e}$ that work. For exponential DR arrays, we believe that there are at least two possibilities. Furthermore, we will explore how ( $k$ )Riordan arrays can be utilized in other areas of combinatorics, such as Ramsey theory. We are interested in using various matrices (first entries, image partition, etc.) to construct Riordan arrays and see if we can deduce combinatorial interpretations from them.
(Received September 01, 2023)

## 1192-20-28650

Matt Clay*, University of Arkansas. An Invitation to Geometric Group Theory.
Geometric group theory studies groups by using the topology and geometry of the spaces they act on. Increasingly diverse techniques and tools are continually being created and answering long standing questions. I will give an overview of a couple of developments in geometric group theory from the last 10 years, in a way that is accessible to mathematicians from outside of the field.
(Received September 01, 2023)

## 1192-20-28708

Aaron W Messerla*, University of Illinois at Chicago. Quasi-isometries of relatively hyperbolic groups with an elementary hierarchy.
Sela introduced limit groups in his work on the Tarski problem, and showed that each limit group has a cyclic hierarchy. We prove that a class of relatively hyperbolic groups, equipped with a hierarchy similar to the one for limit groups, is closed under quasi-isometry. Additionally, these groups share some of the algebraic properties of limit groups. In this talk I plan to present motivation for and introduce the class of groups studied, as well as present some of the results for this class. (Received September 01, 2023)

## 1192-20-28715

Meng-Che Ho*, California State University, Northridge. Coding structures in groups
It has long been recognized that some classes of structures have a lower "complexity" than others. Computability theory provides a formal definition of "coding" a structure A in another structure B, establishing that B is at least as complex as A Using small cancellation theory, we "code" any finitely generated structure within a finitely generated group. This demonstrates that finitely generated groups are as complicated as possible. If time permits, we will discuss how this coding can be applied to answer a variety of questions. (Received September 02, 2023)

1192-20-28717
Michael Kapovich, University of California, Davis, Didac Martinez Granado*, University of Luxembourg. Geodesic currents and bounded backtracking property. Preliminary report.
Consider a hyperbolic group acting on a real tree. On the one hand, we say that it has the "bounded backtracking property" (BBP) if its orbit map is a one-sided quasi-isometry. This notion was introduced for free groups by Gaboriau-Jaeger-LevittLustig in 1998. On the other hand, we say the action is "small" if it has (virtually) cyclic edge stabilizers. In joint work with Misha Kapovich we show that for a large class of hyperbolic groups, small actions have the BBP. Then we relate this to the problem of extending the translation length of the action to geodesic currents, the space of invariant measures on the square of the Gromov boundary.
(Received September 02, 2023)

## 1192-20-28868

Genevieve S. Walsh*, Tufts University. Drilling hyperbolic Groups. Preliminary report.
I will define and discuss drilling a hyperbolic group, which is analagous to drilling a hyperbolic manifold along an embedded geodesic. This is joint work with Groves, Haissinsky, Manning, Osjada, and Sisto.
(Received September 04, 2023)

1192-20-28870
Mark Pengitore*, University of Virginia. Residual finiteness of the mapping class group and solvable covers. Preliminary report.
It is a classical result of Grossman that mapping class groups of finite type surfaces are residually finite. In recent years,
residual finiteness growth functions of groups have attracted much interest; these are functions that roughly measure the complexity of the finite quotients needed to separate particular group elements from the identity. Residual finiteness growth functions detect many subtle properties of groups, including linearity. In this talk, I will discuss some recent joint work with Thomas Koberda on residual finiteness growth for mapping class groups, adapted to nilpotent and solvable quotients of the underlying surface group.
(Received September 04, 2023)

## 1192-20-29020

Karina Behera, Pomona College, Rachael Combes, Biola University, James Kian Howard, San Diego State University, Christopher O'Neill, San Diego State University, Shawn Michael Perry*, St. Joseph's College of Maine, Vadim Ponomarenko, San Diego State University, Brianna Worms, James Madison University. $t$-Elasticity of Numerical Semigroups. Preliminary report.
A numerical semigroup $S$ is an additive semigroup of non-negative integers that contains zero and has a finite complement. We can associate with every $S$ a unique vector a of atoms of $S$, so that every factorization of $n \in S$ is a vector $\mathbf{x}$ satisfying $\mathbf{a} \cdot \mathbf{x}=n$. The length of the factorization $\mathbf{x}$ is the sum of the components of $\mathbf{x}$, or $\|\mathbf{x}\|_{1}$. We may extend our definition of length to $t$-length for arbitrary $t$ by using $t$-norms instead of only 1-norms. Using any $t$-length, we are able to determine the elasticity of $S$ and state if that elasticity is accepted. Moreover, viewing elasticity of a semigroup or element as a function of $t$, we give an analysis of elasticity functions for arbitrary semigroups as well as elements of two-generated semigroups.
(Received September 05, 2023)

## 1192-20-29196

Cris Negron*, University of Southern California. Modularity for quantum groups at arbitrary roots of unity. Preliminary report.
I will discuss constructions of small quantum groups at arbitrary roots of unity. Through a consideration of a number of examples, we illustrate disparate phenomena which can occur at even versus odd order roots of unity, and simply-connected versus non-simiply-connected reductive groups. In the end, we associate a finite (non-semisimple) modular tensor category to any pairing of a simply-connected reductive groups with an even order root of 1 .
(Received September 05, 2023)

1192-20-29239
Gabriel Picioroaga, University of South Dakota, Olivia Roberts*, University of South Dakota. Musical Systems with $\mathbb{Z}_{n}$ Cayley Graphs.
It is well-established that music theory uses mathematics to explain how music is created. The chromatic scale and pitch frequencies can be explained by modulo 12 arithmetic and geometric ratios. Using group theory, we interpret concepts from Western music theory. We show that in $\mathbb{Z}_{12}$, chords, scales, the circle of fifths, and the first species of counterpoint can be explained using a Cayley graph with respect to generators 3 and 4 . Using $\mathbb{Z}_{12}$ as a model, we generalize to $\mathbb{Z}_{n}$ where n is a product of two relatively prime numbers. Most major and minor chords can be constructed using paths on the oriented Cayley graph, beginning on the root of the chord. The circle of fifths can be explained by adding both generators at once. The unoriented Cayley graph gives way to a weaker form of counterpoint, with minimum distance elements forming a set of consonant elements. Then, we create partitions of consonant and dissonant elements using affine transformations to create a full generalization of counterpoint. We assume equal-tempered tuning for various $\mathbb{Z}_{n}$ systems. As application, we have written code in Maple to hear chords, scales, and counterpoint in these musical systems.
(Received September 06, 2023)

## 1192-20-29246

Nic Brody*, UC Santa Cruz. Two-by-two matrices. Preliminary report.
In this talk, we survey the class of all subgroups of $\mathbb{G}=\mathrm{PSL}_{2}(\overline{\mathbb{Q}})$. Upon choosing an embedding of the algebraic numbers in the complex numbers, we can consider those which are discrete in the usual topology. This is precisely the class of Kleinian groups, corresponding to hyperbolic 3-orbifolds. Generalizing from the Kleinian case, we consider groups which are discrete with respect to some (possibly non-Archimedean) topology on $\mathbb{G}$, and we call such a group primary. From a group-theoretic perspective, a primary group is (virtually) a free group, a surface group, or a hyperbolic 3-manifold group. For a subring $A \subseteq \overline{\mathbb{Q}}$, we will call a group commensurable with $\mathrm{PSL}_{2}(A)$ arithmetic. We propose the possibility that every subgroup of $\mathbb{G}$ must be (1) primary, (2) arithmetic, or (3) not Zariski-dense, and we discuss some evidence towards this possibility. (Received September 06, 2023)

Sogol Cyrusian*, UC Santa Barbara, Alex Domat*, Trinity College, Christopher O'Neill, San Diego State University, Vadim Ponomarenko, San Diego State University, Eric Ren*, Arizona State University, Mayla Ward*, Western Washington University. $t$-Delta Sets of Numerical Semigroups. Preliminary report.
A numerical semigroup is a cofinite submonoid of $\left(\mathbb{N}_{0},+\right)$ containing all linear combinations of a finite number of coprime integer generators. These semigroups allow for non-unique factorizations, meaning that elements can often be expressed as sums of the generators in multiple ways. Traditionally, the length of these factorizations has been measured using the 1-norm, with Delta sets consisting of gaps, which are the differences between consecutive lengths of an element when in ascending order. We introduce a method of computing lengths using $t$-norms for various $t$, and identify properties of the associated Delta sets for different families of numerical semigroups. In particular, for $t=0$, the $\Delta_{0}$-sets of all semigroups up to three generators, as well as maximal embedding dimension semigroups, semigroups with generators in generalized arithmetic progression, and semigroups with generators in a compound sequence, are explicitly given. For $t=\infty$, the $\Delta_{\infty}$-sets of semigroups with two generators are explicitly given, and the contents of other generalized families, including semigroups with generators in generalized arithmetic progression, are analyzed. The periodicity of the $\Delta_{0}$ and $\Delta_{\infty}$ sets of individual semigroup elements is also proven, along with general results for $t$-lengths between 1 and $\infty$. We also relate semigroup trade structure, $t$
-catenary degree, and $\Delta_{t}$ sets.
(Received September 06, 2023)

1192-20-29320
Edgar A. Bering, San José State University, Daniel Studenmund*, Binghamton University. Every countable locally finite group lives in $\operatorname{Comm}\left(F_{2}\right)$.
Comm $\left(F_{2}\right)$ is the group of "hidden symmetries" of the free group on two generators. Many hidden symmetries exist that are not "obvious symmetries" (automorphisms). For example, Charney and Crisp showed that every finite-type Artin group contains an infinitely generated free abelian group of hidden symmetries, and Clay, Leininger, and Margalit showed that groups $(\mathbb{Z} / 2 \mathbb{Z})^{k}$ occur as hidden symmetries of every non-abelian right-angled Artin group for arbitrarily large $k$. It is an elementary exercise to check that every finite group can be realized as a group of hidden symmetries of $F_{2}$. We will show much more is true: any union of an increasing sequences of finite groups, such as $\mathbb{Q} / \mathbb{Z}$, can be realized as hidden symmetries of $F_{2}$. To do so, we study the topology of the universal graph solenoid, the inverse limit of all finite-sheeted pointed covering spaces of the figure eight.
(Received September 06, 2023)

## 1192-20-29546

Yandi Wu*, University of Wisconsin, Madison. Marked Length Spectrum Rigidity of Certain Quotients of the Davis Complex. The marked length spectrum of a negatively curved metric space can be thought of as a length assignment to every closed geodesic in the space. A celebrated result by Otal says that metrics on negatively curved closed surfaces are determined completely by their marked length spectra. In my talk, I will discuss my work towards extending Otal's result to a large class of surface amalgams, which can arise as quotients of model geometries of right-angled Coxeter groups, a group of particular interest in geometric group theory.
(Received September 07, 2023)

1192-20-29551
Christopher O'Neill*, San Diego State University, Vadim Ponomarenko, San Diego State University. Numerical semigroups and $t$-norms of factorizations. Preliminary report.
A numerical semigroup is a subset $S$ of the natural numbers that is closed under addition, and a factorization of $n \in S$ is an expression of $n$ as a sum of generators (atoms) of $S$. Since $S$ has only finitely many atoms $n_{1}, \ldots, n_{k}$, we often denote each factorization $n=z_{1} n_{1}+\cdots+z_{k} n_{k}$ as a tuple $z=\left(z_{1}, \ldots, z_{k}\right) \in \mathbb{Z}_{\geq 0}^{k}$. The length of a factorization $z$ is the number $z_{1}+\cdots+z_{k}$ of atoms used, which coincides with the 1-norm of $z$. In this talk, we present on recent work in which the $t$-norm of $z$ is used as a generalized notion of factorization length (here, we consider $t$ in the closed interval $[0, \infty]$, where the 0 -norm of $z$ is the number of nonzero entries and the $\infty$-norm of $z$ is its maximum entry). Popular factorization invariants, such as elasticity and the delta set, are considered in this new setting, as are questions of asymptotic behavior for large $n$. Most results discussed in this talk are from student projects completed at the SDSU 2023 REU.
(Received September 07, 2023)

## 1192-20-29822

Kathryn Van Etten*, University of Nebraska - Lincoln. Introduction to Autostackable Groups.
Automatic groups have desirable geometric and computational properties, and large families of groups have automatic structures. However, several remaining examples of groups are not automatic, such as Baumslag-Solitar groups and certain 3manifold groups. Autostackable groups, introduced by Brittenham, Hermiller, and Holt in 2014, are a generalization of automatic groups that retain some of their nice properties. Most notably, autostackable groups have solvable word problem. In this talk, we introduce the definition of autostackable groups and discuss some of their properties and examples. (Received September 08, 2023)

## 1192-20-29854

Sahana H. Balasubramanya, Lafayette College, Talia Fernos*, UNC Greensboro. The Semi-Simple Theory of Acylindricity in Higher-Rank.
Acylindricity may be viewed as a generalization of being a uniform lattice in a locally compact second countable group. The theory of acylindrical actions on hyperbolic spaces has seen an explosion in recent years. Trees are of course examples of hyperbolic spaces, and by considering products, we start to see new and interesting behaviors that are not present in rank-1, such as the simple Burger-Mozes-Wise lattices, or Bestvina-Brady kernels. In a new joint worth with S. Balasubramanya we introduce a new class of nonpositively curved groups. Viewing the theory of S-arithmetic semi-simple lattices as inspiration, we extend the theory of acylindricity to higher rank and consider finite products of delta-hyperbolic spaces. The category is closed under taking products and subgroups and is therefore quite versatile. Weakening acylindricity to AU-acylindricity (i.e. acylindricity of Ambiguous Uniformity) the theory captures all S-arithmetic semi-simple lattices with rank-1 factors, the examples mentioned above, acylindrically hyperbolic groups, HHGs and many others.
(Received September 08, 2023)

1192-20-29878
Eric C. Rowell*, Texas A\&M University. non-semisimple motion group quotients.
As we lift our eyes from $2+1$ dimensional TQFTs to $3+1$ dimensions, our excuse for focusing on semisimple categories/representations/TQFTs is wearing thin. The role played by the braid group in 2 spacial dimensions is assumed by groups of motions of 1 -manifolds (i.e. knots and links) in 3 dimensions, which provides an accessible corner to investigate. Recently with Damiani and Martin we constructed some non-semisimple finite dimensional quotients of the family of groups of motions of $n$ unlinked circles, the loop braid groups. This has led to several papers in which we generalize these quotients, in joint work with Martin and Torzewska. I will present some case studies of these examples.

Kim E. Ruane*, Tufts University. Connections between CAT(0) and Morse Boundaries. Preliminary report.
I will report on recent work concerning the connections between the topology of all CAT( 0 ) boundaries for a given CAT( 0 ) group G with the topology of its Morse boundary. This is ongoing joint work with A. Karrer, M. Cordes, and M. Mihalik. (Received September 08, 2023)

Eduard Einstein, Swarthmore College, Suraj Krishna MS, Technion, MurphyKate Montee, Carleton College, Thomas Ng*, Brandeis University, Markus Steenbock, Universitat Wien. Random quotients of free products. Preliminary report. Quotients of free products are a rich source of relatively hyperbolic groups with exotic subgroups. I will discuss joint work with Einstein, Krishna MS, Montee, and Steenbock that introduces an analog of the Gromov density model for random quotients of free products. In particular, at density below $1 / 6$ such random quotients act relatively geometrically on a CAT( 0 ) cube complex. (Received September 08, 2023)

## 1192-20-30094

Daniel Alpay*, Chapman University, Ilwoo Cho, Saint Ambrose University. Scaled hypercomplex rings, from quaternions to split-quaternions.
We define a parametrized family of rings, indexed by [-1,1], and connecting the quaternions to the split-quaternions. We study this family from various points of view, including algebraically, analytically, dynamical systems and free probability. We also define and study associated regular functions and Fueter expansions.
(Received September 08, 2023)

## 1192-20-30411

Bjorn Cattell-Ravdal*, Metropolitan State University of Denver, Mandi A. Schaeffer Fry, University of Denver, Nicole Venner*, Metropolitan State University of Denver. Decoding the Structure of $\operatorname{Sp}(4, q):$ A Dive into Character Theory and Defect Groups.
The group is a well-known algebraic object and one of the most fundamental structures within mathematics. For a given group, we may correspond the elements to those of a group of invertible matrices, forming a matrix representation of the group. Associated with these representations are characters, obtained by taking the traces of each matrix. Studying these characters, we find that they contain much information about their underlying group. One of the main goals in character theory is to determine what properties about the group can be determined by looking at its characters. We will discuss several recent conjectures on this topic that study properties of certain subgroups known as defect groups. In particular, we prove some such conjectures for a specific class of groups, the symplectic group, $\operatorname{Sp}(4, q)$, of $4 \times 4$ matrices over a finite field of odd characteristic.
(Received September 10, 2023)

## 1192-20-30452

Russell Philip Stetson*, Rutgers University - New Brunswick. Characterizing Sofic Groups.
For $n \geq 1$, let $S_{n}$ be the symmetric group on $\{1,2, \cdots, n\}$ equipped with the normalized Hamming distance metric $d_{n}$. Then a group $G$ is said to be sofic if for every finite subset $F \subseteq G$, and every real $\varepsilon>0$ there exists an injection $\varphi: F \rightarrow S_{n}$ for some $n \geq 1$ such that whenever $g, h, g h \in F$, then $d_{n}(\varphi(g h), \varphi(g) \varphi(h))<\varepsilon$; if $1_{G} \in F$ then $d_{n}\left(\varphi\left(1_{G}\right), 1\right)<\varepsilon$; and for some fixed $c>0$ if $g \neq h \in F$ then $d_{n}(\varphi(g), \varphi(h)) \geq c$. It is not known whether every group is sofic. In the group theoretic literature, sofic groups are usually characterized in terms of embeddings into metric ultraproducts of finite symmetric groups. It is natural to ask whether there is a characterization in terms of the more concrete notion of a metic reduced product of finite symmetric groups. In more detail, let $P=\prod_{n \geq 1} S_{n}$ be the full direct product and let $N$ be the normal subgroup of elements $\left(\pi_{n}\right) \in P$ such that $d_{n}\left(\pi_{n}, 1\right) \rightarrow 0$ as $n \rightarrow \infty$. Then the metric reduced product is the quotient $P_{0}=P / N$. We have shown that it is neither provable nor disprovable from the $Z F C$ axioms of set theory that if $G$ is a group such that $|G| \leq 2^{\aleph_{0}}$, then $G$ is a sofic group if and only if $G$ embeds into $P_{0}$.
(Received September 10, 2023)

## 1192-20-30570

Paul Baginski*, Fairfield University. Algebraic Properties of Subsemigroups and Semigroup Ideals of Factorial Monoids. Consider a factorial monoid $F=F^{\times} \times \mathcal{F}(P)$, where $P$ is a set of prime elements and $F^{\times}$are the units of $F$. We will discuss the general algebraic and arithmetic properties of subsemigroups $H$ of $F$, determining when they are Krull, root closed, bounded factorization, etc. Attention will be paid to subtle technical details that arise when adapting concepts from the factorization theory of monoids to the setting of semigroups. Additionally, the role of the units $F^{\times}$, particularly the set $F^{\times} \cap H$, will be discussed in detail. We then will specialize to semigroup ideals, namely nonempty subsets $H$ of $F$ satisfying $H F=H$. Here, many of the technical concerns involving units are quickly resolved and we obtain a much clearer picture of the algebra, which is often wild from a factorization-theoretic perspective. In particular, we will exhibit a large class of semigroup ideals which exhibit pathological factorization properties in the "furcus" family of conditions (bifurcus, $m$-furcus, and multifurcus).
(Received September 10, 2023)

Lycan II*, San Diego State University, Sarah Mendoza De La Cruz*, University of Texas at Austin, Christopher O'Neill, San Diego State University, Vadim Ponomarenko, San Diego State University. On Generalized Factorization Lengths in Atomic Monoids. Preliminary report.
We generalize the concept of factorization length within atomic semigroups, and present a study of the factorization lengths of iterated powers of elements. We do so through the investigation of several families of semigroups, including numerical semigroups, block monoids, and arithmetical congruence monoids. Our results range from asymptotic behavior to complete, quasi-polynomial characterizations of factorization length.
(Received September 10, 2023)

## 1192-20-30610

Wolfgang A. Schmid*, University Paris 8. Generalized cross numbers. Preliminary report.
A classical concept in factorization theory is the cross number of a sequence. For $(G,+)$ be a finite abelian group and a sequence $S=g_{1} \ldots g_{l}$ over $G$ the cross number of $S$, denote by $\mathrm{k}(S)$, is defined as $\mathrm{k}(S)=\frac{1}{\operatorname{ord}\left(g_{1}\right)}+\cdots+\frac{1}{\operatorname{ord}\left(g_{1}\right)}$. It is a a key-tool in investigations of monoids of zero-sum sequences over finite abelian groups. The definition extends in a meaningful way to abelian torsion groups and more generally to sequences of elements of finite order in not necessarily commutative groups. However, for other monoids of interest, such as numerical monoids or monoids of weighted zero-sum sequences a direct generalization is not possible or not meaningful. For a certain class of $C$-monoids, which includes monoids of weighted zero-sum sequences and numerical monoids, we present a generalized notion of cross number and discuss some applications in the context of weighted zero-sum sequences. This is joint work with K. Merito and O. Ordaz.
(Received September 10, 2023)

## 1192-20-30647

G Christopher Hruska*, University of Wisconsin-Milwaukee, Genevieve Walsh, Tufts University. Relatively hyperbolic groups with planar boundaries. Preliminary report.
Consider a relatively hyperbolic group pair $(G, \mathbb{P})$ with planar Bowditch boundary $\partial(G, \mathbb{P})$ not homeomorphic to $S^{2}$. We assume that $G$ is one ended and its boundary is connected with no cut points. We also assume that $G$ has a finite index subgroup with no 2 -torsion. However, we do not require that peripheral subgroups be virtually abelian. If the boundary contains no embedded Sierpiński carpet then $G$ is virtually Kleinian. More generally, if every toral relatively hyperbolic group pair with Sierpiński carpet boundary is virtually Kleinian, then $G$ is virtually Kleinian. This theorem generalizes a result proved by Haïssinsky using heavy analytic methods.
(Received September 10, 2023)

## 1192-20-31065

April Chen, Harvard University, Nathan Kaplan*, University of California, Irvine, Liam Lawson, University of California, Irvine, Christopher O'Neill, San Diego State University, Deepesh Singhal, University of California, Irvine. Numerical sets associated to a numerical semigroup.
A numerical set $T$ is a subset of $\mathbb{N}_{0}$ that contains 0 and has finite complement. The atom monoid of $T$ is the set of $x \in \mathbb{N}_{0}$ such that $x+T \subseteq T$. Marzuola and Miller introduced the anti-atom problem: how many numerical sets have a given atom monoid? This is equivalent to asking for the number of integer partitions with a given set of hook lengths. We introduce the void poset of a numerical semigroup $S$ and show that numerical sets with atom monoid $S$ are in bijection with certain order ideals of this poset. We will survey what is known about the anti-atom problem and discuss questions about partitions with a given set of hook lengths.
(Received September 11, 2023)

## 1192-20-31180

Jean-Francois Lafont, The Ohio State University, Lorenzo Ruffoni*, Tufts University. Hyperbolization, cubulation, and applications.
The Charney-Davis strict hyperbolization is a procedure that turns polyhedra into spaces of negative curvature, while preserving some topological features. It has been used to construct examples of manifolds that exhibit unexpected features, despite having negative curvature. One may expect the fundamental groups of these manifolds to display strange features as well. On the other hand, we show that these groups admit nice actions on CAT(0) cube complexes, both in the absolute and relative settings. As an application, we obtain new examples of negatively curved Riemannian manifolds whose fundamental groups are virtually special and algebraically fibered.
(Received September 11, 2023)

## 1192-20-31223

Dylan Schuster*, St. Norbert College. Classifying Character Degree Graphs with Seven Vertices. Preliminary report. In this talk, we discuss graphs with seven vertices in an effort to classify which of them appear as the prime character degree graphs of finite solvable groups. This classification is complete for the disconnected graphs. Of the 853 non-isomorphic connected graphs, we were able to demonstrate that twenty-two occur as prime character degree graphs. Forty-four graphs remain unclassified.
(Received September 11, 2023)

## 1192-20-31514

Georgia Burkhalter*, University of North Georgia, Rachel Lauren Pincus Niebler*, Haverford College. Property (NL) in Coxeter Groups.
A group has Property (NL) if it does not admit a loxodromic action on any hyperbolic space. This property has been studied in torsion and Thompson-like groups. We expand on this research by studying this property in Coxeter groups, a class of groups that are defined by an underlying graph. First, we show that a right-angled Coxeter group has Property (NL) if and only if its
defining graph is complete. Second, we move beyond the right-angled case to show that if a graph is disconnected, its corresponding Coxeter group does not have Property (NL). Third, we classify which triangle groups (Coxeter groups with three generators) have Property (NL) by analyzing their actions on different spaces.
(Received September 11, 2023)

## 1192-20-31674

Bangzheng Li*, Massachusetts Institute of Technology. Hereditarily atomic, underatomic, and ACCP. Preliminary report. A cancellative commutative monoid is atomic if each nonzero nonunit factors into irreducibles. In this talk, we will talk about various stronger versions of atomicity, including ACCPness, hereditarily atomicity, and underatomicity. We will study the intervention between these notions. In particular, we will discuss the theorem that underatomic implies hereditarily atomic, which implies ACCP. In the proof we introduce a partial order on the set of all submonoids of a certain monoid. With the help of this structure, we are able to show that under-BFM also implies hereditarily BFM. Finally, we will state why similar pattern is violated when we are trying to show that under-FFM implies hereditarily FFM. This talk is based on a joint work with Felix Gotti.
(Received September 12, 2023)

## 1192-20-31936

Stian Du Preez*, Rice University, Zachary Alexander Duah, University of Michigan, David Milan, University of Texas at Tyler, Shreyas Ramamurthy, University of California, Berkeley, Lucas Vega, University of Texas at Tyler. Morita equivalence of inverse hulls of Markov shifts. Preliminary report.
We characterize the Morita equivalence of inverse hulls of semigroups associated with Markov shifts. In particular, we demonstrate how to construct a labeled graph that is a complete invariant of the Morita equivalence classes of inverse hulls of semigroups associated with Markov shifts over finite alphabets. (Received September 12, 2023)

1192-20-32369
Robert Boltje, University of California, Santa Cruz, Serge Bouc, CNRS-LAMFA, Universite de Picardie - Jules Verne, Hatice
Mutlu Akaturk*, University of California, Los Angeles. Lefschetz invariants of monomial G-posets and canonical induction. The Euler-Poincaré characteristic of a given poset $X$ is defined as the alternating sum of the orders of the sets of chains $\backslash \operatorname{Sd}_{n}(X)$ with cardinality $n+1$ over the natural numbers $n$. Given a finite gorup $G$, Thévenaz extended this definition to $G$ posets and defined the Lefschetz invariant of a $G$-poset $X$ as the alternating sum of the $G$-sets of chains $\backslash \operatorname{Sd}_{n}(X)$ with cardinality $n+1$ over the natural numbers $n$ which is an element of Burnside ring $B(G)$. Let $A$ be an abelian group. We will introduce the notions of $A$-monomial $G$-posets and $A$-monomial $G$-sets, and state some of their categorical properties. The category of $A$-monomial $G$-sets gives a new description of the $A$-monomial Burnside ring $B_{A}(G)$. We will also introduce Lefschetz invariants of $A$-monomial $G$-posets, which are elements of $B_{A}(G)$. An application of the Lefschetz invariants of $A$ monomial $G$-posets is the $A$-monomial tensor induction. Another application is a work in progress that aims to give a reformulation of the canonical induction formula for ordinary characters via $A$-monomial $G$-posets and their Lefschetz invariants. For this reformulation we will introduce $A$-monomial $G$-simplicial complexes and utilize the smooth $G$-manifolds and complex $G$-equivariant line bundles on them.
(Received September 12, 2023)

1192-20-32423
Karina Behera*, Pomona College, Rachael Combes*, Biola University, James Kian Howard, San Diego State University, Christopher O'Neill, San Diego State University, Shawn Michael Perry*, St. Joseph's College of Maine, Vadim Ponomarenko, San Diego State University, Brianna Worms*, James Madison University. On the t-elasticity of numerical semigroups. Preliminary report.
A numerical semigroup, $S=\left\langle a_{1}, \ldots, a_{k}\right\rangle$, is a subset of the nonnegative integers that is closed under addition and has a finite complement in the nonnegative integers. We call the vector, $\mathbf{a}=\left(a_{1}, \ldots, a_{k}\right)$, the generating vector. Any element $n \in S$ can be written as $n=\mathbf{a} \cdot \mathbf{z}$, for which $\mathbf{z}=\left(z_{1}, z_{2}, \ldots, z_{k}\right)$ is called a factorization of $n$. In the study of semigroups, the notion of the length of a factorization is typically defined as its coordinate sum, or $\ell_{1}$ norm. We extend this notion to all $\ell_{p}$ norms using a real-valued parameter $t \in[0, \infty)$, defining the $t$-length of a factorization in terms of its $\ell_{t}$ norm. With these new definitions of length, we are able to determine both the local $t$-elasticity of $n \in S$ and the global $t$-elasticity of the entire semigroup $S$. Additionally, we can state whether or not that elasticity is accepted. Moreover, by viewing elasticity as a function of $t$, we give an analysis of both the global elasticity for any semigroup and the local elasticity for elements of two-generated semigroups. (Received September 12, 2023)

1192-20-32462
Kailey Perry*, University of Arkansas. Geometry of relatively hyperbolic groups. Preliminary report.
A finitely generated group is hyperbolic if its Cayley graph with regards to any finite generating set is a hyperbolic metric space, meaning that there exists some $\delta$ where triangles in the Cayley graph are $\delta$-thin. A relatively hyperbolic group is a group $G$ along with a finite set of peripheral subgroups $\mathcal{H}$ with two properties. First, the coned-off Cayley graph obtained by coning off cosets of each $H_{i} \in \mathcal{H}$ is a hyperbolic metric space. Also, $(G, \mathcal{H})$ must have bounded coset penetration which is a property on quasi-geodesics in the Cayley graph that penetrate cosets of a peripheral subgroup. I will be talking about relatively hyperbolic groups, particularly free by cyclic groups, semi-direct products $F_{n} \rtimes_{\phi} \mathbb{Z}$ where $\phi$ is an automorphism of the free group $F_{n}$.
(Received September 12, 2023)

Sogol Cyrusian, UC Santa Barbara, Alex Domat*, Trinity College, Christopher O'Neill, San Diego State University, Vadim Ponomarenko, San Diego State University, Eric Ren, Arizona State University, Mayla Ward, Western Washington University. $\Delta_{\infty}$ Sets of Numerical Semigroups.
The numerical semigroup generated by coprime positive integers $a_{1}, \ldots, a_{k}$ is defined to be
$S=\left\langle a_{1}, \ldots, a_{k}\right\rangle=\left\{c_{1} a_{1}, \ldots, c_{k} a_{k}: c_{1}, \ldots, c_{k} \in \mathbb{N} \cup\{0\}\right\}$ and forms a monoid under addition. The vector $\left(c_{1}, \ldots, c_{k}\right)$ is said to be a factorization of the corresponding element of $S$. Numerical semigroups do not admit unique factorization, so we may examine the lengths of different factorization vectors for a single element. This gives rise to the "delta set" invariant for an element of $S$, which is defined to be the set of all gaps between consecutive factorization lengths of an element of $S$. The delta set of a semigroup is the union of the delta sets of all of its elements. Typically, factorization lengths are measured using the 1-norm. In this talk, we will instead measure factorization lengths using the $\infty$-norm and explore what can be said about delta sets using this norm.
(Received September 12, 2023)

## 1192-20-32719

Sogol Cyrusian, UC Santa Barbara, Alex Domat, Trinity College, Eric Ren, Arizona State University, Mayla Ward*, Western Washington University. A Structure Theorem for Infinity-Length Delta Sets of Numerical Semigroups. Preliminary report. A Numerical Semigroup is a cofinite submonoid of $(N,+)$. These semigroups admit non-unique factorization into irreducibles; the factorization set of a single element is conventionally denoted as a set of vectors. Extensive existing literature considers the set of 1-norms of said vectors (the "length set") and the gaps between consecutive lengths (the "delta set"). We alter this conception of length sets by introducing the infinity-norm as a measure of length, and identify properties of the associated delta sets. We prove that the infinity-length delta set for numerical semigroups is periodic, and identify certain values which are guaranteed to lie within the delta set. (Received September 12, 2023)

## 1192-20-32736

Meghan Maureen De Witt*, St. Thomas Aquinas College. Harnessing Group and Group-like Attributes to Explore Phenomena. Preliminary report.
Various examinations of group based structures have been explored and will be presented. We examine the symmetric groups of hypercubes and their interrelationship. We examine a semigroup strucutre inherent in swarms and how this structure might be defined and harnessed for better exploration and understanding of such.
(Received September 12, 2023)

1192-20-32746
Meghan Maureen De Witt*, St. Thomas Aquinas College. Flatlands Experiments that lead to Tesseract Groups. Preliminary report.
Math Circle and outreach activities have the ability to grab a students attention and get them to understand the true beauty of mathematics. We describe an activity of exploring hypercubes and higher dimensions with a Flatlands recreation that led to a research project with undergraduates on symmetries of hypercubes and their interconnections.
(Received September 12, 2023)

1192-20-33223
Kolton Oneal*, Texas State University. Classifying Primitive Solvable Permutation Groups of Rank 5 and 6.
Let $G$ be a finite solvable permutation group acting faithfully and primitively on a finite set $\Omega$. Let $G_{0}$ be the stabilizer of a point $\alpha$ in $\Omega$. The rank of $G$ is defined as the number of orbits of $G_{0}$ in $\Omega$, including the trivial orbit $\{\alpha\}$. In this paper, we completely classify the cases where $G$ has rank 5 and 6 , continuing the previous works on classifying groups of rank 4 or lower.
(Received September 13, 2023)

## 1192-20-33678

Noah Jillson*, University of Wisconsin-Madison, Pramana Saldin*, University of Wisconsin-Madison, Katerina Stuopis*, University of Wisconsin - Madison. Geometry of Horospheres in Right-Angled Coxeter Groups.
This research project aims to find and optimize algorithms over graphs of horospheres in the hyperbolic group defined by a flag triangulation of a torus with no squares. Another goal is to explore visualization methods for these intricate graphs. We first looked at the Cayley graph defined by a right-angled Coxeter group corresponding to specific symmetries of a pentagon in the hyperbolic plane. Using concepts of geometric group theory we can define a notion of distance in our group. We aim to study horospheres of these groups, which are analogies of spheres with center taken to infinity. Using tools such as finite state machines and fiber products we are able to efficiently generate pieces of the horospheres of right-angled Coxeter Groups. The quick computation allows us to visualize the geometry in a manner not allowed for before. After generating sections of the horosphere corresponding to the torus defining graph, we explored other hyperbolic groups such as those defined by a $\Theta$ graph. This gives us insight into the boundaries of such groups.
(Received September 22, 2023)

1192-20-33791
Eitan Marcus, Bowdoin College, Gavin Pettigrew, University of Washington, Saskia Solotko, Tufts University, Lixin Zheng*, Georgia Insitute of Technology. Prime Graphs of $K_{4}$-solvable groups.
We continue the study of prime graphs (Gruenberg-Kegel graphs) of finite groups. For a group $G$, the vertices of the prime graph $\Gamma(G)$ are the primes that divide $|G|$, and two vertices $p$ and $q$ are connected by an edge if and only if there is an element of order $p q$ in $G$. A complete classification of prime graphs of finite groups remains elusive, yet progress has been made for some families of groups. Prime graphs of solvable groups as well as prime graphs of groups with $K_{3}$ groups as
composition factors have been completely classified. In this paper, we classify prime graphs of various $K_{4}$ groups. In particular, we completely classify prime graphs of $A_{7}$-solvable groups, which motivates the development of general techniques for a wide range of other groups. We find that almost all prime graph complements are 3 -colorable and they have few possible triangles, and we give complete graph-theoretical classifications when possible.
(Received September 26, 2023)

1192-20-33795
Kent B. Vashaw, Massachusetts Institute of Technology, Justin Zhang*, Bergen County Academies. On the Summands of Tensor Products of Monomial Modules over Finite 3-groups.
For finite 2 -groups, Dave Benson conjectured that over an algebraically closed field $\mathbb{k}$ of characteristic 2 , for any
indecomposable module $V$ of odd dimension, $V \otimes V^{*}$ has a unique odd dimensional summand $\mathbb{k}$, and thus any tensor power of $V$ has a unique summand of odd dimension. We examine the validity of an extension of Benson's conjecture to finite 3-groups; namely, we study for what modules $W$ with dimension coprime to 3 does $W \otimes W^{*}$ have a unique summand with dimension coprime to 3 , implying that any tensor power of $W$ has a unique summand with dimension coprime to 3 . We study this proposed extension over monomial modules, a type of graded representation over the group $\mathbb{Z}_{3^{a}} \times \mathbb{Z}_{3^{b}}$ for positive integers $a$ and $b$ which correspond to skew Young diagrams. We show that the proposed extension fails for monomial modules corresponding to partitions $\left(a_{1}, a_{2}, \cdots, a_{n}\right)$ with $a_{i} \equiv 0,5(\bmod 9)$ for $1 \leq i \leq n$. We demonstrate that for all such modules, there exists an isomorphic summand of dimension 5 in their decompositions. We then use the syzgy functor to show that the extension also fails for monomial modules corresponding to $\left(a_{1}, a_{2}, \cdots, a_{n}\right)$ with $a_{i} \equiv 0,4(\bmod 9)$ for $1 \leq i \leq n$. We propose that these two cases are the only monomial representations with null inner partition which fail the proposed extension of Benson's conjecture, which we support with abundant computational evidence. (Received September 26, 2023)

## 1192-20-33851

Jessica Carlos*, Troy University, Patrick Rossi, Troy University. Group Tables.
When Is a Group Table NOT a Group Table? The author examines some enigmatic features of a group table, distinguishing between necessary and sufficient conditions for a table to be a group table The enigmatic group table passes the "sniff test" for being a group table, but is that good enough? What easily applied criteria can we apply in order to decide the issue conclusively?
(Received September 26, 2023)

1192-20-33857
Alexandra Lee Bartas*, San Francisco State University, Bruce Leavitt*, University of California, Los Angeles, Ben Vessely*, University of Oregon. The Futurama Theorem: Exploring Solutions to Mind-Swapping Variations Using the Symmetric Group.
A 2010 episode of Futurama features a mind-swapping machine that is unable to swap the same pair's minds more than once. After a chaotic series of mind swaps, the characters need to find a way to swap each of their minds back. The solution presented in the episode, proven by Futurama writer and mathematician Ken Keeler, shows that any permutation of mind swaps in these conditions has an inverse that can be split into distinct transpositions, thus returning all participants to their original bodies. J. Elder's work presents variations of the theorem, such as the case in which the machine swaps an even or prime number of people at once, and restricts the solution to only involve cycles that form distinct cyclic groups. Our research completes the remaining case from Elder's restriction, in which the machine cycles an odd composite number of people. Using visual graph representations and the symmetric group, we explore an additional restriction, namely that the solution cannot involve cycles that permute the same elements. We then present general solutions for three cases in which the machine operates with different cycle lengths, dealing first with 3-cycles; then, with odd-length cycles greater than three; and finally, with even-length cycles. We further hope to present proofs of the optimality of our solutions and the case in which the machine cycles an infinite amount of people. This work was funded by the CC-REU NSF REU grant (DMS-2050692).
(Received September 26, 2023)

## 22 Topological groups, Lie groups

## 1192-22-27897

Michael Zshornack*, University of California, Santa Barbara. The arithmetic of the Hitchin component.
Higher Teichmüller spaces are deformation spaces of surface group representations which generalize the setting of classical Teichmüller theory to higher-rank Lie groups. The Hitchin component was one of the first examples of such a space to be studied. The representations on these components are all Anosov, and as a result, have many interesting dynamical and coarse-geometric properties. Recently, interest has been drawn towards the arithmetic properties of these components because of their connection with understanding "thin" subgroups of lattices in semisimple Lie groups. In this talk, we will survey some of the ways, old and new, in which the geometry of these components can be leveraged into an understanding of their arithmetic.
(Received August 22, 2023)

## 1192-22-29785

Raul Quiroga-Barranco*, Cimat, Mexico. Lie Theory, Symplectic Geometry and Hypercomplex Analysis. Complex bounded symmetric domains are well known to be closely related to Lie theory. This leads to interesting unitary representations of simple Lie groups on Bergman spaces over those domains. Similarly, the study of classical Fock spaces benefits from the unitary representations of the Heisenberg group. These tools together with those coming from symplectic geometry and the computation of moment maps have shown to be very useful in the understanding of operator theory on classical analytic function spaces. We will explore the application of corresponding tools from Lie theory and symplectic geometry to the setup of hypercomplex analysis. Our goal is to pave the way to a systematic development of such tools.

## 1192-22-29890

Hang Xue*, The University of Arizona. Applications of theta lifts to restriction problems for unitary groups.
We explain how to use theta lifts to prove the local Gan-Gross-Prasad conjecture for real unitary groups.
(Received September 08, 2023)

## 1192-22-30960

Robert Dolan, Grand Valley State University, Amelia Geotzinger, Grand Valley State University, Evan Henning, Grand Valley State University, Firas Y Hindeleh*, Grand Valley State University, Duc Phan, Grand Valley State University, Samuel
Webb, Grand Valley State University. On the Classification of seven-dimensional solvable Lie algebras with five-dimensional nilradical. Preliminary report
In this talk we give an update on the classification problem of the seven-dimensional solvable Lie algebras. We highlight our contribution to the case where nilradical is isomorphic to $A_{5,1}, A_{5,2}$, and $A_{5,3}$.
(Received September 11, 2023)

## 1192-22-31683

Michael Eddy, Swarthmore College, Alpha C Recio Valerio*, University of Massachusetts, Amherst, Juan Rosete,
University of California, Channel Islands. Topological Decoupling of Quasiperiodic Videos. Preliminary report.
This research presents a method for analyzing quasiperiodic videos by leveraging the principles of topological decoupling. We employ persistent cohomology to identify and track persistent topological structures over time. Our findings demonstrate the efficacy of topological decoupling in extracting meaningful spatiotemporal features, which can be harnessed for various applications including de-noising, anomaly detection, and video synthesis. Specifically, the focus of our work today is on a video consisting of two oscillators that continuously move back and forth. The objective is to investigate the manipulation of these oscillators, aiming to stop one while allowing the other to continue its motion.
(Received September 12, 2023)

## 1192-22-32334

Zsuzsanna Dancso, University of Sydney, Iva Halacheva*, Northeastern University, Marcy Robertson, University of Melbourne. Extending solutions of the Kashiwara-Vergne equations degree by degree. Preliminary report. Solutions to the Kashiwara-Vergne equations in Lie theory have been shown to play an important role in topology, describing finite-type invariants for a family of knotted objects known as welded foams. These solutions come from the degree-completed free Lie algebra on 2 generators, which has a grading, so a natural question is whether, given a solution up to degree n it can be extended to degree $n+1$. We show this is indeed the case and leads to further interesting properties of the KashiwaraVergne groups acting on this set of solutions.
Received September 12, 2023)

## 1192-22-32905

Cemile Kurkoglu*, Denison University. Duality in Category $O^{\infty}$ and Locally Analytic Representations.
My research involves understanding representations of $p$-adic reductive groups on $p$-adic vector spaces, the construction of such representations using Lie algebra methods, and certain operations on those representations called 'dualities'. A representation of a $p$-adic reductive group $G$ leads to a representation of the associated Lie algebra $\mathfrak{g}=\operatorname{Lie}(G)$. In the other direction, going from $\mathfrak{g}$-modules to representations, S. Orlik and M. Strauch have exhibited a functor $\mathcal{F}$ from the Bernstein-Gelfand-Gelfand category $\mathcal{O}$ to the category of locally analytic representations, which was later generalized in work of S . Agrawal and M. Strauch to the extension closure $\mathcal{O}^{\infty}$. The derived category of category $\mathcal{O}^{\infty}$ carries an Ext-duality functor $\mathbb{D} \mathfrak{g}$. On the other hand, P. Schneider and J. Teitelbaum have defined and studied a duality functor $\mathbb{D}^{G}$ for locally analytic representations of $G$. The main result explains how, under certain technical assumptions, the functor $\mathcal{F}$ interacts with the duality functors $\mathbb{D}^{\mathfrak{g}}$ and $\mathbb{D}^{G}$.
(Received September 13, 2023)

## 1192-22-33348

Bob Palais*, University of Utah. Rotation Matrices and Quaternions Revisited. Preliminary report.
There are two opposite unit quaternions corresponding to each 3D rotation matrix. There are several advantages of using the former for composition, interpolation, and other computations. We will compare some of the more efficient approaches in each of the two paradigms, and relate their geometric, algebraic, analytical, and historical significance
(Received September 13, 2023)

## 26 Real functions

## 1192-26-30000

Katharine A. Ott*, Bates College. Estimates for Brascamp-Lieb forms in $L^{p}$-spaces with power weights.
I will present on ongoing work to study a family of Brascamp-Lieb forms acting on families of weighted $L^{p}$-spaces where the weight is a power of the distance to the origin. We aim to find the largest set of indices for which we have weighted estimates for these multilinear forms. In this talk I will discuss a set of necessary and sufficient conditions for a special class of forms and then discuss the state of the problem in a more general setting.
(Received September 08, 2023)

Fred Halpern*, Preservation theorems via Smullyan clashing tableau. Integration by Guessing.
ass\{article\}\usepackage\{amsmath,amsthm,amssymb\}\documentIntegrationbyGuessingstreamlinestwostandardintegrationtechniques,SubstitutionandIntegrationbyParts.Weprovidesomeexamplesillustratingitspowerandalsoshowitcanleadtonewresults,e.g.,thebelowextensionofPease'stheorem.Thefirstandsecondderivativesof$f$aredenotedby$\dot{f}$and$\ddot{f}$.Guessing$\dot{f}gh,f\dot{g}h,fg\dot{h}$,and$\dot{f}\dot{g}\dot{h}$:TheoremFor$\ddot{f}=af,\ddot{g}=bg,\ddot{h}=ch,\intfgh=$$\frac{(a-b-c)\dot{\mathrm{f}}\mathrm{g}h+(-a+b-c)f\dot{g}h+(-a-b+c)fg\mathrm{~h}+2\mathrm{f}\dot{\mathrm{g}h}}{}{}^{2}+b^{2}+c^{2}-2(ab+ac+bc)$.$\backslash$parSo,$\int\expx\sin2x\cosh3x$iseasy!Letting$g=f$or$g=\operatorname{dot}f$(etc.)yieldsspecialformula.So$\int(\sin2x)^{3}$and$\int\expx\sinx\cosx$arealsoeasy.$\backslash\backslash$par$\backslash$par$\backslash$parIntegrationbyGuessingemphasizesthatSubstitutionandIntegrationbyPartsbeginwithahiddenguess.Itconsistsofthreesteps:Guess,Differentiatetochecktheguess,andthenAdjusttogetanexactfit.Theadjustmentsareoftwokinds:\par1.Iftheguessisoffbyafactor,dividebythefactor.\par2.Ifthederivativeoftheguesshasanextraterm,thensubtracttheintegralofthetermfromtheguess(Oneuseisintegrationbyparts).\par(ReceivedSeptember10,2023)undefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefined

## 28 Measure and integration

## 1192-28-27135

Iqra Altaf, University of Chicago, Ryan Edward George Bushling*, University of Washington, Bobby L. E. Wilson, University of Washington. Distance sets bounds for polyhedral norms via effective dimension.
We prove that, for every norm on $\mathbb{R}^{d}$ and every $E \subseteq \mathbb{R}^{d}$, the Hausdorff dimension of the distance set of $E$ with respect to that norm is at least $\operatorname{dim}_{\mathrm{H}} E-(d-1)$. An explicit construction follows, demonstrating that this bound is sharp for every polyhedral norm on $\mathbb{R}^{d}$. The techniques of algorithmic complexity theory underlie both the computations and the construction. (Received August 10, 2023)

## 1192-28-27712

Haley Broadus*, University of South Alabama, Madolyn Donaghy-Robinson*, Ithaca College, Hope Steen*, New College of Florida. Pruning 3D Fractal H-Trees. Preliminary report.
The construction and topological properties of binary branching fractal trees, originally described by Mandelbrot and Frame, are relatively well understood in two dimensions. Path-connectivity and space-filling criteria are easily established via simple geometric and trigonometric arguments, making the construction and analysis of these fractal sets a fairly straightforward undertaking. Some work has already been done on 'pruned' trees, i.e iterated function systems (IFSs) with memory, which are made by constructing the trees via IFS and then removing specific sequences of transformations. It is even possible, in some cases, to construct some of these pruned trees with a different set of transformations without having to 'manually' prune them. We reproduce some of these results on the 3D "H-Tree," a space-filling tree analogous to the classic 90-degree H-tree that is space-filling in two dimensions. We demonstrate that the tree is space-filling, and that all of the volume is lost for any given pruning. We then provide constructive proofs for some of the simpler pruned trees. We also determine which of these are pathconnected. Moreover, we explore how some of these pruned trees can be constructed using iterated function systems without resorting to pruning sequences, offering new insights into their generation. Our work concludes with suggestions for further research, aiming to deepen the understanding of pruned 3D H-trees, their properties, and their relation to binary strings. (Received August 18, 2023)

1192-28-28476
Darrell Duffie, Stanford University, Lei Qiao, Shanghai University of Finance and Economics, Yeneng Sun*, National University of Singapore. Continuous-Time Random Matching: A General Model. Preliminary report.
We develop a general model for independent random matching of a large population in a continuous-time dynamical system. We work with a general (and possibly infinite) type space for the agents and construct a continuum of independent continuoustime Markov processes that is derived from random mutation, random matching with enduring partnership, random type changing and random break up. It follows from the exact law of large numbers that the deterministic evolution of the agents' realized type distribution for such a continuous-time dynamical system can be determined by a system of measure-valued differential equations. The results provide the first mathematical foundation for a large literature on continuous-time searchbased models of labor markets, money, and over-the-counter markets for financial products.
(Received September 06, 2023)

1192-28-29107
Caleb Z Marshall*, University of British Columbia, Izabella Laba, University of British Columbia. The Favard Length and Cyclotomic Structure of Rational Product Cantor Sets.
We discuss our improvement of the Lam-Leung lower bound on the number of elements in a vanishing sum of $N$-th roots of unity. Using this result, we extend the Favard length estimates due to Bond, Łaba, and Volberg to a new class of rational product Cantor sets in $\mathbb{R}^{2}$. We also discuss our ongoing study of cyclotomic divisors of masks polynomials. Specifically, we highlight the implications of further results in cyclotomic divisibility for Favard length estimates of rational product Cantor sets.
(Received September 05, 2023)

1192-28-29219
Robert M Anderson*, UC Berkeley, Haosui Duanmu, Harbin Institute of Technology, Mohammed Ali Khan, Johns Hopkins University, Metin Uyanik, University of Queensland. General Equilibrium Theory for Measure-Theoretic Economies. Preliminary report.

Hara(2006) provided a counter-example on the existence of equilibrium in exchange economies with bads and a measuretheoretic space of agents. Noguchi \& Zame (2006) established the existence of equilibrium with distributional externalities on production economies with a measure-theoretic space of agents, but their proof depends on strong monotonicity of preferences and free-disposal in production, which are incompatible with the presence of bads. In this paper, we provide sufficient conditions for the existence of equilibrium in a measure-theoretic production economy with bads and externalities on agents' preferences. The proof of our existence result depends heavily on nonstandard analysis. Given a standard measure-theoretic production economy, we construct a corresponding hyperfinite economy and hyperfinite Loeb economy. We first invoke the existence of equilibrium result from finite production economy to ensure the existence of equilibrium in the hyperfinite economy. We propose sufficient conditions to ensure S-integrability of the candidate equilibrium in the Loeb economy. Finally, we push down the equilibrium allocation in the Loeb economy to obtain an equilibrium in the original measure-theoretic production economy by taking standard part with respect to the weak topology.
(Received September 05, 2023)

1192-28-29303
Mohammed Ali Khan*, Johns Hopkins University, Arthur Paul Pedersen, Remote Sensing Earth Systems Institute, City University of New York, Maxwell B Stinchcombe, Department of Economics, University of Texas at Austin. On a Variety of Loeb Measures and the Theory of Continuum Population Games. Preliminary report.
This talk of 20 minutes is structured in three parts: (i) Five minutes devoted to the pre-history of Loeb measure during which the speaker will trace in earlier work of Peter Loeb the origins that led to the culmination of the 1975 final concept; (ii) an overview, also of 5 minutes, lays out an overview of the theory of continuum population (large anonymous and non-anonymous) games but in a generalized context in which beliefs are possibly mis-specified and notions of rationality are only approximate; (iii) a presentation of two results during the remaining time for continuum population games with measures of the degree of misspecification that are equicontinuous across the population: the existence of exactly normative distributional strategies in such games provided that the distribution of the utility functions is neighborhood tight, and an example showing that normatively adequate strategies may fail to exist if neighborhood tightness is violated.
(Received September 06, 2023)

1192-28-29322
Alex Iosevich, University of Rochester, Akos Magyar, University of Georgia, Alex McDonald*, The Ohio State University, Brian McDonald, University of Georgia. The VC-dimension and point configurations in fractal subsets of $\mathbb{R}^{d}$. Preliminary report.
An important class of problems at the intersection of harmonic analysis and geometric measure theory asks how large the Hausdorff dimension of a set $E \subset \mathbb{R}^{d}$ must be to ensure that it contains certain types of geometric point configurations. A classic example is the Falconer distance problem, which asks how large the Hausdorff dimension must be to ensure the set determines a positive measure worth of distances. The techniques used to establish bounds for the Falconer problem can also be applied to find more complex patterns. We apply these tools to study the VC-dimension of a naturally arising class of indicator functions on fractal subsets of $\mathbb{R}^{d}$.
(Received September 06, 2023)

## 1192-28-30586

Kevin Ren*, Princeton University, Hong Wang, UCLA. Furstenberg sets estimate in the plane.
We fully resolve the Furstenberg set conjecture in $\mathbb{R}^{2}$, that a $(s, t)$-Furstenberg set has Hausdorff dimension $\geq \min \left(s+t, \frac{3 s+t}{2}, s+1\right)$. As a result, we obtain an analogue of Elekes' bound for the discretized sum-product problem and resolve an orthogonal projection question of Oberlin.
(Received September 10, 2023)

## 1192-28-30741

David A. Ross*, Department of Mathematics, University of Hawaii, Honolulu, HI 96822. Some little-known properties of Loeb measures.
The emergence in recent years of new applications for Loeb measures has attracted a new set of mathematicians using the machinery. In this talk I'll survey some little-known properties of Loeb measures that might be useful in such applications but are likely unfamiliar to many people using them.
(Received September 10, 2023)

1192-28-31044
Mihai Stoiciu*, Williams College. Families of Indeterminate Solutions of the Classical Moment Problem. Preliminary report. The classical moment problem asks if a given sequence of moments $m_{0}, m_{1}, \ldots$, corresponding to a measure $\mu$ on the real line, uniquely determines the measure $\mu$. If there are multiple measures with the moments $m_{0}, m_{1}, \ldots$, the moment problem is called indeterminate. In this talk we describe and investigate various families of indeterminate moment problems. (Received September 11, 2023)

## 1192-28-31198

Judy A. Holdener, Kenyon College, Leif Erik Schaumann*, Kenyon College. Generalized Thue-Morse Turtle Curves. Preliminary report.
We considered a type of mathematical curve called a Thue-Morse turtle curve. Turtle curves are defined by the movement of a "turtle" in the plane following a sequence of instructions, which tell the turtle how to move and turn relative to its current position and heading. Our focus, Thue-Morse turtle curves, are a specific family of these curves in which the turtle follows instructions encoded by the well known Thue-Morse sequence. Previous research has explored the connection between these Thue-Morse turtle curves and the also well known Von Koch fractal curve. We obtain generalized results on this connection by
considering several families of related curves, showing a precise relationship between Thue-Morse turtle curves and the Von Koch curve. In the process, we show that Thue-Morse turtle curves are also closely related to other lesser known fractal curves.
(Received September 11, 2023)

## 1192-28-31494

Steven Michael Senger*, Missouri State University. Dot product problems on fractal sets.
We discuss a family of problems stemming from dot products determined by points in a fractal set. The stated results are related to the celebrated Falconer Distance Problem.
(Received September 11, 2023)

1192-28-31902
Ran Tao*, Carnegie Mellon University, Anush Tserunyan, McGill University. Hjorth hyperfinite decomposition in the quasipmp setting.
Hjorth's hyperfinite decomposition theorem states that every ergodic probability measure preserving treeable CBER with integer cost $n$ admits a decomposition into $n$ free factors. We generalize this theorem to the quasi-pmp case. Our method is based on a novel technique introduced by Miller and Tserunyan (in 2017).
(Received September 12, 2023)

## 1192-28-31918

Will Hoffer*, University of California, Riverside. Tube Formulae for Generalized von Koch Fractals through Scaling Functional Equations. Preliminary report.
Herein we study the volume of an epsilon neighborhood of a class of generalized von Koch fractals that allow for different scaling ratios and choices of regular $n$-gons in the construction of the standard snowflake fractal. To do so, we construct and analyze an approximate scaling functional equation for the volume by expressing the neighborhood in terms of scaled copies of the original and a residual area. By converting to a functional equation for tube zeta functions, we describe the complex dimensions of these fractals. With further estimates, explicit formulae allow us to explicitly describe the tube function in terms of these complex dimensions.
(Received September 12, 2023)

## 1192-28-32055

Andrew Warren*, University of British Columbia. Recent connections between Loeb measures and the analysis and geometry of metric measure spaces.
Metric measure spaces can be used to play the role of a nonsmooth analogue of Riemannian manifolds: for instance, when considering geometric evolutions of manifolds, if singularities can form in finite time then the evolution "leaves the space" of manifolds and the relevant limit is merely a metric measure space. Accordingly, notions of convergence for metric measure spaces (along the lines of, for instance, Gromov-Hausdorff convergence of metric spaces) have been intensively studied in recent years. In this talk, I report on some recent results connecting ultralimits of metric measure spaces - a construction which involves Loeb measures in an essential way - and the so-called "pointed measured Gromov convergence" of metric measure spaces, and discuss how tools from Loeb spaces and nonstandard analysis more generally can be brought to bear on questions of geometric stability from metric measure space theory.
(Received September 12, 2023)

## 1192-28-32260

Blair Davey, Montana State University, Silvia Ghinassi, University of Washington, Bobby L. E. Wilson*, University of Washington. Lipschitz graphs covering large subsets of fractals.
In this talk we will discuss the extent to which one can cover a large subset of a purely unrectifiable set with a Lipschitz graph. In particular, we will present tools from harmonic analysis (Favard Length) and ergodic theory that make it possible to identify regular arrangements of points lying inside of any given one-dimensional set in the plane.
(Received September 12, 2023)

## 1192-28-32266

Animesh Biswas*, University of Nebraska Lincoln, Mikil Foss, University of Nebraska-Lincoln, Petronela Radu, University of Nebraska-Lincoln. Symmetry of hypersurfaces with ordered nonlocal mean curvature.
We study the prescribed nonlocal mean curvature problem with the definition of nonlocal curvature is given in the following,

$$
H_{\Omega}^{J}(x):=\int_{\mathbb{R}^{n}} J(x-y)\left(\chi_{\Omega^{c}}(y)-\chi_{\Omega}(y)\right) d y,
$$

where $x \in \mathbb{R}^{n}, \Omega \subset \mathbb{R}^{n}, \chi$ is the characteristic function for a set, $J$ is a radially symmetric, nonegative, nonincreasing convolution kernel. Nonlocal curvature of this form appears in many different applications, such as image processing, curvature driven motion, deformations. In this work, we focus on the case when the mean curvature is monotone in a particular direction. There are several works regarding the monotonicity of classical curvature problem of bounded set, where the solution is shown to be symmetric around a hyperplane. We showed that same thing for the nonlocal case.
(Received September 12, 2023)

The paper [GM] presents a proof that every separable Banach space, denoted $\mathcal{B}$, has an isomorphic, isometric embedding in $\mathbb{R}^{\infty}=\mathbb{R} \times \mathbb{R} \times \cdots$. The authors in [GM] used this result and a method due to Yamasaki to construct a sigma-finite Lebesgue measure $\lambda_{\mathcal{B}}$ for $\mathcal{B}$ and define the associated integral $\int_{\mathcal{B}} \cdot d \lambda_{\mathcal{B}}$ in a way that equals a limit of finite-dimensional Lebesgue integrals. In this talk we will construct the Fourier transform on $L^{1}[\mathcal{B}]$ as an integral which equals a limit of Fourier transforms on $L^{1}\left[\mathbb{R}^{n}\right]$ as the dimension $n \rightarrow \infty$. Thus with this theory we may evaluate infinite-dimensional quantities, such as the Fourier transform on $\mathcal{B}$, by means of finite-dimensional approximation. As an application, we will apply the familiar properties of the Fourier transform to solving the heat equation on $\mathcal{B}$. References [GM]T.L. Gill, T. Myers, Constructive Analysis on Banach Spaces, Real Analysis Exchange, 44 (2019) 1-36.
(Received September 12, 2023)

## 1192-28-32485

Tushar Das*, University of Wisconsin La Crosse. Exact dimensions of the prime continued fraction Cantor set.
We describe joint work (arXiv:2305.11829) with David Simmons proving that the Hausdorff measure of the prime Cantor set (comprising the irrationals whose continued fraction digits are prime numbers) cannot be finite and positive with respect to any sufficiently regular dimension/gauge function. In contrast, under a reasonable number-theoretic conjecture we prove that the packing measure of the conformal measure on the prime Cantor set is in fact positive and finite with respect to a particular dimension/gauge function.
(Received September 12, 2023)

## 1192-28-32493

Tushar Das*, University of Wisconsin La Crosse. Old and new in the dimension theory of continued fraction Cantor sets. Continued fractions have provided a natural playground for several developments in number theory, geometry, topology, dynamics, analysis, and probability theory. I will report on two projects (arXiv 2007.10554 and 1910.10259) about the fascinating fractals that arise from studying continued fractions and their conformal cousins. The talk will be accessible to those whose interests intersect a convex combination of functional analysis, dynamical systems, number theory and fractal geometry. Most importantly, I hope to present a sampling of open questions and research directions that await exploration. (Received September 12, 2023)

## 1192-28-32985

Timothy Ira Myers*, Howard University. A Constructive Definition of The Riemann Integral on a Separable Banach Space. The goal of this talk is to construct a Riemann integral on a separable Banach space which possesses all of the fundamental properties of the Riemann integral on $\mathbb{R}^{n}$. Let $\mathcal{B}$ represent a separable Banach space. The paper [GM] presents a proof that $\mathcal{B}$ has an isomorphic, isometric embedding in $\mathbb{R}^{\infty}=\mathbb{R} \times \mathbb{R} \times \cdots$. In this work we will use this embedding to define a Riemann integral on special subsets of $\mathcal{B}$, which makes the derivations of most of its properties virtually identical to those of its finitedimensional analogue. Similar to the Lebesgue integral on $\mathcal{B}$, this Riemann integral has the advantage of equaling a limit of Riemann integrals on $\mathbb{R}^{n}$ as $n \rightarrow \infty$. We will use this convergence to study some probability density functions on $\mathcal{B}$. References [GM] T.L. Gill, T. Myers, Constructive Analysis on Banach Spaces, Real Analysis Exchange, 44 (2019) 1-36. (Received September 13, 2023)

## 1192-28-33035

Timothy Ira Myers*, Howard University. Transitioning Students from Calculus Toward a More Precise Study of the Riemann Integral in an Introductory Analysis Course.
Modern Calculus texts use the classical notion of area under a continuous curve to motivate the definition of the Riemann integral as the limit of a sequence of Riemann sums. Introductory analysis texts, on the other hand, either omit this familiar limit formulation altogether or prove it after showing that Riemann's and Darboux's integrals are equivalent; which involves significant effort. In this talk, we will discuss how to easily develop this convergence definition from Calculus much earlier when discussing integration in an introductory analysis course; which helps students relate Darboux's theoretical definition of the integral to their intuitive understanding of integration. We will also consider strategies that aid students' recall and understanding of Darboux's integral as tools for writing proofs, and we will outline a clearer method for proving the equivalence between Riemann's and Darboux's integrals.
(Received September 13, 2023)

## 30 Functions of a complex variable

## 1192-30-26577

Prasanna Kumar*, Birla Institute of Technology and Science Pilani, India. On the Inequalities Involving Complex Polynomials. In this paper, some results on the sharpening and generalizations of an inequality due to Rivlin ( T. J. Rivlin, On the maximum modulus of polynomials, Amer. Math. Monthly, 37 (1960), 251-253) on the class of complex polynomials having no zeros in the unit disc are presented. Our results give bounds sharper than the bounds given by all the earlier results in this direction.
(Received August 01, 2023)

## 1192-30-27052

Stephen Deterding*, Marshall University. Boundary Smoothness Conditions for Functions in $R^{p}(X)$.
Let $X$ be a compact subset of the complex plane and let $R^{p}(X), 2<p<\infty$, denote the closure of the rational functions with poles off $X$ in the $L^{p}$ norm. We consider three conditions which provide different ways of showing how the functions in $R^{p}(X)$ can have a greater degree of smoothness at a boundary point $x \in X$ than might otherwise be expected: (A) The existence of an approximate Taylor's theorem at $x$, (B) The existence of an $L^{q}$ function $g$ that represents $x$ such that $g /(z-x)$ is also an $L^{q}$ function, and (C) The existence of a bounded point derivation at $x$. We will show that (B) and (C) are equivalent and imply (A)
but (A) does not imply (B) or (C).
(Received August 09, 2023)

## 1192-30-27728

William L Blair*, University of Arkansas. A New Look at the Schwarz Boundary Value Problem.
The Schwarz boundary value problem is a simplification of the famous Riemann-Hilbert problem and is a well-studied boundary value problem in complex analysis. The classical version seeks a holomorphic function on the unit disk whose real part agrees with a prescribed continuous function on the unit circle. In this talk, I will present my recent work on this problem where I generalize the solution formulas for the nonhomogeneous and higher-order Schwarz boundary value problems by lowering the required regularity of the boundary value to a distributional boundary value of a harmonic function. (Received August 18, 2023)

## 1192-30-27995

Pierre-Olivier Parise*, Université Laval. Divergence of Taylor Series in de Branges-Rovnyak Spaces.
In this talk, I will present sufficient conditions on the existence of a function in a Branges-Rovnyak space in which polynomials are dense, whose Taylor series is unbounded in norm or diverges to infinity in norm. I will also show the corresponding conditions for the Cesàro means of the Taylor series. Our results are applications of a refined version of the Uniform Boundedness Principle established by Müller and Vrsovsky. This is a joint work with Thomas Ransford.
(Received August 24, 2023)

## 1192-30-28123

Mehmet Celik, Texas A\&M University-Commerce, Luke Duane-Tessier, Texas A\&M University, Ashley Marcial
Rodriguez, St. Olaf College, Daniel Rodriguez, Texas A\&M University, Aden Parker Shaw*, Rose-Hulman Institute of Technology. Area Differences under Analytic Maps and Operators (Preliminary report). Preliminary report.
We expand upon the idea of the difference in image area between $(z \cdot h)(\mathbb{D})$ and $h(\mathbb{D})$ for square-integrable holomorphic functions $h \in L_{a}^{2}(\mathbb{D})$. Additionally, we modify $h$ through the Toeplitz operator with conjugate holomorphic functions as symbols and obtain a recursive formulation for the image area of the modified function $\widetilde{h} \in L_{a}^{2}(\mathbb{D})$. We observe that $\widetilde{h}$ offers a finite area of $\mathbb{D}$ if the original $h^{\prime}$ 's image area is finite. Via analyzing the relation between $L^{2}(\mathbb{D})$-norms and $\ell^{2}$-norms of the Taylor coefficients of $h$ and $\partial h / \partial z$, we verified the critical fact that $\partial \widetilde{h} / \partial z$ is in $L_{a}^{2}(\mathbb{D})$, an observation also made by others through more complex techniques in more general settings. We have also constructed a method to generate a polynomial symbol whose Toeplitz operator, acting on a holomorphic polynomial $p$ on $\mathbb{D}$, will yield $\widetilde{p}$ equal to $\partial p / \partial z$. Creating such a symbol makes the Toeplitz operator resemble the differential operator on polynomials in the area formulation. We extend the construction of such symbol functions on the unit disk to any two non-zero polynomials, $p$ and $q$, where the Toeplitz operator applied to $p$ yields in $q$. Additionally, we develop the construction for a broad class of polynomials defined on the polydisc. (Received August 26, 2023)

## 1192-30-28206

Walton Green, Washington University In St. Louis, Nathan A. Wagner*, Washington University In St. Louis. Weighted Estimates for the Bergman Projection on Planar Domains.
Let $\Omega \subsetneq \mathbb{C}$ be a simply connected domain. A fundamental object of study in complex analysis is the Bergman projection $B_{\Omega}$, which is the orthogonal projection from $L^{2}(\Omega)$ to the closed subspace of holomorphic functions, $A^{2}(\Omega)$. Unweighted and weighted $L^{p}$ estimates for $B_{\Omega}$ have close connections to duality of Bergman spaces, Toeplitz and Hankel operators on Bergman spaces, conformal function theory, and complex partial differential equations. We generalize well-known weighted inequalities due to Békollè and Bonami by giving a sufficient condition on the domain $\Omega$ such that the Bergman projection $B_{\Omega}$ is bounded on $L^{p}(\Omega, \sigma)$ for $\$ 1$
(Received August 28, 2023)

1192-30-28365
Ali Balooch*, University of Nevada, Las Vegas, Zhijian Wu, University of Nevada, Las Vegas. A New Formalization of Dirichlet-type Spaces.
Applications and findings involving the norm of the weighted Dirichlet space $\mathcal{D}_{\alpha}$ are steadily being discovered. A paper previously published by Wu presents a notable characterization of $\mathcal{D}_{\alpha}$ through an estimation of its norm. Applications include determining the conditions for boundedness of a Bergman type operator, and characterizing functions in the Drury-Arveson space that induce bounded Hankel operators. This paper improves the norm estimation by extending the domain of relevant parameters, and establishing the asymptotic formula with precise constants.
(Received August 30, 2023)

1192-30-28865
Paula Cerejeiras*, University of Aveiro, Portugal. Ternary Grassmann algebras and Fock spaces with a view to hypersymmetry.
Supersymmetry is based $\mathbb{Z}_{2}$-graded algebras, of which Clifford and Grassmann algebras are classical examples as they allows us to consider a Fock space of monogenic function and build most of the necessary ingredients for a theory of entire functions. But more general settings (for example, quarks) require a more general type of supersymmetry. In this talk we shall present the groundwork for an Itô/Malliavin stochastic calculus and a Hida's white noise analysis in the context of a supersymmetry based on $\mathbb{Z}_{3}$-graded algebras. To this end, we establish a ternary Fock space and the corresponding strong algebra of stochastic distributions and present its application in the study of stochastic processes.
(Received September 04, 2023)

Devon N Munger, University of Washington Bothell, Pietro Paparella*, University of Washington Bothell. The converse of the Cowling-Obrechkoff-Thron theorem.
The converse of the Cowling-Obrechkoff-Thron theorem is given. In addition to its theoretical interest, the result fills a gap in the proof of Kellogg's celebrated eigenvalue inequality for $P$ and $P_{0}$ matrices.
(Received September 04, 2023)

1192-30-28964
Russell W Howell, Westmont College, Sam Lewis Tang*, Westmont College. Probabilistic questions relating to a certain type of function that is analytic in the unit disk.. Preliminary report.
A function $f$ analytic in the open unit disk is annular if there is a sequence $\left\{J_{n}\right\}$ of nested Jordan curves converging to the unit circle such that $\lim _{n \rightarrow \infty} \min _{z \in J_{n}}|f(z)|=\infty$. If the $J_{n}$ can be taken to be concentric circles, then $f$ is strongly annular. These functions were typically constructed by creating power series with large gaps, and whose coefficients increased rapidly. It was thus surprising when it was discovered that strongly annular functions exist whose Maclaurin coefficients are all $\pm 1$. The binary representation of numbers $t \in[0,1]$ offers a convenient mechanism for generating coefficients that are $\pm 1$ : if $t=0 . \alpha_{0} \alpha_{1} \alpha_{2} \ldots$, where $\alpha_{n}=0$ or 1 for all $n$, set the $n$th coefficient to +1 if $\alpha_{n}=0$, and to -1 if $\alpha_{n}=1$. The zero-one law implies that the probability is either zero or one of selecting $t \in[0,1]$ so that the resulting series is strongly annular. This paper addresses the question of what that probability might be.
(Received September 04, 2023)

1192-30-29384
Gi-Sang Cheon, Sungkyunkwan university, Tamás Forgács, California State University, Fresno, Khang Duc Tran*, California State University, Fresno. On the zeros of certain Sheffer sequences and their cognate sequences. Let $\left(s_{n}(x)\right)_{n \geq 0}$ be a Sheffer sequence for $(g(z), f(z))$ generated by

$$
\frac{g(z) e^{x f(z)}=\sum_{n \geq 0} s_{n}(x) z^{n}}{n!}
$$

where $g(0) \neq 0, f(0)=0$ and $f^{\prime}(0) \neq 0$. The Sheffer sequence for $\left(\frac{f^{\prime}(z)}{g(z)}, f(z)\right)$ is called the cognate sequence of $\left(s_{n}(x)\right)_{n \geq 0}$ and is denoted by $\left(s_{n}^{c}(x)\right)_{n \geq 0}$. In this paper, we study various relationships between the zeros of Sheffer sequences and their cognate sequences.
(Received September 06, 2023)

1192-30-29680
Rebekah Ottinger*, Brigham Young University. Zeros of Convex Combinations of an Elementary Family of Harmonic Functions.
Brilleslyper et. al. analyzed a one-parameter family of harmonic trinomials and a relationship between the size of the parameter and the number of zeros of the harmonic trinomial. In this presentation, we examine the convex combination of two members of this family. We determine conditions under which the critical curves separating the sense-preserving and sensereversing regions is a circles. We show that the number of zeros of the convex combination can be greater than either the number of zeros of of its parts.
(Received September 07, 2023)

## 1192-30-30330

Daniel Alpay, Chapman University, Kamal Diki*, Chapman University, Mihaela B. Vajiac, Chapman University, Orange, CA. An Extension of the Complex-Real ( $C-R$ ) Calculus to the Bicomplex Setting, with Applications.
In this talk, we extend notions of complex $\mathbb{C}-\mathbb{R}$-calculus to the bicomplex setting leading to some bicomplex gradient operators. As a consequence of this extension we present two bicomplex least mean square algorithms, which extend classical real and complex least mean square algorithms.
(Received September 10, 2023)

## 1192-30-30619

Michael J Miller*, Le Moyne College. Seeking a quadratic refinement of Sendov's conjecture.
A conjecture of Sendov states that if a polynomial has all its roots in the unit disk and if $\beta$ is one of those roots, then within one unit of $\beta$ lies a root of the polynomial's derivative. If we define $r(\beta)$ to be the greatest possible distance between $\beta$ and the closest root of the derivative, then Sendov's conjecture claims that $r(\beta) \leq 1$. In this paper, we conjecture that there is a constant $c>0$ so that $r(\beta) \leq 1-c \beta(1-\beta)$ for all $\beta \in[0,1]$. We find such constants for complex polynomials of degree 2 and 3 , for real polynomials of degree 4 , and for all polynomials whose roots lie on a line. In addition, we conjecture a specific estimate for a value of $c$ that would apply to polynomials of any degree.
(Received September 10, 2023)

Sofia Boudrai*, Ibn Tofail University-Kenitra, Aiad El Gourari, Ibn Tofail University-Kenitra, Allal Ghanmi, Mohammed V University-Rabat. Slice Hyper-meromorphic Bergman Space.
About the construction of a $\beta$-modified Bergman space $\mathcal{A}_{\beta}^{2, \alpha}(\mathbb{B}(0,1))$, which is the space of slice hypermeromorphic functions
on the unit quaternionic ball $\mathbb{B}(0,1)$ expanded in Laurent series at the origin, resulting from the perturbation of the weight function of the slice hyperholomorphic Bergman space $\mathcal{A}_{\text {Slice }}^{2, \alpha}(\mathbb{B}(0,1))$. Moreover, we give its orthonormal basis, and after proving that it concerns a reproducing kernel Hilbert space, we present the explicit expression of its reproducing kernel in terms of the left-sided Gauss hypergeometric function ${ }_{2} F_{1}^{*}$.
(Received September 11, 2023)

1192-30-31308
Swanhild Bernstein*, TU Bergakademie Freiberg. $q$-monogenic and $q$-harmonic Functions in Clifford Analysis. Preliminary report.
Clifford analysis is a higher dimensional function theory studying the null solutions of the Dirac operator. The Dirac operator is closely related to the rotational invariant Laplacian and therefore Clifford analysis can be seen as an refinement of harmonic analysis. In this presentation we consider quantized versions of these operators; $q$-Dirac operators and $q$-Laplacians $\Delta=q^{n-1} \partial_{1}^{2}+q^{n-2} \partial_{2}^{2}+\ldots+\partial_{n}^{2}$ defined on the algebra of $q$-commuting variables, i.e. $\$ \mathrm{x} \_\mathrm{ix} \_\mathrm{j}=\mathrm{qx} \mathrm{jx} \mathrm{i}, \mathrm{i}$ (Received September 11, 2023)

## 1192-30-31441

Catherine Anne Beneteau*, University of South Florida. A survey of recent results on optimal polynomial approximants. Preliminary report.
In this talk, I will discuss optimal polynomial approximants, which are polynomials that approximate (in some sense) inverses of functions in analytic function spaces. In Hilbert spaces, these approximants are connected to weighted reproducing kernels and weighted orthogonal polynomials. I will give a brief survey of what is known, including information about the zeros of these polynomials, and will examine some interesting open problems.
(Received September 11, 2023)

## 1192-30-31501

Daniel Alpay, Chapman University, Zubayir Kazi, West Valley College, Mariana Tecalero, Alvernia University, Dan Volok*,
Kansas State University. Discrete analytic functions on a rhombic lattice: weighted Hardy spaces and rational functions. In this paper, a novel forward shift operator, which plays the role of multiplication by $z$ in the space of discrete analytic functions on a uniform rhombic lattice in the complex plane, is introduced and reproducing kernel Hilbert spaces of discrete analytic power series, analogous to the classical weighted Hardy spaces, are studied. The focus is on the correspondence between shift-invariant subspaces and contractive operator-valued discrete analytic functions (theorems of Beurling-Lax type). In the finite-dimensional case, this investigation leads to the class of discrete analytic functions that are rational with respect to the convolution product.
(Received September 11, 2023)

## 1192-30-31907

Rafael Morales*, Baylor University. Hypergeometric multiple orthogonal polynomials and free finite convolution. Preliminary report.
Some multiple orthogonal polynomials can be wrote explicitly as terminating generalized hypergeometric functions. However, extracting the information about their zeros from this fact is not trivial. In this talk, we address some recently discovered applications of the notion of the free finite convolution of polynomials (developed in the framework of the free probability theory) to the study of properties of zeros of hypergeometric polynomials. In particular, we discuss some consequences for multiple orthogonal polynomials.
(Received September 12, 2023)

1192-30-32012
Jennifer Brooks, Brigham Young University, Alexander Lee*, Brigham Young University. Zeros of a Family of ComplexValued Harmonic Functions with Poles.
We discuss the zeros of a family of complex-valued harmonic functions $f_{c}(z)=z^{n}+\frac{c}{\bar{z}^{k}}-1$ for $c \in \mathbb{C} \backslash\{0\}$. We use the harmonic analogue of the Argument Principle. The critical curve separating the sense-preserving and sense-reversing regions for $f_{c}$ is a circle $\Gamma_{c}$ whose image under $f_{c}$ is an epicycloid. Thus, after studying the geometry of these curves, we can determine the winding number of $f_{c}\left(\Gamma_{c}\right)$ about the origin for all values of $c$ and count the zeros of $f_{c}$ in the complex plane. (Received September 12, 2023)

1192-30-32047
Riccardo Ghiloni, Università di Trento, Caterina Stoppato*, Università di Firenze. A new approach to regularity in one hypercomplex variable.
Over the last century, several notions of regularity have been introduced for quaternion-valued functions of one quaternionic variable: e.g., the theories of Fueter (1934), of Gentili-Struppa (2006) and of Moisil-Teodorescu (1931). The first two theories have been generalized to real Clifford algebras, respectively, with the celebrated monogenic functions and with the slicemonogenic functions introduced by Colombo-Sabadini-Struppa (2009). They have also been generalized to the the real algebra of octonions, respectively, by Dentoni (1973) and Gentili-Struppa (2010). We present a unified notion of regularity that subsumes all of the aforementioned function theories and includes surprising new ones. (Received September 12, 2023)

Preliminary report.
We will consider von Neumann algebras generated by projective representations of discrete groups on Bergman and Fock spaces of analytic and polyanalytic functions. The corresponding von Neumann dimensions will be computed using a method from Vaughan Jones's last paper [Vaughan F. R. Jones, Bergman space zero sets, modular forms, von Neumann algebras and ordered groups. Enseign. Math. 69 (2023), no. 1/2, pp. 5-36], where Bergman spaces have been considered. Complementary information about Jones work can be found in [Pierre de la Harpe, Florin Rădulescu, On Bergman space zero sets. Editors' comments on the article by Vaughan Jones. Enseign. Math. 69 (2023), no. 1/2, pp. 37-50], and a detailed pedagogic text in [Arne Berrens, Von Neumann algebras and zero sets of Bergman spaces]. The computation of von Neumann dimensions includes Fock space settings and the polyanalytic Bergman spaces connected to wavelets and to the eigenspaces of the Maass Laplacian, obtained in [L. D. Abreu, M. Speckbacher, Affine density, von Neumann dimension and a problem of Perelomov. Advances in Mathematics, 407, 108564, (2022)]. Applications to spaces of automorphic forms and in sampling theory and discrete coherent states in hyperbolic Landau levels will be outlined.
(Received September 12, 2023)

## 1192-30-32169

Zubayir Kazi*, West Valley College, Mariana Tecalero, Alvernia University, Dan Volok, Kansas State University. Discreteanalytic Power Series Expansions and Rational Functions on Uniform Rhombic Lattices.
In this paper, a novel forward shift operator, which plays the role of multiplication by in the space of discrete analytic functions on a uniform rhombic lattice in the complex plane, is introduced and reproducing kernel Hilbert spaces of discrete analytic power series, analogous to the classical weighted Hardy spaces, are studied. The focus is on the correspondence between shift-invariant subspaces and contractive operator-valued discrete analytic functions (theorems of Beurling- Lax type). In the finite-dimensional case, this investigation leads to the class of discrete analytic functions that are rational with respect to the convolution product. This is a joint work with Dan Volok (Kansas State University) and Mariana Tecalero (Alvernia University). (Received September 12, 2023)

## 1192-30-32194

Debendra Prasad Banjade*, Coastal Carolina University. Characterization of Sets $K$ for which $H_{K}^{\infty}(D)$ is an Algebra. Let, $K \subset \mathbb{Z}_{+}$and define the set $H_{K}^{\infty}(\mathbb{D})$ to be the collection of all bounded analytic functions on the unit disk $\mathbb{D}$ in the complex plane whose $k^{\text {th }}$ derivative vanishes at zero for all $k \in K$. In this talk, we will present that $H_{K}^{\infty}(\mathbb{D})$ is an algebra precisely when $\mathbb{Z}_{+} \backslash K$ is an abelian semigroup. In particular, we show that if $K$ is a finite set, then $K$ yielding an algebra is equivalent to $\mathbb{N} \backslash K$ being a numerical semigroup. Moreover, an algorithm for constructing $K$ when $K$ is finite will be provided. We then classify some of these sets and give a list of their exact representations.
(Received September 12, 2023)

## 1192-30-32294

Narciso Gomes*, University of Cape Verde. Compressed sensing principles and monogenic wavelets with applications to Deep Learning. Preliminary report
Here we propose to study compressive sensing for monogenic signals using Clifford analytic techniques. In the field of applications, regularization methods based on sparsity constraints have shown to be well-suited for solving inverse problems. However, two major problems arise. First, their practical implementation is hampered by the fact that greedy algorithms have an exponential grow in computational costs. One possible way out is to use compressed sensing principles. In particular, having in mind applications in terms of monogenic signals the principal question is if sampling matrices based on monogenic polynomials can be used for compressed sensing. Closely linked to it is the question of an adequate choice of the sampling grid itself. In general, the chosen grid is the one given by the zeros of the polynomials but in many cases this is difficult to determine. It is planned to investigate other grids that could be used for sampling without computational loss in the implementation. The tackling of these two problems allow the study of application of compressive sensing to inverse problems and its comparison with other methods using sparse representation techniques. (Received September 12, 2023)

## 1192-30-32421

Ryan Alvarado*, Amherst College. Sobolev spaces in the metric setting.
While metric spaces have been around for many years, the area of research, known as Analysis on Metric Spaces, is less than 30 years old. One of the main facets of this now ever-growing field of mathematics concerns the theory of function spaces, and the main aim of this talk is to report on some recent progress in the direction of understanding the (compact) embedding properties of certain classes of Sobolev spaces in the metric setting. At the center of this discussion is how the geometrical characteristics of the metric space intervene in the theory of these function spaces.
(Received September 12, 2023)

## 1192-30-32490

Maria Elena Luna Elizarraras*, Holon Institute of Technology. Some geometric and algebraic facts related with BicomplexMöebius transformations. Preliminary report.
The bicomplex Möbius transformation is a versatile topic and has been explored by some authors, each of them emphasizing different aspects. In this talk new aspects about them will be discussed. One of the aspects that we will discuss is how to take advantage of the decomposition of bicomplex numbers such as $\mathbb{B} \mathbb{C}=\mathbb{D}+\mathbf{i} \mathbb{D}$. This will give us the opportunity to "see" in $\mathbb{B} \mathbb{C}$ some bicomplex circumferences, hyperbolic lines and symmetric points related with them. We will see also that it is possible to give a good description of the bicomplex stereographic projection using the hyperbolic geometric objects defined in $\mathbb{D}^{2}$ and $\mathbb{D}^{3}$
(Received September 12, 2023)

Alea L Wittig*, University at Albany SUNY. Wolff's Ideal Problem on the Multiplier Algebra of the Dirichlet Space. In 1962, Carleson proved the classical Corona Theorem, which provides a sufficient condition on a finite set $\left\{f_{j}\right\}_{j=1}^{n} \in H^{\infty}(\mathbb{D})$ to satisfy $1 \in \mathcal{I}\left(\left\{f_{1}\right\}_{j=1}^{n}\right)$, where $\mathcal{I}\left(\left\{f_{1}\right\}_{j=1}^{n}\right)$ is the ideal generated by $\left\{f_{j}\right\}_{j=1}^{n}$. With the goal of classifying ideal membership, Wolff found a sufficient condition on $\left\{f_{j}\right\}_{j=1}^{n} \in H^{\infty}(\mathbb{D})$ and $H \in H^{\infty}(\mathbb{D})$, to conclude that $H^{3} \in \mathcal{I}\left(\left\{f_{j}\right\}_{j=1}^{n}\right)$.Recently, an analogue to Wolff's ideal problem for the multiplier algebra of the Dirichlet space was proved by Banjade and Trent. It gives sufficient conditions on $H$ and finite set $\left\{f_{j}\right\}_{j=1}^{n}$ in the multiplier algebra of the Dirichlet space so that $H^{3} \in \mathcal{I}\left(\left\{f_{j}\right\}_{j=1}^{n}\right)$. I will discuss the problem of classifying ideal membership $H \in \mathcal{I}\left(\left\{f_{j}\right\}_{j=1}^{n}\right)$ and provide estimates in this case.
(Received September 12, 2023)

## 31 Potential theory

## 1192-31-29010

Naoki Saito*, University of California, Davis, Eugene Shvarts, UC Davis. Discrete integral operators and distance matrices for graph signal processing.
We will discuss a new viewpoint of graph signal processing and analysis using discrete integral operators on graphs instead of popular graph Laplacians. The latter is based on local information on graphs while the former utilizes graph's global information, e.g., the shortest path distances between nodes. In particular, such global information becomes quite important for directed graphs (digraphs) since the connectivity between any two vertices on a digraph is not a local concept; rather it is a global concept (e.g., imagine networks of one-way streets). Finding a shortest path connecting a pair of nodes provides critical information on a digraph. Hence, to utilize such information fully, spectral analysis of the distance matrices and the associated discrete integral operators (often in the form of potential kernels) on digraphs is performed using the singular value decomposition instead of analyzing the graph Laplacians using the eigen decomposition. We plan to demonstrate the usefulness of this strategy using both synthetic and real examples.
(Received September 06, 2023)

## 32 Several complex variables and analytic spaces

## 1192-32-25882

Friedrich Haslinger*, University of Vienna. Basic estimates and the uncertainty principle.
In the Segal-Bargmann space (also Fock space) $A^{2}\left(\mathbb{C}^{n}, e^{-|z|^{2}}\right)$ of entire functions the differentiation operators $a_{j}(f)=\frac{\partial f}{\partial z_{j}}$ (annihilation) and the multiplication operators $a_{j}^{*}(f)=z_{j} f$ (creation) are unbounded, densely defined adjoint operators with $\|f\|^{2} \leq\left\|a_{j}(f)\right\|^{2}+\left\|a_{j}^{*}(f)\right\|^{2}$ for $f \in \operatorname{dom}\left(a_{j}\right)$, which corresponds to the uncertainty principle. We study certain densely defined unbounded operators on the Segal-Bargmann space, related to the annihilation and creation operators of quantum mechanics. We consider the corresponding $D$-complex and study properties of the complex Laplacian $\tilde{\square}_{D}=D D^{*}+D^{*} D$, where $D$ is a differential operator of polynomial type, in particular we discuss the corresponding basic estimates, where we express a commutator term as a sum of squared norms. The basic estimates can be seen as a generalization of the uncertainty principle represented in the Segal-Bargmann space. In addition we show that Kähler manifolds $(M, h)$ admitting a real holomorphic vector field, i.e. for which there exists a smooth real-valued function $\psi: M \longrightarrow \mathbb{R}$ such that $h^{j \bar{k}} \frac{\partial \psi}{\partial \bar{z}^{k}} \frac{\partial}{\partial z^{j}}$ is a holomorphic vector field, have the property that the weighted Bergman space $A^{2}\left(M, h, e^{-\psi}\right)$ exhibits the same duality between differentiation and multiplication as in the Segal-Bargmann space.
(Received July 13, 2023)

1192-32-26776
Adam Christopherson*, The Ohio State University. Weak-type regularity of the Bergman projection on generalized Hartogs triangles.
For power-generalized Hartogs triangles in $\mathbb{C}^{3}$, we show that the Bergman projection satisfies a weak-type estimate at the upper endpoint of $L^{p}$ boundedness but not at the lower endpoint. Our work complements related results obtained recently for rational Hartogs triangles in $\mathbb{C}^{2}$ and the punctured unit ball in $\mathbb{R}^{3}$. This work is joint with K.D. Koenig. (Received August 04, 2023)

1192-32-27184
Yifei Pan, Purdue University - Fort Wayne, Yuan Zhang*, Purdue University Fort Wayne. Optimal Sobolev regularity of $\bar{\partial}$ on the Hartogs triangle.
In this paper, I will discuss recent progress on Sobolev regularity of the $\bar{\partial}$ problem on the Hartogs triangle. We show that for each $k \in \mathbb{Z}^{+} \cup\{0\}, p>4$, there exists a solution operator $\mathcal{T}_{k}$ to the $\bar{\partial}$ problem on the Hartogs triangle that maintains the same $W^{k, p}$ regularity as that of the data. According to a Kerzman-type example, this operator provides solutions with the optimal Sobolev regularity. Part of the results is based on a joint work with Yifei Pan.
(Received August 11, 2023)

1192-32-27190
Dusty Grundmeier, The Ohio State University, Jiri Lebl*, Oklahoma State University. Normal form for proper maps between balls and applications.
I will talk about a normal form for proper holomorphic maps between balls. Using this form Grundmeier and I have obtained
some new results about group invariant mappings.
(Received August 11, 2023)

## 1192-32-27657

Hamed Baghal Ghaffari*, University of Newcastle, Jeff Hogan, University of Newcastle, Joseph D. Lakey, New Mexico State University. Constructing Multidimensional Monogenic Homogeneous Functions.
This talk is dedicated to the construction of multidimensional spherical monogenics. Firstly, we investigate the construction of monogenic functions in dimension 3 by applying the Dirac operator to the spherical harmonics, resulting in orthogonal spherical monogenics. Additionally, we employ the reproducing kernel for monogenic functions and utilize a specialized optimization method to derive various types of m-dimensional harmonic and monogenic homogeneous functions. (Received August 17, 2023)

## 1192-32-27731

Lily Adlin, Cal Poly SLO, Giovani Thai, Cal Poly SLO, Samuel Tiscareno, Cal Poly SLO, Ryan K. Tully-Doyle*, Cal Poly
SLO. Rational inner functions and graphs.
Given a self-adjoint complex matrix $A$ and a complex vector $\alpha$, the map

$$
f(z)=\left\langle(A-z I)^{-1} \alpha, \alpha\right\rangle
$$

an example of a so-called Nevanlinna representation or Cauchy transform, produces a rational function that maps the complex upper half plane into itself. One interesting class of self-adjoint matrices that can be studied via this function representation is adjacency matrices of simple undirected graphs. Given a simple undirected graph $G$ with adjacency matrix $A$, one can view the matrix $A-z I$ as the adjacency matrix of a graph that puts self-loops of weight $z$ onto the existing structure of the graph $G$. In this case, certain choices of the vector $\alpha$ result produce a rational inner function in the Pick class. In this talk, we'll look extending examples studied by Pascoe, by Bickel, Pascoe, and Sola, and in an honors thesis of Hong, connecting graph structures to the behavior of a two variable analogue of the Nevanlinna representation, which allows for more complicated boundary behavior to occur in the representing functions. The main matrix becomes $(A-z Y-w(I-Y))$, where $Y$ is, for example, a diagonal matrix with diagonal entries 0 and 1, and we view the main matrix as an adjacency matrix of a simple undirected graph with self-loops of weight $z$ and $w$ attached to the vertices. Reformulating these function theoretic ideas in linear algebraic terms turns out to provide straightforward arguments that, for example, extend known one-variable ideas about representation of certain graph products into two-variables. We'll also look at the relationship between Bickel, Pascoe, and Sola's notion of contact order, which measures bunching of level curves of a representing function via power series, and the underlying graph.
(Received August 18, 2023)

1192-32-27867
Arni S. R Srinivasa Rao*, Laboratory for Theory and Mathematical Modeling, Medical College of Georgia, Georgia, U.S.A.. A Jordan Curve Theorem on a Ball through the Brownian Motion. Preliminary report.
A Jordan curve theorem on the boundary of a ball within a complex plane bundle was stated and proved by Krantz and Rao (2022). This was done through a new concept of multilevel contours passing through a complex plane bundle developed in 2021 (Rao 2022). In this talk, a Jordan curve theorem on a ball within a complex plane bundle will be proved using the Brownian motion principle. Some points of the Jordan curve in our proof of 2022 do leave the boundary of the ball considered. Unlike the previous proof of the Jordan curve where some points of the curve are outside the boundary of the ball, the new proof will try to accommodate all the points of the curve on the arbitrary ball considered. This is a joint work with S. G. Krantz. Keywords: Random walk, multilevel contours, information geometry (Received August 21, 2023)

1192-32-28146
Shreedhar Bhat*, Texas A\&M University. p-Skwarczyński distance. Preliminary report.
We will discuss some of the new tools to better study the $p$-Bergman space ( $A^{p}$ ). We also introduce a new distance on a bounded domain using the 'minimizer' functions on $A^{p}(\Omega)$. We discuss its invariance, completeness and other aspects related to it.
(Received August 27, 2023)

1192-32-28389
Michael T. Jury, University of Florida, Georgios Tsikalas*, Washington University In St. Louis. Denjoy-Wolff points on the bidisc.
Abstract: Let $f$ denote a holomorphic self-map of the unit disk $\mathbb{D}$ without any interior fixed points. A classical 1926 theorem of Denjoy and Wolff then asserts that the sequence of iterates

$$
f^{[n]}:=f \circ f \circ \cdots \circ f
$$

converges locally uniformly to a boundary fixed point of $f$, termed the Denjoy-Wolff point. The situation changes dramatically when one considers holomorphic fixed-point-free self-maps $F$ of the bidisc $\mathbb{D}^{2}$; the presence of large "flat" boundary components in $\partial \mathbb{D}^{2}$ will, in general, prevent the iterates from converging. The cluster set of the sequence of iterates in this setting was described in a 1954 paper of Hervé. In this talk, we will discuss extensions of the notion of a Denjoy-Wolff point to $\mathbb{D}^{2}$. Further, we will describe how imposing additional regularity assumptions on the behavior of the function $F$ at such points can lead to much greater control over the behavior of the iterates. Certain refinements of Hervé's results will thus be obtained. Joint work with Michael Jury.
(Received August 30, 2023)

## 1192-32-28435

Liding Yao*, The Ohio State University. The Cauchy-Riemann problems via extension operators.
We discuss some new results on $\bar{\partial}$-solution operators for good domains based on the new understanding of extension operators on domains. Specifically on strongly pseudoconvex domains or convex domain of finite type, we construct "universal solution operators", which are operators $T$ with Sobolev boundedness $T: H^{s, p} \rightarrow H^{s+1 / m, p}$ simultaneously for all $\$ 1$ (Received August 30, 2023)

## 1192-32-28758

Jinqi Chen, Tufts University, Dinh Quan Tran*, University of Minnesota - Twin Cities. $L^{p}$ Boundedness of Toeplitz Operators on Bergman Spaces. Preliminary report
The Bergman projection, which maps functions from $L^{p}(\Omega)$ to $A^{p}(\Omega)$, is one of the richest and most interesting objects in multivariable complex analysis and is well-understood in many simpler domains. In this presentation, using tools such as Schur's Test and Bell's transformation formula, we study the $L^{p}$ boundedness of Toeplitz operators, which are weighted Bergman projections. In particular, we obtain $L^{p}$ estimates for Toeplitz operators on the symmetrized bidisc $\mathbb{G}^{2}=\left\{\left(z_{1}+z_{2}, z_{1} z_{2}\right): z_{1}, z_{2} \in \mathbb{D}\right\}$ for two classes of symbols representing the powers of the Jacobian and the distance to the boundary. Additionally, we analyze the Bergman kernel for more complex quotients of the bidisc, including the symmetrized polydisc and the quotient of the bidisc by the dihedral group of order $2 k$, as well as provide preliminary results on the possibly useful classes of Toeplitz symbols in these domains.
(Received September 02, 2023)

## 1192-32-28798

Soumya Ganguly*, University of California, San Diego. Locally algebraic Bergman kernels on two dimensional Stein spaces with finite type boundaries.
On a two dimensional Stein space with isolated, normal singularities, finite type boundary and locally algebraic Bergman kernel, we find an estimate of the local algebraic degree of the Bergman kernel in terms of the type of the boundary. As an application, we characterize two dimensional ball quotients as finite type Stein spaces with rational Bergman kernel. (Received September 03, 2023)

## 1192-32-28820

Xiaojun Huang*, Rutgers University. Bloom conjecture - a brief survey. Preliminary report.
Our talk has two parts. We first survey recent efforts on the Bloom conjecture which still remains open in the case of complex dimension at least 4. Bloom's conjecture concerns the equivalence of three regular types. There is a more general important notion, called the singular D'Angelo type (or simply, D'Angelo type). While the finite D'Angelo type condition is the right one for the study of local subelliptic estimates for Kohn's $\backslash o v \backslash p$ - Neumann problem, regular types are important as their finiteness give the global regularity up to the boundary of solutions of Kohn's $\backslash o v \backslash p-N e u m a n n$ problem. In the second part of the talk, we discribe a proof of a seemingly elementary but also a fundamental property on the vanishing order of smooth functions along a system of non-integrable vector fields. A special case of our Theorem was introduced in a paper of D'Angelo. (Received September 03, 2023)

## 1192-32-28849

Achinta Kumar Nandi*, Oklahoma State University. On perturbations of singular complex analytic curves.
Suppose $V$ is a singular complex analytic curve inside $\mathbb{C}^{2}$. We investigate when a singular or non-singular complex analytic curve $W$ inside $\mathbb{C}^{2}$ with sufficiently small Hausdorff distance $d_{H}(V, W)$ from $V$ must intersect $V$. We obtain a sufficient condition on $W$ which when satisfied gives an affirmative answer to our question. More precisely, we show the intersection is non-empty for any such $W$ that admits at most one non-normal crossing type discriminant point with respect to some proper projection. As an application, we prove a special case of the higher-dimensional analog, and also a holomorphic multifunction analog of a result by Lyubich-Peters.
(Received September 04, 2023)

1192-32-28947
Song Ying Li*, University of California, Irvine. Sup-norm estimates for the solutions of Cauchy-Riemann equations.
In this talk, I will present my recent work on the sup-norm estimates for Cauchy-Riemann operator on the product domain with $C^{1, \alpha}$ boundary. At the same time, I will also present some results in a joint work with Jie Lao. We gave some remarks on the Berndtsson's conjecture on the sup-norm estimates on the some weighted sup norms. (Received September 04, 2023)

## 1192-32-28959

Luke David Edholm*, University of Vienna. The osculation function and the Leray transform at high frequencies.
The Leray (or Cauchy-Leray) transform $\mathbf{L}$ is a higher-dimensional analogue of the planar Cauchy transform that can be defined for $\mathbb{C}$-convex hypersurfaces $\mathcal{S} \subset \mathbb{C P}^{n}$. When $\mathcal{S}$ is smoothly bounded and strongly $\mathbb{C}$-convex, and $\mu$ is a reasonable surface measure, $\mathbf{L}$ is a skew projection from $L^{2}(\mathcal{S}, \mu)$ onto the Hardy space of $L^{2}$-boundary values of holomorphic functions. Isolated examples show that $L^{2}$-boundedness can fail when $\mathcal{S}$ is unbounded or has non-smooth points, but a general description of the behavior of the Leray transform in these wider settings is lacking. Consider the unbounded hypersurface
$\mathcal{S}_{f}:=\left\{\left(\zeta_{1}, \zeta_{2}\right): \operatorname{Im}\left(\zeta_{2}\right)=f\left(\left|\zeta_{1}\right|\right)\right\}$, where $f$ is increasing, convex and twice differentiable on $(0, \infty)$. Let $(r, \theta, s)$ parametrize $\mathcal{S}_{f}$ via $\left(\zeta_{1}, \zeta_{2}\right):=\left(r e^{i \theta}, s+i f(r)\right)$ and define the measure $\mu_{g}:=g(r) d r \wedge d \theta \wedge d s$, where $g$ is continuous on $(0, \infty)$.

Rotational symmetry yields a series decomposition of the Leray transform: $\mathbf{L}=\bigoplus_{k} \mathbf{L}_{k}$. 1. We give necessary and sufficient conditions characterizing when each $\mathbf{L}_{k}$, as well when as the full operator $\mathbf{L}$, is bounded on $L^{2}\left(\mathcal{S}_{f}, \mu_{g}\right)$. These conditions highlight the interplay between the asymptotics of $f$ and $g$. 2. We show that the boundedness results are accompanied by intriguing and explicit calculations that fall out of lengthy asymptotic analysis. 3. We introduce an invariant "osculation function" $\gamma_{f}(r):=1+\frac{r f \prime \prime}{f^{\prime}(r)}$, a sort of curvature associated to $\mathcal{S}_{f}$, and show the "high frequency" behavior of $\mathbf{L}$ is closely related to $\gamma_{f}$. In particular, we show $\lim \sup _{k \rightarrow \infty}\left\|\mathbf{L}_{k}\right\|_{L^{2}\left(\mathcal{S}_{f}, \mu_{g}\right)}=\sup _{r \in[0, \infty]} \sqrt{\frac{\gamma_{f}(r)}{2 \sqrt{\gamma_{f}(r)-1}}}$. (Note that the right side makes no mention of $g$.) This project is joint with Yonatan Shelah.
(Received September 04, 2023)

## 1192-32-29155

Michael R Pilla*, Ball State University. Linear Fractional Self-Maps of the Unit Ball in $\mathbb{C}^{N}$
Determining the range of complex maps in several variables plays a fundamental role in the study of several complex variables and operator theory. In particular, one is often interested in determining when a given holomorphic function is a self-map of the unit ball. In this talk, we discuss a class of maps in $\mathbb{C}^{N}$ that generalize linear fractional maps. We then proceed to determine precisely when such a map is a self-map of the unit ball. In particular, we take a novel approach obtaining numerous new results about this class of maps along the way.
(Received September 05, 2023)

## 1192-32-29157

Tanuj Gupta, Texas A\&M University, Emil Straube, Texas A\&M University, John N Treuer*, University of California San Diego. Modifications of the Levi core.
On a smooth bounded pseudoconvex domain in $\mathbb{C}^{n}$, the Levi core $\mathfrak{C}(\mathcal{N})$ is the maximal perfect subdistribution of the Levi null distribution with respect to operation of derived distributions. Its significance stems from its applications to global regularity and compactness of the $\bar{\partial}$-Neumann operator. In this talk, we construct a family of subdistributions of $\mathfrak{C}(\mathcal{N})$ called modified Levi cores $\left\{\mathcal{M C}_{\mathcal{A}}\right\}_{\mathcal{A}}$ indexed over closed distributions $\mathcal{A}$ that contain the Levi null distribution $\mathcal{N}$ and are contained in the complex tangent bundle $T^{1,0} b \Omega$. We show that Catlin's Property $(P)$ holds on $b \Omega$ if and only if Property ( $P$ ) holds on the support of one, and hence all, of the modified Levi cores. In $\mathbb{C}^{2}$, all of the modified Levi cores coincide. For a smooth bounded pseudoconvex complete Hartogs domain in $\mathbb{C}^{2}$ that satisfies Property $(P)$, we show that its modified Levi core is trivial. This contrasts with $\mathfrak{C}(\mathcal{N})$, which can be nontrivial for such domains. This talk is joint work with Tanuj Gupta (Texas A\&M University) and Emil Straube (Texas A\&M University). (Received September 05, 2023)

## 1192-32-29354

Emil Straube*, Texas A\&M University. Around a curious estimate. Preliminary report.
Denote by $j_{q}$ the imbeding from $\operatorname{dom}(\bar{\partial}) \cap \operatorname{dom}\left(\bar{\partial}^{*}\right) \rightarrow L_{(0, q)}^{2}(\Omega)$, where $\Omega$ is a (smooth bounded) pseudoconvex domain in $\mathbb{C}^{n}$ . Then $j_{q}$ is continuous in $L^{2}$-norms. A special case of the estimate referred to in the title says that on domains in $\mathbb{C}^{2}$, although the $\bar{\partial}$-Neumann operator need not be regular (the worm domains are examples), $j_{1}$ is: $\|u\|_{s} \leq C_{s}\left(\|\bar{\partial} u\|_{s}+\left\|\bar{\partial}^{*} u\right\|_{s}\right), s \geq 0$. I will discuss a few thoughts around this estimate.
(Received September 06, 2023)

## 1192-32-29504

Herve Gaussier, University of Grenoble Alpes, Xianghong Gong*, University of Wisconsin-Madison, Andrew Zimmer,
University of Wisconsin-Madison. Smooth equivalence of families of strongly pseudoconvex domains.
We establish a smoothness result for families of biholomorphisms between smooth families of rigid strongly pseudoconvex domains. This is accomplished by considering the Riemannian geometry of their Bergman metrics and proving a result about the smoothness of families of isometries between smooth families of Riemannian manifolds.
(Received September 07, 2023)

1192-32-30104
Anne-Katrin Gallagher*, Gallagher Tool \& Instrument LLC. On a sufficient condition for a domain to have trivial DiederichForncess index.
Let $D \subseteq \mathbb{C}^{n}, n \geq 2$, be a smoothly bounded, pseudoconvex domain. If $D$ admits a smooth defining function whose complex Hessian is positive semi-definite on $b D$, then the Diederich-Fornæss index is known to be trivial by results of Fornæss and the presenter. Krantz, Liu and Peloso showed for $n=2$, that the hypothesis may be weakened to the domain having a smooth defining function whose complex Hessian is only positive semi-definite at those boundary points at which the domain is weakly pseudoconvex. We will talk about a new proof of the original result for $n \geq 3$ which remains valid under a similarly weaker hypothesis: $D$ admits a smooth defining function $\rho$ such that for any boundary point $p$ and any holomorphic tangent vector $L$ in whose direction the boundary is weakly pseudoconvex at $p$, it follows that $L$ lies in the kernel of the complex Hessian matrix of $\rho$ at $p$.
(Received September 08, 2023)

Weixia Zhu*, University of Vienna. Spectral stability of the Kohn Laplacian on CR Manifolds. Preliminary report. In this talk, I will report my recent work about stability of the spectrum of the Kohn Laplacian on CR manifolds, and how this
work contributes to our understanding of the stability of embeddings for 3-dimensional pseudoconvex CR manifolds of finite type. This talk is based on joint work with Siqi Fu and Howard Jacobowitz.
(Received September 10, 2023)

## 1192-32-30962

Dusty Grundmeier*, The Ohio State University. Gaps and Group-invariant Mappings. Preliminary report.
Given a non-constant group-invariant CR mapping of spheres, we give restrictions on the possible target dimensions for minimal such maps. In particular, we explore gap phenomena in this context.
(Received September 11, 2023)

## 1192-32-31012

Josef Eberhard Greilhuber*, Stanford University. On irregular CR maps into uniformly pseudoconvex hypersurfaces.
We consider CR maps into pseudoconvex real hypersurfaces whose Levi form has a kernel of constant positive dimension. Such hypersurfaces are foliated by complex submanifolds, and therefore admit a host of nowhere smooth CR maps. However, given some finite initial regularity, these maps are highly constrained by the geometry of the target - in some cases even confined to a single leaf of the Levi foliation. If the Levi foliation of the target is one-dimensional, one can show that irregular CR maps are, locally, deformations of smooth CR maps.
(Received September 11, 2023)

## 1192-32-31150

Xin Dong*, University of Connecticut, Ruoyi Wang, University of California, Riverside, Bun Wong, University of California, Riverside. Characterizations of the Euclidean ball by invariant metrics. Preliminary report.
I will mention some recent results on characterizations of the Euclidean ball by invariant metrics and their curvature properties. These include the Bergman metric and the Carathéodory metric on pseudoconvex domains. This talk is based on the joint work with Ruoyi Wang and Bun Wong at UC Riverside.
(Received September 11, 2023)

## 1192-32-31151

Sean N. Curry*, Oklahoma State University, Peter Ebenfelt, University of California, San Diego. $L^{2}$ theory for the $\bar{\partial}_{b^{-}}$ operator on domains in abstract CR manifolds.
I will talk about an intrinsic approach to the $L^{2}$ theory for the tangential Cauchy-Riemann operator on domains in the Heisenberg group, and more generally in strongly pseudoconvex CR manifolds. This is joint work with Peter Ebenfelt. (Received September 11, 2023)

1192-32-31211
Greg Knese*, Washington University in St. Louis. Local square integrability of rational functions in two variables. Preliminary report.
Given a polynomial $p(x, y)$ with no zeros in the bidisk $(|x|,|y|<1)$ we are interested in boundary singularities of rational functions $Q(x, y) / p(x, y)$. There are many ways to study the nature of a boundary singularity but in this talk we will discuss square-integrability on the two torus $(|x|=|y|=1)$ of the rational function. This problem converts to a local square integrability question on $\mathbb{R}^{2}$ near $(0,0)$ after applying conformal maps. After doing this we are able to give a complete characterization of the locally square integrable rational functions in two variables (with denominator non-vanishing on the product upper half plane). Time permitting we will discuss an application to sums of squares decompositions for stable polynomials.
(Received September 11, 2023 )

1192-32-31369
Adam Cohen, Reed College, Yash Rastogi*, The University of Chicago. Analysis of the Kohn Laplacian and Sub-Laplacian on Compact Quotients of Quadric Groups.
Much of the study of non-elliptic operators has been restricted to strongly pseudoconvex CR manifolds, the prototype of which is the Heisenberg group with the standard CR structure. Quadric groups are a generalization of the Heisenberg group and are prototypes for other CR manifolds. An analogue of Weyl's law, that is, relating the asymptotic growth rate of the eigenvalue counting function to the volume of the manifold, for $\square_{b}$ on compact quotients of the strongly pseudoconvex Heisenberg group was discovered on $(p, q)$-forms by Fan, Kim, and Zeytuncu. Fan later computed Sobolev estimates for $\square_{b}$, showing that it is subelliptic on $(p, q)$-forms with $q=0, n$, outside of the range of J . J. Kohn's initial proof. We obtain similar results on compact quotients of quadric groups with maximal CR dimension. In particular we obtain asymptotics in a Weyl's law form for the eigenvalue counting functions of $\square_{b}$ and $\Delta_{b}$ on certain spaces of $(p, q)$-forms, and show that it fails to be subelliptic in the remaining cases, with infinite dimensional eigenspaces for arbitrarily large eigenvalues.
(Received September 11, 2023)

## 1192-32-31581

Adam Cohen, Reed College, Yash Rastogi*, The University of Chicago, Yunus E. Zeytuncu, University of Michigan Dearborn. Spectral Analysis of the Kohn Laplacian on Sphere Quotients.
Spectral analysis of the Laplacian and other elliptic operators on manifolds has resulted in significant developments and extended the Weyl law for Riemannian manifolds. The relationship between the spectra of non-elliptic operators and the intrinsic geometric and topological features of the manifolds on which they are defined is far less well-understood. We analyze
the spectrum of the Kohn Laplacian on CR manifolds of the form $S^{2 n-1} / G$, where $G \subset U(n)$ is a discrete group acting freely on $S^{2 n-1}$. We prove a novel Weyl law for the Kohn Laplacian on CR manifolds, thus demonstrating that we can detect the volume of these quotients entirely from the spectrum of this operator. For the subset of spherical 3-manifolds arising in the $n=2$ case, we also prove that we can determine the isomorphism class of $G$ solely from this spectral information. Finally, we prove Schatten and Sobolev estimates for the complex Green operator $\mathcal{G}$ on these sphere quotients since $\mathcal{G}$ is the solution operator for the Poisson equation for the Kohn Laplacian, and thereby determines the behavior of boundary values of holomorphic functions.
(Received September 11, 2023)

## 1192-32-31584

Timothy George Clos*, Kent State University. On finitely generated ideals in $H^{\infty}$.
For what bounded pseudoconvex domains in $\mathbb{C}^{n}$ is the ideal of $H^{\infty}$ functions vanishing at a point in the domain finitely generated? Surprisingly, it involves the solvability of the $\bar{\partial}$ equation in $L^{\infty}$. I will discuss my recent work on this problem for bounded strongly pseudoconvex domains in $\mathbb{C}^{n}$. I will also discuss some applications to algebras of Toeplitz operators. (Received September 11, 2023)

Tanuj Gupta*, Texas A\&M University, Emil Straube, Texas A\&M University. Restriction Operator between Bergman Spaces. Preliminary report.
Let $\Omega_{1} \subset \Omega_{2} \subset \mathbb{C}^{2}$ be two domains that share a common boundary point $p$. This geometric setup geometry forces $m_{2} \geq m_{1}$, where $m_{j}$ is finite type of $\Omega_{j}$ at $p$ for $j=1,2$. It is known that if the restriction operator $R: A\left(\Omega_{2}\right) \rightarrow A\left(\Omega_{1}\right)$ between the Bergman spaces of these domains is compact, then $m_{2}>m_{1}$. Similar result is true for higher dimensions. We investigate the converse and try to find sufficient condition on the domains $\Omega_{1}$ and $\Omega_{2}$ so that the restriction operator is compact. This is an on-going work with Emil Straube.
(Received September 12, 2023)

## 1192-32-31965

Siqi Fu*, Rutgers University-Camden. Some estimates for the first eigenvalue of the complex Neumann Laplacian. Preliminary report.
In this talk, we study the lower bound of the first non-zero eigenvalue of the $\bar{\partial}$-Neumann Laplacian on bounded pseudoconvex domains in $\mathbb{C}^{n}$. We are particularly interested in optimal estimates under certain constraints.
(Received September 12, 2023)

## 1192-32-32023

Ziming Shi*, University of California - Irvine. Global Newlander-Nirenberg problem on domains with finite smooth boundary in a complex manifold. Preliminary report.
Let $M$ be a relatively compact domain in a complex manifold $\mathcal{M}$ with $H_{\bar{\partial}}^{(0,1)}(\bar{M}, \Theta)=0$ where $\Theta$ is the holomorphic tangent bundle of $\mathcal{M}$. Suppose that $M$ has $C^{k}$ boundary with $k$ sufficiently large. Assume the Levi-form of the boundary of the domain has at least 3 negative eigenvalues or at least $n-1$ positive eigenvalues pointwise. Using local homotopy formulas and the $\bar{\partial}$ Neumann theory, we construct a global homotopy formula. We then apply a modified Nash-Moser iteration scheme to show that when a formally integrable and smooth almost complex structure on $\bar{M}$ is sufficiently close to the complex structure on $\bar{M}$ , then there is a smooth diffeomorphism from $\bar{M}$ into $\mathcal{M}$ transforming the almost complex structure into the complex structure on $\mathcal{M}$. This is joint work with Xianghong Gong. This is a preliminary report.
(Received September 12, 2023)

## 1192-32-32089

Gian Maria Dall'Ara*, Istituto Nazionale di Alta Matematica "Francesco Severi". Spectral gap estimates for $\bar{\partial}$-Laplacians and subellipticity in the $\bar{\partial}$-Neumann problem. Preliminary report.
I will talk about joint work with S. Mongodi (Univ. Milano Bicocca) about the question of determining the optimal gain of regularity in the $\bar{\partial}$-Neumann problem near a D'Angelo finite type point, as measured in the $L^{2}$-Sobolev scale. While the classical methods of Kohn (via multiplier ideals) and Catlin (via potential theory) are effective in establishing some gain of regularity, they do not seem to be suited for optimal results. We introduce a new technique, at the core of which are novel spectral gap estimates for certain $\bar{\partial}$-Laplacians, and show that it indeed allows to obtain sharp subelliptic estimates in a class of low dimensional examples.
(Received September 12, 2023)

1192-32-32518
Lily Adlin*, Cal Poly SLO, Giovani Thai*, Cal Poly SLO, Samuel Tiscareno*, Cal Poly SLO. Graph Theoretic Interpretations of the Nevanlinna Representation. Preliminary report.
Key Words: Nevanlinna Representation, Cauchy Transform, Pick function, Graph products, Contact order, Schur complement Given a self-adjoint matrix $A$ and a vector $\alpha$, the map

$$
f(z)=\left\langle(A-z I)^{-1} \alpha, \alpha\right\rangle
$$

is an example of a so-called Nevanlinna representation or Cauchy transform, producing a rational self-map of the complex
upper half plane. One interesting class of self-adjoint matrices that can be studied via this function representation is adjacency matrices of simple undirected graphs. Given such a graph $G$ with adjacency matrix $A$, one can view the matrix $A-z I$ as the adjacency matrix of a graph that puts self-loops of weight $z$ onto the existing structure of $G$. In this case, certain choices of the vector $\alpha$ produce a rational inner function in the Pick class. In this poster, we investigate extended examples studied by Bickel, Pascoe, and Sola [1]. In addition, based on an honors thesis by Hong [2], we connect graph structures to the behavior of a twovariable analogue of the representation, which allows for more complicated boundary behavior to occur. The central matrix becomes $(A-z Y-w(I-Y)$ ), where $Y$ is, for example, a diagonal matrix with diagonal entries 0 and 1 . Reformulating these function-theoretic ideas in linear-algebraic terms turns out to provide straightforward arguments that, for example, extend known one-variable ideas about representation of certain graph products into two variables. We'll also look at the relationship between Bickel, Pascoe, and Sola's notion of contact order, which measures bunching of level curves of a representing function via power series, and the underlying graph. References [1] Bickel, K., Pascoe, J., and Sola, A. (2018). Level Curve Portraits of Rational Inner Functions. Ann. Sc. Norm. Super. Pisa Cl. Sci., 2020: Vol XXI (Special Issue), 449-494. Submitted 2018, Published 2020. doi: 10.2422/20362145.201804_25. [2] Hong, Y. (2023). Graphs, Adjacency Matrices, and Corresponding Functions. Bucknell University Honors Thesis. (Received September 12, 2023)

## 1192-32-32616

Peter Ebenfelt*, University of California, San Diego. Bergman logarithmically flat and obstruction flat CR manifolds Preliminary report.
The boundary $X$ of a smoothly bounded strictly pseudoconvex domain $\Omega$ is said to be Bergman logarithmically flat if the log term in the Fefferman asymptotic expansion of the Bergman kernel vanishes to infinite order at the boundary. It is obstruction flat if the Cheng-Yau log potential of the Kähler-Einstein metric extends smoothly to $X$. In two dimensions, obstruction flat boundaries are all expected to be spherical (and Bergman logarithmically flat boundaries are known to be), but in higher dimensions their CR structures have more variety. We shall explain and propose some open questions.
(Received September 12, 2023)

## 1192-32-32831

Qi Han*, Texas A\&M University-San Antonio. On partial differential equations of Waring's-problem form in several complex variables. Preliminary report.
In this talk, we discuss when some PDEs of Waring's-problem form can admit entire solutions in $C^{n}$ and further find these solutions for important cases including particularly $u_{z_{1}}^{\ell}+u_{z_{2}}^{\ell}+\cdots+u_{z_{n}}^{\ell}=u^{\hbar}$, which are (often said to be) PDEs of superFermat form if $\hbar=0, \ell$ and an eikonal equation if $\ell=2$ and $\hbar=0$. (Received September 12, 2023)

## 33 Special functions

## 1192-33-30504

Gaurav Bhatnagar*, Ashoka University. Elliptic extensions of elementary identities. Preliminary report.
We provide elliptic extensions of elementary identities such as the sum of the first $n$ odd or even numbers, the geometric sum and the sum of the first $n$ cubes. Many such identities, and their $q$-analogues, are indefinite sums, and can be obtained from telescoping. So we used telescoping in our study to find elliptic extensions of these identities. In the course of our study, we obtained an identity with many parameters, which appears to be new even in the $q$-case. In addition, we recover some $q$ identities due to Warnaar. This is joint work with Archana Kumari and Michael Schlosser.
(Received September 10, 2023)

## 1192-33-31785

Andrew Thomas Hale*, University of Minnesota. The Method of Brackets Applied to Definite Integrals Involving Bessel Functions.
Title: The Method of Brackets Applied to Definite Integrals Involving Bessel Functions Author: Andrew Hale, University of Minnesota In this talk we present an application of the method of brackets for definite integrals involving Bessel functions. This is a procedure for the evaluation of definite integrals over the half line $[0, \infty)$ based on the expansion of the integrand as a power series. The primary object in this method is the so-called bracket series, which is produced and evaluated according to a small number of rules. Bessel functions are a family of functions with important applications in the study of wave propagation. These functions possess a number of properties and a power series representation which is suitable for the application of the method of brackets. We will illustrate the successful application of the method through several examples, based on problems taken from Table of Integrals, Series, and Products. Also, we will present an intriguing example where the method does not lead directly to the value of the integral, but we propose a solution for this complication. This talk is based on joint work with: Zach Zobair, Nicole Newfield, Nathaniel Smith, Aakash Gurung, Isaac Blackburn, Ryosei Sato, Tevia Jabbour, Joshua Lentz, William Sean, Megan McGhee, Maitreyo Bhattacharjee, Navya Tripathi, Shreepad Agrawal, John Lopez, Victor Moll. Developed during the Polymath Jr. Program 2023. (Received September 12, 2023)

## 1192-33-32443

Daniel Alpay, Chapman University, Kamal Diki, Chapman University, Mihaela B. Vajiac*, Chapman University, Orange, CA. The Bicomplex-Real Calculus and Applications to Bicomplex Hermite-Itô Polynomials.
In this work we extend notions of complex C-R-calculus and complex Hermite polynomials to the bicomplex setting and compare the bicomplex polyanalytic function theory to the classical complex case.
(Received September 12, 2023)

Sebastien Bertrand, University of Hawaii, Sarah Post*, University of Hawaii. Bivariate Racah polynomials and their support graphs.
In this talk we will discuss the rank-2 extension of the Racah algebra and its representation theory. In particular, we will realize the algebra generators as acting on 2-D lattice. Bivariate Racah polynomials arise as expansion coefficients from the action of the generators on an analog of lowest weights in this representation. We will also discuss the realization in terms of quotients of the enveloping algebra by an ideal, similar to the traditional Verma modules. We will finish with a brief description of the extension and arising difficulties in extending to arbitrary rank.
(Received September 12, 2023)

1192-33-32984
Drew Barnes*, Lindenwood University, Tom Cuchta, Marshall University. A Discrete Analogue for Jacobi Polynomials.
Classical Jacobi polynomials are a family of orthogonal polynomial sequences that appear in numerous physical sciences. We investigate discrete analogues of Jacobi polynomials through their hypergeometric representation, resulting in "discrete Jacobi polynomials" that satisfy qualitatively similar properties to their continuous counterparts. A difference equation analogue of the classical Jacobi polynomial differential equation and an analogue of its three-term recurrence are derived. This expands the growing subject of discrete analogues of special functions. A crucial part of this work is to generalize the theory of discrete analogues of generalized hypergeometric functions: in the current literature, finding analogues with monomial arguments of the classical hypergeometric has been solved, but we instead consider a linear argument for which existing methods fail. Our new technique can be applied to the existing discrete Legendre and discrete Chebyshev polynomials, which previously used a workaround that altered the final resulting formulas in an unsatisfying way.
(Received September 13, 2023)

## 34 Ordinary differential equations

## 1192-34-25455

Marcel Julmard Ongoumakaa Yanzda*, Marien Ngouabi University. The Function Number Method : Basis and Applications. Preliminary report.
In this paper, we present a new method to solve some mathematics problems such as integral calculus, derivative calculus and differential equations. The method consists to transform an analytic problem or function to a real number. This real number obtained represents the Function Number. After finding the Function Number solution, it is also possible to transform it to a semi-analytic function which represents the definitive solution of the problem. We qualify the solution as semi-analytic solution because to solve the problem, we make some approximations. So, the semi-analytic function obtained is an approximate analytic solution. This method is simple and concise. It gives strong approximate solutions near to the real solutions. (Received June 10, 2023)

## 1192-34-25775

Elif Demirci*, Ankara University. An Optimal Control Problem for COVID-19 Pandemic. Preliminary report.
We present an optimal control problem for an epidemic model regarding the COVID-19 pandemic. Social isolation and vaccination parameters in the model are considered to be control parameters of the problem. Existence of the optimal control is analyzed and simulations for different scenarios are presented by using empirical data.
(Received July 10, 2023)

## 1192-34-26273

Sundar Tamang*, UAB PhD Thesis. A Model For Currency Exchange Rates.
The catastrophic economic events like, oil price shock in 1973, the 9/11 event in 2001, stock market crash in October, 1987 and crash in late 2008 etc, impacted the US economy without warning from sharp downturns to actual market crashes. And the current economic theory and statistical models are not enough to analyze those events. So, for constituting the models to predict and recovery of US economy with major factors: commodities, stock, bond and currency, I am going to predict and recover the currency exchange rates using models "Stochastic Differential Equations" with predicting trend of exchange rates by system of delay differential equations and recovering of volatility of exchange rates by using inverse problem in "Dupire's Equations" which is obtained from "Black-Scholes Equations".
(Received July 24, 2023)

1192-34-26857
Snezhana Hristova*, University of Plovidiv "Paisii Hilendarski". Approximate method for nonlinear boundary value problem for generalized Caputo fractional differential equation - theoretical proofs and computer realization.
A special type of fractional derivative is applied to a nonlinear differential equation. This derivative is called a generalized proportional Caputo fractional derivative. It is studied a boundary condition to the given equation. Initially, the linear fractional differential equation with a boundary condition is considered and its explicit solution is obtained. An appropriate integral fractional operator for the nonlinear problem is defined and applied to mild solutions, upper mild solutions and lower mild solutions of the given problem. A scheme for obtaining the solution is provided. The new defined integral operator with the monotone-iterative technique and the method of lower and upper solutions are applied to obtain the new algorithm for successive approximations to the mild solution. The algorithm is theoretically well ground, computerized and applied to a particular example. The convergence of the sequence of mild lower solutions and the sequence of mild upper solutions to the mild solution of the given problem is illustrated.
(Received August 07, 2023)

Snezhana Hristova*, University of Plovidiv "Paisii Hilendarski". Mittag-Leffler stability in time for fractional differential equations with Riemann-Liouville type fractional derivatives. Preliminary report.
Initially we present the most generalized fractional derivatives of Riemann-Liouville type. We discuss their connections with the recently defined, studied in the literature and applied to differential equations and models classical fractional derivatives. Some inequalities for convex Lyapunov functions with these fractional derivatives are given. It is studied the case when the order of fractional derivative is on the interval $(0,1)$. We set up initial value problems for differential equations with the given fractional derivatives. Note the initial conditions for fractional differential equation with Riemann-Liouville type fractional derivative are totally different than the initial condition to ordinary differential equations. In connection with the applied fractional derivatives and their singularity at the initial time, Mittag-Leffler stability in time of the solution is introduced and studied. The main characteristic of this type of stability is the excluding of the initial time point. The base of the study is the Lyapunov method and the presented inequalities with the applied fractional derivatives.
(Received August 07, 2023)

1192-34-26969
Samer S Habre*, Lebanese American University. Presenting the Third Special Issue of the CODEE Journal. Preliminary report.
The CODEE online Journal, a peer-reviewed, open-access publication, publishes new materials promoting the teaching and learning of ordinary differential equations. After the success of previous Special Issues of the CODEE Journal (Linking Differential Equations to Social Justice and Environmental Concerns, Vol. 12, 2018, and Engaging Learners: Differential Equations in Today's World, Vol. 14, 2020), calls for contributions to a third Special Issue was sent out in April 2023. The theme of the third issue extends the ideas of the previous two to the possibilities of influencing public policy. Public policy is important because it provides structures and strategies by which our society can work to address and resolve some of the current social, economic, environmental, public health, educational, behavioural, and other issues. Differential equations can predict future behaviours and show the effect of varying parameters. Models in Economics, Management of Resources, Epidemiology, Cognition and Learning, Public Health, and Climate Change, among others, can benefit from a thorough investigation using ordinary differential equations. Results can influence public policies of government and non-governmental agencies alike. In this presentation we share the contents of this Spacial Issue, hoping to trigger interest and curiosity among the audience and hence reaching a wider readership of the Journal.
(Received August 08, 2023)

1192-34-27177
Viktoria Savatorova, Central Connecticut State University, Aleksei Talonov*, University of Nevada Las Vegas. Multi-Scale Analysis of Predator-Prey Equations.
This presentation is dedicated to multi-scale analysis of nonlinear ordinary differential equations used to model predator-prey relationships. For clarity, we will focus on a two-equation system similar to the Lotka-Volterra model, wherein coefficients demonstrate periodic behavior with respect to time. This periodicity arises from factors such as seasonal variations in prey growth rate, predation rate, predator mortality, and the interaction of prey presence with predator growth coefficient. Suppose that predator's average lifespan $(t)$ aligns with that of its prey, yet considerably exceeds the characteristic time ( $\tau$ ) governing system parameter changes due to seasonal shifts. This establishes a two-scale temporal framework wherein prey and predator population densities depend on both slow $(t)$ and fast $(\tau)$ scales. We assume the condition of separations of scales $\varepsilon=\frac{\tau}{t} \ll 1$. Our multi-scale method involves presenting solutions and initial conditions as asymptotic serial expansions in terms of small parameter $\varepsilon$ and averaging of resulting equations over the period of oscillations of the fast variable. In case of predator-prey model with two species, the multi-scale approach is analytically tractable and allows to see how equilibrium solutions respond to periodic shifts embedded within coefficients.
(Received August 11, 2023)

## 1192-34-27447

Ahmed Mahdi Moqbel Ahmed*, King Fahd University of Petroleum \& Minerals. On Nonexistence of Solutions for Nonlinear Systems of Fractional Integro-differential Equations.
We investigate the nonexistence of nontrivial global solutions for a system of two strongly coupled fractional differential equations. Each equation involves two fractional derivatives and a nonlinear source term. The fractional derivatives are of Caputo type of sub-first orders. The nonlinear sources are nonlocal in time. They have the form of a convolution of a polynomial of the state with a (possibly singular) kernel. The system under consideration is a generalization of many interesting special systems of equations whose solutions do not exist globally in time. We establish some criteria under which no nontrivial global solutions exist. Several integral inequalities and estimations are derived and the test function method is adopted. Special cases and examples with numerical computations are given to illustrate the results.
(Received August 15, 2023)

1192-34-27802
Zhilan Feng*, National Science Foundation, John W Glasser, The US Centers for Disease Control and Prevention. Mixing and reproduction numbers for a metapopulation model structured by age and spatial location. Preliminary report. Metapopulation models structured by age and spatial location with appropriate mixing between subpopulations can provide an important tool for identifying optimal allocations of limited resources. When such a model also includes partially immune classes, the derivation of metapopulation reproduction numbers and identification of target groups for effective intervention strategies can be very challenging. This will be demonstrated using our model of SARS-CoV-2 transmission, which is stratified by age, $0-4,5-9, \ldots, 75+$ years, and location, the 50 states plus District of Columbia. We will also illustrate the utility of such analytical results by simulating two hypothetical vaccination scenarios, one random and the other determined by the gradient of the meta-population effective reproduction number with respect to possible vaccination rates, which we compared with the actual program estimated from one of the CDC's nationwide seroprevalence surveys conducted from mid-summer 2020 through the end of 2021. To ensure that our experiment was informative, all else had to be equal (i.e., non-pharmaceutical interventions and available vaccine doses). This required us to calculate the gradient each week with the constraint being the doses available at each location that week.
(Received August 20, 2023)

## 1192-34-28428

Byungjae Son*, Ohio Northern University. Analysis of positive solutions to one-dimensional generalized double phase problems.
In this talk, we consider the one-dimensional generalized double phase problems: $\$ \$ \mathrm{q}\left[\left] \backslash\right.\right.$ matrix $-\left(\alpha(\mathrm{t}) \varphi_{-}\{\mathrm{p}\}\left(\mathrm{u}^{\prime}\right)+\beta(\mathrm{t}) \varphi_{-}\{\mathrm{q}\}\right.$ $\left.\left(\mathrm{u}^{\prime}\right)\right)^{\prime}=\lambda \mathrm{h}(\mathrm{t}) \mathrm{f}(\mathrm{u})$, $\mathrm{t} \in(0,1) \mathrm{u}(0)=0=\mathrm{u}(1)$, \endmatrix. $\$ \$$ where $\lambda>0,1<p<q<\infty, \varphi_{m}(s):=|s|^{m-2} s$,
$h \in L^{1}((0,1),(0, \infty)) \cap C((0,1),(0, \infty))$ and $f \in C((0, \infty), \mathbb{R})$. We discuss the existence and multiplicity of positive solutions when the weight functions $\alpha, \beta \in C([0,1],[0, \infty))$ satisfy $\alpha(t)+\beta(t)>0$ almost everywhere and
$\int_{0}^{1} \alpha^{-\frac{1}{p-1}}(s) \beta^{-\frac{1}{q-1}}(s) d s<\infty$. The Krasnoselskii-type fixed point theorem is used to get the results.
(Received August 30, 2023)

1192-34-29053
D. Chloe Griffin*, Brown University, Amanda J. Mangum, Converse University. Fitting a COVID-19 Model Incorporating Senses of Safety and Caution to Local Data from Spartanburg County, South Carolina.
Common mechanistic models include Susceptible-Infected-Removed (SIR) and Susceptible-Exposed-Infected-Removed (SEIR) models. In their basic forms, these models fail to capture the multiple waves of the COVID-19 pandemic. They do not account for public policies such as social distancing, mask mandates, and the "Stay-at-Home" orders implemented in early 2020. While the Susceptible-Vaccinated-Infected-Recovered-Deceased (SVIRD) model only adds two more compartments to the SIR model, the inclusion of time-dependent parameters allows for the model to better capture the first two waves of the COVID-19 pandemic when surveillance testing was common practice for a large portion of the population. We find that the SVIRD model with time-dependent and piecewise parameters accurately fits the experimental data from Spartanburg County, South Carolina. These additions give insight into the changing social response toward the COVID-19 pandemic within Spartanburg County.
(Received September 05, 2023)

1192-34-29362
Kwadwo Antwi-Fordjour, Samford University, Kendall Hope Bearden*, Samford University. Comment on predator-prey dynamical behavior and stability with square root functional response. Preliminary report.
In this talk, we revisit the paper by Pal et al. [Int. J. Appl. Comput. Math (2017) 3:1833-1845] and comment on the claim that global stability of the interior equilibrium point depends on some key parameters. This is not true, and we have provided detailed proof to this effect. The considered model is a modified Lotka-Volterra model where square root functional response is involved. The square root functional response is non-differentiable at the origin and hence cannot be studied with standard local stability tools. Furthermore, our numerical simulations indicate that we can classify the phase portrait into two modes of behavior where some positive initial conditions converge towards the predator axis in finite time.
(Received September 06, 2023)

## 1192-34-29644

Frederick R Adler, University of Utah, Lindsay T Keegan, University of Utah, Matthew H Samore, University of Utah, Theresa Sheets*, University of Utah, Damon JA Toth, University of Utah. Vaccination and Household Epidemics. Households are major transmission venues for infectious diseases, particularly respiratory illness. Through this project we aim to understand the magnitude of COVID-19 vaccination's impact within households by examining four key parameters: vaccine efficacy, infectiousness of children and adults, household configuration (e.g., vaccination status and age of household members), and source of household infection importation. We identify situations in which vaccination makes the biggest impact on household transmission, particularly to vulnerable groups. We build a stochastic SIR model which contains compartments for vaccinated and non-vaccinated adults and children and simulates transmission within a household. We find that vaccination of children has minimal effect on secondary transmission within a household, but a large potential for preventing primary importation. This work provides a framework to evaluate how non-vaccinated household members impact within household transmission when data are limited, and when transmission parameters are unknown. We then extend this approach by developing an analytic solution to model the spread of an infectious disease within a small population and test it against the results from the simulation which offers a dramatic improvement in efficiency as compared to simulation-based methods. We are able to directly solve for transmission probabilities, and can then use this closed-form solution to explore the transmission parameter space and numerically solve for distributions of infected individuals within households. (Received September 11, 2023)

1192-34-29722
Laszlo Goch*, Adelphi University, Anil Venkatesh, Adelphi University. Analysis of COVID-19 Policy Interventions in New Zealand.
With the rise of the COVID-19 pandemic, the role of mathematical epidemiology in advising policy changes has grown massively. On the one hand, mathematical epidemiologists use models to suggest policy changes, while on the other hand, such policy changes are fed back into the modeling process to assess their effectiveness. New Zealand has been heralded as a leader in the control of COVID spread due to its geography a well as its government-enforced policies such as social distancing. This raises the question: How is the country's relatively low infection rate related to its policy changes? Our SIR model has estimated that the infection rate would have been significantly higher if such restrictions had not been placed. (Received September 07, 2023)

Nourridine Of Siewe*, Rochester Institute of Technology. Hybrid discrete-time-continuous-time models and a SARS CoV-2 mystery: Sub-Saharan Africa's low SARS CoV-2 disease burden.
Worldwide, the recent SARS-CoV-2 virus has infected more than 670 million people and killed nearly 67.0 million. In Africa,
the number of confirmed COVID-19 cases was approximately 12.7 million as of January 11,2023 , that is about $2 \%$ of the infections around the world. Many theories and modeling techniques have been used to explain this lower-than-expected number of reported COVID-19 cases in Africa relative to the high disease burden in most developed countries. We noted that most epidemiological mathematical models are formulated in continuous-time interval, and taking Cameroon in Sub-Saharan Africa, and New York State in the USA as case studies, in this paper we developed parameterized hybrid discrete-time-continuous-time models of COVID-19 in Cameroon and New York State. We used these hybrid models to study the lower-thanexpected COVID-19 infections in developing countries. We then used error analysis to show that a time scale for a data-driven mathematical model should match that of the actual data reporting.
(Received September 07, 2023)

## 1192-34-29828

Chidozie Williams Chukwu*, Wake Forest University. Can key factors contributing to Malaria transmission be controlled? a case study West Sumba Indonesia.
This talk discusses a mathematical model for Malaria dynamics, considering multiple recurrent phenomena: relapse, reinfection, and recrudescence. A limitation of hospital bed capacity, which can affect the treatment rate, is modeled using a saturated treatment function. The qualitative behavior of the model, which encompasses the Malaria-free/endemic equilibria, basic reproduction number, and stability analysis of the endemic equilibrium, was rigorously investigated. Our observation on the malaria-endemic equilibrium shows possible multiple endemic equilibria when the basic reproduction number is larger or smaller than one. A non-trivial result appears that the higher probability of recrudescence (treatment failure) does not always result in the appearance of backward bifurcation. The model was calibrated using Malaria incidence data from West Sumba, Indonesia. Results from global sensitivity analysis using a combination of Latin Hypercube Sampling and Partial Rank Correlation Coefficient indicate that human and mosquito infection rates are the most influential parameters in determining the increase in new infections for humans. An optimal control problem was formulated, incorporating Malaria interventions such as bed net use, hospitalization, and fumigation as a time-dependent variable. Also, we carried out a cost-effectiveness analysis, suggesting that hospitalization is the most cost-effective strategy required to control Malaria.
(Received September 08, 2023)

## 1192-34-30161

Erich McAlister*, Fort Lewis College. Applied Category Theory in the ODE Class. Preliminary report.
Stock-Flow diagrams provide a visual representation for systems of ordinary differential equations. Baez et al recently gave a categorical framework for stock-flow diagrams in which they are realized as instances of a certain small category. We will explore this approach as an organizational tool for constructing (end elaborating on) ODE models typically encountered in an undergraduate ODE course.
(Received September 09, 2023)

## 1192-34-30191

Kaitlin Mari Baudier, Arizona State University, Noam Ben-Asher, Boston Fusion, Jennifer Fewell, Arizona State University, Yun Kang*, Arizona State University, Maria Gabriela Navas Zuloaga, Arizona State University, Theodore P Pavlic, Arizona State University. Collective Defense Dynamics of Social Insect Colonies.
To understand how reproduction, development, and behavior integrate at different time scales to balance defensive demands with other biological processes, we developed a demographic Filippov ODE system to study the dynamical effects of these processes on task allocation and colony size. Our results show that colony size peaks at low proportions of majors, but colonies die if minors are too plastic or defensive demands are too high or if there is a high proportion of quickly developing majors. For fast maturation, increasing major production may decrease defenses. This model elucidates the demographic factors constraining collective defense regulation in social insects while also suggesting new explanations for variation in defensive allocation at smaller scales where the mechanisms underlying defensive processes are not easily observable. Moreover, our work helps to establish social insects as model organisms for understanding other systems where the transaction costs for component turnover are nontrivial, as in manufacturing systems and just-in-time supply chains.
(Received September 09, 2023)

## 1192-34-30211

Djamal Benbourenane*, Rose-Hulman Institute of Technology. Schrodinger equations with potentials having eigenvalues and eigenfunctions given explicitly and Mathematica verifiable... Preliminary report
It is well-known that the Schrodinger equation plays a vital role in Quantum Mechanics. However, only a few potentials, like the harmonic, Coulomb, and Poschl-Teller potentials, are known to have exact solutions. In this talk, I will present new classes of potentials of the time-independent Schrodinger equation. The potentials and their eigenvalues and eigenfunctions are obtained explicitly using the factorization method for shape-invariant potentials. To remove any doubt on the validity of the results, I will demonstrate, using Mathematica, the exactness of the first five to ten eigenvalues and eigenfunctions, by showing that the left-hand side equals the right-hand side of the equations. The results were extended to the complex plane and showed exciting results for some periodic and singular potentials with tunneling. These results will pave the way for a better understanding the quantum world of quantized energies and their corresponding wavefunctions.
(Received September 09, 2023)

1192-34-30305
Adam Spiegler*, University of Colorado Denver. Exploring Differential Equations with Interactive Jupyter Notebooks. I will demo a complete set of interactive course materials created using dynamic Jupyter notebooks and Google Colaboratory. Differential equations provide a rich universe to explore models and train students as mathematical experimentalists. Interactive Jupyter notebooks create a flexible learning environment to bridge the divide between theory and practice. Jupyter is a free, open-source platform that provides instructors and students an environment to weave together narrative text, executable code, visualizations, and videos all in one document. Course materials are delivered in Google Colaboratory, a free, cloud-based application where instructors and students interact with course materials. The flexibility to design materials for a variety of course formats and seamlessly integrate everything into one environment help foster an engaging and fun
environment for both students and instructors. Jupyter notebooks help equip students with computational tools to gain further insight and intuition into differential equations. All course materials are Open Education Materials (OER) with a Creative Commons license and are freely available to be reused, revised, and redistributed.
(Received September 09, 2023)

## 1192-34-30307

Asma Azizi*, Kennesaw State University. Effects of Social Distancing on Emerging Infections Viral Evolution. Preliminary report.
We aim to understand the viral evolution process for emerging infections such as COVID-19 and how we can update mitigation or intervention approaches to bring it under control. We created and simulated an ODE differential equation that can predict the spread and mutation of COVID-19 and help guide public health workers in improving the effectiveness of intervention strategies for controlling virus mutations. Our results confirm that practicing social distancing when the infection spreads drastically, even if the vaccination runs, is crucial to maintaining virus mutation.
(Received September 09, 2023)

## 1192-34-30424

Zhilan Feng*, National Science Foundation, John W Glasser, The US Centers for Disease Control and Prevention. Implications for infectious disease models of heterogeneous mixing on control thresholds.
Mixing among sub-populations, as well as heterogeneity in characteristics affecting their reproduction numbers, must be considered when evaluating public health interventions to prevent or control infectious disease outbreaks. In this talk, we model preferential within- and proportional among-group contacts in compartmental models of disease transmission and derive results for the overall effective reproduction number ( Rv ) assuming different levels of vaccination in the subpopulations. Another significant result is that for general mixing schemes, both the basic and the effective reproduction numbers are bounded below and above by their corresponding expressions when mixing is proportionate and isolated, respectively.
(Received September 10, 2023)

## 1192-34-30453

John A. Arredondo*, Fundación Universitaria Konrad Lorenz. Periodic Oscillations in a $2 N$-Body Problem.
Hip-hop solutions of the $2 N$-body problem are solutions that satisfy, at every instance of time, that $2 N$ bodies with the same mass $m$ are at the vertices of two regular $N$-gons, and each one of these $N$-gons is at a plane that are equidistant from a fixed plane $\Pi_{0}$ forming an antiprism. Under this setting in this talk, we will see that: - For every $N$ and every $m$ there exists a family of periodic hip-hop solutions. For every solution in these families the oriented distance to the plane $\Pi_{0}$, which we call $d(t)$, is an odd function that is also even with respect to $t=T$ for some $T>0$. For this reason we call solutions in these families double symmetric solutions. - We numerically show that some of the branches established in our existence theorem have bifurcations that produce branches of solutions with the property that the oriented distance function $d(t)$ is not even with respect to any $T>0$; we call these solutions single symmetric solutions. - We prove that no single symmetric solution is a choreography. We also display explicit double symmetric solutions that are choreographies. (Received September 10, 2023)

## 1192-34-30591

D.D. Hai, Mississippi State University, Dustin Nichols, UNC Greensboro, Ratnasingham Shivaji*, University of North Carolina Greensboro. uniqueness result for a p-Laplacian infinite semipositone problem involving nonlinear boundary conditions.
We study classes of two-point boundary value problems of the form:

$$
\left\{-\left(\phi\left(u^{\prime}\right)\right)^{\prime}=\lambda h(t) f(u) ;(0,1) u(0)=0 u^{\prime}(1)+c(u(1)) u(1)=0, \backslash\right. \text { endcases }
$$

where $\phi(s)=|s|^{p-2} s$ for $p>1, h \in C^{1}((0,1],(0, \infty))$ is decreasing, $c \in C([0, \infty),(0, \infty))$ is non-decreasing and bounded, and $f \in C^{1}((0, \infty), \mathbb{R})$ is increasing on $[L, \infty)$ for some $L>0$, has infinite semipositone structure at 0 , and growth at $\infty$ like $u^{q}$ for $q \in(0, p-1)$. For classes of such $h$ and $f$, we establish the uniqueness of positive solutions for $\lambda \gg 1$.
(Received September 10, 2023)

## 1192-34-30712

Timothy Lucas*, Pepperdine University. Slopes: An Intuitive Mobile App to Enhance Learning in Differential Equations Slopes is a mobile application with an intuitive interface that is designed to visualize solutions to differential equations and support active learning in the classroom. By making slopefields, phase planes and numerical solutions more accessible, students are able to engage in higher level discussions of mathematical models that incorporate differential equations. Slopes is currently available for iPads, iPhones, and Android phones, which are highly portable and feature larger touch screens that allow students to view and manipulate content easily. I will discuss in-class activities that emphasize a visual understanding of mathematical models in order to introduce and reinforce key concepts in differential equations. In a recent study, we found that students used Slopes to visualize solutions, aid in discussion and cooperation, build prototype models, and demonstrate understanding of differential equations concepts. (Received September 10, 2023 )
mode of content delivery uses a paradigm we refer to as ECAP for Explore, Conjecture, Apply, Prove. Student success is measured by their ability to master a list of fundamental standards and their contributions to course discussions and activities used in the ECAP process. We will demonstrate the ECAP methodology, the assessment of student-learning using fundamental standards, and preview the development of an on-line interactive PreTexT platform we are developing for this this course. (Received September 11, 2023)

1192-34-31172
Chris Ahrendt*, University of Wisconsin-Eau Claire. An Overview of Various Discrete Analogs of Differential Equations Using the Time Scale Calculus. Preliminary report.
There is a rich history of examining discrete analogs of various important differential equations, and many different mechanisms have been used to do so. In this work, we utilize the time scale calculus. The time scale calculus provides a framework to generalize and unify differential equations and difference equations. However, it is also a source of many interesting results that do not have a direct analog in these classic cases. An introduction to the key results of the time scale calculus will be provided. We show how the techniques of time scales can be applied to develop discrete analogs of various well-known differential equations. Among others, we will look at the logistic equation, the Clairaut equation, the Bernoulli equation, as well as the Lotka-Volterra system of equations. After establishing the discrete analogs of these differential equations, we will explore the solution behavior in each case. In particular, the bifurcations that occur as parameters are varied will be explored.
(Received September 11, 2023)

## 1192-34-31207

Sergio Branciamore, Department of Computational and Quantitative Medicine, City of Hope National Medical Center, David Frankhouser, Department of Computational and Quantitative Medicine, City of Hope National Medical Center, Yu-Hsuan Fu, Department of Computational and Quantitative Medicine, City of Hope National Medical Center, Ya-Huei Kuo, Department of Hematological Malignancies Translational Science, Hematology and Hematopoietic Cell Transplantation, The Gehr Family Center for Leukemia Research, City of Hope National Medical Center, Guido Marcucci, Department of Hematological Malignancies Translational Science, Hematology and Hematopoietic Cell Transplantation, The Gehr Family Center for Leukemia Research, City of Hope National Medical Center, Denis O'Meally, Center for Gene Therapy, City of Hope National Medical Center, Russell Rockne*, Department of Computational and Quantitative Medicine, City of Hope National Medical Center, Lisa Uechi, Department of Computational and Quantitative Medicine, City of Hope National Medical Center, Bin Zhang, Department of Hematological Malignancies Translational Science, Hematology and Hematopoietic Cell Transplantation, The Gehr Family Center for Leukemia Research, City of Hope National Medical Center. Connecting phenomenological and mechanistic modeling of gene expression state transitions in health and disease.
Several studies have shown that the transcriptome can be used as a biomarker in cancer to refine disease diagnosis and to predict response to treatment. As a collection of several interconnected gene regulatory networks, the transcriptome evolves in a complex, nonlinear fashion. Here we use mouse models of acute and chronic myeloid leukemia (AML, CML) to study the evolution of the transcriptome over the course of disease initiation, progression, and response to treatment. We model the transcriptome as a particle undergoing Brownian motion in a potential energy landscape to characterize disease states corresponding to local minima in the potential. This phenomenological state-transition model is used to predict disease progression and response to chemo therapy in AML and to tyrosine kinase inhibitor (TKI) therapy in CML. Although this modeling approach accurately predicts transcriptome dynamics observed in the experimental data, it lacks a mechanistic explanation for what produces or alters the potential. Here we connect the phenomenological state-transition model with a mechanistic model to study how oncogenic mutations or therapeutic interventions alter the structure of the potential energy landscape and corresponding transcriptome states. The mechanistic model involves a signal, cancer cells, and the transcriptome, connected to each other through switching Hill functions. We show that the mechanistic model recapitulates known behaviors of simple perturbations as well as provides insights into more complex perturbations. Specifically, we find that the introduction of the oncogenic fusion gene BCR-ABL is sufficient but not necessary to produce tristable CML disease states in the transcriptome. Similarly, we find that the fusion cbfb-MYH11 is sufficient to produce bistable disease states in AML. Moreover, we discovered the nontrivial and nonobvious result that TKI therapy for CML alters both BCR-ABL expression and the sensitivity of cancer cells to changes in BCR-ABL levels. By connecting the detailed mechanistic model with the statetransition model, we have a better understanding of how the transcriptome changes over time. This insight can be used to customize treatment plans for individual patients based on their unique disease dynamics.
(Received September 11, 2023)

## 1192-34-31220

Burak Hatinoglu*, Michigan State University. Spectral properties of periodic discontinuous graphs.
Peter Kuchment and Olaf Post explicitly derived dispersion relations and spectra for periodic Schrödinger operators on carbon nano-structures in their "On the Spectra of Carbon Nano-Structures" paper. In this talk I will consider some generalizations of their results. I will discuss spectral properties of periodic Schrödinger operators with self-adjoint vertex conditions that allow discontinuities. Specifically, jumps at the vertices that reflect existence of rigidity and mass at the vertices in some models. This is a joint work with Mahmood Ettehad and Soohee Bae.
(Received September 11, 2023)

1192-34-31521
Naima Naheed*, Benedict College. Writing an Outstanding Proposal and Motivating Differential Equations Learning with SIMIODE Textbook.
Crafting a successful research proposal for securing funding in the highly competitive academic landscape can be a formidable undertaking. One of the initial challenges lies in identifying research topics that align with the expertise and interests of students and faculty, considering that a mismatch between the subject and a student's major can impede progress. Conversely, selecting a too-elementary topic may render the proposal less competitive. Moreover, exceptional students often engage in internships, making it imperative to maintain a list of at least three potential students for recruitment. The subsequent step entails careful selection of a research project influenced by factors such as the geographical location of the institution, current events, and trending subjects. This ensures relevance and resonance with both the academic community and society at large.

Having completed these preliminary steps, the crucial phase of composing a compelling research proposal arises. In the endeavor, the SIMIODE textbook emerges as an invaluable resource, facilitating the development of outstanding proposals. This presentation aims to impart invaluable insights into the art of crafting persuasive research proposals, drawing from realworld experiences. It will also provide a glimpse into the ongoing research projects that have been propelled by the winning proposal, offering a holistic perspective on the research process.
(Received September 12, 2023)

## 1192-34-31841

Julian Bennett, Research Partner, Lauren Eriksen*, Research Partner. Using SIR Models To Study Covid-19 In Florida. Preliminary report.
Within the likes of any highly contagious and unpredictable disease, lies a predictable and attainable growth rate that researchers can find. This is used to make logistical conclusions about that particular disease and its affected regions' counterparts, which for our research is Florida. The main objective of this work is to study and modify the existing Susceptible-Infected-Removed (SIR) model to better analyze the Covid-19 data in Florida. We were able to adapt from this simplistic model using a previous similar study of Covid-19 in North Carolina, which displayed the use of the time-delayed SIR model. We discovered that modifications to the original SIR Model increased the accuracy for our model and for the growth rate of Covid in Florida. With the delayed SIR, we could find good approximations for our Florida data. However, since we wanted a more accurate model, we created the reinfected SIR model (based on previous studies). This SIR model had the most modifications of the three SIR models, and ended up giving us the most accurate and optimal approximations of our Florida data. This is the model that we used to find the growth rate of Covid for Florida.
(Received September 12, 2023)

## 1192-34-31961

John A. Arredondo*, Fundación Universitaria Konrad Lorenz. Periodic oscillations in electrostatic actuators under time delayed feedback controller.
In this talk we will introduce a large class of delayed Liénard equations that governs the motion of several MEMS devices. Our principal result is to present sufficient conditions for the existence of $T$-periodic solutions for an electrostatic actuator modeled by the time delayed Duffing equation

$$
\ddot{\mathrm{x}}+h_{D}(x, \dot{\mathrm{x}})+x=1-\frac{e \mathcal{V}^{2}\left(t, s_{1}\left(x, x_{d}, g_{1}\right), s_{2}\left(\dot{\mathrm{x}}, \dot{\mathrm{x}}_{d}, g_{2}\right)\right)^{2}}{x}
$$

where $x \in] 0, \infty[$ and the position and velocity feedback are denoted by

$$
x_{d}(t)=x(t-d), \operatorname{and}_{d}(t)=\dot{\mathrm{x}}(t-d)
$$

respectively, the feedback voltage is given by

$$
\mathcal{V}\left(t, x(t), x_{d}(t), \dot{\mathrm{x}}(t), \dot{\mathrm{x}}_{d}(t)\right)=v_{0}+\delta v(t)+g_{1}\left(x(t)-x_{d}(t)\right)+g_{2}\left(\dot{\mathrm{x}}(t)-\dot{\mathrm{x}}_{d}(t)\right)
$$

with positive input voltage $V(t) \in C(\mathbb{R} / T \mathbb{Z})$ and $e \in \mathbb{R}^{+}, g_{1}, g_{2} \in \mathbb{R}, d \in\left[0, T\left[\right.\right.$. For the damping force $f_{D}(x, \dot{x})$ we consider two cases, the linear one, i.e., $f_{D}(x, \dot{x})=c \dot{x}, c \in \mathbb{R}^{+}$and the squeeze film type, i.e., $f_{D}(x, \dot{x})=\gamma \dot{x} / x^{3}, \gamma \in \mathbb{R}^{+}$. This approach can be applied to several MEMS devices used in the industry.
(Received September 12, 2023)

1192-34-31997
Matthias Dogbatsey*, The University of Alabama, Yun Kang, Arizona State University, Theophilus Kwofie, Arizona State University, Lucero Rodriguez, Arizona State University. Assessing the Impact of Intervention Programs on Gang Dynamics: A Mathematical Modeling Approach.
Gangs are involved in a wide range of crimes and are quickly spreading around the globe. The issue of street gangs is getting worse all across the world. Gangs continue to increase in their number, membership, and impact across the United States. This paper presents a model for understanding, analysing and simulating mathematical models for gang dynamics as far as incorporating some control measures. A deterministic mathematical model using a system of non-linear ordinary differential equations is employed to study the dynamics of gangs. We illustrated how the model can be used to investigate the role of reformed gang members in the spread of gangs in the population (using an incidence function which incorporates the influence and the fear factor). The model has a global asymptotically stable gang-free equilibrium whenever a certain criminological threshold, known as the effective reproduction number $\mathcal{R}_{\gamma}$, is less than unity. The simulations are carried out to assess the population-level impact of the widespread use of the intervention program, such as rehabilitation programs in the community. We hypothetically start the intervention program rate, $\gamma=0.0017$, and observe that increasing the intervention programs reduces the number of individuals that are moved into the prison community. We observe that an increase of $\gamma$ from 0.0017 to 0.0027 (the baseline value) results in an approximate decrease of the prison population by $11 \%$. It is worth mentioning that when we further increase the intervention programs above the baseline value ( 0.0027 ), there is approximately $1 \%$ additional decrease in the prison community. The global sensitivity analysis, using Latin Hypercube Sampling technique and partial rank correlation coefficients (PRCCs), is carried out primarily to find out the parameters with the most effect on the spread of gangs in the community. It is shown that the effective contact rate for the prison individuals and the treated (or recovered) individuals are the main drivers of gang dynamics.
(Received September 12, 2023)

1192-34-32071
Alexandria Volkening*, Purdue University. Forecasting U.S. Elections Using a Compartmental Republican-UndecidedDemocratic Model.
Election dynamics are a rich complex system, and forecasting U.S. elections is a high-stakes problem with many sources of
subjectivity and uncertainty. In this talk, we take a dynamical-systems perspective on election forecasting, with the goal of helping to shed light on the forecast process and raising questions for future work. By adapting a Susceptible-Infected-
Susceptible model to account for interactions between voters in different states, we show how to combine a compartmental approach with polling data to produce forecasts of senatorial, gubernatorial, and presidential elections at the state level. Our results for the last two decades of U.S. elections are largely in agreement with those of popular analysts. We use our modeling framework to determine how weighting polling data by polling organization affects our forecasts, and we explore how our forecast accuracy changes in time in the months leading up to each election.
(Received September 12, 2023)

1192-34-32193
Adan Baca, University of Arizona, Diego Raul Gonzalez*, University of La Verne, Alonso Ogueda Oliva, George Mason University, Padmanabhan Seshaiyer, George Mason University. Predictive Models in Mathematical and Computational Biology to Understand Drug Addiction.
Infectious diseases have had vital global health concerns that require a greater understanding of dynamics to develop a solution to control and lessen the disease. There has been a significant increase in drug overdoses following the COVID-19 Pandemic. In this work, we examined addiction to drugs as a disease and used differential equations to depict the nonlinear dynamics through a mathematical model. This model built on the SIR (Kermack-Mckendrick) epidemiological model for describing disease dynamics to create a Susceptible-Exposed-Infected-Recovered_\{H\}ospitalized (SEIRH) model. The population dynamics is modeled via a system of governing differential equations, with different parameters that represent transmission and recovery rates. In this talk, we will describe how to model and predict these parameters for given datasets We employed python coding to run our simulations, used synthetic data to visualize our model, and validated results using Physics Informed Neural Networks (PINNs) that accurately predicted our theoretical assumptions for important parameters. (Received September 12, 2023)

## 1192-34-32223

Maila B. Hallare, US Air Force Academy, USAFA CO USA, Beverly H West*, Cornell University. CODEE Discussion. Preliminary report.
CODEE is an inclusive Community of Ordinary Differential Equations Educators. We invite mathematicians and mathematics educators of differential equations to meet with our CODEE Editorial Board as we continue to build upon the significant work, begun 32 years ago, by its founders and supporters. Our online CODEE Journal, now in its 15 th year, has from the beginning, 2/3 of its downloads outside the United States -- it truly has global impact. This year we are attracting more international authors as well. The CODEE Journal welcomes presenters and other colleagues to submit manuscripts for evaluation and potential inclusion. (Received September 12, 2023)

1192-34-32227
Ryleigh Grace Henderson*, Converse University. Compartmental SIR Modeling of the Yersinia pestis Outbreak in Medieval London.
Plague is a disease caused by the bacteria Yersinia pestis. Plague has been the cause of many of history's most detrimental pandemics, including The Black Death. The Black Death traveled nearly the entire continent of Europe during the mid-1300s, destroying populations in its path. This outbreak has always been accredited to the strain known as bubonic plague; however in recent years researchers have begun to question whether another strain was involved. Pneuomic plague begins as bubonic that spreads and infects the respiratory system of its victims. It has a much higher death rate and has the ability to spread during winter months, making it a key contender of causing a pandemic with the severity of The Black Death. In my research, I will be constructing a compartmental SIR model using Python for three different hypothetical outbreak scenarios: bubonic, pnuemonic, and a combination. I will then be comparing these models to actual data from The Black Death and determining if a pnuemonic component is possible.
(Received September 12, 2023)

## 1192-34-32292

Andrew G Bennett*, Kansas State University. Introducing Calculus of Variations in a First Course in Differential Equations. While Calculus of Variations is usually considered an advanced topic, it can be effectively introduced in a first course in differential equations. This provides examples of non-linear equations in an applied context, and also better prepares engineers and others who may see topics in advanced courses in their disciplines. The presenter will go over how he introduces the topic with rigged problems where students can actually solve the Euler-Lagrange equations. (Received September 12, 2023)

## 1192-34-32341

Vinodh Kumar Chellamuthu, Utah Tech University, Matthew Gergley*, Utah Tech University. A Mathematical Model of HPA Dynamics and Impacts of Alcohol Consumption.
Stress is something all mammals experience. The regulation of stress and the differing amounts we might feel at any given moment of the day is an important role of the neuroendocrine system known as the hypothalamic-pituitary-adrenal (HPA) axis. But what is involved with the feeling of stress? When thinking about stress, people usually associate it with cortisol as what causes the feeling of stress, however despite the major role cortisol plays in stress response, there are other hormones that play key roles such as corticotropin-releasing hormone (CRH), adrenocorticotropin hormone (ACTH), and glucocorticoid receptor complexes (GR). These hormones as a "family" are what help maintain homeostatsis in the human body ensuring that we can respond quickly to stressful events and be able to return to a homeostatic state quickly as well. However, as one might suggest, there are certain mental disorders as classified by the WHO that can cause dysfunction in the HPA axis. We developed a mathematical model to study mental disorders such as depression and PTSD as their impacts on the HPA axis are well documented and modeled but also to explore the impacts of varying levels of chronic alcohol consumption and study the dynamics of the HPA axis when these conditions are imposed. Our model also includes circadian and ultradian rhythms. Our simulation results suggests that alcohol impacts the function of the HPA axis differently as well.

Yun Kang, Arizona State University, Theophilus Kwofie*, Arizona State University. Mathematical Modeling of Obesity Epidemic.
labstract Obesity in the United States of America has become an epidemic, which poses numerous challenges. To tackle such an epidemic, we develop SIRP components of nonlinear differential equations by using a disease modeling framework that incorporates genetics, social influence, and the environment, which serve as the driving forces behind obesity. We provide rigorous mathematical analysis as well as bifurcation diagrams. The analysis shows that genetics and the environment lead to the persistence of obesity. The simulations suggest that (1) genetics and the environment could be mechanisms generating backward bifurcation, and (2) the existence of Hopf bifurcation could be generated from the feedback loop. The additional numerical simulations suggest that there is a marked increase in the obesity population with varying genetic factor rates. More commonly, people who are obese have multiple genes that predispose them to gain excess weight. Keywords: Obesity Dynamics, Basic Reproduction Number, Stability Analysis, Sensitivity Analysis. \endabstract
(Received September 12, 2023)

## 1192-34-32694

Julie Blackwood, Williams College, Eli Goldwyn, University of Portland, Uyen Huynh, Bennington College, Kathryn J Montovan*, Bennington College. Understanding the opioid epidemic: social spread of illicit drug use. Preliminary report. Since the late 1990s, the United States has seen steadily increasing numbers of opioid-related deaths. Overprescription of opioids was originally the leading cause of widespread opioid use disorder but in the last decade, increases in opioid-related deaths have been primarily due to the availability of illicitly produced synthetic opioids such as fentanyl. Modeling is being employed in this field to understand the drivers of this public health crisis and to inform decision making but not much is currently known about the best way to model social spread of illicit drug use. In this work, we motivate and explore possible models of social spread of illicit drug use and utilize temporal data to fit model parameters and compare model performance across the suite of models.
(Received September 12, 2023)

1192-34-32702
Nathaniel Smith*, Miami University. Counting eigenvalues of Hamiltonian operators on quantum graphs with localized potentials. Preliminary report.
A quantum graph is a metric graph with differential operators attached. We are particularly interested in the eigenvalues of Hamiltonian operators defined on such structures. In this paper, we give a method for constructing maps between the traces of functions satisfying arbitrary separated boundary conditions (e.g. Dirichlet-to-Neumann). These methods can also be used to prove a relationship between several Evans functions and boundary maps, which can be used to count the eigenvalues of hard problems with localized potentials by instead counting the eigenvalues of smaller, easier problems.
(Received September 12, 2023)

## 1192-34-32726

James S Wolper*, Idaho State University. Model More by Doing Less.
Many students who take Calculus I anticipate careers in which modeling, especially modeling with differential equations, will be important, But, the Calculus curriculum does not include much modeling, and appears too full to model more. This talk presents ways to introduce models earlier. The first depends on using more estimation and qualitative techniques, starting with linearization based on estimating rather than calculating the slope of the tangent. Using the Mean Value Theorem allows development of useful qualitative solutions to differential equations. The second depends on developing and analyzing models throughout the course, including a module on differential equations between differentiation and integration. At this point students don't have the tools for exact solutions, but they do have tools for informative qualitative analysis, as well as Euler's Method. This, plus the utility of exact solutions, motivates the study of integration. Examples come from re-examining Newton's Law of Cooling and the Logistic Equation, among others.
(Received September 12, 2023)

## 1192-34-32960

Noemi Andor, H. Lee Moffitt Cancer Center and Research Institute, Jackson Cole, H. Lee Moffitt Cancer Center and Research Institute, Andriy Marusyk, H. Lee Moffitt Cancer Center and Research Institute, Daria Miroshnychenko, H. Lee Moffitt Cancer Center and Research Institute, Vural Tagal*, H. Lee Moffitt Cancer Center and Research Institute. $A$ mathematical model revealing chemotherapy sensitivity as a function of cellular DNA content in triple-negative breast cancers. Preliminary report.
The polyploid giant cancer cell (PGCC) state is a common response of cancer cells to various stressors including chemotherapy, irradiation, hypoxia and viral infection. Upon stress, PGCCs adopt an endoreplication in which the genome replicates, mitosis is omitted, and cells grow in size, causing drug resistance. How accessible endoreplication is to a cell, therefore, directly translates to its resistance to therapy. In this study, we hypothesized that endoreplication and PGCC state are more accessible to cancer cells that already have a higher DNA content prior to therapy. To test this hypothesis, we developed a comprehensive three-tier framework consisting of i) computational, ii) in-vitro and iii) in silico components. First, we designed a set of ordinary differential equations (ODEs) to mathematically model how cells enter and exit the PGCC state in response to a given stressor. We engineered how this in silico approach interacts with in vitro experiments into a broadly applicable software solution called CLONEID. The software uses computer vision to monitor phenotypic changes in cell size from standard bright-field microscopy and classify cells into PGCC and non-PGCC states. We used CLONEID to test various therapeutic agents for their ability to select for a stable near-tetraploid ( 4 N ) population in a set of near-diploid ( 2 N ) cell lines. Altogether, this framework enabled us to i) monitor the PGCC state experimentally in ii) two sets of matched isogenic cell lines with 2 N and 4 N DNA content to iii) model the successful entry and exit rates to and from the endoreplication state, respectively. As the first application of this framework, we tested the ability of our 2 N and 4 N TNBC lines (SUM159 and MDA-MB-231) to access the PGCC state upon treatment with 18 chemotherapy agents. Among those drugs, we observed that only
gemcitabine caused continued cell growth without cell division in both tetraploid cell lines whereas near-diploid parental lines were hypersensitive to the treatment. Consequently, tetraploid cancer cells continued to safely grow in the presence of gemcitabine. These PGCCs reentered the proliferative cell cycle and grew in cell number when treatment is terminated. We expect our findings and three-component framework strategy to help stratify the TNBC patient population by their response to gemcitabine.
(Received September 13, 2023)

## 1192-34-33039

Stefano Colafranceschi, Eastern Mennonite University, Roger Thelwell*, James Madison University. The Restricted Hill's Problem and the Power Series Method. Preliminary report.
In this talk, we present an approach to recast non-polynomial IVODEs (e.g. elementary and piecewise differentiable) into polynomial differential systems which exploit auxiliary variables, and provide several examples using the Power Series Method (PSM). We also discuss an a priori error bound for the solution to IVODES of polynomial form that is effectively symplectic. We apply this approach to the restricted Hill's Four Body problem of celestial mechanics, and present a comparison of PSM to familiar symplectic integrators. We offer some novel conservation equations for this system, and a general approach to generate others.
(Received September 13, 2023)

## 1192-34-33129

Xiaoxia Xie*, Idaho State University. Experiences of Course-based undergraduate research.
Course-based undergraduate research experiences represent a powerful training tool for preparing future researchers by providing inclusive research experiences for undergraduates. In the talk, I will share my experiences in two courses developed at Idaho State University to expose undergraduates to research in applied mathematics by integrating publicly available datasets. We will discuss the underlying pipeline of the data-driven research projects and assessment strategies for the course as well as the long-lasting effects on students who have taken these courses.
(Received September 13, 2023)

1192-34-33776
Rachel T Bertaud*, Presenter, Richard Finke, Colorado State University, Luke MacHale, Colorado State University, Patrick Shipman, Colorado State University. Mechanism-Enabled Population Balance Modeling For Platinum Nanoparticle Synthesis. Preliminary report.
Using the Law of Mass Action, ODEs were made to track the mass of atoms as they went through the synthesis process of Platinum nanoparticles. The goal of this was to find and confirm the mechanism of synthesis, as well as crafting tactics to synthesize less dispersed particles. A program was written in MATLAB to analyze the data and the programs output for particle sizing.
(Received September 25, 2023)

## 35 Partial differential equations

## 1192-35-25526

Maleafisha Joseph Pekwa Stephen Tladi*, African Scientific Institute. Well-Posedness and Long-Time Dynamics of the Rotating Boussinesq and Quasigeostrophic equations.
The author elucidates in a concrete way dynamical challenges concerning nonlinear analysis of dissipative $\beta$-plane ageostrophic flows. He employs the simplifying assumptions of the Boussinesq and hydrostatic approximations to the governing equations of barotropic and baroclinic viscous compressible fluid flows. As a result he obtains the dissipative $\beta$-plane ageostrophic equations which describe the motion of a viscous incompressible stratified fluid with Coriolis force. The author elucidates in a concrete way dynamical challenges concerning approximate inertial manifolds (AIMS), i.e., globally invariant, exponentially attracting, finite-dimensional smooth manifolds, for nonlinear dynamical systems on Hilbert spaces. The goal of this theory is to prove the basic theorem of approximation dynamics, wherein it is shown that there is a fundamental connection between the order of the approximating manifold and the well-posedness and long-time dynamics of the rotating Boussinesq and quasigeostrophic equations describing the motion of rotating stratified fluid flows. The author discusses recent progress in analytical and numerical methods covering form initial and boundary value problems, long-time dynamics and stability issues. He presents the most recent advances concerning the questions of global regularity of solutions to the 3D Navier-Stokes and Euler equations of incompressible fluids. Furthermore, he also presents recent global regularity (and finitetime blowup) results concerning the 3D quasigeostrophic and rotating Boussinesq equations describing the motion of rotating stratified fluid flows. The results narrow the gap between the mathematical analysis of the ocean primitive equations and the equations underlying numerical ocean models used in ocean and climate science.
(Received June 19, 2023)

## 1192-35-25584

Hongjie Dong, Brown University, Hyunwoo Kwon*, Brown University. Interior and boundary mixed norm derivative estimates for nonstationary Stokes equations. Preliminary report.
We obtain interior mixed norm derivative estimates for solutions to both equations for nonstationary Stokes equations in divergence and non-divergence form with variable viscosity coefficients that are merely measurable in time variable and have small mean oscillation in spatial variables in small cylinders. We also obtain boundary mixed norm Hessian estimates for solutions to equations in nondivergence form under the Lions boundary conditions (or slip boundary conditions).
(Received June 24, 2023)

Ruoyu P. T. Wang*, University College London. Singular damped waves on manifolds.
We will discuss a damped wave semigroup for damping exhibiting Hölder-type blowup near a hypersurface on a compact manifold. We will use vector field methods to prove a sharp energy decay result for singular damping on the torus, where the optimal rate of energy decay explicitly depends on the singularity of the damping. Dynamically, such fine rates result from that the transverse propagation of lower-order regularity fails to penetrate through the hypersurface where the damping potential is singular. This is a joint work with Perry Kleinhenz.
(Received June 30, 2023)

## 1192-35-25656

Ruoyu P. T. Wang*, University College London. Control and damping for internal and water waves.
We will discuss our recent results on the energy decay asymptotics for internal waves and water waves with localised damping, and the related control estimates. We show that under suitable dynamical assumptions (without the geometric control), we can establish polynomial energy decay for such waves using spectral and microlocal tools. These are joint works with Hans Christianson, Perry Kleinhenz and Jian Wang. (Received June 30, 2023)

## 1192-35-25673

Peter Pang*, National University of Singapore. A cancer invasion model involving chemotaxis and haptotaxis
We present a cancer invasion model involving chemotaxis and haptotaxis in two spatial dimensions. This model comprises two parabolic PDEs and an ODE, which describe the degradation of the extracellular matrix ECM by matrix degrading enzymes secreted by tumor cells, and incorporates self-remodeling of the ECM. We establish global existence and uniqueness of classical solutions in the high cell proliferation regime.
(Received July 03, 2023)

## 1192-35-25753

Thomas Wunderli*, The American University of Sharjah. An $L^{\infty}$ time bound of gradient measures for time flows in BV space with $L^{1}$ data.. Preliminary report.
We prove an $L^{\infty}$ bound for gradient measures of weak solutions to the time flow problem $u_{t}=\operatorname{div} \nabla \varphi(x, \nabla u)+\lambda\left(u-u_{0}\right)$
with a Neumann boundary condition on $(0, \infty) \times \Omega, \Omega \subset \mathbb{R}^{N}$ open and bounded, $u_{0} \in L^{\infty}(\Omega) \cap B V(\Omega)$, with the assumption that $\varphi(x, p)$ is convex of linear growth in $p$ with linear term for $|p|>\beta$. Importantly, we only assume $\varphi(\cdot, p) \in L^{1}(\Omega)$ for each $p \in \mathbb{R}^{N}$, with some additional structure conditions on the nonlinear part of $\varphi$.
(Received July 08, 2023)

## 1192-35-26547

Emily Kelting*, Drexel University. Ion-acoustic Wave Dynamics in a Two-Fluid Plasma. Preliminary report Plasma is a medium filled with free electrons and positive ions. Each particle acts as a conducting fluid with a single velocity and temperature when electromagnetic fields are present. This distinction between the roles played by electrons and ions is what we refer to as the two-fluid description of plasma. In this talk, we investigate the dynamics of these particles in both hot and cold plasma using a collisionless "Euler-Poisson" system. Employing analytical and computational techniques from differential equations, we show this system is governed by the dynamics of the Korteweg-de Vries (KdV) equation in the longwavelength limit and possesses solitary wave solutions.
(Received July 31, 2023)

## 1192-35-26985

Yuri D Latushkin*, University of Missouri. The Maslov index and the spectral stability problem for standing waves of the nonlinear Schroedinger equation on an interval.
We use the Maslov index to study the spectrum of a class of linear Hamiltonian differential operators. We provide a lower bound on the number of positive real eigenvalues, which includes a contribution to the Maslov index from a non-regular crossing. A close study of the eigenvalue curves, which represent the evolution of the eigenvalues as the domain is shrunk or expanded, yields formulas for their concavity at the non-regular crossing in terms of the corresponding Jordan chains. This enables the computation of the Maslov index at such a crossing via a homotopy argument. We apply our theory to study the spectral (in)stability of standing waves in the nonlinear Schrödinger equation on a compact interval. We derive stability results in the spirit of the Jones-Grillakis instability theorem and the Vakhitov-Kolokolov criterion, both originally formulated on the real line.
(Received August 08, 2023)

1192-35-27049
Mohammad Mahfouz Algharabli*, KFUPM, Salim A. Messaoudi, University of Sharjah. On the energy decay of a viscoelastic piezoelectric beam model with nonlinear internal forcing terms and a nonlinear feedback. In recent years, there is a large number of published works on piezoelectric materials. Piezoelectric materials represent a class of intelligent materials capable of generating electrical energy from mechanical deformations and vice versa, which originate from direct and indirect piezoelectric effects. In terms of applications, piezoelectric materials have become increasingly useful to society and realizing this, the industry has established strategies to deploy such materials in many diverse sectors of human activity, seeking to utilize the maximum of the mechanical energy produced in the machines operations, movements of the human body and environmental sources such as waves, winds and other. In this talk, we consider a piezoelectric beam model with nonlinear source terms and investigate the interaction between a viscoelastic damping and a nonlinear frictional damping. Under general assumptions on the relaxation function and the nonlinear feedback, we establish explicit formulae for the energy decay rates of this system and prove that the energy decays at a rate dictated by the weaker damping. Our results substantially improve some earlier related results in the literature.

Ahmed Keddi, University of Adrar, Salim A. Messaoudi*, University of Sharjah. On the well posedness and the stability of a thermoelastic Gurtin-PipkinTimoshenko system without the second spectrum.
In this work we consider a linear thermoelastic Gurtin-Pipkin Timoshenko system free of second spectrum. Precisely, we look into the following system lequation* \aligned \cases $\varrho_{-}\{1\} \phi_{-}\{\mathrm{tt}\}-\kappa\left(\phi_{-}\{\mathrm{x}\}+\psi\right)_{-}\{\mathrm{x}\}=0,-\varrho_{-}\{2\} \phi_{-}\{\mathrm{xtt}\}-\mathrm{b}$
$\psi_{-}\{\mathrm{xx}\}+\kappa\left(\phi_{-}\{\mathrm{x}\}+\psi\right)+\delta \theta_{-}\{\mathrm{x}\}=0, \mathrm{c} \theta_{-}\{\mathrm{t}\}-\backslash \operatorname{frac}\{1\} \beta \int_{-}\{0\} \wedge\{\infty\} \mathrm{g}(\mathrm{s}) \theta_{-}\{\mathrm{xx}\}(., \mathrm{t}-\mathrm{s}) \overline{\mathrm{d} s}+\delta \psi_{-}\{\mathrm{xt}\}=0, \backslash \mathrm{endcases} \backslash \mathrm{endaligned}$
lendequation* together with initial and boundary conditions. We give a detailed proof of the well-posedness, using the semigroup theory and with the help of some new and original operators. Then, we prove that the system is exponentially stable irrespective of the parameters. Our result generalizes an earlier result one in [1]. [1] A. Keddi, S. A. Messaoudi and M. Alahyane, On a thermoelastic Timoshenko system without the second spectrum: Existence and stability, Journal of Thermal Stresses, 2023. DOI: 10.1080/01495739.2023.2191662
(Received August 09, 2023)

## 1192-35-27259

Adel M. Al-Mahdi*, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia.. General decay for the Coleman-Gurtin thermal coupling with Timoshenko beam with variable exponents.
In this work, we consider a nonlinear Timoshenko system with Coleman-Gurtin's heat flux. Precisely, we consider \equation* \aligned $\backslash \operatorname{cases} \varrho_{-}\{1\} \phi_{-}\{\mathrm{tt}\}-\kappa\left(\phi_{-}\{\mathrm{x}\}+\psi\right)_{-}\{\mathrm{x}\}=0, \varrho_{-}\{2\} \psi_{-}\{\mathrm{tt}\}-\mathrm{b} \psi_{-}\{\mathrm{xx}\}+\kappa\left(\phi_{-}\{\mathrm{x}\}+\psi\right)+\gamma \theta_{-}\{\mathrm{x}\}+\beta\left|\psi_{-}\{\mathrm{t}\}\right| \wedge\{\mathrm{v}(\mathrm{x})-2\} \psi_{-}\{\mathrm{t}\}=0$, $\varrho_{-}\{3\} \theta_{-}\{\mathrm{t}\}+\tau \mathrm{q}_{-}\{\mathrm{x}\}+\gamma \psi_{-}\{\mathrm{xt}\}=\overline{0}$, \endcases $\backslash$ endaligned $\overline{\text { end }}$ endequan*where the heat flux $q$ given by the following ColemanGurtin's law \align* $\tau \mathrm{q}+(1-\alpha) \theta_{-}\{\mathrm{x}\}+\alpha \int_{-}\{0\}^{\wedge}\{\infty\} \Psi(\mathrm{s}) \theta_{-}\{\mathrm{x}\}(\mathrm{x}, \mathrm{t}-\mathrm{s}) \mathrm{ds}=0, \alpha \in(0,1)$. \endalign*Consequently, the Fourier and Gurtin-Pipkin's laws are special cases. We prove that the system is exponentially and polynomially stable without imposing the equality of the wave speeds unless the system is not fully damped by the thermoelasticity effect. In other words, the thermoelasticity is coupled only the second equation in the system with the coupling constant $\gamma$. By constructing a suitable Lyapunov functional, we establish exponential and polynomial decay rates. We noticed that sometimes the decay depends on the behavior of the thermal kernel and the variable exponent as well. Our results extend and improve some earlier results in the literature.
(Received August 13, 2023)

## 1192-35-27276

Mohammad M. Kafini*, KFUPM. Blow-up to a parabolic equation modeling epitaxial thin film growth with variable exponent. In this paper, we study a parabolic equation modeling epitaxial thin film growth that contains nonlinearity with variable exponents. By using the potential well method and properties of the spaces with variable exponents, we obtain the lower and the upper blow-up time estimation. Moreover, infinite time blow-up result is established.
(Received August 13, 2023)

## 1192-35-27645

Emer De Jesus Lopera*, Universidad Nacional de Colombia. Existence of solutions for a s-fractional p-Laplacian problem. Two interesting approaches.
In recent years, non-local operators as the $s$-fractional $p$-Laplacian have gained much attention among mathematicians, engineers and scientists, either for a mathematical interest as well as for its applications. In this talk we aim to show two approaches to prove the existence of at least one positive solution for a parameter, semipositone problem that involves a nonlocal operator. In the first one, we highlight the importance of the regularity of the solutions obtained through Mountain Pass Theorem, in order to prove its positivity. Such statement was published at the beginning of 2023. Few months later after the publication of this result a group of researchers proved, independently, a very similar result by other means. In fact they proved that their solution is positive by using degree theory. We want to analyze their result and compare it with ours. (Received August 17, 2023)

## 1192-35-27754

Peter A. Perry, University of Kentucky, Camille Schuetz*, University of Wisconsin-Platteville. A Scattering Result for the Fifth-order KP-II Equation.
In this talk we will discuss the proof of scattering for the fifth-order KP-II equation with small inital data in the Sobolev space $\dot{H}^{-1 / 2}$. To prove our result we extend ideas of Hadac, Herr, and Koch, who proved small-data scattering for the third-order KP-II equation, to the fifth-order case. This is joint work with Peter Perry. (Received September 04, 2023)

## 1192-35-27949

Charles Collot, CY Cergy Paris Université, Slim Ibrahim, University of Victoria, Canada, Quyuan Lin*, Clemson University. Stable Singularity Formation of the Inviscid Primitive Equations.
Large scale dynamics of the oceans and the atmosphere are governed by the primitive equations (PEs). While the global wellposedness of viscous PEs has been well established, the smooth solutions to the inviscid PEs (also known as the hydrostatic Euler equations) can form singularity in finite time. In this talk, I will briefly introduce the inviscid PEs, and discuss the stability of a certain type of blowup for smooth solutions.
(Received August 23, 2023)
differential equations with constraints.
Reduced basis (RB) methods have been used to study and reliably solve parametrized models involving PDEs including models of incompressible flow, where the constraint is the incompressibility constraint, and in optimal control, where the constraints themselves are PDEs. Discretizing these models results in large matrix systems of saddle-point form which must satisfy an infsup condition in order to be well-posed and solvable. Several approaches to RB modeling of these problems result in saddlepoint RB matrix systems and ensure inf-sup stability of the reduced system by augmenting the RB spaces and associated matrices. We propose a new approach to constructing the RB matrix, a stacked RB matrix, that does not result in a saddlepoint reduced system and eliminates the need for augmentation. The stacked RB matrix is of smaller order and requires less computational work to use in solving the online problem. We present numerical results to compare this approach to standard approaches of implementing RB methods and prove the efficiency of the proposed approach for solving the parametrized diffusion control problem and the parametrized Stokes equations.
(Received August 25, 2023)

1192-35-28072
Govanni Granados*, Purdue University. Asymptotic analysis applied to small volume inverse shape problems.
We consider two inverse shape problems coming from diffuse optical tomography and the Helmholtz equation. For both problems, we assume that there are small volume subregions that we wish to recover using the measured Cauchy data. We will derive an asymptotic expansion involving their respective fields. Using the asymptotic expansion we derive a MUSIC-type algorithm for the Reciprocity Gap Functional, which we prove can recover the subregion(s) with a finite amount of Cauchy data. Numerical examples will be presented for both problems in two dimensions in the unit circle.
(Received August 25, 2023)

## 1192-35-28186

$\mathbf{L i} \mathbf{L i}$, University of California, Irvine. An inverse problem for the fractionally damped wave equation.
I will start with the classical and fractional Calderon problems. Then I will focus on an inverse problem for a Westervelt type nonlinear wave equation with fractional damping. Our goal is to uniquely determine the smooth coefficient of the nonlinearity based on the knowledge of the source-to-solution map and a priori knowledge of the coefficient, in an arbitrarily small subset of the domain. Our approach relies on a second order linearization as well as the unique continuation property of the spectral fractional Laplacian. This is a joint work with Dr. Yang Zhang.
(Received August 27, 2023)

1192-35-28202
Zachary Lee*, The University of Texas at Austin, Xueying Yu, Oregon State University. On Uniqueness Properties of Solutions of the Generalized Fourth-Order Schrödinger Equations.
In this paper, we study uniqueness properties of solutions to the generalized fourth-order Schrödinger equations in any dimension $d$ of the following forms,

$$
i \partial_{t} u+\sum_{i=1}^{d} \partial_{x_{j}}^{4} u=V(t, x) u, a n d i \partial_{t} u+\sum_{j=1}^{d} \partial_{x_{j}}^{4} u+F(u, \bar{u})=0
$$

We show that a linear solution $u$ with fast enough decay in certain Sobolev spaces at two different times has to be trivial. Consequently, if the difference between two nonlinear solutions $u_{1}$ and $u_{2}$ decays sufficiently fast at two different times, it implies that $u_{1} \equiv u_{2}$.
(Received August 28, 2023)

## 1192-35-28306

Yekaterina Epshteyn*, University of Utah. Grain Boundaries in Polycrystals: Modeling, Analysis and Simulation. Many technologically useful materials are polycrystals composed of small monocrystalline grains that are separated by grain boundaries of crystallites with different lattice orientations. One of the central problems in materials science is to design technologies capable of producing an arrangement of grains that delivers a desired set of material properties. A method by which the grain structure can be engineered in polycrystalline materials is through grain growth (coarsening) of a starting structure. Grain growth in polycrystals is a very complex multiscale multiphysics process. It can be regarded as the anisotropic evolution of a large cellular network and can be described by a set of deterministic local evolution laws for the growth of an individual grain combined with stochastic models for the interaction between them. In this talk, we will present new perspectives on modeling, simulation, and analysis of the evolution of the grain boundary network in polycrystalline materials. Relevant recent experiments will be discussed as well.
(Received August 29, 2023)

1192-35-28390
M. Burak Erdogan, University of Illinois at Urbana Champaign, Michael J. Goldberg, University of Cincinnati, William R. Green*, Rose-Hulman Institute of Technology. Dispersive estimates for higher order Schrodinger operators with scalingcritical potentials.
We prove a family of dispersive estimates for the higher order Schrödinger equation $i u_{t}=(-\Delta)^{m} u+V u$ for $m \in \mathbb{N}$ with $m>1$ and $\$ 2 m$
(Received August 30, 2023)

1192-35-28404
Shohreh Gholizadeh, Wake Forest University, Stephen B. Robinson*, Wake Forest University. On the convergence of the Variational Iteration Method as applied to Klein-Gordon and Goursat problems. Preliminary report.
This is a preliminary report on the application of the Variational Iteration Method (VIM) to a classic Klein-Gordon problem
and the Goursat problem

$$
c u_{x t}=f(x, t, u), x>0, t>t_{0} u\left(x, t_{0}\right)=g(x), x>0, u(0, t)=h(t), t>t_{0}, g(0)=h\left(t_{0}\right) . \backslash \text { endarray }
$$

Both are motivated by black hole models. There is a growing literature on such applications of VIM but limited results proving convergence. For the linear Klein-Gordon problem we prove a convergence theorem for several choices of $V$ and for certain choices of initial data. For the nonlinear Goursat problem we show that VIM reduces to a classic iteration procedure with a standard proof of convergence.
(Received August 30, 2023)

## 1192-35-28447

Manuchehr Aminian*, Cal Poly Pomona. Discovering extremal domains via shape optimization for passive tracers. Preliminary report.
Work in passive tracers investigates how properties of a tracer distribution depend on boundary conditions and properties of the underlying fluid flow. We apply shape optimization to discover extremal domains for Poiseuille flow informed by analytic predictions of spatial moments - such as mean, effective diffusivity, skewness - derived in prior work. With this combination of asymptotic formulas and numerical studies, we find and report on surprising nonlinear behavior depending on shape parameters.
(Received August 30, 2023)

## 1192-35-28462

Connor Mooney, University of California Irvine, Yang Yang*, Johns Hopkins University. The anisotropic Bernstein problem. The Bernstein problem asks whether entire minimal graphs in $\mathbb{R}^{n+1}$ are necessarily hyperplanes. It is known through spectacular work of Bernstein, Fleming, De Giorgi, Almgren, Simons, and Bombieri-De Giorgi-Giusti that the answer is positive if and only if $n<8$. The anisotropic Bernstein problem asks the same question about minimizers of parametric elliptic functionals, which are natural generalizations of the area functional that both arise in many applications, and offer important technical challenges. We will discuss the recent solution of this problem (the answer is positive if and only if $n<4$ ). This is joint work with C. Mooney.
(Received August 30, 2023)

## 1192-35-28501

Pablo Raúl Stinga*, Iowa State University. A PDE approach to the construction of surfaces of minimum mean curvature variation.
The construction of fair surfaces in computer-aided design and manufacturing involves the finding of a minimizer of the quadratic tangential variation of the mean curvature of a surface. Despite its importance in applied problems, solutions have only been found through numerical schemes. We will present two different analytical ways of constructing minimizers. These are the first rigorous results on the existence of surfaces of minimum mean curvature variation. This is joint work with Luis A. Caffarelli (UT Austin) and Hern\'an Vivas (U. Nacional de Mar del Plata, Argentina).
(Received August 31, 2023)

## 1192-35-28502

Pablo Raúl Stinga*, Iowa State University. The De Giorgi theorem for boundary nonlocal equations.
We introduce the fractional normal derivative of a function on the boundary of a bounded domain as the fractional power of the Dirichlet-to-Neumann map for its corresponding harmonic extension to the interior. We develop a De Giorgi type iteration and prove local boundedness and $\mathrm{H} \backslash$ "older regularity of solutions. This is joint work with Luis A. Caffarelli (UT Austin) and Mitchell Haeuser (Iowa State University).
(Received August 31, 2023)

## 1192-35-28515

Ruimeng Hu, University of California, Santa Barbara, Quyuan Lin*, Clemson University. Pathwise Solutions for the Stochastic Hydrostatic Euler Equations.
The hydrostatic Euler equations, also known as the inviscid primitive equations, are utilized to describe the motion of inviscid fluid flow in a thin domain, such as the ocean and atmosphere on a planetary scale. In this talk, I will present some recent progress on the stochastic version of this model. Specifically, I will demonstrate how the local Rayleigh condition can be used to address the issue of ill-posedness, leading to the establishment of the existence and uniqueness of pathwise solutions. (Received August 31, 2023)

## 1192-35-28592

Christian Parkinson*, University of Arizona, Weinan Wang, University of Oklahoma. Analysis of a Reaction-Diffusion SIR Epidemic Model with Noncompliant Behavior.
Recent work from public health experts suggests that incorporating human behavior is crucial in faithfully modeling an epidemic. We present a reaction-diffusion partial differential equation SIR-type population model for an epidemic including behavioral concerns. In our model, the disease spreads via mass action, as is customary in compartmental models. However, drawing from social contagion theory, we assume that as the disease spreads and prevention measures are enacted, noncompliance with prevention measures also spreads throughout the population. We prove global existence of classical solutions of our model, and then perform $R_{0}$-type analysis and determine asymptotic behavior of the model in different parameter regimes. Finally, we simulate the model and discuss the new facets which distinguish our model from basic SIR-
type models.
(Received August 31, 2023)

## 1192-35-28606

Alfio Borzi, University of Wuerzburg, Germany, Souvik Roy*, The University of Texas at Arlington. Numerical approximation of kinetic Fokker-Planck equations.
In this work, we present the numerical approximation of a general kinetic Fokker-Planck (FP) equation in multi-dimensions with specular reflection boundary conditions. Kinetic FP equations finds applications in numerous fields, like plasma physics, aerodynamics, bio-chemical reactions, economics, electric circuits, neurophysiology, and social dynamics. This motivates the need to build an efficient solver for numerically solving these equations. Another important motivation for our numerical investigation of the general kinetic FP equations is that it includes the case of adjoint FP models arising in optimal control problems with ensemble cost functionals. We present a new numerical method, based on a novel combination of finite volume discretization scheme and a Magnus integrator-based splitting scheme, to numerically solve the kinetic FP equations. A comprehensive numerical analysis is presented that demonstrates second order convergence in the $L^{1}$ norm, which is validated with numerical test cases. This is a joint work with Alfio Borzí. (Received September 01, 2023)

## 1192-35-28613

Ugur G. Abdulla, Okinawa Institute of Science and Technology, Daniel Paul Tietz*, Okinawa Institute of Science and Technology. Wiener's Criterion at $\infty$ for Divergence Form Parabolic Operators with $C^{1}$-Dini Continuous Coefficients. For the Laplace operator on bounded domains, an essential question is if the solution of the Dirichlet problem converges to the given boundary values. This question can be answered with the well-known Wiener criterion [Wiener, 1924]. In the case of unbounded domains, an analogous Wiener criterion at $\infty$ gives us a necessary and sufficient condition for the existence of a unique bounded solution of the corresponding Dirichlet problem [Abdulla, 2007]. We will study a general divergence form parabolic operator with $C^{1}$-Dini continuous coefficients on an unbounded domain with the goal of establishing a similar result for this bigger class of operators. To prove a Wiener criterion at $\infty$ we will use suitable estimates for the fundamental solutions and Harnack type estimates. The result is based on [Fabes, Garofalo, Lanconelli, 1989] where the same type of operator for bounded domains and [Abdulla, 2008] where the heat operator for unbounded domains were studied. (Received September 01, 2023)

## 1192-35-28621

Xin Liu*, Texas A\&M University. A revisit to the rigorous justification of the quasi-geostrophic approximation
The quasi-geostrophic approximation is used to model large-scale atmospheric/oceanic flows close to the geostrophic balance, i.e., the Coriolis force, the pressure, and the gravity are in balance. Such an approximation for inviscid flows has been investigated in the case without boundary or without oscillating fast waves. In this talk, I will (1) point out the possible boundary layer when fast rotation is not present, and (2) show that with fast rotation, there is no boundary layer. In particular, we rigorously justify the QG approximation with both boundary and oscillatory fast waves. Our result demonstrates the stabilizing effect of rotation by suppressing the boundary layer.
(Received September 01, 2023)

## 1192-35-28629

Ghder S Aburamyah, Morgan State University, Guoping Zhang*, Morgan State University. Weighted $l^{p}$ global attractor of the initial value problem to $D N L S$ equation with complex potential. Preliminary report.
In this talk, we first present the improved results on the weighted $l^{p}$ global solution of the the initial value problem (IVP) to the discrete nonlinear Schr $\backslash$ "odinger (DNLS) equation in the case $\$ 1$
(Received September 01, 2023)

## 1192-35-28666

Yu Deng*, University of Southerncalifornia. Recent progress on probabilistic theory of PDEs.
We review some recent progress in the probabilistic theory of PDEs, i.e. PDEs with random data. New methods have been introduced in the last few years, such as the paracontrolled calculus, method of random averaging operators and random tensors. By combining these methods with techniques from harmonic analysis, number theory and combinatorics, we have been able to resolve several open problems in the field, including invariance of $\Phi_{2}^{p}$ and $\Phi_{3}^{4}$ measures under Schrödinger and wave dynamics. We also discuss some future directions. This is joint work with Bjoern Bringmann, Andrea R. Nahmod and Haitian Yue.
(Received September 01, 2023)

Claudia Maria Schmidt*, California State University. Weak solutions. Preliminary report.
If solutions for a Partial Differential Equation such as Laplace's Equation are hard to come up with because they might involve not differentiable function, differentiable replacement solutions, called weak solutions, are multiplied with an auxiliary (test) function in order to obtain via Integration by Parts a quasi-derivative to solve the equation. Spaces of weak solutions include ptimes differentiable and k-power integrable functions and constitute Sobolev spaces. A dance piece illustrates the motivation and definition of weak solutions and the properties of the Sobolev spaces and speculates, given the existence of similar constructions in other spaces (for example Moore-Penrose pseudo-inverses in Matrix spaces), to what Category they belong. (Received September 02, 2023)

## 1192-35-28782

Jingni Xiao*, Drexel University. Free boundary problems from the perspective of nonscattering phenomenon.
Nonscattering occurs when a medium is probed by an incident wave and no scattered waves can be observed in the exterior of the medium. I will present recent work concerning the regularity of nonscattering media. I will introduce how nonscattering is related to free boundary problems (FBP), how techniques in FBP can be applied in nonscattering, and what open questions nonscattering brings into the field of FBP.
(Received September 03, 2023)

## 1192-35-28796

Brandon Philip Ashley*, Southern Oregon University. Intransitive Symmetry Groups of 2-Plane Distributions and Darboux Integrable f-Gordon Equations.
We present a new, transformation group theoretic approach to the classification of Darboux integrable equations of the form $u_{x y}=f\left(x, y, u, u_{x}, u_{y}\right)$. Such equations are commonly referred to as $f$-Gordon equations, generalized wave map equations, or equations of Liouville type. The main result of our approach asserts that a complete list of all $f$-Gordon equations which are Darboux integrable at order three can be determined from a complete list of all 2-plane distributions in five dimensions which admit intransitive, 5 -dimensional symmetry groups. Through this correspondence, we have uncovered a new class of $f$-Gordon equations which we highlight here. The addition of this new class completes the classification of $f$-Gordon equations, Darboux integrable at order three. This talk is based upon joint work with Ian M. Anderson, Utah State University. (Received September 03, 2023)

1192-35-28859
Nsoki Mavinga, Swarthmore College, Timothy Ira Myers*, Howard University, M. N. Nkashama, University of Alabama at Birmingham. A Constructive Existence Result for a Nonlinear Elliptic PDE with a Nonlinear Boundary Condition : Monotone Case.
Let $\Omega \subset \mathbb{R}^{N}$ where $N \geq 2$ be a bounded domain. We will consider the nonlinear elliptic pde with a nonlinear boundary condition

$$
\begin{gathered}
-\Delta u+u=f(x, u) i n \Omega \\
\frac{\partial}{\partial} \eta=g(x, u) o n \partial \Omega
\end{gathered}
$$

where $f: \Omega \times \mathbb{R} \rightarrow \mathbb{R}, g: \partial \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ are Carathéodory functions that satisfy a monotonicity property. We will use a monotone iteration technique to construct a weak minimal solution $u_{*}$ and a weak maximal solution $u^{*}$ between a given pair of weak sub- and supersolutions $u$ and $\bar{u}$ satisfying $u \leq \bar{u}$; so that $u \leq u_{*} \leq u^{*} \leq \bar{u}$. Furthermore, this construction will show that if $u$ is any weak solution satisfying $\underline{u} \leq u \leq \bar{u}$, then in fact $u_{*} \leq u \leq u^{*}$.
(Received September 04, 2023)

## 1192-35-28900

Sayonita Ghosh Hajra, California State University Sacramento, Santosh Kandel*, California State University Sacramento, Shiva Pudasaini, Technical University of Munich. Solutions to a Two-phase Mass Flow Model with Generalized Drag. Preliminary report.
In this talk we discuss a study of the generalized two-phase mass flow model, describing a two-phase rapid gravity mass flow as a mixture of solid particles and viscous fluid down a slope, developed by Pudasaini in 2012. We consider the model that includes the generalized drag-one of the dominant and essential components of the interfacial momentum exchange in mixture mass flows. We analyze the model in two special cases: model with pressure and model without pressure gradients. Using the Lie symmetry method, we construct analytical, and numerical solutions to these models. We use these solutions to study the role of generalized drag in two-phase mass flow models. We also analyze the influence of the pressure gradients. (Received September 04, 2023)

## 1192-35-28903

Asma Alghamdi*, The University of Texas at Arlington, Souvik Roy, The University of Texas at Arlington. Fokker-Planck stochastic modeling and simulation of the signaling pathways in esophageal cancer.
In this work, we present a new approach to model aberrant signaling pathways in esophageal cancer. Our approach includes modeling the dynamics of the signaling pathways in a esophageal cancer cell with an Itó stochastic process, whose dynamics is described by the evolution of the associated probability density function through the degenerate Fokker-Planck partial differential equation. To validate the model, we perform a parameter estimation method, where the unknown parameters correspond to each patient's unique tumor features. The parameter estimation method is done through the formulation of a constrained optimization problem, which is numerically solved using the non-linear conjugate gradient (NCG) method. Results of numerical experiments demonstrate the robustness and accuracy of this modeling framework. This is joint work with my advisor Souvik Roy.
(Received September 04, 2023)

## 1192-35-28912

Chutian Ma*, Johns Hopkins University. On the Radial Defocusing Conformal Wave Equations on Hyperbolic Space $\mathbb{H}^{d}$ with Rough Initial Data.
Defocusing nonlinear wave equation with nonlinearity $u^{p}$ is conjectured to be globally wellposed and scatters for initial data in critical Sobolev space. In Euclidean space, a variety of such results were established, relying heavily on Fourier transform. Some of the well-known methods include the Fourier truncation method and I-method. In the hyperbolic space, solutions to the wave equation enjoys stronger dispersive properties due to the geometry of the ambient space. In turn, we are able to rely more heavily on Strichartz and Morawetz estimates than the Fourier transform. Since the latter is not as convenient to use on
manifolds as in Euclidean space. In this talk, we present the global wellposedness and scattering result of defocusing conformal wave equation with radial data on the hyperbolic space $\mathbb{H}^{d}$.
(Received September 04, 2023)

## 1192-35-28917

Jason Carl Murphy*, University of Oregon. The scattering map determines the nonlinearity.
We review some recent work on the problem of recovering an unknown nonlinearity from the (small-data) scattering behavior of solutions, including the case of modified scattering. The talk will discuss joint work with R. Killip and M. Visan, as well as G. Chen.
(Received September 04, 2023)

## 1192-35-28942

Markus De Medeiros, New York University, Daniel Gomez*, University of Pennsylvania, Sean D Lawley, University of Utah, Juncheng Wei, University of British Columbia, Wen Yang, Wuhan Institute of Physics and Mathematics. Asymptotics of Singularly Perturbed Problems with Lévy Flights.
What do reaction-diffusion system where one species has an asymptotically small diffusivity have in common with the problem of finding the mean time for a Brownian particle to first hit a very small target? For a variety of reaction-kinetics the former is a singularly perturbed problem in $N \geq 1$ dimensional domains, while the latter is a singularly perturbed problem only in $N \geq 2$ dimensional domains. In this talk we consider these problems in a one-dimensional domain with classical diffusion replaced by Lévy flights. Due to the discontinuity of Lévy flights, the first-hitting-time problem becomes singularly perturbed in a one-dimensional domain. Moreover, depending on the fractional order of the Lévy flight, the asymptotic analysis of both of these problems share many similarities with one-, two-, and three-dimensional singularly perturbed problems with Brownian diffusion. We will highlight these similarities and also summarize some of the consequences of Lévy flights on first-hittingtimes to small targets, as well as on "spike" solutions to singularly perturbed fractional reaction-diffusion systems. (Received September 04, 2023)

## 1192-35-28967

Baozhi Chu*, rutgers university. Optimal Liouville theorems for fully nonlinear conformally invariant equations For dimension $n \geq 3$, there is a classic result due to Liouville:

$$
\Delta u=0, u>0, \mathbb{R}^{n} \Rightarrow u \equiv \text { constant } .
$$

There is also a renowned Liouville-type theorem for Yamabe equation due to Caffarelli-Gidas-Spruck (under additional conditions, by Obata and Gidas-Ni-Nirenberg)

$$
-\Delta u=u^{\frac{n+2}{n}-2}, u>0, \mathbb{R}^{n} \Rightarrow u \equiv\left(\frac{a}{1}+b^{2}|x-x|^{2}\right)^{\frac{n-2}{2}}
$$

In a joint work with Yanyan Li and Zongyuan Li , we generalize both Liouville-type theorems above along the direction of conformal invariance to the fully nonlinear version. And our both generalized theorems are optimal by realizing the same borderline. We expect they will be useful in treating scalar curvature changing-sign problems in conformal geometry. If time permits, I will also discuss some applications of our Liouville Theorems, including local gradient estimates, Harnack inequalities, etc.
(Received September 05, 2023)

## 1192-35-29078

Diego Ricciotti*, California State University, Sacramento. Convergence of natural p-means to p-harmonic functions. In recent years, there has been a large interest in the study of different types of mean value properties and averages that can characterize $p$-harmonic functions. We consider natural $p$-means based on $L^{p}$ averages that can be used to provide semidiscrete approximations to solutions of the $p$-Laplace equation. We establish existence and uniqueness of such natural approximation schemes, as well as their convergence to $p$-harmonic functions on domains satisfying boundary conditions that include and generalize the exterior corkscrew condition.
(Received September 05, 2023)

## 1192-35-29093

Hongjie Dong, Brown University, Yanyan Li, Rutgers University, Zhuolun Yang*, Rutgers University. Optimal gradient estimates for the insulated conductivity problem.
In this talk, we will describe an elliptic PDE that models electric conduction, and the electric field concentration phenomenon between closely spaced inclusions of high contrast. In joint work with Hongjie Dong (Brown University) and Yanyan Li (Rutgers University), we considered the case when inclusions are insulators, and obtained optimal gradient estimates in terms of the distance between inclusions. This solved one of the major open problems in this area
(Received September 05, 2023)

1192-35-29098
Kazuo Yamazaki*, University of Nebraska, Lincoln. Recent developments on uniqueness and non-uniqueness of stochastic PDEs in fluid mechanics.
We review recent developments concerning the uniqueness and non-uniqueness of the solutions to the stochastic PDEs in fluid mechanics. Examples include non-uniqueness in law of the two-dimensional surface quasi-geostrophic equations forced by white-in-time noise, white-in-space noise, and space-time white noise, as well as the path-wise uniqueness of the twodimensional magnetohydrodynamics system forced by space-time white noise despite the absence of explicit knowledge of its
invariant measure.
(Received September 05, 2023)

1192-35-29116
David Halpern, University of Alabama, Awa Traore*, University of Alabama. Dynamics of a Multi-Fluid System. Preliminary report.
The dynamics of a multi-layer fluid system inside a cylinder with a passive air core is studied. The multi-layer fluid consists of immiscible fluids with different viscosities inside a cylinder with a passive core. Theoretical studies are conducted to analyze the linear stability of the Navier-Stokes equations that govern fluid motion. The impact of viscosity, surface tension and base flow on the linear growth of the interfacial disturbances is investigated, when the disturbances are small. First, a long-wave asymptotic model is used to study the dynamics of a three-layer system with two interfaces between the layers and one free surface between the innermost fluid and the passive air core. Linear stability analysis of the model yields three modes: two "interfacial modes" and one "free-surface mode." The growth rate of the disturbances is computed by solving an eigenvalue problem, which enables me to determine where in parameter space the system is unstable to small amplitude disturbances. Nonlinear evolution equations have also been derived and are used to investigate the dynamics of a multi-layer fluid system when the disturbances are not small. These equations are solved numerically using the method of lines. The purpose here is to determine whether small disturbances grow without bound or they saturate as finite amplitude waves. Both linear and nonlinear analyses will be used to examine the stability of this system.
(Received September 05, 2023)

## 1192-35-29133

Jungang Li, University of Science and Technology of China, Guozhen Lu, University of Connecticut, Jianxiong Wang*,
University of Connecticut. Symmetry of solutions of higher order conformal equations on hyperbolic spaces.
We study some qualitative properties of positive solutions of nonlinear elliptic equations related to GJMS operator in hyperbolic spaces like symmetry, monotonicity etc. Our main techniques include Helgason-Fourier analysis on $\mathbb{H}^{n}$, and the method of moving planes/spheres to the corresponding integral equations on $\mathbb{H}^{n}$.
(Received September 05, 2023)

## 1192-35-29136

Zachary Bradshaw*, University of Arkansas, Patrick Phelps, Temple University. Error growth for Navier-Stokes flows.
The prospect of non-uniqueness in physical classes of Navier-Stokes flows raises the question: How bad can it be? This can be addressed by developing bounds on the error between two flows. If the bounds are strong, then the separation rate is small and one flow is predictable from another. Weaker bounds are, on the other hand, more concerning from a modeling perspective. This talk explores recent progress toward bounding separation rates of Navier-Stokes flows.
(Received September 05, 2023)

## 1192-35-29215

David Halpern, University of Alabama, Awa Traore*, University of Alabama. Dynamics of a Multi-Fluid System: Stability Analysis. Preliminary report.
The dynamics of a multi-layer fluid system inside a cylinder with a passive air core is studied. The multi-layer fluid consists of immiscible fluids with different viscosities inside a cylinder with a passive core. Theoretical studies are conducted to analyze the linear stability of the Navier-Stokes equations that govern fluid motion. The impact of viscosity, surface tension and base flow on the linear growth of the interfacial disturbances is investigated, when the disturbances are small. First, a long-wave asymptotic model is used to study the dynamics of a three-layer system with two interfaces between the layers and one free surface between the innermost fluid and the passive air core. Linear stability analysis of the model yields three modes: two "interfacial modes" and one "free-surface mode." The growth rate of the disturbances is computed by solving an eigenvalue problem, which enables me to determine where in parameter space the system is unstable to small amplitude disturbances. Nonlinear evolution equations have also been derived and are used to investigate the dynamics of a multi-layer fluid system when the disturbances are not small. These equations are solved numerically using the method of lines. The purpose here is to determine whether small disturbances grow without bound or they saturate as finite amplitude waves. Both linear and nonlinear analyses will be used to examine the stability of this system.
(Received September 05, 2023)

## 1192-35-29223

Diana Milena Sanchez Monsalve*, Universidad Nacional de Colombia-Manizales. Aplication of a generalized Pohozaev Identity to prove existence and positivity of solutions to a Dirichlet problem..
The aim of this talk is to present some existence results for a parametric quasilinear problem that involves the phi-Laplacian operator. In order to achieve this goal we use a generalized Pohozaev Identity. In fact, we proved the existence of at least one positive solution when the parameter is positive and small.
(Received September 06, 2023)

## 1192-35-29225

Roberto Triggiani, University of Memphis, Xiang Wan*, Loyola University Chicago. Luenberger Compensator Theory for Heat-Structure Interaction via Boundary/Interface Feedback Controls.
In this talk, we will introduce some recent development of a continuous theory of the Luenberger dynamic compensator (or state estimator or state observer), with applications on a class of heat-structure interaction PDE-models, with structure subject to high Kelvin-Voigt damping, and feedback control exercised either at the interface between the two media or else at the external boundary of the physical domain in three different settings. Three different cases of controls will be discussed: (i) Neumann at the interface; (ii) Dirichlet at the interface; (iii) Dirichlet at the external boundary. Our goal is to reveal how delicate PDE-energy estimates dictate how to define the interface/boundary feedback control in each of the three cases.

Daozhi Han, The State University of New York at Buffalo, Sayantan Sarkar*, State University of New York at Buffalo. A quasi-incompressible Cahn-Hilliard-Darcy system for two-phase flows in porous media.
The Muskat problem describes two-phase flows in porous media, and could be ill-posed. In this talk we introduce a quasiincompressible Cahn-Hilliard-Darcy system with the Flory-Huggins singular potential as a relaxation of the Muskat problem. We establish global existence of weak solutions. We also present a second-order unconditionally stable and bound-preserving scheme for solving the model.
(Received September 06, 2023)

## 1192-35-29342

Roza Aceska, Ball State University, Yeon Hyang Kim*, Central Michigan University, Hiruni Pallage, Central Michigan University. Time-Variant System Approximation for Partial Differential Equations. Preliminary report.
In this talk, we present a model for a time-variant system approximation. We use later time measurements taken at a strategically selected location of the sensing device to determine the unknown initial conditions. By combining spatial samples of different approximations, our framework reconstructs the solution with precision. Furthermore, our model deals with various initial value problems, assuming that the initial function belongs to a specific class of functions.
(Received September 06, 2023)

## 1192-35-29406

V. R. Martinez*, CUNY Hunter College. On well-posedness at critical regularity of mild regularizations of active scalar equations.
We study dissipative perturbations of the 2D generalized surface quasi-geostrophic (gSQG) equations. This family contains the 2D Euler equations in vorticity form at one endpoint, an active scalar equation whose constitutive law relates the velocity with the scalar with a loss of one derivative, and contains the SQG equation at its midpoint. Recent work of Bourgain \& Li, Elgindi \&Masmoudi, Cordoba \& Zoroa-Martinez, and Jeong \& Kim have established ill-posedness of this family at critical regularity This work considers a mild perturbation of the gSQG equation which recovers well-posedness, but instantaneously confers a mild degree of regularity. This work is in contradistinction with strongly dissipative perturbations, which instantaneously confer Gevrey regularity and recover well-posedness at critical regularity (Jolly, Kumar, M 2021), and inviscid regularization, which do not regularize solutions, but nevertheless recover local well- posedness at critical regularity (Chae, Wu 2010). We show that in this intermediate regime that one may recover local well-posedness at borderline Sobolev regularity, as well as a global existence theory at the 2D Euler endpoint. Moreover, we provide a general existence theory for an entire class of such perturbations that is effectively sharp in light of the recent ill-posedness results. This is joint work with A. Kumar (Florida State University).
(Received September 06, 2023)

## 1192-35-29458

Yang Zhang*, University of Washington. Inverse problems arising in nonlinear acoustic imaging.
Inverse problems of recovering the metric and nonlinear terms originated in the work by Kurylev, Lassas, and Uhlmann for a semi-linear wave equation $\square_{g} u+a u^{2}=f$ in a manifold without boundary. The idea is to use the multi-fold linearization and the nonlinear interactions of distorted planes waves to produce point-source-like singularities in an observable set. In this talk, I will discuss the joint work with Gunther Uhlmann, which considers the recovery of the nonlinearity for a quasilinear wave equation arising in nonlinear acoustic imaging. The main difficulty that we need to handle here is caused by the presence of the boundary. We show the Dirichlet-to-Neumann map determines the nonlinearity. Then I will talk about several other inverse problems arising in nonlinear acoustic imaging.
(Received September 06, 2023)

1192-35-29483
Chen-Chih Lai*, Columbia University, Michael I. Weinstein, Columbia University. Thermal relaxation toward equilibrium and periodically pulsating gas bubbles in an incompressible liquid.
We study the thermal decay of bubble oscillation in an incompressible liquid. We discuss two models, both systems of nonlinear PDEs with a moving boundary: the complete mathematical formulation (full model) and an approximate model, first proposed by A. Prosperetti in [J. Fluid Mech. 1991]; see also Z. Biro and J. J. L. Velazquez in [SIAM J. Math. Anal. 2000], derived in the parameter regimes of sonoluminescence experiments. These two models share a one-parameter manifold of spherical equilibria, parametrized by the bubble mass. Within the approximate model, we prove that the manifold of spherical equilibria is an attracting centre manifold, and that solutions approach this manifold at an exponential rate as time advances. We also study the nonlinear dynamics of the bubble-fluid system subject to a small-amplitude, time-periodic external sound field. We prove that this periodically forced nonlinear system admits a unique time-periodic equilibrium that is nonlinearly asymptotically stable by showing the Poincaré map has an asymptotically stable fixed point. Finally, we report on results toward a characterization of all equilibria within each model. For the approximate system, the above family of spherical equilibria captures all equilibria solutions. However, within the full model, this family is embedded in a larger family of spherically symmetric solutions. For the approximate system, we prove that all equilibrium bubbles are spherically symmetric by an application of Alexandrov's theorem on closed surfaces of constant mean curvature. This talk is based on joint work with Michael I. Weinstein (arXiv:2207.04079 and arXiv:2305.03569).
(Received September 06, 2023)

Seshadev Padhi, Birla Institute of Technology, Mesra, Ranchi-835215, India, Jaffar Ali Shahul Hameed*, Florida Gulf Coast University. Positive Solutions for a Derivative Dependent p-Laplacian Equation with Riemann-Stieltjes Integral Boundary

Conditions.
In this talk, we will discuss the existence of two non-trivial positive solutions to a class of boundary value problems (BVP),

$$
\left(\Phi_{p}\left(x^{\prime}\right)\right)^{\prime}+g(t) f\left(t, x, x^{\prime}\right)=0, \quad t \in(0,1)
$$

involving a $p$-Laplacian, of the form:

$$
\begin{aligned}
x(0)-a x^{\prime}(0) & =\alpha[x], & \text { where } \Phi_{p}(x)=|x|^{p-2} x \text { is a one } \\
x(1)+b x^{\prime}(1) & =\beta[x], &
\end{aligned}
$$

dimensional $p$-Laplacian operator with $p>1, a, b$ are real constants. Here $\alpha, \beta$ are given by Riemann-Stieltjes integrals

$$
\alpha[x]=\int_{0}^{1} x(t) d A(t), \beta[x]=\int_{0}^{1} x(t) d B(t)
$$

where $A$ and $B$ are functions of bounded variations. We will use the fixed point index theory to establish our results. (Received September 07, 2023)

## 1192-35-29499

Md Mashud Parvez*, Old Dominion University. Nonlinear evolution equations from epitaxial growth. Preliminary report. Epitaxial growth is a process in which a thin film is grown above a much thicker substrate. However, since the film may potentially have different rigidity constant from the substrate, such growth leads to a nonuniform film thickness. The equations governing epitaxial growth are high order, generally fourth or sixth order, nonlocal, and highly nonlinear, making the study of their solutions quite challenging. In this talk I will present some recent results about the regularity of solutions to several equations arising from epitaxial growth, along with some numerical results.
(Received September 07, 2023)

## 1192-35-29594

Ramesh Karki, Indiana University East, Allison Perry, Indiana University East, Young H You*, Indiana University East. Title: Recovering initial temperature profile of a one-dimensional uniform rod via finite linear time sampling under periodic boundary conditions. Preliminary report.
In this presentation, I will present the inverse problem of recovering an initial temperature profile of 1-dimensional uniform body of finite length under the periodic boundary setting. More specifically, we have considered a 1-dimensional heat equation with the periodic boundary conditions, and an initial condition where the initial datum is not known except the function space where it lies, and instead, temperature measurements taken at a fixed location of the body and n later times are known. Under these assumptions, we approximate $f$ with a desired rate of accuracy. Our current work generalizes and also improves the previous work done under the Dirichlet boundary conditions (by Aryal and Karki) and the Neumann boundary conditions (by Karki, Shawn, and You).
(Received September 07, 2023)

## 1192-35-29611

Nadia Aiaseh*, Western University. Using Physics-informed Neural Networks to Find Soliton Solutions. Preliminary report. Korteweg-de Vries (KdV) and Kadomtsev-Petviashvili (KP) equations are difficult to solve numerically and analytically as they both are nonlinear and dispersive, with the latter equation having the added challenge of an extra spatial dimension. With the recent advances in machine learning and neural networks, Physics Informed Neural Networks (PINNs) - neural networks that use partial differential equations (PDEs) as a regularization term of their loss function - have gained popularity as an alternative way to find solutions to PDEs. In this work, we show how PINNs can be useful for different solutions to the aforementioned equations. Furthermore, we compare the solutions generated by PINNs to the exact ones to illustrate accuracy. We also comment on the hyperparameters that were used to increase the accuracy and convergence rate of our models.
(Received September 07, 2023)

## 1192-35-29662

Daniel Oliveira Da Silva*, California State University, Los Angeles, Achenef Tesfahun, Nazarbayev University. On Mound Formation and Coarseness for a Molecular Beam Epitaxy Model with Slope Selection. Preliminary report.
A global well-posedness result is presented for a fourth-order parabolic equation proposed by Rost and Krug as a model for a manufacturing process known as molecular beam epitaxy. This result, obtained by standard analytical techniques, has farreaching consequences with respect to a phenomenon known as mound formation, which is the formation of pyramidical structures in thin films. In particular, it will be shown that this model does not lead to the observed mounds as claimed, a result which has been incorrectly believed to be true by the condensed matter physics community for decades. (Received September 07, 2023)

## 1192-35-29663

Luda Korobenko*, Reed College, Florian Meister, Reed College, Olive Ross, Reed College. Carnot-Carathéodory metrics for degenerate elliptic operators. Preliminary report.
Metric spaces associated to degenerate elliptic operators provide a powerful tool for studying regularity theory of such operators. In this talk I will present some of the results obtained together with two undergraduate summer research students, concerning the Carnot-Carathéodory metrics associated to a special class of three-dimensional degenerate elliptic operators. (Received September 07, 2023)

Preliminary report.
The Schrodinger and Dirac equations are fundamental models in quantum physics. We study the solutions to these equations when perturbed by an electric potential. We show that solutions to these equations become small as $t \rightarrow \infty$. We do this by relating solutions to oscillatory integrals. Using tools from harmonic analysis, we analyze the related integrals and deduce properties of the solutions.
(Received September 08, 2023)

1192-35-29885
Nancy Rodriguez, University of Colorado at Boulder, Wuyan Wang*, University of Colorado at Boulder, Timothy Wessler, University of Colorado at Boulder. A mathematical analysis of traveling wave solutions in a model for social outbursts with police management. Preliminary report.
While protesting activity is an expression of the first amendment right, the United States as a country has had a fraught relationship with protesters. Through the years there have been various protest policing strategies employed. In this talk we will introduce a system of reaction-diffusion equations to model the interplay between the dynamics of protesters, social tension, and law-enforcement. We study the existence and stability of planar traveling wave solutions to this system. This study is motivated by the "wave-like" dynamics of rioting activity during the French riots in 2005. Changes in a certain parameter cause a qualitative difference in solution to the system, leading to either the "tension-enhancing" or "tension-inhibiting" case. These two cases correspond to the situations where high levels of rioting activity either slows down or accelerates the decay of social tension, respectively. We use a combination of analytical and numerical tools to prove existence and stability of the traveling wave solutions.
(Received September 12, 2023)

## 1192-35-29886

Eduardo V. Teixeira*, University of Central Florida. Nonlocal free boundary models.
I will describe some recent results on free boundary problems arising from non-local models. The analysis of such systems lead to a new family of free boundary problems characterized by either degenerate diffusion operators or unbounded Bernoulli cost functions. Notably, these models exhibit solutions that are merely Holder continuous within their respective phases. I will discuss methods for establishing improved Lipchitz estimates along the free boundary, leading to geometric classification results.
(Received September 08, 2023)

## 1192-35-29956

Chunmei Wang*, University of Florida. Primal Dual Weak Galerkin Methods.
The speaker will discuss the basics of primal dual weak Galerkin methods for solving PDEs.
(Received September 08, 2023)

## 1192-35-29964

Stephen D. Pankavich*, Colorado School of Mines. Large Time Behavior of Collisionless Plasmas.
Collisionless plasmas arise in a variety of settings, ranging from magnetically confined plasmas to study thermonuclear energy to space plasmas in planetary magnetospheres and solar winds. The two fundamental models that describe such phenomena are comprised of complicated systems of nonlinear partial differential equations known as the Vlasov-Maxwell (VM) and Vlasov-Poisson (VP) systems. We will briefly derive these kinetic models and describe recent results concerning the large-time asymptotic behavior of solutions to such systems.
(Received September 08, 2023)

## 1192-35-30022

Alrazi M Abdeljabbar*, Khalifa University. Hirota Direct Method to Nonlinear Partial Differential Equations.
The Hirota Direct Method, devised by Ryogo Hirota, offers a systematic approach to generating exact solutions for nonlinear partial differential equations. This method revolves around bilinear equations, enabling the construction of soliton solutions. This abstract provides a concise overview of the Hirota Direct Method's applications in fields such as fluid dynamics, quantum field theory, and nonlinear optics. Examples and new results will be presented, highlighting its versatility and systematic nature, making it an indispensable tool for understanding soliton interactions and integrable systems in various domains of physics and mathematics.
(Received September 08, 2023)

## 1192-35-30026

Benjamin August Lyons*, Rose-Hulman Institute of Technology, Shouhong Wang, Department of Mathematics, Indiana University, Bloomington, Indiana 47405, USA. Patterns in the Cahn-Hilliard Equation with Long-Range Interactions. Preliminary report.
The Cahn-Hilliard equation is a partial differential equation that governs the behavior of a binary fluid system. In this work, we use a version of the Cahn-Hilliard equation that contains an additional term to account for the long-range interaction of the fluid molecules. We analyze the dynamic transitions and pattern formation of the model as we vary a system control parameter $\lambda$. One of the main goals of this work is to deduce necessary and sufficient conditions (on $\lambda$ and fixed parameters) for the equilibria to form hexagonally packed cylinder (HPC) patterns.
(Received September 08, 2023)

## 1192-35-30369

Brian B. Luczak*, Vanderbilt University. The relativistic Euler equations for an ideal gas with a physical vacuum boundary.

In this paper, we provide several preliminary results on the relativistic Euler equations for an ideal gas equation of state and a physical vacuum boundary. We start by choosing the correct thermodynamic variables and idenitifying a physically relevant boundary condition in the context of an ideal gas. Our main focus is on developing a weighted book-keeping scheme, and then providing energy estimates for the linearized system in Sobolev spaces. To conclude, we discuss next steps in connecting our analysis back to the quasilinear system.
(Received September 09, 2023)

## 1192-35-30485

Mutlu Akar, Yildiz Technical University, Erdogan Mehmet Ozkan*, Yildiz Technical University. A View of Exact Solutions of $(2+1)$-dimensional Time Conformable Schrödinger Equation.
In this paper, exact solutions for the ( $2+1$ )-dimensional time conformable Schrödinger equation with beta-derivative are obtained using the improved sub-equation method. When the suitable parameter values have been applied, the original structures of the solutions are being shown. The results are illustrated using the maple program.
(Received September 10, 2023)

## 1192-35-30488

Mutlu Akar*, Yildiz Technical University, Erdogan Mehmet Ozkan, Yildiz Technical University. A Study of Analytical Solutions of the $(2+1)$-dimensional Time Conformable Maccari System.
Using the improved sub-equation method, analytical solutions of the (2+1)-dimensional time conformable Maccari system with beta-derivative are got in this study. Rational solutions, generalized hyperbolic function solutions, and generalized trigonometric function solutions have all been obtained using this method. The obtained solutions are validated using the maple program.
(Received September 10, 2023)

1192-35-30616
Ananta Acharya*, Utah State University. Analysis of a population when a second species influences its dynamics in the interior and on the boundary.
We analyse positive solutions to the steady state model for a species A given by

## \cases $-\Delta \mathrm{v}=\lambda \mathrm{rv}[1-\mathrm{v}-\mathrm{b} u] \&$ in $\Omega$; $\backslash \operatorname{frac}\{\partial \mathrm{v}\} \partial \eta+\sqrt{ } \lambda \gamma_{-}\{2\} \mathrm{h}(\mathrm{u}, \varepsilon) \mathrm{v}=0 \&$ on $\partial \Omega$ lendcases

where $\Omega$ is a bounded domain in $\mathbb{R}^{N} ; N>1$ with a smooth boundary $\partial \Omega$ or $\Omega=(0,1)$ when $N=1$ and $\frac{\partial v}{\partial \eta}$ denotes the outward normal derivative of $v$. Here $v$ is the population density of species A and $u$, solution of:

$$
\text { |cases }-\Delta u=\lambda u[1-u] \& \text { in } \Omega ; \backslash \text { frac }\{\partial u\} \partial \eta+\sqrt{ } \lambda \gamma_{-}\{1\} u=0 \& \text { on } \partial \Omega \text {, lendcases }
$$

represents the population density of a second species $B$ which acts as a predator in the habitat and also influences the dynamics on the boundary with $A$. We will consider two cases of this dynamics on the boundary:

$$
h(s, \varepsilon)=\left\{1+\varepsilon s(\text { competitiveinteraction }) ; s \geq 0, \varepsilon>0, \text { or } \frac{1}{1}+\varepsilon s(\text { cooperativeinteraction }) ; s \geq 0, \varepsilon>0 . \backslash\right. \text { endcases }
$$

Note that, $\lambda$ here is a measure of the patch size square, $r$ represents intrinsic growth rate of $v, \gamma_{2}$ is a measure of matrix hostility towards $v, b$ is the strength of perdation in the interior and the $\epsilon$ is the strength of interaction on the boundary. (Received September 10, 2023)

## 1192-35-30629

Patrick Sprenger*, University of California Merced. Whitham modulation theory for full-dispersion models. Preliminary report.
In 1967, the Whitham equation was introduced as a mathematical model for water waves affected by gravity. This equation was designed to capture the combination of weak nonlinearity found in the asymptotic Korteweg-de Vries equation and the complete linear dispersion of unidirectional waves. This approach has since been applied to develop similar models for waves with weak nonlinearity and dispersion in various physical systems. Using Whitham's nonlinear wave modulation theory, a general method for deriving a set of modulation equations for different classes of fully dispersive models is presented. This framework results in a set of partial differential equations (PDEs) that describe the gradual changes in the parameters of a nonlinear periodic wave pattern over time. This theory can be applied to numerous full dispersion models: including those of scalar (KdV) type, bi-directional nonlinear Schrödinger type, and Boussinesq type. Once the modulation equations are established, their structure is studied in the weakly nonlinear regime, where a general, yet explicit criteria for determining the stability of a periodic wave pattern is found. This research is a collaborative effort involving Mark Hoefer (CU Boulder) and Boaz Ilan (UC Merced).
(Received September 10, 2023)

## 1192-35-30667

Hongjie Dong, Brown University, Zhuolun Yang, Rutgers University, Hanye Zhu*, Brown University. Recent results on the insulated conductivity problem with p-Laplacian.
We will discuss the insulated conductivity problem with closely spaced insulators embedded in a homogeneous matrix where the current-electric field relation is the power law $J=|E|^{p-2} E$. The electric field may blow up as the distance between insulators approaches zero. We quantitatively analyze the concentration of the electric field $E$ between the inclusions. This is joint work with Hongjie Dong (Brown University) and Zhuolun Yang (Brown University).
(Received September 10, 2023)

## 1192-35-30679

Federico Glaudo*, Princeton University. Stability for fractional inequalities.
The Caffarelli-Kohn-Nirenberg Inequality (CKN) is a generalization both of the classical Sobolev inequality and of Hardy inequality. The sharp constant for CKN, as well as the symmetry and stability of minimizers are well-understood nowadays. A fractional version of the CKN inequality was recently introduced. For this new inequality many questions remain open: Are minimizers radial? Are they non-degenerate? What is their regularity?. In this talk we will present some recent advances on the topic, with a focus on the novel PDE methods that are necessary in the fractional setting.
(Received September 10, 2023)

## 1192-35-30688

Maria Amarakristi Onyido, Northern Illinois University, Rachidi B. Salako, University of Nevada, Las Vegas, Markjoe Olunna Uba*, Northern Illinois University, Cyril I Udeani, Department of Applied Mathematics and Statistics, Comenius University in Bratislava, Mlynska dolina, 84248 Bratislava, Slovakia. Asymptotic limits of the principal spectrum point of a cooperative system of integro-differential equations and applications.
This talk will discuss our study of a stage-structured population model whose dispersal strategy is modeled by integrodifferential equations. Having recently established that the persistence and extinction of such model could be determined solely on the sign of the principal spectrum point, it is pertinent to examine how the dispersal rates influence the principal spectrum point. Thus, we will discuss the asymptotic limits of the principal spectrum point and present some few applications. (Received September 10, 2023)

## 1192-35-30699

Elinor L Velasquez*, University of California, Berkeley. Non-Abelian Toda Lattices on Finite Analogs of Symmetric Spaces. Preliminary report.
In the 1970s, Zakharov and Shabat [3] constructed a scheme for integrating nonlinear partial differential equations (PDEs). In the 1990s, the inverse scattering method for the integration of nonlinear PDEs was supplemented by an algebro-geometric approach [1]. Fix G to be a finite group and S a subset of G. The graph with coset vertices K\G, K the isotropy subgroup of G for a fixed point, and edges produced by a group action, is a finite analog of a symmetric space. One example is the finite analog of hyperbolic space, as described by Terras and others [2]. We briefly describe the periodic non-abelian Toda lattice generalized to a finite analog of a symmetric space, and discuss the inverse scattering approach and the algebro-geometric method for this setting. Also, in the continuous limit of this generalized Toda lattice, we assert that a multi-dimensional version of the Korteweg-de Vries equation can be realized on a hyperbolic space. [1] Belokolos, E.D., Bobenko, A.I., Enol'skii, V.Z., Its, A.R., Matveev, V.B., Algebro-Geometric Approach to Nonlinear Integrable Equations. Springer-Verlag, Heidelberg, 1994. [2] Terras, A.A., Fourier analysis on finite groups and applications, London Mathematical Society Student Texts, vol. 43, Cambridge University Press, Cambridge, 1999. [3] Zakharov, V.E., Shabat, A.B., A Scheme for Integrating the Nonlinear Equations of Mathematical Physics by the Method of the Inverse Scattering Problem. I., Funct. Anal. Appl., vol. 8 (1974): 226235.
(Received September 10, 2023)

## 1192-35-30732

Maria Amarakristi Onyido*, Northern Illinois University, Rachidi B. Salako, University of Nevada, Las Vegas, Markjoe O Uba, Northern Illinois University, Cyril I Udeani, Department of Applied Mathematics and Statistics, Comenius University in Bratislava, Mlynska dolina, 84248 Bratislava, Slovakia. Persistence and asymptotic profiles of positive steady states of a twostage structured population model with nonlocal dispersal. Preliminary report.
Most population models typically investigate species assuming uniformity in population characteristics like birth and death rates. However, most species naturally involve individuals at different maturity stages who exhibit varying characteristics which significantly influence their survival or extinction. This talk will discuss the persistence and extinction dynamics of a two-stage structured nonlocal competition system consisting of mature adults and immature juveniles. We will establish the persistence and extinction of such species based on the spectral bound of its linearization at the trivial solution. Then, we explore the spatial distribution of positive steady states with respect to the dispersal rates. (Received September 10, 2023)

## 1192-35-30733

Nicola Garofalo*, University of Padova. Overdetermined problems in groups of Heisenberg type: conjectures and partial results. Preliminary report.
We formulate some conjectures in sub-Riemannian geometry concerning a characterisation of the Koranyi-Kaplan ball in a group of Heisenberg type through the existence of a solution to suitably overdetermined problems. We prove an integral identity that provides a rigidity constraint for one of the two problems. By exploiting some new invariances of these Lie groups, for domains having partial symmetry we solve these problems by converting them to known results for the classical $p$ Laplacian
(Received September 10, 2023)

1192-35-30782
Alfonso Castro*, Harvey Mudd College. The continuous wavelet inversion formula and regularity of weak solutions to partial differential equations.
We establish regularity properties of weak solutions to linear partial differential equations in terms of the continuous wavelet transform of the data. Our arguments rely on the existence of radial functions that remain radial under the operator defined by the highest order terms of the linear equation and a variant of the inversion formula introduced by Grossmann, Morlet and Paul.
(Received September 11, 2023)

## 1192-35-30783

Ananta Acharya, Utah State University, J Goddard II, Auburn University, Montgomery, Amila Muthunayake, Weber State University, Dustin Nichols*, UNC Greensboro, Ratnasingham Shivaji, University of North Carolina Greensboro. Modeling the effects of trait-mediated dispersal on the coexistence of two species: competitors \& predator-prey.

> We analyze positive solutions $(u, v)$ to the reaction-diffusion system: $\left\{\begin{array}{l}-\Delta u=\lambda u(1-u) ; \Omega \\ -\Delta v=\lambda r v(1-v) ; \Omega \\ \frac{\partial u}{\partial \eta}+\sqrt{\lambda} g(v) u=0 ; \partial \Omega \\ \frac{\partial v}{\partial \eta}+\sqrt{\lambda} h(u) v=0 ; \partial \Omega\end{array}\right.$ where $\Omega \subset \mathbb{R}^{N}$ is a bounded domain (patch) of unit length, area, or volume; $N \in\{1,2,3\}$ with smooth boundary $\partial \Omega, \lambda>0$ is a parameter proportional to the square of patch size and $\frac{\partial}{\partial \eta}$ is the outward normal derivative. Here $u$ and $v$ represent the normalized densities of two species which inhabit the patch, surrounded by a hostile matrix, where the level of hostility is determined by the functions $g, h \in C^{1}([0,1],(0, \infty))$. Finally, $r>0$ compares the two species by the ratio of patch intrinsic growth to patch diffusion rate. We explore two cases: (1) $u$ and $v$ are competing species ( $g$ and $h$ are increasing) and (2) $v$ competes with $u$ while $u$ cooperates with $v$ ( $g$ increasing, $h$ decreasing). We establish coexistence and nonexistence results analytically for certain ranges of $\lambda$ depending on the characteristics of $g$ and $h$. We prove our coexistence results via coupled subsolutions and supersolutions.
(Received September 11, 2023)

1192-35-30791
Ananta Acharya, Utah State University, Gampola Waduge Nalin Fonseka, University of Central Missouri, J Goddard II, Auburn University, Montgomery, Keta Henderson*, UNC Greensboro, Ratnasingham Shivaji, University of North Carolina Greensboro. On the effects of density-dependent emigration on ecological models with logistic and weak Allee type growth terms.
We analyze the structure of positive steady states for a population model designed to explore the effects of habitat fragmentation, density dependent emigration, and Allee effect growth. The steady state reaction diffusion equation is:
$\left\{\begin{array}{l}-\Delta u=\lambda f(u) ; \Omega \\ \frac{\partial u}{\partial \eta}+\gamma \sqrt{\lambda} g(u) u=0 ; \partial \Omega\end{array}\right.$ where $f(s)=\frac{1}{a} s(1-s)(a+s)$ can represent either logistic-type growth $(a \geq 1)$ or weak Allee
affect growth $\left(a \in(0,1)\right.$ ), $\lambda, \gamma>0$ are parameters, $\Omega$ is a bounded domain in $\mathbb{R}^{N} ; N>1$ with smooth boundary $\partial \Omega$ or $\Omega=(0,1), \frac{\partial u}{\partial \eta}$ is the outward normal derivative of $u$, and $g(u)$ is related to the relationship between density and emigration. In particular, we consider three forms of emigration: density independent emigration ( $g=1$ ), a negative density dependent emigration of the form $g(s)=\frac{1}{1+\beta s}$, and a positive density dependent emigration of the form $g(s)=1+\beta s$, where $\beta>0$ is a parameter representing the interaction strength. We establish existence, nonexistence, and multiplicity results for ranges of $\lambda$ depending on the choice of the function $g$. Our existence and multiplicity results are proved via the method of sub-supersolutions and study of certain eigenvalue problems. For the case $\Omega=(0,1)$, we also provide exact bifurcation diagrams for positive solutions for certain values of the parameters $a, \beta$ and $\gamma$ via a quadrature method and Mathematica computations. Our results shed light on the complex interactions of density dependent mechanisms on population dynamics in the presence of habitat fragmentation
(Received September 11, 2023)

1192-35-30811
Yanyan Li, Rutgers University, Xukai Yan*, Oklahoma State University. Stability of homogeneous solutions of stationary incompressible Navier-Stokes equations..
In 1944, Landau discovered a three parameter family of explicit (-1)-homogeneous solutions of 3D stationary incompressible Navier-Stokes equations, with precisely one singularity at the origin. These solutions, now called Landau solutions, are axisymmetric and has no swirl. Sverak proved that all ( -1 )-homogeneous solutions smooth on the unit sphere are Landau solutions. It was proved by Karch and Pilarczyk that Landau solutions are asymptotically stable under any L2-perturbation. This talk focuses on (-1)-homogeneous solutions of 3D stationary incompressible NSE with finitely many singular rays. I will first discuss the existence and singularity behavior of such solutions that are axisymmetric with two singular rays passing through the north and south poles. We classify all such solutions with no swirl and then construct nonzero swirl solutions through perturbation methods. I will then discuss the asymptotic stability of Landau solutions and some of the (-1)homogeneous solutions with singular rays we obtained. I will also talk about some anisotropic Caffarelli-Kohn-Nirenberg type inequalities we derived and applied in the stability problem of Navier-Stokes equations. This is a joint work with Yanyan Li. (Received September 11, 2023)

## 1192-35-30812

Elena Cherkaev, University of Utah, Anwesa Dey*, University of Utah, Johann Rudi, Virginia Tech. A convolutional neural network-based reconstruction framework in magnetic resonance elastography.
An inverse problem in magnetic resonance elastography (MRE) is formulated where the forward problem is governed by a modified stationary Stokes system; we solve the inverse problem using convolutional neural networks (CNN). In MRE, low frequency acoustic pressure waves are sent through the body. These waves propagate rapidly in stiff unhealthy tissues compared to the healthy tissues. The dynamic measurement of the acoustic wave velocity in the tissue is performed using magnetic resonance imaging (MRI), which provides characterization of the tissue's stiffness. A mathematical inverse problem of reconstructing the viscoelastic modulus is formulated as a map from the acoustic wave displacement vector field. This presentation investigates the applicability of CNNs for constructing a direct inverse map that is accurate and computationally efficient. The wave propagation is described by a modified Stokes equation with piecewise constant coefficients, which are the unknown parameters, to model a layered viscoelastic medium. We discuss analytic results concerning the resolution and sensitivity of recovering the viscoelastic modulus. Numerical simulations demonstrate accurate reconstruction of the complex-
valued properties for the case of two-layered, two-dimensional domain.
(Received September 11, 2023)

## 1192-35-30825

Hongjie Dong, Brown University, Junhee Ryu*, Brown University. Nonlocal elliptic and parabolic equations with general stable operators in weighted Sobolev spaces.
We study nonlocal elliptic and parabolic equations on $C^{1, \tau}$ open sets in weighted Sobolev spaces, where $\tau \in(0,1)$. The operators we consider are infinitesimal generators of symmetric stable Lévy processes, whose Lévy measures are allowed to be very singular. Additionally, for parabolic equations, the measures are assumed to be merely measurable in the time variable.
(Received September 11, 2023)

## 1192-35-30830

Chanwoo Kim, University of Wisconsin-Madison, Trinh Tien Nguyen*, University of Wisconsin-Madison. Asymptotics of Helmholtz-Kirchhoff Point-Vortices in the Phase Space.
A rigorous derivation of point vortex systems from kinetic equations has been a challenging open problem, due to singular layers in the inviscid limit, giving a large velocity gradient in the Boltzmann equations. In this paper, we derive the HelmholtzKirchhoff point-vortex system from the hydrodynamic limits of the Boltzmann equations. We construct Boltzmann solutions by the Hilbert-type expansion associated to the point vortices solutions of the 2D Navier-Stokes equations. We give a precise pointwise estimate for the solution of the Boltzmann equations with small Strouhal number and Knudsen number. (Received September 11, 2023)

## 1192-35-30882

Fioralba Cakoni, Rutgers University, New Brunswick, Narek Hovsepyan*, Rutgers University, Michael Vogelius, Rutgers University. Far field broadband approximate cloaking for the Helmholtz equation with a Drude-Lorentz refractive index. We consider an approximate, transformation optics-based cloaking scheme that incorporates a Drude-Lorentz model to account for the dispersive properties of the cloak. We show that on one hand, perfect (far field) cloaking is impossible at any frequency for any incident field, but on the other hand, one can achieve approximate cloaking for any finite band of frequencies, as the resonant frequency of the Drude-Lorentz term approaches infinity.
(Received September 11, 2023)

## 1192-35-30883

Ananta Acharya, University of North Carolina Greensboro, Gampola Waduge Nalin Fonseka*, University of Central Missouri, J Goddard II, Auburn University, Montgomery, Keta Henderson, UNC Greensboro, Victor Munoz, University of North Carolina Greensboro, Ratnasingham Shivaji, University of North Carolina Greensboro. On the occurrence of a $\Sigma$ shaped bifurcation diagram for a reaction diffusion equation with non linear boundary conditions.
We analyze positive solutions to the steady state reaction-diffusion equation:

$$
\begin{gathered}
-\Delta u=\lambda f(u) ; \Omega \\
\frac{\partial u}{\partial} \eta+\gamma \sqrt{ } \lambda g(u) u=0 ; \partial \Omega
\end{gathered}
$$

where $\lambda>0, \gamma>0, \Omega$ is a bounded domain in $\mathbb{R}^{N} ; N>1$ with smooth boundary $\partial \Omega$ or $\Omega=(0,1)$, $\frac{\partial u}{\partial \eta}$ is the outward normal derivative of $u, g \in C([0, \infty),(0,1])$ is a non-increasing function such that $g(0)=1$ and $\lim _{s \rightarrow \infty} g(s)=g_{\infty}>0, f \in C^{2}\left(\left[0, r_{0}\right]\right)$ with $f(0)=0, f^{\prime}(0)=1,0<r_{0}<\infty$ such that $f\left(r_{0}\right)=0$ and $f(s)\left(s-r_{0}\right) \leq 0$ for $s \in[0, \infty)$. We note that the parameter $\lambda$ influences both the equation and the boundary condition. Under additional hypotheses, we establish the existence, nonexistence, and multiplicity results for certain ranges of $\lambda$ via the method of sub-super-solutions, in particular, we establish that the bifurcation curve is at least $\Sigma$-shaped. Further, when $\Omega$ is a ball, for the example (that arises in ecology) where $f(s)=s-\frac{s^{2}}{K}-\frac{M s^{2}}{1+s^{2}}, g(s)=1-m s$ for $0 \leq s \leq \frac{1}{2 m}$ and $g(s)=\frac{1}{2}$ for $s>\frac{1}{2 m}$ with $K, M$, and $m$ are positive parameters, we prove that the bifurcation diagram for positive solutions is at least $\Sigma$-shaped for certain ranges of the parameters. When $\Omega=(0,1)$, via a quadrature method and Mathematica computations we obtain exact bifurcation diagrams for positive solutions of this example. (Received September 11, 2023)

## 1192-35-30900

Leandro L Recova, California State Polytechnic University Pomona, Adolfo J Rumbos*, Pomona College. Existence and multiplicity of solutions for a cooperative elliptic system using Morse theory. Preliminary report.
We study existence of nontrivial solutions of the elliptic system $-\Delta u=a u+b v+\frac{2 \alpha}{\alpha+\beta} u|u|^{\alpha-2}|v|^{\beta}$ in $\Omega$;
$-\Delta v=b u+c v+\frac{2 \beta}{\alpha+\beta}|u|^{\alpha} v|v|^{\beta-2}$ in $\Omega ; u=v=0$ on $\partial \Omega$, for $\alpha>1, \beta>1, \alpha+\beta \leqslant 2^{*}=2 N /(N-2), N \geqslant 3$, $a, b, c \in \backslash \mathrm{R}$, and $\Omega$ an open bounded domain of $\backslash \mathrm{R}^{N}$ with a smooth boundary $\partial \Omega$. We extend the work of Alves et al. (Nonlinear Analysis 42 (2000) 771-787), in the subcritical growth case: $\alpha+\beta<2^{*}$, by proving that the boundary value problem has at least two nontrivial solutions for the case in which the eigenvalues of the matrix $M=\left(\begin{array}{ll}a & b \\ b & c\end{array}\right)$ are higher than the first eignvalue of the Laplacian over $\Omega$ with Dirichlet boundary conditions; $u=v=0$ on $\partial \Omega$. We use variational methods and infinite-dimensional Morse theory to obtain the multiplicity result.
(Received September 11, 2023)

## 1192-35-30908

Hengrong Du*, Vanderbilt University, Yuanzhen Shao, The University of Alabama, Gieri Simonett, Vanderbilt University. On a thermodynamically consistent model for magnetoviscoelastic fluids in 3D.
In this talk, we investigate a system of equations modeling a non-isothermal magnetoviscoelastic fluid. We demonstrate that this model is thermodynamically consistent and that the critical points of the entropy functional with constant energy constraint correspond to the system's equilibria. Our analysis utilizes the maximal $L_{p}$ regularity theory of quasilinear parabolic systems, proving the model to be locally well-posed. Additionally, we will discuss global existence and the asymptotic behavior of the entire dynamics. This is joint work with Yuanzhen Shao and Gieri Simonett.
(Received September 11, 2023)

## 1192-35-30919

Elena Cherkaev*, University of Utah. Quasiperiodic composites: homogenization and spectral properties.
From quasicrystalline alloys to twisted bilayer graphene, materials with a quasiperiodic structure exhibit unusual properties that drastically differ from those with periodic structures. A key feature of quasicrystalline microgeometry is a long-range order in the absence of periodicity. Quasiperiodic geometries can be modeled using the cut-and-projection method that restricts or projects a periodic function in a higher dimensional space to a lower dimensional subspace cut at an irrational projection angle. Homogenized equations for the effective behavior of a quasiperiodic composite can be derived by cutting and projecting a periodic function in a higher dimensional space. Using equations for the local problem in the higher dimensional space established in the homogenization process, we develop the Stieltjes analytic representation of the effective properties of quasiperiodic materials; this representation determines the spectral characteristics of fields in quasicrystalline composites and used to derive bounds for the effective properties. This is a joint work with Niklas Wellander and Sebastien Guenneau.
(Received September 11, 2023)

## 1192-35-30958

Feng-Bin Wang*, Chang Gung University. A nonlocal reaction diffusion model of West Nile virus with vertical transmission. West Nile virus (WNv) is a mosquito-borne infection, causing health challenges to bird and human populations. In this talk, I plan to propose a general nonlocal reaction-diffusion model to explore the combined effect of alternative hosts, vector-host movement and spatial-temporal environmental structure on WNv transmission dynamics. The model is partially degenerate and its solution maps lack compactness due to the absence of diffusion terms in larval equations. For this tightly coupled disease model, we introduce the mosquito reproduction number and WNv reproduction number, respectively, and further show that these two aforementioned reproduction numbers completely determine the threshold behaviour of the disease spread. Numerical simulations support our analytic results and suggest that environmental heterogeneity plays an important role in shaping WNv dynamics. Moreover, bird migration may increase the disease risk.
(Received September 11, 2023)

## 1192-35-30981

Giangvuthanh Nguyen*, Old Dominion University, Xiang Xu, Old Dominion University. Asymptotic expansion of a singular potential near the nematic-isotropic phase transition point in the Landau-de Gennes theory..
The Landau-de Gennes theory is a type of continuum theory that describes nematic liquid crystal configurations in the framework of the Q-tensor order parameter. In the free energy, there is a singular bulk potential which is considered as a natural enforcement of a physical constraint on the eigenvalues of symmetric, traceless Q-tensors. In this talk we shall discuss some analytic properties related to this singular potential. More specifically, we study the asymptotic expansion of this singular potential (up to fourth order) near the nematic-isotropic phase transition point (Received September 11, 2023)

## 1192-35-31016

Michail E. Filippakis*, Department of Digital Systems, Univercity of Piraeus, Greece. Multiplicity of Solutions for Doubly Resonant Neumann Problems.
In this paper, we examine semilinear Neumann problems which at $\pm$ infinity are resonant with respect to two successive eigenvalues (double resonance situation). Using variational methods based on the critical point theory together with Morse theory, we prove two multiplicity results. In the first we obtain two nontrivial solutions and in the second three, two of which have constant sign (one positive, the other negative).
(Received September 11, 2023)

## 1192-35-31019

Rachidi B. Salako*, University of Nevada, Las Vegas. Asymptotic profiles of coexistence endemic equilibria of an epidemic model with respect to small diffusion rates of population.
We examine the dynamics of solutions of a two-strain diffusive epidemic model with no flux boundary conditions. The objective of our study is to explore the asymptotic behaviors of the coexistence endemic equilibrium with respect to the small movement rates of susceptible and infected populations. By doing so, we aim to gain a better understanding of the spatial distribution of infectious diseases. Our results indicate that there is a sharp critical number which depends delicately on the infected groups' diffusion rates such that if the total size of the population is kept below this critical number, then the disease could be eradicated by restricting the susceptible hosts' movement. However, if the total population size exceeds this critical number, the disease may persist no matter how the movement of the susceptible hosts is controlled. (Received September 11, 2023)
equation on a space scale.
Hilger introduced time scale calculus in the 80s as a way of unifying continuous and discrete analysis. Many classical PDEs have been generalized to this context, including the sine-Gordon equation (by Cieslinski, Nikiciuk, and Waskiewicz in 2015). We derive some general compatibility conditions for Lax systems involving two variables, neither of which are assumed to be continuous, and attempt to use our framework to set up Darboux transformations for the generalized sine-Gordon equation. This is unsuccessful in the general case but it does work in the "space scale" situation, where we assume that the time variable is continuous, and enables us to construct Crum-type closed determinantal formulas for $N$-soliton solutions. (Received September 11, 2023)

## 1192-35-31071

Pelin Guven Geredeli*, Clemson University. Nonlinear Semigroup Approach for the Navier Stokes-full Kirchoff Plate Dynamics.
In this work, we analyze the wellposedness of a certain coupled Navier Stokes-plate interaction dynamics. With respect to the particular FSI system, most of the focus has recently been on strictly linearized versions-i.e., Stokes fluid flow, rather than Navier-Stokes- or on nonlinear versions of the PDE model under consideration in which the nonlinearity emanates strictly from the plate component. In this study, existence-uniqueness of the solutions to the said PDE system is shown with the NavierStokes nonlinearity in place in the fluid PDE component. The methodology partly involves the invocation of the nonlinear semigroup theory which was developed by Z. Yoshida, Y. Giga (1984).
(Received September 11, 2023)

## 1192-35-31074

Mark J Ablowitz, University of Colorado Boulder, Ziad H Musslimani, Florida State University, Nicholas James Ossi*, Florida State University. Space-time nonlocal integrable evolution equations. Preliminary report.
The theory of integrable - that is, exactly solvable - nonlinear evolution equations has been an active research area for over 50 years. Such equations possess rich mathematical properties, including the existence of an infinite number of conservation laws and a close association with a linear eigenvalue problem. In recent years, a new class of spatially and/or temporally nonlocal integrable equations has been introduced and studied. In this talk, we will explore this class of equations. We will present and solve several examples of integrable systems whose nonlocality appears as a reflection and shift to the space and/or time coordinate. Continuous, semi-discrete, $(1+1) \mathrm{D}$, and $(2+1) \mathrm{D}$ examples will be considered.
(Received September 11, 2023)

1192-35-31147
Ananta Acharya, University of North Carolina Greensboro, Shalmali Bandyopdhyay, UNC Greensboro, J. T. Cronin,
Louisiana State University, J Goddard II, Auburn University, Montgomery, Amila Muthunayake*, Weber State University,
Ratnasingham Shivaji, University of North Carolina Greensboro. The diffusive Lotka-Volterra competition model in fragmented patches I: Coexistence.
We study the positive solutions to the reaction diffusion model $\backslash$ equation* $q[] \backslash$ matrix $-\Delta u=\lambda u(1-u-b\{1\} v) ; \Omega-\Delta v=\lambda r v(1-v-$ b_\{2\}u) ; $\Omega \backslash \operatorname{frac}\{\partial u\} \partial \eta+\gamma \_\{1\} \sqrt{ } \lambda u=0 ; \partial \Omega \backslash \operatorname{frac}\{\partial v\} \partial \eta+\gamma_{-}\{2\} \sqrt{ } \lambda v=0 ; \partial \Omega \backslash$ endmatrix.$\backslash$ endequation* which describes the steady states of two species $u$ and $v$ competing in a habitat $\Omega$. Here $b_{1}, b_{2}$ represent the strengths of competition, $\lambda$ represents a patch size measure, and $\gamma_{1}, \gamma_{2}$ are related to the hostility of the exterior domain. We analyze the positive solutions of $\left(^{*}\right)$ as the parameters $b_{1}, b_{2}$ and $\gamma_{1}, \gamma_{2}$ vary.
(Received September 11, 2023)

1192-35-31186
Ashok Aryal*, Minnesota State University Moorhead, Ramesh Karki, Indiana University East. Recovering Initial Temperature Profile of a Thin 2D Plate. Preliminary report.
We present an investigation into the inverse problem of recovering the initial temperature profile of a thin, uniform, twodimensional square plate from finite-time observations at a fixed location. This work extends our prior research in the onedimensional case.
(Received September 11, 2023)

1192-35-31327
Camille Carvalho, Institut National des Sciences Appliquées, Lyon, Elsie Cortes*, University of California, Merced,
Chrysoula Tsogka, University of California Merced, Merced. Boundary Integral Equation Methods for Optical Cloaking Models. Preliminary report
Optical cloaking is the act of making an object invisible by preventing the light scattering in some directions as it hits the object. Developing a model to accurately capture cloaking comes with numerical challenges. In our model, we must determine how light propagates through a medium composed by thin layers of materials with different electromagnetic properties. We consider a multi-layered scalar transmission problem in 2D and use boundary integral equation (BIE) methods to compute the total field. The Kress product quadrature rule is used to approximate singular integrals evaluated on the boundaries, while in the layer we employ the Boundary Regularized Integral Equation Formulation (BRIEF) method with the Periodic Trapezoid Rule (PTR) to treat nearly singular ones appearing in the representation formula. Numerical results illustrate the efficiency of this approach, which may be applied to N arbitrary smooth layers.
(Received September 11, 2023)

George Avalos*, University of Nebraska-Lincoln. a inf-sup approach to wellposedness of a Biot-Stokes interaction. In this talk we consider the use of the Babuska-Brezzi Theorem, by way of establishing semigroup wellposedness of a BiotStokes interactive PDE model. In particular, a three dimensional Biot system is coupled to a three dimensional Stokes flow. Each of these distinct PDE's evolves on a distinct geometry, with the coupling between them being enacted via a boundary
interface.The positive virtue of our approach is that it provides a companion mixed finite element method to approximate both the Biot and fluid solution variables
(Received September 11, 2023)

## 1192-35-31403

Stephanie Dodson*, Colby College, Timothy J Lewis, University of California, Davis. Towards traveling waves in biophysical models of cardiac dynamics.
Regular cardiac function is characterized by coherent traveling waves of electrical activity that drive heart beats. When this process goes awry, the ensuing irregular rhythms are known as arrhythmia, which can be life-threatening. Hence, it is crucial to understand conditions that influence arrhythmia onset. In previous work, these traveling waves have been mathematically investigated in qualitative models of excitable media. We investigate traveling wave properties and arrhythmia onset using biophysically realistic models of cardiac dynamics.
(Received September 11, 2023)

## 1192-35-31497

Dennis Belotserkovskiy*, Colgate University, Justin Li*, Colgate University. Nonlinear Neutral Inclusions. Preliminary report.
In this poster, the problem of determining nonlinear neutral inclusions in (electrical or thermal) conductivity is considered. Neutral inclusions, inserted in a matrix containing a uniform applied electric field, do not disturb the field outside the inclusions. The well-known Hashin-coated sphere construction is an example of a neutral inclusion. The project deals with constructing neutral inclusions from nonlinear materials. In particular, assemblages of circular inclusions with spiraling laminate structure inside them with a nonlinear core are studied, and their effective electrical conductivity is found. (Received September 11, 2023)

## 1192-35-31500

Ramesh Karki*, Indiana University East, Chava Shawn, Indiana University East, Young H You, Indiana University East. On recovering initial temperature profile from finite time-observations made under the Neumann boundary setting.
We have studied, under the Neumann boundary setting, an inverse problem of recovering an initial temperature profile of a thin uniform one-dimensional rod of finite length from finite time observations made at a specifically chosen fixed location of the rod and specifically selected finitely many time instances. Our main result expands the result of an analogous problem studied by Aryal and Karki (2022) under the Dirichlet boundary setting.
(Received September 11, 2023)

## 1192-35-31571

Yifeng Mao*, University of Colorado Boulder. Long-time asymptotics and the radiation condition for time-periodic linear boundary value problems.
Initial-boundary value problems (IBVPs) with constant initial and time-dependent boundary data, also known as the wavemaker problem, are fundamental and of significant importance in mathematics and physics. For the linear time-periodic wavemaker problem, the radiation condition selects the unique traveling wave or spatially decaying wave in the neighborhood of the boundary. However, this simple and physically plausible argument has yet to be mathematically justified in a general context. In recent work, the IBVP for some linear and integrable nonlinear evolution equations has been solved using the unified transform method. A related approach, called the Q-equation method, has been introduced to derive the Dirichlet-to-Neumann ( $\mathrm{D}-\mathrm{N}$ ) map for asymptotically time-periodic boundary conditions. This talk will extend and prove the existence of the unique D N map for a general third-order wave model and prove the radiation condition as a consequence. In addition, two representative linear evolution equations with sinusoidal boundary conditions are studied and uniform asymptotic approximations are obtained for large $t$, one of which is shown to give a quantitative description of wavemaker experiments. (Received September 11, 2023)

## 1192-35-31628

Elliott Zachary Hollifield*, University of North Carolina at Pembroke. Positive weak solutions of nonlocal parabolic problems with logistic reaction term. Preliminary report.
We study a parabolic reaction-diffusion equation with logistic reaction term and the fractional Laplacian as the diffusion operator. We discuss existence of a positive weak solution by constructing appropriate ordered sub- and supersolutions. We also discuss the profiles of positive numerical solutions, corresponding to theoretical results, using a Finite Element Method. (Received September 11, 2023)

## 1192-35-31723

N. Mavinga, Swarthmore College, Timothy Ira Myers, Howard University, M. N. Nkashama*, University of Alabama at Birmingham. Nonlinear elliptic equations with growth involving critical Sobolev exponents. Preliminary report
We will present some recent results on the existence of weak minimal and maximal solutions between an ordered pair of suband super-solutions for nonlinear elliptic equations with nonlinearities in the differential equation and on the boundary. No monotonicity conditions (through one-sided Lipschitz condition, a linear shift or otherwise) are imposed on the nonlinearities. Unlike previous results, we allow the growth conditions in the nonlinearities to go all the way to the critical Sobolev exponents in the appropriate Lebesgue spaces in duality. The approach makes careful use of pseudomonotone coercive operators introduced by H. Brezis, the axiom of choice through Zorn's lemma and a Kato inequality up to the boundary along with appropriate estimates
(Received September 12, 2023)

Alejandro Aceves, Southern Methodist University, Brian Jongwon Choi*, United States Military Academy, Austin Marstaller, Southern Methodist University. On Localization of the Fractional Discrete Nonlinear Schrödinger Equation. Preliminary report.
The continuum and discrete fractional nonlinear Schrödinger equations (fDNLS) represent new models in nonlinear wave phenomena with unique properties. In this presentation, we survey various aspects of localization associated to fDNLS featuring modulational instability, asymptotic construction of onsite and offsite solutions, and the role of Peierls-Nabarro barrier. In particular, the localized onsite and offsite solutions are constructed using the map approach. Under the long-range interaction characterized by the Lévy index $\alpha>0$, the phase space of solutions is infinite-dimensional unlike that of the wellstudied nearest-neighbor interaction. We show that an orbit corresponding to this spatial dynamics translates to an approximate solution that decays algebraically. We also show as $\alpha \rightarrow \infty$, the discrepancy between local and nonlocal dynamics becomes negligible on a compact time interval, but persists on a global time scale. Moreover it is shown that data of small mass scatter to free solutions under a sufficiently high nonlinearity, which proves the existence of strictly positive excitation threshold for ground state solutions to fDNLS.
(Received September 12, 2023)

## 1192-35-31819

Valeria Giunta, Swansea University, Thomas Hillen, University of Alberta, Edmonton, Canada, Mark A Lewis*, University of Victoria, Jonathan Potts, University of Sheffield. Nonlocal multispecies advection-diffusion models. Nonlocal advection is a key process in a range of biological systems, from cells within individuals to the movement of whole organisms. Consequently, in recent years, there has been increasing attention on modeling non-local advection mathematically. These often take the form of partial differential equations, with integral terms modeling the nonlocality. One common formalism is the aggregation-diffusion equation, a class of advection-diffusion models with nonlocal advection. This was originally used to model a single population but has recently been extended to the multispecies case to model the way organisms may alter their movement in the presence of coexistent species. Here we prove existence theorems for a class of nonlocal multispecies advection-diffusion models with an arbitrary number of coexistent species. We give methods for determining the qualitative structure of local minimum energy states and analyze the pattern formation potential using weakly nonlinear analysis and numerical methods. (Received September 12, 2023)

1192-35-31843
Nsoki Mavinga*, Swarthmore College, Quinn Alexander Morris, Appalachian State University, Stephen B. Robinson, Wake Forest University. Asymmetric Spectrum and Solvability of Nonlinear Elliptic Equations.
In this talk, we will discuss the Fučik spectrum of a differential operator. This spectrum corresponds to oscillations of an asymmetric system where different reaction forces act in the positive and the negative directions. We will then present the existence results for nonlinear elliptic equations when the nonlinearities interact with the spectrum.
(Received September 12, 2023)

## 1192-35-31858

Julio Cesar Paez*, University of Texas Rio Grande Valley. On Multi-Fission and Fusion Interactions of Traveling Waves for the Two-Dimensional Euler Equations.
In this talk, we consider analytical models of traveling wave interactions derived from the Euler Equations (EEs) in shallow water with two space dimensions. The model rational exponential functions we use are of a wide variety and include most of the known solitonic type traveling waves. Additionally, they allow for better fit and interactions. We discuss the advantage of the approach in modeling lab experiments involving long wavelength, small amplitude waves with speed close to one as well as fission and fusion interactions. Applications include modeling single traveling waves with different shapes, stability of solitons with respect to initial conditions, and head-on collisions of two fission traveling waves, with an arbitrary number of traveling waves after interaction.
(Received September 12, 2023)

## 1192-35-31885

Betul Orcan-Ekmekci*, Rice University. Geometric Analysis on Nonlocal Models. Preliminary report.
Depending on the natural phenomena, the needs of nonlocal mathematical modeling arise naturally. In recent years, many methods are tried to be carried on these nonlocal models to be able to understand the characterization of possible solutions. In this talk, we'll present our recent results about the geometric analysis techniques applied on some nonlocal differential equations related with fluid dynamics.
(Received September 12, 2023)

1192-35-31973
Dorina I. Mitrea*, Baylor University. Characterizing the Regularity of Domains via Riesz Transforms.
Under mild geometric measure theoretic assumptions on a given open subset, we will show that the Riesz transforms on its boundary are continuous mappings on Holder spaces if and only if the domain in question is a Lyapunov domain. In the category of Lyapunov domains we also establish the boundedness on Holder spaces of singular integral operators with rational kernels, exhibiting the proper parity and homogeneity. This family of singular integral operators, which may be thought of as generalized Riesz transforms, includes the boundary layer potentials associated with basic PDE's of mathematical physics, such as the Laplacian, the Lame system, and the Stokes system.
(Received September 12, 2023)

Stochastic Volatility, Change in Equity Premium, and Interest Rates in a Complete Market.
This paper presents a new model for options pricing. The Black-Scholes-Merton (BSM) model plays an important role in financial options pricing. However, the BSM model assumes that the risk-free interest rate, volatility, and equity premium are constant, which is unrealistic in the real market. To address this, our paper considers the time-varying characteristics of those parameters. Our model integrates elements of the BSM model, the Heston model (1993) for stochastic variance, the Vasicek model $(1977)$ for stochastic interest rates, and the Campbell and Viceira model $(1999,2002)$ for stochastic equity premium. We derive a linear second-order parabolic PDE and extend our model to encompass fixed-strike Asian options, yielding a new PDE. In the absence of closed-form solutions for any options from our new model, we utilize finite difference methods to approximate prices for European call and up-and-out barrier options, and outline the numerical implementation for fixed-strike Asian call options.
(Received September 12, 2023)

## 1192-35-32015

Ryan Creedon*, University of Washington. The Transverse Instability of Stokes Waves.
The first proof of the transverse instability of small-amplitude, periodic water waves in infinite depth is presented. Key details of the proof will be emphasized, and the fundamental analytical results will be connected to previous numerical and formal asymptotic investigations.
(Received September 12, 2023)

## 1192-35-32069

Evan Miller*, University of Alabama, Huntsville. Finite-time blowup for an Euler and hypodissipative Navier-Stokes model equation on a restricted constraint space.
In this talk, I will introduce the $\mathcal{M}$-restricted Euler and hypodissipative Navier-Stokes equations. These equations are analogous to the Euler equation and hypodissipative Navier-Stokes equation, respectively, but with the Helmholtz projection replaced by a projection onto a more restrictive constraint space. The nonlinear term arising from the self-advection of velocity is otherwise unchanged. I will prove finite time-blowup when the dissipation is weak enough, by making use of a permutation symmetric Ansatz that allows for a dyadic energy cascade of the type found in the Friedlander-Katz-Pavlović dyadic Euler/Navier-Stokes model equation. The $\mathcal{M}$-restricted Euler and hypodissipative Navier-Stokes equations respect both the energy equality and the identity for enstrophy growth for the full Euler and hypodissipative Navier-Stokes equations. (Received September 12, 2023 )

## 1192-35-32313

Brian E Moore*, University of Central Florida. Constructing dissipation preserving exponential integrators. Preliminary report.
Exponential methods have numerous advantages for solving nonlinear differential equations that have a linear component. This talk presents techniques for constructing exponential methods that preserve properties of the governing equations, such as energy, momentum, mass, and symplecticity, which are damped by linear terms with time-dependent coefficients. In particular, the methods are generally classified as integrating factor methods or as exponential time differencing methods, depending on the method of construction. As time-stepping schemes, these methods have certain advantages for dampeddriven nonlinear equations, because they can preserve dissipation rates up to machine precision. To demonstrate their effectiveness, the methods are applied to several model problems with linear damping and driving forces.
(Received September 12, 2023)

## 1192-35-32325

Nicholas Brubaker*, California State University, Fullerton. Distributions of confined active particles
Active particles have a distinct anterior side that defines a spatial orientation and generates intrinsic self-propulsion velocity. When the particles are confined in a thermally fluctuating environment, the resulting persistent motion can produce regions of accumulation in the vicinity of the bounding perimeter. This behavior makes their non-passive activity readily apparent However, in specific contexts, the aggregation is absent, and the dynamics produce an equilibrium distribution that is smoothed throughout the restricting space, similar to the dispersal of passive particles in the same setting. We explore this dichotomy between clumping and spreading by studying a prototypical model for active particles-namely, Active Brownian particles-with different ambient conditions. Our analysis involves constructing solutions to the Fokker-Planck equation of the full joint stationary distribution using both asymptotic and numerical methods.
(Received September 12, 2023)

## 1192-35-32329

Martin Bauer, Florida State University, Stephen C. Preston*, CUNY Brooklyn College, Justin Valletta, Florida State University. Liouville comparison theory for blowup of Euler-Arnold equations,
We describe a new method for proving blowup of certain Euler-Arnold equations, partial differential equations which represent geodesics on groups of diffeomorphisms under right-invariant metrics. It is based on using momentum conservation to treat the equation as a first-order ODE on a Banach space, then using proving that solutions breakdown based on a comparison theorem using the known exact solution of the classical Liouville equation. Applications are given for the right-invariant $H^{2}$ Sobolev metric on the group of diffeomorphisms of $\mathbb{R}^{n}$, where we show that solutions can break down in finite time if $n \geq 3$. (Received September 12, 2023)

1192-35-32359
Shalmali Bandyopdhyay*, UNC Greensboro, M Chhetri, University of North Carolina at Greensboro, B Delgado,
Universidad Autónoma de Aguascalientes, Nsoki Mavinga, Swarthmore College, R. Pardo, Complutense University of Madrid. Bifurcation of Positive Weak Solution to Elliptic System with Superlinear Subcritical Nonlinearity on the Boundary. We consider a coupled elliptic system with nonlinearity on the boundary. We discuss the existence, local bifurcation, global
bifurcation and multiplicity of positive solutions when the nonlinearity on the boundary is superlinear but subcritical at infinity. Bifurcation theory, topological degree theory and the method of sub- and supersolutions are employed in establishing the results
(Received September 12, 2023)

## 1192-35-32361

Wen Feng*, Niagara University. Long time behavior for the incompressible MHD system.
In this talk, we will discuss the decay results on a 2D magnetohydrodynamic (MHD) system with only horizontal dissipation. We show the stability result on the perturbations near a background magnetic field. In addition, the solution converges to its horizontal average asymptotically.
(Received September 12, 2023)

## 1192-35-32381

Jing An*, Duke University. Reaction rate of the flux-limited chemotaxis system. Preliminary report.
We investigate a chemotaxis system originally introduced by [1] in the context of a flux limitation. [1] quantifies the effect of chemotaxis on reaction rates compared to the pure reaction-diffusion by estimating the half-time of mass, but results in [1] come with serious constraints such as radial symmetry assumptions on initial masses. By introducing a flux-limited chemotactic law that imposes a speed limit on the transport, we eliminate the need for radial symmetry assumptions. With this modified model, we are able to establish precise scaling laws to show how chemotaxis can significantly reduce reaction halftime. Our results demonstrate the quantitative dependency of the reaction rate on various parameters, initial data, geometry of the problem, and dimension. Moreover, our results expand the parameter regime that can be explored beyond the scope of [1]. [1] Alexander Kiselev, Fedor Nazarov, Lenya Ryzhik, and Yao Yao. Chemotaxis and reactions in biology. Journal of the European Mathematical Society, 2022.
(Received September 12, 2023)

## 1192-35-32424

Bobby L. E. Wilson*, University of Washington, Xueying Yu, Massachusetts Institute of Technology. Well-posedness and Scattering for Mass-critical NLS on hyperbolic space.
In this talk, we will discuss a proof of global well-posedness and scattering for the defocusing, mass-critical nonlinear Schrödinger equation defined on three-dimensional hyperbolic space.
(Received September 12, 2023)

## 1192-35-32486

Radu Dascaliuc*, Oregon State University. Stochastic linearization and existence of solutions for some nonlinear evolution equations: a case study of alpha-Riccati and Pantograph Equations.. Preliminary report.
In this talk we use the stochastic cascades that can be associated to certain deterministic evolution PDE models to connect the solutions of the so-called alpha-Riccati equation (a model used to describe ageing, but which can also be viewed as a simplified analogy to the self-similar Navier-Stokes equations) to the solutions of the Pantograph equation (which has applications ranging from combinatorics to cell division modeling, and which functionally represents a linearization of the alpha-Riccati equation). The key observation is that this connection, somewhat reminiscent of the Cole-Hopf transform for Burgers equation, is established for the associated stochastic structures, rather for the equations themselves. As a result, we obtain non-unique solutions to the corresponding Cauchy Problem for the (non-linear) alpha-Riccati equations from a solution of the (linear) Pantograph equations, obtained via the stochastic cascades. Notably, the Pantograph equation solutions that we obtain are consistent with the algebraically decaying solutions mentioned by Kato and McLeod in 1971. Based on the joint work with Tuan Pham, Enrique Thomann, and Edward Waymire.
(Received September 12, 2023)

## 1192-35-32517

Radu Dascaliuc*, Oregon State University. On non-steady-state solutions of the Navier Stokes equations with constant energy and enstrophy..
The question of existence of non-steady-state solutions to the Navier-Stokes equations that have constant global characteristics - the so-called "ghost solutions" - was originally raised by Foias et al. in the context of mathematical theory of turbulence. In this work we use geometry of triangular interactions in Fourier space to establish existence of non-stationary solutions with constant energy or constant enstrophy in the 3D case. Such solutions lie outside the (weak) global attractor and are supported on an infinite number of modes. While existence of ghost solutions in the 2D case remains open, in the case the force is an eigenvalue of the Stokes operator, we show that any hypothetical ghost solution with non-trivial linear interactions must be supported on the infinite number of Fourier modes. Based on the joint work with Sara Hagen. (Received September 12, 2023)

## 1192-35-32585

Beckett Sanchez*, Florida International University. Constructing Solutions to Korteweg-de Vries (KdV) Equation with Higher Order Dispersion and Low Power Nonlinearity. Preliminary report.
The Korteweg-De Vries equation (KdV)

$$
\partial_{t} u+\partial_{x}^{3} u+u \partial_{x} u=0
$$

is a partial differential equation (PDE) modeling shallow water waves with a weak nonlinear interaction. The dispersive nature of waves in combination with the nonlinear effects of this equation gives rise to fascinating phenomena such as solitons: travelling waves that maintain shape. We study a generalized, higher-order KdV equation ( 5 gKdV )

$$
\partial_{t} u+\partial_{x}^{5} u+\partial_{x}^{3} u+|u|^{\alpha} \partial_{x} u=0, \alpha>0 .
$$

In particular, we consider a more difficult case of low power nonlinearity $0<\alpha<1$. It is currently not well known if higherdispersive KdV models with low power nonlinearity have solutions. We present the existence of solutions to this equation for finite time with a slow-decaying initial condition.
(Received September 12, 2023)

## 1192-35-32635

Yiming Ren*, The University of Alabama. FFT accelerated high order finite difference method for solving elliptic BVP and interface problem.
An augmented matched interface and boundary method is introduced for solving elliptic boundary value problems and interface problems. First, A new finite difference method is proposed based on the matched interface and boundary (MIB) and fast Fourier transform (FFT) schemes, which achieves a fourth order convergence and $O(N \log N)$ efficiency. The method involves a ray-casting MIB scheme to handle different types of boundary conditions and an augmented MIB formulation for efficient inversion of the discrete Laplacian using the FFT algorithm. The accuracy and efficiency of the proposed method are numerically examined for various elliptic BVPs in two and three dimensions. Second, A FFT acclerated finite difference method is proposed for solving a three-dimensional elliptic interface problem involving a smooth material interface inside a cuboid domain. The method can handle different types of boundary conditions and complex geometry using a fourth order ray-casting MIB scheme and Cartesian derivative jumps as auxiliary variables. The proposed method is efficient and accurate, achieving an overall efficiency on the order of $O\left(n^{3} \log n\right)$ for a $n \times n \times n$ uniform grid.
(Received September 12, 2023)

1192-35-32664
Beckett Sanchez*, Florida International University. Solutions to a Generalized KdV Equation with Higher Dispersion. Preliminary report.
The famous Korteweg-de Vries equation

$$
(K d V) \partial_{t} u+\partial_{x}^{3} u+u \partial_{x} u=0
$$

models surface waves in a shallow water canal. For other water wave models a variance of dispersion is used, for example, the Benjamin-Ono model replaces $\partial_{x}^{2} u$ with a weaker dispersion $\partial_{x} \mathcal{H} u$ to describe the deep internal water waves. We investigate a higher-order dispersion generalized KdV equation, including the fifth order dispersion,

$$
(5 K d V) \partial_{t} u+a \partial_{x}^{5} u+b \partial_{x}^{3} u+\mu \mathcal{N}(u) \partial_{x} u=0,
$$

allowing the nonlinearity $\mathcal{N}(u)=|u|^{\alpha}$ to include the powers $\alpha>0$, in particular, considering very low powers $0<\alpha<1$. We are interested in understanding global behavior of solutions, including sign-changing solitons, however, even the local theory is poorly understood for this equation, as Strichartz estimates are not applicable in this situation. We establish local solutions for the 5 KdV in a weighted class of Sobolev spaces with slowly decaying initial data $\inf _{x \in \mathbb{R}}\left|\langle x\rangle^{m} u_{0}(x)\right|>0$ and then discuss global perspectives.
(Received September 12, 2023)

## 1192-35-32704

Beckett Sanchez*, Florida International University. Constructing Solutions to Korteweg-de Vries (KdV) Equation with Higher Dispersion. Preliminary report. The Korteweg-de Vries equation

$$
(K d V) \partial_{t} u+\partial_{x}^{3} u+u \partial_{x} u=0
$$

is a partial differential equation that models shallow water waves with weak nonlinear interaction. The dispersive nature of waves in combination with the nonlinear effects of this equation gives rise to fascinating phenomena such as solitons: travelling waves that maintain shape. We study a generalized, higher-order KdV equation

$$
(5 g K d V) \partial_{t} u+\partial_{x}^{5} u+\partial_{x}^{3} u+|u|^{\alpha} \partial_{x} u=0, \alpha>0 .
$$

In particular, we consider the difficult case $0<\alpha<1$. Solutions to the 5 gKdV equation are not well understood with lowpowered nonlinearity, since in such case standard tools are not applicable. We present local well-posedness results, meaning existence, uniqueness and continuous dependence of solutions, for the 5 gKdV with low powered nonlinearity $0<\alpha<1$. (Received September 12, 2023)

## 1192-35-32713

Vedansh Arya, University of Jyväskylä, Agnid Banerjee, TIFR CAM Bangalore, Donatella Danielli*, Arizona State University, Nicola Garofalo, University of Padova. Space-Like Strong Unique Continuation for Some Fractional Parabolic Equations.
In this talk we will present the space-like strong unique continuation property for a class of nonlocal equations, where the leading operator is the fractional heat. The proof of our main result is achieved via a conditional elliptic type doubling property for solutions to the appropriate extension problem, followed by a blowup analysis.
(Received September 12, 2023)

Diana Nguyen Son*, Florida International University. Stability of Solutions for a Class of KdV Equations. Preliminary report. We investigate stability of solitary waves and breather solutions for a wider class of Korteweg-De Vries type equations. We deduce from our results that the key factor in forming and maintaining breather solutions is the absolute value on the nonlinearity term. We further examine the orbital and asymptotic stability by simulating a range of perturbations and inducing interactions between different solutions, i.e. soliton-breather and breather-breather interactions. This research was done as part of the NSF/NSA REU program "AMRPU @ FIU".
(Received September 12, 2023)

## 1192-35-32799

Diana Nguyen Son*, Florida International University. Behavior of Solutions to the Generalized KdV with Low Power Nonlinearity. Preliminary report.
We study solutions to the two versions of the generalized Korteweg-De Vries equation, both with fractional powers and one of them with an absolute value incorporated into the nonlinearity, i.e., $u_{t}+u_{x x x}+u^{\alpha} u_{x}=0$ vs. $u_{t}+u_{x x x}+|u|^{\alpha} u_{x}=0$. We recall the well-posedness of these equations in a certain class of initial data, subset of $H^{1}$. In this talk we analyze and compare numerical simulations of solutions to both of these equations. We look at the soliton resolution and the interactions of solitary waves, considering different types of initial data decay, including polynomial, exponential, Gaussian, and super-Gaussian-types of decay. This research was done as part of the NSF/NSA REU program "AMRPU @ FIU". (Received September 12, 2023)

## 1192-35-32819

Donatella Danielli*, Arizona State University, Alaa Haj Ali, Arizona State University, Arshak Petrosyan, Purdue University. Regularity properties in obstacle-type problems for higher-order fractional powers of the Laplacian.
In this talk we will discuss a sampler of obstacle-type problems associated with higher-order fractional Laplacians. Our goals are to establish regularity properties of the solution and to describe the structure of the free boundary. To this end, we combine classical techniques from potential theory and the calculus of variations with more modern methods, such as the localization of the operator and monotonicity formulas.
(Received September 12, 2023)

## 1192-35-32821

Diana Nguyen Son*, Florida International University. Stability of Solutions for KdV-Type Equation. Preliminary report. We investigate stability of solitary waves and breather solutions for a larger class of Korteweg-De Vries type equations, which model surface waves in a shallow water. We look at the interaction of positive and negative solitary waves and also deduce that the key factor in forming and maintaining breather solutions is the absolute value on the nonlinearity term. We further examine the orbital and asymptotic stability by simulating a range of perturbations and inducing interactions between different solutions, i.e., soliton-breather and breather-breather interactions. This research was done as part of the NSF/NSA REU program "AMRPU @ FIU".
(Received September 12, 2023)

## 1192-35-32901

Mengxuan Yang*, UC Berkeley. Mathematics of twisted multilayer graphene.
In condensed matter physics, when two or more sheets of graphene are twisted by certain angles, a.k.a. magic angles, the resulting material becomes superconducting. In this talk, I will introduce the basic mathematical theory behind it, which is a blend of representation theory, Bloch-Floquet theory and holomorphic line bundles. I will also present some recent development in the field (using many pictures and animations).
(Received September 13, 2023)

## 1192-35-32946

Iryna Petrenko*, Florida International University. NLS with with higher order dispersion: beyond the standard tools. Preliminary report.
We discuss a nonlinear Schrödinger (NLS) equation with higher order dispersion, namely, the $k$-Laplacian $(-\Delta)^{k}, k$-integer, and with the potential term expressed as a power nonlinearity (with any positive power). When $k=1$ we recover the NLS with the standard Laplacian and when $k=2$ we get the bi-harmonic NLS equation. We study solutions to this higher order NLS equation on a subset of a Sobolev space by constructing a weighted space with certain weights. For that we introduce fractional power weights, for which we use Stein's derivative and commutator-type estimates. We then obtain linear estimates with fractional weights, apply them to the nonlinear setting and obtain the local well-posedness.
(Received September 13, 2023)

1192-35-32983
Alex David Rodriguez*, Florida International University. Nonlinear Schrödinger equation with infinitely many nonlinear terms. Preliminary report.
We study the NLS equation, $i u_{t}+\Delta u+\mathcal{N}(u) u=0, x \in \mathbb{R}^{N}$ and $t \in \mathbb{R}$, with nonlinear term $\mathcal{N}(u)$ term that is expressed as a (possibly infinite) sum of powers:

$$
\mathcal{N}(u)=\sum c_{n}|u|^{\alpha_{n}}, \alpha_{n}>0, c_{n} \in \mathbb{C}
$$

We investigate the local well-posedness in a particular weighted class of initial data, a subset of $H^{1}\left(\mathbb{R}^{N}\right)$, depending on the dimension $N$. Then, using the pseudo-conformal transformation, we extend the local result to global for initial data with a quadratic phase. Furthermore, we investigate the asymptotic behavior of such solutions, that is, for $u_{0}=e^{i b|x|^{2}}$ with various
values of $b$ we study the behavior of $u(t)$. In particular, we prove scattering of these solutions for large positive $b$ in 1 D . We also show numerical simulations in the focusing case for various examples of combined nonlinearities, including the exponential one, to confirm our analytical findings and extend them further. This talk is based on the joint work with Gia Azcotia, Oscar Riaño, Svetlana Roudenko and Hannah Wubben, which was initiated during the REU program "AMRPU @ FIU". (Received September 13, 2023)

## 1192-35-33005

Saja Gherri*, University of Michigan, Samantha Roberts, University of Utah. The Doubly Non-Local Cahn-Hilliard Equation with Fractional Time Derivative. Preliminary report.
The Cahn-Hilliard equation was developed to model phase separation in material science. We consider the doubly non-local form, which replaces the traditional Laplacian with two non-local operators. These non-local operators allow more accuracy in the prediction of behavior in the entire system, rather than locally. We then modify the doubly non-local Cahn Hilliard (dnCHE) with the addition of a fractional time derivative. Utilizing time convolution, we add the piecewise $g_{(1-\alpha)}$ function to incorporate the Caputo fractional time derivative. We establish both the existence and uniqueness of a solution to this modified equation. First, we define Picard Iterates and show that they converge to our solution. Then we use these iterates to prove that our solution is continuous in time and bounded in space, employing the Weierstrass M-test and uniform limit theorem, fulfilling requirements for existence in a space $Y$. We prove uniqueness with a standard proof by contradiction. Then, we employ a forward Euler scheme to portray numerical approximations of our solution. We vary time steps and initial conditions. Our results show the convergence of our solution to that of the original dnCHE, as our fractional order approaches 1.
Acknowledgement- The research of this project was started during the Summer 2022 REU program "AMRPU @ FIU" that took place at the Department of Mathematics and Statistics, Florida International University, and was supported by the NSA grant H982302210016 and NSF (REU Site) grant DMS-2050971.
(Received September 13, 2023)

## 1192-35-33036

Pramya Surapaneni*, High Technology High School. Neural Network Prediction of Ocean Wave Behavior Using Frequency Domain Mapping.
As ocean levels rise due to global warming, it is important to understand, visualize, and predict ocean wave behavior. Such predictions are an important component of emergency preparation. In recent years, key advances have been made in the application of neural networks toward weather related predictions. One neural network architecture, the Fourier Neural Operator (FNO), has shown great promise in image to image mapping where the output image represents the solution space of a physical system. In the current research, we use an FNO to map an ocean basin topography to the resulting wave height. We trained the FNO on a dataset generated from 1,600 simulations of the shallow water equations, where each simulation utilized a different arbitrarily-generated ocean basin topography. This type of simulation accurately depicts wave behavior away from the shoreline. Our setup is ideal for parallel computing. By parallelizing both the algorithm for the numerical method and the topography generation, we reduced computation time by 71.20
(Received September 13, 2023)

## 1192-35-33055

Svetlana Roudenko*, Florida International University. Solitary waves in Benjamin-Ono type equations. Preliminary report. One of the well-known water wave models is the Benjamin-Ono (BO) equation, which describes the internal waves in deep water. Coherent structures such as solitary waves in the BO equation are known to be stable. We first turn to the BO generalization such as the mBO (modified BO ) and the generalized version ( gBO ), with various powers of nonlinearity, to investigate solitary waves behavior, stability or instability. We then look at its two dimensional generalization, called Shrira equation, that describes long-wave perturbations in a boundary-layer shear flow in ocean waves, and examine the behavior of solitary waves, including stability and radiation regimes, including our numerical investigations of solitary waves formation and interaction.
(Received September 13, 2023)

## 1192-35-33093

Chandler Haight*, Florida International University. Breather Investigations of Korteweg-de Vries Type Equations. Preliminary report.
We characterize and investigate breather solutions to the Korteweg-de Vries (KdV) type equations, such as mKdV and Gardner, via numerical simulation, ranging from the integrable systems to Hamiltonian systems. Our numerical simulation results suggest that the absolute value in the potential term $|u|^{p-1} u_{x}$ is the key factor in forming the breather solutions in the generalized KdV (gKdV) case. Moreover, these breather solutions are stable under various perturbations. These numerical results are consistent with a few known theoretical stability results for the integrable systems indicating the future studies of the breather solutions for the general Hamiltonian systems.
(Received September 13, 2023)

## 1192-35-33113

Chandler Haight*, Florida International University. Investigations of Korteweg-de Vries Type Equation Breather Solutions. Preliminary report.
We characterize and investigate breather solutions to the Korteweg-de Vries (KdV) type equations via numerical simulation, ranging from the integrable systems to Hamiltonian systems. Our numerical simulation results suggest that the absolute value in the potential term $|u|^{p-1} u_{x}$ is the key factor in forming the breather solutions in the generalized KdV case. Moreover, these breather solutions are stable under various perturbations. These numerical results are consistent with a few known theoretical stability results for the integrable systems indicating the future studies of the breather solutions for the general Hamiltonian systems.
(Received September 13, 2023)

## 1192-35-33142

Lorena Bociu, NC State University, Evangelia Ftaka*, NC State University, Tien Khai Nguyen, North Carolina State University, Jacopo Schino, North Carolina State University. Piecewise smooth solutions to scalar balance laws with singular source terms. Preliminary report
We will present a local well-posed result for piecewise regular solutions with a single shock of scalar balance laws, with singular integral of convolution type kernels. In a neighborhood of the shock curve, a description of the solution is provided for a general class of initial data.
(Received September 13, 2023)

## 1192-35-33161

Kayla D Davie*, University of Maryland College Park. Reduced basis techniques for parameterized partial differential equations with constraints.
Model order reduction techniques have been used to study and reliably solve parametrized models involving PDEs. Discretizing these models results in large matrix systems of saddle-point form which must satisfy an inf-sup condition in order to be well-posed and solvable. Several approaches to reduced basis (RB) modeling of these problems result in saddle-point RB matrix systems. In these methods, the RB spaces and associated matrices are augmented to ensure inf-sup stability of the reduced system. We propose a new approach to constructing the RB matrix, a stacked RB matrix, that does not result in a saddle-point reduced system and eliminates the need for augmentation. The stacked RB matrix is of smaller order and requires less computational work to use in solving the online problem. We present numerical results to compare this approach to standard approaches of implementing RB methods and prove the efficiency of the proposed approach for solving numerous benchmark problems. (Received October 18, 2023)

## 1192-35-33230

Semon Rezchikov*, Harvard University. Jets of Fueter maps
Fueter maps $M^{3} \rightarrow X$ or $N^{4} \rightarrow X$ where $M^{3}$ is a framed 3 -manifold, $N^{4}$ is a framed 4-manifold, and $X$ has an almost quaternionic structure, are the quaternionic analog of pseudoholomorphic curves in almost complex manifold. The latter have a very useful analytic theory when the target is almost-Kahler, giving rise to the rich categorical invariants of symplectic topology. By analogy, Fueter maps are expected to give rise to a very rich algebraic structure associated to $X$ when $X$ is hyperkähler, including a 2 -category categorifying the Fukaya cartegory of $X$. However, their basic analytic properties have not been fully established. We will discuss some basic analytic properties of Fueter maps and their applications. By proving that there is a local Fueter map that is tangent to every 'jet' of a monogenic function, we will describe a quaternionic generalization of the statement that affine complex varieties have half-dimensional homotopy type.
(Received September 13, 2023)

## 1192-35-33474

Mario Javier Bencomo, University of California Fresno, Mikayla Leggett*, California State University, Fresno. A spectral method approach to solving the axisymmetric cylindrical heat equation with time-varying boundary conditions. Preliminary report.
Real-time monitoring and assessment of thermally processed food is a vital component of the processing workflow, especially at industry scales. Current standards overcook to ensure food safety, leading to energy waste. Our goal is to reduce energy consumption by developing a non-intrusive monitoring system that uses surface thermal data coupled with a mathematical model to accurately assess in real time if processed foods are cooked, thus optimizing cooking workflow. We model the thermal process of cooking using the heat equation, with surface data incorporated as boundary conditions, to estimate internal temperatures. In this talk we develop a computationally efficient approach for a simplified problem, the axisymmetric cylindrical heat equation. We apply the spectral method (i.e., separation of variables), yielding approximations that are truncated series expansions of related eigenfunctions and eigenvalues. Additionally, Duhamel's principle is leveraged to accommodate time-varying boundary conditions. To achieve a prescribed level of accuracy, we offer a way of determining the required number of terms in the truncates series of our spectral method solution. MATLAB is used for numerical integration and other spectral method computations. Our results offer valuable insights into the feasibility and potential benefits of this approach.
(Received September 18, 2023 )

## 1192-35-33757

Gia Azcoitia*, Florida International University. Nonlinear Schrodinger Equation with Combined Nonlinearities.
Ideas of quantum mechanics have fascinated scientists and academics for decades. While the jargon of quantum mechanics may seem absurd to the majority, one idea that almost all are familiar with is that of Schrödinger's cat. This ever so famous thought experiment of a cat in a box was proposed in order to demonstrate so-called "particle-wave duality." This was not the only wave phenomena Schrödinger studied, though. He is credited with the discovery of a differential equation that can be used to describe relativistic mechanics, referred to as the linear Schrödinger Equation. In the realm of modern mathematics, we study more complex scenarios with nonlinear terms such as one-dimensional nonlinear Schrödinger equation

$$
i u_{t}+u_{x x}+\mathcal{N}(u)=0, x, t \in \mathbb{R}
$$

with the $\mathcal{N}$ expressed as an infinite power series:

$$
\mathcal{N}(u)=\sum_{n=0}^{\infty} c_{n}|u|^{\alpha_{n}} u, \alpha_{n}>0
$$

This equation appears in various physical applications such as chemical super fluidity, or the description of elementary particles such as bosons and fermions, or other subatomic structures, and in approximations of anisotropic media. We first
investigate existence and uniqueness of solutions of this equation in a certain weighted class of initial data in the energy space. Then, using the pseudo-conformal transformation, we extend the local result to the global. Furthermore, we investigate the asymptotic behavior of global solutions, those that have initial data with a quadratic phase $e^{i b|x|^{2}}$ with sufficiently large positive $b$, and prove scattering in the energy space. One of the advantages of considering an infinite sum in the nonlinearity term is being able to consider exponential nonlinearity $e^{\alpha|u|} u$ or an oscillating one $\sin (|u|)$, and obtain solutions in those cases, the first such result. To conclude, we show numerical simulations for various cases of combined nonlinearities, including the exponential one, and investigate a threshold behavior for the global versus finite time existing solutions, which extends our theoretical results.
(Received September 25, 2023)

## 1192-35-33773

Brent Michael Christian*, Presenting author, Prabhakar Clement, Co-author, Mojdeh Rasoulzadeh, Co-author. Nonlocal transport in layered media: Role of interface of heterogeneities.
We present nonlocal effects in solute transport arising at the interface of a two-layered, finite heterogeneous porous medium. Two-scale method is applied to derive the analytical solutions to the advection-dispersion equation (ADE). Our solution addresses the asymmetrical transport observed in prior experiments. These experiments highlighted significant differences in breakthrough curves at the outlet for solute travel from fine to coarse ( FtC ) layers compared to coarse to fine (CtF) layers-a discrepancy the traditional ADE fails to explain. To account for varied dispersive and diffusive effects between the two layers, we introduce a small parameter, $\epsilon$, as the fine medium dispersion in the ADE. Leveraging this singular perturbation, we formulate both an inner solution, capturing transport behavior near the heterogeneous interface, and an outer solution. (Received September 25, 2023)

## 1192-35-33836

Emily Foley*, Wake Forest University. Noise Induced Tipping in the forced Sine-Gordon Equation. Preliminary report. The sine-gordon equation is a well studied nonlinear system with stationary and traveling wave solutions known as solitons. For this system, analytic solutions can be recovered using the inverse scattering transform. This project explores the dynamics of the sine gordon equation with periodic forcing and stochastic fluctuations. The primary objective is to understand the stability of soliton solutions obtained via the inverse scattering transform in relation to the amplitude and frequency of the periodic forcing, as well as the intensity of the noise. In the non-chaotic regimes, we determine if noise can cause this system to transition between various soliton and breather solutions.
(Received September 26, 2023)

## 37 Dynamical systems and ergodic theory

## 1192-37-26789

Yunied Puig*, Claremont McKenna College. A hierarchy of linear dynamical notions for pseudo-shifts.
Linear dynamics is an emerging field at the intersection of operator theory and functional analysis closely tied to the invariant subspace problem. Within this framework, we establish a strict hierarchy of linear dynamical notions, and provide a criterion that serves to characterize certain of these linear dynamical notions for the family of pseudo-shifts, a generalization of weighted backward shifts. This is joint work with Ozgur Martin and Quentin Menet.
(Received August 05, 2023)

1192-37-27296
Anna Marks*, Wake Forest University, Julio Santiago-Reyes, The College of New Jersey. Observability Analysis and Data Assimilation Design for a Nonlinear Model of Cardiac Alternans.
Life-threatening cardiac arrhythmias can be caused by disruptions in electrical waves that govern the contractions of the heart. Certain arrhythmias are preceded by alternans, which is a beat-to-beat (period-2) alternation in these electrical waves. Alternans arising from instabilities in either the dynamics of the intracellular calcium concentration or in the potential drop across the cell membrane are called, respectively, calcium-driven or voltage-driven alternans. Many technologies, including patch-clamp devices, have been used to gain insight into alternans mechanisms (including the calcium- and voltage-driven cases), but these methods cannot capture all quantities of interest simultaneously. Some researchers have developed data assimilation algorithms to fill in gaps in cardiac data sets, but the impact of alternans mechanisms on data assimilation methods has not yet been addressed in the literature. To address this shortcoming, we examined a nonlinear ODE model that replicates calcium and voltage- driven alternans. Using computational methods from control theory, we studied the effects of different alternans mechanisms on a model property called observability, which indicates how well a system's state variables can be determined from a given measurement. We examined two simulated measurements: cellular membrane potential ( $V$ ) and intracellular free calcium concentration ( $c_{i n t}$ ). Although observability strengths were affected by alternans mechanisms, we found that $c_{i n t}$ was a more informative measurement than $V$, in the sense of maximizing observability, for both mechanisms. We designed linear data assimilators (Kalman Filters or KFs) and confirmed that observability typically predicted KF performance accuracy. Additionally, we tested several successful preliminary nonlinear assimilator designs. Our results could eventually provide researchers with refined data reconstruction algorithms for a more in-depth study of arrhythmia formation.
(Received August 13, 2023)

1192-37-27379
Edward Anthony Beck, University of Florida, Kathryn Grace Cantrell*, Loyola University Chicago. Using Agent-Based Modeling to Understand Biofilm Growth and Eradication by Antibiotics and Phages Respectively Global leaders in public health have recognized the need for alternate treatments against antibiotic-resistant bacteria. Phage therapy, which uses bacteriophages to target specific bacterial pathogens, has become one of the most promising solutions. Bacteria can thrive in micro-communities called biofilms, whose structural importance suggests the need for a model which
incorporates spatial effects. Our objective in this study is to use mathematical and statistical tools to identify optimal treatment strategies as they relate to phage-antibiotic combined intervention. We developed an agent-based model to capture the effect of environmental factors such as temperature, pH value, and resources on bacteria-phage interactions during phage therapy for healthy and immunocompromised patients. Furthermore, we investigated and compared antibiotic-based, phage-based, and combined treatment strategies. Using Escherichia coli (E. coli) as a case study, we found that some of the bacterial cells mutate during growth, resulting in adaptation to changes in temperature, pH , and treatment strategy. Treatment of pathogenic and mutated bacterial cells can be enhanced by controlling application time and dose amount of treatments. Lastly, we explored the role of biofilm structures in phage therapy.
(Received August 14, 2023)

1192-37-27425
Nishant Malik, co-author, Isamar Solorio, co-author, Braden Yates*, co-author. Synchronization in Adaptive Networks of Type I Neurons. Preliminary report.
The theta neuron is a model of Type-I neural excitability based on the normal form for a saddle-node bifurcation involving only a single variable, thus allowing one to study large networks of interacting neurons. A critical neurobiological phenomenon that occurs in such networks is synchronization, which plays various roles in brain functioning. Although a wide range of studies exist on synchronization in neuronal ensembles, there has been little investigation into the effects of dynamic neural connections on synchronization patterns in ensembles of Type-I neurons. Connections among neurons are inherently dynamic, with the ability to dynamically adapt their strengths through synaptic plasticity. Here we study synchronization patterns in networks of theta neurons with adaptive synaptic strengths, exploring various initial network topologies such as the ErdösRényi and Watts-Strogatz graphs. For these adaptive networks of neurons, we carry out simulations for numerous parameter settings and demonstrate a variety of synchronization patterns, including the phenomena of chimera states. Finally, we characterize parameter ranges that lead to each behavior.
(Received August 14, 2023)

## 1192-37-27601

Aimee S A Johnson*, Swarthmore College, David M. McClendon, Ferris State University. Odometer factors of rank-one $\mathbb{Z}^{d}$ actions.
In this talk, we make use of the cutting and stacking construction of a rank one $\mathbb{Z}^{d}$-action to determine whether or not it factors onto a given $\mathbb{Z}^{d}$-odometer. Similar criteria will be discussed for determining conjugacy to this, or in fact any, $\mathbb{Z}^{d}$ odometer. This extends results of Foreman, Gao, Hill, Silva, and Weiss.
(Received August 16, 2023)

## 1192-37-27661

Alejandro Bravo-Doddoli*, University of Michigan. Metric lines in jet space.. Preliminary report.
Given a sub-Riemannian manifold, a relevant question is: what are the metric lines (isometric embedding of the real line)? The space of $k$-jets of a real function of one real variable $x$, denoted by $J^{k}(\mathbb{R}, \mathbb{R})$, admits the structure of a Carnot group, as every Carnot group $J^{k}(\mathbb{R}, \mathbb{R})$ is a sub-Riemannian Manifold. This talk is devoted to provide a partial result about the classification of the metric lines in $J^{k}(\mathbb{R}, \mathbb{R})$. The method to prove the main reasult is to use an intermediate 3-dimensional sub-Riemannian space $\mathbb{R}_{F}^{3}$ lying between the group $J^{k}(\mathbb{R}, \mathbb{R})$ and the Euclidean space $\mathbb{R}^{2} \simeq J^{k}(\mathbb{R}, \mathbb{R}) /\left[J^{k}(\mathbb{R}, \mathbb{R}), J^{k}(\mathbb{R}, \mathbb{R})\right]$. (Received August 17, 2023)

1192-37-27797
Michael Bersudsky, The Ohio State University, Hao Xing*, The Ohio State University. Limiting distribution of dense lattice orbits in a space of discrete subgroups of the Euclidean space.
It is common that a lattice subgroup $\Gamma$ of a Lie group $G$ has dense orbits in a homogeneous space $G / H$. Then, it is natural to ask about the limiting distribution of the dense orbits with respect to a filtration given by increasing "balls" in $\Gamma$. The problem was solved in great generality for many different types of subgroups $H$, but an interesting case which wasn't addressed so far is in the setting that $H$ has infinitely many non-trivial connected components. I will talk about my joint work with Michael Bersudsky where we solve a non-trivial case of this problem. Specifically, in our case $G / H$ describes the space of rank-m discrete subgroups of covolume one of $\mathbb{R}^{m+1}$, where every orbit of a every lattice subgroup is dense. I will describe our main results and give an overview of the main methods. If time permits, I will talk about further questions and generalizations of this problem.
(Received August 20, 2023)

1192-37-27883
Toni Dolph, Columbus Collegiate Academy, Sarah Frick, Furman University, Nicholas Ormes*, University of Denver. Adic Dynamics on the Catalan Graph. Preliminary report.
The Catalan graph is a Bratteli diagram in which the Catalan numbers arise as path counts. This infinite graph is the right half of the Pascal graph, on which the adic map has been well-studied. For the adic map on the Catalan graph, we characterize the invariant measures and prove a number of results about the dynamics and complexity of the associated measure-theoretic dynamical systems. The invariant measures here differ from those in the Pascal and many other well-known examples in that the edge weights for invariant measures depend not only on the level, but also the index of the source and range vertices. (Received August 21, 2023)
matics-ComputationalBiology"Master'sProgramattheDepartmentofBiologyoftheNationalandKapodistrianUniversityofAthensfortheirgeneroussupport(ProjectCode:70/3/15427)InthispaperweconsiderthefollowingstochasticfractionaldissipativeKleinGordonSchrödingertypesystem(SFKGSforshort)throughYukawacouplinginonespacedimensionundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefined

$$
\begin{gathered}
i d \psi-\left((-\Delta)^{\alpha} \psi+i \gamma \psi+\phi \psi\right) d t=f d t+\psi d W_{1} \\
d \phi_{t}+\left((-\Delta)^{\alpha} \phi+\phi+\lambda \phi_{t}+\operatorname{Re}(-\Delta)^{\frac{a}{2}} \psi\right) d t=g+d W_{2} \\
\psi(x, 0)=\psi_{0}, \phi(x, 0)=\phi_{0}, \phi_{t}(x, 0)=\phi_{1}
\end{gathered}
$$

for $\alpha \in\left(\frac{1}{2}, 1\right)$ and where $\psi(x, t)$ is a complex valued function and $\phi(x, t)$ is a real valued function and $\gamma>0, \lambda>0$, with $f, g \in L^{2}$ and $W_{1}, W_{2}$ two independent two-side real valued Wiener processes on a probability space which will be specified later. $(-\Delta)^{a}$ denotes the fractional Laplacian which is defined as $\widehat{(-\Delta)^{a}} u(t, \xi)=|\xi|^{2 a} \hat{u}(t, \xi)$, where ${ }_{\wedge}$ is the Fourier transform with respect to $x$. The variable $\psi$ stands for the dimensionless low frequency electron field, whereas $\phi$ denotes the dimensionless low frequency density. This system describes the nonlinear interaction between high frequency electron waves and low frequency ion plasma waves in a homogeneous magnetic field, adapted to model the UHH plasma heating scheme. The term $R e \psi_{x}$ is a consequence of the different low frequency coupling that was considered, i.e. the polarization drift instead of the ponderomotive force. The system focuses on the vital role of collisions by considering the non-homogeneous polarization drift for the low frequency coupling. A motivation for the noise in these specific equations could be the effect of the space-time varying electric field on the ion channel
(Received August 23, 2023)

## 1192-37-28203

Robert Bland*, UNC Charlotte. An embedding theorem for SFTs over groups with comparison.
Given subshifts $X$ and $Y$ over a group $G$, under what conditions does $X$ embed into $Y$ ? In 1983, Krieger gave an answer to this question in the case where $G=\mathbb{Z}$ (namely, that a subshift $X$ embeds into a mixing SFT $Y$ if and only if $h(X)<h(Y)$ and $\operatorname{Per} \hookrightarrow \operatorname{Per}(Y)$ ). This result has become a cornerstone of the structure theory of SFTs over $\mathbb{Z}$. In 2003, Lightwood gave an answer in the case where $G=\mathbb{Z}^{d}$ for $d \geq 2$ (namely, that a strongly aperiodic $X$ embeds into a square mixing SFT $Y$ when $h(X)<h(Y)$ and $Y$ contains a factor of $X$ and a point with a finite orbit). In this talk, I will present a result extending Lightwood's approach to the case where $G$ is an amenable group with the comparison property. The proof relies on on recent developments in the theory of tilings and quasi-tilings of amenable groups due to Downarowicz and Zhang. (Received August 28, 2023)

## 1192-37-28209

Ty Frazier*, University of Minnesota. Neural Networks Applied to ODE's.
Differential equations (DE) are a great tool for modeling almost anything, however, they are not always analytically solvable. This is where numerical analysis (NA) plays a role to be able to get approximate solutions to DEs. NA has been excellent over the decades, once we obtained the computational power to use it. The issue we are running into now with NA being applied to DEs is that the computational time needed to produce some of these solutions is starting to get in the way of the results we wish to gather insights from. The field of equation learning aims to subvert this problem by using neural networks to become the surrogate model for the solution operator and with the neural network's fixed computational time, this seems like a reasonable path to explore. In this talk we will go through our contribution to this with neural networks and ordinary differential equations. (Received August 28, 2023)

## 1192-37-28412

Ko W Ohm, University of California San Diego, Anthony Sanchez*, University of California San Diego. Quantitative finiteness of hyperplanes in hybrid manifolds. Preliminary report.
The geometry of non-arithmetic hyperbolic manifolds is mysterious in spite of how plentiful they are. McMullen and Reid independently conjectured that such manifolds have only finitely many totally geodesic hyperplanes and their conjecture was recently settled by Bader-Fisher-Miller-Stover in dimensions larger than 3. Their works rely on superrigidity theorems and are not constructive. In this talk, we strengthen their result by proving a quantitative finiteness theorem for non-arithmetic hyperbolic manifolds that arise from a gluing construction of Gromov and Piatetski-Shapiro. The proof relies on a number of ideas including an effective density theorem for certain periodic orbits and a restricted projection theorem. This is joint work with K. W. Ohm. (Received August 30, 2023)

## 1192-37-28594

Joseph H. Silverman*, Brown University. Isotrivial Markoff-type K3 surfaces and orbits over finite fields. Preliminary report. We consider K3 surfaces in $\mathbb{P}^{1} \times \mathbb{P}^{1} \times \mathbb{P}^{1}$ given in affine coordinates by equations of the form

$$
\mathcal{W}_{a, b, c, d, e}: a x^{2} y^{2} z^{2}+b\left(x^{2} y^{2}+x^{2} z^{2}+y^{2} z^{2}\right)+c x y z+d\left(x^{2}+y^{2}+z^{2}\right)+e=0 .
$$

Each of the projections $\pi_{1}, \pi_{2}, \pi_{3}: \mathcal{W}_{a, b, c, d, e} \rightarrow \mathbb{P}^{1}$ gives $\mathcal{W}_{a, b, c, d, e}$ the structure of a fibration of genus 1 curves, which are isomorphic due to the $\mathcal{S}_{3}$-symmetry of the equation defining the surface. Let $\mathcal{J}_{a, b, c, d, e} \rightarrow \mathbb{P}^{1}$ denote the Jacobian of any one of these fibrations. We will characterize the parameters $[a, b, c, d, e] \in \mathbb{P}^{5}$ for which $\mathcal{J}_{a, b, c, d, e}$ is isotrivial, but not split, and we will discuss the automorphism orbit structure of $\mathcal{W}_{a, b, c, d, e}\left(\mathbb{F}_{q}\right)$ for the isotrivial parameters. We note that the isotrivial
$\mathcal{W}_{a, b, c, d, e}$ are natural K3 analogues of classical Markoff surfaces $\mathcal{M}$, for which the three projections $\mathcal{M} \rightarrow \mathbb{A}^{1}$ give $\mathcal{M}$ the structure of a $\mathbb{G}_{m}$-torsor.
(Received August 31, 2023)

## 1192-37-28777

Jayadev S. Athreya*, University of Washington. Counting for Invariant Point Processes. Preliminary report.
Using linear algebra and the ergodic theory of $\operatorname{SL}(2, R)$ actions, we survey how to solve several natural asymptotic counting problems for discrete subsets of the plane using an axiomatic perspective. Applications include counting holonomies of saddle connections, lattice points, and fine scale distribution in various contexts. This is a perspective inspired by work of Veech, and developed further by, among others, Eskin-Masur and Athreya-Ghosh.
(Received September 03, 2023)

## 1192-37-28790

John T. Griesmer*, Colorado School of Mines. A hierarchy of rigidity properties for measure preserving systems.
A sequence of integers $\left(n_{k}\right)_{k \in \mathbb{N}}$ is a rigidity sequence for a probability measure preserving system (PMPS) $(X, \mu, T)$ if for every measurable $A \subset X, \lim _{k \rightarrow \infty} \mu\left(A \triangle T^{-n_{k}} A\right)=0$. A classical result in ergodic theory says that the generic measure preserving transformation of $[0,1]$ with Lebesgue measure is weak mixing and also admits a rigidity sequence. Recent work has focused on understanding exactly which sequences $\left(n_{k}\right)_{k \in \mathbb{N}}$ are rigidity sequences for some weak mixing PMPS, and many interesting open questions remain. In this talk we will introduce two natural variants of rigidity: (i) Given a PMPS $(X, \mu, T)$ and $A \subset X$ we say $\left(n_{k}\right)_{k \in \mathbb{N}}$ is union rigid for $A$ and $T$ if $\lim _{K \rightarrow \infty} \mu\left(\bigcup_{k>K} T^{-n_{k}} A\right)=\mu(A)$. (ii) We say $\left(n_{k}\right)_{k \in \mathbb{N}}$ is summably rigid for $A$ and $T$ if $\sum_{k=1}^{\infty} \mu\left(A \triangle T^{-n_{k}} A\right)$ converges. It is easy to see that every summably rigid sequence for some $A$ and a weak mixing $T$ is union rigid for the same $A$ and $T$, and that every union rigid sequence for some $A$ and a weak mixing $T$ is also a rigidity sequence for a weak mixing PMPS. We will explain how recent results on recurrence properties show that the reverse inclusions are both false. We will also see that all the recently studied open questions on rigidity sequences extend naturally to the above variants, and that resolving such extensions may shed light on the original questions for rigidity sequences.
(Received September 03, 2023)

## 1192-37-28794

Lorenzo A Sadun*, University of Texas, Austin. Dynamics and Topology of the Hat and Spectre Tilings.
The celebrated Hat tilings come in a 1-dimensional family; you can vary the relative length of two types of edges while preserving the combinatorics. This can be extended to a 4 -dimensional family at the cost of breaking reflectional symmetry. All tilings in the extended family give rise to topologically conjugate dynamics, up to an overall linear scale and rotation. All have pure point spectrum. All can be described by a $4: 2$ cut and project scheme with a surprisingly simple window, albeit with different projections from 4 to 2 dimensions. The key to all of this is understanding the first cohomology of the tiling space, which is as simple as it could possibly be. Almost identical comments apply to the Spectre tilings. This is joint work with Michael Baake and Franz Gähler.
(Received September 03, 2023)

## 1192-37-28806

Cristian Ramirez*, University of California, Berkeley, Amy Somers*, University of California, Santa Barbara. ( $S, w$ )-Gap Shifts and Their Entropy.
Given an $S \subset \mathbb{Z}_{\geq 0}$, an $S$-gap shift is defined to be the shift space consisting of all sequences in $\{0,1\}^{\mathbb{Z}}$ such that any two 1 's are separated by a word $0^{n}$ for some $n \in S$. The $S$-gap shifts have a dynamically and combinatorially rich structure. Dynamical properties of the $S$-gap shift can be related to the properties of the set $S$. This interplay is particularly interesting when $S$ is not syndetic such as the example when $S$ is the set of prime numbers or when $S=\left\{2^{n}\right\}$. It is a well known result that the entropy of the $S$-gap shift is given by $h(X)=\log (\lambda)$, where $\lambda>0$ is the unique solution to the equation $\sum_{n \in S} \lambda^{-(n+1)}=1$. Fix a point $w$ of the full shift $\{0,1\}^{\mathbb{Z}}$. We introduce the $(S, w)$-gap shift which is a generalization of the $S$-gap shift consisting of sequences in $\{0,1,2\}^{\mathbb{Z}}$ in which any two 2 's are separated by a subword $u$ of $w$ such that $|u| \in S$. We extend the formula for the entropy of the $S$-gap shift to a formula describing the entropy of this new class of shift spaces. Additionally we investigate the dynamical properties including irreducibly and mixing of this generalization of the $S$-gap shift. (Received September 03, 2023)

## 1192-37-28818

M Michael Boyle*, University of Maryland, David Handelman, University of Ottawa. The ordered cohomology groups of zero dimensional systems. Preliminary report.
Let T be a self homeomorphism of a compact zero dimensioaal metric space X . The group $\mathrm{C}(\mathrm{X}, \mathrm{Z}) /(\mathrm{I}-\mathrm{T}) \mathrm{C}(\mathrm{X}, \mathrm{Z})$ in a natural way becomes a unital ordered group, playing an important role in the theory of orbit equivalence of minimal systems, due to work by Giordano, Herman, Matui, Putnam and Skau. The unital ordered groups which arise in the minimal case are the simple unital dimension groups. I'll discuss a variety of results and constructions in the general case.
(Received September 03, 2023)

Sara M Clifton*, Kenyon College, Dayo Ogunmodede, St. Olaf College. Bystander effect as an emergent property of individual psychological prospects. Preliminary report.
The bystander effect is a sociological phenomenon in which individuals are less likely to help a person in need if there are others present. Sociologists and psychologists have proposed multiple plausible reasons for the bystander effect, from
ambiguity and group cohesiveness to diffusion of responsibility and mutual denial. We build a dynamical systems model based on these sociological and psychological hypotheses, along with ideas borrowed from behavioral economics; in particular, we use prospect theory to predict an individual's decision to take action or not. With this model, we find the conditions under which a bystander effect emerges from these individual decisions.
(Received September 22, 2023)

1192-37-28941
Kimberly Ayers*, California State University, San Marcos, Ami Radunskaya, Pomona College. Stability of the Invariant Distribution of the Random Logistic Map. Preliminary report.
The logistic map $f(x)=\lambda x(1-x)$ is a famous example of a chaotic dynamical system in one dimension, when lambda takes values between 3.87 and 4 . Rather than considering the deterministic system, we will consider the stochastic system where each lambda is iid according to some absolutely continuous distribution on [3.87,4]. It has been previously shown that this new process, which is Markovian, demonstrates Feller continuity and thus has an invariant distribution. We improve upon this result to show that not only, is there an invariant distribution, but the invariant distribution is unique and asymptotically stable. Simulations and visualizations of the invariant distribution are included.
(Received September 04, 2023)

## 1192-37-28965

Casey Lynn Johnson*, University of California, Los Angeles. The Effects of Peer Pressure on Bounded-Confidence Models of Opinion Dynamics. Preliminary report.
As people interact with each other, their opinions will change over time. In a bounded-confidence model (BCM) of opinion dynamics, individuals, represented by the nodes of a network, have continuous-valued opinions and are influenced by their neighboring nodes if and only if their opinions are sufficiently similar. In the standard Hegselmann - Krause (HK) BCM, individuals whose opinions are not similar enough are completely ignored. In a social network, people do not always ignore drastically differing opinions. If a person interacts with enough people who have a drastically different opinion, they are affected by peer pressure and alter their opinion. We study the effects of adding peer pressure to the HK BCM. (Received September 04, 2023)

## 1192-37-29131

Darren Creutz, US Naval Academy, Ronnie Pavlov*, University of Denver. 3/2 as a threshold for linear word complexity of subshifts.
The word complexity function $p(n)$ of a subshift $X$ measures the number of $n$-letter words appearing in sequences in $X$, and $X$ is said to have linear complexity if $p(n) / n$ is bounded. It's been known since work of Ferenczi that a minimal subshift $X$ with linear word complexity function must have highly constrained/structured behavior. I'll discuss several results from recent years which demonstrate that an important transition takes place around $\lim \sup p(n) / n=3 / 2$. In particular, in recent work with Darren Creutz, we have shown that every minimal subshift with $\lim \sup p(n) / n<3 / 2$ has (measurably) discrete spectrum, i.e. is isomorphic to a rotation of a compact abelian group. This bound is tight, as we also showed that $\lim \sup p(n) / n=3 / 2$ can occur for subshifts which are (measurably) weakly mixing, i.e. whose only group rotation factor is trivial. This work answered an open question of Ferenczi on the lowest possible word complexity for weakly mixing subshifts. (Received September 05, 2023)

## 1192-37-29201

Shasha Gao, University of Florida, Maia Nenkova Martcheva, University of Florida, Libin Rong, University of Florida, Mingwang Shen, Xi'an Jiaotong University, China, Jin Wang, UTC, Xueying Wang*, Washington State University. A multistrain model with asymptomatic transmission: Application to COVID-19 in the US.
COVID-19, induced by the SARS-CoV-2 infection, has caused an unprecedented pandemic in the world. New variants of the virus have emerged and dominated the virus population. In this paper, we develop a multi-strain model with asymptomatic transmission to study how the asymptomatic or pre-symptomatic infection influences the transmission between different strains and control strategies that aim to mitigate the pandemic. Both analytical and numerical results reveal that the competitive exclusion principle still holds for the model with the asymptomatic transmission. By fitting the model to the COVID-19 case and viral variant data in the US, we show that the omicron variants are more transmissible but less fatal than the previously circulating variants. The basic reproduction number for the omicron variants is estimated to be 11.15, larger than that for the previous variants. Using mask mandate as an example of non-pharmaceutical interventions, we show that implementing it before the prevalence peak can significantly lower and postpone the peak. The time of lifting the mask mandate can affect the emergence and frequency of subsequent waves. Lifting before the peak will result in an earlier and much higher subsequent wave. Caution should also be taken to lift the restriction when a large portion of the population remains susceptible. The methods and results obtained her e may be applied to the study of the dynamics of other infectious diseases with asymptomatic transmission using other control measures. (Received September 05, 2023)

1192-37-29299
Tom Meyerovitch, Ben Gurion University of the Negev, Israel, Shrey Sanadhya*, Ben Gurion University of the Negev, Israel. Universality for $\mathbb{R}^{d}$-flows. Preliminary report.
A dynamical system is called universal if any system with lower entropy can be embedded into it. In this talk, we will discuss universality for flows both in ergodic and Borel contexts. We will discuss a specification type property that implies universality for flows and provide an example of a tiling dynamical system with this property. This is ongoing work with Tom Meyerovitch. (Received September 06, 2023)

Zalman Balanov, The University of Texas at Dallas, Mathematics Department, Wieslaw Krawcewicz, The University of Texas at Dallas, Mathematics Department, Arnaja Mitra*, The University of Texas at Dallas, Mathematics Department, Dmitry Rachinskiy, The University of Texas at Dallas, Mathematics Department. Equivariant Global Hopf Bifurcation in Abstract Nonlinear Parabolic Equations.
We propose a method for studying symmetric global Hopf bifurcation problems in a parabolic system. The objective is to detect unbounded branches of non-constant periodic solutions that arise from an equilibrium point and describe their symmetric properties in detail. The method is based on the twisted equivariant degree theory, which counts orbits of solutions to symmetric equations, similar to the usual Brouwer degree, but according to their symmetric properties.
(Received September 06, 2023)

## 1192-37-29304

Scott R Kaschner*, Butler University. Geometric Limits for Polynomial Skew Products. Preliminary report.
For a polynomial skew product of two complex variables, $f$, we consider the family of maps given as the sum of $f(z, w)$ and $\left(z^{n}, w^{n}\right)$. I will give a brief history of related problems and outline results regarding the limiting dynamics of this family as $n$ approaches infinity. Specifically, I will give a general description of the limiting set of points with bounded orbit when $f$ is Axiom A.
(Received September 06, 2023)

## 1192-37-29450

Daniel Richard Levy, University of Maryland, Erin Okey, McMaster University, Paige Yeung*, Massachusetts Institute of Technology. Front Dynamics in a Reaction-Diffusion Model for Tumor Growth. Preliminary report.
In reaction-diffusion models of tumor growth, an invading tumor can be represented by a traveling front. While such fronts can be stable when restricted to one spatial dimension, instabilities may develop when viewing the front as a two-dimensional interface. To explore such phenomena, we consider a modified version of the three-component reaction-diffusion model of tumor growth introduced by Gatenby and Gawlinski (1996). We use geometric singular perturbation and numerical methods to study the instability of planar interfaces in this model and understand the impact of phenomenologically relevant parameter values on the resulting front dynamics.
(Received September 22, 2023)

## 1192-37-29481

Taylor McAdam*, Pomona College. Slope gap distributions of translation surfaces: A story of dynamics, geometry, and numbers.
How "random" are the rational numbers? To make sense of this question, let us consider the set of Farey fractions of level $n-$ that is, the rational numbers between 0 and 1 with denominator at most $n$. It turns out that these distribute uniformly in the unit interval as $n$ goes to infinity, which would suggest they appear to be quite random. However, we may consider a finer test of randomness by considering the distribution of gaps between consecutive Farey fractions as $n$ tends to infinity. To investigate this, we will first realize the Farey fractions as the slopes of geodesic paths on the square torus. We will then define the horocycle flow on the space of all flat tori, which will allow us to study our question about gaps between Farey fractions via a dynamical system following the method of Athreya-Cheung. Finally, we will see how this method can be generalized to study the slope gap distributions for paths on a larger class of geometric objects called translation surfaces and discuss original results on collections of surfaces created by gluing together the sides of regular polygons.
(Received September 06, 2023)

## 1192-37-29548

Tung T. Nguyen*, Western University. Broadcasting solutions on multilayer networks of phase oscillators. Networks of nonlinear oscillators have attracted interest in several scientific domains, such as theoretical physics, mathematical biology, power-grid systems, and many more. In this domain, the Kuramoto oscillator has emerged as the central mathematical model for studying these networks. In our examination of the Kuramoto model, multilayer networks appear regularly, providing new and exciting phenomena. The problem of finding a reduced, simpler version of these complicated multilayer systems has sparked a lot of interest due to its wide range of applications. In this talk, we will explain a broadcasting mechanism on a multilayer network that provides a partial answer to this question. Furthermore, by leveraging tools from algebraic graph theory and non-commutative algebra, we will perform an extensive stability analysis of the broadcasted solutions. This talk is based on joint work with Roberto Budzinski, Federico Pasini, Robin Delabays, Ján Mináč, and Lyle Muller.
(Received September 07, 2023)

1192-37-29571
Eleanor Sophia Waiss*, Butler University. Geometric Limits of Julia Sets of Sums of Iterates and Powers of Polynomials. Preliminary report.
For maps of one complex variable, $f$, the geometric limit as $n$ approaches infinity of the set of points that remain bounded under iteration by $f$ has been studied in a variety of contexts. We will provide a summary of these results and some new generalizations. For maps given as the sum of two polynomials of which one is raised to this $n$, we show that the limit is almost always determined by preimages of the unit disk by the indexed polynomial. We will also discuss the limiting dynamics of maps given as the sum of a polynomial iterated $n$ times and a fixed polynomial.
(Received September 07, 2023)

1192-37-29584
Eleanor Sophia Waiss*, Butler University. Geometric Limits of Julia Sets of Sums of Powers of Polynomials. Preliminary report.

For maps of one complex variable, $f$, the geometric limit as $n$ approaches infinity of the set of points that remain bounded under iteration by $f$ has been studied in a variety of contexts. We will provide a summary of these results and some new generalizations. For maps given as the sum of two polynomials of which one is raised to this $n$, we show that the limit is almost always determined by preimages of the unit disk by the indexed polynomial. We will also discuss the limiting dynamics of maps given as the sum of a polynomial iterated $n$ times and a fixed polynomial.
(Received September 07, 2023)

## 1192-37-29645

Michelle Catherine LeMasurier*, Hamilton College. Bratteli Diagrams for Bounded Topological Speedups.
A bounded topological speedup of a Cantor minimal system $(X, T)$ is a minimal system $(X, S)$, where $S(x)=T^{p(x)}(x)$ for some bounded function $p: X \rightarrow \backslash \mathrm{z}^{+}$, or any system topologically conjugate to such an $(X, S)$. Assuming the system $(X, T)$ is represented by a properly ordered Bratteli diagram $\mathcal{B}$, we provide a method for constructing a new, perfectly ordered Bratteli diagram $\widetilde{\mathcal{B}}$ that represents the sped-up system $(X, S)$. The diagram $\widetilde{\mathcal{B}}$ relates back to $\mathcal{B}$ in a manner that enables us to see how certain dynamical properties are preserved under speedup. As an application, in the case that $(X, T)$ is a substitution minimal system, we show how to use $\widetilde{\mathcal{B}}$ to write an explicit substitution rule that generates the sped-up system $(X, S)$, answering an open question from Alvin, Ash, Ormes 2018.
(Received September 07, 2023)

## 1192-37-29789

Thomas Schmidt, Oregon State University, Mesa E Walker*, Oregon State University. Two Distinct Pseudo-Anosov Maps of Odd Degree. Preliminary report.
In 1981, Arnoux and Yoccoz constructed the first known family of odd degree pseudo-Anosov homeomorphisms. In 1985, David Fried constructed another family of pseudo-Anosov homemorphisms using completely different methods: studying cross sections of the mapping torus of a toral automorphism with periodic orbits blown up. Fried's genus 3 homeomorphism and Arnoux and Yoccoz's genus 3 homeomorphism both have the same stretch factor, and Fried asked if these maps are distinct or the same. We show that these maps are in fact distinct by showing that the mapping torus of Arnoux and Yoccoz's pseudoAnosov homeomorphism does not have Fried's blown up torus as a cross section. (Received September 07, 2023)

## 1192-37-29842

Alan Albert Sola*, Stockholm University. Dynamics of rational inner skew-products. Preliminary report.
We discuss dynamics of skew-products of the form $f=\left(f_{1}(x), f_{2}(x, y)\right)$, where $f_{1}$ is a finite Blaschke product in the unit disk, and $f_{2}$ is a rational inner function in the unit bidisk. The transformation $f$ maps the bidisk into itself and fixes the
distinguished boundary $\mathbb{T}^{2}$, which allows us to visualize the action of iterates. The talk will focus on restricted classes of RISPs, in particular the case when $f_{1}=x$ and $f_{2}$ has bidegree ( $n, 1$ ) (joint work with R . Tully-Doyle, Cal Poly), and the case when $f$ has the maximal possible number of fixed points on $\mathbb{T}^{2}$ (joint work with R. Birkett, UIC; J. Raissy, Bordeaux; and L. Vivas, Ohio State.)
(Received September 08, 2023)

1192-37-29849
Xinyao Yang*, Xi'an Jiaotong Liverpool University. Optimizing the allocation of carbon and nitrogen between microbes and plant roots.
Plant root systems release a diverse array of chemicals into the soil, a phenomenon known as root exudation. This process is believed to significantly enhance the activity of microorganisms and the production of exoenzymes, thereby accelerating carbon mineralization and nutrient cycling in rhizosphere soils when compared to bulk soils. However, the microbial biomass and the nitrogen content of these exoenzymes can potentially impose stoichiometric constraints on the efficient utilization of root exudates, especially when the exudates are abundant in carbon but deficient in nitrogen. In light of this ecological theory, we have developed an innovative mathematical model designed to simulate the dynamic allocation of carbon and nitrogen within the soil, involving plant roots and microorganisms. This model has been meticulously optimized using dynamic systems theory to address and mitigate potential limitations arising from the nitrogen content of exudates.
(Received September 08, 2023)

1192-37-29861
Henk Bruin, University of Vienna, Silvia Radinger*, University of Vienna. Interval Translation Maps with Weakly Mixing Attractors.
In 2003 H . Bruin and S. Troubetzkoy studied a renormalization map for a two-parameter family of interval translation maps. For a non-typical subset of the parameter space the interval translation map has a Cantor attractor. The renormalization G is a procedure similar to the Rauzy induction. It acts as dynamics on the parameter space and can be used to find the attractor. In this talk we further study these systems, focusing on weak mixing. We look at the symbolic representation of the interval translation map to define a S-adic subshift and use results about the eigenvalues of Bratteli-Vershik systems to determine whether the interval translation map is weakly mixing. Additionally we characterize the subset of linearly recurrent interval translation maps and their eigenvalues. This is joint work with Henk Bruin.
(Received September 08, 2023)

1192-37-29896
Drew Ash*, Albion College, Andrew T Dykstra, Hamilton College, Michelle Catherine LeMasurier, Hamilton College. Bratteli Diagrams for Bounded Topological Speedups II.

Given a dynamical system $(X, T)$ one can define a speedup of $(X, T)$ as another dynamical system $S: X \rightarrow X$ where $S=T^{p(\cdot)}$ for some $p: X \rightarrow \mathbb{Z}^{+}$. In this talk, we restrict our attention to bounded topological speedups of minimal Cantor systems; That is, $(X, T)$ is a minimal Cantor system and we require that our "jump function" $p$ be bounded and hence continuous. Moreover, our presentation focuses on a novel construction of a perfectly ordered Bratteli diagram for ( $X, S$ ) given a properly ordered Bratteli diagram for $(X, T)$. We will conclude the talk with an brief application of this construction as well as discuss various open problems inspired by this construction. The work presented is joint work with Andrew Dykstra and Michelle LeMasurier, both of Hamilton College.
(Received September 08, 2023)

## 1192-37-30015

Lori Alvin*, Furman University. Unimodal Maps and Substitutions. Preliminary report.
We investigate unimodal maps whose kneading sequences have a nice structure. We say that a kneading sequence $\mathcal{K}(f)$ has substitutive structure if there exists a substitution $\theta: \mathcal{A} \rightarrow \mathcal{A}^{+}$with fixed point $\mathbf{w}=\lim _{n \rightarrow \infty} \theta^{n}(a)$ (for some $a \in \mathcal{A}$ ) and a rule $\varphi: \mathcal{A} \rightarrow\{0,1\}^{+}$such that $\mathcal{K}(f)=\varphi(\mathbf{w})$. We study the relationships between various dynamical properties of the unimodal map and dynamical properties of the substitution shift. In particular, we discuss conditions where $\left.f\right|_{\omega(c)}$ is topologically conjugate to an odometer.
(Received September 08, 2023)

## 1192-37-30055

Emily Dong, Scripps College, Sarah Marzen*, W. M. Keck Science Department. Cognitive biases can move opinion dynamics from consensus to chaos.
Research in the area of how networks of people form consensus opinions has exploded recently, with statistical physics approaches and contributions from economists both suggesting that there is convergence to a fixed point in belief networks. We straightforwardly generalize the model used by economists to describe Bayesian updating so that the likelihood of a piece of data depends on ground truth and, potentially, the alignment of the receiver's beliefs with the sender's beliefs and the data point itself. Confirmation bias occurs when the data point is considered more likely when it aligns with the receiver's beliefs; a version of in-group bias occurs when the receiver further considers the data point to be more likely when the receiver's beliefs and the sender's beliefs are aligned. When the likelihood of the data point only depends on ground truth, so that receivers exhibit no confirmation bias or in-group bias, the network of people always converges to complete consensus. With confirmation bias, there can be polarization in the final state. When in-group bias is added, consensus and polarization are still possible; but when agents do their best to counteract confirmation bias, so is chaos. This is the first work to suggest that chaos might be a feature of opinion dynamics when cognitive biases, or attempts to counteract cognitive biases, are taken into account.
(Received September 08, 2023)

## 1192-37-30192

Jiachen Liu*, Wake Forest University. A Study on Fractional Order Mandelbrot Set.
The Mandelbrot set is a well-known mathematical fractal with a number of interesting properties. In recent years, researchers have considered a modification of this set, called the Fractional Order Mandelbrot set, or FOM. In this paper, we discuss some properties of the FOM. We show that the map which generates the FOM does not have fixed points. We also show that as $q$ goes to 0 , the FOM will approach the ordinary Mandelbrot map. Finally, we approximate the stability region of the FOM map and make some conjectures based on our numerical results.
(Received September 09, 2023)

1192-37-30263
Marian Gidea*, Yeshiva University. Geometric control in nearly integrable Hamiltonian systems. Preliminary report.
We apply techniques from control theory to obtain orbits with prescribed itineraries in nearly integrable Hamiltonian systems. We consider an integrable Hamiltonian system with singularities subject to a small, time-periodic perturbation. More precisely, we assume that the unperturbed system has a normally hyperbolic invariant manifold (NHIM), which persists under small perturbations, and so that the stable and unstable manifolds of the perturbed NHIM have transverse intersections. Associated to each transverse intersection one defines a scattering map, which gives the future asymptotic of a homoclinic orbit as a function of its past asymptotic. We assume that we have a system of such scattering maps. We provide results on the geometric controllability of the system. Under explicit conditions on the inner dynamics (the dynamics restricted to the NHIM) and on the scattering maps, given any two points on the NHIM (or in an open set of it), there is an orbit of the Hamiltonian flow that goes from a small neighborhood of the first point to a small neighborhood of the second point. Also, given any path on the NHIM, there is an orbit of the Hamiltonian flow that shadows that path. Possible applications to space mission design, where one is applying small controls to steer the trajectory of a spacecraft in a desired direction, will be investigated in future work. This is joint work with R. de la Llave and T. M-Seara. (Received September 09, 2023)

## 1192-37-30295

Garam Choi*, Colby College, Clayton James Kelly*, IUPUI, Benjamin Weber*, Willamette University. Periodic Orbit Launch Angles for Billiards on Surfaces of Revolution. Preliminary report.
Mathematical billiards is a dynamical system where a point mass travels inertially inside a table and reflects off a boundary with an angle equal to its angle of incidence. There is an extensive body of literature on planar billiards; in this poster, we consider tables on surfaces of revolution. We first investigate billiard dynamics on regular k-gon tables, both in the plane and on the sphere. On these tables, we present explicit formulae for launch angles yielding periodic orbits. While we have not yet devised a method to map these regular k-gon tables onto surfaces of revolution beyond spheres, we are able to describe a generalized equation derived from isometry arguments and the Gauss-Bonnet theorem that generates launch angles of periodic orbits. As an extension of Cao, Klawitter, and Manogue (2021), we also find the launch angles of certain periodic
orbits on parametric rectangle billiard tables. We additionally investigate phase spaces for spherical regular k-gons. In order to run the simulations for this research, we expanded a Mathematica codebase to allow for the creation of new tables and billiard trajectories.
(Received September 09, 2023)

## 1192-37-30718

Tamara Kucherenko*, The City College of New York, Department of Mathematics, Anthony Quas, University of Victoria, Department of Mathematics and Statistics. Asymptotic behavior of the pressure function for Hölder potentials.
We discuss the behavior of the pressure function for Hölder continuous potentials on mixing subshifts of finite type. The classical theory of thermodynamic formalism shows that such pressure functions are convex, analytic and have slant asymptotes. We provide a sharp exponential lower bound on how fast the pressure function approaches its asymptotes. As a counterpart, we also show that there is no corresponding upper bound by exhibiting systems for which the convergence is arbitrarily slow. However, we prove that the exponential upper bound still holds for a generic Hölder potential. In addition, we determine that the pressure function satisfies a coarse uniform convexity property. Asymptotic bounds and quantitative convexity estimates are the first additional general properties of the pressure function obtained in the settings of Bowen and Ruelle since their groundbreaking work more than 40 years ago.
(Received September 10, 2023)

1192-37-30720
Anush Tserunyan, McGill University, Jenna Zomback*, University of Maryland, College Park. Ergodic theorems along trees. Preliminary report.
In the classical pointwise ergodic theorem for a probability measure preserving (pmp) transformation $T$, one takes averages of a given integrable function over the intervals $\left(x, T x, T^{2} x, \ldots, T^{n} x\right)$ in front of the point $x$. We prove a "backward" ergodic theorem for a countable-to-one pmp $T$, where the averages are taken over subtrees of the graph of $T$ that are rooted at $x$ and lie behind $x$ (in the direction of $T^{-1}$ ). Surprisingly, this theorem yields forward ergodic theorems for countable groups, in particular, one for pmp actions of free groups of finite rank, and can be extended to yield ergodic theorems for pmp actions of free semigroups as well. In each case, the averages are taken along subtrees of the standard Cayley graph rooted at the identity. This is joint work with Anush Tserunyan.
(Received September 10, 2023)

## 1192-37-30744

Elliot Kimbrough-Perry*, The City College of New York, Tamara Kucherenko, The City College of New York, Department of Mathematics. Constructing Multiple Phase Transitions on One-Sided Shifts.
In thermodynamic formalism, the term "phase transition" refers to the simultaneous existence of several equilibrium states, which corresponds to a change in the behavior of the system. We discuss the explicit construction of potentials on one-sided subshifts of finite type which exhibit multiple phase transitions. Notably, this construction has significant differences from the known construction on two-sided shifts.
(Received September 10, 2023)

## 1192-37-30787

Philip Arathoon, University of Michigan, Ann Arbor, Matthew D Kvalheim*, University of Maryland, Baltimore County. Linearizability of dynamical systems by embeddings.
In the rapidly developing field of "modern Koopman theory for dynamical systems", an open problem for the last two decades has been to determine the class of dynamical systems that are globally linearizable in the sense of admitting an embedding into a linear system on a finite-dimensional vector space. This problem has also appeared in the control theory and polynomial flows literatures going back to the 1980s. In this talk I will describe a solution to this problem for the case of continuous-time systems having a compact state space or global attractor. Our results reveal relationships between linearizability, symmetry topology, and invariant manifold theory that impose fundamental limitations on algorithms from the aforementioned literature. The talk is based on arXiv:2305.18288.
(Received September 11, 2023)

## 1192-37-30821

Ricky Martua Simon Simanjuntak*, Indiana University Bloomington. Compactification of Blaschke Product as Invariant Measure.
I will explain a construction done by Curtis McMullen to inject Blaschke product into the space of $z^{d}$-invariant measures using Cumulative Distribution Function. This injection and its inverse are effectively computable. One application is on computing measure of Maximal Entropy of polynomials in a Hyperbolic Component
(Received September 11, 2023)

## 1192-37-30858

Gregory Borissov*, University of California, Irvine. Generalized Bounded Distortion Property. Preliminary report.
We prove the Nonstationary Bounded Distortion Property for $C^{1+\varepsilon}$ smooth dynamical systems on multidimensional spaces. The results we obtain are motivated by potential application to study of spectral properties of discrete Schrödinger operators with potentials generated by Sturmian sequences.
(Received September 11, 2023)

## 1192-37-30964

Rodrigo Treviño*, University of Maryland. The Ruelle spectrum for flat Wieler solenoids.

Flat Wieler solenoids are the Smale spaces which show up in the study of self-similar tilings, and one of their defining characteristics is that their stable sets are Cantor sets. In this talk I will describe how the Ruelle spectrum for these types of systems are recovered through the induced action on cohomology of the hyperbolic map given by the self-similarity. The results are concrete in dimensions 1 and 2, and incomplete for higher dimensions.
(Received September 11, 2023)

## 1192-37-31059

Emma Olivia Harper*, CUNY Graduate Center. Computability of equilibrium measures.
We prove the computability of equilibrium measures for Hölder continuous potentials on a shift of finite type, with the Hölder constant bounded above. Our method is to use a computable version of the Ruelle-Perron-Frobenius theorem. This extends work of Spandl on computability of topological pressure.
(Received September 11, 2023)

1192-37-31079
Sheridan Harding, Brigham Young University, Quinlan Leishman*, Brigham Young University, Whitney Lunceford, Brigham Young University, DJ Passey, University of North Carolina at Chapel Hill, Taylor Pool, Carnegie Mellon University, Ben Webb, Brigham Young University. Global Forecasts in Reservoir Computers.
A reservoir computer is a machine learning model that can be used to predict the future state(s) of time-dependent processes, e.g. dynamical systems. In practice, data, in the form of a an input-signal is fed into the reservoir. The trained reservoir is then used to predict the future state of this signal. We develop a new method for not only predicting the future dynamics of the input-signal but also the future dynamics starting at an arbitrary initial condition of a system. The systems we consider are the Lorenz, Rossler, and Thomas systems restricted to their attractors. This method, which creates a global forecast, still uses only a single input-signal to train the reservoir but breaks the signal into many smaller windowed signals. We examine how well this windowed method is able to forecast the dynamics of a system starting at an arbitrary point on a system's attractor and compare this to the standard method without windows. We find that the standard method has almost no ability to forecast anything but the original input-signal while the windowed method can capture the dynamics starting at most points on an attractor with significant accuracy.
(Received September 11, 2023)

1192-37-31167
Saber Ahmed, Hamilton College, Natasha Crepeau*, University of Washington, Paul R Dessauer, Texas A\&M University, Alexis Edozie, University of Michigan, Odalys Garcia-Lopez, The College of New Jersey, Tanisha Grimsley, Juniata College, Jose Lopez, Fresno State, Jordy Lopez Garcia, Texas A\&M University, Viridiana Jasmin Neri, Columbia University, Anne Shiu, Texas A\&M University. Identifiability and the Singular Locus of Certain Linear Compartmental Models.
A linear compartmental model is identifiable when its parameters can be recovered from data. Given a linear compartmental model, we want to use its inputs, outputs, and leaks to determine its identifiability. We focus on two types of models: a model whose underlying graph is a directed cycle, and catenary models, whose underlying graph is a bidirected path. For directed cycles with one input and one output, we provide a complete characterization of identifiability. For catenary models, we provide an expression of the coefficients of the input-output equations as a step towards understanding the identifiability of individual parameters.
(Received September 11, 2023)

## 1192-37-31247

Ruby Kim*, University of Michigan. Bifurcations of limit cycles in mechanistic models of physiological processes. Mechanistic models of oscillatory biological systems can lead to interesting insights in both biology and mathematics. In this talk, I will discuss some applications of the study of limit cycles and bifurcations to our understanding of circadian rhythms, intercellular signaling, and the menstrual cycle. These oscillatory systems are complicated by interactions with coupled dynamical systems, and this makes them difficult to study without the use of advanced mathematical tools. I will additionally discuss how these applications can be used to motivate undergraduate projects in mathematical biology.
(Received September 11, 2023)

## 1192-37-31276

Hongming Nie, Stony Brook University, Chenxi Wu*, University of Wisconsin At Madison. Topological entropy of p-adic polynomial maps.
This is a collaboration with Hongming Nie. We showed that for polynomial maps on $\mathbb{Q}_{p}$ or its finite covers where the critical values on the Julia set has finite orbit, the Artin-Mazur zeta function must be rational with integer coefficients, which implies that the topological entropy must be logarithms of algebraic integers. I will also present our results on the continuity of entropy with respect of coefficients, and discuss some further questions.
(Received September 11, 2023)

## 1192-37-31296

Behzad Djafari-Rouhani, UTEP, Tung D. Nguyen, Texas A\&M University, Zhisheng Shuai, University of Central Florida, Tingting Tang*, San Diego State University, Amy Veprauskas, University of Louisiana at Lafayette, Yixiang Wu, Middle Tennessee State University, Ying Zhou, Lafayette College. Impact of resource distributions on the competition of species in stream environment
In this talk, a two patch logistic metapopulation model over stream network environment is considered. The metapopulation growth rate and the total biomass (of the positive equilibrium) are used as metrics for different aspects of population persistence. The objective is to find distributions of resources that maximize these persistence measures. It was shown that the species whose resource allocations maximize the total biomass has the competitive advantage. In addition, it was proven that for a single patch model the strategy to maximize the total biomass is to concentrate all the resources in the most upstream
locations. In contrast, when the diffusion rates are sufficiently small, the metapopulation growth rate is maximized when all resources are concentrated in one of the most downstream locations
(Received September 11, 2023)

## 1192-37-31701

Cayden Blake, Brigham Young University, Eric Benson Manner, Student, Benjamin Webb*, Brigham Young University. Improving the Adam Optimizer Using Time Delays. Preliminary report.
The Adam optimizer is an optimization algorithm that has seen extensive use in deep learning applications in computer vision and natural language processing. It uses exponentially decaying averages of both gradients and the second moments of the gradients to produce its results. We propose a time-delayed adaptation of the Adam optimizer as a method for improving optimization on certain high-dimensional loss surfaces which simply wraps a given optimizer, in this case the Adam optimizer allowing us to add a variety of time-delays. We find that adding stochastic time-delays, among others, significantly improves the Adam optimizer's performance on a number of benchmark loss surfaces including the Rastrigen, Ackley, and Zacharov functions. This improvement is especially noticeable in high dimensions. We also show, under mild conditions, that the local and global minima of an arbitrary loss surface remain attracting fixed points of the time-delayed Adam optimizer by extending the theory of intrinsic stability to multistable systems.
(Received September 11, 2023)

## 1192-37-31729

Sarah Day*, College of William and Mary. Topological Data Analysis for quantifying structure in complex systems. Preliminary report.
Complex systems often give rise to high dimensional data that may be difficult or infeasible to simulate at fine scale. In addition, measurements of complex systems are typically sparse and/or prone to error. The field of Topological Data Analysis (TDA) offers a collection of tools designed to track coarse, topological features. These tools are well-suited for dealing with certain types of measurement noise and, as is being shown more recently, recovering important information from the influence of noise on measurements. I will discuss recent work with Laura Storch on developing and applying these techniques to complex spatial systems from ecology, primarily in model simulations and with an eye towards direct application to satellite images and other spatial measurements of populations
(Received September 12, 2023)

## 1192-37-31753

Robert Bland, UNC Charlotte, Kevin McGoff*, UNC Charlotte, Ronnie Pavlov, University of Denver. Subsystem entropies of shifts of finite type and sofic shifts on countable amenable groups.
In this work we study the entropies of subsystems of shifts of finite type (SFTs) and sofic shifts on countable amenable groups. We prove that for any countable amenable group $G$, if $X$ is a $G$-SFT with positive topological entropy $h(X)>0$, then the entropies of the SFT subsystems of $X$ are dense in the interval $[0, h(X)]$. In fact, we prove a "relative" version of the same result: if $X$ is a $G$-SFT and $Y \subset X$ is a subshift such that $h(Y)<h(X)$, then the entropies of the SFTs $Z$ for which $Y \subset Z \subset X$ are dense in $[h(Y), h(X)]$. We also establish analogous results for sofic $G$-shifts.
(Received September 12, 2023)

## 1192-37-31797

Brittany Stephenson, Lewis University, Cara Sulyok*, Lewis University. Quantifying the Transmission of Clostridioides difficile: Mathematical Models of Transmission and Control in Healthcare Settings.
Clostridioides difficile (C. difficile) is the leading cause of infectious diarrhea and the most frequently identified healthcareacquired infection in United States hospitals. C. difficile is typically contracted after antibiotic use, when healthy gut microbiota that prevent colonization is compromised. Colonized patients, both symptomatic and asymptomatic, shed C. difficile endospores that can survive for long periods on surfaces outside the host and are resistant to many commonly-used disinfectants. Transmission pathways can include contact with endospores on fomites, objects likely to carry infection. This talk will present various mathematical models aimed at quantifying the transmission of C. difficile in healthcare settings ranging from systems of ordinary differential equations to agent-based models. Results can be applied by healthcare professionals by focusing on precautionary measures that reduce patient colonization with C. difficile. (Received September 12, 2023)

## 1192-37-31863

Nicholas Mendler*, San Francisco State University. On the boundary of a self-similar plane continuum.
We consider the problem of when the complement of a self-similar planar continuum consists of Jordan domains. We find this to be equivalent to there existing particular arrangements of finitely many polygons - and hence reduce the problem to finite geometry. With similar techniques we show that in other cases the boundary of a complementary domain of a self-similar continuum may contain Jordan arc "segments" without necessarily being a complete Jordan curve. We also find natural parameterizations for the boundaries in terms of the code-space of the self-similar structure, and provide fast algorithms for approximating the boundary.
(Received September 12, 2023)

1192-37-32083
Alexander M. Blokh*, UAB, Genadi Levin, Hebrew Univeristy of Jerusalem, Lex Oversteegen, UAB, Vladlen Timorin, HSE. No a priori bounds for satellite renormalizations of rational functions.
The modulus of a polynomial-like map is an important invariant that controls distortion of the straightening map and, hence, geometry of the corresponding polynomial-like Julia set. Lower bounds on the modulus, called complex a priori bounds, are known in a great variety of contexts. For any rational function we complement this by an upper bound for moduli of
polynomial-like maps in the satellite case that depends only on the relative period and the degree of the polynomial-ike map. Given a polynomial-like map $P: U \rightarrow V$ with connected filled Julia set $K^{*}$, call $U \backslash K^{*}$ a root annulus (of $K^{*}$ ); the degree of $P$ is denoted by $d^{*}$. Theorem. Let $f$ be a rational function of degree $d \geq 2$ with a satellite polynomial-like cycle $\mathcal{K}$ of relative period $s \geq 2$. Then, for any root annulus $A$ of any filled polynomial-like Julia set $K^{*} \in \mathcal{K}$ we have

$$
\bmod (A)<\frac{d^{*} \pi}{l} n(4(s+1)) \leq \frac{2^{2 d-2} \pi}{l} n(4(s+1))
$$

and if $f$ is a polynomial, then

$$
\bmod (A)<\frac{d^{*} \pi}{l} n(4(s+1)) \leq \frac{2^{d-1} \pi}{l} n(4(s+1))
$$

This rules out a priori bounds in the satellite case with unbounded relative periods. The proofs can be found in the preprint below: A. Blokh, G. Levin, L. Oversteegen, V. Timorin, Maps with no a priori bounds, arXiv:2205.03157 (2023)
(Received September 12, 2023)

## 1192-37-32095

Kevin Burke, University of Limerick, Alina Dubovskaya*, University of Limerick, Susan Fennell, University of Limerick, James Gleeson, University of Limerick, Doireann O'Kiely, University of Limerick. Analysis of mean-field approximation for Deffuant opinion dynamics on networks.
The Deffuant-Weisbuch (DW) model is a bounded-confidence opinion formation model where an individual's opinion is influenced by nearby agents within a confidence-bound distance [1]. The confidence bound size strongly affects the numbers and positions of opinion clusters formed in the infinite time limit. Mean-field (MF) approximations enable mathematical analysis for both fully mixed populations and network interactions [2]. In this work, we present a mathematical analysis of the MF DW model on networks composed of two-degree classes as well as a fully-mixed population case. With the use of asymptotic analysis, we explain how opinions evolve on such networks and how opinion clusters form. We consider a small confidence bound limit and derive an approximate model that is independent of the confidence bound parameter. Finally, through linear stability analysis, we estimate the number and positions of final opinion clusters for any given confidence bound value [3]. Comparison with numerical simulations shows that our estimate accurately predicts the location of major clusters for both the network-based model and the model with a fully mixed population. References: [1] G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch: Mixing beliefs among interacting agents. Adv. Complex Syst. 03(01n04):87-98, (2000); [2] S. C. Fennell, K. Burke, M. Quayle, and J. P. Gleeson: Generalized mean-field approximation for the Deffuant opinion dynamics model on networks. Phys. Rev. E. 103(1), (2021); [3] A. Dubovskaya, S. C. Fennell, K. Burke, J. P. Gleeson, and D. O’Kiely: Analysis of mean- field approximation for Deffuant opinion dynamics on networks. SIAM Journal on Applied Mathematics. Apr 30;83(2):436-59 (2023).
(Received September 22, 2023)

## 1192-37-32143

Victor Donnay, Bryn Mawr College, Daniel Visscher*, Ithaca College. Quantifying the genus of an embedded surface with Anosov geodesic flow.
The geodesic flow on a surface of negative curvature serves as a prototype of Anosov systems. Such a surface is not embeddable in $R^{3}$, however, due to the same curvature property that generated the Anosov dynamics. This naturally leads to the question of whether there can be such a physically realizable surface. Donnay and Pugh showed in 2003 that embedded surfaces with Anosov geodesic flow do exist via a limiting argument: for a particular sequence of surfaces with increasing genus, the geodesic flows will eventually be Anosov. In 2018, the present authors provided a different sequence of surfaces for which this limiting argument also applies. In this work, we develop tools for quantifying these methods and present the smallest currently-known genus of an embedded surface with an Anosov geodesic flow (on the order of $10^{9}$ ).
(Received September 12, 2023)

## 1192-37-32248

Jay Whitmon*, Stevenson University. Information Theory Through Games: Optimizing Shut the Box.
Information theory is the scientific study of the quantification and communication of information in a system. We work with systems known as stochastic processes consisting of the different states of the system and the probabilities of moving from each state to each other state. The game Shut the Box can be modeled as a stochastic process where a state is which tiles are still up. Modeling Shut the Box as a stochastic process allows for the computation of a variety of quantities and an analysis of the optimal strategy for the game.
(Received September 12, 2023)

## 1192-37-32320

John Michael Neuberger*, University of Northern Arizona. A Bifurcation Lemma for Invariant Subspaces. Preliminary report.
The Bifurcation from a Simple Eigenvalue (BSE) Theorem is the foundation of steady-state bifurcation theory for oneparameter families of functions. When eigenvalues of multiplicity greater than one are caused by symmetry, the Equivariant Branching Lemma (EBL) can often be applied to predict the branching of solutions. The EBL can be interpreted as the application of the BSE Theorem to a fixed point subspace. There are functions which have invariant linear subspaces that are not caused by symmetry. For example, networks of identical coupled cells often have such invariant subspaces. We present a generalization of the EBL, where the BSE Theorem is applied to nested invariant subspaces. We call this the Bifurcation Lemma for Invariant Subspaces (BLIS). We give several examples of bifurcations and determine if BSE, EBL, or BLIS apply. We extend our previous automated bifurcation analysis algorithms to use the BLIS to simplify and improve the detection of branches created at bifurcations.
(Received September 12, 2023)

## 1192-37-32506

Claire Merriman*, Davidson College. Natural extensions and entropy of $\alpha$-odd continued fractions.
Nakada's $\alpha$-expansions move from the regular continued fractions ( $\alpha=1$ ), Hurwitz singular continued fractions (obtained at $\alpha=\frac{-1+\sqrt{5}}{2}$ ), and nearest integer continued fractions ( $\alpha=1 / 2$ ). This talk will look at similar continued fraction expansions where all of the denominators are odd. I will describe how restricting the parity of the partial quotients changes the Gauss map and natural extension domain. This is join work with Florin Boca as well as Yusef Hartono, Cor Kraaikamp, and Niels Langeveld
(Received September 12, 2023)

## 1192-37-32516

Brooks Emerick*, Kutztown University, Jared Guhl, Kutztown University. Selective competition in host-parasitoid population dynamic models. Preliminary report.
Early mathematical models of the host-parasitoid interaction include the discrete-time Nicholson-Bailey model, which is known to be unstable, i.e. coexistence is impossible. In this research, we explore the stability of the system using both numerical and analytical methods to investigate the response of the host larvae to multiple parasitoid populations. The model under consideration involves three types of parasitoids: two specialist species and one generalist species. The generalist can ovisposit in any type of host, while the specialists can only infect on their respective host. Such selective competition among parasitoid species is common in natural systems. We demonstrate how stability in the system is achieved through selective competition with various forms of parasitic attack.
(Received September 22, 2023)

## 1192-37-32545

Jeffrey John Ventrella*, independent. Particle-hedra: classifying and designing polyheda with inter-particle forces. Preliminary report
There are several ways to classify polyhedra in terms of faces, edges, and vertices. This paper describes an alternative method for describing the Platonic and Archimedian solids that does not rely on prior information about polyhedral geometry, based on 3D particles having attractive and repulsive forces with each other. Using a sparse simulation, particles can be initialized with random positions; the forces cause the particles to converge on the vertices of polyhedra. This paper introduces a novel categorization scheme based on the three symmetry types (tetrahedral, octahedral, and icosahedral). The chiral (snub) polyhedra are not included, as they require rotational forces. Faces and edges can be derived from features based on the relative positions of particles, as vertices. This method is compared and contrasted with the geometric transformations of truncation, cantellation, and others. The relationship of the cuboctahedron to the vector-equilibrium - as a dynamical system is revealed with this scheme. This scheme could be useful as the basis of an educational tool, and as an alternative way to classify polyhedra, with references to molecular modeling and tensegrity.
(Received September 12, 2023)

## 1192-37-32564

Scott Schmieding*, Pennsylvania State University. Chaotic almost minimal systems. Preliminary report.
A classical theorem of Furstenberg shows that the only proper closed subsets of the circle which are invariant under multiplication by both two and three are finite. Since then, there have been several generalizations of this result. I will discuss a class of group actions motivated by Furstenberg's Theorem which we call chaotic almost minimal systems. I'll survey some results we have about such systems, including the existence of Z-actions which are chaotic almost minimal and possess multiple non-atomic ergodic measures. This is joint work with Van Cyr and Bryna Kra.
(Received September 12, 2023)

1192-37-32580
Van Cyr*, Bucknell University, Bryna Kra, Northwestern University, Samuel Petite, Université de Picardie Jules Verne. Subshifts and invariant measures under non-standard constraints.
The Krylov-Bogolyubov theorem guarantees that every topological Z-system has a measure invariant under the dynamics. But what if we seek a measure with additional properties? In this talk I will survey some recent research joint with Kra-Petite that investigates this question and specifically when a subshift has a measure invariant under not only the shift but all automorphisms of the system. A key tool will be the rate of approximation of a shift by the terms of its natural cover by subshifts of finite type, and related covers that arise when we take factors of systems known to have strong approximations. (Received September 12, 2023)

## 1192-37-32847

Mariam Yousif Al-Hawaj*, University of Toronto. Generalized pseudo-Anosov Maps and Hubbard Trees.
The Nielsen-Thurston classification of the mapping classes proved that every orientation preserving homeomorphism of a closed surface, up to isotopy is either periodic, reducible, or pseudo-Anosov. Pseudo-Anosov maps have particularly nice structure because they expand along one foliation by a factor of $\lambda>1$ and contract along a transversal foliation by a factor of $\frac{1}{\lambda}$. The number $\lambda$ is called the dilatation of the pseudo-Anosov. Thurston showed that every dilatation $\lambda$ of a pseudo-Anosov map is an algebraic unit, and conjectured that every algebraic unit $\lambda$ whose Galois conjugates lie in the annulus $A_{\lambda}=\left\{z: \frac{1}{\lambda}<|z|<\lambda\right\}$ is a dilatation of some pseudo-Anosov on some surface $S$. Pseudo-Anosovs have a huge role in Teichmuller theory and geometric topology. The relation between these and complex dynamics has been well studied inspired by Thurston. In this project, I develop a new connection between the dynamics of quadratic polynomials on the complex plane and the dynamics of homeomorphisms of surfaces. In particular, given a quadratic polynomial, we show that one can construct an extension of it which is generalized pseudo-Anosov homeomorphism. Generalized pseudo-Anosov means the foliations have infinite singularities that accumulate on finitely many points. We determine for which quadratic polynomials such an extension exists. My construction is related to the dynamics on the Hubbard tree which is a forward invariant subset of the filled Julia set
that contains the critical orbit.
(Received September 12, 2023)

## 1192-37-32856

Will Thompson*, University of Vermont. The Emergence of Polarization in the Non-Linear Voter Model on Higher Order Networks. Preliminary report.
Polarization is a pervasive societal issue, demanding insight into its origins. The non-linear voter model, can explain polarization through pairwise interactions, neglecting group-level dynamics. This study presents a minimal extension of the non-linear voter model to better understand how groups affect polarization. In our model, nodes belong to social cliques representing interaction groups. As nodes engage with more cliques, interdependence among group dynamics grows. A parameter, $q$, introduces nonlinearity; $q=1$ corresponds to the linear voter model, while $q>1$ amplifies conformity, and $q<1$ diminishes it. To investigate system dynamics, we employ approximate master equations (AMEs) and probability generating functions, unveiling the model's asymptotic behavior in the thermodynamic limit. Through analytical and numerical methods, we delineate phases of the system based on model parameters, q, conformity bias, and group coupling strength. These phases encompass consensus, coexistence, and a unique bimodal consensus. Notably, this study reveals a unique bimodal consensus phase, combining high coupling strength with an anti-conformity bias supports the existence of stable minorities within cliques. We analytically derive the phase transitions and demonstrate that heterogeneous group sizes foster coexistence at lower coupling strengths. Finally, we compare our results with a non-linear voter model on a pairwise network to demonstrate the importance of higher-order effects for our results. This research bridges the gap between voter model theory and higher-order networks, shedding light on the factors facilitating or impeding the coexistence of diverse viewpoints. (Received September 22, 2023)

## 1192-37-32876

Jiranan Kerdboon*, University of California, Irvine, Xiaowen Zhu, University of Washington. Anderson Localization for Schrödinger Operators with Monotone Potentials over Circle Homeomorphisms.
We prove pure point spectrum for a large class of Schrödinger operators over circle maps with conditions on the rotation number going beyond the Diophantine. More specifically, we develop the scheme to obtain pure point spectrum for Schrödinger operators with monotone bi-Lipschitz potentials over orientation-preserving circle homeomorphisms with Diophantine or weakly Liouville rotation number. The localization is uniform when the coupling constant is large enough. (Received September 12, 2023)

1192-37-32882
May Mei*, Denison University. Fall into the Gap.
(Received September 12, 2023)

## 1192-37-32995

Jasmin Mohn*, United States Military Academy, Brian Raines, Baylor University. Distributional Chaos on the Baire Space. In this talk, we consider distributional chaos in a non-compact metric dynamical system. Specifically, we focus on a shift space over a countable alphabet called the Baire space. We prove that on the Baire space a subshift of finite type exhibits dense distributional chaos. We also prove that a subshift of bounded type on the Baire space that is perfect and has a dense set of periodic points exhibits distributional chaos.
(Received September 13, 2023)

1192-37-33009
May Mei, Denison University, Kitty Yang*, UNC Asheville. (Don't) Mind the Gap: Constant-Shape Substitutions in 1 Dimension. Preliminary report.
Substitution sequences are an accessible point of entry for the study of symbolic dynamics. We provide a different point of view by studying a 1-dimensional generalization of constant-length substitutions that goes by many names including digit tiling and constant-shape substitution. In particular, we do not require that the image of letters be contiguous so the image of a letter might contain "gaps." We then investigate properties such as conjugacy, recognizability, and complexity in a specific family of examples.
(Received September 13, 2023)

## 1192-37-33028

Ting Gao*, Huazhong University of Science and Technology. Detecting Transition Pathway in Stochastic Dynamical Systems through Optimal Control and Machine Learning.
Many complex real world phenomena exhibit abrupt, intermittent, or jumping behaviors, which are more suitable to be described by stochastic differential equations under non-Gaussian Levy noise. Among these phenomena, the most likely transition paths between metastable states are important since these rare events may have a high impact in certain scenarios. One of the challenges to calculate the most likely transition path for stochastic dynamical systems under nonGaussian Levy noise is that the associated rate functional cannot be explicitly expressed by paths. For this reason, we formulate the original variational problem into an optimal control problem to obtain the optimal state as the most likely transition path. In this talk, we will present three types of efficient ways to solve this issue and the corresponding numerical analysis on the convergence and computational efficiency, including supervised learning and reinforcement learning as well as FBSDE with Pontryagin's Maximum Principle. Various stochastic dynamical systems in applications will be discussed. (Received September 13, 2023)

Julia Jammalo, Fairfield university, Lingran Zhang*, Fairfield University. Elliptic Islands In Moon Billiards.
Mathematical billiards are central models of dynamical systems in statistical mechanics in which point particles collide elastically with fixed boundaries. This project studies a class of billiard tables called moon billiards, whose boundary comprises two circular arcs, one concave and one convex. One of the primary objectives of this research project is to explore how the dynamics vary as a function of two system parameters: the radius of the larger circle and the distance between the centers of the circles. By studying a family of stable periodic orbits, we are able to identify elliptic islands in the phase space whose existence excludes the possibility of ergodic dynamics. By systematically varying the table parameters, we gain insights into the diverse behavior of the moon billiard system, uncovering regions of stability and hyperbolicity. This research was conducted at Fairfield University in the summer of 2023 with the support of National Science Foundation grant DMS 2055070. (Received September 13, 2023)

1192-37-33143
Bernd Sing*, University of the West Indies. Fourier transform of Rauzy fractals of 1D Pisot inflation tilings in the non-unit case. Preliminary report.
We consider inflation tilings of the real line with an inflation factor $\lambda$ that is a Pisot-Vijayaraghavan (PV) number, but not a unit. More precisely, we consider such inflation tilings where the vertex points are regular model sets and therefore have pure point diffraction and dynamical spectrum. However, since the PV number is not a unit, the internal space is a product of Euclidean and $p$-adic spaces. Additionally, the windows are typically complicated with fractal boundary. In order to calculate the Fourier transforms of the windows - which is needed to determine the diffraction intensities of the tiling system explicitly respectively to calculate the eigenfunctions of the corresponding dynamical system under the translation action of $\mathbb{R}$ - we extend the approach using a closed expression of matrix Riesz product type to this non-unit case. This will be illustrated using a concrete example.
(Received September 13, 2023)

## 1192-37-33345

Qiyue Zhang*, Wake Forest University. Noise-induced pattern selection in the Swift Hohenberg equation. Preliminary report. The Swift Hohenberg (SH) equation is a well-studied partial differential equation (PDE) that models pattern forming systems. Specifically, patterned states such as stripes or hexagons, correspond to equilibrium of this system i.e local minimum of an underlying energy. Adaptations of the SH equation have been used to model vegetation patterns from which the state of the (local) climate can be observed. That is, transitions in vegetation patterns have been proposed as indicators for climate change. In this talk, we will present a study of noise-induced transition between patterned states in the SH equation perturbed by additive noise. Through numerical simulations we illustrate how patterned states can undergo transitions by fluctuating between local minimums. These transitions are also studied mathematically using Freidlin-Wentzell large deviations adapted to discrete approximation of the PDE to predict the most probable transition paths. Knowledge of these most probable transition paths could help elucidate our understanding of early warning signs for tipping in spatially extended systems. (Received September 13, 2023)

## 1192-37-33656

Max Collins*, Harvey Mudd College, Zhuying Gong*, University of California, Los Angeles, Arie Ogranovich*, Rice University, Nicholas White*, University of Nebraska-Lincoln. A Deffuant-Weisbuch Model of Opinion Dynamics with Adaptive Confidence Bounds. Preliminary report.
Agent-based models of opinion dynamics have enabled researchers to examine how opinions spread and interact on a network. One popular agent-based model is the Deffuant-Weisbuch (DW) bounded-confidence model (BCM), in which two neighboring agents update their opinion at each discrete time step if the opinions of the two agents are sufficiently close to each other. The standard DW model has a fixed confidence bound for all agents; however, in real life, people with different opinions tend to possess different amounts of tolerance towards opinions that differ from theirs. One can imagine that individuals with extreme opinions tend to listen less to others, while people with moderate opinions may be open-minded and willing to accept different opinions. To explore this possibility, we incorporate an adaptive confidence bound into the standard DW model. In our model, each agent's willingness to listen to others is a function of its current opinion. We refer to such a function as a "confidencebound function." We prove that our DW model with adaptive confidence bounds reaches a limit state in three cases, and we our model for various graphs and confidence-bound functions. In our simulations, we explore how changes in one or two parameters affect a limit state's behavior and how long it takes the model to terminate. We observe that varying the value of the confidence-bound function around the middle of the opinion space results in more significant changes in entropy than varying values around the boundary of the opinion space.
(Received September 22, 2023)

## 1192-37-33694

Chris Aagaard, Portland State University, Hannah Kravitz, Portland State University, Heather L. Moore*, Portland State University, Isabelle Shankar, Portland State University, J.J.P. Veerman, Portland State University. Birkhoff Summation of Irrational Rotations. Preliminary report.
The ith iterate of the rotation by $\rho$ is given by

$$
f_{\rho}^{i}(x)=(x+i \rho) \bmod 1
$$

where we take a constant irrational $\rho$ and values of $x \in[0,1)$. We interpret this as a function from the circle to the real line. Note that it has a discontinuity at $x=-i \rho(\bmod 1)$. Then we define the Birkhoff summation $S(\rho, N, x)$ as follows.

$$
S(\rho, N, x)=\sum_{i=0}^{N-1}\left(f_{\rho}^{i}(x)-\frac{1}{2}\right)
$$

For fixed $N$ and $\rho$, the range of $S(\rho, N, x)$ as a function of $x$ is an interval symmetric about zero. The length of that interval as a function of both $\rho$ and $N$ is one of our objects of study. For a given value of $\rho$, it is known that this length grows
logarithmically with N . Our numerical data illustrate this beautifully. When $\rho$ is the golden ratio, the range as a function of $N$ displays a remarkable degree of self-similarity. Patterns in local maxima and minima can be explained in terms of the continued fraction expansion of $\rho$, which in the case of the golden ratio are the Fibonacci numbers. Our conjecture is that the maxima are unbounded but increase at a slower rate compared to other irrationals. While much prior research in ergodic theory has concentrated on examining the behavior of these sums at a fixed value of $x$ we extend this inquiry by considering the entire interval $[0,1)$. This allows us to investigate patterns that may not emerge for a constant $x$.
(Received September 23, 2023)

## 39 Finite differences and functional equations

## 1192-39-25895

Turhan Koprubasi*, University of Central Florida, Ram N Mohapatra, University of Central Florida. Some properties to the inverse scattering problem of eigenparameter dependent discrete Dirac operator.
Scattering theory of difference operators are of a great importance in many areas such as mathematical physics, electronics and metallurgy. Particularly, the inverse problems of the Dirac operators occur frequently in the motion structure of particles in quantum mechanics, and in calculating the density of the non-homogeneous wire using its vibrations. In this study, we will give some properties of the zeros of the Jost function and the scattering function for the discrete Dirac system with an eigenparameter dependent boundary condition. After that, we will present the results including the main equation and the scattering data set of this operator. And then, we will mention the uniqueness of the solution of our boundary value problem as an inverse problem of the scattering theory.
(Received July 14, 2023)

## 1192-39-32945

Jerzy Filar, University of Queensland, Australia, Matthew H Holden, University of Queensland, Australia, Manuela Mendiolar, University of Queensland, Australia, Sabrina H Streipert*, University of Pittsburgh. Overcoming the impossibility of age-balanced harvest.
In many countries, sustainability targets for managed fisheries are often expressed in terms of a fixed percentage of the carrying capacity. Despite the appeal of such a simple quantitative target, an unintended consequence may be a significant tilting of the proportions of biomass across different ages, from what they would have been under harvest-free conditions. Within the framework of a widely used age-structured model, we propose a novel quantitative definition of "age-balanced harvest" that considers the age-class composition relative to that of the unfished population. We show that achieving a perfectly age-balanced policy is impossible if we harvest any fish whatsoever. However, every non-trivial harvest policy has a special structure that favors the young. To quantify the degree of age-imbalance, we propose a cross-entropy function. We formulate an optimization problem that aims to attain an "age-balanced steady state", subject to adequate yield. We demonstrate that near balanced harvest policies are achievable by sacrificing a small amount of yield. These findings have important implications for sustainable fisheries management by providing insights into trade-offs and harvest policy recommendations.
(Received September 13, 2023)

1192-39-33785
Julian Mark Kaufmann*, University of Notre Dame. Cluster Alegebras and Polylogarithms - On Quadrangular Polylogarithms..
Polylogarithms-and especially their recent relation to cluster algebras-have been mathematically fascinating far into the past. Polylogarithms can be viewed as an extension of the natural logarithm into the complex plane. Many famous mathematicians such as Euler, Abel, Kummer, and Lobachevsky have worked on them in the past, and have found interesting functional relations between them, hinting at a higher structure. Some of these relations have now been recreated through cluster algebras, a new algebraic structure that was only developed in 2002. Additionally, in 2022, Matveiakin and Rudenko developed a theory, of so-called Quadrangular Polylogarithms, which they claim can be used to find all possible polylogarithm relations. In this project, the relationship between cluster algebras and polylogarithms was studied in-depth, and the Quadrangular Polylogarithms and their relations were explicitly computed, verified, and some low-level relations retrieved. (Received September 25, 2023)

## 40 Sequences, series, summability

## 1192-40-31894

Luke Bridges*, Michigan State University, Max Evan Budnick*, Georgia State University, Zachary Martin*, Willamette University, Janine Wang, Williams College. Combinatorial and Analytic Properties of Higher-Order Recursive Polynomial Sequences. Preliminary report.
We consider two families of polynomial sequences defined by linear recurrences of the form:

$$
F_{n}(x)=x F_{n-1}(x)+F_{n-a}(x)
$$

with initial conditions $F_{i}(x)=x^{i}$ for $0 \leq i \leq a-1$, and:

$$
C_{n}(x)=x C_{n-1}(x)+C_{n-a}(x)
$$

with initial conditions $C_{0}=a$ and $C_{i}(x)=x^{i}$ for $1 \leq i \leq a-1$. Both of these sequences have natural combinatorial interpretations that lead to novel identities, including a closed form for $C_{n}$ given by:

$$
C_{n}=\sum_{k=0}^{\left\lfloor\frac{n}{a}\right\rfloor}\left(a\binom{n-1-(a-1) k}{k}-1+\binom{n-1-(a-1) k}{k}\right) x^{n-a k}
$$

as well as several other identities relating the sequences to each other and to other combinatorial numbers, such as integer compositions. Finally, for both $F_{n}$ and $C_{n}$ with arbitrary $a \geq 2$, we obtain a precise description of where their roots lie in the complex plane using previous work done by Boyer and Tran. We also show that for even $a$, there are no nonzero real roots, while for odd $a$, the sequences of minimum real roots of $F_{n}$ and $C_{n}$ both converge to

$$
\frac{-a}{\sqrt{ }}[a](a-1)^{(a-1)}
$$

\enddocument
(Received September 12, 2023)

## 41 Approximations and expansions

## 1192-41-27837

George A. Anastassiou*, University of Memphis, Dimitra Kouloumpou, Hellenic Naval Academy. Brownian Motion on Simple Graphs. Preliminary report.
The first author recently derived several approximation results by neural network operators see his new monograph. There, the approximation methods derived from the parametrized and deformed neural networks induced by the parametrized error and q-deformed and $\beta$-parametrized half hyperbolic tangent activation functions. The results we apply here are univariate on a compact interval, regular and fractional. The outcome is the quantitative approximation of Brownian motion on simple graphs, in particular over a system $S$ of semiaxes emanating from a common origin radially arranged and a particle moving randomly on S. We derive several Jackson type inequalities estimating the degree of convergence of our neural network operators to a general expectation function of Brownian motion. We give a detailed list of approximation applications regarding the expectation of well known functions of this Brownian motion. Smoothness of our functions is taken into account producing higher speeds of approximation.
(Received August 21, 2023)

## 1192-41-30405

Demetrio Labate*, University of Houston. Sparse dynamic tomography using cylindrical shearlets. Preliminary report.
Reconstructing moving object densities from under-sampled dynamic x-ray tomography poses numerical challenges, especially in the realistic measurement setting where we do not have access to full Radon transforms at each time step but only projections along a few angular directions. This assumption enforces a joint space-time reconstruction that we perform using cylindrical shearlets and applying a regularization jointly over the spatial volume and across time. This approach brings numerical advantages with respect to separable representations and is consistent with the provable approximation properties of cylindrical shearlets.
(Received September 10, 2023)

## 1192-41-30512

Edwin Lu*, Brown University. Polynomial Approximation of $X^{n}$.
Polynomial approximation of functions has been a long-standing question with a large range of applications. A particular example which has been studied in depth is the polynomial and rational approximation of $x^{n}$ on the interval $[0,1]$, and notably the minimax approximation of $x^{n}$, that is, the approximation which minimizes the supremum norm. In their paper "Rational Approximation to $x^{n "}$, Newman and Reddy showed that the best non-negative polynomial (and rational) minimax
approximation of $x^{n}$ of degree at most $k$ is $p(x)=d x^{k}$, where $d$ satisfies the equation
$n(1-d)=(n-k)\left(\frac{k}{n}\right)^{k /(n-k)} d^{n /(n-k)}$, giving the supremum error $1-d$. We observe that this approximation is not optimal
in the case where $x$ is instead an $m$-by- $m$ matrix, and we discuss trends in the minimax error and of the coefficient $d$ as $n, k$, and $m$ vary.
(Received September 10, 2023)

## 1192-41-31542

Javad Mashreghi*, Laval University. Approximation and duality.
In various classical function spaces, including Hardy spaces, Weighted Dirichlet spaces, and Bergman spaces, we have traditionally relied on direct methods to establish suitable polynomial approximation schemes. Specifically, in these families of spaces, both Taylor series and their Cesàro means serve as legitimate approximation techniques. However, when dealing with function spaces such as the Bloch space, VMOA, and the Smirnov class, we encountered the need for alternative methods. Upon investigation, we found that duality techniques could be effectively employed to obtain the desired results in these function spaces. Our explorations culminated in the development of a very general duality theorem, which, in turn, implies a range of specific results. These outcomes encompass both entirely new findings and extensions of previously established results.
(Received September 11, 2023)

## 1192-41-31827

Daniel Ingebretson*, University of Illinois at Chicago. The Hausdorff and packing measure of some sets of digital and Lüroth expansions.
We determine the exact value of the Hausdorff and packing measures of sets of digital and Lüroth expansions that are missing

David Jacob Wildstrom*, University of Louisville. Integer Approximations for Proportion Systems.
The Golden Spiral is constructed by decomposing a rectangle into a square and a rectangle with the same aspect ratio as the original but rotated $90^{\circ}$. This is the simplest of an infinitude of systems built out of squares and self-similar rectangles, which Ed Harriss calls proportion systems. One of the simpler proportion systems produces a shape known as the Harriss Spiral, but there are many others which produce spiraling self-similar shapes of different degrees of complexity and beauty. However, in many proportion systems, including the Golden and Harriss Spirals, the aspect ratio of the underlying rectangle is irrational, which cannot be realized in any art form whose individual units are discrete, such as knitting, crochet, counted cross-stitch, or pixel-art. This work describes a relaxation of the aspect-ratio requirement which enables a near approximation of any proportion system to be crafted using squares and rectangles with integer side lengths.
(Received September 12, 2023)

## 1192-41-33132

Manki Cho*, University of Houston at Clear Lake. Steklov eigenproblems on elliptic PDEs and its applications to boundary value problems. Preliminary report.
This talk will introduce Steklov eigenproblems on elliptic PDEs with the certain boundary condition. Various ways of finding Steklov eigenpairs have been studied either analytically or numerically. This work will feature some studies of explicit Steklov eigenpairs on polygonal domains. Main results are based on the fact that Steklov eigenfunctions may construct bases of spaces of harmonic functions using only boundary conditions. Solutions of mixed boundary value problems are represented by the series of Steklov eigenfunctions where its coefficients are determined by boundary data of the problems. This idea provides pointwise error estimates in the region. Moreover, specific quantities of functions such as the central value of the solution or the magnitude of its gradient are accurately estimated by the Steklov expansion method. From the localization of Steklov eigenfunctions to the Dirichlet-to-Neumann operator in electrostatics, this talk will provide numerical supports for the method. (Received September 13, 2023)

1192-41-33290
Ike Agbanusi*, Colorado College. Extensions and Approximations.
We investigate the order of approximation when certain singular kernels act on the zero-extension of functions. The results can be repurposed to give non trivial estimates on the moduli of continuity of zero-extensions. Our approach avoids the use of Hardy type inequalities and so applies to any function in $L^{p}\left([0,1]^{d}\right)$ regardless of smoothness. In particular, it yields results for Sobolev and Besov spaces when the smoothness index exceeds $1 / p$.
(Received September 13, 2023)

## 42 Fourier analysis

## 1192-42-26432

Aidan Benjamin Backus*, Brown University, James Leng, University of California, Los Angeles, Zhongkai Tao, University of California, Berkeley. The fractal uncertainty principle via Dolgopyat's method in higher dimensions.
The fractal uncertainty principle (FUP) asserts that it is not possible to localize both a function and its Fourier transform to a neighborhood of a fractal. The FUP was established for fractional-dimension Ahlfors-David subsets of $\mathbb{R}$ by Bourgain-Dyatlov and Dyatlov-Jin, but the higher-dimensional FUP remained open. We establish FUP for a wide class of fractal subsets of $\mathbb{R}^{d}$, including Ahlfors-David sets of dimension $\geq d / 2$ which are "not orthogonal to themselves" in a suitable sense. As a corollary we get an explicit spectral gap for convex cocompact hyperbolic manifolds with Zariski dense fundamental group. The key improvement over the method of Dyatlov-Jin is a Christ-type decomposition of the fractal set. This is joint work with James Leng and Zhongkai Tao.
(Received July 28, 2023)

## 1192-42-26493

Zoe Markman*, Swarthmore College, Teresa Sofia Pollard*, New York University, Joshua Darryl Zeitlin*, Yale University. Arbitrary finite intersections of doubling measures and applications.
It is a longstanding question in harmonic analysis as to whether measures which are doubling on any list of bases are doubling overall. In the 2022, Anderson and Hu , constructed a measure $\mu$ by re-weighting the Lebesgue measure which is $p$-adic and $q$ adic doubling for distinct primes $p$ and $q$ but not doubling overall. Leveraging the machinery in their construction, we develop a new and far less restrictive scheme for which we re-weight the Lebesgue measure and construct a measure which is $n_{i}$-adic doubling for any $n_{i}$ in a finite set of natural numbers $\left\{n_{1}, \ldots, n_{k}\right\}$. Our paper uses a fascinating intersection of number theory, topology, geometry and transcendence theory in order to construct such a family of measures. We provide several nontrivial applications to reverse Hölder weights, $A_{p}$ weights, Hardy spaces, BMO and VMO function classes, and connect our results with key principles and conjectures across number theory. (Received July 31, 2023)

1192-42-27088
Xiaolong Han*, California State University, Northridge. Spherical harmonics with extreme Lp norms.
Spherical harmonics are homogeneous and harmonic polynomials restricted onto the sphere. They are the eigenfunctions of the Laplace-Beltrami operator on the sphere with round metric. Therefore, they are subject to Sogge's celebrated Lp norm estimates, that is, the Lp norm of a spherical harmonic is bounded by a power of its eigenvalue. Due to the large dimension of
the eigenspace, there are a great variety of spherical harmonics. In this talk, we present the quest to find the spherical harmonics with the maximal Lp norm (i.e., which saturate Sogge's estimates), as well as the ones with the minimal Lp norm. (Received August 09, 2023)

## 1192-42-27720

Rachel Bailey*, University of Connecticut, Maxim Derevyagin, University of Connecticut. DEK-Type Orthogonal Polynomials. Preliminary report.
In this poster we revisit one of the first known examples of exceptional orthogonal polynomials that was introduced by Dubov Eleonskii, and Kulagin in relation to nonharmonic oscillators with equidistant spectra. We analyze the DEK polynomials from the point of view of discrete Darboux transformations and unravel a characterization bypassing the differential equation that defines the DEK polynomials. This characterization leads to a family of general orthogonal polynomials with finitely many missing degrees. We then investigate which properties the DEK-type polynomials share with exceptional orthogonal polynomials, such as the behavior of zeros and completeness in the corresponding weighted $L^{2}$ space. We also discuss the associated difference operators.
(Received August 18, 2023)

## 1192-42-27795

Christopher Dock, Tufts University, Lucien Petit*, Lewis and Clark College, Kabir Tripathi*, Tufts University. Numerical Stability of Phase Retrieval for Frames.
The problem of phase retrieval is central to signal processing, having wide ranging contexts including x-ray crystallography, quantum information, and much more. In this paper we use the methods developed by Balan and Zou to examine the phase retrieval problem for frames in $\mathbb{C}^{n}$. We prove the existence of a symmetry in their objective function, thereby reducing the dimension of the problem they present and display visualizations for the smallest interesting case, frames in $\mathbb{C}^{2}$, that were previously unobtainable. Finally, we present our work on Gabor (Weyl-Heisenberg) frame design by introducing a new function that, when optimized over, yields the most stable Gabor generator for a given dimension, and our work on said optimization. (Received August 24, 2023)

## 1192-42-27922

Theresa Anderson*, Carnegie Mellon, Angel Kumchev, Towson University, Eyvindur Ari Palsson, Virginia Tech University. From spheres to simplices.
Counting configurations in Euclidean space interests both analysts and number theorists. We will discuss the problem of counting simplices in such a region, which has its beginnings in number theory, while seeing how refined understanding of spherical operators in harmonic analysis can contribute to this counting problem.
(Received August 22, 2023)

## 1192-42-28125

Tainara Gobetti Borges*, Brown University, Alex Iosevich, University of Rochester, Yumeng Ou, University of Pennsylvania. A singular variant of the Falconer distance problem.
In this talk we will discuss the following variant of the Falconer distance problem. Let $E$ be a compact subset of $\mathbb{R}^{d}, d \geq 1$, and define

$$
\square(E)=|(y, z)-(x, x)|: x, y, z \in E, y \neq z \subseteq \mathbb{R}
$$

We showed using a variety of methods that if the Hausdorff dimension of $E$ is greater than $\frac{d}{2}+\frac{1}{4}$, then the Lebesgue measure of $\square(E)$ is positive. This problem can be viewed as a singular variant of the classical Falconer distance problem because considering the diagonal $(x, x)$ in the definition of $\square(E)$ poses interesting complications stemming from the fact that the set $\{(x, x): x \in E\} \subseteq \mathbb{R}^{2 d}$ is much smaller than the sets for which the Falconer type results are typically established. (Received August 26, 2023)

## 1192-42-28769

Tongou Yang*, University of California, Los Angeles. Maximal planar Radon transforms via local smoothing.
We state a multi-parameter cinematic curvature condition for planar curves, and prove $L^{p}$ bounds for related maximal Radon transforms via local smoothing estimates. In particular, we are able to prove a sharp $L^{p}$ bound of the maximal elliptical operator without rotation. This is joint with Mingfeng Chen and Shaoming Guo.
(Received September 02, 2023)

## 1192-42-28890

Krystal Taylor*, The Ohio State University. Davies efficient covering theorem and a prescribed projection theorem in a nonlinear setting.
Davies theorem states that a set of positive Lebesgue measure can be covered by lines in such a way that the union of the set of lines has the same measure as the original set. This surprising and counter-intuitive result has a dual formulation in the form of a prescribed projection theorem. We investigate an analogue of Davies' result where lines are replaced by shifts of a fixed curve using the dual formulation in a nonlinear setting. In particular, we show that a measurable set in the plane can be covered by translations of a fixed open curve, obeying some mild curvature assumptions, in such a way that the union of the translated curves has the same measure as the original set. Our results rely on a Venetian blind construction and extend to transversal families of projections.
(Received September 04, 2023)

Elizabeth J. Hale*, Kansas State University, Virginia M. Naibo, Kansas State University. Fractional Leibniz Rules in QuasiBanach Function Spaces and Bi-parameter Settings.
Fractional Leibniz rules are reminiscent of the product rule learned in calculus classes. These inequalities traditionally give estimates in the Lebesgue norm for fractional derivatives of a product of functions in terms of the Lebesgue norms of each function and its fractional derivative. Such estimates have also been obtained in several other function spaces using a variety of methods. In particular, the use of Nikol'skiĭ representations allows for a flexible approach by first obtaining estimates at the level of Triebel-Lizorkin spaces based on the function spaces of interest. We prove such estimates for Coifman-Meyer multiplier operators in the setting of Triebel-Lizorkin and Besov spaces based on quasi-Banach function spaces. As corollaries, we obtain results in the class of rearrangement invariant quasi-Banach function spaces, which include weighted Lebesgue spaces, as well as non-rearrangement invariant spaces such as weighted mixed Lebesgue spaces and Morrey spaces. We further demonstrate the flexibility of this method by using it to prove bi-parameter fractional Leibniz rules in the setting of weighted Lebesgue spaces.
(Received September 04, 2023)

1192-42-29178
David Cruz-Uribe, University of Alabama, Brandon Sweeting*, University of Alabama. Multiplier Weak-Type Inequalities for Maximal Operators and Singular Integrals. Preliminary report.
We discuss a kind of weak type inequality for the Hardy-Littlewood maximal operator and Calderón-Zygmund singular integral operators that was first studied by Muckenhoupt and Wheeden and later by Sawyer. This formulation treats the weight for the image space as a multiplier, rather than a measure, leading to fundamentally different behavior; in particular, as shown by Muckenhoupt and Wheeden, the class of weights characterizing such inequalities is strictly larger than $A_{p}$. In this talk, I will discuss quantitative estimates obtained for $A_{p}$ weights, $p>1$, that generalize those results obtained by Cruz-Uribe, Isralowitz, Moen, Pott and Rivera-Ríos for $p=1$, both in the scalar and matrix weighted setting. I will also discuss an endpoint result for the Riesz potentials as well as recent work on the characterization of such weights.
(Received September 05, 2023)

1192-42-29206
Robert Fraser*, Wichita State University. The Number Field Technique in Harmonic Analysis. Preliminary report.
In this talk we discuss the number field technique, a new technique in Harmonic analysis. This technique involves equipping $\mathbb{R}^{n}$ with the multiplicative structure of the tensor product algebra $\mathbb{R} \otimes_{\mathbb{Q}} K$, where $K$ is an algebraic number field of degree $N$ . We discuss some problems in real-variable harmonic analysis that can be studied using this technique. This is joint work with various authors, including Gian Maria Dall'Ara, Kyle Hambrook, and James Wright.
(Received September 05, 2023)

1192-42-29298
Rachel Greenfeld*, Institute for Advanced Study. Tiling, spectrality and aperiodicity of connected sets. Preliminary report. Let $\Omega \subset \mathbb{R}^{d}$ be a set of finite measure. The periodic tiling conjecture asserts that if $\Omega$ tiles $\mathbb{R}^{d}$ by translations then it admits at least one periodic tiling. Fuglede's conjecture suggests that $\Omega$ admits an orthogonal basis of exponential functions if and only if it tiles $\mathbb{R}^{d}$ by translations. Both conjectures are known to be false in sufficiently high dimensions, with all the so-far-known counterexamples being disconnected. In the talk I will survey the study of these conjectures, and discuss a joint work with Mihalis Kolountzakis where we construct connected counterexamples to the periodic tiling conjecture, as well as to both directions of Fuglede's conjecture.
(Received September 06, 2023)

1192-42-29452
Alexander Ortiz*, Massachusetts Institute of Technology. Cone restriction theory and geometric duality. Given a function whose Fourier transform is supported near the lightcone, what do its level sets look like? We describe a geometric duality which gives us insight into the shape of level sets. (Received September 06, 2023)

1192-42-30036
Rachel Bailey*, University of Connecticut. A new perspective on an old example. Preliminary report.
In 1992, Dubov, Eleonskii, and Kulagin introduced one of the first known examples of Exceptional Orthogonal Polynomials, which we will refer to as "DEK polynomials", in relation to nonharmonic oscillators with equidistant spectra. It was shown that the DEK polynomials are a perturbation of Hermite polynomials which are eigenfunctions of a differential operator as well as a difference operator. Remarkably, this perturbation preserved these properties in the DEK polynomials. In an effort to better understand how various perturbations can preserve certain properties of classical orthogonal polynomials, we revisit the DEK polynomials and show that they, in fact, belong to a larger class of orthogonal polynomial sequences which do not contain a degree 1 or degree 2 polynomial, and are eigenfunctions of a difference operator. We will describe the construction of such a sequence and provide a concrete example utilizing the Chebyshev polynomials. We will then revisit the classical Christoffel formula and provide a modification which can be applied to this class of orthogonal polynomials. (Received September 08, 2023)

1192-42-30066
Paige Bright*, Massachusetts Institute of Technology. Recent Developments in Radial Projections.
Over the past few years, the study of radial projections in Euclidean space with respect to Hausdorff dimension has been a rapidly developing topic. As has been highlighted in the work of Orponen-Shmerkin and Ren-Wang, radial projections have deep connections to the Furstenberg set problem, exceptional set estimates for orthogonal projections, the $A B C$ sum-product
problem in the plane, and more. In this talk, we present a survey of recent results on this topic, from the (since resolved) conjectures of Lund-Pham-Thu and Bochen Liu to the deeply influential paper of Orponen-Shmerkin-Wang. We will also discuss current work since the Orponen-Shmerkin-Wang paper, including results of Bright, Fu, Ren, and more. As it turns out, problems regarding radial projections are closely related to exceptional set estimates for orthogonal projections. A talk on exceptional set estimates will occur by the same presenter during the AMS special session on "Harmonic Analysis, Geometric Measure Theory, and Fractals".
(Received September 08, 2023)

## 1192-42-30075

Paige Bright*, Massachusetts Institute of Technology. Exceptional Set Estimates for Orthogonal Projections.
A classic result of Marstrand and Mattila in projection theory states that given $A \subset \mathbb{R}^{n}$ Borel, for almost every $k$-dimensional subspace $V$ in $\mathbb{R}^{n}$, we have that $\operatorname{dim} \pi_{V}(A)=\min \{\operatorname{dim} A, k\}$, where $\pi_{V}$ is the orthogonal projection onto the subspace $V$. Given this result, a natural question to ask is how often is the Hausdorff dimension of an orthogonal projection smaller than expected? I.e., for a fixed parameter $0 \leq s \leq \min \{\operatorname{dim} A, k\}$, how many subspaces $V$ are there such that $\operatorname{dim} \pi_{V}(A)<s$ ? Exceptional set estimates give an answer to this question. In this talk, we will briefly discuss some classical results, due to Falconer and Kaufman, towards answering this question, including a recent new proof of these classical results from BrightGan. Then, we will motivate and present the sharp exceptional set estimate in the plane, which was recently fully solved by Ren and Wang. We will conclude with a discussion of current work and further directions regarding exceptional set estimates As it turns out, exceptional set estimates for orthogonal projections are closely related to problems regarding radial projections. A talk on recent developments in radial projections will occur by the same presenter during the AWM special session on "Recent developments in Harmonic Analysis". (Received September 08, 2023)

## 1192-42-30179

Yeonwook Jung*, San Francisco State University, Chun-Kit Lai, San Francisco State University. Classification of Topologically Universal Sets in ZFC.
It is well-known that a finite set is universal, that is, each Lebesgue measurable set with positive measure contains an affine copy of a finite set. The Erdős Similarity Conjecture, which remains open, states that there is no infinite universal set. In 2022, Gallagher, Lai, and Weber considered a topological version of this conjecture, defining a set to be topologically universal if each dense $G_{\delta}$ set contains an affine copy of the set. They conjectured that there are no such uncountable sets. In this talk, we give a full classification of topologically universal sets as a special subfamily of measure zero sets. As a corollary, we prove that the existence of uncountable topologically universal sets is independent in ZFC.
(Received September 09, 2023)

1192-42-30195
Donggeun Ryou*, University of Rochester. Near-optimal restriction estimates for Cantor sets on the parabola. For any $0<\alpha<1$, we construct Cantor sets of Hausdorff dimension $\alpha$ on the parabola such that they are Salem sets and each associated measure $\nu$ satisfies the estimate $\widehat{f d \nu}{ }_{L^{p}\left(\mathbb{R}^{2}\right)} \leq C_{p}\|f\|_{L^{2}(\nu)}$ for all $p>6 / \alpha$ and for some constant $C_{p}>0$ which may depend on $p$ and $\nu$. The range $p>6 / \alpha$ is optimal except for the endpoint. In earlier related works, fractal subsets of $\mathbb{R}^{d}$ were considered, whereas we consider fractal subsets of the parabola.
(Received September 09, 2023)

1192-42-30708
Ben Johnsrude*, UCLA. Fourier decoupling and restricted projections in the p-adic setting. Preliminary report. We discuss the application of Fourier decoupling methods to projection theorems in the $p$-adic context. Many of the basic methods of Fourier decoupling translate directly to the $p$-adics, and are often more efficient than in the real context. In particular, we are able to recover a restricted projection theorem of Gan, Guo, and Wan, transposed into $\mathbb{Q}_{p}^{n}$, by essentially their methods. This is motivated by applications to homogeneous dynamics, particularly recent work of Lindenstrauss and Mohammadi. This work is joint with Zuo Lin, UCSD.
(Received September 10, 2023)

1192-42-30726
Giovanni Alberti, University of Pisa, Alan Chang*, Washington University in St. Louis, Gian Maria Dall'Ara, Istituto Nazionale di Alta Matematica Francesco Severi. Dividing a set in half.
We discuss the following isoperimetric-type problem: Given a set $E$ in $\mathbb{R}^{d}$ with finite volume, is it possible to find an hyperplane $P$ that cuts $E$ in two parts with equal volume, and such that the area of the slice $\mathcal{H}^{d-1}(P \cap E)$ is of the expected order, namely $|E|^{(d-1) / d}$ ? It turns out that this question has some connections with Radon transforms, Sobolev spaces, and Kakeya sets. This is work in progress with Giovanni Alberti and Gian Maria Dall'Ara. (Received September 10, 2023)

1192-42-31218
Ingrid Daubechies, Duke University, Shira Faigenbaum-Golovin*, Duke University. New families of multiscale functions that are easy to learn by Neural Networks.
Refinable functions, which are the solutions of refinement equations, are the building stones in many constructions; including subdivision schemes used in computer graphics, wavelets, B-splines, as well as several fractals. Even though earlier work proved that all refinable functions can be implemented, up to arbitrary high precision, by ReLu-based Neural Networks, it was far from clear how such functions could be learned from data. We propose a different type of refinement that involves not only
translation and rescaling but also mirroring; functions satisfying the resulting reflecto-refinement equations still generate multiresolution hierarchies that provide an excellent approximation for many functional spaces of interest, yet are also adapted to ReLu networks. We will illustrate the proposed methodology to create new function families, many of which have fractal properties.
(Received September 11, 2023)

1192-42-31329
David Cruz-Uribe*, University of Alabama. Convex set-valued functions, Rubio de Francia extrapolation, and the matrix $A_{2}$ conjecture.
In the 1990s, Nazarov, Treil and Volberg introduced a generalization of the scalar Muckenhoupt $A_{p}$ condition to matrix weights. Let $W$ be a $d \times d$ symmetric, positive definite matrix weight function. For $\$ 1$
(Received September 11, 2023)

## 1192-42-31390

Iqra Altaf*, University of Chicago. A One-Dimensional Planar Besicovitch Set.
A $\Gamma$-Besicovitch set is a set which contains a rotated copy of $\Gamma$ in every direction. Our main result is the construction of a nontrivial 1-rectifiable set $\Gamma$ in the plane, for which there exists a 1-dimensional $\Gamma$-Besicovitch set.
(Received September 11, 2023)

## 1192-42-31526

Dongwei Chen*, Clemson University. Paley-Wiener Theorem for Probabilistic Frames.
Paley-Wiener theorem is a classical result about the stability of basis in a Banach space, which claims that if a sequence in a Banach Space is "close" to a basis in some sense, this sequence is also a basis. Paley-Wiener theorem is also generalized to frames in Hilbert space. In this talk, we generalize Paley-Wiener theorem to probabilistic frames for $\mathbb{R}^{d}$. Probabilistic frame is a probability measures on $\mathbb{R}^{d}$ with finite second moment and the support spans $\mathbb{R}^{d}$. We claim that if a probability measure is "close" to a probabilitistic frame, then this probability measure is also a probabilistic frame.
(Received September 11, 2023)

## 1192-42-31837

Kasso A. Okoudjou*, Tufts University. The HRT conjecture for some analytic functions. Preliminary report.
The HRT Conjecture asserts that the (finite) set $\mathcal{G}(g, \Lambda)=\left\{e^{2 \pi i b_{k}} g\left(\cdot-a_{k}\right)\right\}_{k=1}^{N}$ is linearly independent for any non-zero square integrable function $g$ and subset $\Lambda=\left\{\left(a_{k}, b_{k}\right)\right\}_{k=1}^{N} \subset \mathbb{R}^{2}$. It was put forth in 1996 by C. Heil, J. Ramanathan, and P. Topiwala and remains largely unresolved. In this talk, we consider the conjecture for some classes of analytic functions. In particular, we complex analysis methods to re-prove some known instances of the conjecture. (This is a joint work with S . Guan and M. Maslouhi.)
(Received September 12, 2023)

## 1192-42-31869

Lijing Sun*, Uniersity of Wisconsin-Milwaukee. Asymptotic Estimates for Unimodular Fourier Multipliers on alpha-Modulation Space.pdfL.
Recently, asymptotic estimates for the unimodular Fourier multipliers $e^{i \mu(D)}$ have been studied for the function space $\alpha$ -
modulation space. In this talk, using the almost orthogonality of projections and some techniques on oscillating integrals, we obtain asymptotic estimates for the unimodular Fourier multipliers $e^{i t|\Delta|^{\frac{\alpha_{o}}{2}}}$ on the $\alpha$-modulation space. As applications, we give the asymptotic estimate of the solutions for the Cauchy problem for the free Schrödinger equation, the wave equation, and the Airy equation with initial data in a $\alpha$-modulation space. We also obtain a quantitative form about the solution to the Cauchy problem of the nonlinear dispersive equations and the nonlinear wave equation.
(Received September 12, 2023)

1192-42-31959
Marius Mitrea*, Baylor University. Quantifying Flatness in Terms of Singular Integral Operators.
A fundamental question in mathematics is determining what sort of analysis a certain geometric environment can support and, conversely, extracting geometric features from available information on various analytic structures. In this talk I will be elaborating on such a relationship between the "flatness" of a surface and the size of singular integral operators defined on it. (Received September 12, 2023)

Taryn Cristina Flock*, Macalester College, Betsy Stovall, University of Wisconsin-Madison. Fourier Restriction to the sphere is extremizable more often than not.
Inequalities play a central role in harmonic analysis. However, in many cases the fundamental question "Is equality possible?" is left unanswered. Resolving this question is a first step toward proving stronger versions of the inequality. In this talk, we'll consider this question in the context of Fourier restriction to the sphere. In particular, we'll show that if valid, the $L^{p}-L^{q}$ Fourier extension inequality possesses extremizers whenever $\$ \mathrm{p}$ " "
(Received September 12, 2023)

Kaiyi Huang*, University of Wisconsin-Madison, Betsy Stovall, University of Wisconsin-Madison. Inequalities of BrascampLieb type on the Heisenberg group.
We discuss the proof of sharp necessary and sufficient conditions for $L^{p}$-boundedness of certain multilinear generalized Radon transforms that arise as generalizations of the Brascamp-Lieb inequalities on Euclidean space.
(Received September 12, 2023)

1192-42-32577
Wedad Alharbi*, Saint Louis University. Recovery of signals from saturated linear measurements. Preliminary report. We consider the problem of recovering a signal using a collection of linear sensors which have some specified range. The sensors will output the maximum value whenever they measure something above the range, and the sensors will output the minimum value whenever they measure something below the range. We present this as a non-linear problem about frames for Hilbert spaces, and we will provide some of the foundational mathematical theory. In particular, we prove that the frame algorithm can be adapted to this non-linear setting to provide an algorithm which recovers the signal and converges exponentially fast. This is joint work with Daniel Freeman, Dorsa Ghoreishi, and Brody Johnson.
(Received September 12, 2023)

1192-42-32875
Dominique Kemp*, Institute for Advanced Study. Some Applications of Decoupling to Restriction Theory and Related Problems. Preliminary report
Almost two decades ago, decoupling emerged as a developing new perspective on the circle of problems related to Fourier restriction theory, initially focused on local smoothing for the cone. We revisit this special utility of decoupling and produce new results for surfaces intrinsically tied to space-curve geometry.
(Received September 12, 2023)

## 43 Abstract harmonic analysis

## 1192-43-27433

Gestur Olafsson, Louisiana State University, Iswarya Sitiraju*, Louisiana State University. Analytic Wavefront Sets of Spherical Distributions on De Sitter Space.
A De Sitter space $\mathrm{d} \mathrm{S}^{n}$, is a one sheeted hyperboloid with Lorentzian metric. For G the orthochronous Lorentzian group acting linearly on $(n+1)$ Lorentzian space $\mathbb{R}^{1, n}$ and the subgroup $H$, the orthochronous Lorentzian group acting on $\mathbb{R}^{1, n-1}$, the space $\mathrm{d} \mathrm{S}^{n}$ is homogoneous space $\mathrm{G} / \mathrm{H}$. A distribution u is said to be a spherical distribution if it is H -invariant eigendistribution of the Laplace-Beltrami operator on $\mathrm{d} S^{n}$. The dimension of the space of spherical distribution is two. I will construct the basis for this space as follows: The spherical function on the space $G / K$, where K is orthogonal group $\mathrm{O}_{n}(\mathbb{R})$, can be extended as sesquiholomorphic kernel on an open complex domain called as crown domain. The de Sitter space lies on the boundary of this crown domain. The boundary value of this kernel on $\mathrm{dS}^{n}$ turns out to be the spherical distribution. Further, I will characterize the analytic wavefront set of these distributions.
(Received August 15, 2023)

1192-43-30290
Naga Manasa Vempati*, Louisiana State University. Compactness of the Bloom sparse operators and applications.
We discuss the characterization of compactness for the sparse operator (associated with symbol in weighted VMO space) in the two-weight setting on the spaces of homogeneous type. As a direct application we obtain the compactness characterization for the maximal commutators with respect to the weighted VMO functions and the commutator of Calderón-Zygmund operators on the homogeneous spaces. We will look at the applications of this approach to multilinear Bloom setting. (Received September 09, 2023)

## 44 Integral transforms, operational calculus

## 1192-44-25573

Harish Nagar*, Department of Mathematics, University Institute of Sciences, Chandigarh University, Mohali, Punjab, India,
Sonia Sharma, Department of Mathematics, UIS, Chandigarh University, Mohali, Punjab.. Applications of Complex SEE
Integral Transform.
This paper refers to the study of applications of complex SEE (Sadaq-Emad-Emann) integral transform on various special functions. Some properties of this transform also investigated in this paper. The fundamental problem is also reduced to a straightforward algebraic equation using this complex integral transform. Then, by resolving this algebraic equation and using the inverse of this complex integral transform, the solution to this main problem can be found. In this paper, we applied complex SEE transform on some special functions including Mittag-Leffler function, Generalized function, piecewise continuous function and integration function. Also, we introduce some physics and engineering problem.
(Received June 22, 2023)

Earth Sonrod*, Ithaca College. Some Results on Integral Transforms of Dawson's Integral.
Dawson's integral is a special function with interesting derivative properties. This function and its generalization arise in significant physics problems and applications. In this research, we express the Laplace transform of Dawson's integral in terms of exponential function and exponential integral, which can be used for some computations of improper integrals. The idea to evaluate the $\mathscr{L}_{2}$-transform of Dawson's integral inspires the introduction of the $\mathscr{L}_{k}$-transform to obtain the transform of the generalized Dawson's integral.

Lance Nielsen*, Creighton University. An Overview of Feynman's Operational Calculus since 1999.
We will discuss the development of Feynman's operational calculus in the abstract setting initiated by B. Jefferies and G. W. Johnson in the late 1990's. In this setting, the computations required by R. Feynman's heuristic 'rules' are carried out in a mathematically rigorous fashion in the "commutative world" supplied by the disentangling algebra $\mathbb{D}$, a commutative Banach algebra, and the result is mapped to the noncommutative setting of $\mathcal{L}(X)$ by the disentangling map $\mathcal{T}_{\mu_{1}, \ldots, \mu_{n}}$ indexed by the Borel probability measures $\mu_{1}, \ldots, \mu_{n}$ on $[0, T]$ which supply the time-ordering directions. (These measures are called timeordering measures.) The initial setting of Jefferies and Johnson was the time independent setting with continuous timeordering measures; the time-dependent setting (also with continuous time-ordering measures) was first introduced by $L$. Nielsen in 1999. We will discuss the progression of the operational calculus (primarily in the time-dependent setting) through the use of time-ordering measures with finitely supported discrete parts (2001) and to the use of arbitrary measures (2022). We will also discuss certain aspects and properties of the operational calculus over the last 24 years, including stability theory and the establishment of certain integral equations and evolution equations satisfied by the operational calculus. (Received September 13, 2023)

## 1192-44-33883

Brandon Mukadziwashe*, Tufts University. Limited Data Compton Tomograph. Preliminary report.
This research delves into the concept of Compton Tomography, precisely when the photon source and detector move along a circular pathway. The primary objective of this study was to reconstruct objects within the unit circle by integrating circular arc pairs generated from the photon source. The reconstructions were created through the adjustment of the angle between the source and detector, and as a result, different visible singularities and artifacts were produced. This study analyzes the artifacts generated from three different angles between the source and detector, weighing the advantages and disadvantages of each angle.
(Received September 27, 2023)

## 45 Integral equations

## 1192-45-25619

Gnaneshwar Nelakanti, IIT Kharagpur, Ritu Nigam*, INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR. Convergence analysis of projection methods for Non-linear Hammerstein Integral Equations on Unbounded Domain. Preliminary report. Many physical problems represented as initial and boundary value problems are usually solved by transforming them into integral equations on the half-line. Therefore, this paper discusses the Galerkin and collocation methods along with the multiGalerkin and multi-collocation methods for non-linear integral equations with compact and Wiener-Hopf kernel operators on the half-line in the space of piecewise polynomial subspaces. As Hammerstein integral equation on half-line has the unbounded domain, the finite section approximation method is applied and afterward, we find an approximate solution for the finite section integral equation. In order to obtain improved superconvergence rates, we apply Kumar and Sloan method and then compare proposed convergence rates with the existing results of Nahid et al. \cite\{nahid2019projection\} both theoretically and numerically. These results are further improved by applying multi-projection methods. In addition, it has shown that the proposed theory enhances the results of \cite\{nahid2019projection\}. Finally, numerical examples are presented to demonstrate the given theoretical framework.
(Received June 28, 2023)

## 1192-45-27708

Shivam Kumar Agrawal*, IIT Kharagpur, Gnaneshwar Nelakanti, IIT Kharagpur. Superconvergent multi-Galerkin method for nonlinear Fredholm-Hammerstein integral equations. Preliminary report. In this paper, we apply the multi-Galerkin method with Kumar-Sloan technique to improve the superconvergence rates and find the numerical solution of non-linear Fredholm-Hammerstein integral equations for both smooth as well as the weakly singular kernels. Considering piecewise polynomial as basis function of the approximating subspace, we derive the improved superconvergence rates for multi-Galerkin method based on Kumar-Sloan approximations which are exactly the same superconvergence rates for the approximate solution as in the case of iterated multi-Galerkin method. We have shown improved superconvergence results without the need for iterated multi-Galerkin method. Theoretical results are verified using numerical examples.
(Received August 18, 2023)

1192-45-27890
Tadele Mengesha, University of Tennessee, Abner J. Salgado, Department of Mathematics, University of Tennessee, Joshua Siktar*, University of Tennessee. Analysis and Discretization of Optimal Control Problems in Peridynamics.
In this talk, we investigate a non-local optimal control problem in solid mechanics involving a linear, bond-based peridynamics model; this model depends on the horizon parameter, which is the degree of non-locality. In addition to establishing the wellposedness of our problem, we study the behavior as the horizon parameter approaches zero. We then analyze a finite elementbased discretization of this problem, its convergence, and the so-called asymptotic compatibility as the discretization parameter and the horizon parameter tend to zero simultaneously. (Received August 21, 2023)
inequalities for product of two and three functions using Atangana-Baleanu Caputo fractional derivative.
In this talk, we will present some new Ostrowski type inequalities for product of two and three functions using AtanganaBaleanu fractional derivative in the sense of Lioville-Caputo(ABC). The results obtained for both left and right ABC fractional derivatives can be applied to study further fractional inequalities and estimate various nonlocal function problems since the operator consists of non-singular kernel. The obtained results are more generalized in nature.
(Received August 30, 2023)

## 1192-45-28505

Armin Schikorra*, University of Pittsburgh. On s-Stability of $W^{\wedge}\{s, n / s\}$-minimizing maps between spheres in homotopy classes.
We consider maps between spheres $\mathbb{S}^{n}$ to $\mathbb{S}^{\ell}$ that minimize the Sobolev-space energy $W^{s, n / s}$ for some $s \in(0,1)$ in given homotopy class. The basic question is: in which homotopy class does a minimizer exist? This is a nontrivial question since the energy under consideration is conformally invariant and bubbles can form. Sacks-Uhlenbeck theory tells us that minimizers exist in a set of homotopy classes that generates the whole homotopy group $\pi_{n}\left(\mathbb{S}^{\ell}\right)$. In some situations explicit examples are known if $\mathrm{n} / \mathrm{s}=2$ or $\mathrm{s}=1$.
(Received August 31, 2023)

## 1192-45-29940

Melissa De Jesus*, Florida International University. On a New Paradigm for Binary Phase-Separation Processes. Preliminary report.
The classical Cahn-Hilliard equation (CHE) was originally introduced to model phase separation, a phenomenon in which a binary mixture begins to separate after it reaches some critical temperature. With the use of nonlocal operators, we are able to model separation with a less traditional macroscopic approach, as opposed to a microscopic view that is used with differential operators. For this, we consider a doubly nonlocal Cahn-Hilliard equation (dnCHE), where we can allow for additional flexibility in how particles interact with each other and how they move within the domain. To further extend our applications we replace the classical time derivative with a time kernel that allows us to play with the speed of diffusion of particles. In doing so, this modification can be used to model dynamic processes in which particles are thought to have some 'memory'. We establish both the existence and uniqueness of a solution to this modified equation. Then, using a clever mix of an implicit and explicit numerical scheme we approximate the energy in our system in the special case when our new kernel gives us the Caputo Fractional Time Derivative, and show convincing evidence that our scheme is unconditionally energy stable. (Received September 08, 2023)

## 1192-45-30049

Tadele Mengesha*, University of Tennessee. Analysis of a nonlocal equation with variable horizon subject to local boundary condition.
I will present a recent work on the mathematical analysis of a nonlocal equation with variable horizon satisfying a conventional Dirichlet boundary condition. The integral equation is characterized by a nonlocal interaction kernel defined heterogeneously with a special localization feature on the boundary. The associated energy defines a function space that allows functions to be as rough as a measurable integrable function inside the domain but as smooth as a differentiable function near the boundary. As a con- sequence, we can prove trace theorems as well as a Hardy- type inequality which will lead to establishing stability of the energy functional subject to vanishing Dirichlet boundary condition. This is joint work with Q. Du and X. Tian. (Received September 08, 2023)

## 1192-45-31969

Anh Vo*, University of Nebraska-Lincoln. Solutions convergence of the nonlocal conservation laws.
Nonlocal models have been widely studied as alternative methods to solve partial differential equations with discontinuous solutions. The integral operators utilized in these models capture long-range interactions in a finite horizon. In this presentation, I will present the convergence of nonlocal divergence operators with a general flux density function to their local operators. Assuming the flux density function is differentiable, and its derivative is Hölder continuous, the nonlocal divergence of a differentiable function converge to the classical counterpart as the horizon vanishes. Then I will present the convergence of solutions of the nonlocal conservation-diffusion equation to the solutions of its classical equivalent. (Received September 12, 2023)

## 1192-45-32001

Bacim Alali*, Kansas State University, Nathan Albin, Kansas State University, Thinh Dang, Kansas State University. Fourier analysis for Peridynamics.
A Fourier multipliers approach for the analysis and computations of peridynamics is presented. The analysis is provided for linear state-based peridynamic operators for isotropic homogeneous media in any spatial dimension. We present explicit formulas for the eigenvalues of the peridynamic operator in terms of the space dimension, the nonlocal parameters, and the material properties. We use these explicit representations to reveal the asymptotic behavior of the eigenvalues, which is then utilized to develop regularity results for peridynamic equations over the space of periodic distributions. Efficient spectral numerical methods are presented for scalar nonlocal models.
(Received September 12, 2023)

1192-45-32849
Mamikon Gulian*, Sandia National Laboratories. Calibrating Fractional-Order Models for Anomalous Subsurface Flow through Fractures. Preliminary report.
Data from both numerical simulations as well as physical experiments of subsurface flows reveal that contaminant tracer is often anomalous, with breakthrough curves exhibiting heavy, non-Gaussian tails characteristic of Levy processes. The statistical properties of such flows are described by fractional-order models, i.e., systems of nonlocal integral equations. We
leverage optimization to demonstrate the efficacy of fractional-order models in describing coarse-grained subsurface flows in a variety of datasets. We also study correlation between physical properties of fractured media and model parameters. Our results represent a novel application of optimization constrained by nonlocal physics for model discovery in subsurface flows, and shed light on what type of models can be expected to accurately describe subsurface flows in different materials. (Received September 12, 2023)

## 46 Functional analysis

## 1192-46-26372

Ngai Ching Wong*, National Sun Yat-sen University. Linear maps of operator algebras and Hilbert C*-modules preserving angles.. Preliminary report.
Let $x, y$ be two vectors in a (real or complex) Hilbert $C^{*}$-module $\mathcal{H}$ over a $C^{*}$-algebra $\mathcal{A}$. The angle $\angle(x, y)$ between $x$ and $y$ can be defined in several way. When $\mathcal{A}=C_{0}(X)$ is a commutative $C^{*}$-algebra, in other words, $\mathcal{H}$ is a continuous field of Hilbert spaces over a locally compact space $X$, we define the cosine of the angle, $u=\cos \angle(x, y) \in C(X)$, by the equation

$$
|\langle x, y\rangle|=|x||y| u
$$

Given a linear module map $T: \mathcal{H} \rightarrow \mathcal{K}$ between two Hilbert $C_{0}(X)$-modules, and nonvanishing $u, v \in C(X)$. If $T$ preserves (nonflat) angles, i.e.

$$
\angle(x, y)=u \Longrightarrow \angle(T x, T y)=v,
$$

then $T=\alpha J$ for a bounded, strictly positive and continuous scalar function $\alpha$ on $X$ and a module into isometry $J: \mathcal{H} \rightarrow \mathcal{K}$. For a Hilbert $C^{*}$-module $\mathcal{H}$ over a non-commutative $C^{*}$-algebra $A$, we study the cases when $u=v=0$ or $u=v=1$, namely, the orthogonality or the parallelism preservers. While the linear orthogonality preservers are well studied in previous works, the problem of describing linear parallelism preservers seems to be rather difficult. After presenting some intrinsic properties of different versions of parallelism, we show that, considering the example of $\mathcal{H}=A=M_{n}$, any bijective linear map $T: M_{n} \rightarrow M_{n}$ of a matrix algebra preserving various types of parallelism assume the form $T A=\alpha U A V$ for some positive scala $\alpha$ and unitary matrices $U, V$ in $M_{n}$.
(Received July 26, 2023)

1192-46-27557
Rui Wang, Jilin University, Yuesheng Xu, Old Dominion University, Mingsong Yan*, Old Dominion University. Sparse Representer Theorems for Learning in Reproducing Kernel Banach Spaces. Preliminary report.
Sparsity of a learning solution is a desirable feature in machine learning. Certain reproducing kernel Banach spaces (RKBSs) are appropriate hypothesis spaces for sparse learning methods. In this talk, we will discuss what kind of RKBSs can promote sparsity for learning solutions. We consider two typical learning models in an RKBS: the minimum norm interpolation (MNI) problem and the regularization problem. We shall first establish an explicit representer theorem for solutions of these problems, which represents the extreme points of the solution set by a linear combination of the extreme points of the subdifferential set, of the norm function, which is data-dependent. We then propose sufficient conditions on the RKBS that can transform the explicit representation of the solutions to a sparse kernel representation having fewer terms than the number of the observed data. Under the proposed sufficient conditions, we investigate the role of the regularization parameter on sparsity of the regularized solutions. We further show that two specific RKBSs: the sequence space $11(\mathrm{~N})$ and the measure space can have sparse representer theorems for both MNI and regularization models.
(Received August 16, 2023)

1192-46-27755
Victor Bailey*, Georgia Institute of Technology. Frames via Unilateral Iterations of Bounded Operators. Preliminary report. Dynamical Sampling is, in a sense, a hypernym classifying the set of inverse problems arising from considering samples of a signal and its future states under the action of a bounded linear operator. Recent works in this area consider questions such as when can a given frame for a separable Hilbert Space, $\left\{f_{k}\right\}_{k \in I} \subset H$, be represented by iterations of an operator on a single vector and what are necessary and sufficient conditions for a system, $\left\{T^{n} \varphi\right\}_{n=0}^{\infty} \subset H$, to be a frame? In this talk, we will discuss the connection between frames given by iterations of a bounded operator and the theory of model spaces in the HardyHilbert Space as well as necessary and sufficient conditions for a system generated by the orbit of a pair of commuting bounded operators to be a frame. This is joint work with Carlos Cabrelli.
(Received August 18, 2023)

## 1192-46-27842

Shiho Oi*, Niigata University, Japan. Isometries between groups of invertible elements in Fourier-Stieltjes algebras.
We show that locally compact groups $G$ and $H$ are topologically isomorphic if the groups of invertible elements in the FourierStieltjes algebras $B(G)$ and $B(H)$ are isometric as metric spaces.
(Received August 21, 2023)

Qingying Bu, University of Mississippi, Yongjin Li, Sun Yat-Sen University, Apoorva Mate*, University of Mississippi. Pełcyński's Property (V) on Positive Tensor Products of Banach Lattices.
Let $E$ be an atomic reflexive Banach lattice and be any Banach lattice with Pełczyński's property (V). We show that the positive injective tensor product $E \check{\otimes}_{|\varepsilon|} X$ has Pełczyński’s property (V) and the positive projective tensor product $E \hat{\otimes}_{|\pi|} X$ has Pełczyński's property (V) if and only if every positive linear operator from $E$ to $X^{*}$ is compact. As an application, we provide
new examples of non-reflexive Banach lattices with Pełczyński’s property (V). (Received August 28, 2023)

## 1192-46-28352

Marc A Rieffel*, University of California Berkeley. Dirac operators for quantum Hamming metrics. Preliminary report Given the set of words of a given length for a given alphabet, the Hamming metric between two such words is the number of positions where the two words differ. A quantum version of Hamming metrics was introduced in 2021 by De Palma, Marvian, Trevisan and Lloyd. For the quantum version the alphabet is replaced by a full matrix algebra, and the set of words is replaced by the tensor product of a corresponding number of copies of that full matrix algebra. While De Palma et al. work primarily at the level of states, they do obtain the corresponding seminorm (the Hamming metric) on the algebra of observables that plays the roll of assigning Lipschitz constants to functions. A suitable such seminorm on a unital C*-algebra is the current common method for defining a metric on a $C^{*}$-algebra. In many important cases such seminorms can be obtained from spectral triples. I will indicate how Hamming metrics can be obtained from spectral triples, that is, from a representation of the C*-algebra on a Hilbert space together with a self-adjoint operator $D$ on the Hilbert space such that the value of the seminorm on an element $a$ of the algebra is given by the operator norm of the commutator $[D, a]$. One consequence of this is that the seminorm will be strongly Leibniz, i.e. satisfy a Leibniz inequality and more. This is part of a project in progress. (Received August 29, 2023)

## 1192-46-28631

Ghder S Aburamyah*, Morgan State University, Guoping Zhang, Morgan State University. $\ell^{p}$ SOLUTION TO THE INITIAL VALUE PROBLEM OF THE DISCRETE NONLINEAR SCHR "ODINGER EQUATION WITH COMPLEX POTENTIAL Preliminary report.
In this talk, we present our recent results on the global well-posedness of the initial value problem of the time-dependent discrete nonlinear Schrödinger equation with a complex potential and sufficiently general nonlinearity on a multidimensional lattice in weighted $\ell^{p}$ spaces in the case $\$ 2$
(Received September 01, 2023)

1192-46-29059
Sana Kazemi*, University of Memphis. Isometries of Special Classes of Sequence Spaces.
In this talk, I will provide an introduction to a new example of combinatorial Banach spaces, describe its properties and characterize all the isometries on this space. I will define the famous Schreier's space and outline some of its properties. Then, I will give an overview of Tsirelson's spaces and discuss for which case the form of the surjective isometries is known. This has led to open questions which is motivating for future research endeavors
(Received September 05, 2023)

## 1192-46-29120

Lin Lin*, University of California, Berkeley. Linear combination of Hamiltonian simulation for non-unitary dynamics.
We present a novel method for simulating a broad class of non-unitary dynamics through a Linear Combination of Hamiltonian Simulation (LCHS) problems. Our approach circumvents the need for converting the problem into a dilated linear system or relying on the spectral mapping theorem, which underpins many existing quantum algorithms like the quantum singular value transformation (QSVT). Remarkably, the LCHS technique attains optimal cost-efficiency in state preparation. Furthermore, we showcase its applicability in certain open quantum dynamics simulation using the complex absorbing potential method, achieving near-optimal performance across all parameters. (Joint work with Dong An and Jin-Peng Liu) (Received September 05, 2023)

Michael Aizenman*, Princeton University, Giorgio Cipolloni, Princeton University, Department of Mathematics.
Ruminations on Matrix Convexity and the Strong Subadditivity of Quantum Entropy.
As was pointed out by C. Loewner (in 1934), and F. Krauss (in 1936), for a matrix-valued function F(M) ranging over self adjoint matrices the conditions of monotonicity, and correspondingly convexity, pose severe restrictions on F. For related reasons difficulties have been encountered in establishing some of the basic properties of quantum entropy, which were successfully confronted in ground breaking works on the subject. Seeking elementary arguments we demonstrate that the second derivative test for convexity is a useful tool also in the context of matrix-valued functions. We show how it can be deployed for the derivation of joint convexity principles which played an essential role in the Lieb-Ruskai (' 73 ) proof of the strong subadditivity of quantum entropy.
(Received September 05, 2023)

## 1192-46-29254

Scott McCullough, University of Florida, Georgios Tsikalas*, Washington University In St. Louis. Commutant lifting and Nevanlinna-Pick interpolation for pairs of spaces. Preliminary report.
Let $X_{1}$ and $X_{2}$ be Hilbert spaces of (vector-valued) functions defined on the unit disk $\mathbb{D}$, both invariant with respect to the shift operator

$$
S_{i}: f(z) \mapsto z f(z), f \in X_{i}, i=1,2
$$

Assume also that $M_{i}(i=1,2)$ are $S_{i}^{*}$-invariant subspaces of $X_{i}$ and $T \in \mathcal{B}\left(M_{1}, M_{2}\right)$ is a bounded operator intertwining $\left.P_{M_{1}} S_{1}^{*}\right|_{M_{1}}$ and $\left.P_{M_{2}} S_{2}^{*}\right|_{M_{2}}$. When can we find an operator $R \in \mathcal{B}\left(X_{1}, X_{2}\right)$ that intertwines $S_{1}^{*}$ and $S_{2}^{*}$ and satisfies $T=\left.R\right|_{M_{1}}$ and $\|T\|=\|R\|$ ? The celebrated commutant lifting theorem of Sz.-Nagy and Foiaş (1968) asserts that this is always possible if we take $X_{1}$ and $X_{2}$ to be (vector-valued) Hardy spaces. Treil and Volberg (1992) showed that the existence of $R$ is
guaranteed even if $S_{1}$ is assumed only to be a contraction on $X_{1}$ and $S_{2}$ is expanding on $X_{2}$. Another generalization was given by Ball, Trent and Vinnikov (1997), who allowed for $X_{1}$ and $X_{2}$ to be vector-valued versions of a complete Pick space on some set (the operators $S_{i}$ replaced by the associated algebra of multipliers). Then, in 2004, Shimorin achieved a simultaneous generalization of both of these latter results, showing that a commutant lifting theorem holds for any pair of spaces $\left(X_{1}, X_{2}\right)$ with reproducing kernels satisfying a certain factorization property. An analogue of the Nevanlinna-Pick interpolation theorem for multipliers $\phi: X_{1} \rightarrow X_{2}$ was also obtained as a corollary. We will discuss recent progress concerning the following question: for which pairs of spaces does a commutant lifting theorem hold? In particular, our theorems assert that, for a large family of pairs of kernels, Shimorin's sufficient factorization condition is also necessary, even if we only ask for liftings of Nevanlinna-Pick type. This is joint work with Scott McCullough
(Received September 06, 2023)

1192-46-29612
Naga Manasa Vempati*, Louisiana State University. Schatten Classes of Commutators Operators and Besov Spaces. The commutator operators plays a crucial role in harmonic analysis, which connects closely to complex analysis, noncommutative analysis and operator theory. In this talk we look at the characterization of the Schatten class singular value estimates for the commutators of some singular integral operators of interest via Besov spaces.
(Received September 07, 2023)

1192-46-30007
William Johnston*, Butler University, Sara Moore, Butler University, Rebecca Wahl, Butler University. Harmonic Functions on Four Dimensions. Preliminary report.
This presentation newly defines a bicomplex hyperbolic harmonic function with four real-dimensional inputs, in a way that generalizes the connection between real harmonic functions with two real-dimensional inputs and complex analytic functions. For example, every bicomplex hyperbolic harmonic function appears as this presentation's newly defined hyperbolic real part of a bicomplex analytic function, just as every real harmonic function with two real-dimensional inputs is the real part of a complex analytic function. In addition, this connection produces a unique (up to additive constant) and newly defined hyperbolic harmonic conjugate function, just as every real harmonic function has a unique (up to additive constant) real harmonic conjugate. Finally, the presentation determines a bicomplex Poisson kernel function that produces a corresponding integral representation for bicomplex harmonic functions, one that generalizes the complex harmonic function Poisson integral representation.
(Received September 08, 2023)

## 1192-46-30335

Lakshay Jatin Patel*, University of California Berkeley. Quantum Metric Spaces Associated to Cyclic Groups. Preliminary report.
In his 1989 paper, Connes gave examples of metric spaces associated to group C*-algebras, later viewed as 'quantum metric spaces' by Marc Rieffel. Let $G$ be a discrete group and $l$ a length function on $G$. Connes defined a metric on the state space $S\left(C^{*}(G)\right)$ by first defining a Lipschitz seminorm on $C^{*}(G)$, and then defining $d_{G}$ in a way analogous to the C*-algebra of functions on a metric space $C(X)$ and its Lipschitz seminorm. Rieffel has generalized the notion of the Gromov- Hausdorff distance for compact metric spaces to the quantum Gromov - Hausdorff distance, which is a metric on the set of isometric isomorphism classes of order unit spaces. We investigate these quantum metric spaces for cyclic groups $C_{n}=\mathbb{Z} / n \mathbb{Z}$ with the length function given by $l_{n}(z)=\inf \{|x|: x \in z+n \mathbb{Z}\}$, with an end goal of understanding whether the sequence of quantum metric spaces converges with respect to the quantum Gromov-Hausdorff distance. Because $C^{*}\left(C_{n}\right)$ is commutative and finitedimensional, our approach involves studying the smaller finite metric subspace ( $\widehat{C_{n}}, d_{n}$ ) obtained by identifying pure states of $C^{*}\left(C_{n}\right)$ with the dual group $\widehat{C_{n}}$. We prove uniform bounds on the pairwise distances of points in $\left(\widehat{C_{n}}, d_{n}\right)$. We also show that the geometry of $\left(C^{*}\left(C_{n}\right), d_{n}\right)$ is different than that of the usual identification of $\widehat{C_{n}}$ as the $n$th roots of unity in the complex plane. We conclude with a conjecture regarding the relationship between restricting seminorms and group algebra inclusions. (Received September 25, 2023)

## 1192-46-30630

Ioana Ghenciu, University of Wisconsin-River Falls, Roxana Popescu*, University of Pittsburgh. On some classes of operators on $C(K, X)$.
Let $T: C(K, X) \rightarrow Y$ be a strongly bounded operator with representing measure $m: \Sigma \rightarrow L(X, Y)$ and let
$\hat{T}: B(K, X) \rightarrow Y$ be its extension operator. In this presentation, we will show that several properties of the operator $T$ are also transferred to its extension operator $\hat{T}$, as well as to its representing measure $m(A)$, for all $A \in \Sigma$. In addition, some equivalence cases will be discussed.
(Received September 10, 2023)

1192-46-30704
Nazli Dogan*, Fatih Sultan Mehmet Vakif University. ON OPERATORS DEFINED BY TRIANGULAR TOEPLITZ MATRICES BETWEEN KÖTHE SPACES.
Fréchet spaces are one of the leading classes of locally convex spaces and include most of the important examples of nonnormable locally convex spaces. Dynin Mitiagin Theorem states that nuclear Fréchet spaces with a Schauder basis have a Köthe space representation. Within the Köthe spaces, power series spaces hold significant importance and have been extensively examined and well-studied. In this talk, after introducing these spaces, conditions regarding the continuity and compactness of operators defined by a triangular Toeplitz matrix between Köthe spaces, one of which is a power series space will be presented.
(Received September 10, 2023)

Sofya Sharipovna Masharipova*, University of Pittsburgh at Johnstown, Shukhrat M Usmanov, University of Pittsburgh at Johnstown. Morita equivalence for real operator algebras. Preliminary report
We define an equivalence of Morita for real operator algebras. Some theorems of isomorphisms for real $\mathrm{W}^{*}$-algebras and real C*-algebras equivalent by Morita are proven.
(Received September 11, 2023)

## 1192-46-30804

Dimitri Y. Shlyakhtenko*, UCLA. A notion of free dimension based on Wasserstein distance. Preliminary report. We discuss a notion of dimension, with properties similar to that of free entropy dimension, whose definition is based on the notion of Wasserstein distance between non-commutative laws introduced by Biane and Voiculescu. (Received September 11, 2023)

## 1192-46-30949

J. E. Pascoe*, Drexel University. Payoff minus half norm-squared penalty allocation problems.

The kernel embedding of distributions represents a measure as the corresponding linear functional on some reproducing kernel Hilbert space. The optima of payoff minus half norm-squared penalty type allocation objectives over distributions essentially embed to a function taking maximum real value on its support. Iterative methods are given for obtaining explicit approximations with law of large numbers type convergence in the worst case. We give some obstructions to Herglotz type representations arising from kernels.
(Received September 11, 2023)

## 1192-46-30973

Michael Ray Montgomery*, Dartmouth College. A category of reproducing kernel Hilbert algebras.
In the talk we will develop a natural definition, in the categorical sense, of an algebra structure on a reproducing kernel Hilbert space. This definition is also equivalent to subconvolutivity of weight functions in examples of reproducing kernel Hilbert spaces from harmonic analysis. We will then show the category of reproducing kernel Hilbert algebras is closed under orthogonal sums, tensor products, pushouts, and pullbacks such that the spectrum is a functor compatible with these constructions. Furthermore, the image of the spectrum as a functor from RKHAs to the category of compact Hausdorff topological spaces contains all compact subspaces of $\mathbb{R}^{n}, n>0$.
(Received September 11, 2023)

## 1192-46-31098

Chi-Fang Anthony Chen, Caltech, Hsin-Yuan Huang*, MIT, John Preskill, Caltech, Leo Zhou, Caltech. Local minima in quantum systems.
Finding ground states of quantum many-body systems is known to be hard for both classical and quantum computers. As a result, when Nature cools a quantum system in a low-temperature thermal bath, the ground state cannot always be found efficiently. Instead, Nature finds a local minimum of the energy. In this work, we study the problem of finding local minima in quantum systems under low-temperature thermal perturbations. While local minima are much easier to find than ground states, we show that finding a local minimum is computationally hard for classical computers. In contrast, we prove that a quantum computer can always find a local minimum efficiently using a proposed thermal gradient descent algorithm that mimics the cooling process that occurs in Nature. To establish the classical hardness of finding local minima, we consider a family of two-dimensional Hamiltonians such that any problem solvable by a quantum computer in polynomial time can be reduced to finding an approximate ground state of a Hamiltonian in the family, and we show that for such Hamiltonians all approximate local minima are also approximate global minima. Therefore, assuming that quantum computation is more powerful than classical computation, finding a local minimum is classically hard.
(Received September 11, 2023)

## 1192-46-31192

Marius Junge*, University of Illinois, Urbana and Champaign. Complexity and Hamiltonian Simulation. Preliminary report. It appears to be extremely difficult to certify that a given quantum circuit is complex enough to perform certain quantum operations. In this talk, based on joint work with Y. Chen, R Araiza, and Peixue Wu, we use techniques from Wasserstein distances to identify simple certificates of complexity. The 'logarithmic' complexity used here is a convex functions which only grows linearly with the number of qubits, motivated by the work of Bu, Koh, Jaffe and Lloyd. The flexibility of the new notion of Lipschitz complexity applies extremely well to Hamiltonian simulation. (Received September 11, 2023)

## 1192-46-31239

Ioana Ghenciu*, University of Wisconsin-River Falls. A note on some classes of operators on $C(K, X)$.
Suppose $X$ and $Y$ are Banach spaces, $K$ is a compact Hausdorff space, $\Sigma$ is the $\sigma$-algebra of Borel subsets of $K, C(K, X)$ is the Banach space of all continuous $X$-valued functions (with the supremum norm), and $T: C(K, X) \rightarrow Y$ is a strongly bounded operator with representing measure $m: \Sigma \rightarrow L(X, Y)$. We show that if $T$ is a strongly bounded operator and $\hat{T}: B(K, X) \rightarrow Y$ is its extension, then $T^{*}$ is pseudo weakly compact (resp. limited completely continuous, limited $p$ convergent, $1 \leq p<\infty$ ) if and only if $\hat{T}^{*}$ has the same property. It is shown that if $T^{*}$ is pseudo weakly compact (resp. limited completely continuous, limited $p$-convergent), then for each $A \in \Sigma, m(A)^{*}: Y^{*} \rightarrow X^{*}$ is pseudo weakly compact (resp. limited completely continuous, limited $p$-convergent). If we additionally assume that $K$ is a dispersed compact Hausdorff space and $1<p<\infty$, then $T^{*}: Y^{*} \rightarrow C(K, X)^{*}$ is pseudo weakly compact (resp. limited completely continuous, limited p-
convergent) if and only if for each $A \in \Sigma, m(A)^{*}: Y^{*} \rightarrow X^{*}$ is pseudo weakly compact (resp. limited completely continuous, limited $p$-convergent).
(Received September 11, 2023)

1192-46-31496
Samantha Brooker*, Arizona State University. Spectral triples on a non-standard presentation of Effros-Shen AF algebras. Preliminary report.
The Effros-Shen algebra corresponding to an irrational number $\theta$ can be described by an inductive sequence of direct sums of matrix algebras, where the continued fraction expansion of $\theta$ encodes the dimensions of the summands, and how the matrix algebras at the $n$th level fit into the summands at the $(n+1)$ th level. In recent work, Mitscher and Spielberg present the Effros-Shen algebra as the $C^{*}$-algebra of a category of paths determined by the continued fraction expansion of $\theta$. With this approach, the algebra is realized as the inductive limit of a sequence of infinite-dimensional, rather than finite-dimensional, subalgebras. Drawing on a construction by Christensen and Ivan, we use this inductive limit structure to define a spectral triple, trading the advantages of working with finite-dimensional approximants for the techniques provided by the category of paths. This is joint work with Konrad Aguilar and Jack Spielberg.
(Received September 11, 2023)

## 1192-46-31740

Kaifeng Bu*, Harvard, Weichen Gu, University of New Hampshire, Arthur M. Jaffe, Harvard. Magic from a quantum convolutional approach. Preliminary report.
In this talk, I will introduce a new framework of quantum convolution to study stabilizer states and magic states in discretevariable systems. Within this convolutional framework, we propose a protocol, called the quantum convolution-swap test, to perform stabilizer testing for quantum states and gates. Based on this protocol, we introduce "magic entropy" to quantify magic in quantum states and gates, in a way which may be measurable experimentally. This talk is based on the joint works with Weichen Gu, and Arthur Jaffe(PNAS120(25)2023, arXiv: 2302.08423, arXiv:2306.09292). (Received September 12, 2023)

## 1192-46-31786

Terje Hill*, Florida Atlantic University, David A Robbins, Trinity College (CT) - Ret'D. Character amenability of vector-valued algebras. Preliminary report.
Let $\left\{A_{x}: x \in X\right\}$ be a collection of complex Banach algebras indexed by the compact Hausdorff space $X$. We investigate the character amenability of certain algebras $A$ of $A_{x}$-valued functions in relation to the character amenability of the $A_{x}$. (Received September 12, 2023)

## 1192-46-31849

Weichen Gu*, University of New Hampshire. Quantum Convolution and Central Limit Theorem. Preliminary report.
We introduce a quantum convolution and a conceptual framework to study states in discrete-variable (DV) quantum systems, and establish a quantum central limit theorem. All our results suggest that stabilizer states play a role in DV quantum systems similar to the role Gaussian states play in continuous-variable systems. For example, the convolution of two stabilizer states is stabilizer, and the stabilizer states extremize both quantum entropy and Fisher information. This is a joint work with Kaifeng Bu and Arthur Jaffe.
(Received September 12, 2023)

## 1192-46-32053

Eric A Carlen*, Rutgers University. Duality and stability for some quantum entropy inequalities.
We present an approach to stability bounds for quantum entropy inequalities based on Legendre transforms.
(Received September 12, 2023)

## 1192-46-32306

Jun Yang*, Harvard University. A Level-Depth Correspondence between Verlinde Rings and Subfactors.
We establish a correspondence between the levels of Verlinde rings and the depths of subfactors. Given the $l$-level Verlinde ring $R_{l}(G)$ of a simple compact Lie group $G$, the tensor products of fundamental representations give us the inclusion of a pair of $\mathrm{II}_{1}$ factors $N \subset M$. For the depth $d$ of $N \subset M$, we first prove $d=l$ for type $A_{n}, C_{n}$ and $B_{2}$. More generally, the depth $d$ is shown to satisfy $\beta \cdot l \leq d \leq l$ with $\beta \in(0,1)$, where $\beta$ is uniquely determined by the simple type of $G$. We also show that the simple $N$ - $N$-bimodules contained in $L^{2}(M)$ generate the Verlinde ring $R_{l}(G)$ as its fusion category.
(Received September 12, 2023)

1192-46-32394
M. Eugenia Celorrio Ramirez*, Lancaster University. Arens regularity of weighted convolution algebras that arise from totally ordered semilattices.
In 1951, Arens introduced two products in the bidual $A / \prime$ of a given Banach algebra $A$. These products are such that the space
$A \prime \prime$ is a Banach algebra with respect to both products and $A$ is a closed subalgebra of both these algebras. In this talk, we shall introduce these products and we shall talk about the Arens regularity of weighted semigroup algebras that arise from a family of totally ordered semilattices.
(Received September 12, 2023)

Daniel Willem Van Wyk*, Fairfield University, Dana P Williams, Dartmouth College. Properties preserved by groupoid equivalence..
Morita equivalence between two C*-algebras essentially implies that these C*-algebras share the same representation theory. Consequently, many representation-theoretic properties, such as being GCR or CCR, are preserved under Morita equivalence. Another related concept is that of groupoid equivalence. Equivalent groupoids have Morita equivalent C*-algebras. Often, representation-theoretic characterizations of groupoid $C^{*}$-algebras involve the isotropy and orbit structure of the underlying groupoid. In this presentation, I will explore isotropy and orbit structure properties that are preserved under groupoid equivalence. My goal is to provide an introductory talk on these topics. Before delving into groupoid equivalence, I will briefly describe what a groupoid and its associated C*-algebra is.
(Received September 12, 2023)

## 1192-46-32733

Daniel Logan Blevins*, University of Missouri - Columbia. Computer - Assisted Proof of Khinchin Inequality.
We aim to compute sharp constants for the Khinchin inequality in high dimension through novel numerical approach. Sampling techniques are employed to create finite nets on the space, over which we apply the computation. We explore a variety of methods in computing the expectation, optimizing for efficiency in different regimes of the parameter. We believe this method can be applied to explore sharp bounds in other inequalities, and may shed light on the harder task of computing Grothendieck's constant.
(Received September 12, 2023)

## 1192-46-33138

Oleg Friedman*, Lander College for MEN/Touro University, NY, Alexander A Katz, Department of Mathematics and Computer Science, St. John's College of LAS, St. John's University, NY. Projections from a locally JB-algebra onto its factors.. A version of Apostol theorem on surjectivity of projections from projective limits onto factors is established for locally JBalgebras (projective limits of projective families of JB-algebras).
(Received September 13, 2023)

## 1192-46-33141

Cassie Ding, Brown University, Mattie Ji*, Brown University, Kun Meng, Brown University. Euler Calculus of Definable Sublevel Sets with Applications to Topological Data Analysis.
Euler calculus is an integral calculus with respect to the Euler characteristic as a finitely additive "measure". Motivated originally by sheaf theory and tame topology, Euler calculus now provides the foundational frameworks behind the Euler characteristic transform (ECT), proposed by Turner et al., and the smooth Euler characteristic transform (SECT), proposed by Crawford et al., in topological data analysis. In this work, given a continuous definable function $f: S \rightarrow \mathbb{R}$ on a definable set $S$ , we study sublevel sets of the form $S_{t}^{f}=\{x \in S: f(x) \leq t\}$ for all $t \in \mathbb{R}$. Using tame topology and Euler calculus, we prove that the Euler characteristic of $S_{t}^{f}$ is right continuous with respect to $t$, which is a generalization of Curry, Mukherjee, and Turner's result on piecewise-linearly embedded simplicial complexes. Furthermore, when $S$ is compact, we use a mapping cylinder argument to show that $S_{t+\delta}^{f}$ deformation retracts to $S_{t}^{f}$ for all sufficiently small $\delta>0$. Applying these results, we show that the ECT of any constructible function is right continuous, which suggests that the sample paths of this stochastic process are right continuous with left limits (RCLL). We also show that one can recover ECT from SECT completely. (Received September 13, 2023)

## 1192-46-33758

Davit Harutyunyan*, University of California, Santa Barbara. Function space identification for peridynamics. In the model of linearized peridynamics the associated function space is a subspace of $L^{p}(\Omega)$ that contains the fractional Sobolev space $W^{s, p}(\Omega)$. The question is whether for Lipschitz domains $\Omega \subset \mathbb{R}^{n}$ the function space is in fact $W^{s, p}(\Omega)$. The question has been answered positively by Mengesha and Scott in the case when $\Omega$ is either the entire space $\mathbb{R}^{n}$ or the half space. We answer the question affirmatively for bounded Lipschitz domains $\Omega$ with small Lipschitz constant (in particular $C^{1}$ domains).
(Received September 25, 2023)

## 1192-46-33876

James Henry Harbour*, University of Virginia. Discrete measured groupoid von Neumann algebras via malleable deformations and 1-cohomology. Preliminary report.
Given a probability measure preserving groupoid $\mathcal{G}$, we study properties of the corresponding von Neumann algebra $L(\mathcal{G})$ using the techniques of deformation-rigidity theory. Building on work of Sinclair and Hoff, we extend the Gaussian construction for equivalence relations to general measured groupoids. Using Popa's spectral gap argument, we then obtain structural properties about $L(\mathcal{G})$ including primeness and lack of property $(\Gamma)$. We also generalize results of de Santiago, Hayes, Hoff, and Sinclair to characterize maximal rigid subalgebras of $L(\mathcal{G})$ in terms of the corresponding groupoid $L^{2}$ cohomology.
(Received September 27, 2023)

## 47 Operator theory

## 1192-47-26788

Yunied Puig*, Claremont McKenna College. From Operator Theory to Additive Combinatorics and Ergodic Theory..

We will discuss our work on Kriz's theorem, which asks about the existence of a subset of positive integers with specific combinatorial properties. While the theorem was first proven by Kriz in 1987 and has since been proven by different methods, all existing proofs rely on Lovasz's theorem concerning the chromatic number of Kneser graphs and give the same example originally exhibited by Kriz. Our approach involves adopting a new perspective, dynamics of linear operators acting on infinite dimensional separable Banach spaces, and as a result, we have generalized Kriz's theorem and obtained a new proof that does not rely on Lovasz's theorem. Additionally, our example is genuinely different from the one originally exhibited by Kriz. This talk will showcase the power of dynamics of linear operators and its potential to make significant contributions to longstanding problems in additive combinatorics and ergodic theory.
(Received August 05, 2023)

1192-47-27118
Himanshu Singh*, University of South Florida. Provable convergence guarantee in Dynamical Systems. Preliminary report. Traditional topics of operator theory and complex analysis has been proved to be boon for the development of learning architecture for mathematical modelling of dynamical systems in Machine Learning \& Artificial Intelligence. One such topic is Koopman operator or simply composition operator ' $C_{\varphi}$ ' $\left(C_{\varphi} g=g \circ \varphi\right)$ interacting over the underlying reproducing kernel Hilbert spaces (RKHSs). Results of Ikeda, Ishikawa \& Sawano (JMAA ${ }^{\dagger}$ 2022) demonstrates that Koopman operator acting on RKHS (in general) is able to capture only an affine dynamics i.e. $\varphi(\mathbf{x}(t))=a \mathbf{x}(t)+b$. Here, 'capture' implies the boundedness of Koopman operator over the underlying RKHS. It is worth mentioning that many real-life physical phenomena (dynamical systems) are almost-surely not affine. Moreover, Koopman operators often fails to be compact over RKHSs as well; one such RKHS is Paley-Wiener Space, as demonstrated by Chacón, Chacón \& Giménez (PAMS ${ }^{\dagger} 2007$ ). Due to this, practitioner often encounters with the mathematical modelling failure of dynamical systems by Koopman operators. In this session, we will learn about the Liouville weighted composition operator $A_{\mathbf{f}, \varphi}$; defined as $A_{\mathbf{f}, \varphi} g=\nabla(g \circ \varphi) \cdot \mathbf{f}$, where $\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x}(t))$ describes the dynamical system. We will demonstrate how the Liouville weighted composition operator can help in capturing non-affine dynamics when it interacts over the Bergman-Segal-Fock Space. Furthermore, the essential norm estimates and compactness of $A_{\mathbf{f}, \varphi}$ will also be addressed which helps in establishing provable convergence guarantee phenomena. Time permitting, new development on weighted composition operator acting over Paley-Wiener Space will also be discussed. These results are crucial to the issue for handling the compactness characterization over the Paley-Wiener Space as (already mentioned) composition operator fails to act compactly over the Paley-Wiener Space.
(Received August 10, 2023)

## 1192-47-27822

Konrad Aguilar*, Pomona College, Stephan Ramon Garcia, Pomona College, Elena Kim, Massachusetts Institute of Technology, Frederic Latremoliere, University of Denver. Frobenius-Rieffel norms on matrix algebras and noncommutative metric geometry.
Frobenius-Rieffel norms were introduced by Rieffel to form seminorms that satisfy a form of the Leibniz/product rule and quotient rule on certain algebras including matrix algebras. Recently, we have found an application of these norms in the realm of noncommutative metric geometry for certain algebras called approximately finite-dimensional algebras (AF algebras), which are inductive/direct limits of matrix algebras in a suitable sense. However, in order to utilize these Frobenius-Rieffel norms, we needed to find explicit equivalence constants between them and the operator norm on matrix algebras that have certain desirable properties. In this talk, we discuss the application to noncommutative metric geometry of our findings as well as the matrix theory involved in finding these desirable equivalence constants. (This talk includes joint work with Stephan Ramon Garcia, Elena Kim, and Frédéric Latrémolière).
(Received August 21, 2023)

1192-47-28103
Dhruba R. Adhikari*, Kennesaw State University, Ashok Aryal, Minnesota State University Moorhead, Ghanshyam Bhatt, Tennessee State University, Ishwari Kunwar, Fort Valley State University, Rajan Puri, Wake Forest University, Min Ranabhat, University of Delaware. Solvability of Inclusions Involving Perturbations of Positively Homogeneous Maximal Monotone Operators.
Let $X$ be a real reflexive Banach space and $X^{*}$ be its dual space. Let $G_{1}$ and $G_{2}$ be open subsets of $X$ such that $\bar{G}_{2} \subset G_{1}$, $0 \in G_{2}$, and $G_{1}$ is bounded. Let $L: X \supset D(L) \rightarrow X^{*}$ be a densely defined linear maximal monotone operator, $A: X \supset D(A) \rightarrow 2^{X^{*}}$ be a maximal monotone and positively homogeneous operator of degree $\gamma>0, C: X \supset D(C) \rightarrow X^{*}$ be a bounded demicontinuous operator of type $\left(S_{+}\right)$with respect to $D(L)$, and $T: \bar{G}_{1} \rightarrow 2^{X^{*}}$ be a compact and uppersemicontinuous operator whose values are closed and convex sets in $X^{*}$. We first take $L=0$ and establish the existence of nonzero solutions of $A x+C x+T x \ni 0$ in the set $G_{1} \backslash G_{2}$. Secondly, we assume that $A$ is bounded and establish the existence of nonzero solutions of $L x+A x+C x \ni 0$ in $G_{1} \backslash G_{2}$. We remove the restrictions $\gamma \in(0,1]$ for $A x+C x+T x \ni 0$ and $\gamma=1$ for $L x+A x+C x \ni 0$ from such existing results in the literature. We also present applications to elliptic and parabolic partial differential equations in general divergence form satisfying Dirichlet boundary conditions.
(Received August 26, 2023)

1192-47-28147
Pan-Shun Lau*, University of Nevada, Reno. Generalized Numerical Ranges of Commuting Matrices.
We study the generalized numerical ranges of commuting matrices. We show that the joint numerical range of a family of 3-by3 commuting matrices is always convex. In addition, the convexity fails to hold in general for $n$-by- $n$ matrices with $n \geq 4$. We extend our techniques to examine the convexity of a generalized numerical range as defined by Asplund and Ptak.
(Received August 27, 2023)

Javad Mashreghi, Laval University, William Verreault*, University of Toronto. Nonlinear expansions in reproducing kernel Hilbert spaces.
Over the last few years, many mathematicians became interested in a nonlinear analogue of Fourier series that allows them to approximate a signal by a sum of terms whose components represent frequency and amplitude. It is the Blaschke unwinding series introduced by Coifman, or adaptive Fourier decomposition. Because it has many advantages over the classical Fourier series, this series expansion has been used in several other problems since. Yet, the question of convergence of the series has remained a major problem for a few decades. We only know that it converges in certain weighted subspaces of $H^{2}$ and, by recent work, in Hardy spaces. I will introduce an expansion scheme in reproducing kernel Hilbert spaces which is motivated by operator theory and de Branges-Rovnyak spaces, and which as a special case covers the Blaschke unwinding series. The expansion scheme can also be generalized to cover certain reproducing kernel Banach spaces. I will discuss convergence results for this series expansion and present a few applications and examples.
(Received August 31, 2023)

## 1192-47-28626

Douglas T. Pfeffer*, University of Tampa. Automorphisms of some subalgebras of the disc algebra.. Preliminary report. Let $\mathscr{A}$ denote the disc algebra - the algebra of functions that are holomorphic on the unit disc and extend to be continuous on the unit circle - and consider the subalgebras $\mathscr{A}_{0}=\{f \in \mathscr{A}: f(0)=0\}$ and $\mathscr{A}_{1}=\left\{f \in \mathscr{A}: f^{\prime}(0)=0\right\}$. If $T$ is an automorphism of either space, we show that $T: f \mapsto f \circ \phi$, where $\phi$ is some rotation of the disc. We establish this result by way of endomorphisms, where we carefully highlight the role that the fixed set of $\phi$ plays. We then generalize this result to yield the same conclusion for a broad class of subalgebras of $\mathscr{A}$ characterized by this same fixed set. Finally, we will discuss what can be said about these algebra's endomorphisms, and we'll look at other common subalgebras of $\mathscr{A}$. (Received September 01, 2023)

## 1192-47-28659

Chafiq Benhida, University of Lille, France, Raul E Curto, University of Iowa, George R. Exner*, Bucknell University. Geometrically Regular Weighted Shifts.
We exhibit a class of weighted shifts on Hilbert space parameterized by points in the unit square in $\mathbb{R}^{2}$. For pleasing regions in the parameter space, these geometrically regular weighted shifts have instances of membership in the classes of moment infinitely divisible, subnormal, $k$ - but not $(k+1)$-hyponormal, and completely hyperexpansive operators. The proofs of these memberships use interpolation by Bernstein functions or log Bernstein functions and signed representing measures (Berger charges).
(Received September 01, 2023)

## 1192-47-28719

Dan Han*, University of Louisville, Stanislav Molchanov, University of North Carolina at Charlotte. Spectral Analysis of Schrödinger-Type Operators Linked to Nonstationary Anderson Models on Lattices. Preliminary report.
This study delves into the behavior of solutions in the non-stationary Anderson parabolic problem, which is governed by the equation:

$$
\begin{aligned}
& \frac{\partial u}{\partial} t=\kappa^{\mathcal{L}} u(t, x)+\xi_{t}(x) u(t, x) \\
& u(0, x) \equiv 1,(t, x) \in[0, \infty) \times \mathbb{Z}^{d}
\end{aligned}
$$

Here, $\varkappa \mathcal{L}$ represents a non-local Laplacian operator, and $\xi_{t}(x)$ is a correlated white noise potential. The moment analysis of $u(t, x)$ leads to the Hamiltonian $H_{2}=2 \varkappa \mathcal{L}+B(x)$, where $B(x)$ is a positive definite function that characterizes the spatial correlation of $\xi_{t}(x)$ in the lattice $\mathbb{Z}^{d}$. This paper focuses on the spectral properties of $H_{2}$, specifically, the essential spectrum $S p_{\text {ess }}\left(H_{2}\right)$ and the discrete positive spectrum $S p_{d}\left(H_{2}\right)$. Under natural assumptions, we establish the existence of positive principal eigenvalues $\lambda_{0}\left(H_{2}\right)$, which play a crucial role in the intermittency structure of the field $u(t, x)$ as time approaches infinity. These findings contribute to a comprehensive understanding of the non-stationary Anderson parabolic problem and its associated spectral analysis.
(Received September 02, 2023)

## 1192-47-28762

John T Anderson, College of the Holy Cross, Linus Bergqvist, Stockholm University, Kelly Bickel*, Bucknell University, Joseph Cima, University of North Carolina, Alan Albert Sola, Stockholm University. Clark Measures for Rational Inner Functions.
Clark measures associated with one-variable inner functions are closely connected to a number of topics in classical operator theory and complex function theory on the unit disk. In this talk, we'll use an analogous definition to introduce Clark measures on the polydisk associated to d-variable rational inner functions. For the two-variable setting, we'll give exact formulas for these Clark measures, characterize when associated Clark embeddings are unitary, and time permitting, connect the vanishing of the Clark measure weights to the behavior of the rational inner function at a particular singularity.
(Received September 02, 2023)

## 1192-47-28815

Kelly Bickel, Bucknell University, Pamela Gorkin*, Bucknell University. Blaschke products and the Crouzeix Conjecture. Let $A$ be an $n \times n$ matrix and $p$ be a polynomial. Let

$$
W(A)=\langle A x, x\rangle: x \in \mathbb{C}^{n},|x|=1
$$

denote the numerical range of $A$. Michel Crouzeix conjectured that for every polynomial $p$ the following is true:

$$
|p(A)| \leq 2 \sup _{z \in W(A)}|p(z)|
$$

In this talk, we discuss some of the history of this conjecture and explain why we consider it in the model space setting. We then turn to Blaschke products with a focus on the properties that we hope will aid our understanding of the conjecture.
(Received September 03, 2023)

1192-47-28816
Flavia Colonna*, George Mason University, Nacir Hmidouch, Department of Mathematical Sciences, Talladega College, Talladega, AL 35160, USA. Essential norm of intrinsic operators from Banach spaces of analytic functions into weighted-type spaces
In this work we characterize the boundedness of a large class of operators, mapping a general Banach space of analytic functions defined on the open unit disk into a weighted-type Banach space of analytic functions and obtain estimates on the essential norm. The results show that the boundedness of such operators depends only on the behavior of the kernel functions. The results we obtain are extensions of previous work dealing with several specific classes of operators: the multiplication operator, the composition operator, the weighted composition operator, and a certain integral operator. We present applications to various choices of the domain space $X$, including, the Hardy space $H^{p}$, the space $S^{p}$ consisting of the functions whose derivatives are in $H^{p}$, the Bloch-type spaces $\mathcal{B}_{\alpha}$ (for $\alpha \geq 11$ ), $B M O A$, and the weighted Bergman space $A_{\alpha}^{p}$, (for $p \geq 1, \alpha>-1$ ).
(Received September 03, 2023)

## 1192-47-28860

Uwe Kahler*, Universidade de Aveiro. Triangular decompositions of quaternionic non-self-adjoint operators.
One of the principal problems in studying spectral theory for quaternionic or Clifford-algebra-valued operators lies in the fact that due to the noncommutativity many methods from classic spectral theory are not working anymore in this setting. For instance, even in the simplest case of finite rank operators there are different notions of a left and right spectrum. Hereby, the notion of a left spectrum has little practical use while the notion of a right spectrum is based on a nonlinear eigenvalue problem. In the present talk we will recall the notion of S-spectrum as a natural way to consider a spectrum in a noncommutative setting and use it to study quaternionic non-selfadjoint operators. To this end we will discuss quaternionic Volterra operators and triangular representation of quaternionic operators similar to the classic approaches by Gohberg, Krein, Livsic, Brodskii and de Branges. Hereby we introduce spectral integral representations with respect to quaternionic chains and discuss the concept of P- triangular operators in the quaternionic setting. This will allow us to study the localization of spectra of non-selfadjoint quaternionic operators and presented triangular decompositions of non-selfadjoint operators with respect to maximal quaternionic eigenchains.
(Received September 04, 2023)

1192-47-28907
Tomas Miguel Rodriguez*, University of Toledo, Sönmez Şahutoğlu, University of Toledo. Compactness of Toeplitz operators with continuous symbols on pseudoconvex domains in $\mathbb{C}^{n}$.
Let $\Omega$ be a bounded pseudoconvex domain in $\mathbb{C}^{n}$ with Lipschitz boundary and $\phi$ be a continuous function on $\bar{\Omega}$. We show that the Toeplitz operator $T_{\phi}$ with symbol $\phi$ is compact on the weighted Bergman space if and only if $\phi$ vanishes on the boundary of $\Omega$. This material comes from the presenter's PhD thesis written at University of Toledo under the supervision of Sönmez Sahutoğlu.
(Received September 04, 2023)

## 1192-47-28938

Priyadarshi Dey*, Kenyon College. Projections in the combination of powers of operators of finite order.
An interesting problem in Banach space theory is to study the projections which are in the convex hull of surjective isometries. The study was initiated by Fernanda Botelho for the space of all continuous functions with values in a strictly convex space and it was shown that for a strictly convex Banach space $X$, a projection which is in the convex hull of 2 surjective isometries is a generalized bicircular projection. The study was further generalized to three projections and for projections which lie in the convex hull of $n$-surjective isometries. This problem has also been studied in several settings. In this talk, I will give a classification result of projections which belong to the combination of powers of finite order operators. In specific, I will mention results for the case of operators upto order 4 . Some open problems and unsolved cases will also be mentioned. This is a joint work with Fernanda Botelho \& Zachary Easley.
(Received September 04, 2023)

1192-47-29281
Vishwa Dewage*, Clemson University, Mishko Mitkovski, Clemson University. Quasi-radial Toeplitz algebras via quantum harmonic analysis on the Fock space.
The norm density of Toeplitz operators in the Toeplitz algebra over the Fock space was proved by Xia. The Toeplitz algebra over the Fock space coincides with a certain algebra of bounded uniformly continuous operators in quantum harmonic analysis by recent results of Fulsche. As a consequence, he also proved that Toeplitz operators with bounded uniformly continuous symbols are norm dense in the Toeplitz algebra as well as in translation invariant subalgebras. We prove that this extends to G-invariant Toeplitz algebras where G is a subgroup of the affine unitary group. In particular, this includes quasi-radial Toeplitz algebras which are not translation invariant. We revisit the Gelfand theory of quasi-radial Toeplitz algebra via the tools from quantum harmonic analysis, providing a simpler proof of the fact that this Toeplitz algebra is isomorphic to the algebra of all bounded multi-sequences that are uniformly continuous with respect to the square-root metric. This is a joint work with Mishko Mitkovski.
(Received September 06, 2023)

Boo Rim Choe*, Korea University. Compact difference of composition operators on the Hardy space.
Answering to a question raised by Shapiro and Sundberg in 1990, we obtain two characterizations for compact differences of composition operators acting on the Hilbert-Hardy space over the unit disk. We first obtain a characterization in terms of certain Bergman-Carleson measures involving derivatives of the inducing maps. Based on such a result, we take one step further to obtain a completely new derivative-free characterization, which is more intuitive and much simpler. The talk is based on two joint works with my colleagues Koeun Choi, Hyungwoon Koo, Inyoung Park and Jonho Yang.
(Received September 06, 2023)

## 1192-47-29933

Marius Beceanu, NYS, Jiho Hong, Seoul National University, Hyun-Kyoung Kwon*, University At Albany, SUNY, Mikyoung Lim, Korea Advanced Institute of Science and Technology. Spectral optimization and the Gohberg-Sigal Theory. We consider the eigenvalue problem for the Laplacian in two dimensions with the Dirichlet boundary condition. We generalize the simple eigenvalues result by the second and the fourth authors to cover the higher multiplicity cases. The proof uses the layer potential characterization of the eigenvalues and the Gohberg-Sigal theory for operator-valued functions. (Received September 08, 2023)

## 1192-47-30165

Brooke Randell*, Bill and Linda Frost Fund. Exploring the Numerical Range of Block Toeplitz Operators.
This poster will discuss the numerical range of a family of Toeplitz operators with symbol function $\phi(z)=A_{0}+z A_{1}$, where $A_{0}$ and $A_{1}$ are $2 \times 2$ matrices with complex-valued entries. A special case of a result proved by Bebiano and Spitkovsky in 2011 states that the closure of the numerical range of the Toeplitz operator $T_{\phi(z)}$ is the convex hull of $\{W(\phi(z)): z \in \partial \mathbb{D}\}$. Here, $W(\phi(z))$ denotes the numerical range of $\phi(z)$. We combine this result with the envelope algorithm to describe the boundary of the convex hull of $\{W(\phi(z)): z \in \partial \mathbb{D}\}$. We also place specific conditions on the matrices $A_{0}$ and $A_{1}$ so that $\{W(\phi(z)): z \in \partial \mathbb{D}\}$ is a set of potentially degenerate circular disks. The convex hull of $\{W(\phi(z)): z \in \partial \mathbb{D}\}$ takes on a wide variety of shapes, including the convex hull of limalc \{c\}ons. \par (Received September 09, 2023)

## 1192-47-30261

Joseph A. Ball*, Virginia Tech, Haripada Sau, Indian Institute of Science Education and Research, Maharashta, INDIA. Dilation and Model Theory for Pairs of Commuting Contraction Operators.
The Sz.-Nagy dilation theorem asserts that any Hilbert-space contraction operator $T$ on $\mathcal{H}$ can be lifted (minimally) to an isometric operator $V$ on a Hilbert space $\mathcal{K} \supset \mathcal{H}\left(\right.$ so $\left.T^{*}=\left.V^{*}\right|_{\mathcal{H}}\right)$. Later work of Sz.-Nagy and Foias identified the characteristic function $\Theta_{T}$ (an explicit contractive-operator analytic function on the unit disk determined by $T$ ) as a complete unitary invariant for $T$ and then used $\Theta_{T}$ to construct a unitarily equivalent functional-model version of $T$ and its minimal isometric lift $V$. Furthermore one can start with an arbitrary "pure" contractive operator function $\Theta$ to construct a functionalmodel operator $T(\Theta)$ whose characteristic function $\Theta_{T(\Theta)}$ coincides with the original contractive-operator analytic function $\Theta$. In this way the study of completely non unitary contraction operators $T$ is equivalent to the study of pure contractive operator functions $T$. A natural next step is to try to extend this program to the setting of a commuting pair of contraction operators $\left(T_{1}, T_{2}\right)$. A well-known result of Ando provides the analogue of the Sz.-Nagy dilation theorem: any commuting contractive pair $\left(T_{1}, T_{2}\right)$ can be lifted to a commuting isometric pair $\left(V_{1}, V_{2}\right)$. The goal of this talk is to present recent work on extending the second step (the Sz.-Nagy-Foias functional model and unitary classification theory for a single contraction operator $T$ ) to the commuting contractive-pair setting.
(Received September 09, 2023)

## 1192-47-30267

Alberto Takase*, Michigan State University. Spectral estimates of dynamically-defined and amenable operator families. We consider kernel operators defined by a dynamical system. The Hausdorff distance of spectra is estimated by the Hausdorff distance of subsystems. We prove that the spectrum map is $\frac{1}{2}$-Hölder continuous provided the group action and kernel are Lipschitz continuous and the group has strict polynomial growth. Also, we prove that the continuity can be improved resulting in the spectrum map being Lipschitz continuous provided the kernel is instead locally-constant. This complements a 1990 result by J. Avron; P.H.M.v. Mouche; B. Simon establishing that one-dimensional discrete quasiperiodic Schrödinger operators with Lipschitz continuous potentials, e.g., the Almost Mathieu Operator, exhibit spectral $\frac{1}{2}$-Hölder continuity. Also, this complements a 2019 result by S. Beckus; J. Bellissard; H. Cornean establishing that $d$-dimensional discrete subshift Schrödinger operators with locally-constant potentials, e.g., the Fibonacci Hamiltonian, exhibit spectral Lipschitz continuity. Our work exposes the connection between the past two results, and the group, e.g., the Heisenberg group, needs not be the integer lattice nor abelian. This is joint work with Siegfried Beckus.
(Received September 09, 2023)

1192-47-30375
Christopher Felder, Indiana University Bloomington, Douglas T. Pfeffer, University of Tampa, Benjamin Peter Russo*, Oak Ridge National Laboratory. Spectra for Toeplitz Operators Associated with a Constrained Subalgebra. A two-point algebra is a set of bounded analytic functions on the unit disk that agree at two distinct points $a, b \in \mathbb{D}$. This algebra serves as a multiplier algebra for the family of Hardy Hilbert spaces $H_{t}^{2}:=\left\{f \in H^{2} \mid f(a)=t f(b)\right\}$. In this talk, we will show various spectra of certain Toeplitz operators acting on these spaces are connected. (Received September 09, 2023)

## 1192-47-30425

Linda J. Patton*, Cal Poly San Luis Obispo. Operator numerical ranges determined by finite matrices. Preliminary report The numerical range of a linear operator on a Hilbert space is a bounded convex subset of the complex plane. The numerical range of an operator on a finite dimensional Hilbert space is compact and it is known to be the convex hull of a certain algebraic curve. In some cases, when $T$ is an operator on an infinite dimensional Hilbert space, there exists an operator (given by a matrix $M$ ) on a finite dimensional space such that $W(T)$ or its closure is equal to $W(M)$. For example, Tso and Wu showed that for any quadratic operator $T$ on a Hilbert space, there exists a 2 -by- 2 matrix $M$ such that $W(M)$ is the closure of $W(T)$. In addition, if $T_{\phi}$ is a Toeplitz operator with quadratic polynomial symbol, the closure of $W\left(T_{\phi}\right)$ is the numerical range of a finite matrix exactly when the root of $\phi$ is in the closed unit disk. Generalizations of this last statement will be discussed, as will examples of some block Toeplitz operator numerical ranges that are determined by finite matrix numerical ranges. (Received September 10, 2023)

1192-47-30778
J. E. Pascoe, Drexel University, Ryan K. Tully-Doyle*, Cal Poly SLO. Matrix convexity, generating functions, and graphs Consider the set of balanced brackets (e.g. "[[][]]"). One can view this set $\mathcal{L}$ as the collection of words $\omega$ formed from the alphabet $\mathcal{G}$ consisting of the symbol "[" and its adjoint "]". This collection of words is called a formal language $\mathcal{L}$ in $\mathcal{G}$ (in this case, the Dyck paths). Supposing an alphabet $\mathcal{G}=\left\{a_{1}, \ldots, a_{d}\right\}$ and a collection $\mathcal{L}$ of words $\omega$ in $\mathcal{G}$, we can create a function that completely enumerates the language, i.e. the formal power series

$$
f\left(a_{1}, \ldots, a_{g}\right)=\sum_{\omega \in \mathcal{L} .} \omega
$$

It turns out that if the language $\mathcal{L}$ satisfies certain structural properties (e.g. closure under the involution) then the enumeration function $f$ is matrix convex and hence analytic by way of the royal road theorem. In this case, a butterfly realization of $f$ (an analogue of the classical Kraus integral representation) arises from a natural graph with chromatic edges. We'll look at some examples that arise in the study of boundary regularity of functions in several complex variables. (Received September 11, 2023)

## 1192-47-30983

Iris T Emilsdottir*, Rice University. Johnson-Schwartzman gap labelling and applications.
Gap labelling involves identifying a set of labels for the spectral gaps on dynamically defined operators. Our focus will be on one-dimensional Schrödinger operators, where Johnson's theorem can be used to find those labels via the Schwartzman homomorphism. After a brief introduction we will discuss several recent classifications of Schwartzman groups and their applications including answering a question posed by D-F-Z regarding the Jacobi operator defined by the doubling map. (Received September 11, 2023)

1192-47-31083
Juntao Liu*, St. Olaf College, David Walmsley, St. Olaf College. Super Strongly Hypercyclicity for Weighted Backward Shifts. Preliminary report.
Recently in the theory of linear operators, two stronger versions of hypercyclicity have been introduced and investigated: strong hypercyclicity (SH) and hypermixing (HM). For the family of weighted backward shifts on $\ell^{p}$ or $c_{0}$, these new properties have been characterized in terms of the action of the forward shift. We introduce a new property for the family of weighted backward shifts, called super strong hypercyclicity (SSH), and characterize it in terms of the forward shift. We show that HM $\Rightarrow \mathrm{SSH} \Rightarrow \mathrm{SH}$ and $\mathrm{SSH} \nRightarrow \mathrm{HM}$, and ask whether $\mathrm{SH} \Rightarrow$ SSH for the family of weighted backward shifts.
(Received September 11, 2023)

1192-47-31254
Fernanda Botelho, University of Memphis, Richard Fleming, Central Michigan University, Fnu Monika*, Stevens Institute of Technology. Dr.
The motivation comes from two different but related topics of interest since ages. First Lindenstrass and Tzafriri lifting property (LP) and secondly from the problem of extension of bounded operators. We say that a subspace $Y$ has the lifting property (LP) if, for every operator $\psi$ from a Banach Space $X$ to a Banach Space $W$ and for every bounded linear operator $S: Y \rightarrow W$, there is a bounded linear operator $\hat{S}: Y \rightarrow X$, such that $S=\psi \circ \hat{S}$. We note that the space $\ell_{1}$ has LP and in addition to that, any space isomorphic to $\ell_{1}$ also has LP. In this talk, we put attention on $W$ rather than $Y$ to present an alternate lifting property(ALP). We consider the case where $W$ is the quotient of $X$ by a closed subspace $J$ with $\psi=\pi$, where $\pi$ is the quotient map. We are interested in the existence of norm preserving lifts of a bounded operator $S: Y \rightarrow X / J$, if for every $Y$ and $S$, such a lift exists then we say that the pair $(X, J)$ has quotient lifting property (QLP). We will discuss conditions for the existence of norm preserving lifts of operators (QLP) and its interconnections with metric projections, linear selections and proximinality. Some recent developments will also be mentioned as well. This is a joint work with Fernanda Botelho and Richard Fleming.
(Received September 11, 2023)

1192-47-31449
Deborpita Biswas*, Clemson University. Near-Riesz Bases
James R. Holub, in one of his papers in 1994, introduced the idea of Near-Riesz bases from the perspective of frames. We develop the definition of near- Riesz bases for the sequences which are not frames. We give a characterization of near- Riesz bases using the index theory for the associated synthesis operator.
(Received September 11, 2023)

Christopher Felder, Indiana University Bloomington, Brittney R Miller*, Coe College. Quadratic Rational Self-maps of the Disk. Preliminary report.
Let $\varphi$ be a rational map from $\mathbb{D}$ to itself that is analytic on $\mathbb{D}$. The composition operator $C_{\varphi}$, with symbol $\varphi$, is defined by
$C_{\varphi} f=f \circ \varphi$ for $f$ in a Hilbert space of analytic functions on $\mathbb{D}$. Motivated by classifying the adjoints of these composition operators and their kernels, we are led to investigate when quadratic rational maps are self-maps of the unit disk. (Received September 11, 2023)

## 1192-47-31541

Maxim S. Derevyagin*, University of Connecticut. A reflection on perfect state transfer and related problems in algebra and analysis. Preliminary report.
We will start by presenting the interplay between orthogonal polynomials, spectral theory of Jacobi matrices, and continued fractions and then show how it can be used for perfect state quantum transfer problems. After that we will consider some new results and discuss perspectives of the methods.
(Received September 11, 2023)

## 1192-47-31595

Ana Colovic, Washington University in St. Louis, Carson Givens, Iowa State University, Muhammad Haashir Ismail,
Virtual University of Pakistan, Aathreya Kadambi*, University of California, Berkeley, Nathan A. Wagner, Washington University In St. Louis, Isaac Wu, Carnegie Mellon University, Jiahui Yu, Pomona College. Boundedness of Hankel Operators on the Hartogs Triangle. Preliminary report.
Hankel operators defined with respect to the Bergman projection form an important class of operators on $L^{p}$ spaces, whose boundedness is well understood on the unit ball. In our work, we consider the problem of classifying the boundedness and compactness of Hankel operators on the Hartogs triangle, a pseudoconvex domain that is not smooth and exhibits interesting analytic behavior. We obtain specific examples of symbols for which the Hankel operator is unbounded on $L^{p}$ for certain values of $p$, and also generalize results on $p$-Carleson measures which may be effective in showing the boundedness of Hankel operators under other conditions.
(Received September 11, 2023)

## 1192-47-31610

Anna H. Kaminska*, University of Memphis. Geometric properties of noncommutative symmetric spaces of measurable operators.
In this talk we consider noncommutative symmetric spaces of measurable operators $E(\mathcal{M}, \tau)$ and unitary matrix ideals $C_{E}$, where $\mathcal{M}$ is a von Neumann algebra with a semi-finite, faithful and normal trace $\tau$, and $E$ is a (quasi)Banach function and a sequence lattice, respectively. It appears that under some standard assumptions on $E$, a number of geometric properties of $E$ is inherited by $E(\mathcal{M}, \tau)$ and/or $C_{E}$. For instance it is well known that if $E$ is uniformly convex then the corresponding spaces $E(\mathcal{M}, \tau)$ and $C_{E}$ have the same properties. We will present here how other geometric properties are related between $E$ and $E(\mathcal{M}, \tau)$ or $C_{E}$, for instance (complex) extreme points, (strong) smooth points, Radon-Nikodým property.
(Received September 11, 2023)

## 1192-47-31673

Trevor Camper*, Clemson University. A Szegö-Type Limit Theorem and Weyl's Law for Berezin-Toeplitz Operators.
Preliminary report.
Szegő Limit Theorems have been used to obtain semi-classical Weyl Laws for Gabor-Toeplitz Operators. In this talk, a new Szeg"o-Type Limit Theorem for a semiclassical setting will be presented. From this, semi-classical Weyl Laws for BerezinToeplitz Operators will be presented for a variety of function spaces and classes of symbols. Finally, some applications to Gabor systems and Bergman spaces will be presented.
(Received September 11, 2023)

1192-47-31744
Palak Arora*, Williams College. An optimal approximation problem for noncommutative polynomials and rational functions. Motivated by recent work on optimal approximation by polynomials in the unit disk, we consider the following noncommutative approximation problem: for a polynomial $f$ in $d$ noncommuting arguments, find an nc polynomial $p_{n}$, of degree at most $n$, to minimize

$$
c_{n}:=\left|p_{n} f-1\right|^{2}
$$

(Here the norm is the $\ell^{2}$ norm on coefficients.) We show that $c_{n} \rightarrow 0$ if and only if $f$ is nonsingular in a certain nc domain (the row ball). As an application, we prove the same result for nc rational functions, $f$. (Received September 12, 2023)

Gamal Mograby*, University of Maryland. Jacobi operators on graphs: Applications to almost Mathieu operators and Grover's quantum walk.
Jacobi operators can be viewed as the discrete analog of Sturm-Liouville operators and appear in various applications. This talk introduces an approach to studying such operators from a graph theoretical perspective. We present a class of Jacobi operators ("piecewise centrosymmetric Jacobi operators") defined on certain substitution graphs. We show that their spectral analysis can be explicitly related to the spectral theory of graph Laplacians using certain orthogonal polynomials. Depending
on time, we will discuss applications to almost Mathieu operators and Grover's quantum walk on graphs. This talk is based on joint work with Radhakrishnan Balu, Kasso A. Okoudjou, and Alexander Teplyaev.
(Received September 12, 2023)

## 1192-47-31840

Matthew H Faust*, Texas A\&M University, Jordy Lopez Garcia, Texas A\&M University. On Irreducibility of the Bloch Variety.
Given a $\mathbb{Z}^{2}$-periodic graph G, the Schrodinger operator associated to G is a graph Laplacian with a potential. After a Floquet transform our operator can be represented by a finite matrix whose entries are Laurent polynomials. The vanishing set of this characteristic polynomial is the Bloch variety. Questions regarding the algebraic properties of this object are of interest in mathematical physics. We will focus our attention on the irreducibility of this variety. Understanding the irreducibility of the Bloch variety is important in the study of the spectrum of discrete periodic operators, providing insight into quantum ergodicity. In this talk we will present results on preserving irreducibility of the Bloch variety after changing the period lattice. This is joint work with Jordy Lopez.
(Received September 12, 2023)

Meric Augat*, Bucknell University. The noncommutative Implicit Function Theorem and the L'vov-Kaplansky Conjecture. Preliminary report.
Free Analysis is a burgeoning subfield of complex analysis and operator theory that aims to understand noncommutative functions by studying their evaluations on matrices of any size. An interesting tool in Free Analysis is the Free Implicit Function Theorem, however, not all functions admit a point at which the Free IFT can be applied. An exploration into when we can apply the Free Implicit Function Theorem leads us to questions about the images of multilinear polynomials on the $n \times n$ matrices. Specifically, we encounter the L'vov-Kaplansky Conjecture: if $f$ is a multilinear polynomial, then is its image on the $n \times n$ matrices a linear subspace? We present the basic ideas behind the conjecture as well as questions about a weakened form of the conjecture with applications in Free Analysis.
(Received September 12, 2023)

## 1192-47-32139

David F Benson*, University of New Hampshire. Extensions of the Mandelbrot Set and Filled Julia Sets to $C^{*}$-algebras. Preliminary report.
In this talk, we define an extension of the Mandelbrot set and filled Julia sets for $C^{*}$-algebras. We explore similarities and differences between the traditional Mandelbrot set and filled Julia sets and their generalizations. We give sufficient and necessary conditions on the $\mathrm{C}^{*}$-algebra for the associated Mandelbrot set to be compact. We explore properties of Mandelbrot sets in terms of constructions with $C^{*}$-algebras. Visualizations of the obtained sets are shown. (Received September 12, 2023)

## 1192-47-32168

Jose Perea, Northeastern University, Matt Piekenbrock*, Northeastern University. Spectral relaxations of the persistent rank invariant.
Using a duality result between persistence diagrams and persistence measures, we introduce a framework for constructing families of continuous relaxations of the persistent rank invariant for parametrized families of persistence vector spaces indexed over the real line. Like the rank invariant, these families obey inclusion-exclusion, are derived from simplicial boundary operators, and encode all the information needed to construct a persistence diagram. Unlike the rank invariant, these spectrally-derived families enjoy a number of stability and continuity properties typically reserved for persistence diagrams, such as smoothness and differentiability over the positive semi-definite cone. Leveraging a connection to combinatorial Laplacian operators, we find the non-harmonic spectra of our proposed relaxation encode valuable geometric information about the underlying space, prompting several avenues for geometric data analysis. Finally, after reducing our proposed relaxation to a sum of a trace-class operators, we introduce a randomized algorithm based on the stochastic Lanczos quadrature method to efficiently approximate its value in quadratic time.
(Received September 12, 2023)

## 1192-47-32234

Miklós Pálfia*, Corvinus University of Budapest. Free functions preserving partial orders of operators.
Recently free analysis has been a very active topic of study in operator and function theory. In particular free functions that preserve partial orders of operators have been studied by a number of authors, in connection with Loewner's theorem. Also operator concave and convex free functions naturally get into the picture as we study the positive definite order preserving free functions like operator means, more generally means of probability measures on positive operators. We will go through recent results and open problems in this field, and we will cover some joint work with M. Gaál on real operator monotone functions.
(Received September 12, 2023)

1192-47-32913
Ken Dykema, Texas A\&M University, Amudhan Krishnaswamy Usha*, National Institute of Standards and Technology. On spectral operators in finite von Neumann algebras.
The Jordan canonical form of a matrix $T$ allows us to express $T$ as $A^{-1}(N+Q) A$, where $N$ is diagonal, $Q$ is nilpotent, and $Q, N$ commute. For bounded operators on a Hilbert space, such a decomposition exists (with $N$ normal and $Q$ quasinilpotent) precisely when $T$ is a spectral operator, as defined by Dunford. Operators which live in a finite von Neumann algebra possess a lattice of invariant subspaces known as the Haagerup-Schultz subspaces. We will present equivalent criteria for the spectrality of an operator in terms of the angles between these subspaces. Further, we will discuss a variety of classes of
operators which satisfy said conditions, as well as some intriguing open problems in this regard. (Received September 13, 2023)

1192-47-33013
Catherine Beneteau, University of South Florida, Fernanda Botelho, University of Memphis, Maria Cueto Avellaneda, University of Kent, Jill E Guerra*, Harvard University, Dijana Ilisevic, University of Zagreb, Sana Kazemi, University of Memphis, Shiho Oi, Niigata University, Japan. Surjective Isometries of $C^{*}$-algebras. Preliminary report.
We will discuss the complex spectrum of real linear surjective isometries of $\mathrm{C}^{*}$-algebras. We develop a decomposition for such isometries and use this representation to find the complex spectrum for a particular class of operators.
(Received September 13, 2023)

## 1192-47-33753

Bernard Akwei, University of Connecticut, Rachel Bailey, University of Connecticut, Tonya Patricks*, University of South Florida, Luke G Rogers, University of Connecticut, Genevieve Romanelli*, Tufts University, Alexander Teplyaev, University of Connecticut. Approximation of Laplacian Operators. Preliminary report.
Algorithms involving Laplacian eigenmaps are commonly used for dimension reduction in big data and machine learning settings. However, despite their wide-ranging applications, the theory behind these dimension-reduction algorithms is not complete. Using the methods of Giné and Koltchinskii (2006), inspired by Belkin and Niyogi (2005), we analyze the behavior of several graph Laplacian operators on points chosen from the interval $[-1,1]$. In particular, we compare these operators with the continuous averaging operator $\left(\mathcal{L}_{\varepsilon} f\right)(M)=\frac{1}{2 \varepsilon} \int_{M-\varepsilon}^{M+\varepsilon} f(M)-f(x) d x$. We then expand our study's results to consider other generalized measures and their associated properties of convergence.
(Received September 25, 2023)

1192-47-33892
Bernard Akwei, University of Connecticut, Rachel Bailey, University of Connecticut, Jonathan A. Kerby-White, Indiana University Bloomington, Luke G Rogers, University of Connecticut, Yiheng Su*, Colby College, Alexander Teplyaev, University of Connecticut. Laplacian Eigenmaps and Orthogonal Polynomials. Preliminary report.
In this talk, we investigate the nature of eigenmaps of graph Laplacians, in particular approximating the eigenmaps using Chebyshev polynomials with controlled errors. We begin by defining several types of discrete Laplacian operators on graphs whose vertices only connect to their nearest neighbors. We define eigen-coordinates $\left(f_{1}(x), f_{\ell}(x)\right) \in \mathbb{R}^{2}$ where $f_{\ell}$ are the eigenfunctions corresponding to the $\ell^{t h}$ smallest nonzero eigenvalues for the Laplacian operators. Next, we derive a general formula for eigenvalues and eigenfunctions for three specific discrete Laplacians: regular, probabilistic, and periodic. We then prove the eigen-coordinates for these operators are exactly Chebyshev polynomials of the first kind $T_{\ell}(x)$. Lastly, we analyze discrete Robin problems on the probabilistic Laplacian and show it converges to the continuous Robin Problem on $[-1,1]$. Eigenmaps are important in analysis, geometry, and machine learning, in particular, in nonlinear dimension reduction. However, few studies focus on the relationship between eigen-coordinates of graph Laplacians and Chebyshev and other polynomials. To this end, we study the error between eigen-coordinates and Chebyshev polynomials,
$\operatorname{Err}(x)=\left|f_{\ell}(x)-T_{\ell}\left(f_{1}(x)\right)\right|$. We have constructed graph Laplacian operators on $[-1,1],[-1,1]^{2}$, the sphere $S^{2}$, and the Sierpinski gasket. We also have explored these operators when the points are selected randomly with various distributions, such as uniform and Gaussian. We have developed algorithms capable of generating eigenmap plots for the aforementioned sample spaces. In our next step, we would also like to compare the eigen-coordinates of graph Laplacian operators constructed from those intervals with families of orthogonal polynomials, such as the Hermite polynomials and possibly exceptional orthogonal polynomials.
(Received September 27, 2023)

## 49 Calculus of variations and optimal control; optimization

## 1192-49-26328

Miho Kasai*, University of Michigan. Solutions to a Linear Equation on the Set of Probability Vectors on Graphs. Preliminary report.
Recent focus on the theory of calculus of variations has been the study of Hamilton-Jacobi equations in various measures. There has been increased interest in PDEs in the space of probability measures due to its connections in mean field games, however, not much is known about the HJEs in a discretized setting. In this talk we investigate solutions to a linear HamiltonJacobi equations in the Wasserstein space of probability vectors on a finite simply connected graph, motivated by Nash Equilibria with finitely many states in Game Theory. We prove that there exists a solution under the assumption that the initial value function $u_{0}: \mathcal{P}(G) \rightarrow \mathbb{R}$ is Fréchet continuously differentiable.
(Received July 25, 2023)

1192-49-26433
Aidan Benjamin Backus*, Brown University. The p-Laplacian, minimal laminations, and the max flow/min cut theorem. We study the limiting behavior of the $p$-Laplacian and its convex dual problem as $p \rightarrow 1$. The solution of the 1 -Laplacian, known as a function of locally least gradient, induces a laminations of minimal surfaces. We prove that the lamination has Lipschitz regularity, and that the property of inducing minimal laminations completely characterizes functions of locally least gradient. The dual problem is a system of PDE analogous to the $\infty$-Laplacian. We show that the solutions of the dual problem are "bottlenecked" by the constraint that they must have flux density 1 through the minimal surfaces, establishing a continuous analogue of the max flow/min cut theorem. This generalizes a theorem of Daskalopolous and Uhlenbeck concerning the $\infty$-Laplacian, and is progress towards a conjecture of Thurston on the role of the max flow/min cut theorem in Teichmueller theory.
(Received July 28, 2023)

Michail E. Filippakis*, Department of Digital Systems, Univercity of Piraeus, Greece. Nodal Solutions for Neumann Problems with a Nonhomogeneous Differential Operator. Preliminary report.
lusepackagesfs\}\usepackage\{textcomp\}\usepackage\{authblk\}WeconsideranonlinearellipticNeumannproblemdrivenbyanonhomogeneousdifferentialoperator,whichisstrictlymonotoneandincorporatesasspecialcasesthe$p$-Laplacian,the$(p-q)$-differentialoperatorandthegeneralized$p$meancurvaturedifferentialoperator.UsingvariationalmethodscoupledwithsuitabletruncationandcomparisontechniquesandMorsetheory(criticalgroups),weshowthattheproblemhasatleastthreenontrivialsmoothsolutions,onepositive,thesecondnegativeandthethirdnodal.Alsoweshowthattheproblemhasextremalnontrivialconstantsignsolutions.(ReceivedAugust23,2023)undefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefined

1192-49-28261
Ahmed Attia*, Argonne National Laboratory, Arthur Barnes, Los Alamos National Laboratory, Russell Bent, Los Alamos National Laboratory, Sven Leyffer, Argonne National Laboratory, Adam Mate, Los Alamos National Laboratory, Minseok Ryu, Arizona State University. Efficient Heuristic Approaches to Binary Optimization: a Sensor Placement Application. We present new heuristic approaches aimed at resolving the challenge of determining optimal placements for geomagnetically-induced currents blocking devices on electrical grids. Traditionally, these determinations are approached by formulating the problem as mixed-integer nonlinear programming (MINLP) models and solving them using established MINLP solvers grounded in the spatial branch-and-bound algorithm. However, the pursuit of an optimal solution often demands substantial computational time due to its inability to leverage the inherent problem structure. In this talk, we present two heuristic approaches based on a stochastic learning algorithm for binary optimization, and a three-block alternating direction method of multipliers algorithm, both of which exploit the structure of the problem of interest and show superior performance compared to conventional MINLP solvers.
(Received August 28, 2023)

## 1192-49-28358

Clara Pitkins*, Rochester Institute of Technology. Comparison of Modern Optimization Algorithms: Application to Image Deblurring. Preliminary report.
In this work, we present an exploration of modern optimization methods of stochastic gradient type and their applications to the image deblurring problem in particular. Comparison of existing algorithms and new approaches are addressed and numerical examples are provided. Finally, we discuss the applicability of the methods for nonlinear parameter identification problems.
(Received August 29, 2023)

## 1192-49-28468

Edward D Huynh*, University of Arizona, Christian Parkinson, University of Arizona. Optimal Path Planning on Manifolds. Preliminary report.
Optimal path-planning is of crucial importance for robotics, autonomous vehicles, and many other engineering applications. In this work, we consider an agent on a regular surface embedded in $R^{3}$ that seeks to navigate from some initial point to another point on the manifold. The problem may be modeled as an optimal control problem and can be solved using the associated Hamilton-Jacobi-Bellman equation. We develop a novel model and then we design an algorithm that finds the solution efficiently based on the generalized Hopf-Lax formula. We report our preliminary results in this presentation. (Received August 30, 2023)

1192-49-29181
Sharmin Afroz*, University of Alabama, Brendan Ames, University of Alabama. Deflation Free Sparse Optimal Scoring Problem. Preliminary report.
Linear Discriminant Analysis (LDA), an important supervised learning method and a tool for classification, dimension reduction, and data visualization, is vastly used in face recognition, information retrieval etc. A significant challenge of LDA in a high dimensional setting, is the need for many training observations. However, much recent research has shown that elastic net regularization facilitates discriminant analysis in the under sampled high-dimensional setting; in particular, the use of an $l_{1}$ -norm penalty forces sparsity and enables feature selection. In this work, we propose a new iterative approach for the Sparse Optimal Scoring (SOS) problem. SOS reformulates penalized LDA as an equivalent regression task with feature selection via elastic net regularization. Earlier heuristics for SOS rely on deflationary approaches, where discriminant vectors are found in sequence, i.e., we use previously calculated discriminant vectors to calculate the next. In order to avoid propagation of errors while sequentially calculating the family of discriminant vectors, we instead consider SOS as an orthogonality constrained optimization problem. We use a specialization of the Bregman iteration splitting method proposed for orthogonality constrained optimization proposed by Lai and Osher, to calculate all scoring and discriminant vectors simultaneously. We compare our Bregman iteration approach with existing deflationary algorithmic frameworks (e.g., Least Angle Regression, Proximal Gradient methods) for the SOS problems using randomly generated data from a family of Gaussian distributions. (Received September 05, 2023)

1192-49-29197
Deepanshu Verma*, Emory University. Deep Learning Meets Optimal Control and Dynamical System.
Deep neural networks, or DNNs, have become a popular tool in various fields due to their universal approximation properties. This talk delves into the intricacies of designing efficient DNN algorithms and architectures. These architectures are known to be difficult to design and train, often resulting in challenges such as the exploding or vanishing gradients issue. These networks also exhibit a large number of equivalent optimal solutions in the parameter space. A neural network architecture designed using fractional derivatives will be introduced to tackle the former challenge. Additionally, the notion of bias ordering, with theoretical guarantees, will be introduced to narrow down the parameter search space.

Michael J Miller, Le Moyne College, Alain Trouvé, Ecole Normale Supérieure Paris-Saclay, Laurent Younes*, Johns Hopkins University. Mesh-Based Image Varifolds Registration for Mapping Spatial Transcriptomics.
Advances in the development of largely automated microscopy methods (spatially resolved transcriptomics) such as MERFISH for imaging cellular structures in mouse brains are providing spatial detection of micron resolution gene expression. While there has been tremendous progress made in the field Computational Anatomy to perform diffeomorphic mapping at the tissue scales for advanced neuroinformatic studies, the development of such techniques for micro-scale data remains largely unexplored. We introduce algorithms mapping mesh-based representations of spatially resolved transcriptomics data through the computation of geodesics in the space of diffeomorphism, s extending the family of large deformation diffeomorphic metric mapping (LDDMM) algorithms. The data representation is adapted to discrete and non-smooth structures with the introduction of "image varifolds" (or space-feature measures) and their associated diffeomorphic action. The implementation solves a particle-based optimal control with end cost provided by an RKHS norm in the space of measures. We also provide an extension of this approach to cross-modality mapping, and in particular to the alignment of a spatial transcriptomics image to a labelled atlas. The proposed algorithms are illustrated by (preliminary) experimental results.
(Received September 06, 2023)

1192-49-29316
Laurent Younes*, Johns Hopkins University. Shape Spaces: Construction and Algorithms.
The talk will review the general approach underlying the construction of shape spaces as Riemannian or metric spaces, focusing on several examples, such as elastic metrics on curves, or diffeomorphic and "diffeo-elastic" metrics on curves, surfaces, volumes or images. This construction, which includes most of the shape spaces introduced in the literature, relies on the definition of an invariant metric (with respect to suitable group actions) on a top (or pre-shape) space, followed by a projection on the quotient space. The calculation of the distance, which can be formulated an an optimal control problem, is performed using the adjoint method, and the relevant algorithms will be described and illustrated by experimental results (Received September 06, 2023)

## 1192-49-29858

Tan Bui-Thanh*, Oden Institute for Computational Sciences and Engineering. Model-constrained deep learning methods for forward, inverse and UQ problems.
Deep Learning (DL), in particular deep neural networks (DNN), by default is purely data-driven and in general does not require physics. This is the strength of DL but also one of its key limitations when applied to science and engineering problems in which underlying physical properties and desired accuracy need to be achieved. DL methods in their original forms are not capable of respecting the underlying mathematical models or achieving desired accuracy even in big-data regimes. However, many data-driven science and engineering problems, such as inverse problems, typically have limited experimental or observational data, and DL would overfit the data in this case. Leveraging information encoded in the underlying mathematical models, we argue, not only compensates missing information in low data regimes but also provides opportunities to equip DL methods with the underlying physics, hence promoting better generalization. In this talk we will present two real time approaches: 1) mcTangent (a model-constrained tangent slope learning) approach for learning dynamical systems; and 2) TNet (a model-constrained Tikhonov network) approach for learning inverse solutions. Both theoretical and numerical results for various problems including transport, heat, Burgers and Navier-Stokes equations will be presented.
(Received September 08, 2023)

1192-49-30322
Leo Chang*, Northwestern University, Stanley Jian, Columbia University, Aren Martinian*, University of California, Berkeley, Adam Moubarak, Stevens University. Notions of Convexity and Ellipticity Conditions for Anisotropic Minimal Surfaces. Preliminary report.
In the study of regularity theory for anisotropic minimal surfaces, several ellipticity conditions have been posed, including the atomic condition (AC), the scalar atomic condition (SAC), the scalar atomic condition 1 (SAC1), and the uniform scalar atomic condition (USAC). Our work explores the relationships between and the properties of these ellipticity conditions to better understand the classes of integrands satisfying each condition and the corresponding stationary varifolds. In particular, we show that the condition (SAC1), a condition which implies (AC1), cannot be weakened further to be implied by (SAC). For topological properties of the ellipticity conditions, we prove in general that (AC1) is open in the $C^{1}$ topology, and in codimension 1 we show that (SAC) and (AC), which have been shown to be equivalent to strict convexity of $C^{1}$ integrands, are not open in the $C^{2}$ topology. Additionally, to include a broader class of varifolds which are minimizers for integrands, we construct a weaker condition ( wAC ) which is equivalent to convexity of $C^{1}$ integrands in codimension 1 and leads to a weakened rectifiability theorem. Analogously, we also demonstrate equivalence between (USAC) and strong convexity in the codimension 1 setting.
(Received September 09, 2023)

## 1192-49-30663

Fedor Manin*, UC Santa Barbara. Filling random cycles.
Construct a random knot by selecting a sequence of $N$ uniformly random points in the cube and connecting them in cyclic order. This model was proposed by Millett for studying polymer molecules in a constrained volume. What is the expected area of the minimal area Seifert surface of this knot? The answer turns out to be on the order of $\sqrt{N \log N}$. While this result is purely geometric, it is inspired by the following topological question to which I still don't have a good answer: what is the Hopf invariant of a random $L$-Lipschitz map $S^{3} \rightarrow S^{2}$ ? On the other hand, the proof methods combine combinatorial probability and geometric measure theory.
(Received September 10, 2023)

Sara-Grace Lien, University of California, Irvine, Vishvas Ranjan*, UM DAE Centre for Excellence in Basic Sciences, Mumbai. Theory of Optimization and Applications to Finance. Preliminary report.
In the first part of the presentation, we explore portfolio optimization, a fundamental problem in finance and investment. We focus on Markowitz' theory of mean-variance optimization, a cornerstone in modern portfolio management. We introduce the portfolio optimization problem, which involves selecting a combination of assets to achieve the optimal trade-off between expected returns and risks. The portfolio optimization problem requires us to apply quadratic optimization theory using optimization software like GAMS. Our presentation of the material illustrates the utility of quadratic and linear optimization in decision making. In the second part of the presentation, we investigate the Gradient Descent Method. To illustrate its relevance, we begin by presenting real-life scenarios, such as optimizing hospital locations for three buildings, selecting the ideal location for a telecom point to connect three airports, and determining the shortest routes to traverse all US cities. Next, we explore examples of unconstrained optimization in both one and two dimensions. We introduce the concept of convex sets and functions, delving into their fundamental properties. The presentation further delves into iterative algorithms and the rationale behind selecting initial points within the minimizing sequence. Lastly, through visual examples, including a car navigating hilly terrain, we demystify the Gradient Descent Method's mechanics. This presentation is based on two independent projects in the Surveys in Theory CS subgroup of Polymath Jr.
(Received September 11, 2023)

1192-49-32869
Francisco Acosta*, UC Santa Barbara, Khanh Dao Duc, University of British Columbia, Manu Madhav, University of British Columbia, Nina Miolane, UC Santa Barbara, Sophia Sanborn, UC Santa Barbara. Quantifying Extrinsic Curvature in Neural Manifolds.
The neural manifold hypothesis postulates that the activity of a neural population forms a low-dimensional manifold whose structure reflects that of the encoded task variables. In this work, we combine topological deep generative models and extrinsic Riemannian geometry to introduce a novel approach for studying the structure of neural manifolds. This approach (i) computes an explicit parameterization of the manifolds and (ii) estimates their local extrinsic curvature-hence quantifying their shape within the neural state space. Importantly, we prove that our methodology is invariant with respect to transformations that do not bear meaningful neuroscience information, such as permutation of the order in which neurons are recorded. We show empirically that we correctly estimate the geometry of synthetic manifolds generated from smooth deformations of circles, spheres, and tori, using realistic noise levels. We additionally validate our methodology on simulated and real neural data, and show that we recover geometric structure known to exist in hippocampal place cells. We expect this approach to open new avenues of inquiry into geometric neural correlates of perception and behavior. (Received September 12, 2023)

1192-49-33139
Ouayl Chadli*, IBN Zohr University, RAM N. Mohapatra, Department of Mathematics, University of Central Florida, Orlando, FL, USA.. Generative Adversarial Networks via a descent method for Nikaido Isoda function. Preliminary report. In this paper, we propose a descent method for Generative Adversarial Networks (GAN) via a regularized merit function associated to a Nikaido-Isoda function for the Nash Equilibrium. Our method does not require the computation of the Hessian of the objective function, and hence permits to avoid prohibitive computations in recent methods in literature on training GAN. (Received September 13, 2023)

1192-49-33730
Youssef N. Raffoul, University of Dayton, Ethan W. Shade*, University of Dayton. River Crossings and Hanging Chains: Applying Calculus of Variations to Solve Boundary Value Problems in Navigation and Nature.
The calculus of variations is a branch of mathematics that deals with the optimization of functionals, which are a type of function that assigns a scalar value to an input function. In order to find an optimal input function of the functional, one uses the Euler-Lagrange equation to derive a candidate function and then verifies that the candidate function satisfies the Legendre necessary condition to confirm it is in fact the optimal input function. Boundary conditions specific to a functional's input function create additional constraints that must be satisfied for the candidate function to be the optimal input function. To illustrate the applicability and effectiveness of the technique, this project investigated the following real-world boundary value problems: 1) finding the quickest path to cross a non-uniformly moving river; and 2) deriving the representative equation of a hanging chain (sometimes referred to as the catenary curve) whose terminal links are free to slide along poles. Dynamic models in Desmos and comparisons to real-world construction are provided for further discussion. These problems demonstrate how the calculus of variations can be used to solve for both optimal paths of navigation and the representative equations of phenomena already found in nature.
(Received September 24, 2023)

## 51 Geometry

## 1192-51-28012

Richard H Hammack*, Virginia Commonwealth University. Mathematical pop-ups.
I describe, explain and demonstrate some of my recent mathematical pop-up card constructions. I am especially interested in creating pop-ups that are three-dimensional projections of higher-dimensional objects. Many of these creations are page studies for a longer-term project, Pop-up Book of the Hypercube.
(Received August 24, 2023)

## Correspondence.

The input to most representations of object shape is the object boundary, and as a result many representations have been of the boundary geometry. Others have been of the deformations from a base boundary to the target boundary produced by LDDMM methods. An improvement in producing correspondence of shape instances in a population, at least for anatomic shapes, comes from recognizing that the shape of the object interior, mediated by skeletal representations, yields important features provided by the deformation from a base shape, an ellipsoid, into the target shape. This yields a shape space made using fitted frames in the closure of the object interior in the form of vector lengths and directions and local rotations, each of which benefits from Euclideanization before standard statistical methods are applied.
(Received August 24, 2023)

## 1192-51-28059

Katherine M. Goldman*, Graduate Student, Ohio State University. CAT(0) and cubulated Shephard groups.
Shephard groups are common generalizations of Coxeter groups, Artin groups, and graph products of cyclic groups. Their definition is similar to that of a Coxeter or Artin group, but generators may have arbitrary order rather than strictly order 2 or $\infty$. We extend a well known result that Coxeter groups are CAT(0) to a class of Shephard groups that have "enough" finite parabolic subgroups. We also show that in this setting, if the associated Coxeter group is type FC, then the Shephard group is cubulated (i.e., acts properly and cocompactly on a CAT(0) cube complex). This provides many new non-trivial examples of CAT(0) and cubulated groups. Our method of proof combines the landmark works of Charney-Davis on the Deligne complex for an Artin group and of Coxeter on the classification and properties of regular complex polytopes.
(Received August 25, 2023)

## 1192-51-28113

R. Amzi Jeffs*, Carnegie Mellon University. Quantitative upper bounds on the Gromov-Hausdorff distance between spheres. The Gromov-Hausdorff distance between two metric spaces measures how far the spaces are from being isometric. It has played an important and longstanding role in geometry and shape comparison. More recently, it has been discovered that the Gromov-Hausdorff distance between unit spheres equipped with the geodesic metric has important connections to BorsukUlam theorems and Vietoris-Rips complexes. We develop a discrete framework for obtaining upper bounds on the GromovHausdorff distance between spheres, and provide the first quantitative bounds that apply to spheres of all possible pairs of dimensions.
(Received August 26, 2023)

## 1192-51-28114

Henry Hugh Adams*, University of Florida. Gromov-Hausdorff distances, Borsuk-Ulam theorems, and Vietoris-Rips complexes.
The Gromov-Hausdorff distance between two metric spaces is an important tool in geometry, but it is difficult to compute. I will show how to provide new lower bounds on the Gromov-Hausdorff distance between unit spheres of different dimensions by combining Vietoris-Rips complexes with Borsuk-Ulam theorems. This is joint work with Johnathan Bush, Nate Clause, Florian Frick, Mario Gomez, Michael Harrison, R. Amzi Jeffs, Evgeniya Lagoda, Sunhyuk Lim, Facundo Memoli, Michael Moy, Nikola Sadovek, Matt Superdock, Daniel Vargas, Qingsong Wang, Ling Zhou, available at https://arxiv.org/abs/2301.00246. Many questions remain open!
(Received August 26, 2023)

## 1192-51-28296

Marshall A. Whittlesey*, California State University San Marcos. A quaternion proof of a theorem about transversals in a spherical triangle. Preliminary report.
Quaternions can be used to prove theorems in spherical geometry. Here we use quaternions to provide an interesting proof of a theorem relating the measure of one side of a spherical triangle with an arc connecting two points selected from the other two sides of the triangle. This technique is featured in the speaker's book "Spherical Geometry and its Applications," with CRC Press.
(Received August 29, 2023)

## 1192-51-28635

Daniel Levitin*, University of Wisconsin-Madison. Metric Spaces of Arbitrary Finitely-Generated Scaling Group.
For a metric space $X$ with a compatible measure $\mu$, Genevois and Tessera defined the Scaling Group of $(X, \mu)$ as the subgroup $\Gamma$ of $\mathbb{R}_{>0}$ of positive real numbers $\gamma$ for which there are quasi-isometries of $X$ coarsely scaling $\mu$ by a factor of $\gamma$. We show that for any finitely generated subgroup $\Gamma$ of $\mathbb{R}_{>0}$ there exists a space $N_{\Gamma}$, bi-Lipschitz equivalent to a graph of finite degree, with scaling group $\Gamma$.
(Received September 01, 2023)

## 1192-51-29057

Thomas Weighill*, University of North Carolina at Greensboro. Coarse homotopies and coarse fundamental groups.
Various versions of coarse homotopy theory have been around since the beginning of coarse geometry, and served important roles in early proofs of the coarse Baum-Connes Conjecture for certain spaces. More recently, coarse fundamental groups were formally defined by Mitchener, Norouzizadeh and Schick. In this talk I will define coarse homotopies and coarse fundamental groups, and prove a coarse lifting lemma. This lifting lemma allows us to prove results about the coarse fundamental groups of some interesting spaces, including warped cones.
(Received September 05, 2023)

Joel Ireta, Universidad Autonoma Metropolitana, David Orbe*, Universidad Autonoma Metropolitana. DNA and RNA structure: An approach through dual quaternions. Preliminary report.
DNA and RNA are linear biopolymers fundamental for any living system. The monomers are composed of pentose, phosphate and a side chain. These are units linked in a specific sequence. The pentose in DNA and RNA is ribose and deoxyribose, respectively. Each pentose is attached to a particular side chain (a nitrogen base). The structure of DNA and RNA are different, DNA shows double-helical structures with base pairs, while RNA is single-stranded with complex and intricate basepair interactions. Despite the pentose, the phosphate unit and nitrogen base are usually distorted, one may ask if one can use rigid-body displacements to characterized the local conformations of the units along the main chain. Motivated by the above, in this work we implicate two elements: conformations of the main and side chain of the DNA and RNA and dual quaternions. Geometrically speaking, the basic idea is to apply the dual quaternion in the representation of two displacements: rotation and translation. Our methodology and algorithm provide a natural and intrinsic characterization of the entire main and side chain of DNA and RNA. Our examples show that it is possible to use the RNA and DNA main and side chain and dual quaternions to provide geometric and conformational characterization of the DNA and RNA chains.
(Received September 07, 2023)

1192-51-29914
Xiangxiang Wang*, University of Nevada, Reno. Some Inequalities of Geometric Means in Grassmannians.. Preliminary report.
We obtain some inequalities for the geometric means of elements in the Grassmannians, for example, semi-parallelogram law, geodesic triangle inequalities, majorization for geodesic triangle and their related inequalities. These inequalities reflect the elliptic geometry of the Grassmannians as Riemannian manifolds.
(Received September 08, 2023)

## 1192-51-30471

Paul R Hurst*, BYU-Hawaii. The Centroid Solid Angle and Probability Models of Square Prism Dice Rolls.
We explore the connection between the centroid solid angle and probabilities for square prism dice rolls and propose a mathematical probability model. The model is based on the principle that the probability of ending up on a particular resting aspect is proportional to the centroid solid angle of that aspect and inversely proportional to a power of the centroid height in that aspect. Using a power of 2.427 as the singular parameter, this proposed model fits our data of over 60,000 square prism dice rolls of varying height-to-width ratios with the largest magnitude Z-score of 1.01. Different powers can potentially describe other situations; e.g. different surfaces, larger dice, heavier dice, etc.
(Received September 10, 2023)

1192-51-30924
Khanh Dao Duc*, University of British Columbia. Application of non linear metrics for biological shape analysis. Recent advances in microscopy and techniques for extracting biological shapes, from the atomic to the cellular scales, have led to a surge of data with new mathematical and computational challenges associated with it. In this context, my group recently developed several computational methods that take advantage of non-linear metrics in shape space. First, I will present various applications of optimal transport-based distances for comparing 3D voxelized maps of biological atomic structures from single particle cryogenic electron microscopy (cryo-EM). I will especially focus on registration types of problem, for which we introduced several algorithms that have important key applications in cryo-EM. In particular, by optimizing the Wasserstein distance between representative point clouds over rigid body transformations, our methods significantly improve results obtained from standard linear methods. At the cellular scale, I will similarly present how using a Riemannian elastic metric over cell shape 2D contours yields a better clustering and separation in comparison with the linear metric, when we apply this metric and on various datasets of cancer cell images across different conditions. Finally, I will summarize how we integrate these metrics within a larger computational effort to process and represent complex biological datasets and objects, and produce end-to-end pipelines, databases and softwares for visualizing shape heterogeneity from raw biological data. (Received September 11, 2023)

1192-51-31125
Eric S Brussel, Cal Poly San Luis Obispo, Madeleine E Goertz*, Cal Poly San Luis Obispo, Elijah J Guptill, Cal Poly San Luis Obispo, Kelly J Lyle, Cal Poly San Luis Obispo. Geometry of the moduli spaces of similarity classes of triangles defined by three classical theorems. Preliminary report.
In his Pillow Problems [2], Lewis Carroll asked, "what is the probability that a random triangle is obtuse?" To answer this question requires defining a "random" triangle, which many authors have explored [3, 4, 5]. We consider labelled triangles and examine the geometry of the moduli space of their similarity classes under three different classical similarity theorems: side-side-side, side-angle-side, and angle-angle-angle. Each theorem results in a topologically distinct moduli space: a 2 -sphere, Klein bottle, and Clifford torus of triangles [1], respectively, some of which appear in the literature. We contrast the group structure and how each theorem gives rise to distinct types of degenerate triangles. To answer Carroll's question, we put a metric and a uniform probability measure on each compact space. References [1] Brussel, E., and Goertz, M. E. (2023) The Torus of Triangles. arXiv:2303.11446. [2] Carroll, L. (1893) Curiosa Mathematica, Part II: Pillow Problems, MacMillan, London. [3] Edelman, A. and Strang, G. (2015) Random Triangle Theory with Geometry and Applications, Found. Comput. Math. 15 681-713, DOI 10.1007/s10208-015-9250-3. [4] Kendall, D.G. (1989) A Survey of the Statistical Theory of Shape, Stat. Sci. $487-$ 120. [5] Portnoy, S. (1994) A Lewis Carroll Pillow Problem: Probability of an Obtuse Triangle, Stat. Sci. 9 279-284. (Received September 11, 2023)

Ingrid Daubechies, Duke University, Shira Faigenbaum-Golovin*, Duke University, Shan Shan, University of Southern Denmark, Alexander Winn, Duke University, Rui Xin, University of Washington. An iterative approach to learn and correct the connection in the fibre bundle model of families of shapes.
Earlier work had modeled collections of holonomic morphological shapes as a nonlinear fiber bundle in which the individual
shapes are the fibers, and the correspondence maps between them the connection. Combining geometric pairwise correspondence maps, known to be an a priori only coarse approximation of the true correspondence, with spectral analysis and denoising of diffusion over the full fiber bundle, we show that the correspondence maps can be improved, and then used for a more detailed study of the geometry of the underlying collection.
(Received September 11, 2023)

## 1192-51-31248

Olivia Sylvester*, SUNY Fredonia. A Shortcut to Cutting Triangles and Tetrahedra.
Given a 3, 4, 5 right triangle, what is the shortest cut which will divide this triangle into two pieces of equal area? We will provide the answer to this question and present how this can be generalized to any triangle. The method used to obtain this answer can be generalized to dividing any triangle into two pieces in which the ratio of their areas is any fraction between 0 and 1 . Furthermore, we derived a formula that will produce the length of the shortest cut given only the side lengths of a triangle. We will also investigate how this result can be applied to tetrahedra, and finding a minimal area plane that divides the shape into regions whose volumes have a given ratio.
(Received September 11, 2023)

1192-51-31270
Sathyanarayanan N. Aakur, Oklahoma State University, Shenyuan Liang*, Florida State University, Sudeep Sarkar, University of South Florida, Mauricio Pamplona Segundo, University of South Florida, Anuj Srivastava, Florida State University. Shape-Graph Matching Network (SGM-net): Registration for Statistical Shape Analysis. Preliminary report. This paper focuses on the statistical analysis of shapes of data objects called shape graphs, a set of nodes connected by articulated curves with arbitrary shapes. In tasks such as object detection, tracking, and recognition, shape graphs offer a more complex and nuanced representation than landmarks, capturing inherent variability across objects and measurement errors. They are critical for the statistical analysis of shapes in applications like 3D Microglia, 3D Brain Arterial, and 2D Blood Vessel Networks. A critical need here is a constrained registration of points (nodes to nodes, edges to edges) across objects, requiring optimization over the permutation group. This optimization is especially challenging due to differences in nodes (in terms of numbers, locations) and edges (shapes, placements, and sizes) across shape graphs. This paper tackles this registration problem using a novel neural-network architecture and involves an unsupervised loss function developed using the elastic shape metric for curves. The architecture is designed to efficiently address quadratic assignment problems (QAP), known for their computational complexity. It results in (1) state-of-the-art matching performance and (2) an order of magnitude reduction in computational cost relative to baseline approaches. We demonstrate the effectiveness of the proposed approach using both simulated and real-world 2D and 3D shape graphs.
(Received September 11, 2023)

1192-51-31280
Victor Ahona, Stanford University, Pranav Kulkarni*, Independent Researcher. Kulkarni-Ahona Corollary for the NewtonGauss Theorem.
We give a new corollary approach to the Newton-Gauss Line, showing collinearity using coordinate geometry and an algebraic expansion method in the Cartesian coordinate plane. Building upon principles from the Newton-Gauss line and properties of a complete quadrilateral, we prove that the midpoints of the three diagonals of a complete quadrilateral are collinear. Notable methods of proving said theorem include proof by similarity, similar triangles, comparing areas, ratios, and proof by other geometric approaches. We formulate a corollary approach to the Newton-Gauss Line using coordinate geometry by mapping the Newton-Gauss diagram on the Cartesian coordinate plane with one quadrilateral vertex at the origin. We then assign coordinates to the other vertices, find the midpoints of the diagonals, and show the collinearity of the three key points, considering the shared point at the intersection of both lines. Furthermore, we build upon ideas of a complete quadrilateral through a right-angle quadrilateral on the base of the Cartesian coordinate plane, satisfying the definitional ideas of the Newton-Gauss line and its geometric makeup.
(Received September 11, 2023)

## 1192-51-31444

Emmy Murphy*, University of Toronto. Weinstein Kirby calculus and Fukaya categories.
Weinstein manifolds are a class of symplectic manifolds whose geometry is compatible with a Morse-like Liouville vector field.
This in turn allows one to describe these manifolds with the data of Legendrian links inside simple contact manifolds. We'll discuss how one can alter these Legendrian links without changing the underlying symplectic manifold. We'll also discuss how relevant pseudo-holomorphic invariants relate in this picture, namely Legendrian contact homology and (partially) wrapped Floer cohomology.
(Received September 11, 2023)

## 1192-51-31532

Vitaliy A Kurlin*, University of Liverpool (UK). Continuous maps of the high-dimensional universe of protein structures The structures of most molecules are determined in a rigid form. Hence it is practically important to distinguish different rigid shapes (conformations) of the same molecule. Biomolecules such as proteins can continuously change their shape, for example, due to flexible torsion angles. These continuous changes are best captured by a continuous metric that should only slightly change under atomic displacements due to noise. Each protein structure in the Protein Data Bank (PDB) consists of chains whose rigid backbones are conveniently represented by a sequence of ordered alpha-carbons whose simplest complete invariant is the matrix of pairwise distances between the atoms. One simpler metric between backbones is the maximum deviation of inter-atomic distances, not on atomic positions, hence not requiring a slow optimal alignment. This metric was computed for all $325+$ billion pairs of all $800 \mathrm{~K}+$ chains in the PDB within 49 hours on a modest laptop. Among only highquality structures with a maximum resolution of 3 Angstroms, 2892 pairs have zero distance, and 761 pairs have identical coordinates of all alpha-carbon atoms. Some of these geometric duplicates differ by amino acid residues, which requires urgent corrections. After detecting thousands of unexpected duplicates, we visualized the PDB universe of protein backbones considered as rigid clouds of alpha-carbons. A continuous map of this protein universe in invariant coordinates revealed
evolutionary preferences for a few frequently appearing shapes.
(Received September 11, 2023)

1192-51-32017
Khaled Abdel-Ghaffar, University of California, Davis, Juane Li, Micron Technology, Inc., Shu Lin*, University of California, Davis. A Class of Partial Geometries and Their Descendants for LDPC Code Constructions. Preliminary report. Partial geometries (PaGs) form a branch in combinatorial mathematics. They have been shown to be effective mathematical tools in constructing LDPC codes with distinct geometric and algebraic structures. PaG-based LDPC codes perform well over the additive white Gaussian noise channel (AWGNC) with various iterative decoding algorithms based on belief propagation. Long high-rate PaG-LDPC codes can achieve very low error-rates without visible error-floors, as needed in flash memories and high-speed satellite and optical communications. Iterative decoding of PaG-LDPC codes converges rapidly in comparison with LDPC codes constructed using other methods. Besides LDPC codes, partial geometries can also be used to construct constantweight (CW) codes. CW codes find applications in communication and computer systems. This talk presents a class of PaGs and their descendant geometries. Based on this class of PaGs and their descendants, quasi-cyclic LDPC and CW codes are constructed and they have duality relationship. CW codes constructed based on a class of descendant PaGs are cyclic and optimal.
(Received September 12, 2023)

## 1192-51-32084

Zhiqin Lu, UC Irvine, Yufei Ren*, School of Physical Sciences University of California, Irvine. A linear algebraic proof of the Viviani's Theorem. Preliminary report.
Viviani's theorem, attributed to the Italian mathematician and scientist Vincenzo Viviani, establishes a fundamental relationship within equilateral triangles. According to the theorem, the sum of distances from an interior point to the three sides of an equilateral triangle is equal to the length of the triangle's altitude. The introduction of the concept of signed distance enables its extension to the entire plane. Consequently, the theorem holds for any point in the Cartesian coordinate system, regardless of whether it lies inside the triangle, outside of it, or on the triangle itself. Additionally, the converse Viviani's theorem also holds true. If the sum of distances from any point within an arbitrary triangle to its three sides is constant, then the triangle must be an equilateral triangle.
(Received September 12, 2023)

## 1192-51-33032

William Geller*, IUPUI, Michal Misiurewicz, IUPUI. Holes, nonconvexity, and curvature in metric spaces.
We introduce a way of measuring nonconvexity of a metric space. We apply it to define a broad generalization and refinement of the classical curvature of a curve, and compute it for a variety of examples. We also use it to introduce a natural new notion of a fractal set. This is joint work with Michal Misiurewicz.
(Received September 13, 2023)

Nathan Fisher*, University of Wisconsin-Madison. On horofunction boundaries of homogeneous groups.
We investigate general properties of horofunction boundaries of homogeneous metrics on graded groups. In particular, for a general family of metrics on Carnot groups, we show that all horofunctions are piecewise linear, i.e., piecewise Pansu, and that all horofunctions are functions of coordinates in the first layer of the grading. We also explore infinite families of groups which generalize the 3-dimensional Heisenberg group, namely, filiform groups and the higher Heisenberg groups, and answer questions about the dimension and topology of their horofunction boundaries.
(Received September 13, 2023)

1192-51-33701
Georgia Soo Frances Corbett*, Bucknell University. Centers of n-degree Poncelet Circles. Preliminary report.
In 1822, Poncelet introduced what we now know as Poncelet's porism: Given two plane conics, one inside of the other, if there exists a polygon that is inscribed in one conic and circumscribes the other conic, then there are infinitely many such polygons. Our work focuses on Poncelet circles and packages of Poncelet circles. We provide elementary geometric proofs for circles inscribed in polygons with low numbers of vertices (e.g. 4,6 ) to show that if one circle in the package is centered at 0 , then all circles in the package must be centered at 0 as well. This was proven in general by Spitkovsky and Wegert using elliptic integrals. However, our proof only uses elementary geometry making this problem more easily accessible. (Received September 23, 2023)

## 1192-51-33824

Tyler Michael Acton*, Troy University. Some Congruence Criteria for Quadrilaterals and n-gons.
We give the definition of congruence for convex quadrilaterals and higher n-gons. Then we give congruence criteria and their proofs. For those that are not congruence criteria, we construct counterexamples. We also consider whether these criteria are valid for when the quadrilaterals are not convex.
(Received September 26, 2023)

## 1192-51-33828

Abigayle Dirdak*, Arizona State University. The Pentagram Map and Gale Transform.
The Pentagram map as introduced by R. Schwartz in 1992 takes any n-gon P to a corresponding n-gon P' formed from intersecting diagonal lines connecting alternating vertices of P. Since the pentagram map results in a new polygon we can take this map multiple times which results in a sequence of polygons $\mathrm{P}, \mathrm{P}^{\prime}, \mathrm{P}^{\prime \prime} \ldots$. I am interested in determining the types of
polygons for which P and $\mathrm{P}(\mathrm{i})$ are projectively equivalent. I will explore several cases and conjectures in which this holds true, including generalizations of the pentagram map to higher dimensions. Additionally, I will introduce the Gale transform and its relationship to the pentagram map in the process
(Received September 26, 2023)

1192-51-33833
Kieran Wolfgang Layne*, University Of North Carolina At Asheville. Geometric Dissections of the squares that can demonstate the Pythagorean Thereom for any right triangle. Preliminary report.
The Pythagorean Theorem is a fundamental theorem in classical geometry. Rather than exploring the many proofs that exist for this theorem, I will focus on the geometry. Specifically addressing the question: How can the combined areas of the squares on the two legs be dissected, so that these pieces will exactly fit into the square that is the area of the hypotenuse? As the size and shapes of these pieces will differ for right triangles with different values for the legs, the solution for each triple of values satisfying the Pythagorean Theorem will yield a unique diagram for that right triangle. However, there are certain algorithmic ways to approach the dissection of the areas which will lead to a class of solutions which exhibit a similar pattern. Some algorithmic pattern solutions might require more subdivisions for some triangles than others. An ultimate algorithmic solution might be defined as one which yields exactly the same number of pieces for any right triangle. Some of these ultimate algorithmic pattern dissections that I will explain are well known. Others that I will present are totally new to my knowledge. For each pattern solution presented, I will show why the dimensions of the pieces will necessarily work to satisfy the Pythagorean Theorem for every possible right triangle. I will also endeavor to include actual physical puzzles of the various dissection patterns and demonstrate how this fun exploration of an essential mathematical theorem can enhance a classroom hands-on approach to learning geometry.
(Received September 26, 2023)

## 1192-51-33837

Taran Chandra Mellacheruvu*, Illinois Institute of Technology. Maximizing the Volume of a Hyperrectangle With Fixed Diagonal Length.
Given any hyperrectangle, there are infinitely many others that can be constructed with the same diagonal length. Consider that set of each such shape, oriented with a vertex at the origin and the entire hyperrectangle in non-negative Cartesian space. The objective of this presentation is to determine the hyperrectangle that produces the maximum possible volume under the stipulated diagonal length constraint. We present two proofs, each at a different level of complexity. For the first proof, we use hyperspherical coordinates and find a condition on all $\theta_{i}$ such that volume is maximized. For the second proof, we use a set of vectors to derive formulas which allow volume calculation and maximization. While this creates multivariate functions, we design an approach to limit complexity and lower the barrier to understanding. Multivariate concepts are hence distilled into familiar algebra, allowing us to probe higher-dimensional space without the need for higher-level mathematical knowledge. (Received September 27, 2023)

## 52 Convex and discrete geometry

## 1192-52-25404

Terence Tao*, UCLA. Translational Tilings of Euclidean Space.
Translational tilings of Euclidean space are a partition of Euclidean space (up to null sets) by translates of one or more tiles. For instance, the famous Penrose tilings are a translational tiling by a finite number of triangles which can only tile the plane non-periodically. One of the outstanding problems in this field is the "periodic tiling conjecture" - if a tile can tile space by translations, must there exist a way to tile space periodically with this tile? Recently, Rachel Greenfeld and I were able to disprove this conjecture in high dimensions, by encoding a certain "Sudoku puzzle" as a tiling problem. We survey this result and other recent progress in this talk.
(Received June 05, 2023)

## 1192-52-27444

Boris Bukh, Carnegie Mellon University, Robert Amzi Jeffs*, Carnegie Mellon University. Enumeration of interval graphs and d-representable complexes.
How many different ways can we arrange $n$ convex sets in $\mathbb{R}^{d}$ ? One answer is provided by counting the number of $d$ representable complexes on vertex set $[n]$. We show that there are $\exp \left(\Theta\left(n^{d} \log n\right)\right)$-many such complexes, and provide bounds on the constants involved. As a consequence, we show that $d$-representable complexes comprise a vanishingly small fraction of the class of $d$-collapsible complexes. In the case $d=1$ our results are more precise, and improve the previous best estimate for the number of interval graphs. (Received August 15, 2023)

## 1192-52-27586

Doris J Schattschneider*, Moravian University. Marjorie Rice's pursuit of convex pentagons and their tilings.
It is amazing how intense curiosity and ingenuity can propel a person with little or no higher mathematical background to investigate mathematical problems and make surprising discoveries. Beginning in her middle age, American homemaker Marjorie Rice (1923-2017), having no math background beyond a high school general math course, pursued for years a selfassigned task of finding all types of convex pentagons that can tile the plane. Her many discoveries and unique methods will be highlighted.
(Received August 16, 2023)

Christ's College Cambridge, Amogh Kumar Mishra, Krea University, Adam Sheffer, Baruch College, CUNY, Minh-Quan Vo, University of Science, Vietnam National University, Bernardus Adriaan Wessels, Stellenbosch University, Edward Yu*, Massachusetts Institute of Technology. New Results Towards the Erdős-Fishburn Problem. Preliminary report. The famous Erdős distinct distances problem asks for the minimum number of distinct distances spanned by $n$ points in the plane. Its inverse problem is known as the Erdős-Fishburn problem: determine $g(k)$, the size of the largest point set with $k$ distances. While it has been shown that $g(6)=13$, the optimal configurations have not been classified. Working towards such a classification, we prove that all 11-point convex sets with 6 distances are subsets of the regular 12- or 13 -gon. Part of our proof relies on an algorithmic approach that successively applies simple geometric observations to deduce distance relations. (Received August 24, 2023)

## 1192-52-28556

Toby Aldape, University of Texas at Austin, Jingyi Liu*, Princeton University, Gregory Pylypovych, Massachusetts Institute of Technology, Adam Sheffer, Baruch College, Minh-Quan Vo, University of Science, Vietnam National University. Distinct Distances in $\mathbf{R}^{3}$ Between Quadratic and Orthogonal Curves.
We study the minimum number of distinct distances between point sets on two curves in $\mathbf{R}^{3}$. Assume that one curve contains $m$ points and the other $n$ points. Our main results: (a) When the curves are conic sections, we characterize all cases where the number of distances is $O(m+n)$. This includes new constructions for points on two parabolas, two ellipses, and one ellipse and one hyperbola. In all other cases, the number of distances is $\Omega\left(\min \left\{m^{2 / 3} n^{2 / 3}, m^{2}, n^{2}\right\}\right)$. (b) When the curves are not necessarily algebraic but smooth and contained in perpendicular planes, we characterize all cases where the number of distances is $O(m+n)$. This includes a surprising new construction of non-algebraic curves that involve logarithms. In all other cases, the number of distances is $\Omega\left(\min \left\{m^{2 / 3} n^{2 / 3}, m^{2}, n^{2}\right\}\right)$.
(Received August 31, 2023)

## 1192-52-28682

Bryson Kagy*, North Carolina State University, Seth Sullivant, North Carolina State University. A Description of the Polyhedral Geometry of Equidistant Phylogenetic Networks. Preliminary report.
Phylogenetic trees record the evolutionary relationship between species, however in practice some cycle edges can be introduced through processes like reticulation events and hybridization. Thus phylogenetic networks are studied which allow got some limited non-tree edges. One such natural type of phylogenetic networks is the circular split network Split networks give a useful way to construct networks with intuitive distance structures induced from the associated split graph. We explore the polyhedral geometry of distance matrices built from circular split systems which have the added property of being equidistant. We give a characterization of the facet defining inequalities and the extreme rays of the cone of distances that arises from an equidistant network associated to any circular split network. We also explain a connection to the Chan-RobbinsYuen polytope from geometric combinatorics.
(Received September 01, 2023)

1192-52-28825
Satyan L. Devadoss*, University of San Diego, Matthew Steven Harvey, University of Virginia, Wise. Unfolding Puzzles of Regular 4D Polytopes.
A decade ago, every edge unfolding of the Platonic solids was shown to flatten beautifully onto the plane, without any overlap. We explore what happens in 4D (and beyond), proving what works and puzzling over what doesn't. This talk is heavily infused with visual imagery.
(Received September 03, 2023)

## 1192-52-29386

Satyan L. Devadoss, University of San Diego, Diane Hoffoss*, University of San Diego. Unfolding Humanity: Return to Burning Man. Preliminary report
"Unfolding Humanity" is a massive interactive metal dodecahedral sculpture standing 12' tall, which debuted at Burning Man 2018 and returned again as a honored guest in 2023. Its exterior faces are etched with characters illuminated by 17,000 programmed LEDs, with interior faces fully-lined with reflective acrylic mirrors. It alludes both to Dürer's 500-year old unsolved problem on unfolding polyhedra, as well as pointing towards the Poincaré dodecahedral sphere and a possible shape of our universe. We discuss lessons learned, adaptations made, and the joys and pains from both 2018 and 2023. Wisdom is given for anyone interested in creating Burning Man art projects.
(Received September 06, 2023)

## 1192-52-29682

Vitaliy A Kurlin*, University of Liverpool (UK). A complete and continuous isometry invariant of Euclidean clouds of unordered points.
Rigid structures such as cars or any other solid objects are often represented by finite clouds of ordered points. The most natural equivalence on these point clouds is rigid motion or isometry maintaining all inter-point distances. Rigid patterns of point clouds can be reliably compared only by complete isometry invariants that can also be called equivariant descriptors without false negatives (isometric clouds having different descriptions) and without false positives (non-isometric clouds with the same description). Noise and motion in data motivate a search for invariants that are continuous under perturbations of points in a suitable metric. We propose the first continuous and complete invariant of unordered clouds under rigid motion in any Euclidean space, called the Simplexwise Centered Distribution (SCD). This invariant extends the distribution of pairwise distances by Boutin and Kemper (Adv Appl Math 2004), which is generically complete but cannot distinguish 2D clouds of 4 points depending on four free parameters, and the more recent Pointwise Distance Distribution (PDD) by Widdowson and Kurlin (NeurIPS 2022), which was proved to be generically complete in the much harder case of periodic point sets but cannot distinguish 3D clouds of 6 points depending several free parameters. For a fixed dimension, the new metric for the SCD invariant is computable in a polynomial time in the number of points. All results were published in the paper at

## 1192-52-29856

Ruriko Yoshida*, Naval Postgraduate School. Tropical Geometric Tools for Machine Learning: the TML package. Preliminary report.
In the last decade, developments in tropical geometry have provided a number of uses directly applicable to problems in statistical learning. The TML package is the first R package which contains a comprehensive set of tools and methods used for basic computations related to tropical convexity, visualization of tropically convex sets, as well as supervised and unsupervised learning models using the tropical metric under the max-plus algebra over the tropical projective torus. Primarily, the TML package employs a Hit and Run Markov chain Monte Carlo sampler in conjunction with the tropical metric as its main tool for statistical inference. In addition to basic computation and various applications of the tropical HAR sampler, we also focus on several supervised and unsupervised methods incorporated in the TML package including tropical principal component analysis, tropical logistic regression and tropical kernel density estimation.
(Received September 08, 2023)

1192-52-30173
Thomas William Murphy, CSU Fullerton, David Weed*, CSU Fullerton. A Characterization of Archimedean Solids. Preliminary report.
In studying any family of mathematical objects, a fundamental issue is to understand how one object can "sit inside" another object in the family, preserving the mathematical structure. We are concerned with convex uniform polyhedra. Two famous families of polyhedra live in this class: the Platonic and Archimedean solids, as well as the prisms and antiprisms. Our main result geometrically characterizes the famed Archimedean solids among the convex uniform polyhedra by studying how they sit inside a regular tetrahedron.
(Received September 11, 2023)

1192-52-30952
Tim Ophelders, TU Eindhoven, Anna Schenfisch*, Eindhoven University of Technology, Willem Sonke, Eindhoven University of Technology, Bettina Speckmann, Eindhoven University of Technology. Volume Filtrations and Discrete Morse Theory to Study Intertidal Regions.
When studying water flow through the lens of persistence, a first approach might be to use a height-based filtration on the surface under the flow - after all, water flows along the path of steepest descent, right? In fact, assumption about steepest descent are often not true due to factors such as inertia or tides. To better capture such nuances, we instead propose using a volume-based filtration. After first defining a discrete Morse function on a cubulation of the surface, we filter the set of descending manifolds and saddle edges (representing water channels) by removing saddle edges that have low neighboring volume. Similarly, we filter out leaf nodes of the remaining trees if the surrounding volume is below the filtration threshold. Conceptually, we remove edges if one would not need to dredge much sand/land to alter the corresponding channels. After first exploring the theory and mathematical set-up of our methods, we will then investigate an application to the Wadden Sea, an intertidal region located along the northwest coast of the Netherlands characterized by channels of water with a tree-like or braided structure.
(Received September 11, 2023)

## 1192-52-30970

Elizabeth Ann Dinkelman*, George Mason University. Combinatorial Properties of the Alternating Sign Matrix Polytope Preliminary report.
The Alternating Sign Matrix Polytope, $A S M_{n}$ is the convex hull of $n \times n$ matrices whose entries are 0 , 1 , and -1 , whose nonzero entries alternate in sign, and whose row and column sums are 1. These polytopes were initially defined by Striker, and by Behrend and Knight. Brualdi and Dahl, as well as Lascoux, initiated the study of paths in the graph of the $A S M_{n}$ polytope. As with the earlier authors, we are looking for an upper bound on the distance in the graph from an $A S M$ to the nearest permutation matrix. We will also investigate combinatorial properties of faces of the $A S M$ polytope.
(Received September 11, 2023)

## 1192-52-31208

Florian Frick, Carnegie Mellon University, Zoe Wellner*, Carnegie Mellon University. Colorful Borsuk-Ulam Theorems. The classical Borsuk-Ulam theorem states that for any continuous map from the sphere to Euclidean space, $f: S^{d} \rightarrow R^{d}$, there is a pair of antipodal points that are identified, so $f(x)=f(-x)$. We prove a colorful generalization of the Borsuk-Ulam theorem. The classical result has many applications and consequences for combinatorics and discrete geometry and we in turn prove colorful generalizations of these consequences such as the colorful ham sandwich theorem, which allows us to prove a recent result of Bárány, Hubard, and Jerónimo on well-separated measures as a special case, and Brouwer's fixed point theorem, which allows us to prove an alternative between KKM-covering results and Radon partition results. (Received September 11, 2023)

## 1192-52-31724

Laura Felicia Matusevich, Texas A\&M University, Alexander Ruys De Perez*, Georgia Institute of Technology, Anne Shiu, Texas A\&M University. Wheels: A New Criterion for Nonconvexity in Neural Codes.
A neural code on $n$ neurons is a combinatorial code which describes the intersections of $n$ subsets of some Euclidean space. The problem of convexity, that is determining which neural codes can describe a collection of convex subsets, remains a significant open question in this field. Here, we introduce a type of structure that can occur in neural codes, which we refer to as a "wheel". We show how the presence of a wheel guarantees the nonconvexity of a code, as well as how to scan a neural
code for potential wheels.
(Received September 12, 2023)

1192-52-32723
Vitaliy A Kurlin*, University of Liverpool (UK). Continuous metrics on moduli spaces of lattices.
In a Euclidean space, a lattice is defined as the infinite set of all integer linear combinations of basis vectors. Unfortunately, the same lattice can be generated by infinitely many different bases. Crystallography tried to resolve this ambiguity by introducing a uniquely reduced basis. Theorem 15 in MATCH 2022 proved that any such reduction is discontinuous in the sense that slightly different bases can have very distant reduced bases. Motivated by the rigidity of crystal structures, the strongest equivalence on lattices is rigid motion, which is a composition of translations and rotations. The slightly weaker equivalence is isometry also allowing mirror reflections. Practical comparisons of periodic crystals motivate the problem to define continuous metrics on the moduli spaces lattices under rigid motion and isometry. For isometry in dimension 2 , the metric problem was essentially solved by Lagrange in the 18 th century by using quadratic forms. However, the metric problem remained open for rigid motion and for isometry even in dimension 3. The first unknown case was completed in the paper "Mathematics of 2-dimensional lattices" (Foundations of Computational Mathematics 2022) with practical applications to large crystal databases in the journals Acta Cryst A and Chirality. The topological space of 2-dimensional lattices under rigid motion composed with uniform scaling is a punctured 2-dimensional sphere. All papers are linked at http://kurlin.org/research-papers.php\#Geometric-Data-Science
(Received September 12, 2023)

## 1192-52-32940

James Kian Howard*, San Diego State University, Christopher O'Neill, San Diego State University. On Graver bases of shifted families of numerical semigroups..
A numerical semigroup $M$ is a cofinite subset of the non-negative integers that is closed under addition. A factorization of $n \in M$ is an expression of $n$ as a sum of generators of $M$, and the Graver basis of $M$ is a collection $G r(M)$ of trades between the generators of $M$ that allows for efficient movement between factorizations. Given positive integers $r_{1}, \ldots, r_{k}$, we consider the family \$M_t = \of $\backslash$ shifted $/$ nnumericalsemigroupswhosegeneratorsareobtainedbytranslatingr_1, ···, $\mathrm{r}_{-} \mathrm{k}$ byanintegerparametert. Inourwork, wecharacterizetheGraverbasis $\mathrm{Gr}_{\mathrm{M}}(\mathrm{H})$ of $\mathrm{M}_{-} \mathrm{t}$ forsufficientlylargetinthecasek $=2$ , intheformofarecursiveconstructionof $\mathrm{Gr}\left(\mathrm{M}_{-} \mathrm{t}\right)$ fromthatofsmallervaluesoft
. Asaconsequenceo fourresult, thenumberoftradesin $\mathrm{Gr}_{\left(\mathrm{M}_{-} \mathrm{t}\right) \text {, whenviewedasafunctionoft\$, is eventually quasilinear, }}^{\text {, }}$ with a sharp lower bound on the start of quasilinear behavior. We also present partial results for numerical semigroups of higher embedding-dimension.
(Received September 13, 2023)

1192-52-33515
Timothy Joseph Edwards*, Gannon University, Pablo Soberón, Baruch College, City University of New York. Helly Type Results with Discrete Sets and Axis Parallel Boxes.
A recent Helly type result of Halman states that a family of axis parallel boxes in $\mathbb{R}^{d}$ contains a point from some discrete set $S$ in its intersection whenever every subfamily of size $2 d$ contains a point of $S$ in its intersection. We prove a fractional variation: if any positive fraction of the subfamilies of size $d+1$ contain a point of $S$ in their intersection, then some positive fraction of the entire family contains a point of $S$ in its intersection. Thus, the discrete fractional Helly number for axis parallel boxes is $d+1$. We also prove colorful and quantitative variations of Halman's result and show that these results can be applied to families of $H$-convex sets.
(Received September 19, 2023)

1192-52-33539
Pablo Soberón, Baruch College, City University of New York, Christina Yu*, Massachusetts Institute of Technology. Highdimensional envy-free partitions. Preliminary report.
The problem of fair division is one central to both mathematical economics and topological combinatorics. Given a resource to be shared between $n$ people, can we partition and distribute it so that each person receives a "fair" share according to their subjective measure of value? We study the problem of finding envy-free partitions by splitting $\mathbb{R}^{d}$ into convex pieces. In an envy-free partition, no person thinks anyone else got a better share (though they may not agree that everyone received the same value). Such partitions are known to exist for one-dimensional resources between an arbitrary number of people. We generalize this result and prove the existence of envy-free partitions in higher dimensions. Specifically, we show the existence of a partition that equiparts one measure and is an envy-free partition for $d-1$ sets of $n$ measures, for $n$ a prime power. For general $n$, envy-freeness is replaced by a weaker fairness constraint, where each person receives at least $1 / n$ of their total valuation of the resource. (Received September 20, 2023)

## 1192-52-33675

R. Amzi Jeffs, Carnegie Mellon University, Henry Siegel*, Carnegie Mellon University, David D. Staudinger*, Carnegie Mellon University, Yiqing Wang, Carnegie Mellon University. Embedding dimension gaps in sparse codes.
We discuss the open and closed embedding dimensions of a convex code $\mathcal{F} \mathcal{P}$ arising from the Fano plane. The motivation for this is the following question: Do there exist convex codes whose open dimension is very large but have small codewords? Past work of Cruz, Giusti, Itskov, and Kronholm tells us that the open dimension of intersection complete codes has an upper bound which grows with respect to the number of maximal codewords, whereas the closed dimension of intersection complete codes has an upper bound which grows with respect to the size of the codewords. Hence, 3-sparse intersection complete codes all have closed embedding dimension at most 5 . There is no known example of an 3-sparse intersection complete code with open embedding dimension larger than 5 even though there is no known upper bound prohibiting it. Our research was to understand whether or not the open embedding dimension of $\mathcal{F P}$ exceeds 5 .
(Received September 22, 2023)

## 1192-52-33677

R. Amzi Jeffs, Carnegie Mellon University, Henry Siegel*, Carnegie Mellon University, David D. Staudinger*, Carnegie Mellon University, Yiqing Wang, Carnegie Mellon University. Embedding dimension gaps of sparse codes.
We discuss the open and closed embedding dimensions of a convex code $\mathcal{F} \mathcal{P}$ arising from the Fano plane. The motivation for this is the following question: Do there exist convex codes whose open dimension is very large but have small codewords? Past work of Cruz, Giusti, Itskov, and Kronholm tells us that the open dimension of intersection complete codes has an upper bound which grows with respect to the number of maximal codewords, whereas the closed dimension of intersection complete codes has an upper bound which grows with respect to the size of the codewords. Hence, 3-sparse intersection complete codes all have closed embedding dimension at most 5 . There is no known example of a 3-sparse intersection complete code with open embedding dimension larger than 5 even though there is no known upper bound prohibiting it. Our research was to understand whether or not the open embedding dimension of $\mathcal{F} \mathcal{P}$ exceeds 5 .
(Received September 22, 2023)

## 1192-52-33689

Srinivas Arun*, Cherry Creek High School. Improved Bounds on Helly Numbers of Product Sets. Preliminary report. The Helly number $h(S)$ of a set $S \subseteq \mathbb{R}^{d}$ is defined as the smallest positive integer $h$, if it exists, such that the following statement is true: For any finite family $\mathcal{F}$ of convex sets in $\mathbb{R}^{d}$, if every subfamily of $h$ sets intersect at a point in $S$, then all sets in $\mathcal{F}$ intersect in $S$. We study Helly numbers of product sets, which are sets of the form $A^{d}$ for some one-dimensional set $A$. Inspired by Dillon's research on the Helly numbers of product sets, Ambrus, Balko, Frankl, Jung, and Naszódi recently obtained the first upper and lower bounds for Helly numbers of exponential lattices in two dimensions, which are sets of the form $S=\left\{\alpha^{n}: n \in \mathbb{N}\right\}^{2}$ for some $\alpha>1$. We use a different, simpler method to obtain better upper bounds for exponential lattices. In addition, we generalize their lower bound construction to higher dimensions. Secondly, we investigate sets $A \in \mathbb{Z}$ whose consecutive elements differ by at most 2 such that $h\left(A^{2}\right)=\infty$. We slightly strengthen a theorem of Dillon that such sets exist while also providing a shorter proof. Finally, we obtain bounds for Helly numbers of certain sets defined by arithmetic congruences.
(Received September 23, 2023)

## 1192-52-33738

Sasha Sluis-Cremer, University of Pittsburgh, Lark Song*, University of Pittsburgh. The Smoothed Decagon Problem. We conjecture that among all balanced disks in the Euclidean plane, the smoothed decagon has the highest packing density. We give the definition of balanced disks and formulate an optimal control problem on a manifold closely related to the Lie group $\mathrm{SL}_{2}(\mathbb{R})$. We highlight its remarkable structures and connections with hyperbolic geometry and Hamiltonian mechanics. We construct explicit solutions to the optimal control problem, generating a smoothed polygon given by a bang-bang control with finitely many switches. We prove that for each integer $k>1$, the smoothed $(6 k-2)$-gon is a Pontryagin extremal trajectory. We further discuss the possible existence of a counterexample in the degenerate case $k=1$.
(Received September 25, 2023)

## 53 Differential geometry


#### Abstract

1192-53-25660 Tse-Yu Lin*, National Taiwan University, Yen-lung Tsai, National Chengchi University. Residual Net Aspect of Disk stitching-based Manifold Reconstruction Method. Manifold learning is a branch of nonlinear dimensionality reduction based on the assumption that data of interest lies in a lowdimensional manifold embedded in a higher-dimensional space. The goal of manifold reconstruction is to reconstruct a Riemannian manifold for Euclidean and non-Euclidean datasets. One differential geometry-based framework of manifold reconstruction proposed by Fefferman et al (2020) state a reconstruction process from both of theoretical and computational aspects. Algorithm of this type is called the disk stitching method since multiple tangent affine spaces will be glued smoothly. In this work, we aim to provide a new point of view to rephrase this process as a learning process of a residual neural network model. Each local affine orthogonal projection can be viewed as a residual block satisfying some functional equations. This idea offers a new insight to study the interdisciplinary relationship of differential geometry and deep learning in the future. (Received July 01, 2023)


## 1192-53-25662

Jiayin Pan*, UC Santa Cruz. Nonnegative Ricci curvature, nilpotency, and asymptotic geometry
Let $M$ be a complete and noncompact manifold with $\mathrm{Ric} \geq 0$ and escape rate not $1 / 2$. It is known that its fundamental group $\pi_{1}(M)$ has a torsion-free nilpotent subgroup $N$ of finite index. We show that the nilpotency step of $N$ must be reflected in the asymptotic geometry of the universal cover $\widetilde{M}$ in terms of the Hausdorff dimension of some $\mathbb{R}$-orbit: there exist an asymptotic cone $(Y, y)$ of $\widetilde{M}$ and a closed $\mathbb{R}$-subgroup $L$ of the isometry group $\operatorname{Isom}(Y)$ such that its orbit $L y$ has Hausdorff dimension at least the nilpotency step of $N$.
(Received July 02, 2023)

Qixing Huang*, UT Austin. Geometric Regularizations for Shape Generative Modeling.
Parametric generative models, which map a latent parameter space to instances in an ambient space, enjoy various applications in 3D Vision and related domains. A standard scheme of these models is \textsl \{probabilistic\}, which aligns the induced ambient distribution of a generative model from a prior distribution of the latent space with the empirical ambient distribution of training instances. While this paradigm has proven to be quite successful on images, its current applications in

3D shape generation encounter fundamental challenges in the limited training data and generalization behavior. The key difference between image generation and shape generation is that 3D shapes possess various priors in geometry, topology, and physical properties. Existing probabilistic 3D generative approaches do not preserve these desired properties, resulting in synthesized shapes with various types of distortions. In this talk, I will discuss recent works in my group that seek to establish a novel geometric framework for learning shape generators. The key idea is to view a generative model as a sub-manifold embedded in the ambient space and develop differential geometry tools to model various geometric priors of 3D shapes through differential quantities of this sub-manifold. We will discuss the applications in deformable shape generation, manmade shape generation, and joint shape matching.
(Received July 18, 2023)

## 1192-53-26410

Ryad Ghanam*, Virginia Commonwealth University in Qatar. Lie symmetries of the canonical connection on Lie groups Preliminary report.
In this talk we will present our results about Lie symmetries of the geodesic equations of the canonical connection on Lie groups. We will focus on the case of co-dimension one and co-dimensional two abelian nilradical. In particular, we will consider lower dimensional Lie algebras in dimensions less than or equal to six.
(Received July 27, 2023)

## 1192-53-26956

Brian Daniel Allen*, Lehman College, CUNY, Edward Bryden, Universiteit Antwerpen, Demetre Kazaras, Michigan State University. On the Stability of Llarull's Theorem in Dimension Three.
Llarull's Theorem states that any Riemannian metric on the $n$-sphere which has scalar curvature greater than or equal to $n(n-1)$, and whose distance function is bounded below by the unit sphere's, is isometric to the unit sphere. Gromov later posed the Spherical Stability Problem, probing the flexibility of this fact, which we give a resolution of in dimension 3 . The argument is based on a proof of Llarull's Theorem due to Hirsch-Kazaras-Khuri-Zhang using spacetime harmonic functions and a characterization of Sormani-Wenger Intrinsic Flat convergence given by Allen-Perales-Sormani.
(Received August 08, 2023)

## 1192-53-27766

Richard W Montgomery*, UC Santa Cruz. Sone integrable subRiemannian geodesic flows. Preliminary report.
What are the completely integrable subRiemannian geometries 'over' the Euclidean plane? A recent quite interesing member of this list was published on the arXiv in August 2023 by Perline and Tabachnikov and suggests searching for other members among the N -trailer systems (aka Semple-Monster tower) and its variants. We report on any progress made up to showtime. (Received August 19, 2023)

## 1192-53-28220

Edmund O. Harriss*, University of Arkansas, Steve Trettel, University of San Fransisco. Feeling geometry in images and models. Preliminary report.
The notions of straightness and angle lie at the heart of all levels of modern geometry. For example, the straight lines in Euclidean geometry become the geodesic of differential geometry. Yet this is an intuitive idea but gets quite complicated in the full differential picture. The result is an apparent distinction between straight lines in euclidean geometry, seen as geometric objects with an algebraic description, and geodesics on surfaces that can appear to the solution of differential equations with geometric meaning. In this talk we will discuss the artistic process of creating models and experiences that can develop the intuition of geometric ideas beyond Euclidean geometry, from geodesics to holonomy, through the Gauss-Bonnet theorem to curvature.
(Received August 28, 2023)

## 1192-53-28283

Sumeyra Sakalli*, University of Arkansas, Jeremy Van Horn-Morris, University of Arkansas. Singular Fibers in Algebraic Fibrations of Genus 2 and Their Monodromy Factorizations.
Kodaira's classification of singular fibers in elliptic fibrations and its translation into the language of monodromies and Lefschetz fibrations has been a boon to the study of 4-manifolds. In this article, we begin the work of translating between singular fibers of genus 2 families of algebraic curves and the positive Dehn twist factorizations of Lefschetz fibrations for a certain subset of the singularities described by Namikawa and Ueno in the 70s. We look at four families of hypersurface singularities in $\mathbb{C}^{3}$. Each hypersurface comes equipped with a fibration by genus two algebraic curves which degenerate into a single singular fiber. We determine the resolution of each of the singularities in the family and find a flat deformation of the resolution into simpler pieces, resulting in a fibration of Lefschetz type. We then record the description of the Lefschetz as a positive factorization in Dehn twists. This gives us a dictionary between configurations of curves and monodromy factorizations for some singularities of genus two fibrations.
(Received August 29, 2023)

1192-53-28293
Andrew Alaniz*, University of Texas Rio Grande Valley. A Conjecture on the Irregularity Function for Geometric Local Langlands Parameters. Preliminary report.
For a simple complex algebraic group $G$, it is known that the adjoint irregularity of an irregular singular flat $G$-bundle on the formal punctured disc is bounded from below by the rank of G. Moreover, the rank is realized by the formal Frenkel-Gross connection. This fact is a geometric analog of a conjecture of Gross and Reeder on the Swan conductor of arithmetic local Langlands parameters. We discuss the current state of an interesting combinatorial problem which arises when minimizing the irregularity function but with respect to an arbitrary representation of G.
(Received August 29, 2023)

## 1192-53-28379

Tanushree Shah*, University of Glasgow. Classification of tight contact structures.
Odd dimensional counterparts of symplectic structures are contact structures. They come in two flavors: tight and overtwisted. Classification of overtwisted contact structures is well understood as opposed to tight contact structures. We will in particular look at Seifert fibered manifold with 4 exceptional fibers. These are interesting manifolds because they have incompressible tori, which allows for infinitely many tight contact structures. We will look at some techniques like contact surgery and convex surface theory that are used for classification. I will end by stating what more classification results we can hope to get using the same techniques and what is far-fetched. If time permits I would like to hint at what new techniques can be used to get these far-fetched contact structures.
(Received August 30, 2023)

## 1192-53-28451

Marcelo S. Atallah, Universite de Montreal, Han Lou*, University of Georgia. On the Hofer Zehnder Conjecture for Semipositive Symplectic Manifolds.
Arnold conjecture says that the number of 1-periodic orbits of a Hamiltonian diffeomorphism is greater than or equal to the dimension of the Hamiltonian Floer homology. In 1994, Hofer and Zehnder conjectured that there are infinitely many periodic orbits if the equality doesn't hold. In this talk, I will show that the Hofer-Zehnder conjecture is true for semipositive symplectic manifolds with semisimple quantum homology. This is a joint work with Marcelo Atallah
(Received August 30, 2023)

## 1192-53-28530

Jacob Arthur Van Hook*, University of Pennsylvania. On the Geometry of Conullity Two Manifolds
If $\Gamma$ is the nullity space of the curvature tensor of a Riemannian manifold $M^{n}$, it is well known that if its dimension is constant and if $M^{n}$ is complete then the distribution $\Gamma$ is completely integrable with flat leaves. The case of $\operatorname{dim} \Gamma=n-2$ are the so called conullity two manifolds which naturally arise in various geometric contexts. The obstruction to the metric splitting isometrically is a $2 x 2$ matrix which is either nilpotent or invertible. We study the case where it is nilpotent, or equivalently where the scalar curvature is constant along the leaves of $\Gamma$. When $M^{n}$ is locally irreducible we show that $M^{n}$ admits a Lipschitz foliation $F$ by totally geodesic flat hyperplanes and determine the metric on a naturally defined open dense subset in terms of $n-1$ functions, uniquely determined up to isometry. We furthermore show that the fundamental group is either trivial or infinite cyclic. (Received August 31, 2023)

## 1192-53-28718

Ivo Terek Couto*, The Ohio State University. On compact Cotton-parallel three-manifolds
Conformal flatness of a three-dimensional pseudo-Riemannian manifold is controlled by its Cotton tensor C (as opposed to the Weyl tensor in dimensions higher than three). The next natural condition to consider is $\nabla \mathrm{C}=0$. The local structure of threedimensional pseudo-Riemannian manifolds with nonzero parallel Cotton tensor was fully described in 2014. Using such result and considering the action of deck isometries of the universal covering on the Cotton tensor, we show that a compact threedimensional Cotton-parallel pseudo-Riemannian manifold is always conformally flat.
(Received September 02, 2023)

## 1192-53-28732

Xianzhe Dai, UC Santa Barbara, Shouhei Honda, Tohoku University, Jiayin Pan, UC Santa Cruz, Guofang Wei*, UC Santa Barbara. Singular Weyl's Law with Ricci curvature bounded below.
The classical Weyl's law describes the asymptotic behavior of eigenvalues of the Laplace Beltrami operator in terms of the geometry of the underlying space. Namely, the growth order is given by (half of) the dimension and the limit by the volume The study has a long history and is important in mathematics and physics. In a recent joint work with X. Dai, S. Honda and J. Pan, we find two surprising types of Weyl's laws for some compact Ricci limit spaces. The first type could have power growth of any order (bigger than one). The other one has an order corrected by logarithm as some fractals even though the space is 2 dimensional. Moreover the limits in both types can be written in terms of the singular sets of null capacities, instead of the regular sets. These are the first examples with such features for Ricci limit spaces. Our results depend crucially on analyzing and developing important properties of the examples constructed by J. Pan and G. Wei (GAFA 2022). (Received September 02, 2023)

## 1192-53-28753

Lorenzo Sarnataro, Princeton University, Douglas Stryker*, Princeton University. Existence of closed embedded curves of constant curvature
The existence of a closed embedded curve of any prescribed constant curvature in any metric on the 2-sphere was conjectured in the 1980s by Novikov. I will discuss recent progress towards this conjecture (and some related problems) using min-max techniques, based on joint work with Lorenzo Sarnataro.
(Received September 02, 2023)

1192-53-29012
Patrick Shipman*, Colorado State University. Patterns on Surfaces and Geometry of the Mean Curvature Equation.
Preliminary report.
The mean-curvature equation provides a condition for a graph of a surface to have a prescribed mean curvature. Solutions for constant mean curvature include surfaces such the unduloid and nodoid which appear in liquid bridges (liquid sustained by surface tension between two plates). Geometric methods yield solutions to this equation, and analogies to the Cross-Newell
equation which governs pattern formation far from threshold provide a pattern-formation interpretation of the geometry of surfaces of prescribed mean curvature. These methods result in conformal surface parameterizations which aid studies of pattern formation on these surfaces, such as color patterns formed by plant pigments called anthocyanins. This is work together with Nick Ercolani and Stephen Thompson.
(Received September 05, 2023)

## 1192-53-29063

Devashi Gulati*, University of Georgia, Athens, Peter Lambert-Cole, University of Georgia. Searching for Triple Grid diagrams.
Grid diagrams have emerged as a powerful combinatorial tool for encoding knots and links, finding applications in Grid Homology and relevance in Contact topology. This talk focuses on Triple Grid diagrams, as defined by Blackwell, Gay, and Lambert-Cole, which encode embedded surfaces in $\mathbb{C P}^{2}$, with applications to obstructing Lagrangian fillings. Specifically, we explore the motivation and role of Triple Grid diagrams in representing Lagrangian surfaces within $\mathbb{C P}^{2}$. Furthermore, we discuss the challenges encountered when searching for examples of Triple Grid diagrams and highlight the significance of cubic Tait-colored graphs in the spine of the bridge trisection, which gives rise to a moduli space of Geometric Triple Grid diagrams.
(Received September 05, 2023)

## 1192-53-29150

Ipsita Datta, Institute for Advanced Study, Oleg Lazarev, University of Massachusetts Boston, Chindu Mohanakumar, Duke University, Angela Wu*, University College of London and Louisiana State University. Weinstein presentations for highdimensional antisurgery.
A Legendrian knot in the boundary of a Weinstein domain of dimension at least 6 which bounds a Lagrangian disk can be considered the boundary of the co-core of a handle. A Weinstein anti-surgery amounts to carving out this handle from the Weinstein domain. In this talk, I'll explain an algorithm which constructs explicit handle decompositions of many of these highdimensional Weinstein anti-surgery manifolds using a new high-dimensional Legendrian isotopy. I'll give a specific application of this algorithm to Lazarev and Sylvan's class of Weinstein manifolds which they called P-flexible, formed from handle attachment along P-loose Legendrians. This talk is based on work in progress with Ipsita Datta, Oleg Lazarev, and Chindu Mohanakumar.
(Received September 05, 2023)

## 1192-53-29394

Karen Butt*, University of Chicago. Closed geodesics and stability of negatively curved metrics.
The marked length spectrum of a closed Riemannian manifold of negative curvature is a function on the free homotopy classes of closed curves which assigns to each class the length of its unique geodesic representative. It is known in certain cases that the marked length spectrum determines the metric up to isometry, and this is conjectured to be true in general. In this talk, we explore to what extent the marked length spectrum on a sufficiently large finite set approximately determines the metric. (Received September 06, 2023)

## 1192-53-29558

Pranav Kulkarni, Independent Researcher, Harmanjot Singh*, Independent Researcher. Transcendental Equations for Nonlinear Optimization in Hyperbolic Space. Preliminary report
We present a novel application of transcendental equations for nonlinear distance optimization in hyperbolic space. Through asymptotic approximations using Fourier and Taylor series expansions, we obtain approximations for the transcendental equations with non-zero real values on the boundary $\lambda$. The series expansion of the logarithmic form of our equations around two arbitrary points $P_{1}$ and $P_{2}$ can be used to find values close to definite coordinates on $\lambda$. Applying principles from the Poincare hyperbolic disk - a non-Euclidean space with constant negative curvature, we construct optimization methods following $\lambda$ of our transcendental equations. Given the Euclidean, Mader, and Poincaré distance metrics $d_{E}, d_{M}$, and $d_{P}$ respectively, we establish bijective transformations allowing us to construct a hyperbolic distance optimization model and applications in 6-dimensional thermodynamic phase space. \par
(Received September 07, 2023)

## 1192-53-29608

Jo Nelson, Rice, Morgan Weiler*, Cornell University. The embedded contact homology of positive torus knots.
We compute the embedded contact homology (ECH) chain complex for $S^{1}$-invariant perturbations of ellipsoidal contact forms via the standard open book decompositions which fiber the positive torus knots $T(p, q)$ in $S^{3}$. We use our chain complex to compute the ECH knot filtration for any contact form adapted to one of these open books which satisfies a boundary twist condition and a bound on its Calabi invariant (based on $p$ and $q$ ). We apply our knot filtration result to prove quantitative existence of periodic orbits of symplectomorphisms isotopic to the monodromies of these open books. (Received September 07, 2023)

## 1192-53-29678

Lee Kennard, Syracuse University, Lawrence Mouillé*, Syracuse University. Positive intermediate Ricci curvature and maximal symmetry rank.
In foundational work for studying positively curved spaces with symmetries, Grove and Searle established a diffeomorphism classification of $n$-manifolds with positive sectional curvature that have maximal symmetry rank (i.e. isometry group with rank equal to that of $\mathrm{O}(n+1)$ ). In this talk, I will present work on generalizing their rigidity result to manifolds with positive intermediate Ricci curvature, a condition that interpolates between positive sectional curvature and positive Ricci curvature. The key tools that I will describe are generalizations of the Berger-Sugahara isotropy rank lemma and Wilking's connectedness
lemma for fixed point sets in this setting of partially positive curvature. This talk is based on joint work with Lee Kennard. (Received September 07, 2023)

## 1192-53-29851

Christoph von Tycowicz*, Zuse Institute Berlin. Riemannian Spline Models for Analyzing Shape Trajectories. Time-varying shapes exist in abundance, especially in the life sciences, where shape changes within and between individuals are tracked over time to gain insights into dynamic processes such as aging or disease progression. Such data can be considered as curves in shape space that are often given by a small set of observations. In this talk, we will discuss recent advancements in shape analysis that build upon the parametric modeling of these curves via Riemannian spline models. A particular focus will be put on the space of splines that forms a manifold on its own. We will introduce Riemannian structures for these spaces, paving the way for group-wise analysis of shape trajectories in the form of generative hierarchical models. The effectiveness of the resulting approaches will be demonstrated on tasks such as the disentanglement of shape trends, prediction of shape developments, and classification.
(Received September 08, 2023)

## 1192-53-29998

## Emmanuel L Hartman*, Florida State University. Numerical Frameworks for Elastic Shape Analysis using Second Order

 Sobolev Metrics.This talk introduces a set of numerical methods for Riemannian shape analysis of 3D surfaces within the setting of invariant (elastic) second-order Sobolev metrics. More specifically, we address the computation of geodesics and geodesic distances between parametrized or unparametrized immersed surfaces represented as 3D meshes. Building on this, we develop tools for the statistical shape analysis of sets of surfaces, including methods for estimating Frechet means and performing tangent PCA on shape populations, and for computing parallel transport along paths of surfaces. Additionally, we will discuss an extension of this model where a latent space representation is equipped with a Riemannian metric associated to second-order Sobolev metric on the space of surfaces.
(Received September 08, 2023)

## 1192-53-30092

Dan Cristofaro-Gardiner, University of Maryland, Nicole Magill*, Cornell University, Dusa McDuff, Barnard College, Columbia University, Ana Rita Pires, University of Edinburgh, Morgan Weiler, Cornell University. Symplectic Embeddings of Ellipsoids into Toric Domains.
This talk will focus on symplectic embeddings of four dimensional ellipsoids into toric domains. For certain toric domains, there are infinitely many relevant obstructions to these embeddings, often referred to as an infinite staircase. Classifying which toric domains have infinite staircases results in an intricate structure. We will discuss examples where these intricate structures arise. Furthermore, we show that we don't expect a generic toric domain to have an infinite staircases. This includes various work joint with Dan Cristofaro-Gardiner, Dusa McDuff, Ana Rita Pires, and Morgan Weiler. (Received September 08, 2023)

## 1192-53-30209

Shuli Chen*, Stanford University. Stable submanifolds in the product of projective spaces.
We consider stable compact minimal immersions in the product of a projective space with another Riemannian manifold. We prove that there do not exist odd-dimensional stable compact minimal immersions in the product of two complex projective spaces. We also prove that the only stable compact minimal immersions in the product of a quaternionic projective space with any other Riemannian manifold are the products of quaternionic projective subspaces with compact stable minimal immersions of the second manifold in the Riemmanian product. In addition, we prove that the only stable compact minimal immersions in the product of a octonionic projective plane with any other Riemannian manifold are the products of octonionic projective subspaces with compact stable minimal immersions of the second manifold in the Riemmanian product. This is joint work with Alejandra Ramirez-Luna.
(Received September 09, 2023)

## 1192-53-30233

Erin Griffin*, Seattle Pacific University, William Wylie, Syracuse University. Expanding our understanding of ambient obstruction solitons. Preliminary report.
In this talk we will discuss the recent work that defines and investigates what we call generalized ambient obstruction solitons. This talk will focus on how this new notion enables us to further the study the traditional notion of ambient obstruction solitons as examined in the speaker's 2021 paper. We conclude by looking at more explicit examples of Bach solitons and generalized Bach solitons in dimension $n=4$.
(Received September 09, 2023)

## 1192-53-30401

Ailana M Fraser*, U British Columbia. Stable minimal surfaces and applications.
I will discuss questions about stable minimal surfaces in Euclidean space and in Riemannian manifolds, and give new perspectives and new results about manifolds with positive isotropic curvature.
(Received September 09, 2023)

Xinrui Zhao*, MIT. Unique continuation problem on $R C D$ spaces
In this talk we will sketch the proof of the unique continuation property of harmonic functions and caloric functions on any
$\operatorname{RCD}(\mathrm{k}, 2)$ spaces and a counterexample for the strong unique continuation property of harmonic function on an $\operatorname{RCD}(\mathrm{k}, 4)$ space. This characterizes one of the significant differences between RCD spaces and smooth manifolds. We will also talk about some related open problems. The talk is based on joint works with Qin Deng. Unique continuation of harmonic functions on RCD spaces is a long-standing open problem, with little known even in the setting of Alexandrov spaces. In the paper "Failure of strong unique continuation for harmonic functions on RCD spaces. J. Reine Angew. Math.795(2023), 221-241", we establish the weak unique continuation theorem for harmonic functions on $\operatorname{RCD}(\mathrm{K}, 2)$ spaces and give a counterexample for strong unique continuation in the setting of $\operatorname{RCD}(\mathrm{K}, \mathrm{N})$ space for any $N \geq 4$ and any $K \in \mathbb{R}$. Also in "Unique Continuation Problem on RCD Spaces. I arXiv:2212.14237" we establish the weak unique continuation theorem for caloric functions on compact $R C D(K, 2)$ spaces and show that there exists an $R C D(K, 4)$ space on which there exist non-trivial eigenfunctions of the Laplacian and non-stationary solutions of the heat equation which vanish up to infinite order at one point. We also establish frequency estimates for eigenfunctions and caloric functions on the metric horn. In particular, this gives a strong unique continuation type result on the metric horn for harmonic functions with a high rate of decay at the horn tip, where it is known that the standard strong unique continuation property fails.
(Received September 10, 2023)

## 1192-53-30566

Peter Rock*, University of Colorado Boulder. Computation of Infinitesimal Symmetries on a Multispace of One Independent Variable.
In his paper Geometric Foundations of Numerical Algorithms and Symmetry, Peter Olver applies a moving frames approach to examine the action of a group on a curve within a generalization of jet space known as multispace. This generalization arose from the desire to prescribe multiple contact conditions (of possibly different orders) at several points along a given curve. The resulting multi-contact equivalence classes can then be represented using a polynomial of sufficiently high degree which passes through some set of interpolated points and which meets a desired set of contact conditions at those points. A natural question for us to ask, then, is what properties of jet space remain when we expand to this larger multispace? One of the main ways that we use jet space commonly is to study symmetries of differential equations under a particular group action; to do this, we generally take some action on the base space and prolong it to the jet space. So, if multispace is meant to be a generalization of jet space, we may then ask what prolongations to this mutispace look like and if they satisfy a similar set of prolongation formulae to those that arise when we prolong to jets? And the answer to this question is "Yes!" The goal of this talk will be to give a brief overview of the derivation of these prolongation formulae and to examine their implications when it comes to the discrete approximation of differential equations.
(Received September 10, 2023)

1192-53-30583
Sergio Zamora Barrera*, Max Planck Institute for Mathematics at Bonn. The global shape of compact universal covers. Given a compact geodesic space $X$, it is well understood how the geometry of its universal cover $\tilde{X}$ influences the qualitative algebraic behavior of the fundamental group $\pi_{1}(X)$. When $\tilde{X}$ is compact (or equivalently, $\pi_{1}(X)$ is finite), it is also possible to obtain, under natural geometric assumptions, quantitative information about $\pi_{1}(X)$ such as Cayley diameter or Kazhdan constant with respect to suitable geometrically chosen generators. I will talk about how this can be done, and the problem of understanding the shape of $\tilde{X}$.
(Received September 10, 2023)

## 1192-53-30587

Sergio Zamora Barrera*, Max Planck Institute for Mathematics at Bonn. Pointed Gromov-Hausdorff convergence and fundamental groups.
The fundamental group satisfies a lower-semi-continuity property under Gromov-Hausdorff convergence: if a sequence of geodesic spaces $X_{n}$ converges in the Gromov-Hausdorff sense to a compact semi-locally-simply-connected space $X$, then for $n$ large enough, the group $\pi_{1}(X)$ is a quotient of $\pi_{1}\left(X_{n}\right)$. This clearly fails if one drops the asumption that $X$ is compact and uses pointed Gromov-Hausdorff convergence instead of standard Gromov-Hausdorff convergence, as one can build a sequence of ellipsoids converging to $S^{1} \times R$. I will discuss under which conditions one can recover the above lower-semi-continuity in the non-compact setting. (Received September 10, 2023)

## 1192-53-30788

Emily Autumn Windes*, University of Rochester. Hyper-holomorphic connections. Preliminary report.
In this talk, I discuss a joint project with Jesse Madnick which explores gauge theory on hyperkahler manifolds. We study the relationships between various notions of instantons on hyperkahler manifolds. These instantons are related to special submanifolds in hyperkahler manifolds via the real Fourier-Mukai transform.
(Received September 11, 2023)

## 1192-53-30793

Emily Autumn Windes*, University of Rochester. The Moduli Space of Graphical Associative Submanifolds. Preliminary report.
In this talk, I discuss an infinite-dimensional Lagrange-multipliers problem that first appeared in Donaldson and Segal's paper
"Gauge Theory in Higher Dimensions II". The longterm goal is to apply Floer theory to a functional whose critical points are generalizations of three (real) dimensional, special Lagrangian submanifolds. I will discuss recent progress on transversality problems related to the moduli space of solutions to the Lagrange multiplers problem.
(Received September 11, 2023)

## 1192-53-30907

Dahye Cho*, Yonsei University. Relative Symplectic Cohomology and Applications to Singularities. Preliminary report.
We introduce some versions of relative symplectic cohomologies and the related spectral sequences, following ideas and works of Umut Varolgunes and Mark McLean. Using the spectral sequences, we explain how to understand isolated hypersurface singularities as well as algebraic knots.
(Received September 11, 2023)

## 1192-53-30928

Joseph Ansel Hoisington*, Max Planck Institute for Mathematics. Metric Lower Bounds for the Energy of Maps
We will present several theorems which give lower bounds for the energy of mappings between Riemannian manifolds in terms of various metric properties. Time permitting, we will then apply them to establish metric properties of some mappings which minimize energy in their homotopy classes.
(Received September 11, 2023)

## 1192-53-31043

Tianyue Liu*, University of Pennsylvania. Classification of $T^{2}$ invariant Einstein 4-manifolds with nonnegative sectional curvature
Four dimensional Einstein manifolds arise naturally in general relativity, but they are also of significant interest in mathematics. The goal of my research is to classify Einstein metrics on four dimensional manifolds under the assumption that they are invariant under $T^{2}$ actions. In my thesis I proved that the only $T^{2}$ invariant Einstein metrics with nonnegative sectional curvature on closed 1-connected four manifolds are the known examples: the round metric on $S^{4}$, the Fubini-Study metric on $\mathbb{C} P^{2}$, or the product of two round $S^{2}$ of the same radii.
(Received September 11, 2023)

## 1192-53-31048

John B Etnyre, Georgia Institute of Technology, Caitlin Leverson*, Bard College. A Friendly Introduction to Lagrangian Realizations of Ribbon Cobordisms. Preliminary report.
Given two knots, a ribbon cobordism is a particularly nice surface with boundary the disjoint union of the two knots. Similarly to how every smooth knot has a Legendrian representative (in fact, infinitely many different representatives), in this talk we will discuss why every ribbon cobordism has a Legendrian representative. Meaning, if $C$ is a ribbon cobordism in $[0,1] \times S^{3}$ from the link $K_{0}$ to $K_{1}$, then there are Legendrian realizations $\Lambda_{0}$ and $\Lambda_{1}$ of $K_{0}$ and $K_{1}$, respectively, such that $C$ may be isotoped to a decomposable Lagrangian cobordism from $\Lambda_{0}$ to $\Lambda_{1}$. Along the way we will give a brief introduction to Legendrian knots and Lagrangian cobordisms. This is joint work with John Etnyre.
(Received September 11, 2023)

## 1192-53-31096

Thomas William Murphy*, CSU Fullerton. Rigidity of symmetric spaces.
We prove that the biinvariant Einstein metric on $S U_{2 n+1}$ is isolated in the moduli space of Einstein metrics, even though it admits infinitesimal deformations. This gives a non-Kaehler, non-product example of this phenomenon adding to the famous example of $\mathbb{C} P^{2 n} \times \mathbb{C} P^{1}$ found by Koiso. We apply our methods to derive similar solitonic rigidity results for the KaehlerEinstein metrics on 'odd' Grassmannians. We also make explicit a connection between non-integrable deformations and the dynamical instability of metrics under Ricci flow. (Received September 11, 2023)

## 1192-53-31552

Edmund O. Harriss*, University of Arkansas. Gradient of Grain.
When using digital manufacturing to render geometry the mathematics is often imposed on the material. As a material, however, wood talks back revealing patterning of grain, whatever the geometry does. Yet the grain itself is a geometric structure that also reveals the history of the wood itself. Using the grain of a disk of wood, as an input allows for an artwork that takes its material into account in creating geometry. Furthermore, by carving always at right angles to the grain of the wood the cuts are as cleans as possible, allowing for a carving with dramatic height shifts between the cuts made into the wood. In this talk I will discuss artwork made using these ideas, and more generally the ways to create surfaces and other geometry in wooden pieces using CNC.
(Received September 11, 2023)

1192-53-31699
Renato G. Bettiol, CUNY, McFeely Jackson Goodman*, Colby College. Curvature Operators and Rational Cobordism. We give new conditions on positivity of certain linear combinations of eigenvalues of the curvature operator of a Riemannian manifold which imply the vanishing of the indices of Dirac operators twisted with bundles of tensors. The vanishing indices in turn have topological implications in terms of the Pontryagin classes, rational cobordism type, and Witten genus of the manifolds. To prove our results we generalize new methods developed by Petersen and Wink to apply the Bochner technique to Laplacians on bundles of tensors. We give preliminary results on topological conditions which imply the existence of metrics satisfying these curvature conditions. This is joint work with Renato Bettiol.
(Received September 11, 2023)

Matemáticas, UNAM. Asymptotic dimension and geometric decompositions in dimension 4.
Consider a closed orientable 4-manifold that is geometrizable in the sense of Thurston. In this talk we will show that the asymptotic dimension of the fundamental group of such manifolds is at most 4 . We will also explain some consequences that follow from the finiteness of the asymptotic dimension, such as the coarse Baum-Connes and Novikov conjectures.
(Received September 12, 2023)

1192-53-31754
Alexander Nabutovsky*, U Toronto. Boxing inequalities in higher codimension and related inequalities. Preliminary report Let $M^{n}$ be a closed Riemannian manifold in a finite or infinite-dimensional Banach space, and $m \leq n$ a positive number. We prove that there exists a pseudo manifold $W^{n+1}$ such that $\partial W^{n+1}=M^{n}$ and the $m$-dimensional Hausdorff content $H C_{m}\left(W^{n+1}\right)$ of $W^{n+1}$ does not exceed $c(m) H C_{m}\left(M^{n}\right)$. Recall that the $m$-dimensional Hausdorff content $H C_{m}(X)$ is, by definition, the infimum of $\Sigma_{i} r_{i}^{m}$ over all covering of $X$ by metric balls, where $r_{i}$ denote the radii of these balls. When $M^{n}$ is a closed hypersurface in $\mathbb{R}^{n+1}=B$ this result implies that that for each $m \in(0, n] H C_{m}(\Omega) \leq c(m) H C_{m}\left(M^{n}\right)$, where $\Omega$ is the domain bounded by $M^{n}$. (The case of $m=n$ is the well-known and widely used boxing inequality first proven by W. Gustin.) We will also discuss further generalizations of this result, its connections with analysis, systolic geometry, and with Urysohn width-volume inequalities first pioneered by L. Guth. This is a joint work with Sergey Avvakumov. (Received September 12, 2023)

## 1192-53-31855

Ivko Dimitric*, Pennsylvania State University Fayette, Srdjan Vukmirovic, Faculty of Mathematics, University of Belgrade, Serbia. Subgrassmannian of Lagrangian planes. Preliminary report.
In the paper "A note on equivariant embeddings of Grassmannians" (1996), the first-named author considered a natural equivariant embedding of Grassmannian $G_{k}\left(\mathbb{F}^{m}\right)$ of $k$-dimensional planes in the number space $\mathbb{F}^{m}, \mathbb{F} \in\{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$ via projection operators, which embedds the Grassmannian into a suitable Euclidean space of matrices. Given a hermitian almost complex structure $J$ on $\mathbb{R}^{2 n}$, an $n$-dimensional plane $\pi \in G_{n}\left(\mathbb{R}^{2 n}\right)$ is said to be Lagrangian if $g(X, J Y)=0$ for all $X, Y \in \pi$. The set of all Lagrangian planes $\mathcal{L}_{J}$ is naturally a submanifold of $G_{n}\left(\mathbb{R}^{2 n}\right)$, which is also a symmetric space $\mathcal{L}_{J}=U_{J}(n) / O(n)$. Using the embedding $\phi: G_{n}\left(\mathbb{R}^{2 n}\right) \rightarrow H(2 n) \subset M_{2 n}(\mathbb{R})$ into a Euclidean space by projection operators, we determine some basic properties of $\mathcal{L}_{J}$ as a submanifold of $G_{n}\left(\mathbb{R}^{2 n}\right)$. In particular, we show that $\mathcal{L}_{J}$ is of 1-type via $\phi$ in the context of Chen's theory of submanifolds of finite type. Certain generalizations are also produced for submanifold $\mathcal{L}_{J}^{k} \subset G_{k}\left(\mathbb{R}^{2 n}\right)$ of Lagrangian planes of dimension $k$.
(Received September 12, 2023)

1192-53-32052
Bo-Jui Chang, UT Southwestern Medical Center, Bingying Chen, UT Southwestern Medical Center, Gaudenz Danuser, UT Southwestern Medical Center, Meghan Driscoll, University of Minnesota, Gabriel M. Gihana, UT Southwestern Medical Center, Andrew Weems, UT Southwestern Medical Center, Felix Y Zhou*, UT Southwestern Medical Center. Surface-guided computing to study 3D subcellular morphology and signal dynamics across space and time.
Form is function. Just as Darwin's finches have beaks adapted to their ecological niche, so too do cell morphology associate with function. Neurons have stereotypically long axons and dendrites to connect to other neurons and for long-range information transmission. The distortion of normal cellular morphology is often the first clinical indicator of disease. Metaplasia, for example is a key tissue marker of cancer progression. Yet, beyond morphological measurements as a readout of a cell's functional response, the mechanisms by which morphology could regulate cell signalling, function and fate and their contribution to pathogenesis is not well understood. Molecularly, cell morphology, specifically the cell membrane is indispensable for extra- and intra- cellular signaling. Not only does the membrane contain the receptors to sense and trigger signal transduction, but is crucially the interface for cell-to-cell contact and a catalyst for biochemical reactions. How do signalling molecules spatially distribute relative to subcellular morphological structure like blebs, lamellipodia or filopodia? How does this distribution change over time or in response to microenvironmental changes? To systematically study such questions quantitatively from observational data we must be able to compute on complex 3D cell surfaces across space and time. Here, I will present a surface-computing framework, u-Unwrap3D for analyzing arbitrary cell morphologies and to sample both external and internal surface-proximal signals including surface contacts, structures, and molecular intensity. uUnwrap3D generalizes the idea of surface unwrapping prevalent in map-making, differential geometry and developmental biology in a manner applicable to cells. Specifically, u-Unwrap3D develops distortion-minimizing and topology-preserving bidirectional mappings that enables the 3D cell surface at every video timepoint to be projected into shared lower dimensional topographic and 2D image coordinate spaces. This enables us to perform computations using the most suited surface coordinate representation including the decomposition of local surface structures from their underlying global geometry and their tracking over time. I demonstrate this on videos of 3D cell blebbing, ruffling and cancer-immune cell interaction. (Received September 12, 2023)

## 1192-53-32066

Anastasiia Sharipova*, Pennsylvania State University. Convex Bodies with all Planar Characteristics.
I will show that in symplectic space smooth strongly convex bodies with all characteristics planar or all outer billiard trajectories planar are affine symplectic images of a ball.
(Received September 12, 2023)

## 1192-53-32146

Nicklas Charles Day*, Texas A\&M University. Local Geometry of Rank 2 Distributions: symplectification, Cartan prolongations, and Cartan connections II.
In 1970, N. Tanaka gave a method for obtaining a canonical frame for distribution with a constant Tanaka symbol. In 2009, B. Doubrov and I. Zelenko utilized a symplectification procedure to obtain a canonical frame for rank 2 distributions independent
of their Tanaka symbol under an additional assumption called maximality of class. For a rank 2 distribution on an n dimensional manifold, the symplectification can be interpreted as the result of $(n-4)$ iterative Cartan prolongations and the canonical frame in the method of Doubrov-Zelenko can be arranged to be a Cartan connection associated to the rank 2 distribution obtained from the original one by $(n-4)$ Cartan prolongation. The talk is devoted to the question: what is the minimal $i$ such that to the $i$ th Cartan prolongation of any rank 2 distribution one can assign a canonical Cartan connection. For $n=5$ it is well known due to E. Cartan (1910) that this minimal i is equal to 0, i.e. the canonical Cartan connection can be assigned to the original distribution without any Cartan prolongation. We proven a generalization: for $n \geq 5$, the minimal such $i$ is equal to $(n-5)$, i.e. occur one step before the symplectification. The talk is based on joint work with Igor Zelenko. (Received September 12, 2023)

1192-53-32235
Roger Casals, University of California Davis, Honghao Gao, Yau Mathematical Sciences Center, Linhui Shen, Michigan State University, Daping Weng*, University of California, Davis. Cluster Ensemble for Legendrian Links. In recent years, the discovery of a connection between cluster theory and Legendrian links in contact geometry has lead to breakthroughs in infinitely-many-filling problems of Legendrian links. In this talk, I will introduce the two dual cluster structures associated with Legendrian links, one on the augmentation variety and the other on the Kashiwara-Schapira moduli space of microlocal sheaves, and discuss how these cluster structures help solve the infinitely-many-filling problems for rainbow closures of positive braids. This talk is based on my joint works with H. Gao and L. Shen (2008.10793 \& 2009.00499) and my joint work with R. Casals (2204.13244). (Received September 12, 2023)

## 1192-53-32250

Yevgeny Liokumovich, University of Toronto, Davi Maximo, University of Pennsylvania, Regina Rotman*, University of Toronto. Length of a shortest closed geodesic on a closed 3-manifold.. Preliminary report.
Let M be a closed Riemannian 3-manifold with the scalar curvature bounded below by some positive constant k . We will prove that there exists a closed non-trivial geodesic on M of length at most $\frac{c}{\sqrt{k}}$. (Joint with Y. Liokumovich, D. Maximo.)
(Received September 12, 2023)

## 1192-53-32373

Isabel Beach*, University of Toronto. Short Simple Geodesic Loops on a 2-Sphere. Preliminary report.
The classic Lyusternik-Schnirelmann theorem states that on any Riemannian 2 -sphere $M$, there are three distinct simple periodic geodesics. It has been proven by Y. Liokumovich, A. Nabutovsky and R. Rotman that the shortest three such curves have lengths bounded in terms of the diameter of $M$. In this talk, we will show that at any point $p$ on $M$ there exist at least two distinct simple geodesic loops (geodesic segments that start and end at $p$ ) whose lengths are bounded in terms of the diameter of $M$.
(Received September 12, 2023)

## 1192-53-32390

Xinle Dai*, Harvard University. Sectorial decompositions of symmetric products of surfaces. Preliminary report. Symmetric products of Riemann surfaces play a crucial role in symplectic geometry and low dimensional topology; for example, they are essential ingredients for defining Heegaard Floer homology and important examples of Liouville manifolds when the surfaces are open. In this talk, I will discuss work in progress on the symplectic topology of these spaces via Liouville sectorial methods.
(Received September 12, 2023)

## 1192-53-32427

Katie Gittins, Durham University, Carolyn Gordon, Dartmouth College, Ingrid Membrillo-Solis, University of Westminster, Juan Pablo Rossetti, Universidad Nacional de Córdoba, Mary R Sandoval, Trinity College, Elizabeth Stanhope*, Lewis \& Clark College. Examining orbifold singular structure using Hodge spectra. Preliminary report.
A Riemannian orbifold is a mildy singular generalization of a Riemannian manifold. We examine cases when the spectrum of the Hodge Laplacian acting on $p$-forms allows us to detect the presence of singular points in an orbifold, including conditions on the codimension of an orbifold's singular set which guarantee that the volume of the singular set is determined. Examples where the Hodge spectrum fails to detect aspects of the structure of an orbifold will also be discussed. Heat invariants and zeros of Krawtchouk polynomials are important tools in this work.
(Received September 12, 2023)

## 1192-53-32446

Catherine Kendall Asaro Cannizzo*, UC Berkeley, Sara Venkatesh, Stanford University. Wrapping Lagrangians in the Fukaya category of a symplectic fibration.
In symplectic geometry, the Fukaya category is an algebraic invariant of symplectic manifolds, where Lagrangian submanifolds are objects, and their intersections are morphisms. Mirror symmetry connects the symplectic geometry of one manifold to the complex geometry of its mirror. We will outline how a blow-up of a complex manifold along a submanifold is mirror to wrapping of Lagrangians around a puncture on the symplectic side. This is joint work with Sara Venkatesh. (Received September 12, 2023)

Smith College, Grace Pepperman, Smith College, Xavier Ramos Olive, Smith College. Curvature and eigenvalues on graphs. Preliminary report.
Ricci curvature is a concept that originated in the world of Differential Geometry. In that field, Ricci curvature can constrain the diameter of a manifold, it controls the volume growth of geodesic balls, and it interacts with the natural frequencies of oscillation of a wave in that space (the eigenvalues of the Laplacian). For instance, a famous result by Obata tells us that the positively curved spaces that produce the lowest sounds are round spheres. In recent years, new notions of Ricci curvature have been developed on metric measure spaces that are not necessarily smooth. Of particular interest are notions of curvature in discrete spaces like graphs, where these notions have applications to Network and Data Sciences. Many results from Differential Geometry involving Ricci curvature can be translated to results on Graph Theory with these new curvature notions, including diameter bounds for positively curved graphs and eigenvalue estimates. Using the Lin-Lu-Yau Ricci curvature on graphs, we explore the following question: what are the positively curved graphs that produce the lowest sounds? We will see that, contrasting with what happens in the smooth setting, there is a large family of graphs satisfying this condition.
(Received September 12, 2023)

## 1192-53-33023

Kieran Favazza*, Saint Louis University. The Denseness of Canonical Algebraic Curvature Tensors and a Revision to the Signature Conjecture.
The structure of the space of algebraic curvature tensors over a vector space is of great interest and captures information about the behavior of the Riemann curvature tensor on a manifold. Studying this structure in itself and underlying substructures can help in the determination of invariants of algebraic curvature tensors. These are of importance in understanding how algebraic curvature tensors can be distinguished from one another; we investigate invariants of algebraic curvature tensors to further our understanding of this topic in connection with the structure of the space of algebraic curvature tensors. Further, we study the denseness of canonical algebraic curvature tensors and their behavior overall. We prove basic results and state conjectures for future research.
(Received September 13, 2023)

## 1192-53-33166

Nicklas Charles Day, Texas A\&M University, Igor Zelenko*, Texas A\&M University. Local Geometry of Rank 2 Distributions: symplectification, Cartan prolongations, and Cartan connections, I. Preliminary report.
In 1970, N . Tanaka gave a method for obtaining a canonical frame for distribution with a constant Tanaka symbol. In 2009 , B . Doubrov and I. Zelenko utilized a symplectification procedure to obtain a canonical frame for rank 2 distributions independent of their Tanaka symbol under an additional assumption called maximality of class. For a rank 2 distribution on an ndimensional manifold, the symplectification can be interpreted as the result of $n-4$ iterative Cartan prolongations and the canonical frame in the method of Doubrov-Zelenko can be arranged to be a Cartan connection associated to the rank 2 distribution obtained from the original one by $n-4$ Cartan prolongation. The talk is devoted to the question: what is the minimal $i$ such that to the $i$ th Cartan prolongation of any rank 2 distribution one can assign a canonical Cartan connection. For $n=5$ it is well known due to E. Cartan (1910) that this minimal is equal to 0 , i.e. the canonical Cartan connection can be assigned to the original distribution without any Cartan prolongation. We proven a generalization: for $n \geq 5$, the minimal such $i$ is equal to $n-5$, i.e. occur one step before the symplectification. The talk is the first part of two talks based on joint work with Nicklas Day.
(Received September 13, 2023)

## 1192-53-33229

Enrique Guadalupe Alvarado*, UC Davis, Stephan Wojtowytsch, Texas A\&M. Random covers of data manifolds]Random covers of low regularity data manifolds in high-dimensional ambient spaces.
In many applications in data science, data is sampled from a low-dimensional region of a much higher-dimensional ambient space. We develop an extrinsic perspective which allows us to study a coarse geometric invariant - covers by small balls Euclidean balls - under minimal regularity assumptions. Namely, we show that if data is sampled from a distribution which is close to a measure theoretic surface (rectifiable varifold) of variable dimension, then under an optimal mean curvature bound the covering properties are comparable to those of a Euclidean space of the highest-dimensional component of the varifold, independently of the dimension of the ambient space.
(Received September 13, 2023)

1192-53-33461
Du Anh Tran*, California State University, Fullerton. Inequalities Between Intrinsic and Extrinsic Quantities for ThreeDimensional Smooth Hypersurfaces.
It might seem surprising that some elementary inequalities could provide an insight into the problem of thebest possible immersion of a space into another ambient space. By J.F. Nash's Theorem, any Riemannian manifold can be embedded into a Euclidean ambient space with dimension sufficiently large. S.-S. Chern pointed out in 1968 that a key technical element in applying Nash's Theorem effectively is finding useful relationships between intrinsic and extrinsic elements that are characterizing immersions. After 1993, when a groundbreaking work written by B.-Y.Chen on this theme was published, many explorations pursued this important avenue. Bearing in mind this historical context, in our present project we obtain new relationships involving intrinsic and extrinsic curvature invariants, under natural geometric conditions.
(Received September 18, 2023)

Zofia Ewa Adamska*, California Institute of Technology, Yakov Berchenko-Kogan, Florida Institute of Technology, Pedro Estrada Gallegos, National Autonomous University of Mexico, Ricardo Garcia, California Institute of Technology. Variational Discretization Methods for Curvature Flows on Riemannian Manifolds.
The study of geometric flows has several applications that span a wide variety of scientific disciplines. For example, these geometric evolution equations were used to model processes such as the annealing of metal sheets, evolution of reaction
diffusion systems, or the behavior of cellular automata. Furthermore, geometric flows are applied to problems in computer vision and image processing. We present a variational method to compute curve-shortening and curve-straightening flows on arbitrary Riemannian manifolds. We apply our method to manifolds with different metrics, and verify convergence to expected theoretical results. Furthermore, we introduce an algorithm that uses partitions of unity to evolve curves in manifolds covered by multiple charts, an example being noncontractible loops in tori. The methods developed in this project could have the potential to deepen our understanding of the geometry of non-trivial Riemannian manifolds, such as their geodesics and minimal submanifolds. These insights can open new avenues of investigation and inspire further developments in the field of discrete and numerical differential geometry.
(Received September 23, 2023)

## 1192-53-33729

Isaac Marcelo Lopez*, Massachusetts Institute of Technology, Daniel Alejandro Santiago*, Massachusetts Institute of Technology. Positive Mass Theorems for Asymptotically Euclidean Smooth Metric Measure Spaces. Preliminary report. We prove a positive mass theorem for asymptotically Euclidean smooth metric measure spaces, which are generalizations of weighted manifolds. In a special case, we recover the weighted positive mass theorem proven by Baldauf and Ozuch. Our result is proven in two different ways: by applying spinorial techniques on certain warped products and by making a conformal change of metric. Our proof methods yield results of independent interest, including eigenvalue bounds for the Dirac operator on closed manifolds and a characterization of the Dirac operator on warped products with manifolds admitting parallel spinors. (Received September 24, 2023)

## 1192-53-33778

Jesse Coy Wallace*, Oklahoma State University. Controlled Local Isometric Embeddings of Riemannian Surfaces into $\mathbb{R}^{4}$. Preliminary report.
It has been well established that Riemmanian surfaces can be locally isometrically embedded into $\mathbb{R}^{4}$; further, it has been shown that for specific cases, such embeddings are possible into $\mathbb{R}^{3}$. However, there remain cases where the existence of a local isometric embedding into $\mathbb{R}^{3}$ isn't clear. We describe a strengthening of the classical result on embeddability into $\mathbb{R}^{4}$ by showing that the embedding can be made to lie in certain 3-dimensional submanifolds of $\mathbb{R}^{4}$ that can be taken to be arbitrarily close to an $\mathbb{R}^{3}$ subspace.
(Received September 25, 2023)

## 54 General topology

## 1192-54-28215

Carolyn Otto, University of Wisconsin-Eau Claire, Janee Schrader*, University of Wisconsin-Eau Claire. Exploring the combinatorics of pretzel links through grid diagrams
The representation of links by grid diagrams naturally involve combinatorial aspects of pretzel links. Using grid diagrams, we establish a methodology to encode and analyze pretzel links, enabling a systematic approach of their topological and algebraic properties. By employing a combination of knot theory, graph theory, and combinatorial techniques, we uncover relationships between the structure of pretzel links and their corresponding grid diagram representations. Additionally, we present computational techniques for efficiently generating and analyzing grid diagrams of pretzel links.
(Received August 28, 2023)

## 1192-54-29509

Emanuel Ayala Lopez, The University of Utah, Jillian Cervantes*, University of Wisconsin - Milwaukee, Katherine Elizabeth Lovelace, The Ohio State University. Optimizing Gravitational Wave Detection Using Topological Data Analysis. Preliminary report.
The interactions between black holes create decaying periodic "chirp" signals known as gravitational waves. The detection of gravitational waves is an important problem in physics, but this task proves difficult as the signals are embedded in background noise. We investigate the optimal topological feature extraction methods and machine learning models in detecting gravitational waves. In 2019, Bresten and Jung determined that using persistence vectors as topological features for input to a convolutional neural network (CNN) results in a better classifier than the traditional CNN method. We evaluate persistence vectors in comparison with other topological features, including persistence landscapes, persistence images, and persistence entropy. Our aim is to determine whether a CNN is truly necessary, or if better topological features with a simpler classifier (such as logistic regression, random forest, support vector classifier, and multi-layer perceptron) can achieve similar results. We determine that Bresten and Jung's persistence vectors feature extraction method with a support vector classifier achieved the best results without using a CNN, at 0.943 AUC. This classifier is much easier to train and interpret compared to a CNN, and is competitive even with a significantly smaller sample size.
(Received September 07, 2023)

## 1192-54-29999

P Christopher Staecker*, Fairfield University. A higher homotopy group for digital images.
We define a second homotopy group in the setting of abstract digital images. Namely, we construct a functor from digital images to abelian groups, which closely resembles the ordinary second homotopy group from algebraic topology. We illustrate that our approach can be effective by computing the second digital homotopy group of a digital 2-sphere. We also discuss some natural extensions of this work. This is a report on work begun at the 2023 AIM workshop on "Discrete and combinatorial homotopy theory" and completed in collaboration with Gregory Lupton, Oleg Musin, Nicholas Scoville, and Jonathan Treviño. (Received September 08, 2023)

Lori Alvin*, Furman University. Triodic Set-Valued Functions. Preliminary report.
We investigate the family of set-valued functions $F:[0,1] \rightarrow 2^{[0,1]}$ whose graphs are obtained by connecting the point $(a, b) \in(0,1) \times(0,1)$ via straight lines to the points $(0,0),(0,1)$, and $(1,1)$, forming a triod. We study the dynamical behavior of these functions and classify specific properties of interest in terms of the location of the branch point ( $a, b$ ) within the unit square. This is preliminary work motivated by an undergraduate research project with Furman student Sam Housand during Summer 2023.
(Received September 08, 2023)

## 1192-54-31030

David Boothe, Army Research Lab, Piotr Franaszczuk, Johns Hopkins University, Vasileios Maroulas, The University of Tennessee, Knoxville, Edward Mitchell, Joe Gibbs Human Performance Institute, Brittany Story*, University of Tennessee, Knoxville. Simplicial convolutional recurrent neural networks for neural decoding.
The brain's spatial orientation system uses different neuron types, such as head direction and grid cells, to aid in environmentbased navigation. Head direction cells provide orientation information whereas grid cells consist of layers of decked neurons that overlay to provide environment-based navigation. These neurons fire in ensembles where several neurons fire at once to activate a single head direction or grid. We want to capture this firing structure and use it to decode head direction and grid cell data. Understanding and using these underlying neural structures requires models that encompass higher order connectivity, more than the 1-dimensional connectivity that traditional graph-based models provide. Simplicial complexes, topological spaces that use not only vertices and edges but also higher-dimensional objects, naturally generalize graphs and capture more than just pairwise relationships. Thus, we use unsupervised simplicial complex discovery paired with the power of deep learning to construct the simplicial convolutional recurrent neural network combines. We demonstrate the effectiveness and versatility of the simplicial convolutional neural network on head direction and trajectory prediction via head direction and grid cell datasets.
(Received September 11, 2023)

## 1192-54-32205

Erica Choi*, Columbia University, Katerina Stuopis*, University of Wisconsin - Madison. Cubiquitous Lattices and $\chi$ sliceness. Preliminary report.
It is a classical result that any knot or link in $\mathbb{R}^{3}$ bounds a surface embedded in $\mathbb{R}^{3}$. It turns out that knots and links also bound surfaces embedded in $\mathbb{R}^{4}$. An active area of research explores knots and links that bound surfaces in $\mathbb{R}^{4}$ with simple topology, or in other words, are slice and $\chi$-slice, respectively. A lattice is cubiquitous if it admits an embedding into $\mathbb{Z}^{n}$ in such a way that its image $\lambda$ intersects a point in each unit cube with integer vertices. Greene and Ownes have proved that if an alternating nonsplit link $L$ is $\chi$-slice, then there exists an associated cubiquitous sublattice. We show various properties that a basis $B=\left\{v_{1}, \ldots, v_{n}\right\}$ that generates a cubiquitous lattice has, and prove that $B^{T} B$ has 1 's, 2 's, and $4^{\prime}$ 's on the diagonal. This finding proves the Greene and Owens' conjecture that the converse of their theorem is true for torus links. (Received September 12, 2023)

1192-54-33020
Shreya Arya, Duke University, Arnab Auddy, Columbia University, Ranthony A Clark*, The Ohio State University, Sunhyuk LIM, Max Planck Institute for Mathematics in the Sciences, Facundo Mémoli, The Ohio State University, Daniel Packer, The Ohio State University. Gromov-Wasserstein distance between spheres.
In this talk we introduce a two parameter family of Gromov-Wasserstein distances between metric measure spaces. We focus our discussion on the novel result that we are able to compute the exact value of our Gromov-Wasserstein distance between unit spheres (equipped with the Euclidean distance and uniform measure) for particular choices of parameters. In addition, we exhibit a hierarchy of lower bounds in the case of spheres (equipped with the geodesic distance and uniform measure). (Received September 13, 2023)

## 1192-54-33159

Daniel Gonzalez, The University of Arizona, Tania A Gonzalez*, Sam Houston State University, Alberto Magana, Vanderbilt University. Data Sets Resulting in Relatively Compact Sets of Persistence Diagrams. Preliminary report. Persistence diagrams contain vital topological information, which traditional machine learning techniques are ill-equipped to leverage effectively. Template functions are a promising tool for bridging this gap by facilitating the featurization of persistence diagrams, thus making them more amenable as inputs to machine learning algorithms. The featurization and application of template functions, however, is only possible on relatively compact sets of persistence diagrams. In this work, we establish conditions and restrictions on families of data sets that ensure the relative compactness of the set of their persistence diagrams. Specifically, we show that relative compactness under Gromov-Hausdorff distance implies relative compactness at the level of persistence diagrams for the Vietoris-Rips filtration. These results are further extended to general Lipschitz continuous filtrations.
(Received September 13, 2023)

1192-54-33771
Ori Salim Friesen*, Macalester College, Lori Ziegelmeier, Macalester College. Using Persistent Homology to Study the Shape of Racial Segregation in United States Cities. Preliminary report.
Racial segregation is a social phenomenon that is ubiquitous in United States cities. Because racial segregation is so common, different methods, such as diversity indices, have been used to quantify and classify different levels and structures of segregation. In our research, we seek to understand structures of racial segregation on a deeper level by looking at the topology of segregation for U.S. cities. To do this, we utilize topological data analysis, namely persistent homology, to measure the shape of each city. We utilize 2020 racial demographic data at the census tract level for 100 U.S. cities, and we explore various topological methods as inputs for machine learning algorithms to categorize the different shapes of segregation in
cities across the United States. By using this topological approach to cluster segregation structures of U.S. cities, we can find underlying patterns that might not have been previously apparent.
(Received September 25, 2023)

## 55 Algebraic topology

## 1192-55-27605

Robyn Kaye Brooks*, University of Utah. Computing the Rank Invariant and the Matching Distance for Mulit-Parameter Persistence Modules using Discrete Morse Theory.
Persistent Homology is a tool of Computation Topology which is used to determine the topological features of a space from a sample of data points. In this talk, I will introduce the (multi-)persistence pipeline, as well as some basic tools from Discrete Morse Theory which can be used to better understand the multi-parameter persistence module of a filtration. In particular, the addition of a discrete gradient vector field consistent with a multi-filtration allows one to exploit the information contained in the critical cells of that vector field as a means of enhancing geometrical understanding of multi-parameter persistence. I will present results from joint work with Claudia Landi, Asilata Bapat, Barbara Mahler, Elizabeth Stephenson, and Celia Hacker, in which we are able to show that the rank invariant for nD persistence modules can be computed by selecting a small number of values in the parameter space determined by the critical cells of the discrete gradient vector field. These values may be used to reconstruct the rank invariant for all other possible values in the parameter space. I will then present forthcoming results on how these critical parameter values may be used to compute the matching distance between two 2-D persistence modules. (Received August 16, 2023)

## 1192-55-27610

Jose Perea*, Northeastern University. Vector bundles for alignment and dimensionality reduction.
Vector bundles have rich structure and arise naturally when trying to solve dimensionality reduction and synchronization problems in data science. I will show in this talk how the classical machinery (e.g., classifying maps, characteristic classes, etc) can be adapted to the world of algorithms and noisy data, as well as the insights one can gain.
(Received August 16, 2023)

## 1192-55-27888

Jose Perea*, Northeastern University. Topological Time Series Analysis.
Time series are ubiquitous in biomedical applications and machine learning tasks such as classification, recurrence quantification, data imputation and anomaly detection. In this talk, I will describe how tools from applied topology and dynamical systems can be leveraged to answers some of these questions, and in particular, their relevance in biological applications
(Received August 21, 2023)

## 1192-55-28078

Thomas Read*, University of Warwick. Generalised Witt vectors and the Hill-Hopkins-Ravenel norm.
Witt vectors are an algebraic construction first introduced in Galois theory in the 1930s, but later finding applications in many areas of mathematics, including stable equivariant homotopy theory. The norm is an important construction on equivariant spectra, most famously playing a key role in the solution of the Kervaire invariant one problem by Hill, Hopkins and Ravenel. In this talk I will describe a new generalisation of Witt vectors, the $G$-typical Witt vectors with coefficients, that can be used to compute the zeroth equivariant stable homotopy groups of the norm of a connective spectrum.
(Received August 25, 2023)

## 1192-55-28228

Yariana Diaz*, Macalester College. Hom-size identifiable subcategories of quiver representations for use in multi-parameter persistence theory. Preliminary report.
Persistence theory draws on quiver representations theory and homology to generate an algebraic summary, called a persistence diagram, which details the appearance and disappearance of topological features in a filtration of a topological space. It has applications in topological data analysis, where topological spaces may be be inferred from a sampling of data points. Once the information about the life of topological features is computed, their meaning can be inferred within the setting from which the original data was drawn. Topological data analysis has applications to a breadth of subjects and disciplines, including music theory, urban planning, machine learning, microbiology, physics, and economics. When filtering a topological space by a single parameter, the theory of quiver representations completely describes how to decompose the resulting representation. The complexity increases significantly when filtering by two or more parameters. In particular, multiparameter persistence yields quivers whose indecomposable representations are more complicated to describe. The theme of this work is to provide examples of subcategories of quiver representations (which will be determined by their indecomposable objects) whose objects can be distinguished from one another in a computationally feasible manner. (Received August 28, 2023)

## 1192-55-28266

Daniel Tolosa*, Purdue University. An algebraic model for the free loop space as an $S^{1}$-space. Preliminary report.
The free loop space of a topological space has a canonical circle action given by rotating loops, making it an $S^{1}$-space. The work of Jones, Goodwillie, and others, relates the equivariant homology of the free loop space to the cyclic homology of the algebra of singular chains on the topological monoid of based loops. Recently, M. Rivera described a construction that models the free loop space in terms of the chains on the underlying space considered as a categorical coalgebra, a notion Koszul dual to a non-negatively graded dg category. This construction is "as small as possible", has no hypotheses on the underlying space, and is suitable for computations in (non-simply connected) string topology. I will present a cyclic theory for categorical
coalgebras and dg-categories extending the theory of cyclic homology for (dg) algebras and coalgebras. In particular, the cyclic chains of the categorical coalgebra of chains on a simplicial set provides a model for the $S^{1}$-equivariant chains on the free loop space that is suitable for computations. Time permitting, we establish a comparison between Goodwillie's and Rivera's models, which can be understood in terms of combinatorics of certain polytopes.
(Received August 28, 2023)

## 1192-55-28302

Erin Wolf Chambers*, St. Louis University. Bounding the Interleaving Distance for Geometric Graphs with a Loss Function. Preliminary report.
A geometric graph is generally defined as an abstract graph along with a well behaved embedding of the graph into the Euclidean plane. Such graphs are a fundamental object used to model a wide range of data sets, ranging from maps and trajectories to commodity networks (i.e. electrical grids) to skeletons for shape recognition. The ability to compare and cluster such objects is required in a data analysis pipeline, leading to a need for distances or metrics on these objects. In this work, we utilize the idea of the interleaving distance, where functor representations of data can be compared by finding pairs of natural transformations between them. However, in many cases, particularly those of the set-valued functor variety, computation of the interleaving distance is NP-hard. For this reason, we take inspiration from the work of Robinson to find quality measures for families of maps that do not rise to the level of a natural transformation. Using the structure of the embedded graphs, we define a loss function which measures how far the required diagrams of an interleaving are from commuting. We can use this loss function to determine a bound on the interleaving distance as follows, as well as begin to develop an algorithmic approach to finding locally optimal interleavings. We expect these ideas are not only useful in our particular use case of embedded graphs, but can be extended to a larger class of interleaving distance problems where computational complexity creates a barrier to use in practice. This paper is joint work with Elizabeth Munch, Sarah Percival and Bei Wang (Received August 29, 2023)

## 1192-55-28337

Erin Wolf Chambers*, St. Louis University. On Complexity of Computing Bottleneck and Lexicographic Optimal Cycles in a Homology Class.
Homology features of spaces which appear in applications, for instance 3D meshes, are among the most important topological properties of these objects. Given a non-trivial cycle in a homology class, we consider the problem of computing a representative in that homology class which is optimal. We study two measures of optimality, namely, the lexicographic order of cycles (the lex-optimal cycle) and the bottleneck norm (a bottleneck-optimal cycle). We give a simple algorithm for computing the lex-optimal cycle for a 1 -homology class in a closed orientable surface. In contrast to this, our main result is that, in the case of 3-manifolds of size $n^{2}$ in the Euclidean 3-space, the problem of finding a bottleneck optimal cycle cannot be solved more efficiently than solving a system of linear equations with an $n \times n$ sparse matrix. From this reduction, we deduce several hardness results. Most notably, we show that for 3-manifolds given as a subset of 3-space of size $n^{2}$, persistent homology computations are at least as hard as rank computation (for sparse matrices) while ordinary homology computations can be done in $O\left(n^{2} \log n\right)$ time. This is the first such distinction between these two computations. Moreover, it follows that the same disparity exists between the height persistent homology computation and general sub-level set persistent homology computation for simplicial complexes in 3-space. This is joint work with Salman Parsa and Hannah Schreiber.
(Received August 29, 2023)

1192-55-28411
Amit Patel, Colorado State University, Tatum Rask*, Colorado State University. Poincaré Duality for Generalized Persistence Diagrams of (co)Filtrations.
We dualize previous work on generalized persistence diagrams for filtrations to cofiltrations. When the underlying space is a manifold, we express this duality as a Poincaré duality between their generalized persistence diagrams. A heavy emphasis is placed on the recent discovery of functoriality of the generalized persistence diagram and its connection to Rota's Galois Connection Theorem.
(Received August 30, 2023)

## 1192-55-28438

David Mehrle*, University of Kentucky, J.D. Quigley, University of Virginia, Michael Stahlhauer, .. Koszul Complexes for free $C_{p^{n}}$-Tambara functors.
Koszul complexes are well-understood resolutions of certain quotients of commutative rings. They are used ubiquitously in homological algebra, for example to define Lie algebra cohomology or to compute Tor and Hochschild homology. We might hope to use such resolutions to compute Mackey-functor-valued Tor for free $C_{p^{n}}$-Tambara functors, but naively transporting the definition of a Koszul complex to equivariant algebra does not yield a resolution - it has nontrivial homology in positive degree. In this talk, we will see how to fix this problem by adding dimensions to the naive Koszul complex to produce free resolutions of certain free $C_{p^{n}}$-Tambara functors. As an application, we compute Mackey-functor-valued Hochschild homology of these free $C_{p^{n}}$-Tambara functors over the Burnside functor.
(Received August 30, 2023)

Nicholas Wawrykow*, University of Michigan. Disk Configuration Spaces and Representation Stability. One can generalize the idea of the configuration space of points in a manifold by fixing a metric on the manifold and replacing the points with open unit-diameter disks. In this talk we discuss one of the simplest of these disk configuration spaces, $\operatorname{conf}(n, w)$, the ordered configuration space of $n$ open unit-diameter disks in the infinite strip of width $w$. We show that when the width of the strip is at least 2 , the rational homology groups of $\operatorname{conf}(\bullet, w)$ stabilize as $n$ increases in a sense similar to Church-Ellenberg-Farb, Miller-Wilson's representation stability for ordered configuration spaces of points. (Received August 31, 2023)

Abhishek Rathod*, Ben Gurion University of the Negev, Israel. Cup Product Persistence and Its Efficient Computation. It is well-known that cohomology has a richer structure than homology. However, so far, in practice, the use of cohomology in persistence setting has been limited to speeding up of barcode computations. Some of the recently introduced invariants, namely, persistent cup-length, persistent cup modules and persistent Steenrod modules, to some extent, fill this gap. When added to the standard persistence barcode, they lead to invariants that are more discriminative than the standard persistence barcode. In this work, we devise an $O\left(d n^{4}\right)$ algorithm for computing the persistent k-cup modules for all $k \in\{2, \ldots, d\}$, where d denotes the dimension of the filtered complex, and n denotes its size. Moreover, we note that since the persistent cup length can be obtained as a byproduct of our computations, this leads to a faster algorithm for computing it. The talk is based on joint work with Tamal Dey.
(Received August 31, 2023)

## 1192-55-28691

Florian Frick, Carnegie Mellon University, Jacob Lehmann Duke*, Williams College, Arianna Meenakshi McNamara*, Purdue University, Hannah Park-Kaufmann, Bard College, Steven Raanes, Vassar College, Steven Simon, Bard College, Zoe Wellner, Carnegie Mellon University. Generalizations of the Erdős-Ginzburg-Ziv Theorem via Topology. Erdős, Ginzburg, and Ziv showed that any sequence of $2 n-1$ numbers in $\mathbb{Z} / n$ has a subsequence of length $n$ that sums to zero. This zero-sum subsequence may be unique, which is prohibitive to proving constrained or quantitative generalizations of the Erdős-Ginzburg-Ziv theorem. However, for a prime $p$, a sequence of $p$ numbers in $\mathbb{Z} / p$ sums to zero if and only if it is a difference of two permutations of $\mathbb{Z} / p$. We develop a novel topological approach, based on the non-existence of continuous maps that commute with certain symmetries, to prove that the permutations whose difference gives the zero-sum subsequence may be strongly restricted. More specifically, we argue that $\mathbb{Z} / p$-equivariant maps from the chessboard complex $\Delta_{p, 2 p-1}$ must hit the barycenter of properly chosen codomains, guaranteeing a zero-sum subsequence. We then use this to provide exponential lower bounds (in $p$ ) to the number of relevant permutations, thus establishing a quantitative generalization of the Erdős-Ginzburg-Ziv theorem. Our topological approach naturally lends itself to proving fractional strengthenings of the theorem as well, a special case of which is the following extension: For any sequence of $2 p-1$ sets $A_{1}, \ldots, A_{2 p-1} \subset \mathbb{Z} / p$, each of cardinality $k$, there is a subsequence $A_{i_{1}}, \ldots, A_{i_{p}}$ such that $A_{i_{1}}+1, A_{i_{2}}+2, \ldots, A_{i_{p}}+p$ is a balanced family of subsets of $\mathbb{Z} / p$, in the sense that these translated sets determine a hypergraph that admits a perfect fractional matching. The $k=1$ case recovers the classical Erdős-Ginzburg-Ziv theorem. (Received September 01, 2023)

## 1192-55-28722

Daniel C. Cohen*, Louisiana State University. Topological complexity and configuration spaces.
Topological complexity is a homotopy-type invariant motivated by the motion planning problem from robotics. I will discuss this invariant, and related ideas, largely in the context of ordered configuration spaces of surfaces. (Received September 02, 2023)

1192-55-28768
Antonio Rieser*, CONACYT-CIMAT. Recent Developments in the Algebraic Topology of Mesoscopic Spaces.
We will discuss recent work on the development of algebraic topology for mesoscopic spaces, or metric spaces decorated with a preferred scale, which we argue are the natural spaces of interest for applied topology. We will describe how doing homotopy theory for these spaces requires working in a category more general than topological or uniform spaces, and we will discuss relationships between this work and discrete homotopy theory.
(Received September 02, 2023)

1192-55-28817
Deborah Ajayi, University of Ibadan, Agnese Barbensi, University of Oxford, Heather A Harrington, University of Oxford, Christian Degnbol Madsen, University of Melbourne, Michael P.H. Stumpf, University of Melbourne, Iris H. R. Yoon*, University of Delaware. HyperTDA: Hypergraphs \& persistence diagrams for multiscale topological features in structured data.
Complex spatial structure is a hallmark of many real-world complex systems. Topological data analysis provides a window on the connectivity and shape of such systems across multiple scales. But constructing and analysing topological summaries remains challenging. We introduce hyperTDA, an interpretable, automated, and holistic perspective that navigates these challenges and traverses the panorama of topological data analysis by encoding all local topological information into a hypergraph. HyperTDA enables flexible choices of homology generators (e.g. matroid or minimal) and seamless integration with network science. We showcase hyperTDA on spatial curve datasets arising in ecology, physics, and biology. This approach selects between synthetic trajectories from the 2020 AnDi challenge and quantifies species movements, even when data is noisy or limited.
(Received September 03, 2023)

## 1192-55-28866

Katherine Benjamin*, University of Oxford. Topology of spatiotemporal data.
In recent years, multiparameter persistent homology (MPH) has been developed to address the limitations of traditional singleparameter persistence. While much progress has been made on the theoretical aspects of MPH, practical applications have mostly been limited to the special case of two parameters. Building on recent developments in the computation of MPH in more than two parameters, we propose and implement a new Gröbner basis algorithm for the computation of multiparameter persistence landscapes in this setting. We apply this work to compute landscapes of three-parameter persistence modules built on spatiotemporal data arising in biology.

Guchuan Li, Peking University, Sarah Petersen*, University of Colorado Boulder, Elizabeth Ellen Tatum, Stockholm University. A Thom Spectrum Model for $C_{2}$-Integral Brown-Gitler Spectra.
We establish a Thom spectrum model for a $C_{2}$-equivariant analogue of integral Brown-Gitler spectra and show these have a multiplicative property. The $C_{2}$-equivariant spectra we construct enjoy properties analogous to classical nonequivariant integral Brown-Gitler spectra and thus may prove useful for producing splittings of $B P\langle 1\rangle \wedge B P\langle 1\rangle$ and bo $\wedge b o$ in the $C_{2^{-}}$ equivariant setting.
(Received September 04, 2023)

## 1192-55-28921

Daniel Carranza*, Johns Hopkins University, Krzysztof R. Kapulkin, University of Western Ontario. Nonexistence of colimits in naive discrete homotopy theory.
During the 2023 AIM workshop "Discrete and Combinatorial Homotopy Theory", the question was raised whether the category of graphs can be equipped with a model structure whose weak equivalences are the A-homotopy equivalences. This talk will report on joint work with C. Kapulkin and J. Kim (arXiv: 2306.02219), which settles this question in the negative. In the talk, we will explain the significance of this problem, e.g. how such a model structure can be used to define homotopy pushouts as a tool to compute discrete homotopy and homology groups. We briefly introduce the language of $\infty$-categories in order to provide an underlying structure for when homotopy pushouts exist. By studying the $\infty$-category of graphs, recently introduced in joint work with C. Kapulkin and Z. Lindsey (arXiv: 2306.02218), we can give an explicit diagram of graphs and graph maps which fails to admit a homotopy pushout. We conclude by discussing the consequences of this result as well as giving a revised conjecture.
(Received September 04, 2023)

## 1192-55-28929

Zhengchao Wan*, University of California San Diego. Persistent Laplacians: Theoretical Foundations, Efficient Algorithms, and Applications in Data Science.
Persistent homology is arguably the most successful technique in Topological Data Analysis. It combines homology, a topological feature of a data set, with persistence, which tracks the evolution of homology over different scales. The persistent Laplacian is a recent theoretical development that combines persistence with the combinatorial Laplacian, the higher-order extension of the well-known graph Laplacian. In this talk, we will explore the theoretical foundations of the persistent Laplacian, highlighting its relationship with persistent homology and extending its applicability to cubical complexes. In addition, we will present efficient algorithms for calculating its matrix representations and discuss its connections to spectral graph theory and circuit network theory. To conclude, we will assess the effectiveness of the persistent Laplacian as a tool for data embedding, with a focus on its capabilities in downstream classification and regression tasks. The talk aims to emphasize the advantages of the persistent Laplacian, demonstrating its potential to surpass traditional persistent homology techniques in applications.
(Received September 04, 2023)

## 1192-55-28940

Qingsong Wang*, University of Utah. Generalized ultrametricity and vanishing result of persistent homology. Preliminary report.
In this talk, we introduce the notion of $k$-ultrametric spaces as a generalization of ultrametric spaces. Our primary aim is to present a vanishing result for higher-dimensional persistent homology groups of the Vietoris-Rips complexes. This involves studies the discrete curvature of the $k$-ultrametric spaces. In conclusion, we discuss its implications for higher-order clustering techniques.
(Received September 07, 2023)

## 1192-55-29101

Sunhyuk LIM*, Max Planck Institute for Mathematics in the Sciences, Facundo Memoli, The Ohio State University (columbus, OH, US)", Osman Okutan, Florida State University. Vietoris-Rips persistent homology, injective metric spaces, and the filling radius.
In the applied algebraic topology community, the persistent homology induced by the Vietoris-Rips simplicial filtration is a standard method for capturing topological information from metric spaces. In this paper, we consider a different, more geometric way of generating persistent homology of metric spaces which arises by first embedding a given metric space into a larger space and then considering thickenings of the original space inside this ambient metric space. In the course of doing this, we construct an appropriate category for studying this notion of persistent homology and show that, in a category theoretic sense, the standard persistent homology of the Vietoris-Rips filtration is isomorphic to our geometric persistent homology provided that the ambient metric space satisfies a property called injectivity. As an application of this isomorphism result we are able to precisely characterize the type of intervals that appear in the persistence barcodes of the Vietoris-Rips filtration of any compact metric space and also to give succinct proofs of the characterization of the persistent homology of products and metric gluings of metric spaces. Our results also permit proving several bounds on the length of intervals in the Vietoris-Rips barcode by other metric invariants, for example the notion of spread introduced by M. Katz. As another application, we connect this geometric persistent homology to the notion of filling radius of manifolds introduced by Gromov and show some consequences related to (1) the homotopy type of the Vietoris-Rips complexes of spheres which follow from work of M. Katz and (2) characterization (rigidity) results for spheres in terms of their Vietoris-Rips persistence barcodes which follow from work of F. Wilhelm. Finally, we establish a sharp version of Hausmann's theorem for spheres which may be of independent interest. (Received September 05, 2023)

Sidhanth Raman*, UC-Irvine. A smooth Birman-Hilden theory for 4-manifolds. Preliminary report.
The classical Birman-Hilden theory links braid groups and the mapping class groups of Riemann surfaces: when two surfaces are related by a sufficiently nice cover, the Birman-Hilden isomorphism ensures that one can seamlessly transfer information about their respective mapping class groups through the cover. We will discuss a smooth analog of the Birman-Hilden theory for certain 4-manifolds (hyperkahler), and its applications to Nielsen realization problems. For example, we prove that Dehn twists are not realizable by any finite order diffeomorphism on an Enriques surface. Time permitting, we will discuss the emergence of actual braid group actions on these 4-manifold mapping class groups, and some open problems around these actions.
(Received September 05, 2023)

## 1192-55-29214

Henry Hugh Adams, University of Florida, Johnathan Bush, University of Florida, Florian Frick*, Carnegie Mellon University. Topology and the positivity locus of odd maps.
An odd map is a map that commutes with multiplication by -1 . Thus an odd map from the $d$-sphere to $\mathbb{R}$ splits the sphere into two symmetric regions of constant positive, respectively negative, sign. The Borsuk-Ulam theorem states that a $d$-dimensional vector space of odd maps defined on the $d$-sphere has a common zero. I will present more general results that bound the diameter of regions, where higher-dimensional vector spaces of odd maps cannot have constant sign. This is related to the topology of a certain space of probability measures on the sphere. I will mention applications.
(Received September 05, 2023)

## 1192-55-29235

Gregory Lupton*, Cleveland State University, Oleg R Musin, University of Texas Rio Grande Valley, Nicholas A. Scoville, Ursinus College, P Christopher Staecker, Fairfield University, Jonathan Treviño, CIMAT. Higher Homotopy Groups in Digital Topology.
Digital topology refers to the use of notions and methods from (algebraic) topology to study digital images. A digital image in our sense is an idealization of an actual binary digital image, which consists of pixels in the plane, or higher dimensional analogues of such. One aim of digital topology is to provide useful theoretical background for certain steps of image processing, such as feature extraction or recognition. An extensive literature on digital topology includes several treatments of the fundamental group. I will report on our recent work in which we develop a notion of a second (higher) homotopy group and calculate its value to be $\mathbb{Z}$ for a digital image that may reasonably be interpreted as a digital 2 -sphere. This calculation involves some interesting combinatorial ingredients. Our development and calculation may equally well be applied in the (graphtheoretic) settings of tolerance spaces and X-homotopy theory. The definitions readily extend to higher homotopy groups of any dimension. I will also speculate briefly about possible applications of our work. (Received September 06, 2023)

## 1192-55-29268

Erin Wolf Chambers, St. Louis University, Elizabeth Munch, Michigan State University, Sarah Percival, Michigan State University, Elena Wang*, Michigan State University. A Distance for Geometric Graphs via the Labeled Merge Tree Interleaving Distance. Preliminary report.
Geometric graphs appear in many real-world datasets, such as road networks, sensor networks, and molecules. We investigate the notion of distance between graphs and present a semi-metric to measure the distance between two geometric graphs via the directional transform combined with the labeled merge tree distance. We introduce a way of rotating the sub-level set to obtain the merge trees via the idea of the directional transform. We represent the merge trees using a surjective multi-labeling scheme and then compute the distance between two representative matrices. Our distance is not only reflective of the information from the input graphs, but also can be computed in polynomial time. We illustrate its utility by implementation on a Passiflora leaf dataset.
(Received September 06, 2023)

## 1192-55-29411

Ziqiang Li*, Virginia Polytechnic Institute and State University, Jindi Sun, University of Arizona. Winding-Number PointInclusion Tests for Boundary Antipode-Excluding Spherical Polygons.
Spherical polygons used in practice are nice, but the spherical point-in-problem (SPiP) has long eluded solutions based on the winding number (wn). That a punctured sphere is simply connected is to blame. As a workaround, we prove that requiring the boundary of a spherical polygon to not intersect the antipode of said boundary is sufficient to reduce its SPiP problem to the planar, point-in-polygon (PiP) problem without using known interior points (KIP). Such a spherical polygon satisfies the socalled boundary antipode-excluding (BAE) property. Moreover, spherical polgyons fully contained within an open hemisphere is automatically BAE. We document two linear transformations, one successful and one unsuccessful in SPiP-to-PiP reduction, and discuss their fundamental differences. A combined algorithm solves SPiP correctly and efficiently for BAE spherical polygons.
(Received September 06, 2023)

## 1192-55-29482

Beren Sanders*, University of California, Santa Cruz. The Balmer spectrum of Goodwillie calculus.
In this talk, I'll discuss joint work with Greg Arone, Tobias Barthel and Drew Heard, in which we compute the Balmer spectrum of the category of $n$-excisive functors from spectra to spectra. There are interesting similarities (both metaphysical and real) between these categories of $n$-excisive functors and the equivariant stable homotopy categories of symmetric groups. In any case, equivariant homotopy theory plays an essential role in our work and we are able to obtain a complete and satisfying description of the Balmer spectrum of $n$-excisive functors for all $n$.
(Received September 06, 2023)

Robin Belton*, Smith College, Robyn Kaye Brooks, University of Utah, Lisbeth Fajstrup, Aalborg University, Brittany Terese Fasy, Montana State University, Elizabeth Aurora Vidaurre, Molloy College. "Directed Collapsibility and its Connections to Topological Data Analysis".
While collapsibility of CW complexes dates back to the 1930s, collapsibility of directed Euclidean cubical complexes has not been well studied. We study the question on how to collapse Euclidean cubical complexes while preserving spaces of directed paths, where directed paths are paths that are nondecreasing in all coordinates. We then use this theory to provide a new perspective for studying multiparameter persistence modules.
(Received September 07, 2023)

1192-55-29706
Maxine Elena Calle, University of Pennsylvania, David Chan*, Michigan State University, Andres Mejia, University of Pennsylvania. Algebraic K theory of coefficient rings.
Geometric invariants of a space X , like the Wall finiteness obstruction, are often encoded in the algebraic K theory of the group ring $\mathbb{Z}\left[\pi_{1}(X)\right]$. In this talk we will discuss some changes which must be made to this approach when considering spaces with an action by a finite group $G$. Specifically, the group ring should be replaced with a different algebraic structure, known as a coefficient ring and we will discuss how to define a version of algebraic K theory for these objects. As an application, we show how the equivariant Wall finiteness obstruction naturally takes values in the algebraic K theory of a coefficient ring. This is joint work with Maxine Calle and Andres Mejia.
(Received September 07, 2023)

1192-55-29736
Anna Marie Bohmann, Vanderbilt University, Teena Gerhardt, Michigan State University, Cameron Krulewski*, Massachusetts Institute of Technology, Sarah Petersen, University of Colorado Boulder, Lucy Z Yang, Columbia University. Equivariant Algebra and Hochschild Homology. Preliminary report.
Classically, the algebraic structure of topological Hochschild homology ( $T H H$ ) has played a key role in the trace methods approach to algebraic $K$-theory computations. A topological analog of Hochschild homology, THH assigns to a commutative ring spectrum $A$ an $S^{1}$-equivariant, cyclotomic ring spectrum $T H H(A)$. The homotopy groups of the $C_{p^{n}}$-fixed points of this spectrum carry the structure of a Witt complex: a prodifferential graded ring that in differential level zero receives a map from the $p$-typical Witt vectors of $A$, and which carries an elaborate set of operations and relations that may be leveraged toward computations. Recent work has developed equivariant notions of both Hochschild homology and topological Hochschild homology. There are theories of twisted Hochschild homology for Green functors, and given a finite cyclic group $G$ and a $G$ ring spectrum $R$, there is an equivariant notion of $T H H$ called the $G$-twisted topological Hochschild homology of $R$. We study the structure of this equivariant version of $T H H$ and show that its homotopy Green functors carry the structure of what we term an equivariant Witt complex.
(Received September 07, 2023)

1192-55-29738
Anh Trong Nam Hoang*, University of Minnesota. Configuration spaces and applications in arithmetic statistics. In the last dozen years, topological methods have been shown to produce a new pathway to study arithmetic statistics over function fields, most notably in Ellenberg-Venkatesh-Westerland's work on the Cohen-Lenstra conjecture. More recently, Ellenberg, Tran and Westerland proved the upper bound in Malle's conjecture for function fields by studying stability of the homology of braid groups with certain exponential coefficients. In this talk, we will give an overview of their framework and extend their techniques to study other questions in arithmetic statistics. As an example, we will demonstrate how this extension can be used to study character sums of the resultant of monic square-free polynomials over finite fields, answering and generalizing a question of Ellenberg and Shusterman.
(Received September 07, 2023)

1192-55-29791
Nikolas Schonsheck*, University of Delaware, Stefan Schonsheck, University of California Davis. Spherical coordinatizations of data via persistent cohomology.
The circular coordinates algorithm of de Silva-Vejdemo-Johansson-Morozov, later strengthened by Perea, extracts circular parameterizations of data from a persistent cohomology calculation. In other words, given a data set $X$, one can obtain coordinate functions from $X$ to the unit circle $S^{1}$ via fist-degree persistent cohomology. Since these results were first published a little over a decade ago, the question of whether spherical coordinates, i.e., maps from $X$ to the unit sphere $S^{2}$, could be similarly (and directly) obtained from second-degree persistent cohomology remained open. In this talk, I will discuss work in which we obtain such spherical parameterizations and describe examples of our algorithm on various data sets. This is joint work with Stefan Schonsheck.
(Received September 07, 2023)

1192-55-29818
Hannah Alpert, Auburn University, Fedor Manin*, UC Santa Barbara. Topology of the configuration space of disks in a strip. The topology of the configuration space of points in the plane is well-understood. In statistical physics and topological robotics, one may instead want to understand the configuration space of thick particles in a constrained space. The topology of such spaces is quite complicated, and several models have been used to simplify it. One such model is that of $n$ unit disks in an infinitely long strip of radius $w$, introduced by Alpert, Kahle, and MacPherson. After introducing the background, I will discuss joint work with Hannah Alpert studying the homology of this family of spaces and how it changes as we add more disks or widen the strip.
(Received September 08, 2023)

Alyson Bittner, Pacific Northwest National Laboratory, Peter Bubenik, University of Florida, Vladimir Itskov, Pennsylvania State University, Helen Jenne, Pacific Northwest National Laboratory, Emilie Purvine*, Pacific Northwest National Laboratory. Exploring homology for hypergraphs. Preliminary report.
Core to the theory of homology, a method used by topologists to study multi-dimensional holes in topological spaces, is the idea of continuous deformation. In surfaces and topological spaces this is clearly understood. However, in some scientific applications a topological space may not be immediate from data. Instead, graphs and hypergraphs may be more readily available, especially when dealing with relational data. Given the great insights that homology and related concepts like persistent homology and cohomology have provided to many application areas our team was interested in applying homology to the hypergraphs that arise naturally from our data sets. But it is not immediately clear what continuous deformation would mean in the case of a hypergraph. While we can draw a hypergraph on a page, it is not something we can hold in our hands and twist or bend. In this talk we will give some background on homology for hypergraphs and will discuss the work of our AIM working group to explore homology of associated simplicial complexes including the polar complex and order complex of related topological spaces. We will provide examples and intuition behind the definitions and explorations. (Received September 08, 2023)

## 1192-55-29850

Henry Hugh Adams, University of Florida, Johnathan Bush*, University of Florida, Florian Frick, Carnegie Mellon University. Connections between Vietoris-Rips complexes, packings and coverings of projective spaces, and zeros of odd maps. If a dataset is sampled from a manifold, then, as more and more samples are drawn, the Vietoris-Rips (VR) persistent homology of the dataset will converge to the VR persistent homology of the underlying manifold. However, very little is known about the VR persistent homology of manifolds. Toward better understanding the persistent homology of data sampled from an underlying, unknown manifold I will describe relationships between the following three problem areas: (i) understanding the topology of VR complexes of spheres, (ii) finding optimal packings and coverings of projective spaces, and (iii) bounding the diameter of zero sets of odd maps from spheres to euclidean spaces. This is joint work with Henry Adams and Florian Frick. (Received September 08, 2023)

## 1192-55-29897

Kimberly Herrera, UC Berkeley, Martin Martinez, University of Washington Bothell, Sofia Rose Rose Martinez Alberga*, Purdue University, Austin MBaye, Vassar College. Cup Products Detect Quasi-Periodicity. Preliminary report. The latest methods of studying data involve the tools of topological data analysis (TDA). Even though the theoretical foundation of TDA comes from areas of math as abstract as algebraic topology, linear algebra, and homological algebra, the implementation of these tools can be done in a jupyter notebook with a few external packages. Given an audio file, we create a data set and use TDA to extrapolate information about the data set in particular its shape. By looking at the highly persistent features that appear in the persistence diagram, we develop an algorithm for determining and detecting the periodicity in the audio file. In short this talk will discuss how using the structure of persistent cohomology can help distinguish two data sets that give rise to the same persistence diagrams. This is joint work with Kimberly Herrera, Martin Martinez, and Austin MBaye and advised by Dr. Jose Perrea. (Received September 08, 2023)

## 1192-55-29925

Ling Zhou*, Duke University. Decoding zero-length bars.
Barcodes are valuable tools in TDA that originate from the decomposition of persistent homology. Traditionally, only non-zerolength barcodes are considered. However, recent research has shown that zero-length barcodes encode additional information to what is captured at the homology level. In light of this new development in the field, the AIM working group formed by Barbara Giunti, Arnur Nigmetov, Bei Wang, Alyson Bittner, Emilie Purvine and myself, further explores the topic. I will present a review of previous works, my participation in this working group and other related AIM activities. (Received September 08, 2023)

## 1192-55-29935

Bridget Schreiner*, University of Notre Dame. Cross effects and stability. Preliminary report.
A sequence of groups (or spaces) $G_{1} \hookrightarrow G_{2} \hookrightarrow G_{3} \hookrightarrow \ldots$ satisfies homological stability if $H_{i}\left(G_{n}\right) \cong H_{i}\left(G_{n+1}\right)$ for $n \gg i$.
There are notions of higher stability introduced by Miller-Wilson and Galatius-Kupers-Randal-Williams that define meta-stable ranges for such sequences. This talk will introduce a generalization of the cross effects of Eilenberg and MacLane that is suited to answer questions about homological stability, with the goal of finding a more standard approach to higher stability (Received September 08, 2023)

1192-55-29936
Henry Hugh Adams, University of Florida, Tia Suzanne Karkos*, Colorado State University. Time-Varying Spaces and Mobile Sensor Networks.
We consider mobile sensor networks in which each sensor covers a ball and wanders continuously within a bounded domain. Sensors do not know their locations, but can detect whether their covered regions overlap; intruders cannot pass undetected between overlapping sensors. An evasion path exists if it is possible for a moving intruder to avoid being detected by the sensors. We generalize notions of topological equivalence to time-varying spaces and apply these notions to mobile sensor networks. We then consider two mobile sensor networks, which were originally constructed by Adams and Carlsson in order to show that the time-varying homotopy type of the covered region does not determine the existence of an evasion path. Even though the uncovered regions of these two sensor networks are not time-varying homotopy equivalent, their covered regions are, and we explain this proof in detail. Furthermore, we prove that the two covered regions are not time-varying homeomorphic.
(Received September 08, 2023)

Danika Van Niel*, Michigan State University. Algebraic structures of twisted topological Hochschild homology. Preliminary report.
Topological Hochschild homology (THH) is an invariant of ring spectra and is a key component of the trace method approach to algebraic K-theory. One of the main computational tools for THH is the Bökstedt spectral sequence. The study of the algebraic structure of THH and the Bökstedt spectral sequence have advanced our computational ability. In recent years, a generalization of THH for equivariant ring spectra called twisted THH has been developed along with an equivariant version of the Bökstedt spectral sequence. In this talk we introduce THH, twisted THH, and discuss work in progress on the algebraic structure of twisted THH and the equivariant Bökstedt spectral sequence.
(Received September 08, 2023)

## 192-55-30010

William Balderrama*, University of Virginia. The equivariant J-homomorphism and $R O(G)$-graded periodic phenomena. For a $G$-representation $V$, the representation sphere $S^{V}$ is defined as one-point compactification of $V$. The $G$-equivariant stable stems are comprised of equivariant homotopy classes of maps between representation spheres, suitably stabilized. When $G$ is the trivial group, these are the classical stable stems, and a significant amount of work has gone into their computation and into finding regular patterns within them. The first example of such a pattern comes from the $J$ homomorphism, which produces infinite " $v_{1}$-periodic" families in the stable stems. Early work of Bredon, built on by Landweber, Araki and Iriye, and more recently by Behrens and Shah, has highlighted the existence of certain periodic behavior in $C_{2}$-equivariant homotopy theory, not seen nonequivariantly. In this talk, I will describe how a version of the equivariant $J$-homomorphism can be used to construct these equivariant periodicities, in a way that works in $G$-equivariant homotopy theory for an arbitrary finite (and to an extent compact Lie) group $G$. (Received September 08, 2023)

## 1192-55-30027

Lucy Yang*, Columbia University. Normed $\mathbb{E}_{\infty}$-rings in genuine equivariant $C_{p}$-spectra.
Genuine equivariant homotopy theory is equipped with a multitude of coherently commutative multiplication structures generalizing the classical notion of an $\mathbb{E}_{\infty}$-algebra. Our work concerns the $C_{p}-\mathbb{E}_{\infty}$-algebras of Nardin-Shah with respect to a cyclic group $C_{p}$ of prime power order. We show that many of the higher coherences inherent to the definition of parametrized algebras collapse; in particular, they may be described more simply and conceptually in terms of ordinary $\mathbb{E}_{\infty}$-algebras as a diagram category which we call normed algebras. Our main result provides a relatively straightforward criterion for identifying $C_{p}-\mathbb{E}_{\infty}$-algebra structures. We visit some applications of our result to real motivic invariants.
(Received September 08, 2023)

## 1192-55-30084

Ling Zhou*, Duke University. Persistent Cup Product Structures and Related Invariants.
One-dimensional persistent homology is arguably the most important and heavily used computational tool in topological data analysis. Additional information can be extracted from datasets by studying multidimensional persistence modules and by utilizing cohomological ideas. In this work, given a single parameter filtration, we investigate a certain 2-dimensional persistence module structure associated with persistent cohomology. This new persistence structure, called the persistent cup module, is induced by the cohomological cup product and adapted to the persistence setting. We show that this persistence structure is stable. In addition, we consider a generalized notion of a persistent invariant, which extends both the rank invariant (also referred to as persistent Betti number), Puuska's rank invariant induced by epi-mono-preserving invariants of abelian categories, and the recently-defined persistent cup-length invariant, and we establish their stability. This generalized notion of persistent invariant also enables us to lift the Lyusternik-Schnirelmann (LS) category of topological spaces to a novel stable persistent invariant of filtrations, called the persistent LS-category invariant.
(Received September 11, 2023)

## 1192-55-30108

Kimberly Herrera, UC Berkeley, Martin Martinez*, University of Washington Bothell, Austin MBaye, Vassar College. Using Persistent Cup Products for Dissonance Detection. Preliminary report.
Quasiperiodic time series are those whose sliding window point clouds are dense in high-dimensional tori. This topological structure can be captured via persistence diagrams, thus providing algorithms for quasiperiodicity detection. There are, however, time series whose sliding window point clouds are not dense in tori - and hence not quasiperiodic - but have the persistence diagrams of one. In this study, we propose an algorithm that incorporates both persistent cohomology and cup products, introducing a persistent cup product that distinguishes between quasiperiodic and non-quasiperiodic time series. By utilizing this algorithm, we achieve a more precise detection of quasiperiodicity. Additionally, we apply this algorithm to the tritone interval to display its use for dissonance detection, as well as add levels of noise to a tritone audio file to demonstrate the robustness of the algorithm.
(Received September 08, 2023)

## 1192-55-30153

Gregory Henselman-Petrusek, University of Oxford, Christian Joseph Lentz*, Macalester College, Xintan Xia, Macalester College, Lori Ziegelmeier, Macalester College. A computational approach for persistent relative homology. Preliminary report.
A central problem in data-driven scientific inquiry is how to interpret structures in large data sets uncovered by modern tools. The field of topological data analysis (TDA) provides a potential solution via the language of homology, which encodes features of interest as holes. A particularly important task in TDA is to compute topological holes in a dataset and identify the original data generating them. Recently new matrix factorizations have been proposed to place this calculation in a more flexible form, which relates the standard persistence calculation to several variants. We will discuss one such technique, which puts the boundary operators of a chain complex into a form where bases have been matched, making extraction of generators
expeditious. We discuss this technique in the context of persistent homology and extend it to an algorithm for persistent relative homology, which can identify features relative to some subset of the original data.
(Received September 08, 2023)

## 1192-55-30225

Greg Alexander Depaul*, University of California, Davis. Adding Differential Topology and Statistical Robustness to TDA. Preliminary report.
Vietoris Rips is a widely used filtration, despite being a rigid topology as well as susceptible to outliers. Instead, we propose to utilize density estimation and classicial linear fitting methods to build a novel filtered complex from multidimensional Gaussian balls that are meant to approximate a tangent space. Because this construction leverages density estimation, it is often more robust against outliers than traditional TDA methods. One application of this technique can be used for resolving singularities present in point cloud data in the hopes of recognizing intrinsic dimensionality of a dataset.
(Received September 09, 2023)

## 1192-55-30237

Giulia De Pasquale, ETH Zurich, Fabiana Ferracina, Washington State University, Rebecca Hardenbrook, Dartmouth College, Jiajie Luo*, UCLA, Molly Lynch, Hollins University, Juan Carlos Martinez Mori, Cornell University, Anna Nelson, Duke University, Mason A Porter, UCLA, William Thompson, University of Delaware. Persistent Homology for Assessing Facility Placement.
It is important to choose the geographic distribution of resource facilities in a way that fairly and equitably serves the underlying population. One way to systematically choose resource locations is by considering facility-location problems, which are optimization problems whose solutions give resource-facility distributions that minimize cost of access. Different facilitylocation problems seek to produce optimal solutions differently (e.g., minimize average vs. worst-case cost), which complicates the assessment and comparison of solutions. In this talk, we discuss persistent homology, a tool derived from algebraic topology, as a way to analyze resource facility placement. In particular, we use persistent homology as a means to assess and compare solutions to different facility-location problems by identifying and quantifying regions of poor resource access. (Received September 09, 2023)

## 1192-55-30270

Liam Keenan*, University of Minnesota - Twin Cities. Filtrations and topological Hochschild homology.
We discuss filtered variants of topological Hochschild homology. In particular, we will introduce the May filtration on THH of a ring spectrum, and explain how this filtration can be used to establish faithfully flat descent for THH of connective $\mathbf{E}_{2}$-ring spectra, generalizing a result of Bhatt-Morrow-Scholze.
(Received September 09, 2023)

## 1192-55-30301

Liam Keenan*, University of Minnesota - Twin Cities. A chromatic vanishing result for topological restriction homology.
In joint work with Jonas McCandless, we established that topological restriction homology satisfies a chromatic vanishing result for connective $\mathbf{E}_{1}$-rings, identical to the one enjoyed by algebraic K-theory, which was established by Land-Mathew-Meier-Tamme. In this talk, I'll give a very brief introduction to the interaction of trace methods and chromatic homotopy theory and explain the proof of our vanishing result.
(Received September 09, 2023)

## 1192-55-30321

Vitaliy A Kurlin*, University of Liverpool (UK). Geometric Data Science: old challenges and new solutions.
Geometric Data Science studies moduli spaces of data objects up to important equivalences. The key example is a finite or periodic set of unordered points up to isometry preserving inter-point distances in a metric space. In the finite Euclidean case, a continuous and complete invariant of unordered point clouds under rigid motion and isometry is published in CVPR 2023. Periodic sets of points model all crystalline materials and cannot be reduced to finite clouds because the smallest pattern (a unit cell) of a periodic crystal is discontinuous under perturbations of atoms. This ambiguity was resolved by generically complete and continuous Pointwise Distance Distributions (PDD). The PDD has a near-linear time (ICML 2023) and was tested via $200+$ billion pairwise comparisons of all $660 \mathrm{~K}+$ periodic crystals in the world's largest collection of real materials. This experiment took only two days on a modest desktop and detected five pairs of geometric duplicates. In each pair, the crystals are truly isometric to each other but one atom is replaced with a different atom type, which seems physically impossible without perturbing geometry. The corresponding publications are being investigated by five journals for data integrity. The more important conclusion is the Crystal Isometry Principle meaning that all real periodic crystals have unique geographicstyle locations in a common continuous Crystal Isometry Space (NeurIPS 2022), see all papers at http://kurlin.org/research-papers.php\#Geometric-Data-Science
(Received September 10, 2023)

1192-55-30347
Morgan Peck Opie*, UCLA. Applications of higher real K-theory to classification of vector bundles. Preliminary report. I will discussion applications of higher real $K$-theories - homotopy fixed point spectra of Morava $E$-theories - the the study of vector bundles on projective spaces. The focus will be on how unstable bundles on complex projective spaces can be detected using higher real $K$-theories and alternative stabilization techniques, and on what the scope of these techniques might be. This report will include joint work with Yang Hu and Hood Chatham.
(Received September 09, 2023)

Morgan Peck Opie*, UCLA. Homotopical approaches to topological vector bundles on projective spaces.
Vector bundles are classical objects that naturally arise in geometry and topology. The stable classification of topological vector bundles is the study of topological $K$-theory, which is well-understood and computable in many cases. In seeking to answer questions beyond the scope of $K$-theory using the techniques of stable homotopy theory, more ingenuity is required. In this talk, I will discuss a few aproaches to alternative stabilization of vector bundle problems, focusing on classification problems for complex topological vector bundles on complex projective spaces.
(Received September 09, 2023)

## 1192-55-30374

Sucharita Mallick*, PhD student, University of Florida. Chromatic numbers of Bursuk graphs using anti-Vietoris-Rips thickenings. Preliminary report.
In this talk, I will introduce Borsuk graphs, (circular) chromatic numbers, and anti-Vietoris-Rips thickenings. I will discuss the homotopy types of the anti-VR thickenings built on $S^{n}$ in a range of scales. Using topological obstructions, I will show that for $k>n$, a graph homomorphisms from the Borsuk graph on $S^{k}$ to the Borsuk graph on $S^{n}$ can exist only if the scale for the latter Borsuk graph is sufficiently relaxed. Can similar techniques be used to provide new lower bounds on the chromatic number of Borsuk graphs? This is joint work with Henry Adams and Alex Elchesen.
(Received September 09, 2023)

## 1192-55-30393

Quincy Alston, University of Pennsylvania, Elise Alvarez-Salazar*, University of California Santa Barbara, Kiyanna Porter, University of Utah. Manifold Modeling of Pentagon Spaces Using Laplacian Eigenfunctions. Preliminary report.
The algorithmic sampling of conformation spaces is a problem with applications across computational chemistry and biology. Often these spaces represent the configurations of a molecule at different energies. We present an algorithm for sampling the moduli space of pentagons with mathematical justification. Furthermore, we use persistent cohomology and EilenbergMacLane spaces to identify the underlying manifold of said moduli spaces. We use said identification as well as eigenfunctions of the Laplacian to create best fit manifolds. In the process we recover a 2 D visualization of our space.The developed pipeline allows for the visualization of complicated high dimensional spaces in lower dimension. (Received September 09, 2023)

## 1192-55-30417

Kristen Mazur, Elon University, Angélica M Osorno, Reed College, Constanze Roitzheim, University of Kent, Rekha Santhanam, IIT Bombay, Danika Van Niel*, Michigan State University, Valentina Zapata Castro, University of Virginia. $C_{p^{n} q^{m}}$ compatible transfer systems. Preliminary report.
Transfer systems are combinatorial objects that encode information about equivariant operations. More precisely, a transfer system encodes the transfers (or wrong-way maps) carried by algebras over certain equivariant operads. Thus, transfer systems allow us to use combinatorial tools to study equivariant homotopy theory. Compatible pairs of transfer systems, which are a pair of transfer systems satisfying certain conditions, correspond to multiplicative structures compatible with an underlying additive structure. In particular, compatible pairs are closely related to bi-incomplete Tambara functors. In this talk we introduce transfer systems, compatible pairs, and discuss when a transfer system is only compatible with at most two other transfer systems. The work discussed in this talk began as a collaboration through the Women in Topology workshop and is joint with Kristen Mazur, Angélica M. Osorno, Constanze Roitzheim, Rekha Santhanam, and Valentina Zapata Castro. (Received September 10, 2023)

## 1192-55-30476

Christin Bibby, Louisiana State University, Nir Gadish*, University of Michigan. A Serre spectral sequence for moduli spaces of tropical curves.
The moduli space of genus $g$ tropical curves with $n$ marked points is a fascinating topological space, with a combinatorial flavor and deep algebro-geometric meaning. In the algebraic world, forgetting the n marked points gives a fibration whose fibers are configuration spaces of a surface, and Serre's spectral sequence lets one compute the cohomology "in principle". In joint work with Bibby, Chan and Yun, we construct a surprising tropical analog of this spectral sequence, manifesting as a graph complex and featuring the cohomology of compactified configuration spaces on graphs.
(Received September 10, 2023)

1192-55-30524
Kelly Spry Maggs, EPFL, Markus Kirolos Kirolos Youssef*, EPFL. Circular structures in high dimensional gene expression data. Preliminary report.
Single-cell RNA sequencing (sc-RNAseq) technology allows us to represent a tissue (e.g., a piece of skin) as a high-dimensional point cloud, comprising $n$ cells in a $g$-dimensional gene expression space. This point cloud characterizes the "state" of each cell, allowing us to establish a link between geometrical and biological concepts. Notably, circular patterns (i.e., non-trivial 1homologies) can be associated with "cyclic cellular programs" (CCPs) such as the cell cycle. In practice, we often deal with around 5,000 cells in a 20,000 -dimensional space taken from extremely noisy measurements, which renders the identification of circular structures a highly ill-posed problem. In this talk, we introduce a robust algorithm to identify circular patterns in lower-dimensional projections of this gene expression space, leveraging data diffusion and persistent homology. This approach allows us to connect geometrical and topological attributes to biological concepts. Specifically, we can associate 1-dimensional homology groups with the cyclic nature of a cellular program, and the circular coordinates of a homology group with the "intrinsic clock" of these programs. We will conclude with insights into potentially new discoveries in biology. (Received September 10, 2023)

Dmitriy Morozov*, Lawrence Berkeley National Laboratory, Amit Patel, Colorado State University. Output-sensitive Computation of Generalized Persistence Diagrams for 2-filtrations. Preliminary report.
When persistence diagrams are formalized as the Möbius inversion of the birth-death function, they naturally generalize to the multi-parameter setting and enjoy many of the key properties, such as stability, that we expect in applications. The direct definition in the 2-parameter setting, and the corresponding brute-force algorithm to compute them, require $\Omega\left(n^{4}\right)$ operations. But the size of the generalized persistence diagram, $C$, can be as low as linear (and as high as cubic). We elucidate a connection between the 2-parameter and the ordinary 1-parameter settings, which allows us to design an output-sensitive algorithm, whose running time is in $O\left(n^{3}+C n\right)$.
(Received September 10, 2023)

## 1192-55-30607

Dmitriy Morozov*, Lawrence Berkeley National Laboratory, Arnur Nigmetov, Lawrence Berkeley National Laboratory. Topological Optimization with Big Steps.
Using persistent homology to guide optimization has emerged as a novel application of topological data analysis. Existing methods treat persistence calculation as a black box and backpropagate gradients only onto the simplices involved in particular pairs. We show how the cycles and chains used in the persistence calculation can be used to prescribe gradients to larger subsets of the domain. In particular, we show that in a special case, which serves as a building block for general losses, the problem can be solved exactly in linear time. This relies on another contribution of this paper, which eliminates the need to examine a factorial number of permutations of simplices with the same value. We present empirical experiments that show the practical benefits of our algorithm: the number of steps required for the optimization is reduced by an order of magnitude. (Received September 10, 2023)

## 1192-55-30608

Robert W Ghrist*, University of Pennsylvania. Optimal Global Sections of Cellular Sheaves.
Cellular sheaves over a simplicial complex are an excellent data structure for systems with local constraints. Combining optimization problems over vertices with the sheaf as a constraint over simplices leads to what one could call "Homological Programming" problems. This talk will outline practical examples of such problems and detail a suite of methods for distributed solution to optimal global sections using a Hodge Laplacian.
(Received September 10, 2023)

1192-55-30705
Dhananjay Bhaskar*, Yale University, Feng Gao, Columbia University, Valentina Greco, Yale University, Firas Khasawneh, Michigan State University, Smita Krishnaswamy, Yale, Jessica Moore, Duke University, Elizabeth Munch, Michigan State University, Bastian Rieck, Technical University of Munich. Analyzing complex spatiotemporal patterns with geometric scattering trajectory homology.
Coordinated signaling mechanisms play a critical role in orchestrating the complex interactions and processes necessary for cells to perceive and respond to their ever-changing environment. However, the dynamics of signaling processes, including their initiation, propagation, termination, and adaptation, are not yet fully understood. To facilitate quantitative analysis of spatially and temporally coordinated signaling activity, we developed Geometric Scattering Trajectory Homology (GSTH), a general framework that integrates geometric scattering and topological data analysis (TDA) to capture local and global patterns in the spatial and temporal domains. We tested this framework using various computational models and experimental data, ranging from calcium signaling in the Drosophila wing imaginal disc to neuroimaging data from mice and human brains. Our findings demonstrate that the GSTH-generated trajectory is related to the degree of synchrony, speed, and quasiperiodicity of the underlying signaling pattern. Additionally, we recover model parameters and experimental conditions using GSTH, highlighting its potential as a signature of cellular and neuronal activity in response to various stimuli and drug treatments.
(Received September 10, 2023)

1192-55-30710
Gabriel Angelini-Knoll, Universite Sorbonne Paris Nord, Mona Merling, University of Pennsylvania, Maximilien Peroux*, Michigan State University. Homology of twisted G-rings. Preliminary report.
Topological Hochschild homology (THH) is an important variant for ring spectra. It is built as a geometric realization of a cyclic bar construction. It is endowed with an action of circle. This is because it is a geometric realization of a cyclic object. The simplex category factors through Connes' category $\Lambda$. Similarly, real topological Hochschild homology (THR) for ring spectra with anti-involution is endowed with a $O(2)$-action. Here instead of the cyclic category $\Lambda$, we use the dihedral category $\Xi$. From work in progress with Gabe Angelini-Knoll and Mona Merling, I present a generalization of $\Lambda$ and $\Xi$ called crossed simplicial groups, introduced by Fiedorwicz and Loday. To each crossed simplical group G, I define THG, an equivariant analogue of THH. Its input is a ring spectrum with a twisted group action. THG is an algebraic invariant endowed with different action and cyclotomic structure, and generalizes THH and THR.
(Received September 10, 2023)

1192-55-30933
Vin de Silva, Pomona College, Chad Giusti, University of Delaware, Vladimir Itskov, Pennsylvania State University, Michael Robinson, American University, Radmila Sazdanovic, NC State University, Nikolas Schonsheck, University of Delaware, Melvin Vaupel*, Norwegian University of Science and Technology, Iris H. R. Yoon, University of Delaware. Extending Dowkers theorem to multiway relations. Preliminary report.
Given a relation between two sets $I$ and $J$ one may form two simplicial complexes. The first one has elements of $I$ as vertices and the second one elements of $J$. That the realisations of these two complexes are always homotopy equivalent is known as Dowker's duality theorem. Its relevance for applications has recently lead to a surge of renewed interest. We discuss extensions of this classical construction to multiway relations between more than two sets. The available information may now be represented in many different ways. We prove that all so obtained structures realise to weakly equivalent spaces, thus
generalising Dowkers duality theorem to multiway relations. As an example consider a multiway relation between three sets $I$, $J$ and $K$. We may form a cuboid complex, three cosheaves of rectangle complexes and six cosheaves of Dowker complexes. All the respective projection maps are weak equivalences.
(Received September 11, 2023)

## 1192-55-30948

Daniel Berwick-Evans*, University of Illinois Urbana Champaign. Supersymmetric field theories and elliptic cohomology. Since the mid 1980s, there have been hints of a deep connection between 2-dimensional field theories and elliptic cohomology. This lead to Stolz and Teichner's conjectured geometric model for the universal elliptic cohomology theory of topological modular forms (TMF) in which cocycles are 2-dimensional supersymmetric field theories. Basic properties of these field theories lead to expected integrality and modularity properties, but the abundant torsion in TMF has always been mysterious. In this talk, I will describe deformation invariants of 2-dimensional field theories that realize some of the torsion in TMF. (Received September 11, 2023)

## 1192-55-30975

Nathaniel Clause, The Ohio State University, Woojin Kim*, Duke University, Facundo Mémoli, The Ohio State University. The discriminating power of the generalized rank invariant.
In topological data analysis, the rank invariant is one of the best known invariants of persistence modules over posets. The rank invariant of a persistence module $M$ over a given poset $P$ is defined as the map that sends each comparable pair $p \leq q$ in $P$ to the rank of the linear map $M(p \leq q)$. The recently introduced notion of generalized rank invariant acquires more discriminating power than the rank invariant at the expense of enlarging the domain of rank invariant to a collection $\mathcal{I}$ of intervals of $P$ that contains all segments of $P$. In this talk, we discuss the tension that exists between computational efficiency and the discriminating power of the generalized rank invariant, depending on its domain $\mathcal{I}$. The Möbius inversion formula will assume a significant role in clarifying the discriminating power, even in cases where the domain $\mathcal{I}$ is not locally finite. Along the way, we show that the possibility of encoding the generalized rank invariant of $M$ over a non-locally-finite $\mathcal{I}$ into a multiset of signed intervals of $P$ depends on how "tame" $M$ is. Such a multiset, if it exists, is obtained via Möbius inversion of the generalized rank invariant over a suitable locally finite subset of $\mathcal{I}$.
(Received September 11, 2023)

1192-55-31004
Manu Aggarwal*, National Institutes of Health, Vipul Periwal, National Institutes of Health. The functional significance of topological features from large-scale biological networks.
The arrangement of biomolecules in structures across different spatial scales is often critical for biological function. For example, effects of chromatin structure on gene expression. Persistent homology (PH) has been used to robustly determine topological features that are stable over multiple scales in noisy datasets. However, it has usually been applied to small datasets due to high computational cost. Further, to analyze functional significance of topological features, it is crucial to estimate their location with precision. However, representative boundaries of topological features are not unique and can be geometrically imprecise. We developed a novel algorithm that solves both the problems of scale and representation. It enabled us to study gene interactions in the human genome by analyzing genome-wide Hi-C frequencies at 1 kilobase resolution ( 3 million points). Additionally, we introduce an application of PH to find biologically significant structural features by incorporating user-defined rules in the homology computation. As a case study, we analyzed the arrangement of cells in human pancreatic islets. Our work enables applications of PH to relate data driven discovery of structural features with functional properties of underlying biological systems.
(Received September 11, 2023)

## 1192-55-31066

Russell Funk, University of Minnesota, Jingyi Guan*, Macalester College, Jason Owen-Smith, University of Michigan Adam Schroeder, Macalester College, Lori Ziegelmeier, Macalester College. Topological Data Analysis of Knowledge Networks. Preliminary report
Knowledge networks can organize complex data by constructing graphs where nodes are concepts or ideas and edges represent connections of significance. Understanding the structure of these knowledge networks to uncover how science progresses over time is of interest to researchers studying the "Science of Science." In this project, we are interested in understanding cycles or holes within a network, which can be thought of as gaps in knowledge. We use topological data analysis, and in particular, persistent homology filtered through time where the nodes represent scientific concepts and edges between two nodes are added at the time when they appear together in an abstract of a scientific paper. We study properties of these knowledge gaps in multiple dimensions such as when they form, when they no longer remain, and the concepts and papers that make up the cycles. We observe that papers involved in the knowledge gaps are cited more frequently than papers that are not.
(Received September 11, 2023)

## 1192-55-31179

David Chan*, Michigan State University. Compatible transfer systems.
Transfer systems of a finite group encode various flavors of additive and multiplicative structure which arise in equivariant algebra. For a fixed additive transfer system, a natural question to ask is which multiplicative transfer systems are compatible in the sense that there are appropriate distributive laws relating the additive and multiplicative operations. In this talk, we will discuss a combinatorial characterization of compatible pairs of transfer systems and implications for ring structures in equivariant algebra.
(Received September 11, 2023)

Emilie Purvine*, Pacific Northwest National Laboratory. Exploring Topological Features in Cyber Hypergraphs. Preliminary report.
Applied topology has found success in a variety of application areas. One such recent area is cyber security. Our team has been developing a novel framework based in the theory of hypergraphs and topology to understand cyber data through topological signatures, which are both flexible and can be traced back to the data. The complexity of cyber data manifests as multi-way relationships among individual entities being observed. For example, groups of ports can be related by virtue of the processes that utilize them; or groups of IPs can be related based on the protocol they employ for their various
communications. Hypergraphs are a mathematical model of multi-way relationships that allow for analysis of data using a higher level perspective. Implied by that perspective is the ability to perform topological analysis of cyber data which allows for the definition and identification of flexible and interpretable patterns of behavior. In this talk we present our framework for hypergraph and topological analysis of cyber log data, both theory and example, to demonstrate the value of applying methods from the burgeoning fields of hypernetwork science and applied topology to understand relationships among behaviors in cyber data.
(Received September 11, 2023)

## 1192-55-31205

Aziz Burak Gülen*, The Ohio State University. Orthogonal Möbius Inversion.
We introduce the notion of orthogonal Möbius inversion, which can be applied to functions that take inner product spaces as values. When applied to the birth-death spaces, or to the space of harmonic persistent cycles (i.e. the kernel of the persistent Laplacian), we prove that one produces canonical representatives for each bar in the barcode of a filtration. Furthermore, we establish that these representatives are stable with respect to a suitable notion of distance. (Received September 11, 2023)

## 1192-55-31242

Firas Khasawneh, Michigan State University, Elizabeth Munch, Michigan State University, Sarah Tymochko*, UCLA. Time Series Analysis using Zigzag Persistent Homology.
Topological data analysis (TDA) is a field with tools to quantify the shape of data in a manner that is concise and robust using concepts from algebraic topology. Persistent homology, one of the most popular tools in TDA, has proven useful in applications to time series data, quantifying features like periodicity. We consider an problem inspired by bifurcation detection in dynamical systems where a time series changes depending on some choice of input parameter. While standard persistent homology has been used in this type of setting, it usually requires analyzing a collection of persistence diagrams, which in turn drives up the computational cost. Using zigzag persistence, we can capture topological changes in the data in only one persistence diagram. From this zigzag persistence diagram, we can translate back to the original input system and identify parameter values where the time series behavior changed significantly. (Received September 11, 2023)

## 1192-55-31644

Ben Spitz*, University of California Los Angeles. Generalized Mackey and Tambara Functors.
In $G$-equivariant homotopy theory (for $G$ a finite group), the basic algebraic invariants of interest are $G$-Mackey and $G$ -
Tambara functors, generalizing abelian groups and commutative rings, respectively. "Generalizing" here means that these respective notions coincide when $G=1$. In this talk, I'll introduce more general definitions of "Mackey functor" and "Tambara functor" which can be interpreted in a wide variety of contexts. This subsumes the classical $G$-equivariant notions, as well as the more recent notion of motivic Mackey/Tambara functors introduced by Bachmann. The aim of this approach is to translate theorems between contexts, enriching the theory and providing cleaner proofs of essential facts. To this end, I'll discuss recent progress in boosting a foundational result about norms from equivariant algebra to this more general setting.
(Received September 11, 2023)

## 1192-55-31712

Thomas Brazelton*, University of Pennsylvania. Equivariant complex oriented cohomology.
In classical algebraic topology, characteristic classes for topological vector bundles can be decomposed and studied under the presence of a structured enough cohomology theory. Complex oriented cohomology theories, for example, support a theory of Chern classes which provide a toolkit for the study complex vector bundles. In this talk we will discuss how complex orientations behave in the equivariant setting, and how equivariant characteristic classes allow one to study equivariant vector bundles. We will provide specific applications to classical results in enumerative geometry.
(Received September 11, 2023)

## 1192-55-31734

Richard Wong*, UCLA. Endotrivial modules for groups with periodic cohomology via Galois descent. Preliminary report. Let $G$ be a finite group, and $k$ be a field of characteristic $p$, where $p$ divides the order of the group (i.e. the setting of modular representation theory). One object of interest is the stable module category of $G$, which has a homotopy-theoretic interpretation as a stable $\infty$-category. This allows us to use homotopy-theoretic methods to compute invariants of representation-theoretic significance. In this talk, we will demonstrate how to use Galois descent to compute the Picard spectrum of the stable module category for groups with periodic cohomology, like $S_{3}$ or $S L_{2}\left(\mathbb{F}_{3}\right.$. This provides new, homotopocal, insights into the classical computation of the group of endotrivial modules. (Received September 12, 2023)

## 1192-55-31745

Eva Belmont, Case Western Reserve University, J.D. Quigley, University of Virginia, Chase Vogeli*, Cornell University. Homological Stability for Equivariant Configuration Spaces.
In an open manifold, an unordered configuration of $n$ points can be extended to a configuration of $n+1$ points by "adding a
point near infinity." A classical result of McDuff and Segal shows this induces a phenomenon known as homological stability: after adding sufficiently many points, the low-dimensional homology groups of the unordered configuration space stabilize. In this talk, I'll discuss an equivariant analogue of this phenomenon for manifolds equipped with an action by a finite group. In an appropriate sense, the Bredon homology of the resulting equivariant configuration spaces exhibits stability with respect to "adding orbits at infinity."
(Received September 12, 2023 )

## 1192-55-31747

Richard Wong*, UCLA. The Tate spectral sequence and Endotrivial modules for groups with periodic cohomology. Preliminary report.
Let $G$ be a finite group, and $k$ be a field of characteristic $p$, where $p$ divides the order of the group (i.e. the setting of modular representation theory). One object of interest is the stable module category of $G$, which has a homotopy-theoretic interpretation as a stable $\infty$-category. This allows us to use homotopy-theoretic methods to compute invariants of representation-theoretic significance. In this talk, we will demonstrate how to use the Tate spectral sequence to compute the Picard spectrum of the stable module category for groups with periodic cohomology, like $S_{3}$ or $S L_{2}\left(\mathbb{F}_{3}\right.$. This provides new, homotopocal, insights into the classical computation of the group of endotrivial modules.
(Received September 12, 2023)

## 1192-55-31767

Christy Hazel*, Grinnell College. The cohomology of equivariant configuration spaces. Preliminary report.
Given a space X we can consider the configuration space of n distinct points from X . When X is a Euclidean space, the singular cohomology of these configuration spaces has rich structure. If we instead consider configurations of points in Grepresentations where G is a finite group, then the configuration space inherits an action of the group G. In this talk we'll review some of the classical computations by Arnold and Cohen to compute the singular cohomology, and then discuss new techniques used to compute the Bredon G-equivariant cohomology. This is joint work with Dan Dugger. (Received September 12, 2023)

## 1192-55-31808

Nikola Milicevic*, Pennsylvania State University, Nicholas A. Scoville, Ursinus College. Homotopy and singular homology groups of finite graphs from the closure space perspective. Preliminary report.
We verify analogues of classical results for higher homotopy groups and singular homology groups of (Čech) closure spaces. Closure spaces are a generalization of topological spaces that also include graphs and directed graphs and are thus a bridge that connects classical algebraic topology with the more applied side of topology, such as digital topology. More specifically, we show the existence of a long exact sequence for homotopy groups of pairs of closure spaces and that a weak homotopy equivalence induces isomorphisms for homology groups. Our main result is the construction of a weak homotopy equivalences between the geometric realizations of (directed) clique complexes and their underlying (directed) graphs. This implies that singular homology groups of finite graphs can be efficiently calculated from finite combinatorial structures, despite their associated chain groups being infinite dimensional. This work is similar to the work McCord did for finite topological spaces, but in the context of closure spaces. Our results also give a novel approach for studying (higher) homotopy groups of discrete mathematical structures such as digital images.
(Received September 12, 2023)

## 1192-55-31926

Mira Wattal*, Boston College. Morse-Smale functions and Morse-Smale categories: Recovering the Morse complex from a homotopy-theoretic construction.
Over the last several decades, homotopy theorists and low-dimensional topologists have worked independently and in tandem to develop constructions that better approximate geometric data. In this talk, we narrate a piece of these paralleling histories by way of the flow category $C_{f}$ associated to a Morse-Smale function $f: M \rightarrow \mathbb{R}$ defined on a closed Riemannian manifold. It turns out that this flow category is smooth, compact, Morse-Smale, and of finite type-in a homotopy-theoretic sense, that is. It meets the sufficient amount of (pointedly named) criteria necessary to define a functor whose geometric realization realizes the associated Morse complex of $f$.
(Received September 12, 2023)

## 1192-55-31976

Kyle M Ormsby*, Reed College. $N_{\infty}$ operads and the combinatorics of model structures.
Model structures underpin the modern enterprise of abstract homotopy theory and form presentations of $(\infty, 1)$-categories. Despite their fundamental nature, model structures have historically been studied en masse or applied in specific cases, and very little is known about the totality of model structures on a given (bicomplete) category. Homotopical combinatorics is an emerging field that remedies this situation by studying the enumerative combinatorics and structural properties of model structures on finite lattices. Specialized to a finite chain, we find rich connections with Catalan combinatorics, including (intervals in) the Tamari and Kreweras lattices. I will sketch homotopical combinatorics as it currently stands, including the surprising way in which the theory of equivariant $N_{\infty}$-operads has enabled recent advances.
(Received September 12, 2023)

1192-55-31989
Chad Giusti, University of Delaware, Gregory Henselman-Petrusek, University of Oxford, Lu Li, University of Pennsylvania, Connor Thompson, Purdue University, Lori Ziegelmeier*, Macalester College. Minimal Cycle Representatives in Persistent Homology using Linear Programming.
Cycle representatives of persistent homology classes can be used to provide descriptions of topological features in data. However, the non-uniqueness of these representatives creates ambiguity and can lead to many different interpretations of the
same set of classes. One approach to solving this problem is to optimize the choice of representative against some measure that is meaningful in the context of the data. In this work, we provide a study of the effectiveness and computational cost of several $\ell_{1}$-minimization optimization procedures for constructing homological cycle bases for persistent homology with rational coefficients in dimension one, including uniform-weighted and length-weighted edge-loss algorithms as well as uniform-weighted and area-weighted triangle-loss algorithms. We conduct these optimizations via standard linear programming methods, applying general-purpose solvers to optimize over column bases of simplicial boundary matrices. Our key findings are: (i) optimization is effective in reducing the size of cycle representatives, though the extent of the reduction varies according to the dimension and distribution of the underlying data, (ii) the computational cost of optimizing a basis of cycle representatives exceeds the cost of computing such a basis, in most data sets we consider, (iii) the choice of linear solvers matters a lot to the computation time of optimizing cycles, (iv) the computation time of solving an integer program is not significantly longer than the computation time of solving a linear program for most of the cycle representatives, using the Gurobi linear solver, (v) strikingly, whether requiring integer solutions or not, we almost always obtain a solution with the same cost and almost all solutions found have entries in $\{-1,0,1\}$ and therefore, are also solutions to a restricted $\ell_{0}$ optimization problem, and (vi) we obtain qualitatively different results for generators in Erdős-Renyi random clique complexes than in real-world and synthetic point cloud data.
(Received September 12, 2023)

## 1192-55-32034

Mathieu John Yves Chabaud*, University of Washington, Sean Hadley, San Francisco State University, Solís McClain, Reed College. Circular Coordinates for Non-Uniform Distributions: Introducing Weights to Nonlinear Topological Dimensionality Reduction. Preliminary report.
Our work examines and improves a topological dimensionality reduction algorithm which constructs an explicit formula for circle-valued functions on data with persistent $H^{1}$ features. This algorithm, developed by Dr. Jose A. Perea, is reflected in the Python library DREiMac or Dimension Reduction with Eilenberg-MacLane Coordinates, yet presently this code assigns near constant circular coordinates to datasets with nonuniform density. We investigate solutions to this problem involving weights on landmark data points, harmonic cocycle representatives, and coverings in sparsely populated areas such that circular coordinates are appropriately assigned. Finally, we explore alternative routes to further improve the circular coordinates algorithm, including using a different metric space based on the relative density of data. Our work utilizes a geometric explanation for the linear algebra contained within the algorithm, as well as synthetic datasets to demonstrate the utility of this approach. These findings highlight obstacles in the current DREiMac algorithm, and provide valuable insights for enhancing dimensionality reduction techniques for non-uniform data.
(Received September 12, 2023)

## 1192-55-32043

Chad Giusti*, University of Delaware, Nikolas Schonsheck, University of Delaware. Learning to represent topological information in feed-forward neural networks.
The stimulus space model for neural population activity describes the firing rates of neurons using the structure of a metric "stimulus" space. In this model, each neuron corresponds to a point in the stimulus space, as do individual stimuli, and the firing rate of a neuron in response to a stimulus is given by a monotonically decreasing function of the distance in between the corresponding points. Under this model, which has been experimentally observed in a variety of both real and artificial neural networks, we can infer topological structure in the stimulus space from observed activity using the nerve theorem and persistent homology. Applying the method of analogous cycles developed in (Yoon, Ghrist, and Giusti 2023), we can further compare such structure across neural populations to understand how topological information propagates through neural networks. In this work, we utilize these tools to investigate how the connectivity of neural networks admits or inhibits the transmission of topological structure between neural populations, and how population size modulates learning of these structures.
(Received September 12, 2023)

## 1192-55-32058

Matthias Kawski*, Arizona State University, Kim Klinger-Logan, Kansas State University, Shay Allen Logan, Kansas State University. From PVC pipes and couplings to algebraic topology.
PVC pipes and couplings are inexpensive and widely available for irrigation, plumbing, to build shade structures and even furniture. In an interactive hands-on session, we demonstrate how to engage students (and teachers) to build simple and more complicated (compact) "tubular" surfaces and to investigate their topological properties and invariants, with primary focus on the Euler characteristic. These tubular surfaces are closely related to finite graphs that are imbedded in 3-space. A wonderful initial example is the surface built from tubes about the edges of a cube in 3 -space. Its genus is 5 , not 6 . Continuous deformations such as "unfolding" preserve many features and make a connection to the Euler characteristics of planar graphs. However, this does not work for tubular surfaces made up of tubes about imbeddings of (subdivisions of) the complete graph $K_{5}$ on 5 vertices, or the of (subdivisions of) the bipartite graph $K_{3,3}$, as these graphs are not planar by Kuratowski's Theorem. As icing on the cake, these surfaces made of PVC pipes and couplings, even provide very simple verifications of the GaussBonnet theorem which relates the geometric notion of curvature to the topological concept of genus. This activity was premiered at the 2023 Dine (Navajo) Math Summer Camp with both middle and high school aged students, and also at a teachers workshop. While easily accessible to school students, this activity beautifully leads to deeper explorations at the level of 4th year university level courses (graph theory, topology, differential geometry), and further to problems in algebraic topology to be explored.
(Received September 12, 2023)

## 1192-55-32101

Miguel O'Malley*, ScaDS.AI. Methods in clustering utilizing (alpha) magnitude.. Preliminary report.
Alpha magnitude is a new invariant utilizing the alpha complex and persistent magnitude (due to Govc and Hepworth) to derive valuable information from point cloud data. Strong connections exist between alpha magnitude and magnitude, an existing invariant introduced by Tom Leinster which evaluates the effective diversity of a metric space. Both alpha magnitude and magnitude strongly encode the extent to which a point cloud is clustered, but as yet this behavior has yet to be leveraged.

In this talk, we will cover new methods utilizing both invariants to derive and refine clustering results for point cloud data. (Received September 12, 2023)

## 1192-55-32181

Juan C. Moreno*, University of Colorado Boulder. The Spanier-Whitehead duals of some higher real K-theory spectra. Preliminary report.
Higher real K-theory spectra are spectra obtained from Morava E-theory by taking the homotopy fixed points with respect to finite subgroups of the Morava stabilizer group. They are central objects in chromatic homotopy theory. Work of Beaudry-Goerss-Hopkins-Stojanoska shows that their K(n)-local Spanier-Whitehead duals can be obtained by suspending by a certain representation sphere before taking homotopy fixed points. We identify the dualizing representation in some cases, then focus on some examples in which the result can be simplified further to an integer shift.
(Received September 12, 2023)

## 1192-55-32192

Keith Mills*, University of Maryland, College Park. The Structure of the Spin ${ }^{h}$ Bordism Spectrum.
Spin ${ }^{h}$ manifolds are the quaternionic analogue to spin ${ }^{c}$ manifolds. We compute the spin ${ }^{h}$ bordism groups at the prime 2 by proving a structure theorem for the cohomology of the spin ${ }^{h}$ bordism spectrum MSpin ${ }^{h}$ as a module over the mod 2 Steenrod algebra. This provides a 2-local splitting of MSpin ${ }^{h}$ as a wedge sum of familiar spectra. We also compute the decomposition of $H^{*}\left(\mathrm{MSpin}^{h} ; \mathbb{Z} / 2 \mathbb{Z}\right)$ explicitly in degrees up through 30 via a counting process. (Received September 12, 2023)

## 1192-55-32212

Juan C. Moreno*, University of Colorado Boulder. An inductive approach to quotients of norms of Real bordism. Preliminary report.
As part of their solution to the Kervaire invariant one problem, Hill-Hopkins-Ravenel introduced the theories $B P{ }^{((G))}\langle m\rangle$.
These theories are connective models of Lubin-Tate theories together with the action of a finite subgroup of the Morava Stabilizer group. In this talk we will outline an inductive approach to computing their homotopy. At the heart of this approach is a Bockstein-like spectral sequence computing $B P^{((G))}\langle m\rangle$ starting from knowledge of $B P^{((G))}\langle m-1\rangle$.
(Received September 12, 2023)

## 1192-55-32249

Shreya Arya, Duke University, Barbara Giunti, SUNY Albany, Abigail Hickok*, Columbia University, Lida Kanari, EPFL, Sarah McGuire, Michigan State University, Katharine Turner, Australian National University. A Geometric Interpretation of Monodromy in the Persistent Homology Transform. Preliminary report.
In earlier work, I introduced the concept of a persistence diagram (PD) bundle, a framework developed for analyzing the topological features of a data set that is parameterized by some "base space" B (e.g., n-dimensional Euclidean space or a manifold). More specifically, a PD bundle is the space of PDs for a set of filtrations parameterized by B. Important special cases of PD bundles include vineyards, where $B=\mathbb{R}$, and the persistent homology transform (PHT), where $B=S^{n}$. Unlike a vineyard, it is possible for a PD bundle to have "monodromy"- that is, the "total space" $E$ of PDs may not decompose into the union of "global sections" $B \rightarrow E$. In this talk, I will discuss our result that if a shape $M$ is star-shaped, then there is no monodromy in the PHT of $M$, thus proving a connection between the geometry of $M$ and the geometry of the PD bundle $\mathrm{PHT}(M)$. We also show that monodromy can and does occur for other shapes. Lastly, we will discuss the algebraic decomposition of the " $S^{n}$-persistence modules" (PD bundles with additional algebraic structure) that arise from PHT. (Received September 12, 2023)

## 1192-55-32354

Ángel Javier Alonso, TU Graz, Michael Kerber, TU Graz, Tung Lam, University at Albany, SUNY, Michael Lesnick*, University at Albany, SUNY. Delaunay Bifiltrations of Functions on Point Clouds. The Delaunay filtration $\mathcal{D}(X)$ of a point cloud $X \subset \mathbb{R}^{d}$ is a central tool of topological data analysis. Its use is justified by the topological equivalence of $\mathcal{D}(X)$ and the offset (i.e., union-of-balls) filtration of $X$. Given a function $\gamma: X \rightarrow \mathbb{R}$, we introduce a Delaunay bifiltration $\mathcal{D C}(\gamma)$ which satisfies an analogous topological equivalence, ensuring that $\mathcal{D C}(\gamma)$ topologically encodes the offset filtrations of all sublevel sets of $\gamma$, as well as the topological relations between them. $\mathcal{D C}(\gamma)$ is of size $O\left(|X|^{\left.\frac{d+1}{2}\right\rceil}\right)$, which for $d$ odd matches the worst-case size of $\mathcal{D}(X)$. Adapting the Bowyer-Watson algorithm for computing Delaunay triangulations, we give a simple, practical algorithm to compute $\mathcal{D} \mathcal{C}(\gamma)$. Our implementation, based on CGAL, computes $\mathcal{D C}(\gamma)$ with modest overhead compared to computing $\mathcal{D}(X)$, and handles tens of thousands of points in $\mathbb{R}^{3}$ within seconds. (Received September 12, 2023)

## 1192-55-32397

Conrad Plaut*, University of Tennessee. Dicing the Shadow. Preliminary report.
I introduce a process called "dicing" that subdivides a finite union of cells (polytopes) in a Euclidean space into a polyhedral complex. I apply this process to the "shadow" of a Euclidean data set, namely the image of the Vietoris-Rips (VR-)complex with scale s under the natural map induced by the vertices. A cell in the shadow is the convex hull of a maximal set in the data of diameter at most s, and a primary intersection between cells is one that is contained in a face of each. After dicing the shadow, one can create a new complex by gluing together the diced cells along their primary intersections. This newly assembled complex has dimension at most equal to the dimension of the Euclidean space in which the original data lies, generally much lower than the dimension of the VR-complex. This space also has the structure of a Euclidean polyhedral complex with its natural geodesic metric. Chambers, de Silva, Erickson and Ghrist showed in 2010 that the shadow need not have the same
homotopy type as the VR-complex, but for their hexagon example, the complex that I construct is simply a subdivision of the VR-complex, and hence is homeomorphic to it. I will describe the construction in more detail and state what I know about its relationship to the VR-complex
(Received September 12, 2023)

## 1192-55-32556

Katharine Adamyk*, Hamline University. Stability for Compressions of Multi-Parameter Clusterings. Preliminary report. Computing persistent 0-th homology for a Vietoris-Rips complex produces a hierarchical clustering-essentially, the information presented in a barcode, plus a record of which data point lies in which cluster and which clusters are related to each other through merges. In this talk, we will discuss results that hold for hierarchical clusterings more generally, with persistent 0 -th homology as a motivating example. Layers partition information about a hierarchical clustering into sets of parameter values pertaining to the same cluster. Various types of layer points can be used to identify when a cluster first appears, either along a single-parameter path, or in a more global way. The purpose of layer points is to compress the information required to describe a hierarchical clustering by recording only these initial occurrences of clusters. For layer points to be useful, similar data sets should yield similar layer point structures-i.e., layer points should be stable. In this talk, we will present stability theorems for several versions of layer points (including stability with respect to subsampling) and discuss criteria for "good" compressions of hierarchical clusterings. (Received September 12, 2023)

## 1192-55-32626

Jerome Roehm*, Doane University. Anti-geometric Persistence - Theory and Applications.
The input to many persistent homology computation algorithms is a square symmetric dissimilarity matrix - a generalization of a distance matrix, where the rows and columns correspond to points in space and the matrix entries describe the dissimilarity between the points. However, many data analysis pipelines such as correlation of time series produce similarity measures rather than dissimilarity measures. If a transformation from similarity to dissimilarity is not performed before computing the diagram, the resulting persistence diagram is often wildly different from the expectation. In this talk, I present theoretical results concerning "anti-geometric" or "reverse" persistence diagrams and discuss applications of this tool. (Received September 12, 2023)

## 1192-55-32639

Chad Giusti, University of Delaware, Melinda Kleczynski*, National Institute of Standards and Technology. Topological Descriptors of Plant-Pollinator Communities. Preliminary report.
Healthy plant-pollinator communities are critical for biodiversity and agriculture. A plant-pollinator community consists of a set of floral resources, a set of animals providing pollination services, and a set of plant-pollinator interactions. From datasets of observed interactions, we generate a sequence of resource complexes. We utilize the resulting persistent homology to obtain a topological summary of resource use in the community. This provides a method for monitoring structural changes in the face of external stressors such as extreme heat. We demonstrate the utility of these techniques by analyzing an extensive plant-pollinator dataset.
(Received September 12, 2023)

1192-55-32744
Nathaniel Clause*, The Ohio State University. Meta-Diagrams for 2-Parameter Persistence and Applications. In this talk, we introduce the notion of meta-rank for a 2 -parameter persistence module, an invariant that captures the information behind images of morphisms between 1D slices of the module. We then define the meta-diagram of a 2-parameter persistence module to be the Möbius inversion of the meta-rank, resulting in a function that takes values from signed 1parameter persistence modules. We show that the meta-rank and meta-diagram contain information equivalent to the rank invariant and the signed barcode. This equivalence leads to computational benefits, as we introduce an algorithm for computing the meta-rank and meta-diagram of a 2-parameter module $M$ indexed by a bifiltration of $n$ simplices in $O\left(n^{3}\right)$ time. This implies an improvement upon the existing algorithm for computing the signed barcode, which has $O\left(n^{4}\right)$ time complexity. Further, we demonstrate an intuitive visualization of meta-diagrams as a persistence diagram of diagrams, which generalizes the well-understood persistence diagram in the 1-parameter setting. To conclude, we overview an application of metadiagrams arising when viewing other applicable functors as 2-parameter persistence modules. This talk is based on joint work with Tamal K. Dey, Woojin Kim, Facundo Mémoli, and Bei Wang. (Received September 12, 2023)

## 1192-55-32969

Herng Yi Cheng*, University of Toronto. The Geometry of Steenrod Squares. Preliminary report.
The Steenrod algebra is central to computing the cohomology of Eilenberg-Maclane spaces. It also plays an important role in the construction of Whitehead towers and their analogues. We will present a study of the geometry of the Steenrod Squares, with an eye to applications in quantitative topology.
(Received September 13, 2023)

## 1192-55-32976

Hannah Housden*, Vanderbilt University. Orbits and Model Structures in Diagram Equivariance.
The notion of a continuous $G$-action on a topological space readily generalizes to that of a continuous $D$-action, where $D$ is any small category. The key to understanding $D$-actions is via " $D$-orbits," a generalization of $G$-sets of the form $G / H$. In this talk, we give a quick overview of the theory of $D$-orbits and discuss how they encode the homotopical data of $D$-spaces. This is made precise via the Quillen equivalence $\operatorname{Top}^{D} \cong \operatorname{Top}^{\operatorname{Orb}(D)^{o p}}$. Time permitting, we will then briefly discuss how to use $D$ orbits to define stable $D$-actions and $D$-spectra.

Rowan J Barker-Clarke*, Theory Division, Cleveland Clinic Lerner Institute, Cleveland, OH. Topology and robustness in the fully enumerated HP $6 \times 6$ and HP $3 \times 3 \times 3$ protein genotype-phenotype maps.. Preliminary report.
Evolution can be broadly described in terms of mutations of the genotype and the subsequent selection of the phenotype. The full enumeration of a given genotype-phenotype (GP) map is, therefore, a powerful technique for examining the existing evolutionary bias within phenotypic space. The topology of these spaces can also be useful in determining the nature of evolution, robustness and evolvability. However, because the number of genotypes typically grows exponentially with genome length, such calculations rapidly become intractable. Here I apply graphics processing unit (GPU) techniques to the simplistic hydrophobic-polar (HP) model for protein folding. The HP model is a simple and well-studied model for the complex process of protein folding and involves the on-lattice folding of proteins. The folds are calculated by enumerating all Hamiltonian paths on the lattice. The phenotypes (folds) are calculated by finding the ground state fold for protein sequences that consist of residues labeled as either hydrophobic or polar. Prior studies on relatively small 2D and 3D lattices have been exclusively carried out using conventional central processing unit (CPU) approaches. By using GPU techniques, we were able to reproduce prior calculations (Li et al., 1996) with a speedup of 580-700 fold over a CPU. I was also able to perform the first full enumeration of this model on the $6 \times 6$ and $3 \times 3 \times 3$ lattices. The enumeration also allowed us to assess the evolvability of proteins within these models, and their mutational robustness, and to question the topology of the HP model's evolutionary space.
(Received September 13, 2023)

## 1192-55-33192

Millie Rose*, University of Kentucky. Bialgebras, Partitions, and Class Functions. Preliminary report.
I will introduce partition functors as an intermediate between bialgebras and global Mackey functors. Partition functors capture the combinatorics of global functors applied to wreath products with symmetric groups and have a close relationship to power operations. In this language, I will describe a kind of "class functions" for partition functors along with a universal property and rational isomorphism. When applied to the representation theory of symmetric groups, this recovers the usual integer valued class functions and a universal property for them.
(Received September 13, 2023)

## 1192-55-33362

Chad Giusti, University of Delaware, Gregory Henselman-Petrusek*, University of Oxford, Lori Ziegelmeier, Macalester College. Open Applied Topology: A fast, flexible, user-friendly tool for matrix algebra in TDA. Preliminary report. Many problems in TDA have solutions in homological - and indeed, linear - algebra. However, it's challenging to harness these solutions, computationally. Matrices are often large, having millions or billions of rows and columns. They are indexed by simplices, rather than integers. They have coefficients in abstract fields and require exact numerical accuracy. They have unusual sparsity patterns. This talk will introduce an open-source library to address some of these problems, Open Applied Topology (OAT). OAT is a high-performance linear algebra solver with a user-friendly front end. It allows the user to perform mathematical operations including matrix/vector addition, multiplication, and factorization ( $\mathrm{R}=\mathrm{DV}, \mathrm{RU}=\mathrm{D}$, U-match), and to compute persistence diagrams, barcodes, (optimal) (co)cycle representatives, and induced maps. Users can easily link the library to new types of chain complexes (simplicial, cubical, etc.), and to Python libraries such as SciPy. In sum, OAT is a userfriendly tool for matrix algebra in TDA. If time remains, we will examine broader trends in TDA software development across the research landscape (government, private, academic), and opportunities for future growth and cooperation. (Received September 14, 2023)

## 1192-55-33364

Richard Kyung*, CRG-NJ, Zimo Li, United World College of South East Asia. Study on Topological and Spectral Characteristics of Networks Using Gershgorin Circle Theorem and Computational Simulations.
In graph theory, the adjacency matrix or Laplacian matrix has eigenvalues that help provide insights into the topological and structural properties of the graph or network. The largest eigenvalue of the adjacency matrix refers to the network's connectivity, and the spectrum of the Laplacian matrix is linked to properties such as the number of connected components and clustering coefficients. In this paper, the Gershgorin Circle Theorem was used to gain a deeper understanding of the topological features and behaviors of complex networks. The locations of eigenvalues of different matrices associated with various networks and structural properties, such as the number of connected components and graph diameter, were found. Bounding the eigenvalues within some areas of the complex plane, the spectrum of the matrix and the network's properties, such as structural characteristics that describe the shape and topology of the network, were found by extension. (Received September 14, 2023)

## 1192-55-33646

Yuyuan Luo*, Massachusetts Institute of Technology. On the classification of rank-2 vector bundles. Preliminary report. We describe a systematic approach to find the classification of rank 2 vector bundles using Postnikov tower arguments, and demonstrate this approach to find such a classification on $\mathbb{C} P^{3}$. We begin with an expository on Postnikov towers, then we describe the general technique for computing this, and we proceed with the computation of the Postnikov tower in low degrees for $B U(2)$. Following this, we describe the general method of using the Postnikov tower of the classifying space to compute the classification of bundles. Finally, we present progress on applying this method to classify rank- 2 bundles over $\mathbb{C} P^{3}$. (Received September 22, 2023)

The goal was to find similarities between songs which would otherwise seem dissimilar. For this work, we chose the coding language python, and we utilized python's music21 package to handle translating music files into data usable for our analysis. Topological data analysis techniques were then applied to that data to arrive at persistence diagrams for each song. Following that, we grouped songs together and analyzed them by using their persistence diagrams to compute the bottleneck distances between the songs allowing dendrograms to be created. Finally, we evaluated the similarity of the songs using those dendrograms.
(Received September 25, 2023)

## 1192-55-33781

Gregory Henselman-Petrusek, University of Oxford, Christian Joseph Lentz*, Macalester College, Xintan Xia, Macalester College, Lori Ziegelmeier, Macalester College. A computational approach for persistent relative homology. Preliminary report.
A central problem in data-driven scientific inquiry is how to interpret structures in large data sets uncovered by modern tools. The field of topological data analysis (TDA) provides a potential solution via the language of homology, which encodes features of interest as holes. A particularly important task in TDA is to compute topological holes in a dataset and identify the original data generating them. Recently new matrix factorizations have been proposed to place this calculation in a more flexible form, which relates the standard persistence calculation to several variants. We will discuss one such technique, which puts the boundary operators of a chain complex into a form where bases have been matched, making extraction of generators expeditious. We discuss this technique in the context of persistent homology and extend it to an algorithm for persistent relative homology, which can identify features relative to some subset of the original data. (Received September 25, 2023)

## 1192-55-33835

Jingyi Guan*, Macalester College. Topological Data Analysis of Knowledge Networks.. Preliminary report.
Knowledge networks can organize complex data by constructing graphs where nodes are concepts or ideas and edges represent connections of significance. Understanding the structure of these knowledge networks to uncover how science progresses over time is of interest to researchers studying the "Science of Science." In this project, we are interested in understanding cycles or holes within a network, which can be thought of as gaps in knowledge. We use topological data analysis, and in particular, persistent homology filtered through time where the nodes represent scientific concepts and edges between two nodes are added at the time when they appear together in an abstract of a scientific paper. We study properties of these knowledge gaps in multiple dimensions such as when they form, when they no longer remain, and the concepts and papers that make up the cycles. We observe that papers involved in the knowledge gaps are cited more frequently than papers that are not.
(Received September 26, 2023)

1192-55-33884
Khush Agrawal*, Grove City College, Hannah Proctor*, Grove City College. Using TDA to Analyze the Swimming Movement of Fish.
In the children's movie Finding Nemo, Dori (a rather forgetful fish) imparts the inspiring words of wisdom - "Just keep swimming!" For the past several years, Dr. Erik Anderson, a professor of Mechanical Engineering at Grove City College, has put her catch phrase to the test with the help of a specially crafted flow tank at Woods Hole Oceanographic Institute. This research project centers around the question: does the tailbeat of fish exhibit a resonance allowing the fish to find a tail beat frequency that achieves a higher efficiency? By using a camera taking repeated pictures beneath the fish swimming in the flow tank, both the position of the fish and its shape in each frame are recorded. Using python, we processed this data and created animations that mirror the original fish movement from the original photos. Inspired by recent work concerning the motion of C. elegans, we used the tools of topological data analysis to generate persistence diagrams and compared the results of three differently sized fish of the same species. Ultimately, the goal of our research is to analyze differences of these diagrams and to identify hidden patterns that lie within the data to inform Dr. Anderson's research.
(Received September 27, 2023)

## 57 Manifolds and cell complexes

## 1192-57-25528

Ethan Dlugie*, UC Berkeley. The Burau representation and shapes of polyhedra.
The Burau representation of braid groups has been around for almost a century. Still we don't know the full answer to whether this representation is faithful. The only remaining case is for the $n=4$ braid group, and faithfulness here has intimate connections to the question of whether the Jones polynomial detects the unknot. We are able to identify evaluations of the Burau representation at roots of unity with monodromy representations of complex hyperbolic structures on moduli spaces of flat cone metrics on spheres explored by W. Thurston. Leveraging this connection, I will explain how one can identify the kernels of some of these evaluated representations as normal subgroups generated by appropriate powers of Dehn twists. This places strong restrictions on the kernel of the $n=4$ Burau representation.
(Received June 19, 2023)

George Domat, Rice University, Hannah Lynn Hoganson, University of Maryland, Sanghoon Kwak*, University of Utah. Mapping class groups of Infinite graphs - "Big $\operatorname{Out}\left(F_{n}\right)$ ".
Surfaces and graphs are closely related; there are many parallels between the mapping class groups of finite-type surfaces and finite graphs, where the mapping class group of a finite graph is the outer automorphism group of a free group of (finite) rank. A recent surge of interest in infinite-type surfaces and their mapping class groups begs a natural question: What is the mapping class group of an "infinite" graph? In this talk, I will explain the answer given by Algom-Kfir and Bestvina, and present recent work, joint with George Domat and Hannah Hoganson, on the coarse geometry of such groups.
(Received July 21, 2023)

## 1192-57-26651

Nicholas Cazet*, UC Davis. The length of quandle 3-cocycles. Preliminary report.
Shin Satoh and Akiko Shima '03 used the quandle 3-cocycle invariant to calculate the triple point number of the 2-twist-spun trefoil. They showed that Mochizuki's 3-cocycle of the order 3 dihedral quandle has a length of 4 , meaning the quandle 3 cocycle invariant using this cocycle is trivial for any surface-link whose triple point number is less than 4 . Hatakenaka ' 04 and Satoh '16 calculated the length of Mochikzuki's 3-cocycle of the order 5 dihedral quandle and showed that the triple point number of the 2 -twist-spun figure-eight is 8 . I will show that their method generalizes to many more cocycles and that any such cocycle has length at least 3 . This also allows for a simple method of finding 3-cocycles.
(Received August 02, 2023)

## 1192-57-27701

Sergei Gukov*, California Institute of Technology. Quantum groups at generic $q$ and non-semisimple TQFTs.
I will review the status - theorems and conjectures - of the 3d TQFT associated to quantum groups at generic values of the parameter $q$ that assigns a $q$-series convergent in $|q|<1$ to every closed oriented 3-manifold equipped with a choice of Spin ${ }^{c}$ structure. Specializing $q$ to roots of unity recovers the familiar non-semisimple TQFT due to Blanchet, Costantino, Geer and Patureau-Mirand. While the original motivation for studying this TQFT is rooted in categorification of Reshetikhin-Turaev invariants, it exhibits a number of unexpected properties and connections to other areas of mathematics. For instance, $q$-series invariants turn out to be characters of logarithmic vertex algebras. This aspect will be discussed in a companion talk at the AMS Special Session SS40A on Mock Modular forms, Physics, and Applications.
(Received August 18, 2023)

## 1192-57-27707

Renaud Detcherry*, University of Burgundy. Kauffman bracket skein module of small 3-manifolds.
Combining the work of Bonahon-Wong and Frohman-Kania-Bartoszyǹska-Lê with arguments from algebraic geometry, we develop a novel method for computing the Kauffman bracket skein modules of closed 3-manifolds with coefficients in $\mathbb{Q}(A)$. We show that if the skein module $S\left(M, \mathbb{Q}\left[A^{ \pm 1}\right]\right)$ of $M$ is finitely generated over $\mathbb{Q}\left[A^{ \pm 1}\right]$ and the $\mathrm{SL}(2, \mathbb{C})$-character variety is reduced, then $\operatorname{dim}_{\mathbb{Q}(A)} S(M, \mathbb{Q}(A))$ is the number of closed points in the $\mathrm{SL}(2, \mathbb{C})$-character variety. As an application, we compute skein modules of Dehn fillings of the figure-eight knot and of $(2,2 n+1)$-torus knots. (Received August 18, 2023)

## 1192-57-27709

Nick Salter*, University of Notre Dame. The equicritical stratification and stratified braid groups.
One of the many guises of the braid group is as the fundamental group of the space of monic squarefree polynomials. From this point of view, there is a natural "equicritical stratification" according to the multiplicities of the critical points. These equicritical strata form a natural and rich class of spaces at the intersection of algebraic geometry, topology, and geometric group theory, and can be studied from many different points of view; their fundamental groups ("stratified braid groups") look to be interesting cousins of the classical braid groups. I will describe some of my work on this topic thus far, which includes a partial description of the relationship between stratified and classical braid groups, and some progress towards showing that the equicritical strata are $K(\pi, 1)$ spaces.
(Received August 18, 2023)

## 1192-57-27923

Nathan Mankovich*, Colorado State University. Averaging and Dimensionality Reduction Using Flag Manifolds. Preliminary report.
In this talk we will introduce Grassmann and flag manifolds, robust averaging in these spaces, and finally discuss relationship between these tools Principal Component Analysis (PCA). Grassmann manifolds (a.k.a. Grassmannians) are parameterized by the set of subspaces of a vector space. Flag manifolds, on the other hand, generalize Grassmannians because they are parameterized by the set of nested sequences of a vector space. These manifolds have broad applications in computer vision including interpretable and robust dimensionality reduction, clustering, and other basic tasks. This presentation will highlight the utility of robust averaging on Grassmannians and generalize these averages to flag manifolds. Then we will show how subspace averaging and dimensionality reduction are closely related. In fact, we will use the nested structure of Flag manifolds as a lens for understanding simple dimensionality reduction algorithms like PCA.
(Received August 22, 2023)

## 1192-57-27974

Connor W Malin*, University of Notre Dame. Homotopy invariance results for configuration spaces.
The Fulton-MacPherson compactifications of configuration spaces assemble into an operad. We discuss how the geometry of the Fulton-MacPherson compactifications influences the homotopy type of the configuration spaces of a framed manifold. In particular, we discuss application to the Browder bracket on configuration spaces.
(Received August 23, 2023)

Connor W Malin*, University of Notre Dame. Configuration spaces with labels in a spectral Lie algebra. Preliminary report. We recount the recent history of spectral Lie algebras and their application to configuration spaces of manifolds. We describe a new approach to studying the factorization homology of of the higher enveloping algebras of spectral Lie algebras which
uses the Koszul self duality of the $E_{n}$ operad.
(Received August 23, 2023)

1192-57-28131
Sujoy Mukherjee, University of Denver, Daniel Scofield*, Francis Marion University. Predicting self-distributive algebraic structures through machine learning. Preliminary report.
Multiplication tables for algebraic structures, such as shelves and racks, are easily verified but expensive to enumerate. We show that a machine learning algorithm, given partially completed tables as prompts, can predict valid examples of shelves with a high degree of accuracy.
(Received August 26, 2023)

## 1192-57-28132

Geunyoung Kim*, University of Georgia. $n$-knots in $S^{n} \times S^{2}$ and contractible $(n+3)$-manifolds.
In 1961, Mazur constructed a contractible, compact, smooth 4-manifold with boundary which is not homeomorphic to the standard 4 -ball, using a 0 -handle, a 1 -handle and a 2 -handle. In this talk, for any integer $n \geq 2$, we construct a contractible, compact, smooth $(n+3)$-manifold with boundary which is not homeomorphic to the standard ( $n+3$ )-ball, using a 0 -handle, an $n$-handle and an $(n+1)$-handle. The key step is the construction of an interesting knotted $n$-sphere in $S^{n} \times S^{2}$
generalizing the Mazur pattern.
(Received August 26, 2023)

## 1192-57-28292

Jordan Incledon*, California State University, Fullerton, Matt Rathbun, California State University, Fullerton. Hopf plumbings and generalized Hopf bandings.
Fibered links are a special class of links that admit a particular kind of decomposition of the complement of the link into fiber surfaces. Hopf plumbings and generalized Hopf bandings are two operations on fiber surfaces that preserve fiberability. In our efforts to understand generalized Hopf bandings as Hopf plumbings, we found that performing a pair of specific Hopf plumbings results in the same surface as performing a pair of specific generalized Hopf bandings. We develop a relation in the mapping class group coming from the way the monodromies change via these two operations.
(Received August 29, 2023)

1192-57-28349
Priyam Patel*, University of Utah, Samuel J. Taylor, Temple University. Mapping class groups of infinite-type surfaces and their actions on hyperbolic graphs.
Given a finite-type surface, there are two important objects naturally associated to it. The first is the mapping class group and the second is the curve graph, which the mapping class group acts on via isometries. This action is well understood and has been extremely useful in understanding the algebraic and geometric properties of mapping class groups. For instance, the elements acting loxodromically on the curve graph and precisely the pseudo-Anosov homeomorphisms. In this talk I'll discuss recent joint work with Sam Taylor regarding infinite-type mapping classes that act as loxodromic isometries on graphs associated to infinite-type surfaces. The aim of these projects is to work towards a Nielsen-Thurston type classification of mapping classes for infinite-type surfaces and to understand which homeomorphisms are the generalizations of pseudoAnosovs is in this setting.
(Received August 29, 2023)

## 1192-57-28351

Christopher Jay Leininger, Rice University, Jacob Russell-Madonia*, Rice University. Purely pseudo-Anosov subgroups of 3-manifold groups.
Farb and Mosher's convex cocompact subgroups are some of the geometrically, dynamically, and algebraically richest subgroups of the mapping class group. A major open question about these subgroups asks if they are characterized by each element acting with pseudo-Anosov dynamics on the surface. We show the answer is 'yes' when you restrict to subgroups of fibered 3-manifold groups included into the mapping class group via the Birman exact sequence. Joint work with Chris Leininger
(Received August 29, 2023)

## 1192-57-28469

Tarik Aougab, Haverford College, David Futer*, Temple University, Samuel J. Taylor, Temple University. Counting fixed points of pseudo-Anosov maps. Preliminary report.
Let $S$ be a hyperbolic surface and $f$ a pseudo-Anosov map on $S$. I will describe a result that predicts the number of fixed points of $f$, up to constants that depend only on the surface $S$. If $f$ satisfies a mild condition called "strongly irreducible," then the logarithm of the number of fixed points of $f$ is coarsely equal to its translation length on the Teichmüller space of $S$. Without this mild condition, there is still a coarse formula in terms of curve complexes. This result and its proof has some applications to the search for surface subgroups of mapping class groups, to the hyperbolic volume of mapping tori, and to the knot Floer invariants of fibered hyperbolic knots. This is joint work with Tarik Aougab and Sam Taylor.
(Received August 30, 2023)

Kai Ishihara, Yamaguchi University, Koya Shimokawa*, Ochanomizu University. Spatial graphs confined to tube regions in the simple cubic lattice. Preliminary report.
We will consider trivalent spatial graphs confined in tube regions in the simple cubic lattice. In particular, we will characterize
theta curves and spatial handcuff graphs in the $2 \times 1$-tube region.
(Received August 31, 2023)

1192-57-28513
Jessica S Childress*, Gonzaga University, Kate Kearney*, Gonzaga University. A Component Upper Bound for Link Mosaic Diagrams. Preliminary report.
A link mosaic is a link diagram made using tiles on a $n \times n$ square grid. In this project we produce a formula for an upper bound on the number of components in a mosaic diagram given the number of crossings and a fixed grid size. In particular, on an $n \times n$ mosaic with $x$ crossings, $c \leq 2\left\lfloor\frac{x}{4}\right\rfloor-x+\frac{n^{2}}{2}-n+\varepsilon$, where $\varepsilon=1$ when $n$ is even, and $\varepsilon=\frac{1}{2}$ when $n$ is odd. (Received August 31, 2023)

## 1192-57-28546

David Futer, Temple University, Emily Hamilton*, California Polytechnic State University, San Luis Obispo, Neil Hoffman, Oklahoma State University. Infinitely Many Virtual Geometric Triangulations.
We prove that every cusped hyperbolic 3 manifold has a finite cover admitting infinitely many geometric ideal triangulations. Furthermore, every long Dehn filling of one cusp in this cover admits infinitely many geometric ideal triangulations. This cover is constructed in several stages, using results about separability of peripheral subgroups and their double cosets, in addition to a new conjugacy separability theorem that may be of independent interest.
(Received August 31, 2023)

## 1192-57-28674

Eleni Panagiotou*, Arizona State University. Topological metrics of structural complexity in biopolymers. Preliminary report. Biopolymers, such as DNA and proteins, can be represented by mathematical curves in 3-space, which may have distinct endpoints. Novel mathematical methods in knot theory enable the rigorous characterization of such mathematical curves, without any closure or other approximation schemes. In this talk we discuss these methods and introduce novel metrics of entanglement of collections of open curves in 3-space. We apply these methods to experimental structures of proteins and DNA and show how these methods can thus help us understand biopolymer function and biological material properties in many contexts with the goal of their prediction and design.
(Received September 01, 2023)

## 1192-57-28675

Ciprian Manolescu*, Stanford University, Michael Willis, Texas A\&M University. A Rasmussen invariant for links in $\mathbb{R P}^{3}$. We construct an analogue of Rasmussen's s-invariant for links in $\mathbb{R}^{1} \mathbb{P}^{3}$. We show that the s-invariant gives constraints on the genera of link cobordisms in the cylinder $I \times \mathbb{R P}^{3}$. As an application, we give examples of freely 2-periodic knots in $S^{3}$ that are concordant but not standardly equivariantly concordant.
(Received September 01, 2023)

## 1192-57-28716

Gabriel Islambouli, University of California, Davis, Laura P. Starkston*, University of California, Davis. Decomposing Weinstein manifolds.
Just as trisections and multisections provided a way to encode smooth 4-manifolds through collections of curves on a surface, we will show how this can be enhanced with a dividing set to encode the symplectic topology of a Weinstein 4 -manifold. We show that any Weinstein 4-manifold admits a decomposition as a "multisection with divides" and demonstrate how to encode this diagrammatically.
(Received September 01, 2023)

## 1192-57-28724

Huizheng Guo*, GWU. The Generalized Kauffman-Harary Conjecture.
For a reduced alternating diagram of a knot with a prime determinant $p$, the Kauffman-Harary conjecture states that every non-trivial Fox $p$-coloring of the knot assigns different colors to its arcs. We prove a generalization of the conjecture stated nineteen years ago by Asaeda, Przytycki, and Sikora: for every pair of distinct arcs in the reduced alternating diagram of a prime link with determinant $\delta$, there exists a Fox $\delta$-coloring that distinguishes them. Additionally, we study the properties of incompressible surface from the GKH conjecture.
(Received September 02, 2023)

## 1192-57-28750

Mladen Bestvina, University of Utah, Alexander James Rasmussen*, Stanford University. Transverse measures to infinite type laminations.
Geodesic laminations are crucial tools for studying mapping class groups and three-manifolds. However, little is understood about laminations on infinite type surfaces. In this talk, we consider the cone of transverse measures to a geodesic lamination on an infinite type surface. We show that this cone admits an explicit description as an inverse limit of finite-dimensional cones. We use this explicit description to illustrate exotic new behaviors exhibited by laminations on infinite type surfaces. This talk is joint work with Mladen Bestvina.
(Received September 02, 2023)

Tamunonye Cheetham-West*, Yale University. Finite quotients of fibered hyperbolic 3-manifold groups.
The finite quotients of the fundamental group of a 3-manifold are the deck groups of its finite regular covers. We often pass to these finite-sheeted covers for different reasons, and these deck groups are organized into a topological group called the profinite completion of a 3-manifold group. In this talk, we will discuss how to leverage certain properties of mapping class groups of finite-type surfaces to study the profinite completions of the fundamental groups of fibered hyperbolic 3-manifolds of finite volume.
(Received September 03, 2023)

## 1192-57-28807

Puttipong Pongtanapaisan*, Arizona State University. Polynomial invariants of knotoidal graphs. Preliminary report. Many biological structures can be represented as spatial graphs in 3-space. However, without extra care on how open ends of the graphs are treated, numerous structures will be classified as untangled. An approach that researchers take when the graph has degree at most two is to project a given embedding of a graph in various directions to obtain a collection of diagrams. Polynomial invariants are then computed for these diagrams. In this talk, I will discuss an analogous method for graphs with higher degree vertices. I will also talk about several methods that give the same polynomial when applied to a given open graph. Some methods are more suitable for practical calculations than others.
(Received September 03, 2023)

## 1192-57-29177

Rhea Palak Bakshi*, ETH Institute for Theoretical Studies, Zurich. Skein Modules, Connected Sums, and Torsion. Skein modules were introduced by Józef H. Przytycki as generalisations of the Jones and HOMFLYPT polynomial link invariants in the 3-sphere to arbitrary 3-manifolds. The Kauffman bracket skein module (KBSM) is the most extensively studied of all. However, computing the KBSM of a 3-manifold is known to be notoriously hard, especially over the ring of Laurent polynomials. With the goal of finding a definite structure of the KBSM over this ring, several conjectures and theorems were stated over the years for KBSMs. We show that some of these conjectures, and even theorems, are not true. In this talk I will briefly discuss a counterexample to Marche's generalisation of Witten's conjecture. I will show that a theorem stated by Przytycki in 1999 about the KBSM of the connected sum of two handlebodies does not hold. I will also give the exact structure of the KBSM off of the connected sum of two solid tori and show that it is isomorphic to the KBSM of a genus two handlebody modulo some specific handle sliding relations. Moreover, these handle sliding relations can be written in terms of Chebyshev polynomials. We also compute the skein module of the connected sum of $S^{1} \times S^{2}$ with the solid torus and
$S^{1} \times S^{2} \# S^{1} \times S^{2}$.
(Received September 05, 2023)

## 1192-57-29213

Anna Cepek*, University of Oregon. Higher categorical combinatorics of configuration spaces of Euclidean space.
We approach manifold topology by examining configurations of finite subsets of manifolds within the homotopy-theoretic context of $\infty$-categories by way of stratified spaces. Through these higher categorical means, we identify the homotopy types of such configuration spaces of Euclidean space $\mathbb{R}^{n}$ in terms of the category $\boldsymbol{\Theta}_{n}$. (Received September 05, 2023)

1192-57-29218
Anna Cepek*, University of Oregon. The geometry of Milner's link invariants. Preliminary report.
We discuss Milnor's link invariants through a geometric lens using intersections of Seifert surfaces. Our work is thus of a similar flavor as that of Cochran from 1990, who based his work on particular choices of Seifert surfaces. But like Mellor and Melvin in 2003, who considered only the first invariant (after linking number), we allow for more arbitrary choices. We conjecture that Milnor's invariants can be recovered geometrically using the work of Monroe and Sinha on linking of letters and Sinha and Walters on Hopf invariants. We expect our approach to recover Cochran's work and to extend work of Polyak, Kravchenko, Goussarov, and Viro on Gauss diagrams. (Received September 05, 2023)

## 1192-57-29296

Léo Schelstraete*, UCLouvain. Odd Khovanov homology and higher representation theory. Preliminary report.
Khovanov homology can be understood via higher representation theory: this mimicks the construction of the Jones polynomial using the representation theory of $U_{q}\left(\mathfrak{s l}_{2}\right)$, and has lead for instance to a proper functorial construction of Khovanov homology. Odd Khovanov homology is a homological invariant of links that also categorifies the Jones polynomial: it is distinct from Khovanov homology, and much less understood. In this talk, we present work giving a construction of odd Khovanov homology via higher representation theory, using a supercategorification of the representation theory of $U_{q}\left(\mathfrak{s l}_{2}\right)$ (j.w.w. Pedro Vaz).
(Received September 06, 2023)

## 1192-57-29346

Marissa Kawehi Loving*, University of Wisconsin Madison. Volumes of end-periodic mapping tori.
In this talk, I will briefly introduce the notion of end-periodic homeomorphisms of infinite-type surfaces. My goal will be to illustrate the ways these homeomorphisms mimic the behavior of pseudo-Anosov homeomorphisms of finite-type surfaces by displaying interesting geometric, dynamical, and topological behavior. As part of this discussion, I will describe some joint work with Elizabeth Field, Autumn Kent, Heejoung Kim, and Chris Leininger (in various configurations) on the volume of endperiodic mapping tori.
(Received September 06, 2023)

Manpreet Singh*, University of South Florida. Knot invariants from biquandles and virtual biquandles. Preliminary report. Quandles are known to provide a complete algebraic characterization of oriented knots, up to the orientation of the knot and the ambient space. Biquandles are generalizations of quandles and yield stronger invariants in the context of virtual links. However, in both quandles and biquandles, the labeling of the arcs at virtual crossings remains unchanged. A virtual biquandle is defined as a biquandle $Q$ equipped with an automorphism $f: Q \rightarrow Q$. This additional structure allows for the alteration of labels on the semi-arcs at virtual crossings, offering the potential for more information about virtual knots. In this talk, we will delve into the connections between virtual knot invariants originating from biquandles and virtual biquandles. Please note that this work is still in progress.
(Received September 07, 2023)

## 1192-57-29490

Hannah-Elsie Meit*, Rhodes College. Computing Formulae for $\Gamma$-Orbifold Euler Characteristics of $\mathrm{O}(2)$ Representations. Given a compact Lie group $G$, a finitely presented discrete group $\Gamma$, and a representation $X$ of $G$, the $\Gamma$-orbifold Euler characteristic $\chi_{\Gamma}(G \ltimes X)$ is a topological invariant computed by determining the orbit types of $X$, each of their orbit spaces, the spaces of conjugacy classes of each isotropy, and the Euler characteristic of each of these components. We will discuss recent results in computing formulae for $\chi_{\Gamma}(G \ltimes X)$ with $G$ as the group $\mathrm{O}(2)$ of $2 \times 2$ orthogonal matrices acting on an arbitrary representation of $\mathrm{O}(2)$, as well as the consequences of certain restrictions to $G$-invariant subsets of $X$ and changes of $\Gamma$.
(Received September 06, 2023)

## 1192-57-29493

Francis Bonahon, University of Southern California, Vijay Higgins*, University of California, Santa Barbara. Central elements in the $S L_{d}$ skein algebra of a surface. Preliminary report.
The skein algebra of a surface is spanned by link diagrams on the surface subject to local skein relations coming from a quantum group. Multiplication of two diagrams is given by superimposing one diagram over the other. The skein algebra of a surface is generally non-commutative, and its center depends on the parameter $q$. For the quantum group associated to $S L_{2}$ the skein relations are the Kauffman bracket skein relations. When the parameter $q$ is a root of unity, the work of BonahonWong produced a family of central elements in the $S L_{2}$ skein algebra related to Chebyshev polynomials. In this work, we find elements in the $S L_{d}$ skein algebra related to analogues of these polynomials, and use the $S L_{d}$ skein relations to show that these elements are central at roots of unity.
(Received September 07, 2023)

## 1192-57-29521

David Cates, Texas A\&M University, Ansel Goh*, University of Washington, Minyi Liang, Jilin University, Samuel Lowery, Slippery Rock University, Maxwell Natonson, University of Michigan. Ribbon numbers for 12-crossing knots. Preliminary report.
A ribbon disk is an immersed disk $D \subset \mathbb{R}^{3}$ that only admits a particular type of self-intersection called a ribbon intersection. A knot $K$ is ribbon if it bounds a ribbon disk, and the minimum number of ribbon intersections of any such disks is the ribbon number of the knot. In this presentation, we explore the ribbon number of 12 -crossing knots by utilizing the Alexander polynomial, the determinant of the knot, and the symmetric union presentation of the knot. Any ribbon disk can be represented by a combinatorial graph called a ribbon code, from which we can compute the Alexander polynomial of any knot that bounds it. We enumerate all ribbon codes with at most 4 ribbon intersections, thus finding all possible Alexander polynomials of ribbon knots with ribbon number 4 . This allows us to improve the existing lower bounds on the ribbon number of knots up to 12 crossings, successfully identifying the ribbon number of 56 knots. Additionally, we consider $r_{g}(K)$, a generalization of the ribbon number for knots bounding genus $g$ ribbon surfaces. In general, for a ribbon knot $K$ we expect $r_{g}(K)=r_{g-1}(K)-1$. When $r_{g}(K)<r_{g-1}(K)-1$, we say that the knot has a jump. We provide a lower bound on $r_{g}(K)$ which gives examples of knots that exhibit at least two jumps.
(Received September 07, 2023)

## 1192-57-29622

Giulio Belletti, Université Paris Saclay, Tian Yang*, Texas A\&M University. Quantum 6j-symbols and generalized hyperbolic tetrahedra.
Volume Conjecture relates the asymptotic behavior of various quantum invariants of a 3-manifold with the geometric information of the manifold. In this talk I will focus on the building blocks of the Turaev- Viro invariants, the quantum $6 j$ symbols. We observed that the geometric quantity underlying the asymptotic behavior of quantum 6 j -symbols is the volume of a suitably generalized hyperbolic tetrahedron.
(Received September 07, 2023)

Melody Molander*, UC Santa Barbara. Skein Theory of Index 4 Subfactor Planar Algebras. Preliminary report.
Subfactor planar algebras first were constructed by Vaughan Jones as a diagrammatic axiomatization of the standard invariant of a subfactor. Planar algebras can be conveniently encoded by diagrams in the plane. These diagrams satisfy some skein relations and have an invariant called an index. The Kuperberg Program asks to find all diagrammatic presentations of subfactor planar algebras. This program has been completed for index less than 4 . In this talk, I will introduce subfactor planar algebras and find presentations for subfactor planar algebras of index 4 associated with the affine A Dynkin diagram. (Received September 07, 2023)

Masahico Saito*, University of South Florida, Emanuele Zappala, Idaho State University. Invariants for surface ribbons from group heaps and braided Frobenius algebras.
A surface ribbon is a compact orientable surface with boundary embedded in 3-space in a ribbon form. They are represented by diagrams of framed graphs. Reidemeister type moves for such framed graphs are known. A group with the ternary operation defined by $(x, y, z) \mapsto x y^{-1} z$ is called a group heap, and the operation is ternary self-distributive. We define invariants of surface ribbons using group heaps, by assigning elements on boundary arcs of their framed diagrams. Analogues of the fundamental group and cocycle invariants are defined. Constructions of braided Frobenius algebras using this idea, and an approach to use them for invariants are also discussed.
(Received September 07, 2023)

## 1192-57-29743

Elizabeth Denne*, Washington \& Lee University. 3D Printed Knots.
In Spring 2023, I taught an Introduction to Knot Theory class. This was an intense class with two hour class meetings four days a week for four weeks. I will discuss how the students engaged with knot theory using physical models. Also how we incorporated a 3D printing project into the course.
(Received September 07, 2023)

## 1192-57-29803

Yuanan Diao*, University of North Carolina Charlotte. Ropelengths of alternating knots.
A long standing conjecture states that the ropelength of any alternating knot is at least proportional to its crossing number. In this talk I will present a recent result that shows this conjecture is true. That is, there exists a constant $b_{0}>0$ such that $R(K) \geq b_{0} C r(K)$ for any alternating knot $K$, where $R(K)$ is the ropelength of $K$ and $C r(K)$ is the crossing number of $K$. (Received September 07, 2023)

## 1192-57-29814

Dionne Ibarra*, Monash University, Daniel Mathews, Monash University, Jessica S. Purcell, Monash University. Triangulations of the complements of double twist knots $K_{p, p}$ and computing the $A$-polynomial.
A. A. Champanerkar introduced a geometric way to compute the $A$-polynomial of any triangulated knot complement by way of the Neumann-Zagier matrix. T. Dimofte outlined a process of symplectic reduction to compute the $A$-polynomial. J. A. Howie, D. V. Mathews, and J. S. Purcell discovered a way to convert Champanerkar's equations into simpler equations with a Ptolemylike structure that can be read off almost entirely from the Neumann-Zagier matrix. In this talk we will present triangulations of the complements of double twist knots of the form $K_{p, p}$ obtained by Dehn filling the crossing circles of the Borromean rings, then we will show how to use Howie, Mathews, and Purcell's work to obtain the Ptolemy-like equations used to calculate the $A$ -polynomial. This is joint work with D. V. Mathews and J. S. Purcell.
(Received September 08, 2023)

## 1192-57-29817

Hiroaki Karuo, Gakushuin University, Han-Bom Moon*, Fordham University, Helen M Wong, Claremont McKenna
College. Cluster algebras and generalized skein algebras.
For each punctured surface admitting a triangulation, we may associate two algebras. One is the cluster algebra of surfaces, and the other is the generalized skein algebra from quantum topology. In this talk, I will explain their compatibility and some consequences in the Teichmuller theory and the structure of cluster algebra.
(Received September 08, 2023)

## 1192-57-29844

Atzimba Martinez*, Washington University in St. Louis. Taut foliations for Montesinos knots.
In this talk we consider orientable 3-manifolds that arise from Dehn surgery along Montesinos knots. We will discuss the classification of Montesinos knots (in particular those of odd type), described through continued fractions, and study their Seifert surfaces of minimal genus. We then use this information to find persistent foliations.
(Received September 08, 2023)

## 1192-57-29908

Ian Sullivan, University of California, Davis, Melissa Zhang*, University of California, Davis. Computations of the skein lasagna module for $S^{2} \times S^{2}$. Preliminary report
Morrison, Walker, and Wedrich's skein lasagna modules are 4-manifold invariants defined using Khovanov-Rozansky homology. Computations and elaborations by Manolescu and Neithalath support the hope that this intricate invariant may be sensitive to 4D nuances; for example, they show that the skein lasagna module distinguishes $\mathbb{C P}^{2}$ and $\overline{\mathbb{C P}^{2}}$. One especially important conjecture states that the skein lasagna module of $S^{2} \times S^{2}$ is 0 (or infinite-dimensional). This is a necessary condition for the invariant to be able to detect exotic 4-manifold pairs. We have computed significant portions of the skein lasagna module of $S^{2} \times S^{2}$, and I will report on these. This is joint work with Ian Sullivan.
(Received September 08, 2023)

Zhang*, University of California, Davis. Bridge multisections for symplectic surfaces in Weinstein 4-manifolds. Preliminary report.
Islambouli and Starkston describe two algorithms for encoding Weinstein 4-manifolds using multisections with divides. In this talk, we report on progress toward encoding embedded symplectic surfaces in their construction, by adapting Meier and Zupan's bridge trisection techniques to the symplectic setting. This is joint work with Román Aranda, Patricia Cahn, Agniva Roy, James Hughes, Marion Campisi, and Daniela Cortes Rodriguez.
(Received September 08, 2023)

## 1192-57-30002

Lillian E Whitesell*, Rhodes College. Integrating $O(n)$-invariant Functions via the Hilbert Embedding.
Let $G$ be a collection of $n \times n$ invertible complex matrices and define $\mathbb{C}\left[x_{1}, \ldots, x_{n}\right]^{G}$ to be the set of polynomials in $n$ variables that are invariant under $G$, meaning that they don't change when elements of $G$ are applied to the variables. If we suppose $G$ acts on a vector space $V$ and assume that $G$ is either finite or one of the classical groups, then a theorem of Hilbert guarantees that $\mathbb{C}\left[x_{1}, \ldots, x_{n}\right]^{G}$ is finitely generated; we are primarily interested in the case where $G$ is isomorphic to an orthogonal group and the representation is $k$ copies of its defining representation. This means that it is the subalgebra of $\mathbb{C}\left[x_{1}, \ldots, x_{n}\right]$ generated by a finite set of polynomials $f_{1}, \ldots, f_{k}$. Such a generating set is called a Hilbert basis. Given a Hilbert basis, the Hilbert embedding (a map $\boldsymbol{f}: V \rightarrow \mathbb{R}^{k}$ ) can be defined. This allows us to think of the orbit space as a subset of $\mathbb{R}^{k}$. The goal of my research is to integrate functions that are invariant with respect to a group action over the orbit space via the Hilbert embedding. By the Schwarz-Mather Theorem, smooth invariant functions on $V$ can be expressed as smooth functions of the invariant polynomials, and we will discuss progress on the question of how to integrate such functions on the image of the Hilbert embedding. This is known as the "canonical measure" or "canonical volume form" on the orbit space that pulls back via the Hilbert embedding to the ordinary volume form. I will present computations of this canonical measure in several cases where the action is coregular.
(Received September 08, 2023)

## 1192-57-30056

Andreas Stavrou*, University of Chicago. Configuration spaces and the Johnson filtration.
The mapping class group of a surface acts naturally on the homology of the configuration spaces of the surface. The representations that arise vary in complexity with the number of configuration points and with the flavour of the configuration spaces. In this talk, we will relate this complexity with the Johnson filtration of the mapping class group by presenting recent results by the author and others.
(Received September 08, 2023)

1192-57-30059
Mark Hughes, Brigham Young University, Seungwon Kim, Sungkyunkwan University, Maggie Miller*, Clay Mathematics Institute. Splitting spheres in $S^{4}$.
In this talk, I'll explain why, in the complement $X$ of an unlinked genus- $m$ and genus- $n$ surface in $S^{4}$, there are two smooth splitting 3 -spheres that are not topologically isotopic in $X$ (provided $m \geq 4$ ). This contradicts intuition from classical knot theory, as a 2 -component split link in $S^{3}$ always admits a unique splitting sphere up to smooth isotopy in the link complement. (Received September 08, 2023)

## 1192-57-30076

Marissa Masden*, University of Oregon. Level set topology for piecewise linear functions: From ReLU neural networks to more general polyhedral domains.. Preliminary report.
A (ReLU) neural network is a type of piecewise linear (PL) function $F$ which induces a canonical polyhedral subdivision, $\mathcal{C}(F)$, on its input space (Grigsby and Lindsey, 2022). We will explore how combinatorial properties of the face poset of this polyhedral subdivision may be used to compute topological properties of the function such as its level set topology, critical points, and (most recently) a discrete gradient vector field agreeing with the function, among other useful measures, and demonstrate how this may be used to understand ReLU neural networks as a class of functions. We finally discuss the extent to which differential topology and combinatorial techniques for this motivating example can be extended for use within a larger category of polyhedral complexes under PL functions. (Received September 08, 2023)

## 1192-57-30088

Krishnendu Kar, Louisiana State University, Moses Samuelson-Lynn*, University of Utah. Glimpses on Symmetric Union Presentations and p-Colorings \par.
Ribbon knots are knots which bound a disk which is allowed to intersect itself in 3-space only at ribbon singularities. These objects hold interest to mathematicians due to the infamous Slice-Ribbon Conjecture. A natural question to ask is how to construct ribbon knots. This question was answered by Christoph Lamm in 1998 with the introduction of symmetric union presentations. It turns out that all symmetric union knots are ribbon knots, but we don't know whether the converse is true or not. Two knots are called symmetrically related if they are both partial knots for symmetric union presentations of the same knot. Lamm showed that two symmetrically related knots must share the same knot determinant, and he proposed the question for converse, i.e. whether two knots that share the same determinant are symmetrically related. In this project, we have found an upper bound for the number of $p$-colourings of a symmetric union in terms of the partial knot, which can be used to answer the proposed question. $\backslash \mathrm{par}$
(Received September 08, 2023)

Isidora Dare Bailly-Hall*, Grinnell College, Karina Dovgodko, Columbia University, Akash Ganguly, Carleton College,
Jiachen Kang, University of Michigan, Jishi Sun, University of Michigan. Detecting boundary slopes of two-bridge knots via intersections in the character variety arising from epimorphisms. Preliminary report.
The $\mathrm{SL}(2, \mathbb{C})$ character variety has long been an important tool in the study of 3-manifolds. In particular, intersection points in character varieties of knot groups detect essential surfaces via $\mathrm{SL}(2, \mathbb{C})$-trees. We describe intersection points in the character varieties of a family of hyperbolic two-bridge knot groups that have epimorphisms onto the trefoil knot. Using the technique of Farey recursion, we show that these intersection points correspond to algebraic non-integral representations. We also determine the boundary slopes detected by these intersection points.
(Received September 09, 2023)

## 1192-57-30316

Isidora Dare Bailly-Hall*, Grinnell College, Karina Dovgodko*, Columbia University, Akash Ganguly*, Carleton College, Jiachen Kang*, University of Michigan, Jishi Sun*, University of Michigan. Detecting boundary slopes in two-bridge knot complements via intersections in the character variety arising from epimorphisms. Preliminary report.
The SL( $2, C$ ) character variety has long been an important tool in the study of 3-manifolds. In particular, intersection points in character varieties of knot groups detect essential surfaces via SL(2,C)-trees. We describe intersection points in the character varieties of a family of hyperbolic two-bridge knot groups that have epimorphisms onto the trefoil knot. Using the technique of Farey recursion, we show that these intersection points correspond to algebraic non-integral representations. We also determine the boundary slopes detected by these intersection points.
(Received September 09, 2023)

## 1192-57-30478

Nir Gadish*, University of Michigan. Letter-braiding invariants of words in groups.
How can one tell if a group element is a k-fold commutator? A computable invariant of words in groups that does not vanish on k -fold commutators will help. For free groups this is achieved by Fox calculus, whose geometric applications include Milnor invariants of links, and there are generalizations for braid groups and RAAGs, but beyond that little is known. We introduce a complete collection of such invariants for any group, using (higher) linking numbers of letters in words due to Monroe and Sinha.
(Received September 10, 2023)

## 1192-57-30636

Sunghyuk Park*, Harvard University. 3-manifolds and $q$-series.
Since the work of Lawrence and Zagier, quantum invariants of 3-manifolds have been a primary source of quantum modular forms. Most notably, in 2017, Gukov, Pei, Putrov and Vafa conjectured the existence of a q-series invariant of 3-manifolds equipped with a spin-c structure, and since then various people have shown that this invariant exhibits quantum modularity (of possibly higher depth). In this talk, I will give a review of this q-series invariant of 3-manifolds, with particular emphasis on their modular properties.
(Received September 10, 2023)

## 1192-57-30696

Sam Gunningham*, Montana State University, David Jordan, University of Edinburgh, Monica Vazirani, UC Davis. Quantum Character Theory.
I will describe a dictionary between skein theory on a 2 -torus and the theory of q-character sheaves on a reductive group. This gives rise to a fruitful exchange of results and ideas between the two sides.
(Received September 10, 2023)

## 1192-57-30785

Moshe Cohen*, State University of New York At New Paltz, Adam M. Lowrance, Vassar College. The genus distribution of 2bridge knots is asymptotically normal.
The average genus of a 2-bridge knot with crossing number approaches c/4 $+1 / 12$ as approaches infinity, as proven by Suzuki and Tran and independently Cohen and Lowrance. In this talk, we prove that the distribution of genera of all 2-bridge knots with a given crossing number approaches a normal distribution. We also determine the median, mode, and variance. This is joint work with undergraduate students Abigail DiNardo, Steven Raanes, Izabella Rivera, Andrew Steindl, and Ella Wanebo. (Received September 11, 2023)

1192-57-30789
Bodie Beaumont-Gould*, Lewis \& Clark College, Erik Brodsky*, Michigan State University, Vijay Higgins, University of California, Santa Barbara, Alaina Hogan*, Grand Valley State University, Joseph M Melby, Michigan State University, Joshua Piazza*, Wheaton College. Power Sum Elements of the $G_{2}$ Skein Algebra.
Given two knots on a surface, we can multiply them by overlaying their respective diagrams. Extending this operation linearly and applying rules for reducing a knot into a linear combination of simpler diagrams creates an algebraic structure called a skein algebra. This skein algebra is a non-commutative algebra accompanied by a parameter $q$, and is related to quantum groups. We are interested in studying the center of the skein algebra built from Kuperberg webs that is associated with the exceptional Lie group $G_{2}$. Our work extends results from Bonahon-Wong and Le, who found central elements of the skein algebra for the easier case of $S L_{2}$. To do this, we establish relationships between elements of the $G_{2}$ skein algebra and the traces of $G_{2}$ matrices. These traces can be described by power sum polynomials. In our main result, we find that the elements corresponding to $n$th degree power sum polynomials are central when $q$ is a $2 n$th root of unity.
(Received September 11, 2023)

Orsola Capovilla-Searle*, UC Davis. On exact Lagrangian fillings of Legendrian links in $S^{\wedge}\{3\}$ with the standard contact structure.. Preliminary report.
Exact Lagrangian fillings of Legendrian submanifolds in the standard contact Darboux ball are an important object of study in contact and symplectic topology. The Legendrian differential graded algebra ( also known as the Chekanov-Eliashberg dgalgebra) is an important legendrian invariant that is functorial over exact Lagrangian cobordisms. In particular, exact Lagrangian fillings of a legendrian $\Lambda$ induce augmentations of the Legendrian dg-algebra of $\Lambda$. We will discuss results on distinguishing exact Lagrangian fillings using augmentations.
(Received September 11, 2023)

## 1192-57-30874

Audrey Baumheckel, California State University, Fresno, Carmen L Caprau*, California State University, Fresno, Conor Righetti, California State University, Fresno. On an invariant for colored classical and singular links. Preliminary report A colored link, as defined by Francesca Aicardi, is an oriented classical link together with a 'coloration', which is a function defined on the set of link components and whose image is a finite set of 'colors'. An oriented classical link can be regarded as a colored link with its components colored with a sole color. Aicardi constructed an invariant $F(L)$ of colored links $L$ defined via skein relations. When the components of a colored link are colored with the same color or when the colored link is a knot, $F(L)$ is a specialization of the HOMFLY-PT polynomial. Aicardi also showed that $F(L)$ is a stronger invariant than the HOMFLY-PT polynomial when evaluated on colored links whose components have different colors. In this talk, we provide a state-sum formula for the invariant $F(L)$ of colored links using a graphical calculus for colored 4 -valent planar graphs. We also extend $F(L)$ to an invariant of colored singular links.
(Received September 11, 2023)

## 1192-57-30875

Orsola Capovilla-Searle*, UC Davis. Legendrian links in the standard contact 3-sphere. Preliminary report.
A knot is an embedding of a circle into three dimensional space and can be visualized by taking a piece of string, tangling it and gluing the endpoints together. A link is a disjoint union of knots in three dimensional space that may be knotted together. Contact 3-manifolds $(Y, \xi)$ arise naturally in the study of constrained dynamics and consist of a 3-manifold $Y$, and maximally non-integrable hyperplane field $\xi$. A Legendrian link $\Lambda$ in $(Y, \xi)$ is a link with whose tangent vectors $T_{p} \Lambda$ must lie in the contact structure $\xi_{p}$ for $p \in \Lambda$. Legendrians show up in many contexts such as wavefronts in optics, and juggling. One can consider Legendrian links up to Legendrian isotopy, a smooth isotopy where at each step the link is Legendrian. For each smooth link there are infinitely many distinct Legendrians up to Legendrian isotopy. We will discuss new results on Legendrians in the standard contact 3-sphere.
(Received September 11, 2023)

## 1192-57-31020

Thang Tu Quoc Le*, Georgia Insitute of Technology. Skein modules and Habiro ring.
We show that the Kauffman bracket skein module of an integral homology sphere can be mapped to the Habiro ring. There is a similar map for the skein module of knot complements. The first part is joint with S. Garoufalidis
(Received September 11, 2023)

## 1192-57-31118

Wout Moltmaker*, University of Amsterdam. Planar Knotoid Invariants. Preliminary report.
Knotoids are a generalization of knots given by considering knot diagrams modelled on the interval rather than the circle. The endpoints of this interval may lie anywhere in the diagram, but strands may not pass over or under them. I will discuss the applications and tabulation of knotoids. In particular, we will see that it makes an appreciable difference whether these knotoids are drawn on the sphere or plane: many inequevalent knotoids on the plane become equivalent when a point at infinity is added to the plane. Most known knotoid invariants factor through this one-point compactification and so cannot distinguish such planar knotoids, making their tabulation a difficult problem even for low crossing numbers. I will define several knotoid invariants that can 'detect planarity', and show how they do so. These include quantum invariants and invariants based on quandles. Finally I will mention how these invariants tie into the applications and tabulation efforts. (Received September 11, 2023)

## 1192-57-31136

Carmen L Caprau, California State University, Fresno, Victoria Lynne Wiest*, California State University, Fresno. An extension of the $\operatorname{sl}(n)$ polynomial to knotted 4 -valent graphs. Preliminary report.
We consider a special type of knotted oriented 4 -valent graphs with rigid vertices and construct an invariant of such graphs. The resulting invariant is an extension of the well-known $s l(n)$ polynomial invariant for knots and links. Our approach uses a version of the Murakami-Ohtsuki-Yamada state model for the $s l(n)$ polynomial via 4 -valent planar graphs and graphical relations among such graphs.
(Received September 11, 2023)

1192-57-31251
Louis H. Kauffman*, UIC. Penrose Evaluations, Perfect Matching Polynomials and Invariants of Multiple Virtual Knots and Links. Preliminary report.
We discuss the Penrose evaluation for counting the proper edge three colorings of planar trivalent maps and the author's generalization of this evaluation for arbitrary planar maps. We then describe a further generalization as a perfect matching polynomial and its application in joint work with Scott Baldridge and Ben McCarty to the Total Coloring Polynomial in a graph
homology theory. The perfect matching polynomial generalization of the Penrose evaluation is described in this talk in terms of a (tensor) state summation. The color count associated with a perfect matching of a trivalent graph is a natural count for $n$ colors such that identical distinct colors occur at the ends of each perfect matching edge. This is a generalization of the original three color stipulation for Penrose evaluations. We then explain how these ideas can be transferred to virtual knot theory with multiple virtual crossings and discuss the structure of this virtual theory. (Received September 11, 2023)

## 1192-57-31333

Melanie Matchett Wood*, Harvard University. An Application of Probability Theory for Groups to 3-Dimensional Manifolds. The fundamental group of a 3-dimensional manifold captures information about loops in the manifold. One naturally asks what kind of groups are possible as the fundamental groups of 3-manifolds. For any finite group G, it is well-known that there exists a 3-manifold M with G as a quotient of the fundamental group of M. However, we can ask more detailed questions about the possible finite quotients, e.g. for G and H finite groups, does there exist a 3-manifold group with G as a quotient but not H as a quotient? We answer all such questions. To prove existence of 3-manifolds with certain finite quotients but not others, we use a probabilistic method, by first proving a formula for the distribution of the fundamental group of a random 3-manifold, in the sense of Dunfield-Thurston. This formula comes from our recent work on the moment problem for random groups. This is joint work with Will Sawin.
(Received September 11, 2023)

## 1192-57-31345

Ziqin Feng*, Auburn University. Homotopy Types of Vietoris-Rips Complexes related to Certain Graphs.
We'll discuss the topology of simplicial complexes associated with hypercube graphs and Kneser graphs. For any metric space $X$ and a scale parameter $r \geq 0$, the Vietoris-Rips complex, $V R(X ; r)$, has $X$ as its vertex set and a finite subset $\sigma \subseteq X$ is a simplex if and only if the diameter of $\sigma$ is less or equal to $r$. The topology of the Vietoris-Rips complex over hypercube graph $Q_{m}$ with small scales, $r=0,1$, or 2 , is well-studied by Adamaszek and Adams. We'll describe the homotopy types of VietorisRips complexes of hypercube graphs at scale 3 . We represent the vertices in the hypercube graph $Q_{m}$ as the collection of all subsets of $[m]=\{1,2, \ldots, m\}$ and equip $Q_{m}$ with the metric using symmetric difference distance. Adamaszek and Adams proved that the Vietoris-Rips complexes of hypercube graphs $Q_{m}$ at scale $2, V R\left(Q_{m} ; 2\right)$, is homotopy equivalent to $c_{m}$-many spheres with dimension 3 where $\$ \mathrm{c}$ _m $=$ \sum _\{ $0 \leq j$ (Received September 11, 2023)

## 1192-57-31409

Nipun Vibodha Amarasinghe*, California State University, Fresno, Carmen L Caprau, California State University, Fresno. An invariant for oriented tangles motivated by the sl(3) polynomial for links. Preliminary report.
An $(m, n)$-trivalent web is an oriented planar graph in $\mathbb{R} \times[0,1]$ with trivalent vertices that are either sources or sinks and with $m$ and $n$ boundary points in $\mathbb{R} \times\{0\}$ and $\mathbb{R} \times\{1\}$, respectively. We define the category of trivalent webs Web and provide a presentation of this tensor category via generators and relations. We also define the category LWeb as the free $\mathbb{Z}\left[q, q^{-1}\right]$-linear category generated by Web modulo additional relations motivated by the Kuperberg bracket. By constructing a functor from the category OTa of oriented tangles to LWeb, we obtain an invariant for oriented tangles. We also explain how to use this functor to recover the $\operatorname{sl}(3)$ polynomial of links.
(Received September 11, 2023)

## 1192-57-31436

Dipali Swain*, University of South Florida. Invariants using Idempotents in Quandle Rings. Preliminary report.
We will review quandle rings and discuss recent work on invariants of knots using idempotents in quandle rings. We will show how to obtain some knot invariants from idempotents in quandle rings. This is a joint work with (M. Elhamdadi). (Received September 11, 2023)

## 1192-57-31589

Seraphina Eun Bi Lee*, University of Chicago. Mapping class groups of rational 4-manifolds and isotropic homology classes.
Consider the 4-manifold $M_{N}=\mathbb{C P}^{2} \# N \overline{\mathbb{C P}^{2}}$. In this talk we study the stabilizer of a primitive, isotropic class $w \in H_{2}\left(M_{N}\right)$ of minimal genus 0 under the natural action of the topological mapping class group $\operatorname{Mod}\left(M_{N}\right)$. In doing so, we discuss the Nielsen realization problem for a certain maximal torsion-free, abelian subgroup of $\operatorname{Mod}\left(M_{N}\right)$ and its relation to genus- 0 Lefschetz fibrations on $M_{N}$.
(Received September 11, 2023)

## 1192-57-31650

Uwe Kaiser*, Boise State University. Skein theory and diffeomorphisms of 3-manifolds. Preliminary report. Let $K \subset M$ be a link in a 3-manifold. Let $\mathcal{L}(M ; K)$ denote the set of links in $M-K$ up to isotopy by diffeomorphisms of $M$, $\alpha_{t}$, such that $\alpha_{0}$ is the identity of $M$ and $\alpha_{1}(K)=K$ but not necessarily $\alpha_{t}(K)=K$ for all $t$. This set of isotopy classes is in general distinct from the usual set of isotopy classes of links in $M-K$ (defined by isotopy in $M-K$ ). In studying the difference one considers the evaluation map from diffeomorphisms of $M$ isotopic to the identity to links in $M$, $\alpha \mapsto \alpha(K) \subset M$, on the level of $\pi_{1}$. Skein theory based on $\mathcal{L}(M, K)$ forms a bridge between skein theory of $M-K$ and skein theory of $M$ in an obvious way. We discuss in particular the example of $K$ an unlink in a 3-ball contained in $M$. (Received September 11, 2023)

Nathan Broaddus, Ohio State University, Lindsey-Kay Lauderdale, Southern Illinois University, Emille Davie Lawrence*, University of San Francisco, Anisah Nabilah Nu'Man, Spelman College, Robin T Wilson, Cal Poly Pomona. A new presentation of the Steinberg module of $B_{n}$. Preliminary report.
Harer proved that the braid group $B_{n}$ is a duality group with a dualizing module called the Steinberg module $\operatorname{St}\left(B_{n}\right)$. As such it is an important cohomological object associated with the braid group. We will discuss recent joint work which builds upon work of Harer, et. al to decrease the number of generators and relations in the Harer presentation of $\operatorname{St}\left(B_{n}\right)$. This work was generously supported by the 2021 ADJOINT program at Simons Laufer Mathematical Sciences Institute.
(Received September 11, 2023)

## 1192-57-31675

Carmen L Caprau, California State University, Fresno, Isaiah Alfred Martinez*, California State University, Fresno. An extension of Khovanov homology to singular links and tangles. Preliminary report.
We define a polynomial invariant $P(L)$ for singular links $L$, which is an extension of the $s l(2)$ polynomial for classical links. Given a diagram $D$ of a singular link $L$, we associate a (co)chain complex $[D]$ of graded vector spaces and show that the (co)homology of $[D]$ is an invariant of $L$, and its graded Euler characteristic is $P(L)$. That is, we construct a bi-graded (co)homology theory for singular links that categorifies our polynomial invariant $P$ and which can be regarded as an extension of the original Khovanov homology to singular links. (Received September 11, 2023)

## 1192-57-31789

Dylan Skinner*, Brigham Young University. Leveraging Deep Reinforcement Learning and Braid Representations to Explore Knot Theory.
Deep reinforcement learning (DRL) continues to demonstrate remarkable efficacy in pattern recognition and problem-solving, particularly in domains where human intuition falls short. In the realm of knot theory, an important challenge revolves around constructing minimal-genus slice surfaces for knots of varying complexity. In this presentation, I will outline a novel approach that leverages the power of DRL to tackle this difficult problem. Using braid representations of knots, we train reinforcement learning agents to construct minimial genus slice surfaces by finding sequences of braid transformations that are optimal with respect to a given objective function. This provides a template for attacking other computationally difficult problems in topology and pure mathematics using reinforcement learning.
(Received September 12, 2023)

## 1192-57-31861

Kasturi Barkataki*, arizona state university, Louis H. Kauffman, UIC, Eleni Panagiotou, Arizona State University. Open curves in 3-space, Linkoids and Virtual Knots.
The entanglement of open curves in 3-space appears in many physical systems and affects their material properties and function. A new framework in knot theory was introduced recently that enables to characterize the complexity of open curves in 3-space without any closure or diagram approximation schemes. The method relies in using appropriate tools from the theory of knotoids (open arc diagrams) and integrating over the sphere. Even though many invariants of knotoids exist, linkoids, which are multi-component open arc diagrams, are less well understood. In this talk we will introduce a new collection of invariants of linkoids. This is done via a new rigorous connection between linkoids and virtual knots and links, which is an extension of that of knotoids and virtual knots. This leads to a collection of novel measures of entanglement of open curves in 3-space which are continuous functions of the curve coordinates and tend to their corresponding classical invariants when the endpoints of the curves tend to coincide. (Received September 12, 2023)

## 1192-57-31878

Kaitlin Tademy*, University of Nebraska - Lincoln. Tea Time with Virtual Torus Knots.
Virtual knots are a somewhat mysterious type of knot. Given the oriented Gauss code of a knot diagram, one can sometimes (but not always!) reconstruct said knot diagram. We can think of virtual knots as those knots with diagrams which cannot be reconstructed from their oriented Gauss code. On the other hand, classical torus knots are a well-understood type of knot, completely determined by coprime integers p and q. In this talk, we will combine these two topics for a tea time with virtual torus knots: who they are, what they're about, and what it takes to unravel them.
(Received September 12, 2023)

1192-57-31896
Colin C. Adams, Williams College, Joye Chen*, Massachusetts Institute of Technology. Hyperbolicity of alternating links in surfaces with boundary.
Given $F$ a compact orientable surface with nonempty boundary other than a disk, and $L$ a link in $F \times I$ with a connected weakly prime cellular alternating projection to $F$, we provide simple conditions that determine exactly when $(F \times I) \backslash N(L)$ is hyperbolic. We also consider suitable embeddings of $F \times I$ in an ambient manifold $Y$ with boundary and provide conditions on links $L \subset F \times I$ which guarantee hyperbolicity of $Y \backslash N(L)$. These results provide many examples of hyperbolic links in handlebodies and other manifolds. They also provide many examples of staked links that are hyperbolic. (Received September 12, 2023)

## 1192-57-31904

Kaitlin Tademy*, University of Nebraska - Lincoln. A Tea Time with Virtual Torus Knots,
Virtual knots are a somewhat mysterious type of knot. Given the oriented Gauss code of a knot diagram, one can sometimes (but not always!) reconstruct said knot diagram. We can think of virtual knots as those knots with diagrams which cannot be
reconstructed from their oriented Gauss code. On the other hand, classical torus knots are a well-understood type of knot, completely determined by coprime integers $p$ and $q$. In this talk, we will combine these two topics for a tea time with virtual torus knots: who they are, what they're about, and what it takes to unravel them.
(Received September 12, 2023)

## 1192-57-31919

Petr Vojtechovsky*, University of Denver. Cycle structure of translations in connected quandles.
All translations in a given connected quandle have the same cycle structure. We will show that, somewhat surprisingly, certain cycle structures force the connected quandle to be latin. This is true for instance when any two distinct cycles have distinct lengths. We will also discuss connections to Hayashi's Conjecture. This is joint work with Pedro Lopes and Antonio Lages. (Received September 29, 2023)

## 1192-57-31927

Tushar Pandey*, Texas A \& M University, Ka Ho Wong, Yale University. Generalized Bonahon-Wong-Yang volume conjecture of quantum invariants of surface diffeomorphisms I. Preliminary report.
Recently, Bonahon-Wong-Yang defined a quantum invariant for surface diffeomorphisms by using the representation theory of the Kauffman bracket skein algebra. They proposed a volume conjecture for this invariant, stating that the asymptotics of the invariant captures the volume of the mapping torus with the complete hyperbolic structure. The construction of the invariant depends on data called puncture weights. Joint with Ka Ho Wong, we generalize the conjecture to be such that, by choosing the puncture weights appropriately, the asymptotics of the invariants can capture the volume of the cone structure of the manifold as well. We call it the generalized Bonahon-Wong-Yang volume conjecture. We will present the construction of the invariant and state our result for the figure eight knot complement. We then provide more ideas for proving our conjecture for one-punctured torus bundles in general.
(Received September 12, 2023)

## 1192-57-32079

Kyle A Miller*, University of California, Santa Cruz. The homological arrow polynomial for virtual links.
The arrow polynomial is an invariant of framed oriented virtual links that generalizes the virtual Kauffman bracket. We generalize this further to define the homological arrow polynomial, which is an invariant of framed oriented virtual links with labeled components. The key observation is that, given a link in a thickened surface, the homology class of the link defines a functional on the surface's skein module, and by applying it to the image of the link in the skein module this gives a virtual link invariant. This leads to a convenient graphical calculus for the homological arrow polynomial by taking the usual diagrams for the Kauffman bracket and including labeled "whiskers" that record intersection numbers with each labeled component of the link. We use the homological arrow polynomial to study $(\mathbb{Z} / n \mathbb{Z})$-nullhomologous virtual links and checkerboard colorability, giving a new way to complete Imabeppu's characterization of checkerboard colorability of virtual links with up to four crossings. We also prove a version of the Kauffman-Murasugi-Thistlethwaite theorem that the breadth of an evaluation of the homological arrow polynomial for an "h-reduced" diagram $D$ is $4(c(D)-g(D)+1)$. The arrow polynomial is implemented in KnotFolio
(Received September 12, 2023 )

## 1192-57-32115

Yuanan Diao*, University of North Carolina Charlotte. An algorithm for a better estimation of the braid index of a random knot.
Like the ACN, the average braid index is another topological property one can use to study a random knot system. However, a naive approach can lead to significant over estimates. In this talk, I will discuss an algorithm that will mitigate the over estimate problem.
(Received September 12, 2023)

## 1192-57-32134

Sarah Heuss*, University of Wisconsin-Eau Claire, Allison Versaskas*, University of Wisconsin-Eau Claire. Randomly Generating the Unknot. Preliminary report.
\documentclass \{article\} \document A Mathematical knot is an embedding of a circle in 3-dimensional space. Two knots are called isotopic if they can be deformed to each other without cutting or passing a strand through another. Random knot theory was first introduced in the 1960s by chemists Frish-Wasserman, and Biologists Delbruck-Fuller. We study a new perspective on the theory of random knots based on the universal knot diagrams introduced in a recent paper by Even-Zohar-Hass-LinialNowik. We ask what is the probability that a randomly generated potholder knot is unknotted. A potholder diagram for a knot is a diagram that is given by starting with an $n \times n$ grid in the plane, assigning crossings at each intersection point, and finally connecting adjacent edges. As the parameter $n$ goes to infinity every knot admits such a diagram. We use potholder diagrams to provide a new notion of random knot theory. By studying small potholder diagrams, we present some numerical evidence toward the conclusion that the probability of a randomly generated potholder knot being unknotted should be zero. We present explicit computations of the expansion of these potholder diagrams in one direction. By studying the process of randomly generating a potholder knot in terms of a Markov chain, we present forward progress towards proving this result. lenddocument (Received September 12, 2023)

Rhea Palak Bakshi*, ETH Institute for Theoretical Studies, Zurich. On the structure of the skein module of connected sums of manifolds.
Skein modules were introduced by Józef H. Przytycki as broad generalisations of the Jones and HOMFLYPT polynomial link invariants in the 3-sphere to arbitrary 3-manifolds. The Kauffman bracket skein module (KBSM) is the most extensively studied
of all. However, computing the KBSM of a 3-manifold is known to be notoriously hard, especially over the ring of Laurent polynomials. With the goal of finding a definite structure of the KBSM over this ring, several conjectures and theorems were stated over the years for KBSMs. We show that some of these conjectures, and even theorems, are not true. In this talk I will briefly discuss a counterexample to Marche's generalisation of Witten's conjecture. I will show that a theorem stated by Przytycki in 1999 about the KBSM of the connected sum of two handlebodies does not hold. I will also give the exact structure of the KBSM off of the connected sum of two solid tori and show that it is isomorphic to the KBSM of a genus two handlebody modulo some specific handle sliding relations. Moreover, these handle sliding relations can be written in terms of Chebyshev polynomials.
(Received September 12, 2023)

## 1192-57-32476

Tushar Pandey, Texas A \& M University, Ka Ho Wong*, Yale University. Generalized Bonahon-Wong-Yang volume conjecture of quantum invariants of surface diffeomorphisms II. Preliminary report.
Joint with Tushar Pandey, we propose a generalization of the Bonahon-Wong-Yang volume conjecture of quantum invariants of surface diffeomorphisms. In this talk, we discuss the geometry behind the proof of our conjecture of the figure eight knot complement, under some assumption about the volume of the manifold. In particular, we will talk about how geometric quantities, such as the hyperbolic volume and the adjoint twisted Reidemeister torsion, naturally show up in the asymptotics of the invariants
(Received September 12, 2023)

## 1192-57-32495

Kenneth Baker, University of Miami, Dorothy Buck, Duke University, Allison H Moore*, Virginia Commonwealth
University, Danielle O'Donnol, Marymount University, Scott Taylor, Colby College. Unknotting and invariants of trivalent spatial graphs.
This talk addresses unknotting numbers of spatial trivalent planar graphs, particularly $\theta$-curves. The study of such objects is externally motivated by modeling problems arising in molecular biology. Generalizing the theorem of Scharlemann that unknotting number one knots are prime, we prove that if a composite $\theta$-curve has unknotting number one, then it is the order 2 sum of an unknotting number one knot and a trivial $\theta$-curve. We also will discuss some recent results in which we bound the unknotting numbers of spatial trivalent planar graphs by their signature and a certain slice orbifold Euler characteristic. This is joint work with Baker, Buck, O’Donnol and Taylor.
(Received September 12, 2023)

## 1192-57-32544

Sierra Knavel*, Georgia Institute of Technology. Lefschetz fibrations with fixed finitely presented fundamental groups.. In this talk we will discuss Korkmaz's proof that every finitely presented group is the fundamental group of a symplectic Lefschetz fibration. This result was originally proved by Amoros-Bogomolov-Katzarkov-Pantev but re-proven with an explicit construction using Matsumoto's relation. We will also discuss some preliminary results on what fundamental groups a lowgenus Lefschetz fibration over the 2-sphere may have.
(Received September 12, 2023)

## 1192-57-32574

Casandra D. Monroe*, University of Texas at Austin. Flexing and Branched Bending. Preliminary report.
For a hyperbolic n-manifold, bending along a totally geodesic hypersurface is a well-studied method of producing deformations of the original hyperbolic structure. However, there are instances where certain computations show that deformations of a hyperbolic structure exist, even in the absence of totally geodesic hypersurfaces to bend along. In a paper of Cooper, Long, and Thistlethwaite, a manifold is called "flexible" if it has this property. But what explains when a manifold is flexible? In this talk, we will explore one potential answer: a generalization of bending, where instead we bend along totally geodesic branched complexes.
(Received September 12, 2023)

## 1192-57-32637

Ana Wright*, Davidson College. Complete Alexander Neighbors and Unknotting Number.
We call a knot $K$ a complete Alexander neighbor if every possible Alexander polynomial is realized by a knot one crossing change away from $K$. It is unknown whether there exists a complete Alexander neighbor with nontrivial Alexander polynomial. We will eliminate infinite families of knots with nontrivial Alexander polynomial from having this property and discuss possible strategies for unresolved cases. Additionally, we use a condition on determinants of knots one crossing change away from unknotting number one knots to improve KnotInfo's unknotting number data on 11 and 12 crossing knots. Lickorish introduced an obstruction to unknotting number one, which proves the same result. However, we show that Lickorish's obstruction does not subsume the obstruction coming from the condition on determinants.
(Received September 12, 2023)

1192-57-32707
Emily Armstrong*, Smith College, Anisha Jain, Smith College, Elle Obrochta, Smith College. Branched Covering Invariants for Knot Families. Preliminary report.
A Fox coloring of a knot is a labelling of the arcs a knot diagram by the integers $1, . .$, p, satisfying certain criteria. Every Fox coloring of a knot corresponds to a 3-manifold, the dihedral branched cover of the 3-dimensional sphere along the knot. Illig, Karani, and Song implemented an algorithm of Fox for computing algebraic-topological invariants of this branched cover. We build on their work to study the algebraic topology of the branched cover for various families of knots, particularly families of slice knots.
(Received September 12, 2023)

Sanjay Kumar*, The University of California, Santa Barbara, Joseph M Melby, Michigan State University. Turaev-Viro invariants and cabling operations.
The Chen-Yang volume conjecture states that the asymptotics of the Turaev-Viro invariants of a compact oriented 3-manifold recover its simplicial volume. In this talk, we discuss the variation of the Turaev-Viro invariants for 3-manifolds, with toroidal boundary, under the operation of attaching a $(p, q)$-cabling space. For $p$ and $q$ coprime integers, our main result demonstrates that the Chen-Yang volume conjecture is stable under this $(p, q)$-cabling operation. This is joint work with Joseph Melby. (Received September 12, 2023)

## 1192-57-33196

Deep Kundu*, University of Florida. Minimum number of Critical points and Lusternik Schnirelmann Category.
Let $\backslash \operatorname{crit} M$ denote the minimal number of critical points (not necessarily non-degenerate) on a closed smooth manifold $M$. We are interested in the evaluation of $\backslash$ crit. It is worth noting that we do not know yet whether $\backslash \mathrm{crit} M$ is a homotopy invariant of $M$. This makes the research of $\backslash$ crit a challenging problem. In particular, we pose the following question: given a map
$f: M \rightarrow N$ of degree 1 of closed manifolds, is it true that $\backslash \operatorname{crit} M \geq \backslash \operatorname{crit} N$ ? We prove that this holds in dimension 3 or less. Some high dimension examples are considered. Note also that an affirmative answer to the question implies the homotopy invariance of $\backslash$ crit; this simple observation is a good motivation for the research.
(Received September 13, 2023)

## 1192-57-33218

Semon Rezchikov*, Harvard University. Localization Theorems in Heegard Floer Homology via Floer Homotopy Theory. The Heegard Floer Homology groups $\hat{H F}(M, K)$ are important invariants of a knot $K$ inside an oriented 3-manifold $M$. If $K$ is null-homologous, then for every positive natural number $n$ there is a 3 -manifold $\hat{Y}_{n}(K)$ given by taking the $n$-fold branched cover along $K$. We show that for $n=p$ a prime and $M=S^{3}$ that $\operatorname{dim}_{\mathbb{F}_{\mathrm{p}}} \hat{H F}\left(\hat{Y}_{p}(K), K\right) \geq \operatorname{dim}_{\mathbb{F}_{\mathrm{p}}} \hat{H F}\left(S^{3}, K\right)$. This is achieved by constructing a genuine $C_{p}$-equivariant spectrum which lifts the Heegard Floer Homology $\hat{H} F\left(\hat{Y}_{p}(K), K\right)$. This generalizes previous branch-cover inequalities established by Large, Lipshitz-Treumann, and Hendricks which focus on $p=2$, and is loosely related to results for rational homology spheres obtained by Lidman-Manolescu. The generalization from $p=2$ to other primes has previously proven to be difficult, but becomes straightforward when utilizing the construction of Heegard Floer homology with coefficients in the sphere spectrum.
(Received September 13, 2023)

## 1192-57-33581

Jessica J Zhang*, Harvard University. The Lee Spectral Sequence for Band Sums. Preliminary report.
There is a spectral sequence, known as the Lee spectral sequence, whose zeroth page is the Khovanov complex CKh $(K)$, whose first page is the Khovanov homology $\operatorname{Kh}(K)$, and which converges to $\mathbb{Q} \oplus \mathbb{Q}$. Past the zeroth page, this is a knot invariant. The higher differentials $d_{r}$ for $r \geq 2$ of this spectral sequence are not well-understood; in fact, it was only recently shown in Manolescu and Marengon's disproof of the Knight Move Conjecture that $d_{2}$ can be nontrivial. Our work involves understanding these higher differentials in the case that $K$ is a band sum. In particular, given a band sum $K_{b}$ and letting $K_{b+n}$ be the band sum obtained by adding $n \in \frac{1}{2} \mathbb{Z}$ half-twists to $b$, Wang proved that there is a bigraded vector space $H_{b}$ such that $\operatorname{Kh}\left(K_{b+n}\right)=\operatorname{Kh}\left(K_{\#}\right) \oplus h^{n} q^{2 n} \operatorname{Kh}\left(K_{b}\right)$. We prove a naturality property of these higher differentials, so that $E_{r}\left(K_{b+n}\right)=E_{r}\left(K_{\#}\right) \oplus h^{n} q^{2 n} E_{r}\left(H_{b}\right)$ for every $r$. Furthermore, we adapt an argument by Alishahi and Dowlin to show that the differential $d_{r}$ vanishes on $H_{b}$ if the "band uncrossing number" $b u(K)$ is less than $n$; we define the "band uncrossing number" to be analogous to the typical uncrossing number, but where one of the "strands" in each changed crossing must be the band.
(Received September 21, 2023)

## 1192-57-33688

Anton Levonian*, MIT PRIMES-USA. Existence of Circle Packings on Certain Translation Surfaces.
A translation surface is a surface formed by identifying edges of a collection of polygons in the complex plane that are parallel and of equal length using only translations. Every translation surface is in a stratum determined by the order of its singular points. A circle packing is a collection of interiorwise disjoint discs on a translation surface which can be represented by a contacts graph. We examined the realizability of equivalent circle packings on varying translation surfaces in the $\mathcal{H}(2)$ stratum. We also investigated the possible complexity of contacts graphs in $\mathcal{H}(2)$ and $\mathcal{H}(1,1)$, providing a bound on this complexity in the $\mathcal{H}(2)$ stratum. Finally, we established the possibility of certain contacts graphs' complexities in the generalized genus $g$ strata $\mathcal{H}(2 g-2)$ and $\mathcal{H}(g-1, g-1)$.
(Received September 23, 2023)

## 1192-57-33695

Elizabeth Eileen High*, Williams College, Jake Malarkey*, Williams College. Spherical Braidoids.
V. Turaev introduced knotoids in 2010 to extend classical knot theory, and N. Gügümcü and S. Lambropoulou developed a theory of planar braidoids, which are analogous to braided planar knotoids. In this paper, we consider spherical knotoids, define spherical braidoids, and present an algorithm that uses Seifert diagrams and a sequence of Type II Reidemeister Moves to prove every spherical multi-knotoid has a spherical braidoid projection. We then define a new invariant called spherical braidoid index, and show it can be found by minimizing a feature of the Seifert diagram over all projections.
(Received September 23, 2023)

## 58 Global analysis, analysis on manifolds

## 1192-58-28507

Karen Habermann*, University of Warwick, Philipp Harms, NTU Singapore, Stefan Sommer, University of Copenhagen
Long-time existence of Brownian motion on configurations of two landmarks.
In computational anatomy and, more generally, shape analysis, the Large Deformation Diffeomorphic Metric Mapping framework models shape variations as diffeomorphic deformations. An important shape space within this framework is the space consisting of shapes characterised by $n \geq 2$ distinct landmark points in $\mathbb{R}^{d}$. In diffeomorphic landmark matching, two landmark configurations are compared by solving an optimisation problem which minimises a suitable energy functional associated with flows of compactly supported diffeomorphisms transforming one landmark configuration into the other one. The landmark manifold $Q$ of $n$ distinct landmark points in $\mathbb{R}^{d}$ can be endowed with a Riemannian metric $g$ such that the above optimisation problem is equivalent to the geodesic boundary value problem for $g$ on $Q$. Despite its importance for modelling stochastic shape evolutions, no general result concerning long-time existence of Brownian motion on the Riemannian manifold $(Q, g)$ is known. I will present joint work with Philipp Harms and Stefan Sommer on first progress in this direction which provides a full characterisation of long-time existence of Brownian motion for configurations of exactly two landmarks, governed by a radial kernel.
(Received August 31, 2023)

1192-58-30397
Susan J. Colley, Oberlin College, Gary P. Kennedy*, Ohio State University, Corey Shanbrom, California State University, Sacramento. A new observation about the small growth vector of a Goursat distribution.
A space in the monster tower, also known as the Semple tower, has these three distinct interpretations: (1) In algebraic geometry, it is a natural compactification of the space of curvilinear data in the plane. (2) In differential geometry, it is a universal parameter space for Goursat distributions. (3) In mechanics, it is the configuration space for a truck pulling multiple trailers. It is an ongoing project to relate features of the space that are apparent in one interpretation to their counterparts (perhaps less apparent) in the other interpretations. For example, in each of the interpretations there is a notion of singularity, as I will explain. In current work, we have found a hitherto unobserved connection between the small growth vector of a Goursat distribution and the multiplicity sequence of a corresponding plane curve.
(Received September 09, 2023)

1192-58-31334
Abby Brauer*, Lewis \& Clark College, Andrew Ferris, Lewis and Clark College, Ben Lattes, Lewis \& Clark College, Elizabeth Stanhope, Lewis \& Clark College. Can You Hear the Shape of a Tetrahedron?.
An orbifold is a mildly singular generalization of a manifold. A singular point of a 3-dimensional orbifold is a point that is fixed by a set of 3 -dimensional symmetries. Further, for a 3-orbifold we may compute spectral invariants known as the heat invariants, which are impacted by the singular points of that orbifold. A tetrahedral orbifold is an orbifold whose underlying space is a solid 3-dimensional tetrahedron. We use heat invariants to show that, with the exception of an infinite family, given curvature, the tetrahedral orbifolds are spectrally distinct.
(Received September 11, 2023)

## 1192-58-31836

Eric Bahuaud, Seattle University, Christine Guenther*, Pacific University, James Isenberg, University of Oregon,
Emeritus, Rafe Mazzeo, Stanford University. Stability of higher order geometric flows. Preliminary report.
We discuss recent work on the linear and dynamical stability of a class of higher order geometric flows.
(Received September 12, 2023)

1192-58-32379
E. Kwessi*, Trinity University. Topological Comparison of Some Dimension Reduction Methods Using Persistent Homology on EEG Data. Preliminary report.
In this talk, we will explore how to use topological tools to compare dimension reduction methods. We first make a brief overview of some of the methods often used in dimension reduction such as isometric feature mapping, Laplacian Eigenmaps, fast independent component analysis, kernel ridge regression, and t-distributed stochastic neighbor embedding. We then give a brief overview of some of the topological notions used in topological data analysis, such as barcodes, persistent homology, and Wasserstein distance. Theoretically, when these methods are applied on a data set, they can be interpreted differently. From EEG data embedded into a manifold of high dimension, we discuss these methods and we compare them across persistent homologies of dimensions 0,1 , and 2 , that is, across connected components, tunnels and holes, shells around voids, or cavities. We find that from three dimension clouds of points, it is not clear how distinct from each other the methods are, but Wasserstein and Bottleneck distances, topological tests of hypothesis, and various methods show that the methods qualitatively and significantly differ across homologies. We can infer from this analysis that topological persistent homologies do change dramatically at seizure, a finding already obtained in previous analyses. This suggests that looking at changes in homology landscapes could be a predictor of seizure (Received September 12, 2023)
moreover, the extension splits as the product of manifolds two simpler differential systems both of Lie type. Each solution of the DI system thus arises as the image of a pair of solutions to the simpler systems under a 'superposition' map involving the group action. In this preliminary report on joint work with Mark Fels, we outline a construction of a canonical integrable extension for elliptic DI systems. By contrast, in this case the the extension doesn't split but is a holomorphic system on product of complex manifolds (one factor of which is the complexified Vessiot group). In several examples the extension is contact-equivalent to a prolongation of the Cauchy-Riemann equations, leading to solution formulas for the PDE in terms of arbitrary holomorphic functions and their derivatives.
(Received September 12, 2023)

## 60 Probability theory and stochastic processes

## 1192-60-25444

Mous-Abou Hamadou*, University of Mississippi, Martial Longla, University of Mississippi. Estimation problems for some perturbations of the independence copula.. Preliminary report.
This work provides central limit theorems for estimators of parameters based on functions of Markov chains generated by some perturbations of the independence copula. We provide a study of maximum likelihood estimators and confidence intervals for copula parameters for several families of copulas introduced in Longla (2023). We investigate the particular case Markov chains generated by cosine copulas, sine-cosine copulas and the extended Farlie-Gumbel-Morgenstern copula family. Some tests of independence are proposed. A simulation study is provided for the three copula families. A comparative study of the introduced estimators and other known estimators such as that of Longla and Peligrad (2021) and MLE is proposed, showing advantages of the proposed work.
(Received June 08, 2023)

## 1192-60-25556

Joseph P. Stover*, Gonzaga University. Stochastic domination for Markov-modulated Poisson processes.
Markov-modulated Poisson processes (MMPP) are popular in queuing literature. A Poisson process is a point process where arrivals occur randomly in time. MMPPs are Poisson processes where the arrival rate varies randomly according to an underlying continuous-time homogeneous Markov process. We derive the optimal domination of a standard single-rate Poisson process. The maximal arrival rate is shown to be an eigenvalue of part of the quasi-birth-death process matrix formulation generator matrix. A particular type of monotonicity property is shown to be a sufficient condition for this result to hold. We touch on the more general case as well.
(Received June 21, 2023)

## 1192-60-25803

Martial Longla, University of Mississippi, Sahifa Siddiqua*, Department of Mathematics, University of Mississippi. Parameter estimation of some reversible Markov chains generated by a new family of copula. Preliminary report. This talk explores some copula based reversible Markov chains that are generated by a new family of absolutely continuous symmetric copulas with square integrable densities and the maximum coefficient of correlation that can grow up to 1 . In this study, we explore the properties of these Markov chains along with a maximum likelihood estimation study. A comparative simulations study was done in R.
(Received July 11, 2023)

1192-60-25857
Martial Longla, University of Mississippi, Mathias NTHIANI Muia*, Student. A point on discrete versus continuous statespace Markov chains.
This talk highlights the impact of discrete marginals on copula-based Markov chains. We consider a stationary Markov chain model for Bernoulli trials based on a copula from the Frechet(Mardia) family and show that it is $\psi$-mixing, and hence $\phi, \rho, \beta$ and $\alpha$-mixing. We use this result to emphasize the difference between continuous and discrete state-space Markov chains. The Maximum likelihood approach is applied to derive estimators for model parameters. The asymptotic distributions of parameter estimators are provided. A simulation study showcases the performance of different estimators for the Bernoulli parameter of the marginal distribution.
(Received July 12, 2023)

1192-60-26335
Fidel Djongreba Ndikwa*, University of Maroua, Martial Longla, University of Mississippi. Estimation under parametric assumptions on copula-based Markov chains. Preliminary report.
In this work, we estimate the parameters of a stationary Markov chain whose copula $C(u, v)$ is given by $C(u, v)=u v+\frac{2 \theta_{1}}{4 \pi^{2}}(\cos 2 \pi u-1)(\cos 2 \pi v-1)+\frac{2 \theta_{2}}{4 \pi^{2}}(\sin 2 \pi u)(\sin 2 \pi v)$. In particular, the cases where the marginal distributions are uniforms on $(0,1)$, exponential with parameters $\lambda$ and absolutely continuous with density $f(x)$ are examined, then the asymptotic properties of the estimators are provided for each case studied. We use approaches such as the Maximum Likelihood method and the two-stage pseudo-Maximum Likelihood method to estimate the parameters of this copula. An approach that relies on both Kendall's Tau and Spearman's Rho is proposed to solve the problems that may be encountered by the two others estimation's methods. Finally, we conduct a simulation's study to compare the results of the differents estimation's methods used.
(Received July 25, 2023)

1192-60-26573
Chuntian Wang*, The University of Alabama. On the impact of spatially heterogeneous human behavioral factors on $2 D$ dynamics of infectious diseases.

It is well observed that human natural and social behavior have non-negligible impacts on spread of contagious disease. For example, large scaling gathering and high level of mobility of population could lead to accelerated disease transmission, while public behavioral changes in response to pandemics may reduce infectious contacts. In order to understand spatial characteristics of epidemic outbreaks like clustering, we formulate a stochastic-statistical epidemic environment-humaninteraction dynamic system, which will be called as SEEDS. In particular, a 2D agent-based biased-random-walk model with SEAIHR compartments set on a two-dimensional lattice is constructed. Two environment variables are taken into consideration to capture human natural and social behavioral factors, including population crowding effects, and public preventive measures in the presence of contagious transmissions. These two variables are assumed to guide and bias agent movement in a combined way. Numerical investigations imply that controlling mass mobility or promoting disease awareness can impede a global-scale spatial population aggregation to form, and consequently suppress disease outbreaks. Importance of coordinated public-health interventions and public compliance to these measures are explicitly demonstrated. A mechanistic interpretation of spatial geometric traits in progression of epidemic transmissions is provided through these findings, which may be useful for quantitative evaluations of a variety of public-health policies.
(Received August 01, 2023)

## 1192-60-26779

Kathleen Curtius, University of California San Diego, Brian Johnson, University of California San Diego, Jason R Schweinsberg*, University of California San Diego, Yubo Shuai, University of California San Diego. Estimating the growth rate of a birth and death process from the genealogy of a sample.
Consider a birth and death process in which each individual gives birth at rate $\lambda$ and dies at rate $\mu$, so that the population size grows at rate $r=\lambda-\mu$. Lambert (2018) and Harris, Johnston, and Roberts (2020) came up with methods for describing the exact genealogy of a sample of size $n$ taken from this population after time $T$. We use the construction of Lambert, which is based on the coalescent point process, to come up with point and interval estimates for the growth rate $r$, which are asymptotically valid when $T$ and $n$ are large. We apply this method to the problem of estimating the growth rate of clones in blood cancer.
(Received August 04, 2023)

## 1192-60-27126

Quentin Dubroff*, Rutgers University, Jeff Kahn, Rutgers University. Linear cover time is exponentially unlikely.
Proving a 2009 conjecture of Itai Benjamini, we show: For any $C$, there is $a>0$ such that for any simple random walk on an $n$ vertex graph $G$, the probability that the first $C n$ steps of the walk hit every vertex of $G$ is at most $\exp [-a n]$. A first ingredient of the proof is a similar statement for Markov chains in which all transition probabilities are less than a suitable function of $C$. Joint with Jeff Kahn.
(Received August 10, 2023)

## 1192-60-27205

Tom L Lindstrøm*, University of Oslo. Loeb measures, cylindrical measures, and infinite dimensional Lévy processes. Preliminary report.
One of Peter Loeb's first applications of Loeb measures was a construction of Poisson processes. Soon afterwards, Robert M. Anderson used Loeb's techniques to construct Brownian motion and Itô integrals. Poisson processes and Brownian motions are the basic examples of Lévy processes, and today we have a good understanding of how general Lévy processes in finite dimensions can be obtained as standard parts of hyperfinite random walks. In this talk we shall take a look at the extra complications that appear in the infinite dimensional case.
(Received August 12, 2023)

## 1192-60-27829

David P Herzog, Iowa State University, Jina Kim*, Trinity University. Functional Inequalities for Linear Diffusions with Degenerate Noise. Preliminary report.
We introduce quantitative functional inequalities for specific classes of hypoelliptic stochastic differential equations (SDEs). In particular, these inequalities include the reverse log-Sobolev inequality and Wang-type Harnack inequality for a large class of linear SDEs with degenerate noise. The inequalities are obtained from gradient bounds of the semigroup of the process and by making use of the generalized version of the carré du champ operator. From these results, we obtain quasi-invariance of the same form of SDEs that take values in an infinite-dimensional Hilbert space, deducing a form of hypoellipticity in infinite dimensions.
(Received August 21, 2023)

## 1192-60-27909

Elizabeth W Collins-Woodfin, McGill University, Han G Le*, University of Michigan. Free energy of the bipartite spherical SK model at critical temperature.
The spherical Sherrington-Kirkpatrick (SSK) model and its bipartite analog both exhibit the phenomenon that their free energy fluctuations are asymptotically Gaussian at high temperature but asymptotically Tracy-Widom at low temperature. This was proved in two papers by Baik and Lee, for all non-critical temperatures. The case of critical temperature was recently computed for the SSK model in two separate papers, one by Landon and the other by Johnstone, Klochkov, Onatski and Pavlyshyn. In this presentation, we show the critical temperature result for the bipartite SSK model. In particular, we find that the free energy fluctuations exhibit a transition when the temperature is in a window of size $n^{-1 / 3} \sqrt{\log n}$ around the critical temperature, the same window for the SSK model. Within this transitional window, the asymptotic fluctuations of the free energy are the sum of independent Gaussian and Tracy-Widom random variables. (Received August 22, 2023)

Jacopo Borga*, Stanford University. Universal limits of large random permutations.
Consider a large random permutation satisfying some constraints or biased according to certain statistics. What does it look like? In this talk, we make sense of this question by introducing the notion of permutons. Extensive research has led to the establishment of permuton convergence across various models of random permutations. Our discussion provides an overview of some new recent results in this area. The main goal of the talk is to introduce a new family of universal random limiting permutons called skew Brownian permutons. We will discuss their universal properties and how they relate to several previously studied limiting permutons.
(Received August 24, 2023)

## 1192-60-28065

Persi W Diaconis*, Stanford University. ENUMERATIVE THEORY FOR THE LUCE MODEL.
The Luce model is a widely used non-uniform distribution on permutations. It is used in psychology and taste testing experiments (rank these seven tones), for betting on horse racing (Luce-Plackett model) and in settling poker tournaments (ICM). Each of $n$ items is given a weight $w(i)$ and a permutation is generated by sampling these weights without replacement (each time picking from the remainder with probability proportional to weight). Basic enumerative questions about cycles, descents and inversions are all open. In joint work with Sourav Chatterjee and Gene Kim we determine limiting distributions for the top k and bottom k cards. Joint work with Borga and Chatterjee determines permutation limits.
(Received August 25, 2023)

## 1192-60-28207

Pushpi Paranamana*, Saint Mary's College. Modeling and simulations of belief evolution in social networks-including undergraduate students in research.
Data driven research provides an approach to analyze physical and social systems by exploring data and/or applying data into theoretical frameworks, and gain interesting insights. Involving data into the modeling has its own challenges but also has a unique appeal when it comes to engaging undergraduate students. In this talk, I will highlight an undergraduate research project: modeling belief evolution in social networks and using data to understand the dynamics. Moreover, I will discuss challenges and benefits with respect to student preparation, participation and project outcomes.
(Received September 05, 2023)

## 1192-60-28216

Eliot Hodges*, Harvard University. The Distribution of Sandpile Groups of Random Graphs with their Pairings. In 2015, Clancy, Leake, and Payne observed that the distribution of sandpile groups of Erdős-Rényi random graphs did not seem to obey a Cohen-Lenstra heuristic-a natural first guess for how a family of random finite abelian groups should be distributed. Sandpile groups of connected graphs come canonically equipped with a perfect symmetric pairing; Clancy, Leake, and Payne noticed that this pairing seemed to be affecting the distribution of the groups. They conjectured that a finite abelian $p$-group $G$ equipped with a perfect symmetric pairing $\delta$ appears as the Sylow $p$-part of the sandpile group and its pairing with frequency inversely proportional to $|G||\operatorname{Aut}(G, \delta)|$, where $\operatorname{Aut}(G, \delta)$ is the set of automorphisms of $G$ preserving the pairing $\delta$. We fully resolve this conjecture, generalizing the result by Wood from 2017 on the groups.
(Received August 28, 2023)

1192-60-28217
Isaac Grosof, Carnegie Mellon University, Ziv Scully, MIT/Harvard/Cornell, Runhan Xie*, University of California, Berkeley. Heavy-Traffic Optimal Size-Aware Dispatching
Dispatching systems, where arriving jobs are immediately assigned to one of multiple queues, are ubiquitous in computer systems and service systems. A natural and important model is one in which each queue serves jobs in FCFS (First-Come FirstServed) order, and the dispatcher learns the size (i.e. service time) of each job as it arrives. While this size-aware dispatching model has been extensively studied, little is known about optimal size-aware dispatching for the objective of minimizing mean delay. A major obstacle is that no nontrivial lower bound on mean delay is known, even in heavy traffic (i.e. the limit as load approaches capacity). This makes it difficult to prove that any given policy is optimal, or even heavy-traffic optimal. In this work, we propose the first size-aware dispatching policy that provably minimizes mean delay in heavy traffic. Our policy, called CARD (Controlled Asymmetry Reduces Delay), keeps all but one of the queues short, then routes as few jobs as possible to the one long queue. We prove an upper bound on CARD's mean delay, and we prove the first nontrivial lower bound on the mean delay of any size-aware dispatching policy. Both results apply to any number of queues and allow for heterogeneous server speeds. Our bounds match in heavy traffic, implying CARD's heavy-traffic optimality. In particular, CARD's heavy-traffic performance improves upon that of LWL (Least Work Left), SITA (Size Interval Task Assignment), and other policies from the literature whose heavy-traffic performance is known. (Received August 28, 2023)

## 1192-60-28218

Jacopo Borga, Stanford University, William Da Silva*, University of Vienna, Ewain Gwynne, University of Chicago. The length of the longest increasing subsequence in the Brownian separable permutons..
The Brownian separable permutons are a family of universal limits of random constrained permutations, depending on some parameter $p \in(0,1)$. We provide explicit polynomial bounds for the length of the longest increasing subsequence in the Brownian separable permutons, and present simulations suggesting that the lower bound is close to optimal for all $p$. The strategy relies on a connection to fragmentation processes that I will highlight in the talk. The talk is based on joint work with Jacopo Borga (Stanford University) and Ewain Gwynne (University of Chicago).
(Received August 28, 2023)

Yanni Bills*, University of California, Los Angeles, Feng Cheng*, University of California, Berkeley, Eric Han*, CUNY Baruch College, Quoc Viet Le*, University of Kansas, Scott Hai Wynn*, University of Washington, Seattle, Eric Yu*, University of Pennsylvania. Monotonicity for the Frog Model with Drift on Trees.
The frog model with drift starts with an active particle at the root of the infinite $d$-ary tree and dormant particles at each nonroot site. In discrete time, active particles move towards the root with probability $p$, and otherwise move to a uniformly sampled child vertex. When an active particle moves to a site containing dormant particles, all the particles at the site become active. The critical drift $p_{d}$ is the infimum over all $p$ for which infinitely many particles visit the root almost surely. We give improved bounds on $\sup _{d>m} p_{d}$ and prove monotonicity of critical values associated to a self-similar variant.
(Received September 10, 2023 )

## 1192-60-28369

Yanni Bills, University of California, Los Angeles, Feng Cheng, University of California, Berkeley, Eric Han, CUNY Baruch College, Quoc Viet Le, University of Kansas, Scott Hai Wynn, University of Washington, Seattle, Eric Yu*, University of Pennsylvania. Improved Critical Drift Estimates for the Frog Model on Trees.
The frog model with drift starts with an active particle at the root of the infinite $d$-ary tree and dormant particles at each nonroot site. In discrete time, active particles move towards the root with probability $p$, and otherwise move to a uniformly sampled child vertex. When an active particle moves to a site containing dormant particles, all the particles at the site become active. The critical drift $p_{d}$ is the infimum over all $p$ for which infinitely many particles visit the root almost surely. We give improved bounds on $\sup _{d>m} p_{d}$ and prove monotonicity of critical values associated to a self-similar variant.
(Received August 30, 202 $\overline{3}$ )

## 1192-60-28372

Bhargav Narayanan, Rutgers University, Corrine Yap*, Georgia Institute of Technology. Reconstructing Random Pictures Reconstruction problems ask whether or not it is possible to uniquely build a discrete structure from the collection of its substructures of a fixed size. This question has been explored in a wide range of settings, most famously with graphs and the resulting Graph Reconstruction Conjecture due to Kelly and Ulam, but also including geometric sets, jigsaws, and abelian groups. In this talk, we'll consider the reconstruction of a random picture (n-by-n grid with binary entries) from the collection of its $k$-by- $k$ subgrids and prove a nearly-sharp threshold for $k=k(n)$. Our main proof technique is an adaptation of the Peierls contour method from statistical physics.
(Received August 30, 2023)

## 1192-60-28386

Barbara Haas Margolius*, Cleveland State University. An eigenfunction eigenvalue approach to time-varying queues with periodic transition rates. Preliminary report.
We provide eigenfunction expansions for the asymptotic periodic solutions to a variety of ergodic queues with time-varying periodic transition rates. An asymptotic periodic solution for an ergodic queueing system with time-varying periodic rates is the limiting distribution as a function of time within the period. Examples include the single-server queue, queues with catastrophe and repair, level-independent quasi-birth-death processes, and fluid queues.
(Received August 30, 2023)

1192-60-28627
Dylan Altschuler*, Carnegie Mellon University. Sharp Thresholds for Integer Feasibility Problems.
A classical and often powerful approach to combinatorial optimization problems is to rephrase them as geometric questions about the distance between a domain and a feasible region. Motivated by this, we introduce a general random model of a combinatorial optimization problem with geometric structure that encapsulates both linear programming and integer programming. Let $Q$ and $E$ be closed sets called the domain and the target (feasible region) respectively, and let $A$ be a random linear transform. We introduce and study the $\ell^{q}$-margin,

$$
\mathcal{M}_{q}:=d_{q}(A Q, E)
$$

Our contribution is to show that the $\ell^{q}$-margin is highly concentrated for any $q \in(2, \infty]$ as long as $E$ has permutation symmetry, a combinatorial notion of regularity. The case of $q=\infty$ is of particular interest in applications-specifically to combinatorial "balancing" problems-and is markedly out of the reach of the classical isoperimetric and concentration-ofmeasure tools that suffice for $q \leq 2$. As a result, we establish a number of new results and also recover greatly simplified proofs of a number of key known results on sharp thresholds for random versions of: the closest vector problem, integer linear feasibility, perceptron-type problems, $\ell^{q}$-combinatorial discrepancy for $2 \leq q \leq \infty$, and matrix balancing. Our results also hold for a variety of generalizations of these models.
(Received September 01, 2023)

1192-60-28637
Richard W Kenyon*, Yale University. Restricted permutation matrices.
Given a set S in the unit square, we study the set of permutations whose rescaled permutation matrix avoids S . (Received September 01, 2023)
prove that the non-equilibrium steady state of such system is

$$
\mu=\sum_{I \subset \mathcal{P}([N-1])} F(I)\left(\otimes_{x \in I} \operatorname{Bernoulli}\left(\rho_{R}\right) \otimes_{y \in[N-1] \backslash I} \operatorname{Bernoulli}\left(\rho_{L}\right)\right)
$$

In the formula above $\mathcal{P}([N-1])$ denotes the power set of $[N-1]$ while the numbers $F(I)>0$ are such that $\sum_{I \subset \mathcal{P}([N-1])} F(I)=1$ and given in terms of absorption probabilities of the absorbing stochastic dual process. Via probabilistic arguments we compute explicitly the factors $F(I)$ when the graph is a homogeneous segment. Based on the paper: https://arxiv.org/abs/2307.02481
(Received September 02, 2023)

## 1192-60-28740

Darrell Duffie*, Stanford University. Search and matching in large financial markets.
This talk will explain how recent advances in the modeling of random search and matching (in collaboration with Lei Qiao and Yeneng Sun) can be applied to financial markets and the valuation of traded assets. A key step is to use Sun's 2006 exact law of large numbers to obtain an ordinary differential equation governing the measure-valued process describing the types of agents and their portfolios of assets.
(Received September 02, 2023)

## 1192-60-28766

Justin Liu*, University of Michigan Ann Arbor. Transition Density of Brownian Motions on Metric Graphs. Preliminary report. Metric graphs are graphs where each edge is assigned a length and associated with a closed interval of that length. The study of Brownian motions on metric graphs has garnered significant interest in recent years due to wide-ranging applications in fields like mathematical physics and biology. This talk will provide a gentle introduction to the study of Brownian motions on metric graphs from the ground up, work through some simple examples of metric trees, and compute transition densities of Brownian motions on them. Finally, I will give an overview of some important results in the study of Brownian motions on infinite symmetric metric trees and outline some minor results pertaining to Brownian motions on metric trees that were obtained during my time at the Indiana University Mathematics REU in Summer 2023.
(Received September 02, 2023)

## 1192-60-28789

Yizao Wang, University of Cincinnati, Jacek Wesolowski, Warsaw University of Technology, Zongrui Yang*, Columbia University. Askey-Wilson signed measures and open ASEP in the shock region.
The open asymmetric simple exclusion process (ASEP) is a paradigmatic model for non-equilibrium systems with open boundaries and for KPZ universality. The phase diagram consists of the fan region and the shock region. In this talk, we present a new method for studying the stationary measure of open ASEP in the shock region. We introduce a family of multidimensional Askey-Wilson signed measures and describe the joint generating function of the stationary measure in terms of integrations with respect to these Askey-Wilson signed measures. As an application, we offer a rigorous derivation of the density profile and limit fluctuations of open ASEP in the shock region, confirming the existing physics postulations. (Received September 03, 2023)

1192-60-29014
Nhu N. Nguyen*, University of Rhode Island. Hybrid stochastic epidemic sir models with hidden states.
This paper focuses on realistic hybrid SIR models that take into account stochasticity. The proposed systems are applicable to most incidence rates that are used in the literature including the bilinear incidence rate, the Beddington-DeAngelis incidence rate, and a Holling type II functional response. Given that many diseases can lead to asymptomatic infections, this paper looks at a system of stochastic differential equations that also includes a class of hidden state individuals, for which the infection status is unknown. Assuming that the direct observation of the percentage of hidden state individuals being infected, $\alpha(t)$, is not given and only a noise-corrupted observation process is available. Using nonlinear filtering techniques in conjunction with an invasion type analysis, this paper shows that the long-term behavior of the disease is governed by a threshold $\lambda$ that depends on the model parameters. It turns out that if $\lambda<0$ the number $I(t)$ of infected individuals converges to zero exponentially fast (extinction). However, if $\lambda>0$, the infection is endemic and the system is persistent. We showcase our theorems by applying them in some illuminating examples. (Received September 05, 2023)

## 1192-60-29016

Nhu N. Nguyen*, University of Rhode Island. Stochastic nutrient-plankton models.
We analyze plankton-nutrient food chain models composed of phytoplankton, herbivorous zooplankton and a limiting nutrient. These models have played a key role in understanding the dynamics of plankton in the oceanic layer. Given the strong environmental and seasonal fluctuations that are present in the oceanic layer, we propose a stochastic model for which we are able to fully classify the longterm behavior of the dynamics. In order to achieve this we had to develop new analytical techniques, as the system does not satisfy the regular disspativity conditions and the analysis is more subtle than in other population dynamics models.
(Received September 05, 2023)

Sumit Mukherjee*, Columbia University. Permutons and permutation statistics.
Using the theory of permutation limits (permutons), we study limiting properties of various permutation statistics for random
permutations, such as number of fixed points, number of small cycles, and degree distribution of permutation graphs. Our results apply to many non uniform distributions on permutations, including the the celebrated Mallows model and $\mu$-random permutations. This is based on joint work with Bhaswar Bhattacharya.
(Received September 05, 2023)

## 1192-60-29154

Sayan Das*, University of Chicago. Large deviation principle for $\mu$-random permutations.
In this talk, we will describe a large deviation principle for $\mu$-random permutations. This will allow us to study a general class of probability measures on permutations, where we demonstrate the existence of phase transition (unlike what happens in the standard Mallows model). Time permitting, we shall discuss a new notion of permutons, which we call conditionally constant permutons, that arises naturally from our results. Joint work with Jacopo Borga, Sumit Mukherjee, and Peter Winkler. (Received September 05, 2023)

Jordan Banks*, Howard University, Cassi Chen*, University of California, Berkeley, Stefan Inzer*, University of California, Berkeley, Dylan Marchlinski*, University of Pennsylvania. Gauss-Markov Modeling of GPS Ephemeris and Clock Error. Preliminary report.
The Global Positioning System (GPS) is a global navigation satellite system that consists of a constellation of satellites that broadcast signals to users so that they may compute estimates of their positioning, navigation, and timing (PNT). Additionally, the GPS ground control segment monitors the real-time position, velocity, and clocks of the satellites themselves. These estimates of GPS satellite ephemeris and clock obtained from the control segment play a crucial role in determining the accuracy of GPS user PNT solutions. In our work, we develop a Gauss-Markov model of the broadcast ephemeris and clock error. This statistical error model is generated from and validated against real errors of broadcast GPS ephemeris and clock information and can be used for uncertainty quantification of users' PNT. We initially employ an expectation maximization algorithm to refine our model parameters. However, recognizing the challenges of capturing the nonlinear relationships in the data, we implemented a Gauss-Markov model that utilizes observed periodicity of the data and state augmentation. Moreover, our study extends its focus beyond Earth's orbit by investigating the position accuracy of GPS satellites relative to the proposed lunar space station, Gateway.
(Received September 05, 2023)

## 1192-60-29269

Thuy-Duong Vuong*, Stanford University. Optimal mixing of down-up walk on independence sets of a given size. Let $G$ be a graph on $n$ vertices of maximum degree $\Delta$. We show that, for any $\delta>0$, the down-up walk on independent sets of size $k \leq(1-\delta) \alpha_{c}(\Delta) n$ mixes in time $O_{\Delta, \delta}(k \log n)$, thereby resolving a conjecture of Davies and Perkins in an optimal form. Here, $\alpha_{c}(\Delta) n$ is the NP-hardness threshold for the problem of counting independent sets of a given size in a graph on $n$ vertices of maximum degree $\Delta$. Our mixing time has optimal dependence on $k, n$ for the entire range of $k$; previously, even polynomial mixing was not known. In fact, for $k=\Omega_{\Delta}(n)$ in this range, we establish a log-Sobolev inequality with optimal constant $\Omega_{\Delta, \delta}(1 / n)$.
(Received September 06, 2023)

## 1192-60-29271

Thuy-Duong Vuong*, Stanford University. Learning to Generate Multimodal Distributions via Langevin Diffusions with Databased Initialization.
Is there a natural way to sample multimodal distributions using just the vanilla score function i.e. the gradient of the loglikelihood? Inspired by a long line of related experimental work in Contrastive Divergence training, we prove that the Langevin dynamics with early stopping, initialized at the empirical distribution, and run on a score function estimated from data can successfully learn natural multimodal distributions (mixtures of log-concave distributions from parametric families) with sample complexity polynomial in the dimension. Access to the empirical distribution is natural in the context of generative modeling. Previous works show that given only the vanilla score function but no samples from the empirical distribution, no algorithm can learn a mixture of two Gaussians in sub-exponential time in the dimension.
(Received September 06, 2023)

## 1192-60-29282

James Allen Fill*, The Johns Hopkins University, Department of Applied Mathematics and Statistics, Ao Sun, The Johns Hopkins University, Department of Applied Mathematics and Statistics. On the probability of a Pareto record. Preliminary report.
Given a sequence of independent random vectors taking values in $\mathbb{R}^{d}$ and having common continuous distribution $F$, say that the $n$th observation sets a (Pareto) record if it is not dominated (in every coordinate) by any preceding observation. Let $p(F) \equiv p_{n, d}(F)$ denote the probability that the $n$th observation sets a record. There are many interesting questions to address concerning $p$, but this short talk will focus on showing that for fixed $d \geq 2$ and $n \geq 1$ the image of the mapping $p$ on the domain of all continuous distributions is precisely the interval $\left[n^{-1}, 1\right]$, irrespective of $d$. Our proof relies largely on consideration of two classes of distributions, one being a subclass of the Dirichlet distributions and the other being a certain class of distributions with positively associated coordinates. This is based on ongoing joint work with my Ph.D. advisee Ao Sun. (Received September 06, 2023)

1192-60-29350
Adina Oprisan*, New Mexico State University. Functional limit theorems for a time-changed Brownian motion. Preliminary report.
Additive functionals and the time-changed method of constructing Markov semigroups is used in the case in which the Laplace
operator is multiplied by a state-dependent intensity coefficient. Functional limit theorems, including a large deviation principle on the space of continuous functions, for the normalized time changed Wiener process generated this way will be presented, with an emphasis on the duality between the large deviations theory and that of the weak convergence. (Received September 06, 2023)

## 1192-60-29515

Frederic Koehler*, University of Chicago. Trickle-Down in Localization Schemes and Applications.
Trickle-down is an important phenomena in high-dimensional expanders with many important applications - for example, it is a key ingredient in the proof of rapid mixing for the basis exchange walk in matroids and in the analysis of log-concave polynomials. We formulate a generalized trickle-down equation in the abstract context of linear-tilt localization schemes. Building on this general principle, we improve the best known mixing time results for several different problems - for example, we improve the threshold up to which Glauber dynamics is known to mix rapidly in the Sherrington-Kirkpatrick spin glass model. We also prove strong guarantees for a natural dynamics for antiferromagnetic Ising models on expanders, by building upon the phenomena of polarization from the geometry of polynomials.
(Received September 07, 2023)

1192-60-29573
James Allen Fill, The Johns Hopkins University, Department of Applied Mathematics and Statistics, Ao Sun*, The Johns Hopkins University, Department of Applied Mathematics and Statistics. The Pareto Record Frontier on the Simplex. Preliminary report.
For i.i.d. observations $\mathbf{X}^{(1)}, \mathbf{X}^{(2)}, \ldots$ from the $d$-dimensional simplex

$$
\mathcal{S}_{d}:=\mathbf{x}=\left(x_{1}, \ldots, x_{d}\right): x_{j} \geq 0 \text { forall } 1 \leq j \leq \operatorname{dand} \mathbf{x}_{+} \leq 1
$$

where $\mathbf{x}_{+}:=\sum_{j=1}^{d} x_{j}$, each having the $\operatorname{Dirichlet}(1,1, \ldots, 1, a)$ distribution for a given $a>0$, consider the boundary (relative to $\mathcal{S}_{d}$ ), or "frontier", $F_{n}$ of the closed Pareto record-setting (RS) region

$$
R S_{n}:=\mathbf{x} \in \mathcal{S}_{d}: \mathbf{x} \prec \mathbf{X}^{(i)} \text { forall } 1 \leq i \leq n
$$

at epoch $n$, where $\mathbf{x} \prec \mathbf{y}$ means that $x_{j}<y_{j}$ for $1 \leq j \leq d$. (When $a=1$, the sampling is uniform from $\mathcal{S}_{d}$.) With $\mathbf{x}_{+}:=\sum_{j=1}^{d} x_{j}$, let

$$
F_{n}^{-}:=\min \mathbf{x}_{+}: \mathbf{x} \in F_{n} a n d F_{n}^{+}:=\max \mathbf{x}_{+}: \mathbf{x} \in F_{n}
$$

We describe almost sure behavior of $F_{n}^{+}$and $F_{n}^{-}$as $n \rightarrow \infty$. As a rough summary for $a=1$, up to log factors we almost surely have $1-F_{n}^{+}=\Theta\left(n^{-1}\right)$ while $1-F_{n}^{-}=\Theta\left(n^{-1 / d}\right)$. Fill and Naiman (Electron. J. Probab., 2020) studied the corresponding problem where the coordinates of each observation are independent unit-mean Exponential random variables. The proof techniques for this study are largely similar, but the results are quite different. This is based on ongoing joint work with my Ph.D. dissertation advisor, Jim Fill.
(Received September 07, 2023)

## 1192-60-29576

Chenyang Zhong*, Department of Statistics, Columbia University, New York, NY, 10027. The length of the longest increasing subsequence of Mallows permutation models with $L^{1}$ and $L^{2}$ distances.
Introduced by Mallows in statistical ranking theory, Mallows permutation model is a class of non-uniform probability measures on the symmetric group. The general model depends on a distance metric on the symmetric group. In this talk, I will focus on
Mallows permutation models with $L^{1}$ and $L^{2}$ distances. These two models are also known as "spatial random permutations" in the mathematical physics literature. A natural question from probabilistic combinatorics is: Picking a random permutation from either of the models, what does it look like? This may involve various features of the permutation, a prominent one being the length of the longest increasing subsequence. In this talk, I will explain how multi-scale analysis and the hit and run algorithm-a Markov chain for sampling from both models-can be used to establish law of large numbers for this length. (Received September 07, 2023)

## 1192-60-29766

T E Duncan*, University of Kansas. Stochastic Analysis with Rosenblatt Processes. Preliminary report.
Rosenblatt processes are a family of continuous non-Gaussian processes that have a long range dependence. A stochastic calculus has been developed for these processes that is useful for many stochastic questions with Rosenblatt processes. Two problems of stochastic analysis are considered that are the prediction of Rosenblatt processes and some variants of these processes and some Radon-Nikodym derivatives for Rosenblatt measures arising from stochastic differential equations. Explicit results are given for both of these problems.
(Received September 07, 2023)

1192-60-29787
Ron Peled*, Princeton University. Euclidean random permutations.
We discuss the cycle structure of Euclidean random permutations: random permutations of points in $\mathrm{R}^{\wedge}\{\mathrm{d}\}$ which are biased towards the identity in the underlying geometry. We identify sub-critical, critical and super-critical regimes and the limiting distributions of the cycle lengths in all cases. We also highlight analogies with models of random band matrices. Based on joint works with Dor Elboim and Alexey Gladkich.
(Received September 07, 2023)

## 1192-60-30063

Dmitri Krioukov, Northeastern University, Gabor Lippner*, Northeastern University, Carlo Trugenberger, SwissScientific Techonologies, Pim van der Hoorn, Eindhoven University of Technology. Ollivier-Ricci curvature convergence in random geometric graphs.
In recent years various notions of curvature have been studied for graphs. It is natural to ask if there is a direct relationship between such discrete definitions and the curvature of Riemannian manifolds. We give a positive answer in the case of OllivierRicci curvature, which is a notion based on optimal transport. In particular, we show that under suitable ranges of parameters, the Ollivier-Ricci curvature of a random graph sampled from a manifold is a good approximation of the manifold's curvature. (Received September 08, 2023)

## 1192-60-30234

Emily Beatrice Crawford Das*, University of Georgia, Phong Luu, University of North Georgia, Jingzhi Tie, University of Georgia, Qing Zhang, University of Georgia. Pairs Trading Under a Mean Reversion Model with Regime Switching.
This talk is about an optimal pairs trading rule. A pairs position consists of a long position in one stock and a short position in the other. The problem is to find stopping times to open and then close the pairs position to maximize expected reward functions. In this talk, we consider the optimal pairs trading rule under a mean reversion model with regime switching. The optimal policy is characterized by threshold levels obtained by solving the associated HJB equations (quasi-variational inequalities). Moreover, numerical examples are provided to illustrate the results.
(Received September 09, 2023)

## 1192-60-30262

Guy Bresler*, MIT, Dheeraj Nagaraj, Google AI, Eshaan Nichani, Princeton. Metastable Mixing of Markov Chains: Efficiently Sampling Low Temperature Exponential Random Graphs.
We show how to efficiently generate samples within small total variation distance of the low-temperature exponential random graph model (ERGM). The usual approach to sampling is via Markov chain Monte Carlo. Bhamidi et al. (2008) showed that any local Markov chain suffers from an exponentially large mixing time due to metastable states. We instead consider metastable mixing, a notion of approximate mixing relative to the stationary distribution, for which it turns out to suffice to mix only within a collection of metastable states. We show that the Glauber dynamics for the ERGM at any temperature - except at a lower-dimensional critical set of parameters - when initialized at $G(n, p)$ for the right choice of $p$ has a metastable mixing time of $O\left(n^{2} \log n\right)$ to within total variation distance $\exp (-\Omega(n))$ of the stationary distribution.
(Received September 09, 2023)

## 1192-60-30360

Olena Blumberg, -, Ben Morris, University of California, Davis, Hans Oberschelp*, University of California, Davis. Mixing Time of the Overlapping Cycles Shuffle.
The overlapping cycles shuffle mixes a deck of $n$ cards by moving either the $n$th card or the $m$ th card (for some fixed $m<n$ ) to the top of the deck, with probability one half each. Angel, Peres, and Wilson showed the following surprising fact: If we let $m=\lfloor c n\rfloor$ for any rational number $c \in(0,1)$ then the mixing time of a single card is on the order of $n^{2}$. However if we let $m=\left\lfloor\varphi^{-1} n\right\rfloor$ where $\varphi$ is the golden ratio then the mixing time of a single card is on the order of only $n^{3 / 2}$. We show that these mixing times also apply, up to logarithmic factors, to the mixing of the entire deck. (Received September 09, 2023)

1192-60-30410
Aksheytha Chelikavada*, Saint Louis University. Limit Theorems for Fixed Point Biased Permutations Avoiding a Pattern of Length Three. Preliminary report.
The study of the number of fixed points occurring in a uniformly random permutation has a long history going back to Montmort in the early 1700's. In this paper, we prove limit theorems to help understand the asymptotic behavior of pattern avoiding permutations biased by their number of fixed points. In particular, one case we study features a phase transition where the limiting distribution of fixed points changes abruptly from negative binomial to Rayleigh to normal depending on the bias parameter.
(Received September 10, 2023)

1192-60-30416
Heather Z Brooks, Harvey Mudd College, Philip Chodrow, Middlebury College, Thomas Gebhart, University of Minnesota, Linh Huynh*, Dartmouth College, Vicki Modisette, University of Louisville, Will Thompson, University of Vermont, Moyi Tian, Brown University, Alexander Wiedemann, Randolph-Macon College. Inferring Interaction Kernels for Stochastic Agent-Based Opinion Dynamics.
How individuals influence each other plays an important role in the evolutionary opinion dynamics of the whole population However, information about such interactions is usually not explicit. Instead, we can only observe the dynamics of each individual in the population over time. This calls for the need to develop methods to infer the underlying microscopic mechanisms (i.e. interactions) that give rise to the observed population dynamics. In this project, we consider an asynchronous, discrete-time, continuous-state stochastic model of opinion dynamics in which the change in each individual's opinion over a fixed time period follows a normal distribution. The mean of this distribution is defined with an interaction kernel. We develop likelihood-based methods to infer this interaction kernel from opinion time series data. We will discuss our comparison of different inference approaches on different types of kernels. This work is contributed equally by co-first authors (Tom Gebhart, Linh Huynh, Vicki Modisette, Will Thompson, Moyi Tian, Alex Wiedemann) and co-senior authors (Phil Chodrow, Heather Z. Brooks).
(Received September 10, 2023)

Theodore De Santos*, University of California, Riverside, Alan Krinik, California State Polytechnic University, Pomona, Hubertus Von Bremen, California State Polytechnic University, Pomona. Explicit Transient Probabilities of Markov Chains Having Mainly Transition Step Sizes of 0 or $\pm t$.
For integer $t \geq 2$, we consider two types of Markov Chains with a state space of $S=\{0,1,2 \ldots, H+1\}$ where $H \geq t+1$. The first type of Markov Chain is a Gambler's Ruin Markov Chain having 0 and $H+1$ as the absorbing states. When the process is at one of transient states, i.e. $i=1,2 \ldots, H$, the process can transition from state $i$ back to state $i$ ("stay"), transition from state $i$ to $\min \{H+1, i+t\}$ ("up"), transition from state $i$ to $\max \{0, i-t\}$ ("down"), transition from state $i$ to state 0 ("catastrophe"), or transition from state $i$ to state $H+1$ ("windfall"). The groups of states that communicate with each other can each have their own unique set of probabilities for these transitions. The second type of Markov Chain will have the same state space $S=\{0,1,2 \ldots, H+1\}$ and similar structure, but there are no absorbing states nor any "windfall" and "catastrophe" transitions. When the process is at state $i$, the process can transition from state $i$ back to state $i$ ("stay"), transition from state $i$ to $\min \{H+1, i+t\}$ ("up"), or transition from state $i$ to $\max \{0, i-t\}$ ("down"). Furthermore, there is only one set of "up", "stay", and "down" probabilities. The goal of our talk is to find an explicit formula for the $k$-step transition probabilities of these Markov Chains, where $k>1$. We seek a computationally simpler alternative to taking powers of the onestep transition matrices. For the Gambler's Ruin Markov Chains, the one-step transition matrix has a center matrix $C$ that is a $H \times H$ matrix with non-zero entries on the main diagonal, $t$-th super-diagonal, and $t$-th sub-diagonal. Permutation matrices and known results are used to calculate the eigenvalues, right eigenvectors, and left eigenvectors for these $C$ matrices. The eigenvalues and eigenvectors of $C$ may be substituted into a Sylvester's Eigenvalue Formula for $C^{k}$. In this formula only the eigenvalues of $C$ are raised to the $k$-th power. The formula for $C^{k}$ is used to calculate the $k$-th step transition probabilities by block matrix multiplication. A similar process can be done for the non-absorbing Markov after calculating the Siegmund Dual for the one-step transition matrix and applying Duality Theorem.
(Received September 10, 2023)

## 1192-60-30516

Rongchang Liu*, University of Arizona. Limit theorems of quasi-periodically forced $2 D$ stochastic Navier-Stokes Equations in the hypoelliptic setting.
We consider the incompressible 2D Navier-Stokes equations on the torus driven by a deterministic time quasi-periodic force and a noise that is white in time and degenerate in Fourier space. We show that the asymptotic statistical behavior is characterized by a quasi-periodic invariant measure that exponentially attracts the law of all solutions. The result is true for any value of the viscosity $\nu>0$ and does not depend on the strength of the external forces. By utilizing this quasi-periodic invariant measure, we establish a quantitative version of the strong law of large numbers and central limit theorem for the continuous time inhomogeneous solution processes with explicit convergence rates. It turns out that the convergence rate in the central limit theorem depends on the time inhomogeneity through the Diophantine approximation property on the quasiperiodic frequency of the quasi-periodic force.
(Received September 10, 2023)

## 1192-60-30576

Ahmed El Alaoui*, Cornell University. Fast mixing of Glauber dynamics for the RFIM under exponential decay of correlation. Preliminary report. In this talk we present results of convergence of Glauber dynamics for the ferromagnetic Random Field Ising Model (RFIM) on a cube of volume $n$ in $\mathbb{Z}^{d}, d \geq 2$. We show two main results: \enumerate - At high temperature or strong external field, the chain satisfies a Poincaré inequality with constant proportional to \(n^{o(1)}\) with high probability. This is known to imply total variation mixing form a "cold start" in \(n^{1+o(1)}\) steps of the chain. Prior to this, the only available result was by Bauerschmidt and Dagallier (2022) who showed a Poincaré inequality with constant \(O(1)\) for inverse temperatures \(\beta<\beta_{\text {pure }}(d)\), the critical inverse temperature of the pure Ising model on \(\mathbb{Z}^{d}\). This result was a consequence of a recently discovered correlation inequality by Ding, Song and Sun (2022).
- When the model has exponential decay of correlation, the chain satisfies a weak Poincaré inequality in the sense of Liggett (1991) with constant proportional to \(n^{C}\) for some constant \(C\) with high probability. Such an inequality implies a lower bound on the conductance of large sets of the hypercube \(\{-1,+1\}^{n}\), which by an adaptation of the techniques of Lovasz and Simonovits (1993) implies total variation mixing from a "warm start" in polynomially many steps of the chain. A sampler from a cold start can then be constructed in an iterative manner. To our knowledge no mixing result was previously known for \(d \geq 3\) only assuming exponential decay of correlation. \endenumerate This is based on joint work with Ronen Eldan, Reza Gheissari, and Arianna Piana.
(Received September 10, 2023)


1192-60-30671
Bikram Bhusal*, Bikram Bhusal. Stability analysis of a stochastic logistic growth model with multiplicative $\alpha$-stable Lévy noise.. Preliminary report.
In this talk, we discuss one dimensional stochastic processes defined as solutions to stochastic logistic differential equations driven by Brownian motion and $\alpha$-stable Lévy motion. We present the stability analysis of the solution of a stochastic logistic growth model with multiplicative $\alpha$-stable Lévy noise. We establish the existence of a unique global solution of this model under certain conditions. Then, we find the sufficient conditions for the almost sure exponential stability of the trivial solution of the model.
(Received September 10, 2023)

1192-60-30685
Gerardo Rubino*, INRIA, France. Two algebraic matrix transformations with applications to Markov chain analysis. Preliminary report.
In this talk we introduce two transformations of an arbitrary square matrix $M$ into two new matrices, its power-dual, pd( $M$ ),
and its exponential-dual, $\operatorname{ed}(M)$. Their interest is that the powers of $M$ (resp. the exponential $\exp (M)$ - or the function $\exp (M t$ ) with $t$ scalar) can be obtained by evaluating the powers of its power-dual (resp. the exponential of its exponential-dual). The cost of finding, for instance, $\exp (M)$ in terms of $\exp (\operatorname{ed}(M)$ ), is low (linear). There are examples where these transformations can make the evaluation of powers or of exponentials easier than working with the original matrix. When $M$ is the probability transition matrix - p.t.m. (resp. the infinitesimal generator - i.g.) of a Discrete (resp. of a Continuous) Time Markov Chain $X$, and when the Sigmund dual $X^{*}$ of $X$ exists, then the p.t.m. (resp. the i.g.) of $X^{*}$ is the power-dual (resp. the exponential-dual) of $M$. So, they somehow generalize Sigmund duality to arbitrary matrices, showing that their properties are of algebraic nature. We describe the main properties of these transforms, and provide some examples of their applications. In particular, we show their use in queueing theory, and in analyzing gambler ruin problems.
(Received September 10, 2023)

1192-60-30723
Chenyang Zhong*, Department of Statistics, Columbia University, New York, NY, 10027. Stochastic vertex models with Uturn boundary.
Stochastic vertex models form an important class of solvable lattice models in statistical mechanics and integrable probability. In this talk, I will present a class of stochastic vertex models with U-turn right boundary. The models can be interpreted as interacting particle systems in which particles jump alternately between left and right. In the talk, I will discuss how algebraic and probabilistic properties of such models can be derived using the Yang-Baxter equation and the reflection equation. (Received September 10, 2023)

## 1192-60-30775

Ivan Specht*, Harvard College, Faculty of Arts and Sciences, Harvard University, Cambridge, MA 02138.. Analyzing Generalized Pólya Urn Models using Martingales, with an Application to Viral Evolution.
The randomized play-the-winner (RPW) model is a generalized Polya Urn process with broad applications ranging from viral genomics to clinical trials. We derive an exact expression for the variance of the RPW model by transforming the Polya Urn process into a martingale, correcting an earlier result of Matthews and Rosenberger (1997). We then use this result to approximate the full probability mass function of the RPW model for certain parameter values relevant to genetic applications. Finally, we fit our model to genomic sequencing data of SARS-CoV-2, demonstrating a novel method of estimating the viral mutation rate that delivers comparable results to existing scientific literature.
(Received September 11, 2023)

## 1192-60-30892

Edmund Fosu Agyemang*, School of Mathematical and Statistical Sciences, University of Texas Rio Grande Valley, Mrinal Kanti Roychowdhury, School of Mathematical and Statistical Sciences, University of Texas Rio Grande Valley. Quantization for Probability Distributions.
Quantization for probability distributions refers to the idea of estimating a given probability by a discrete probability supported on a set with no more than $n$ points. It has broad applications in signal processing and data compression. Quantization dimension gives the speed how fast the specified measure of the error goes to zero as n approaches to infinity. I will talk about it. (Received September 12, 2023)

## 1192-60-30903

Sheldon M Ross*, University of Southern California. Finding the best player. Preliminary report.
There are n players, with player i having an unknown strength $\mathrm{v}(\mathrm{i})$, where these strengths are assumed to be the values of independent random variables from a specified distribution. The objective is to find the player having the largest strength. To do so, at each stage a group of players $S$ is selected to play a game, which is won by a player in $S$ with a probability proportional to its strength. Assuming that the strategy employed is required to yield the correct answer with a specified guaranteed confidence, and that subject to that constraint we want a strategy whose expected number of games played before a decision is made is relatively small, we present some heuristic strategies.
(Received September 11, 2023)

## 1192-60-30909

Yunbum Kook, Georgia Institute of Technology, Yin Tat Lee, Microsoft, Ruoqi Shen*, University of Washington, Santosh S Vempala, Georgia Tech. Sampling with Riemannian Hamiltonian Monte Carlo in a Constrained Space. We demonstrate for the first time that ill-conditioned, non-smooth, constrained distributions in very high dimension, upwards of 100,000 , can be sampled efficiently in practice. Our algorithm incorporates constraints into the Riemannian version of Hamiltonian Monte Carlo and maintains sparsity. This allows us to achieve a mixing rate independent of smoothness and condition numbers. On benchmark data sets in systems biology and linear programming, our algorithm outperforms existing packages by orders of magnitude. In particular, we achieve a 1,000-fold speed-up for sampling from the largest published human metabolic network (RECON3D). Our package has been incorporated into the COBRA toolbox.
(Received September 11, 2023)

1192-60-30910
Adam Quinn Jaffe*, UC Berkeley. A Strong Duality Principle for Equivalence Couplings and Total Variation. Classical results of ergodic theory show that, if a sufficiently nice group $G$ acts sufficiently nicely on a Polish space $X$, then for all Borel probability measures $\mu_{1}, \mu_{2}$ on $X$, following properties are equivalent: (i) $\mu_{1}$ and $\mu_{2}$ agree on the $G$-invariant $\sigma$ algebra $\mathcal{I}_{G}$, and (ii) there exists a probability measure $\tilde{\mu}$ on the product space $X \times X$ satisfying $\tilde{\mu} \circ \pi_{i}^{-1}=\mu_{i}$ for $i=1,2$ as well as $\mu\left(E_{G}\right)=1$. In analogy with a fundamental principle of optimal transport theory, we say in this case that the Borel equivalence relation $E_{G}$ satisfies "strong duality". In this work we pose the question of understanding when a general Borel
equivalence relation (not necessarily induced by a group action) satisfies strong duality. We prove that all hypersmooth Borel equivalence relations satisfy strong duality, and we apply this result to determine an exact characterization of the so-called "Brownian germ coupling problem" which has recently been studied in stochastic calculus.
(Received September 11, 2023)

1192-60-30932
Pawel Lorek*, University of Wroclaw. On some interpretation of negative probabilities in the theory of duality of Markov chains. Preliminary report.
The primary application of Siegmund duality is as follows. Assume that a Markov chain $X^{*}$ with a transition matrix (t.m) $P^{*}$ defined on a finite state space with two absorbing states 0 and $M$, is a Siegmund dual of an ergodic chain $X$ with t.m. $P$ and stationary distribution $\pi$. Then, the probability of $X^{*}$ being absorbed in $M$ (aka \textsl\{winning prob.\}) when starting in $j$ may be expressed in terms of $\pi$. This application works in two ways: A1) starting with an ergodic $X$, compute its Siegmund dual and its winning prob. to obtain formula for $\pi$; A2) starting with $X^{*}$, compute its Siegmund anti-dual $X$ and its $\pi$ to find the winning prob. of $X^{*}$. If computing $\pi$ or winning prob. exactly is infeasible, we may estimate it from simulations. The computation of the (anti)-dual comes down to algebraic operation between $P$ and $P^{*}$. To have the method fully working - the resulting t.m. must have non-negative entries. In case of A2, the problem of having negative entries in $P$ was partially mitigated: often, it is enough to find $\pi$ such that $\pi P=\pi$ (to obtain winning prob. of $X^{*}$ ). It is important to note that we cannot estimate $\pi$ via simulations, as $P$ does not correspond to a Markov chain. In this talk we will present some
ltextsl\{interpretation\} of negative probabilities, a stochastic process, where (among others): for each state $i$ we consider \textsl \{particles\} of type $i^{+}$and $i^{-}$; if the ( $i, j$ ) entry is negative, then a particle $i^{+}\left(i^{-}\right)$may appear as $j^{-}\left(j^{+}\right)$; one particle may split into several particles; we also had to introduce sampling from "distribution" having mass greater than 1 . The process remains useful for both A1 and A2. If $P$ has negative entries, we can simulate it and approximate its \textsl \{"stationary distribution" . Similarly, if $P^{*}$ has negative entries, we can simulate it and estimate its \textsl\{ "winning probabilities"\}. We emphasize that this is a preliminary report on ongoing work. We will present examples of chains on two or three states and simulations of their corresponding (anti-)dual with negative entries confirming that it fits into the theory of Siegmund duality. We will also mention the challenges of applying it to strong stationary duality. The talk is based on joint work with Bartłomiej Błaszczyszyn (INRIA/ENS).
(Received September 11, 2023)

## 1192-60-30944

Jonas Sjöstrand*, Mälardalen University. Increasing subsequences in locally uniform random permutations.
The Robinson-Schensted correspondence gives a bijection between permutations of $1,2, \ldots, n$ and pairs of standard Young tableaux of the same shape. A celebrated theorem by Vershik and Kerov and, independently, by Logan and Shepp in the 1970s states that for a uniformly random permutation the corresponding Young diagram converges to a limit shape (under a suitable scaling) as $n$ tends to infinity. By a theorem of Greene, the sum of the lengths of the longest $k$ rows in the Young diagram equals the largest possible cardinality of a union of $k$ increasing subsequences in the permutation, so the Vershik-Kerov-Logan-Shepp result can be viewed as a statement about sizes of increasing subsequences. We will present generalizations of this result in two directions: First, we will allow for random permutations that are only locally uniform: Draw $n$ points independently from some absolutely continuous distribution $\rho$ on the plane and interpret them as a permutation by mapping $i$ to $j$ if the $i$ th point from the left is the $j$ th point from below. Second, we will care not only about the size of the increasing subsequences but also their location. As $n$ tends to infinity, increasing subsequences in the permutation will appear as curves in the plane, and by interpreting these as level curves, a union of increasing subsequences gives rise to a surface. It turns out that, under a suitable scaling, this surface approaches a limit, and the limit surface is a solution to a specific variational problem. As a corollary, the Young diagram under Robinson-Schensted converges to a limit shape. In the special case where $\rho$ is the uniform distribution on the diamond $|x|+|y|<1$ we conjecture that the limit shape is triangular, and assuming the conjecture is true we find an explicit formula for the limit surfaces of a uniformly random permutation and recover the famous limit shape of Vershik, Kerov and Logan, Shepp.
(Received September 11, 2023)

## 1192-60-30971

David P Herzog*, Iowa State University. Explicit rates of convergence to equilibrium for Langevin dynamics.
The goal of this work is to understand how a stochastic differential equation converges to its stationary distribution as key parameters are varied in the system. We begin by reviewing related results for underdamped Langevin dynamics as the friction parameter is either taken to zero or infinity. Even in this simple setting, such results are relatively rare and lack key dynamical understanding. We will then connect these known results with recent results of J. Bedrossian and K. Liss on finitedimensional projections of 2d stochastic Navier-Stokes as the viscosity is taken to zero. Again, in this setting, convergence bounds are deduced as they depend on small viscosity, but lack dynamical understanding. The long-term goal is to find effective dynamics for all of these systems (as viscosity or the friction are taken to zero) that may shed light on the phenomenon of anomalous dissipation.
(Received September 11, 2023)

## 1192-60-31085

Jay Mack Newby*, University of Alberta. Extreme first passage trajectories for populations of identical rare events. A collection of identical and independent rare event first passage times is considered. The problem of finding the fastest out of $N$ such events to occur is called an extreme first passage time. The rare event times are singular and limit to infinity as a positive parameter scaling the noise magnitude is reduced to zero. In contrast, previous work has shown that the mean of the fastest event time goes to zero in the limit of an infinite number of walkers. The combined limit is studied. In particular, the mean time and the most likely path taken by the fastest random walker are investigated. Using techniques from large deviation theory, it is shown that there is a distinguished limit where the mean time for the fastest walker can take any positive value, depending on a single proportionality constant. Furthermore, it is shown that the mean time and most likely path can be approximated using the solution to a variational problem related to the single-walker rare event. (Received September 11, 2023)

## 1192-60-31122

Puja Pandey*, University of Florida. Equivalence of Statistical Distances for Discrete Log-Concave Measures. Preliminary report.
We establish how standard statistical distances are equivalent for discrete log-concave distributions. Distances include total variation distance, Wasserstein distance, Kullback-Leibler distance, Levy-Prokhorov distance and f-divergences. This extends a result of Meckes and Meckes (2014)
(Received September 11, 2023)

## 1192-60-31215

Andrea Ottolini*, University of Washington. Curvature via resistance distance
In a joint work with Karel Devriendt and Stefan Steinerberger, we define a notion of curvature on the vertices of a graph with some desirable properties: it is easy to compute, and it implies bounds on the diameter and the spectral gap. In this talk, I will overview strengths and weaknesses of this curvature, show some examples, and address some natural open questions.
(Received September 11, 2023)

Anant P Godbole*, ETSU. Expected Number of Distinct Subsequences or Patterns in Uniform and Markov Generated Strings or Permutations. Preliminary report.
Consider the expected number of distinct subsequences in random strings. This expected value is known for random binary strings where each letter in the string is, independently, equally likely to be a 1 or a 0 . Biers-Ariel et al. generalized this result to random strings where the letter 1 appears independently with probability $\alpha \in[0,1]$. Also, they made some progress in the case of random strings from an arbitrary alphabet as well as when the string is generated by a two-state Markov chain. In this talk, we generalize to the expected number of patterns in uniform permutations, both when the patterns are consecutive or not. This work is joint with several undergraduates. Markov-versions of these have not been considered as yet, nor have sizebiased permutations and preliminary versions of such results will be presented.
(Received September 11, 2023)

## 1192-60-31382

Bowen Xie*, Washington University in St. Louis. Multi-Component Matching Queues in Heavy Traffic.
We consider multi-component matching queue systems in heavy traffic consisting of $K \geq 2$ distinct perishable components, which arrive randomly over time to their respective queues at high speed at the assemble-to-order station and wait until an instantaneous match or their patience runs out. For a sequence of such systems parameterized by n, when the arrival rates tend to infinity in concert as $n \rightarrow \infty$, we obtain a heavy traffic limit of the appropriately scaled queue length vector characterized by a coupled stochastic integral equation with a scalar-valued non-linear term under mild assumptions. We demonstrate some crucial properties. Motivated by the cost structure of blood bank drives, we formulate an infinite-horizon discounted cost functional and show that the expected value of the cost for the nth system converges to that of the heavy traffic limit as $n$ tends to infinity. Additionally, we exhibit a matching queue model with buffers and some new findings of the coupled Skorokhod problem.
(Received September 11, 2023)

## 1192-60-31402

Mohammed Ali Khan, Johns Hopkins University, Arthur Paul Pedersen, Department of Computer Science, City College of New York, CU, Maxwell B Stinchcombe*, Department of Economics, University of Texas at Austin. Additivity and EvidenceBased Agreement for Bayesians.
Drawing on results primarily from Loeb A non-standard representation of measurable spaces, $L_{\infty}$, and $L_{\infty}^{*}$, (1972), we examine conditions under which finitely additive Bayesian prior beliefs are swamped by experimental data. (1) When Bayesians with countably additive priors learn about a stochastic process based on different countably additive priors, mutual absolute continuity of their priors is necessary and sufficient for "merging," the norm distance between their opinions about the future of the process vanish asymptotically as they learn from experience. But prior opinions that are not countably additive can merge even if they disagree about what will be observed, and can fail to merge even if they completely agree about what wil be observed. (2) When Bayesians learn about the distribution of i.i.d. samples, data swamps any finitely additive prior opinion precisely when the data swamps its nearest Prohorov countably additive counterpart, and should no such counterpart exist, the finitely additive prior has given positive credence to unobservable phenomena.
(Received September 11, 2023)

1192-60-31434
Erik Slivken*, University of North Carolina, Wilmington. Scaling Limits of Restricted Permutations. Preliminary report. Suppose we take a large permutation that is chosen uniformly at random and conditioned to satisfy some restriction. What does this permutation look like? The answer depends on the choice of restriction and how one decides to scale the permutation. We will mainly focus on restrictions related to pattern avoidance. We will discuss some basic permuton limits for various pattern-avoiding classes. When the permuton limit is well understood, we can show some examples where the fluctuations can be nicely described through some appropriately scaled path limits that converge to some form of conditioned Brownian motion.
(Received September 11, 2023)

1192-60-31464
Irfan Alam*, Department of Mathematics, University of Pennsylvania. Using Loeb measures to understand probabilistic symmetries. Preliminary report.

We will give a broad overview of results and ideas in several settings of probability theory where "symmetry" plays a part. In particular, we mention here three such directions: (1) A sequence of random variables is called exchangeable if the joint distribution of any finite subcollection is invariant under permutations. Any sequence of independent and identically distributed (iid) random variables is also exchangeable. Conversely, an exchangeable sequence is only identically distributed but is not necessarily independent. Important for its foundational role in Bayesian statistics, de Finetti's theorem (whenever it holds) allows us to view an exchangeable sequence as a mixture of iid sequences. Prior to the use of Loeb measures in this theory, this theorem was known for exchangeable sequences of random variables taking values in certain types of topological spaces. The general framework of Loeb measures allows us to understand this theorem as a consequence of the common (marginal) distribution of the random variables. In particular, we show that it holds for exchangeable sequences of random variables taking values in any Hausdorff space, as long as their common distribution is Radon. (2) A sequence of real-valued random variables is called spherically symmetric if the joint distribution of any finite subcollection is invariant under rotations in the relevant Euclidean space. All sequences of spherically symmetric random variables are exchangeable, but the converse is not true. Loeb measures on spheres in hyperfinite dimensions were recently used by the speaker to obtain some new asymptotic results in large (standard) dimensions. We will describe those existing results, and also explain some ongoing work in this connection. (3) The previous two items loosely involved probability theory on groups (exchangeability concerns the symmetric group, while spherical symmetry is about the orthogonal group). There is a classical result of Weil and Kodaira that allows one to assign a compatible topological structure on any measurable group with an invariant probability measure. We describe how this leads to interesting topological structures on some natural Loeb spaces arising out of internal groups equipped with invariant internal probabilities. This last item is based on joint work with Ambar Sengupta. (Received September 11, 2023)

## 1192-60-31518

Santanu Chakraborty*, University of Texas Rio Grande valley. Study of Invariant Measures based on Convolution of i.i.d probability measures on $2 \times 2$ Stochastic Matrices.
In 1979, Arunava Mukherjea proved that if $\mu$ is a probability measure on a set $S$ of $2 \times 2$ stochastic matrices, then the sequence of convolution products of $\mu$ converges if and only if $\mu$ is not the point mass at the matrix whose first row consists of 0,1 and the second row consists of 1,0 . There were more studies before and after this work of Mukherjea, particularly on the nature of the invariant measure based on such a convolution sequence. In this talk, we try to compile a summary of such works including some interesting contribution by the current author himself.
(Received September 11, 2023)

## 1192-60-31574

Wuchen Li, University of South Carolina, Shu Liu*, Department of Mathematics, UCLA, Bohan Zhou, Department of Mathematics, UCSB, Xinzhe Zuo, Department of Mathematics, UCLA. Acceleration for MCMC methods on discrete states. Preliminary report.
In this research, we propose a Nesterov type method to accelerate the Markov Chain Monte Carlo (MCMC) algorithm on finite graphs. The MCMC method on a finite graph can be viewed as the gradient flow of a divergence functional. By leveraging the idea from the Nesterov acceleration method, we introduce "momentum" to the MCMC algorithm and propose a second order ODE in the probability space, which can be treated as the accelerated version of MCMC process. We provide Lyapunov analysis to justify the convergence of the algorithm. Some numerical examples are also provided to verify the efficiency of the method.
(Received September 11, 2023)

## 1192-60-31639

Kimberly Marilyn Affeld*, Vassar College, Christian Michael Dean*, Baruch College, Connor Michael Panish*, University of Flordia. Coalescing Ballistic Annihilation. Preliminary report.
During the 1980's, physicists introduced a model called ballistic annihilation that mimics chemical reactions. Particles move throughout the real line at fixed velocities and annihilate upon collision. We generalize a recent breakthrough from Haslegrave, Sidoravicius, and Tournier to a variant in which particles sometimes survive collisions. Specifically, we characterize the initial conditions for all particles to be annihilated. Our arguments make use of recursion and hidden symmetries within infinite sums.
(Received September 11, 2023)

## 1192-60-31662

Johannes Krebs, Catholic University of Eichstätt-Ingolstadt, Wolfgang Polonik*, University of California, Davis, Benjamin Roycraft, University of California, Davis. Inference in Topological Data Analysis.
This talk presents some novel contributions to statistical inference in Topological Data Analysis. The presented inference methods are bootstrap based confidence regions for (persistent) Betti numbers and Euler characteristic curves which come with large sample guarantees. This is joint work with J. Krebs and B. Roycraft.
(Received September 11, 2023)

## 1192-60-31834

Dan Han, University of Louisville, Vicki Modisette*, University of Louisville. Ridge Penalized Bayesian Exponential Random Graph Models for Reducing Multicollinearity. Preliminary report.
Large-scale networks pose problems for analysis with Exponential Random Graph Models (ERGMs). Although these historic models provide critical information about network structure, they suffer from degeneracy and multicollinearity. The Bayesian Exponential Random Graph Model (BERGM) has significantly improved the issue of degeneracy but has left the issue of multicollinearity. To address this weakness, this paper develops a novel penalized BERGM that addresses multicollinearity in large and complex networks. This paper presents the Gibbs sampling theory with penalized priors for the Markov Chain Monte Carlo estimating. The results show significant improvement to past models and provide promising advantages. (Received September 12, 2023)

## 1192-60-31879

Douglas Rizzolo*, University of Delaware. Local limits of random permutations avoiding a pattern of length three. Recently there has been considerable interest in the fine structure of random pattern avoiding permutations. In this talk we will discuss local limits of uniformly random permutations avoiding a pattern of length three that give detailed information about how a random pattern avoiding permutation of $\{1, \ldots, n\}$ acts on $\{1, \ldots, k\}$ for fixed $k$ as $n$ tends to infinity. Local limits in this setting have been considered previously in the literature, but whether or not the limit existed in the case of 321avoiding permutations remained open. In this talk we present a unified approach that allows us to establish convergence for 321-avoiding permutations and, in all cases, gives insight into the regenerative structure of the limiting objects.
(Received September 12, 2023)

## 1192-60-31966

Linsen Liu*, Department of Mathematics, University of Maryland College Park. Conditions for Applying Central Limit Theorem in Real-world with Confidence. Preliminary report.
The central limit theorem (CLT) is one of the most startling theorems in statistics and plays a central role in probability theory. According to the CLT, the sampling distribution of a sample mean of any independent, identically distributed variables with finite expected value and variance is approximately normal given large enough sample size $n$. Since there is little theoretical or statistical inference research on how large $n$ should be and under what conditions the CLT holds in practice, applications of the CLT often use a rule-of-thumb $n \geq 30$, which requires scientific evidence. Uncertainties in applying the CLT may lead to inappropriate inferences and data interpretation. The aim of this study is to work on the challenge via simulation. Given a distribution and a sample size, the sampling distribution of the sample mean of a random variable is evaluated using 8 common tests of normality: Shapiro-Wilk, D'Agostino, Anderson-Darling, Jarque-Bera, Cramer-von Mises, Lilliefors (based on the Kolmogorov-Smirnov test), Pearson Chi-Square, and Shapiro-Francia. To build confidence in how well the normal model fits the data, select a percentile cutoff of p-values from iterated normality tests. A sample size threshold $n_{c}$ is determined by normality tests such that at or above the percentile cutoff, all p-values are greater than or equal to 0.05 for differently distributed data with similar skewness corresponding to $n_{c}$. One can confidently apply the CLT to datasets with $n \geq n_{c}$, where $n_{c}$ is the sample size threshold corresponding to the skewness of the datasets. From a practical perspective, this study provides a confidence framework for the application of the CLT on data sets based on sample size and skewness. (Received September 18, 2023)

## 1192-60-31983

Omer Angel*, UBC, Alexander Holroyd, University of Washington, Tom Hutchcroft, Caltech. Cycle structure in Mallows permutations. Preliminary report.
we study the scaling limit of the cycles of the Mallows permutation of $\mathbb{Z}$, as $q=1-\varepsilon \rightarrow 1$. It was shown by Gladkich and Peled that the cycle of 0 typically has length and diameter of order $\varepsilon^{2}$. Associate to each cycle $C$ a measure on $\mathbb{R}$ defined by

$$
\mu_{C}:=\epsilon^{2} \sum_{n \in C} \delta_{\epsilon^{2} n}
$$

so that the sum of the measures over all cycles converge to Lebesgue measure. We study the scaling limit of this partition of Lebesgue measure. For each $t \in \mathbb{R}$, the densities of the measures at $t$ give a partition $X_{t}$ of unity with Poisson-Dirichelet law. Moreover, the evolution of the partitions $X_{t}$ is a special case of the Ethier-Kurtz diffusion arising in population dynamics. (Received September 12, 2023)

## 1192-60-32096

Heba Ayeda, California State Polytechnic University, Pomona, Corey Bangi, California State Polytechnic University, Pomona, David Beecher*, California State Polytechnic University, Pomona, Alan Krinik, California State Polytechnic University, Pomona. Visualizing The Space Of 1-step Transitions Of Simple Somewhat Stochastic Matrices That Have Steady State Distributions.
Somewhat stochastic matrices, S, are real square matrices whose individual rows sum to 1 while some entries of S may be negative. We develop an eigenvalue characterization of when $S^{k}$ converges to a constant limit matrix L as $\mathrm{k} \rightarrow \infty$ where the rows of $L$ are all equal to a steady-state distribution $n$ that also may have some negative entries. This result generalizes wellknown convergence properties of $S^{k}$ to L for stochastic matrices to somewhat stochastic matrices. For somewhat stochastic matrices S of dimension 2 X 2 and 3X3, we explore various geometric conditions on the 1-step transitions that imply that lim $S^{k}$ $=\mathrm{L}$ as $\mathrm{k} \rightarrow \infty$.
(Received September 12, 2023)

## 1192-60-32176

Kelvin Rivera-Lopez*, Gonzaga University. Up-down chains on permutations and their scaling limits.
An up-down chain is a Markov chain in which each transition can be decomposed into a growth step followed by a reduction step. Generally, these two steps are unrelated, but when they satisfy a natural commutation relation, the up-down chain is particularly amenable to analysis (e.g. one can describe the spectrum of the transition matrix). In the first part of this talk, we will discuss a general framework for analyzing these special up-down chains. This approach will mainly be algebraic and will lead to scaling limits. Afterwards, we will demonstrate this approach by considering some up-down chains on permutations. This example will give rise to a permuton-valued process that we describe explicitly through its generator. Finally, we offer a description of the stationary distribution of that process. This will lead to a new family of (random) permutons. Based on joint work with Valentin Féray. (Received September 12, 2023)

Tatyana Shcherbina*, University of Wisconsin-Madison. Supersymmetric approach to the non-Hermitian random matrices. Supersymmetric approach is based on the representation of the determinant as an integral over the Grassmann (anticommuting) variables. Combining this representation with the representation of an inverse determinant as an integral over the Gaussian complex field, SUSY allows to obtain an integral representation for the main spectral characteristics of random matrices such as limiting density of eigenvalues, correlation functions, the resolvent's elements, etc. This method is widely (and successfully) used in the physics literature and is potentially very powerful but the rigorous control of the integral representations, which can be obtained by this method, is quite difficult. In this talk we will discuss some recent progress in the application of SUSY to the non-Hermitian random matrices.
(Received September 12, 2023)

1192-60-32262
Emma Qiu*, Stephen F Austin High School. Markov Chain Modelling for Complex Chemical Reactions. Preliminary report. A Markov Chain is one special type of stochastic process with the Markov property of which the conditional probability distribution of future states of the process depends only upon the present state and not on any past states. This property makes it possible to predict the future state of a system from its present state ignoring its history. Due to Markov property, Markov Chains have found broad applications in simulating complex systems in physics, chemistry, economics, finance, information theory, etc. The most important advantage of using Markov chain is that the physical models can be presented in a unified description via state vector and a one-step transition probability matrix. This work is the mathematical study of this type of stochastic model for complex chemical reactions by treating reactions as possible transition of Markov Chain. We developed a stochastic equation of Markov chain model for a general bimolecular reaction. Further, we explored the reaction rate equations and reaction mixture composition profile using this model. As an example of a complex reaction system, a reversible reaction in series of CSTR reactors was investigated to illustrate how Markov chain can be applied to determine the transient behavior of open systems undergoing simultaneously heat and mass transfer processes as well as chemical reactions. This work is supervised by Dr. Zheyan Qiu, Staff Scientist at SABIC. Zheyan.Qiu@sabic.com (Received September 22, 2023)

1192-60-32305
Benoit Corsini*, Eindhoven University of Technology. Mallows trees.
In this presentation, I will talk about some recent developments on Mallows trees, that is binary search trees drawn from Mallows permutations. I will first introduce these objects and state some of the early results on their structure, as proven by Evans, Grübel, and Wakolbinger. Then, I will explain through a simple heuristic how these results allowed Addario-Berry and myself to characterize their asymptotic height. Finally, and if time allows, I will provide some extra results regarding their limit under various topologies.
(Received September 12, 2023)

## 1192-60-32350

Owen Biesel*, Southern Connecticut State University. Sheaves of Probability. Preliminary report
What does it mean for multiple agents' credence functions to be consistent with each other, if the agents have distinct but overlapping sets of evidence? Michael Titelbaum's rule, called Generalized Conditionalization (GC), sensibly requires each pair of agents to acquire identical credences if they updated on each other's evidence. However, GC allows for paradoxical arrangements of agent credences that we would not like to call consistent. We interpret GC as a gluing condition in the context of sheaf theory, and show that a slightly stronger version of GC eliminates these paradoxes and gives a satisfying equivalent interpretation of consistency.
(Received September 12, 2023)

1192-60-32402
Na Zhang*, Towson University. On the local limit theorems for linear sequences of lower psi-mixing Markov chains. In this paper we investigate the local limit theorem for partial sums of linear sequences of the form $X_{j}=\sum_{i \in \mathbb{Z}} a_{i} \xi_{j-i}$. Here $\left(a_{i}\right)_{i \in \mathbb{Z}}$ is a sequence of constants satisfying $\sum_{i \in \mathbb{Z}} a_{i}^{2}<\infty$ and $\left(\xi_{i}\right)_{i \in \mathbb{Z}}$ are functions of a stationary Markov chain, centered at zero and have finite second moment. The Markov chain is assumed to satisfy one-sided lower psi-mixing condition. (Received September 12, 2023)

## 1192-60-32469

Saeed Ghahramani*, Western New England University. Relationship Between Remaining Full Busy periods of GI/G/c Queues and Stationary Point Processes.
In a $G I / G / c$ queuing system, a full busy period begins when an arrival encounters exactly $c-1$ customers already in the system and concludes when a departure subsequently results in exactly $c-1$ customers remaining. We will explore the distribution of the remaining full busy period when, at a random epoch, such a period is found to be in progress and demonstrate its relationship with stationary point processes.
(Received September 12, 2023)

1192-60-32511
Xue Bai*, West Virginia University. Using the Instantaneous Reproduction Number $R_{t}$ for Prediction of COVID Incidence in a Rural Setting. Preliminary report.
The instantaneous reproduction number $R_{t}$ is at the center of a widely used framework developed by Cori and coworkers \citeCori2013, and provided in the EpiEstim package. The central assumption of the Cori model is that the incidence on a given day $t, I_{t}$, is a random variable that follows a Poisson distribution with rate $\Lambda_{t} R_{t}$, where the infection potential $\Lambda_{t}$ is constructed from past incidences. The Poisson distribution is conditioned on the value of $R_{t}$, which itself is a random variable that follows a Gamma distribution. During the COVID pandemic, we use $R_{t}$ to make short-term predictions of expected COVID
incidence by county in the state of West Virginia, USA. We provide a recursive definition of the sequence of expected future incidences $\hat{I}_{t}, \hat{I}_{t+1}, \cdots, \hat{I}_{t+\ell}$ that can be inferred from the known incidence $I_{t-1}, I_{t-2}, \cdots, I_{1}$, the estimated reproduction number $R_{t}$ that applies over the interval $t \cdots t+\ell$, and the serial interval $w_{s}$. Here we discuss improvements to our original approach, aimed at including the effect of recurring patterns of test availability, and the impact of larger population centers on rural counties with small population
(Received September 12, 2023)

## 1192-60-32534

Brody Michael Miller*, Appalachian State University. Bringing The Buffon Needle Problem Down To Earth
In the late 18th century, a French mathematician by the name of Buffon posed the question "If we were to drop a needle of some length uniformly onto a piece of wide-ruled paper, what is the probability that the needle will land on at least one of the lines on our paper?". This problem is now known as the Buffon Needle problem. The problem has been solved and actually has a quite accessible solution for those with a calculus and trigonometry background. However, the original Buffon Needle problem is unrealistic due to the force of gravity making the needle more likely to land in certain spots compared to others. Within this exploration, we will discuss why the original problem is unrealistic to experiment with in the real world, how we can bring the Buffon Needle problem "down to Earth" and make it more realistic by applying other probability distributions to the needle drops, and finally, we will discuss how changing the probability distribution of the needle drops alters the probability observed within the original problem.
(Received September 12, 2023)

1192-60-32572
Sebastian J. Schreiber*, University of California, Davis. A dynamical classification of three species, competitive LotkaVolterra stochastic differential equations. Preliminary report.
In 1993, Mary Lou Zeeman identified 33 equivalence classes for three species, competitive Lotka-Volterra differential equations. For 26 of these equivalence classes, the dynamics are completely characterized. For the remaining 7 equivalence classes, the dynamics have yet to be fully characterized. In contrast, I will show that the asymptotic behavior of the stochastic differential equation (SDE) counterparts of these Lotka-Volterra equations generically have only 25 equivalence classes. Moreover, within each equivalence class, the long-term behavior can be fully characterized in terms of the convergence of empirical measures to stationary distributions. As for the deterministic models, each equivalence class is characterized by algebraic inequalities of the model's parameters. Beyond Lotka-Volterra systems, I will show that three species, competitive SDEs can be generically classified into 30 equivalence classes and will illustrate how these additional 5 equivalence classes arise.
(Received September 12, 2023)

1192-60-32654
Yann Frezouls*, Maryville University of St. Louis, Yuanjin Liu, Maryville University of St. Louis. Asset allocation Optimization within a Mean-Variance framework enhanced by Conditional Value at Risk (CVaR). Preliminary report. In portfolio management, achieving an optimal equilibrium between returns and risk appears of fundamental importance. Traditional portfolio optimization methods might not be fully optimized regarding the uncertainty of dynamic financial markets when solely relying on historical data. This paper addresses this challenge by introducing an innovative framework that marries forward-looking simulations with historical data. The primary purpose is to introduce an innovative portfolio optimization methodology by integrating Monte Carlo Simulation outcomes into a newly defined Mean-Variance Optimization type method, enhanced with Conditional Value at Risk (CVaR), to provide a more pertinent approach. Firstly, we leverage Monte Carlo Simulation to simulate diverse future return estimations. Then, we integrate these insights into a dynamic MeanCVaR optimization framework. The objective is to introduce a more comprehensive portfolio optimization approach by considering historical and potential future expected values. This new perspective enables the determination of optimal asset allocation, ultimately leading to enhanced returns.
(Received September 12, 2023)

## 1192-60-32720

Thomas Ratliff, Wheaton College, Stephanie Somersille*, Somersille Math Consulting Services, Ellen Veomett, University of San Francisco. Gaming Districting Metrics. Preliminary report.
There are many metrics in use that attempt to quantify how fair a districting map is. Some popular ones are the Efficiency Gap, Mean Median and Partisan Bias metrics, and now a newer metric, the Geography and Election Outcome metric (The GEOmetric) uses both partisan data and the geographic location of districts to evaluate maps. We discuss methods for using Markov Chain Monte Carlo to create gerrymandered maps that favor one party yet still fall within acceptable bounds for various metrics and we show preliminary results.
(Received September 12, 2023)

1192-60-32996
Subas Acharya*, Johns Hopkins University, Naveen K. Vaidya, San Diego State University. Modeling Options Price Influenced by Disease Pandemic.
The emergence of novel diseases causing pandemics is known to affect stock and options pricing. While many models to predict options pricing are available, existing ones often fail to perform during the pandemic. In this talk, I will present a novel model for option pricing that takes into account the transmission pattern of pandemic diseases. We implemented a sophisticated technique to solve our stochastic models using a machine learning approach. We validate our model using various pricing data, including S\&P 500. Using the sensitivity of our model, we identified disease transmission-related parameters that have a high impact on pricing. Our model may provide a valuable tool to predict option pricing during disease outbreaks under various intervention programs.
(Received September 13, 2023)

Yuansi Chen*, Duke University. Localization Schemes: A Framework for Proving Mixing Bounds for Markov Chains. We introduce localization schemes for analyzing the mixing time of Markov chains on continuous space and discrete hypercube. A localization scheme assigns every probability measure a martingale of probability measures which localize in space as time evolves. The use of localization schemes allows us to reduce the mixing time analysis on the original target distribution to that on many simpler transformed distributions. After reviewing its connection to the existing literature on high dimensional concentration and convex geometry, we first explain Eldan's stochastic localization on Euclidean space. Next, we demonstrate how stochastic localization can be used for sampling Ising models in the uniqueness regime on the discrete hypercube, and discuss the differences between working on discrete and continuous spaces. This observation regarding the differences motivates us to introduce new Poisson-process-driven negative fields localization schemes for analyzing Glauber dynamics for sampling the hardcore model.
(Received September 13, 2023)

## 1192-60-33429

Evan Chang*, MIT PRIMES-USA, Neel Kolhe*, MIT PRIMES-USA. Upper bounds on the 2 -colorability threshold of random $d$ regular $k$-uniform hypergraphs for $k \geq 3$.
For a large class of random constraint satisfaction problems (csp), deep but non-rigorous theory from statistical physics predict the location of the sharp satisfiability transition. The works of Ding, Sly, Sun $(2014,2016)$ and Coja-Oghlan, Panagiotou (2014) established the satisfiability threshold for random regular $k$-NAE-SAT, random $k$-SAT, and random regular $k$-SAT for large enough $k \geq k_{0}$ where $k_{0}$ is a large non-explicit constant. Establishing the same for small values of $k \geq 3$ remains an important open problem in the study of random csps. In this work, we study two closely related models of random csps, namely the 2 -coloring on random $d$-regular $k$-uniform hypergraphs and the random $d$-regular $k$-NAE-SAT model. For every $k \geq 3$, we prove that there is an explicit $d_{\star}(k)$ which gives a satisfiability upper bound for both of the models. Our upper bound $d_{\star}(k)$ for $k \geq 3$ matches the prediction from statistical physics for the hypergraph 2-coloring by Dall'Asta, Ramezanpour, Zecchina (2008), thus conjectured to be sharp. Moreover, $d_{\star}(k)$ coincides with the satisfiability threshold of random regular $k$-NAE-SAT for large enough $k \geq k_{0}$ by Ding, Sly, Sun (2014).
(Received September 16, 2023)

1192-60-33436
Thomas Griffin, Iowa State University, Bailey Hall, Westmont College, Jackson Scott Hebner*, University of Connecticut, David P Herzog, Iowa State University, Denis Selyuzhitsky*, Michigan State University, Kevin Wong*, University of California, Los Angeles, John Wright*, The Ohio State University. Cutoff in the Bernoulli-Laplace Model With Unequal Colors and Urn Sizes.
We consider a generalization of the Bernoulli-Laplace diffusion model in which there are two urns, two colors, and $n$ total balls, of which $r$ are red and where the left urn holds $m$ balls. At each time increment, $k$ balls are chosen uniformly at random from each urn and then swapped. This system can be used to model phenomena such as gas particle interchange between containers or card shuffling. Under a reasonable set of assumptions, we bound the mixing time of the resulting Markov chain asymptotically in $n$ with cutoff at $\log n$ and constant window. Among other tools, we employ spectral analysis on the Markov transition kernel and chain coupling techniques. This extends the results of prior mixing time literature to a wider range of diffusion model parameters.
(Received September 16, 2023)

Roger J Fan*, High School Student, Nitya Mani, Massachusetts Institute of Technology. Multidisperse Random Sequential Adsorption and Generalizations. Preliminary report.
We present a unified study of the limiting density in one-dimensional random sequential adsorption (RSA) processes where segment lengths are drawn from a given distribution. In addition to generic bounds, we are also able to characterize specific cases, including multidisperse RSA, in which we draw from a finite set of lengths, and power-law RSA, in which we draw lengths from a power-law distribution.
(Received September 22, 2023)

## 62 Statistics

## 1192-62-25392

Kimberly Sellers*, North Carolina State University. Dispersed Methods for Handling Dispersed Count Data.
While the Poisson distribution is a classical statistical model for count data, it hinges on the constraining equi-dispersion property (i.e. that the mean and variance equal). This assumption, however, does not usually hold for real count data; overdispersion (i.e. when the variance is greater than the mean) is a more common phenomenon for count data, however data under-dispersion has also been prevalent in various settings. It would be more convenient to work with a structure that can effectively model data (over- or under-) dispersion because it can offer more flexibility (and, thus, more appropriate inference) in the statistical methodology. This talk introduces the Conway-Maxwell-Poisson distribution along with several associated statistical methods motivated by this model to better analyze count data under various scenarios (e.g. distributional theory, generalized linear modeling, control chart theory, and count processes). This talk will likewise acquaint the audience with available associated tools for statistical computing.
(Received June 05, 2023)

## 1192-62-25550

Rachid Belhachemi*, Le Moyne College. A Stochastic Representation Of The Hidden Truncated Normal Distribution. Suppose ( $\mathrm{X}, \mathrm{Y}$ ) has a bivariate normal distribution. Consider the conditional distribution of X given $\mathrm{Y}>\mathrm{yo}$, for some fixed real number yo. Azzalini (1985) considered the special case in which $y_{0}=E(Y)$ while the general case $y_{0} \in \mathbb{R}$ was treated in Arnold et al. (1993). The resulting nontruncated marginal distribution of the observable X is obtained by integrating out the truncated
bivariate normal with respect to the unobserved variable Y. A stochastic representation when $\mathrm{y}_{0}=\mathrm{E}(\mathrm{Y})$ was given by Azzalini (1986) and Henze (1986). The purpose of this paper is to extend this result to any yo $\in \mathbb{R}$. A stochastic representation for such hidden truncated normal (HTN) model allows us to simulate data from the HTN distribution by only generating samples from the normal and the truncated normal variates. Central moments of HTN distribution are easily obtained using this stochastic representation. A two-component HTN mixture distribution is presented and its corresponding stochastic representation and central moments are also provided.
(Received June 20, 2023)

## 1192-62-25911

Philip B Stark*, University of California, Berkeley. Supermartingales and Election Integrity. Yes, really..
Any method of counting votes-by computer or by hand-can err or be subverted. If there is a trustworthy paper record of the votes, a risk-limiting audit (RLA) can provide arbitrarily high probability of correcting the results if the reported winners did not really win, limiting the risk that wrong outcomes will be certified. RLAs are recommended by the National Academies of Science, Engineering, and Medicine. They are required or authorized in 15 U.S. states. When outcomes are correct, well designed random samples can confirm the outcomes of hundreds of partially and completely overlapping contests by inspecting only a small fraction of cast ballot cards, for almost every social choice functions used in political elections: plurality (single and multi-winner), supermajority, instant-runoff voting (IRV), and all scoring rules. The most efficient known methods reduce checking correctness to multiple instances of the problem of (sequentially) testing whether the mean of a finite population of bounded, nonnegative numbers is less than or equal to $1 / 2$, using Ville's inequality applied to test statistics that are supermartingales under the null (i.e., $E$-values). For many social choice functions, the outcome is correct iff every null is false; IRV is more complicated, leading to adaptively reweighted supermartingale tests of unions of intersection nulls. (Received July 15, 2023)

## 1192-62-25981

Martial Longla*, University of Mississippi. On continuous exchangeable Markov chains. Preliminary report
labstract\{New copula families are constructed using orthogonality. Subclasses of idempotent copulas with square integrable densities are derived. It is shown that these copulas generate exchangeable Markov chains that behave like independent and identically distributed random variables conditionally on the value of the initial variable. We prove that the extracted copula family is the only set of symmetric idempotent copulas with square integrable densities. We extend these copula families to asymmetric copulas with square integrable densities with special dependence properties. One of our extensions includes the Farlie-Gumbel-Morgenstern copula family (FGM). Mixing properties of Markov chains generated by these copulas are presented. The Spearman's correlation coefficient $\rho_{S}$ is provided for each of these copula families. Some graphs are provided to illustrate the properties of the copula densities and a statistical study is proposed.\} (Received July 17, 2023)

1192-62-26235
Maimouna Diarra, Coppin State University, Abigail Eck*, Monmouth University, Saba Khanmohammadi, East Carolina University, Jeffrey Liebner, Lafayette College. Using Bayes' Theorem to Analyze Polygraph Tests. Preliminary report. Polygraphs, better known as lie detector tests, yield scores for a person's response to certain questions based on their anxieties. The test relies on the person's respiration, cardiovascular activity, and electrodermal activity to make a judgment regarding the truthfulness or deceit of their answer in response to the questions asked. The Empirical Scoring System, created by polygraph expert Raymond Nelson, scores the different sensors with a -1,0, or 1 corresponding to an increase or decrease in the measured physiological activity. This corresponds to a belief that the person is either lying or telling the truth. The composite score of all the sensors of all the questions can be used as an assessment of whether the subject is lying. The underlying question is then, "What is the probability of telling the truth given a total polygraph score?" Bayes' Theorem is a conditional probability equation that allows us to calculate the probability of telling the truth given a certain polygraph score. Bootstrapping the results of this procedure yields similar answers to Nelson's calculations in his 2011 paper, a remarkable result considering the misapplication of Bayes' Theorem in the original paper.
(Received July 21, 2023)

1192-62-26239
Maimouna Diarra, Coppin State University, Abigail Eck, Monmouth University, Saba Khanmohammadi*, East Carolina University. A Statistical Examination of the ESS Polygraph Scoring System. Preliminary report.
Polygraph tests have long been used in interrogations as a method to determine whether or not an individual is being truthful. In Raymond Nelson's "Five Minute Science Lesson: Bayes' Theorem and Bayesian Analysis", he studies the question, "Given a certain polygraph score, what is the probability that the person is telling the truth?" We use Bayes' Theorem, logistic regression, and bootstrapping to better understand Nelson's results. A meta study collected results where subjects were administered a polygraph and were later determined to be truthful or deceptive. We used this study and parametric bootstrapping to obtain distributions of the probability of truthfulness for a particular polygraph score. Bayes' Theorem allows us to study the probabilities of the scores individually and logistic regression imposes a linear model on the conditional probabilities. We produce an estimate of the probability of truthfulness for various scores with associated credible intervals, which are in agreement with Nelson's recommendations for polygraph cutoffs in his 2011 paper, despite a different mathematical approach.
(Received July 21, 2023)

## 1192-62-26407

Louis Aimé Fono, Associate Professor, Regine Constella Imandi*, PhD Student, Martial Longla, University of Mississippi. A Note on Economic Scenario Generator for Central Africa based on Time Series and Copulas.
Using eight variables (three Cameroon macroeconomic variables: GDP, GDP deflator inflation, discount rates and five financial variables coming from The Central Africa Stock Exchange), we propose an Economic Scenario Generator (ESG) (tool that provides the economic agent with information on the probable evolution of economic and financial variables) tailored to Central African countries in order to improve risk management in the economic sector. Notice that to the best our knowledge
there is no ESG based on variables for Central African countries. More precisely, in our current contribution we study our variables using time series (ARIMA models) and we analyze residuals generated by the obtained models. Due to the inadequacy of correlation coefficient in the context of non-elliptical distributions of residuals of some variables, we study their dependencies using copulas. Based on results of this dependency, we used the Cholesky decomposition (specific to elliptical distributions) to project the macroeconomic variables. Given the inability to project all our variables, we adress the problem of projection of variables whose residuals do not follow a normal distribution. Key Words: Economic Scenario Generator, Time Series, Copulas, Financial Variables, Macro-economic Variables, Non-elliptical Distribution.
(Received July 28, 2023)

## 1192-62-27644

Tahmineh Azizi*, University of Wisconsin-Madison. Altered Structure of EEG signals in patients with Epileptic Seizure. Preliminary report.
Recently, characterizing the dynamics of brain functional networks at task free or cognitive tasks has developed different research efforts in the field of neuroscience. Epilepsy is an electrophysiological brain disease which is accompanied by recurrent seizures. Seizure and epilepsy detection is a main challenge in the field of neuroscience. Understanding the underling mechanism of epilepsy and transition from a normal brain to epileptic brain crucial for the diagnosis and treatment purposes. To understand the organization of epileptic brain network functions at large scales, electroencephalogram (EEG) signals measure and record the changes in electrical activity and functional connectivity. Time frequency analysis and continuous spectral entropy are well developed methods which reveal dynamical aspects of brain activity and can analyze the transitions in intrinsic brain activity. In this work, we aim to model the dynamics of EEG signals of epileptic brain and characterize their dynamical patterns. We use Time frequency analysis to capture the alterations in the structure of EEG signals from patients with seizure. Continuous spectral entropy is used to detect the start of seizures. The main purpose of the current is to explore the changes in the organization of epileptic brain networks. Using time frequency techniques, we are able to draw a big picture of how the brain functions before and during seizure and step forward to classify seizure and corresponding brain activity during different stages of epilepsy. The present study may contribute to characterizing the complex non-linear dynamics of EEG signals of epileptic brain and further assists with biomarker detection for different clinical applications. This finding helps towards effective diagnosis and better treatment of epilepsy. (Received September 25, 2023)

## 1192-62-27656

Rehan Mehta*, Department of Mathematics, Manhattan College. Modeling human observer detection and localization for varying data acquisition in MRI.
Data acquisition times for MRI can be decreased by undersampling, i.e. fully sampling low Fourier frequencies and sparsely sampling high frequencies. We determine the number of low frequencies between $0 \%$ to $20 \%$ to fully sample in order to optimize performance on two-alternative forced choice ( $2-\mathrm{AFC}$ ) and forced localization tasks for 5 x undersampling. For the 2AFC task, observers decided which image out of the two provided had a signal, and their performance remained constant across all conditions. For the forced localization task, observers had to locate a signal in a region, and their performance improved from $0 \%$ to $2.5 \%$ and remaining fairly constant all other conditions. Since human observer trials are time consuming, we used a sparse-difference of Gaussians (SDOG) observer model that projects images into a small number of channels and uses a linear discriminant to predict human performance. We varied the SDOG model and found that both our model templates, symmetric and asymmetric, reasonably predicted the 2-AFC performance, but the 3-channel symmetric model fit slightly better. A symmetric 4 -channel SDOG model, using spatial domain convolution on the possible signal locations, reasonably predicted the forced localization human observer results. (Received August 17, 2023)

## 1192-62-27756

Noah Jeremy Bergam*, Columbia University, Abani Patra, Tufts University. A graph-theoretic approach to altimetry-based surface modeling of the Greenland ice sheet.
The dynamics of the Greenland ice sheet are of keen societal importance as climate change progresses. Satellite laser altimeters like IceSAT and CryoSAT provide us with a wealth of data about these dynamics, but converting this data into a proper digital elevation model (DEM) is a highly non-trivial statistical problem. In this paper, we explore a number of new methods to improve existing DEMs of the Greenland ice sheet. Our main contribution involves using a triangulated graph, as opposed to a raster grid (as is commonly used), in order to discretize the surface. The benefit of the graph data structure over the grid is that it allows for a more flexible incorporation of spatially uneven measurements. We show, both theoretically and empirically, that accounting for uneven measurement distribution in this manner can decrease the error of volume change rate estimates. This error is further reduced in our model by our use penalized splines, as opposed to polynomials, for altitude time series regressions. In addition to presenting this data-driven model--which we call the Graph-theoretic Approach to Greenland Altimetry--we also pose and study a theoretical statistical process which mimics the fundamental problem at play in making predictions based on satellite measurements. Simulations of this process suggest that it is not necessarily beneficial to prioritize crossover points, as previous altimetry surface models do.
(Received August 24, 2023)

1192-62-27814
Holland Ann Bill*, The University of Notre Dame, Elke Doby, James Madison University, Jacob Steger, Vassar College, Daniel Trudell, Rhodes College, Dhanuska Wijesinghe, James Madison University, Prabhashi Withana Gamage, James Madison University. Evaluating the Impact of Riparian Buffer Systems in Enhancing Water Quality in the Shenandoah Valley, Virginia.
The Shenandoah Valley is a scenic region nestled in western Virginia that contains a large portion of the state's agricultural operations. Due to farming practices, the area's ecosystem health has been impacted by extensive land conversion, grazing activity, and other forms of agronomic land use. Consequently, waterways along farmland often experience direct influxes of fertilizers, pesticides, and manure as a result of runoff and movement of groundwater. To counteract such water pollution, the U.S. Department of Agriculture has suggested that landowners install riparian buffers - vegetated areas between the pollutant source and the body of water - to prevent flow of harmful chemicals through nutrient absorption. There are many factors that influence the effectiveness of riparian buffer systems in removing pollutants on farms, including buffer and farm size, land
slope, and other best management practices (BMPs) adopted by landowners. The present study focuses on farms in Shenandoah Valley to determine the efficacy of riparian buffers on improving water quality. The water and sediment analysis includes pH levels and chemical concentrations (such as chloride, sulfate, nitrate, magnesium, etc.), with samples collected from three distinct buffer locations. Geographic Information System (GIS) software also quantifies environmental variables including land slope, farm elevation, and buffer size. By use of linear models, the study determines the impact of riparian buffer and farm characteristics on water quality and ecosystem health. These findings help determine which environmental factors most influence the water quality of these farming systems. This work is supported by an NSF-REU grant (\# 1950370) at James Madison University in Summer 2023.
(Received August 21, 2023)

## 1192-62-28178

Felix M Pabon-Rodriguez*, Indiana University School of Medicine. Advancing Infectious Diseases Research via the HostPathogen Interplay.
The host immune system plays a vital role in controlling and eliminating pathogens during an infection. However, modeling the intricacies of the immune response presents numerous challenges. While there are many mathematical and statistical models that account for various immune processes, a critical gap remains in understanding how within-host disease progression interacts with population-level transmission dynamics. Bridging this gap is paramount to fully grasping the complexities of infectious diseases. To begin, we present a joint model of longitudinal and survival data for Leishmania infection. The model incorporates key drivers of disease progression, including pathogen load, antibody levels, and disease status. We also adapted data on CD4+ and CD8+ T cells to represent inflammatory and regulatory immune factors. The data come from a cohort study of dogs naturally exposed to Leishmania infantum. The model characterizes the relationships between longitudinal biomarkers and time to death from progressive infection. It also predicts individual trajectories of Canine Leishmaniosis (CanL) progression. This within-host model provides a foundation to address critical research questions about complex chronic diseases like Visceral Leishmaniasis (VL). Next, we propose a framework linking a within-host model and a transmission model using Bayesian statistical methods. This novel approach unravels the intricate interplay between within-host and between-host dynamics. Connecting these distinct scales has been hindered by practical and computational hurdles. Closing this gap can uncover new strategies against infectious diseases and greatly benefit societies worldwide. To summarize, this presentation introduces an integrative modeling approach to translate within-host immunological processes to the population level. The proposed framework has the potential to transform our understanding of infectious disease dynamics
(Received August 27, 2023)

## 1192-62-28370

Bhikhari PRASAD Tharu*, Spelman College. Forecasting rainfall of the USA by sequential modeling.
Rainfall is the most important factor in the hydrological system, and it plays a vital role in managing and planning water resources and associated issues. Recurrent neural network (RNN) and long short-term memory (LSTM) deep learning models were applied to forecast monthly rainfall. Historically observed monthly rainfall data from 1950 to 2022 for the USA was obtained from the Historical Climatology Network (HCN) and used for analysis. The results show that both methods perform well and can be used for hydrological study.
(Received August 30, 2023)

## 192-62-28603

Ramadha Piyadi Gamage, Western Washington University, Lucas Takayoshi*, Western Washington University. Modified Information Criterion for Detecting Changes in Linear Regression Models.
We are interested in detecting change point(s) in the coefficients of a linear regression model. The problem may be viewed as a two-sample test adjusted for the unknown change location, where the null hypothesis of no change is tested against the alternative of one change. A model selection technique such as an information criterion can be used to perform this test. As in model selection, this is typically the Schwarz information criterion (SIC). However, the SIC method ignores the contribution of the change location. When the suspected change location is near the first or last data observations, it is difficult and undesirable to confirm a change has occurred. The modified information criterion (MIC) adapts the SIC to the change point problem by including the change location in the penalty term. The test statistic based on the MIC follows an asymptotic chisquare distribution with degrees of freedom equal to the number of beta coefficients in the null model. Monte Carlo simulations show the power of the MIC method is higher than the SIC method at different sample sizes, change sizes, and change locations. Additionally, the method is applied to several real world data sets in various fields of study to further illustrate the testing procedure.
(Received September 01, 2023)

R W R Darling*, National Security Agency, Will Grilliette, National Security Agency, Adam Logan, Tutte Institute for Mathematics and Computation, Carleton University. Rank-based linkage: triplet comparisons and oriented simplicial complexes.
Rank-based linkage is a new tool for summarizing a collection $S$ of objects according to their relationships. These objects are not mapped to vectors, and "similarity" between objects need be neither numerical nor symmetrical. All an object needs to do is rank nearby objects by similarity to itself, using a Comparator which is transitive, but need not be consistent with any metric on the whole set. Call this a ranking system on $S$. Rank-based linkage is applied to the $K$-nearest neighbor digraph derived from a ranking system. Computations occur on a 2-dimensional abstract oriented simplicial complex whose faces are among the points, edges, and triangles of the line graph of the undirected $K$-nearest neighbor graph on $S$. In $|S| K^{2}$ steps it builds an edge-weighted linkage graph $(S, \mathcal{L}, \sigma)$ where $\sigma(\{x, y\})$ is called the in-sway between objects $x$ and $y$. Take $\mathcal{L}_{t}$ to be the links whose in-sway is at least $t$, and partition $S$ into components of the graph $\left(S, \mathcal{L}_{t}\right)$, for varying $t$. Rank-based linkage is a functor from a category of out-ordered digraphs to a category of partitioned sets, with the practical consequence that augmenting the set of objects in a rank-respectful way gives a fresh clustering which does not "rip apart" the previous one. (Received September 01, 2023)

Mahmoud Aldeni*, Western Carolina University. On the Exponential-Gumbel Distribution: Regression and Application.
The primary focus of most lifetime data studies is to examine the connection between the independent variables (or covariates) and the time-to-event variable. In this talk, we introduce a novel regression model that utilizes the Gumbel distribution within the $T-R\{Y\}$ framework methodology. We apply this model to right-censored lifetime data. When compared to existing regression models documented in the literature, our newly proposed model demonstrates enhanced flexibility, indicating its suitability for explaining and characterizing lifetime data.
(Received September 04, 2023)

## 1192-62-28894

Raid Al-Aqtash*, Marshall University. On the T-Exponentiated Exponential \{Cauchy\} Family of Distributions; Properties and Application.
A member of the $T-R\{Y\}$ framework, namely the T-Expoenetiated Exponential \{Cauchy\} family of distributions, is defined and studied. Statistical properties, such as mean deviations, moments and entropy are studied. The method of maximum likelihood is used for parameter estimation and a simulation study is conducted to assess its performance. Real data is used in application of the new family and the fit is compared to other distributions in literature.
(Received September 04, 2023)

## 1192-62-29069

Roger G. Ghanem*, University of Southern California, Nicole Jackson, Sandia National Laboratories, Cosmin Safta, Sandia
National Laboratories, Asmita Shrestha, University of Southern California, Jean-Paul Watson, Lawrence Livermore National Laboratory. Characterization of power grid failure as extremes in diffusion coordinates on a graph. Preliminary report
We demonstrate our statistical approach using the NREL-118 test system which represents three regions in California and consists of 118 buses, 186 transmission lines and 327 generators with 9 different generation technologies. We use Pyomo to calculate optimal generation schedules under specified demand. Data for the NREL-118 is available in the form of hourly demand and generation over a period of 1 year. We construe daily demand as independent samples from a random variable (rv) in $R^{2832}$. For each of these demand samples we evaluate, using Pyomo, with standard industry constraints, the optimal generation over each of the 327 generators, together with the associated cost. We thus have a rv in $R^{10681}\left(10681=24^{*} 118\right.$ $+24 * 362+1$ ). We set a threshold for extreme scenarios based on critical cost. Our task is to characterize the statistics of these extremes and their dependence on geographic and weather conditions. For that we pursue a nonlinear dimension reduction using diffusion on the graph in $R^{10681}$ with 365 vertices. We observe a localization onto a vector space spanned by the dominant 38 evectors of the diffusion matrix. We then carry-out statistical sampling on this manifold. We also identify outliers based on their manifold distance to their neighbors.
(Received September 05, 2023)

1192-62-29090
Dan Han, University of Louisville, Pamela Linares*, University of Louisville. Bayesian Exponential Random Graph Models Under the Horseshoe Prior. Preliminary report.
Exponential Random Graph Models (ERGMs), a widely used family of models for analyzing network data, have recently gained popularity for their wide range of applications and ability to capture the complex dependence structure of networks. However, ERGMs face many statistical challenges, and available models are limited. This paper proposes a Bayesian penalized exponential graph model under the Horseshoe Prior using the Markov Chain Monte Carlo (MCMC) algorithm. Moreover, the paper develops tools for parameter estimation, model and variable selection, and good-fit diagnostics, thereby increasing the applicability of ERGMs.
(Received September 25, 2023)

## 1192-62-29145

Emil M Constantinescu, Argonne National Laboratory, Roger G. Ghanem, University of Southern California, Nicole Jackson, Sandia National Laboratories, Aditi Krishnapriyan, Berkeley University, Rasmus Malik Hoeegh Lindrup, Berkeley University, Xihaier Luo, Brookhaven National Laboratory, Cosmin Safta, Sandia National Laboratories, Jean-Paul Watson, Lawrence Livermore National Laboratory, Wei Xu, Brookhaven National Laboratory, Shinjae Yoo*, Brookhaven National Laboratory. Probabilistic Impact Scenarios for Extreme Weather Event Resilience.
Proactive grid management in the face of an unknown threat remains a challenge for power system operators. Extreme weather events, for example, are typically forecasted hours or days ahead of impact, but pre-event planning to minimize consequences is typically rudimentary at best. However, there are uncertainties that need to be quantified, taking into account the expected timing, location, strength, and duration of the weather threat, as well as the vulnerabilities of the electric power grid. Stochastic optimization algorithms for optimal system redispatch have demonstrated the ability to reduce adverse consequences in the face of an unknown threat. However, more rigorous methods for constructing realistic probabilistic scenarios that serve as the key input to stochastic optimization models are still needed. Modern statistical, artificial intelligence (AI), and machine learning (ML) techniques have enormous potential to assist with scenario generation. However, these models can exhibit unexpected behavior due to a lack of reliable uncertainty/error estimates. To address these gaps, our overall objectives are to: 1) develop new statistical methods to sample tails of temperature and wind distributions; 2) apply AI/ML to generate forecast errors; and 3) provide a demonstration visualization to facilitate model comparisons and explainability.
(Received September 05, 2023)

Ruizhe Qian*, University of California, Santa Barbara, Alexander Shkolnik, University of California, Santa Barbara. Singular vector estimation for random matrices with non stationary columns. }\usepackage\{amsmath\}\usepackage\{amssymb\}\documentConsideranobserved$p\timesn$datamatrix$Y$.Themodelissetupasaclassicalsignal-noiseseparationproblemwith$Y=\thetaX^{\top}+E$,where$X$isalatentfactorand$E$isan$p\timesn$noisematrix.Thegoalistorecover$u=\theta/|\theta|$fromtheobserveddata$Y$sincethelengthof$\theta$isunidentifiable.Thisproblemiswellstudiedforthenoisematrixforwhich$\left|E^{\top}E/p-\nu^{2}I_{n\timesn}\right|\rightarrow0$as$p\rightarrow\infty$forconstant$\nu\in(0,\infty)$.Herelimittheoremsforquantitiessuchas$|u-v|$maybederivedfor$v$theleftsingularvectorof$Y$withthelargestsingularvalue.Weanalyzethecasewhen$\left|E^{\top}E/p-\Delta\right|\rightarrow0$where$\Delta$isanarbitrarydiagonalmatrixand$|\theta|\sqrt{p/n}$convergesin$(0,\infty)$.Thisformulationfindsnumerousapplicationsinstatisticalsettingswherethedatalacksstationarity.WealsomakeprogresstowardsthedevelopmentofaJames-Steinestimator$v^{\mathrm{JS}}$forthissettingthatprovablyachievesasymptoticallyasmallerdistanceto$u$relativetothatforthesingularvector,$|u-v|$.lenddocument(ReceivedSeptember06,2023)undefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefined

1192-62-29381
Xavier Pennec*, Université Côte d'Azur and Inria. Advances in Geometric Statistics for Submanifold Learning.
Geometric statistics aims at developing a consistent statistical framework for objects living in non-linear spaces such as Riemannian manifolds and Lie groups. Beyond the Fréchet mean, several generalization of Principal Component Analysis (PCA) generating geodesic subspaces at the mean point have been proposed for manifolds, such as tangent PCA (tPCA) or Principal Geodesic Analysis (PGA). To better capture the non-linearity of the manifold, we have also proposed a more general family of barycentric subspaces, implicitly defined as the locus of weighted means of reference points. The talk will illustrate two recent extensions for dimension reduction of manifold valued data (submanifold learning). We will first detail an extension of barycentric subspaces to a relaxation of the graph space that exhibits interesting results for the analysis of population of graphs. Then, we will use local tPCA to generate a low dimension principal subbundle of rank k. The geodesics of the resulting sub-Riemannian metric starting from a given point locally generate a submanifold which is radially aligned with the estimated subbundle, even in the non-integrable case. Numerical experiments illustrate the power of our framework by showing that we can achieve astonishingly large range reconstructions even in the presence of quite high levels of noise. This is a joint work with Morten Akhoj, Stefan Sommer, James Benn, Elodie Maignant, Anna Calissano in the context of the ERC project GStatistics.
(Received September 06, 2023)

1192-62-29579
Sachith Eranga Dassanayaka Mudiyanselage*, Wittenberg University. Creating a Framework to Classify Actors and Expose Misleading Identifications in Russian Troll Networks on Twitter through Natural Language Processing. Russian Internet trolls employ fake personas to disseminate disinformation across various social media platforms. Given the escalating frequency of this threat across social media platforms, comprehending troll operations is crucial in mitigating their impact. Drawing upon existing research on the inner workings of influence networks on social media, we propose a novel approach to charting these types of operations. Using Twitter content identified as part of the Russian influence network, we have developed a predictive model to map out these network operations. We classify account types based on their authenticity function within a sub-sample of accounts. This classification is achieved by introducing logical categories and training a predictive model to identify similar behavioral patterns across the network. Tweet content and hashtags represent the core components of Twitter. Therefore, we leverage an investigation into these fundamental elements to study and comprehend the authenticity function of actors and their behaviors. To achieve this, we employ Natural Language Processing to cluster text data, allowing us to map tweets to our logical categories. Validation is conducted by comparing the similarities between the logical categories and their corresponding clusters. The results of this comparison demonstrate that text data clustering accuracy, exceeding $90 \%$, can be improved as an independent model separate from the predictive model. The prediction and validation results suggest that our predictive model can assist with mapping the actors in such networks. Furthermore, the visualization and study of activities and patterns on Twitter present attractive tasks for understanding Russian trolls' behaviors. Given the higher dimensionality of activity-related data, we utilize dimensional reduction techniques to visualize Twitter actions and study their patterns, aiming to identify intriguing relationships and behaviors. (Received September 07, 2023)

## 1192-62-29777

Isaac Aaron Leiterman*, St. Norbert College. Predicting Low-Probability River Floods using Extreme Value Theory. Preliminary report.
In this talk we use historical data on Wisconsin rivers to predict low-probability river flooding events. By employing Extreme Value Theory, we can estimate 1-percent and 0.2-percent annual flooding events and compare them to the current United States Geological Survey estimates for these floods.
(Received September 07, 2023)

## 1192-62-30005

Enrique Guadalupe Alvarado, UC Davis, Robin Belton, Smith College, Emily Fischer, Wheaton College, Kang-Ju Lee, Seoul National University, Sourabh Palande, Department of Computational Mathematics, Science \& Engineering, Michigan State University, Sarah Percival*, Michigan State University, Emilie Purvine, Pacific Northwest National Laboratory. Adaptive Covers for Ball Mapper. Preliminary report.
Ball Mapper, introduced by Dłotko in 2019, takes as input a point-cloud $X$ and produces as output a one-dimensional graph that captures the underlying topology of $X$. The Ball Mapper graph is created by taking as input a parameter $\varepsilon$, generating an $\varepsilon$-net of balls on the points of $X$, and taking the nerve of this open cover. However, the Ball Mapper construction is dependent on not just the choice of $\varepsilon$, but also the algorithm used to construct the $\varepsilon$-net; two different $\varepsilon$-nets on the same input data set may result in Ball Mapper graphs with different topology. To produce a graph more stable with respect to the choice of $\varepsilon$-net, we use the idea of the Adaptive Mapper algorithm introduced by Chalapathi, Zhou, and Wang in 2021. Adaptive Mapper works by initializing a cover, then iteratively splitting intervals based on information criteria to obtain an optimal open cover. We begin by applying Ball Mapper to a data set, then use information criteria to iteratively split the $\varepsilon$-balls in the cover. We then introduce a novel method to merge open sets in the split cover, resulting in a new open cover that produces a graph that is more stable to the initial choice of $\varepsilon$-net and which better reflects the topology of the input data set.
(Received September 08, 2023)

Rasitha R Jayasekare*, Butler University, Mark W Kimpel, Indiana University School of Medicine(retired), Lynne A Kvapil, Butler University, Kim Shelton, University of California Berkeley. Exploring Ancient Vessel Morphology using Model Based Clustering.
How often do statisticians get to work on ancient pottery data from a 14th century archeological site in Greece? As a statistician at a PUI, I was able to collaborate with a group of archeologists to mine data on ancient ceramic vessels which were retrieved from a sealed well deposit found within the archeological site. A model-based cluster analysis method, Gaussian Mixture Models Clustering, was applied to vessel dimensions to identify clusters, and tested using a series of non-parametric testes. The clusters were used to verify the morphology of the ceramic vessels conforming to the standard archeological vessel shapes identified by archeologists. This presentation will discuss the statistical modeling and the results, in application to uncovering clusters in the ancient ceramic vessel data.
(Received September 08, 2023)

1192-62-30210
Hyenkyun Woo*, Logitron X. Bregman-divergence-guided Legendre exponential dispersion model with finite cumulants (kLED). Preliminary report.
Exponential dispersion model is a useful framework in statistics. Primarily, thanks to the additive structure of the model, it can be achieved without difficulty to estimate parameters including mean. However, tight conditions on cumulant function, such as analyticity, strict convexity, and steepness, reduce the class of exponential dispersion model. In this work, we present relaxed exponential dispersion model $k$-LED (Legendre exponential dispersion model with k cumulants). The cumulant function of the proposed model is a convex function of Legendre type having continuous partial derivatives of $k$-th order on the interior of a convex domain. Most of the $k$-LED models are developed via Bregman-divergence-guided log-concave density function with coercivity shape constraints. The main advantage of the proposed model is that the first cumulant (or the mean parameter space) of the 1-LED model is easily computed through the extended global optimum property of Bregman divergence. An extended normal distribution is introduced as an example of 1-LED based on Tweedie distribution. On top of that, we present 2 -LED satisfying mean-variance relation of quasi-likelihood function. A typical example is a regular 2-LED model with power variance function, i.e., a variance is in proportion to the power of the mean of observations. The connection between this model and quasi-likelihood function with power variance function will be presented within the Tweedie distribution framework. (Received September 09, 2023)

## 1192-62-30271

Ernesto Jose Ugona Santana*, Liberty University. Testing Hypothesis of Independence and Identicality of Distribution in Tennis Points; In Search of a More Realistic Method. Preliminary report.
Most research on the probability of winning a tennis match is based on the assumption that the points in a match are independent and identically distributed (iid), usually treating each point as a Bernoulli trial with fixed probability of success. This assumption, however, seems to contradict experience. Players' performance appears to fluctuate as the match progresses, not only due to physical exhaustion, but also as a result of the psychological effect caused by their success - or lack thereof - in previous points or the importance of the point in dispute. To assess the validity of this counterintuitive yet central assumption in tennis analytics, previous research has attempted to model the probability of winning a tennis point with the goal of determining the importance of momentum, or the streak of success in recent points, in helping the player win her next point in the match, hence testing the independence hypothesis. However, current research shows a gap in testing the identicality-ofdistribution hypothesis, a question of broader scope than that of independence. Hence, the purpose of the present study is to test the hypothesis that tennis points are identically distributed throughout a server's match. This objective is accomplished by initially identifying, through appropriate homogeneity tests for sparce data, deviations from the base distribution, with the goal of developing a forecasting model that accounts for perturbations in the distribution. The different models are tested against the iid model and a real dataset through Monte Carlo simulations.
(Received September 09, 2023)

## 1192-62-30402

Isabella Kemajou-Brown*, Morgan State University, Zhiyu Lin, Capital One, Evelyn Sander, George Mason University, Serges Love Teutu Talla, Morgan State University. Estimating Environmental Damages Cost of Cryptocurrency Mining using Statistical Methods.
Mining cryptocurrency uses significant amounts of power involving heavy calculations for transactions verification. In fact, an analysis by researchers at Cambridge University suggests that Bitcoin uses more electricity annually than the whole of Argentina. In this work, we build upon previous calculations of energy used for mining cryptocurrencies to estimate the per coin/dollar economic damages of air pollution emissions associated to climate impacts in the USA for mining ten prominent cryptocurrencies (Bitcoin, Bitcoin Cash, Bitcoin SV, Ethereum, Ethereum Classic, Litecoin, Dash, Zcash, Dogecoin, and Monero). Our results show that the environmental cost of mining can sometimes be as high or even higher than the amount of currency mined. With each cryptocurrency, the rising electricity requirements to produce a single coin can lead to an almost inevitable set of negative net social benefits, without any price increase to the miners. In this paper, we give a detailed analysis of the environmental damages (CO2 released and cost associated) of mining ten different cryptocurrencies, compared to the value of the coin value created. We close with discussion of policy implications.
(Received September 25, 2023)

## 1192-62-30569

Durga Hari Kutal*, Augusta University, Khyam Paneru, The University of Tampa. Estimation of Mixture and Non-mixture Cure Models for Survival Data. Preliminary report.
This project considers mixture and non-mixture cure models for time-to-event data. We are concerned with Bayesian estimation method and the maximum likelihood method to estimate model parameters in both cure models with time-to-event data. The simulation study is based on a non-mixture cure model with modified Gompertz susceptible distribution to evaluate the performance of the model. Furthermore, we apply real data sets to compare the performance of Bayesian and maximum likelihood methods.

Ahmad Alzaghal*, The State University of New York at FSC. New Extensions of the Gumbel Distribution and Their Application.. Preliminary report.
In this research, we employ theoretical principles to introduce new extensions of the Gumbel distribution using the $T-R\{Y\}$
framework. In practice, real-life datasets are utilized to illustrate the usefulness and flexibility of our new extensions.
(Received September 11, 2023)

Yulia Alexandr, University of California, Berkeley, Serkan Hosten*, San Francisco State University. Maximum information divergence from linear and toric models.
We study the problem of maximizing information divergence from a new perspective using logarithmic Voronoi polytopes. We show that for linear models, the maximum is always achieved at the boundary of the probability simplex. For toric models, we present an algorithm that combines the combinatorics of the chamber complex with numerical algebraic geometry. We pay special attention to reducible models and models of maximum likelihood degree one.
(Received September 11, 2023)

## 1192-62-30984

Huimin Cheng*, Boston University. Network ANOVA using Riemannian Manifold. Preliminary report.
The prevalence of unlabeled network data, where each observation corresponds to an unlabeled network, has increased with the abundance of network data. In this context, developing network ANOVA to determine the homogeneity of groups of unlabeled networks becomes crucial. Comparing the unlabelled networks poses great challenges for conceptualization of within-group and between-group variation for groups of unlabelled networks. To address these challenges, we utilize Riemannian manifold tools. Specifically, each node of the network is analogous to a point on a Riemannian manifold, and pairs of connected nodes in a network are analogous to two points connected by a geodesic on a manifold. We employ the OllivierRicci curvature distribution to measure the similarity between networks. We show that two networks with the same structure have the same OR curvature distribution. Comparing groups of networks is thus translated into the problem of comparing groups of curvature distributions. We then employ the well-developed SSNOVA methods to investigate the main effect of these curvature distributions in the reproducing kernel Hilbert space (RKHS). In the RKHS, addition and inner product operations are well-defined, we thus surmount the aforementioned challenges. Our theoretical investigations as well as simulation studies show that our proposed method is a promising tool for analyzing unlabeled networks.
(Received September 11, 2023)

## 1192-62-31047

Sami M. Hamid*, University of North Florida. Asymptotic Properties of MLE's of Parameters of Exponentiated Exponential Lifetime Distributions
We consider distributions generated from exponential-class lifetime distributions by taking positive real powers (greater than 1) of the exponential-class c.d.f. We then consider the asymptotic behavior of the MLE's for the parameters of these distributions.
(Received September 11, 2023)

## 1192-62-31072

Nabil Kahouadji*, Northeastern Illinois University. Chicago structural violence and its effect on predicting colorectal adenoma. Preliminary report.
Adenoma is a type of non-cancerous tumor that may affect various organs. What differentiates an adenoma from a cancer is its slower growth and lower propensity to spread. Thus, adenomas are much less aggressive than adenocarcinomas, which are a type of cancer affecting glands that line organs. Common forms of adenocarcinomas include breast, stomach, prostate, lung, pancreatic and colorectal cancers. Evidence is emerging that cancers of all types are influenced by societal and neighborhoodlevel factors that can elicit toxic and sustained stress responses. These responses promote negative changes and dysregulation of cellular pathways associated with the development of cancers. Colorectal cancer (CRC) is the third most prevalent cancer and the third leading cause of cancer deaths in the United States. Among all racial/ethnic groups, the African American (AA) population exhibits the highest CRC incidence and mortality, for reasons that remain poorly understood. Moreover, the male population exhibits a higher incidence and mortality than the female population with and without controlling for race/ethnicity. With equity in mind, our study used electronic medical records of 4818 patients who received a direct access colonoscopy at The University of Illinois Health System in Chicago from 2015 to 2018, together with two neighborhood factors at the census tract level. We performed a robust and equitable machine learning scoring methodology to 1) predict colorectal adenoma, 2) extract a hierarchy of the biological and socio-environmental predictors, 3) track and compare prediction performances, 4) show the effect of a chronic exposure to violence on predicting colorectal adenoma, and, lastly, 5) identify the optimal model for predicting colorectal adenoma, taking into account the sex and the race/ethnicity of a patient. By carefully avoiding the perpetuation of racial biases embedded in some data-driven and algorithmic approaches, this work has the ultimate goal of empowering individuals to effectively assess their cancer and health risks, and thus enable us to be proactive in detecting cancer and diseases at earlier stages.
(Received September 11, 2023)

1192-62-31249
Shan Shan*, University of Southern Denmark. Exploring statistical shape analysis with manifolds and fiber bundles. In this talk, we will explore how geometric objects such as manifolds and fiber bundles can aid in studying the statistics of shape space. We will begin by discussing the concept of shape space as a manifold and review the use of Diffusion Maps (DM) in analyzing the underlying structure of the space. We will also present a new parameter tuning method for DM that takes
advantage of the semigroup properties of the diffusion kernel. Next, we will consider the shape space as approximately a fiber bundle and review Horizontal Diffusion Maps (HDM). We will introduce a probabilistic model for learning the generative process of data on fiber bundles, and define Gaussian processes on fiber bundles. Throughout the talk, we will use examples from geometric morphometrics to illustrate the use of these methods.
(Received September 11, 2023)

1192-62-31287
Javier Arsuaga, University of California, Davis, Maxime Pouokam*, University of California Davis, Radmila Sazdanovic, NC State University. Topological Analysis of Cancer Genomes.
Cancer is a polygenic disease in which genomic events are selected in order to produce a sophisticated and coordinated outcome. Determining when two events are co-occurring is an important open question in data science. This work focuses on addressing that question in breast cancer genomics by developing topology-based methods for analyzing how Copy Number Aberrations (CNAs) relate to the breast cancer types and prognosis. Our results help identified potential genes that may be related to Lumina A patients, namely PAX8, FGF10, RICTOR, NIPBL, DROSHA, and GLI2. Furthermore, we used differing statistical-based methods which include parametric and nonparametric approached for validation. Our statistical based analysis help validate our findings and narrow down potentials genes to RICTOR, NIPBL, DROSHA, and GLI2. (Received September 11, 2023)

1192-62-31513
Sam Reed Burnett, Western Washington University, Nicholas Jacob Chandler*, Western Washington University, Fiona Cleary, Western Washington University, Kimihiro Noguchi, Western Washington University. Application of a New Class of Statistical Tests to Reaction Times Data. Preliminary report.
We explore an application of a new class of statistical tests, which simultaneously assesses the equality of means and stochastic equality, to reaction times data. The data we examine are reaction times of a two-choice task where participants, those with ADHD and those without, are asked to select one of two buttons after an image appears on their screen. Each participant completed this task 120 times, and we look at the distribution of the 90 th percentiles of the reaction times of these participants. The Kolomogorov-Smirnov test and Q-Q plot suggest that the 90th percentiles are approximately gamma distributed with the shape parameter value of 2 and a possible location shift. Note that the null hypothesis of the new class of tests is rejected when there is statistically significant evidence against either the equality of means or stochastic equality. In addition to examining variations of the new class of tests including those based on the asymptotic normality and Max-T bootstrap, we compare the results to the popular Student's t-test and Brunner-Munzel test. Here, Student's t-test and the Brunner-Munzel test are used for assessing the equality of means and stochastic equality, respectively. Interestingly, the MaxT bootstrap test indicates a statistically significant difference in means but no statistically significant evidence against stochastic equality at the 5 percent significance level. On the other hand, both Student's t-test and the Brunner-Munzel test show no statistically significant evidence against the equality of means and stochastic equality, respectively. We analyze possible causes of the difference in the conclusion regarding the equality of means between the new test and Student's t-test. In addition, we discuss implications of the results on characterizing ADHD based on the upper percentiles of reaction times data.
(Received September 12, 2023)

## 1192-62-31540

Lorin Crawford*, Brown University. Probabilistic Generative Frameworks for Sampling 3D Complex Shapes and Images. The recent curation of large-scale databases with 3D surface scans of shapes has motivated the development of computational tools that better detect global patterns in morphological variation. Recent studies have focused on developing methods for the task of sub-image selection which aims at identifying physical features that best describe the variation between classes of 3D objects. A large piece in assessing the utility of these approaches is to demonstrate their performance on both simulated and real datasets. However, when creating a model for shape statistics, real data can be difficult to access and the sample sizes within these data are often small due to expensive collection procedures. Meanwhile, the landscape of current shape simulation methods has been mostly limited to approaches that use black-box inference-making it difficult to systematically assess the power and calibration of sub-image models. In this talk, we present a new statistical framework for simulating realistic 2D and 3D shapes based on probability distributions which can be learned from real data. We demonstrate this framework in two applications within computational biology: (1) cellular imaging of neutrophils and (2) mandibular molars from four different suborders of primates. (Received September 11, 2023)

1192-62-31706
Tommie A Catanach*, Sandia National Laboratories. Variational Inference and Bayesian Optimal Experimental Design in Scientific Machine Learning.
Uncertainty quantification (UQ) is a critical aspect of Scientific Machine Learning (SciML). In this talk, we will present work on four central aspects of Bayesian UQ for SciML: 1) representing uncertainty, 2) updating uncertainty with data, 3) propagating uncertainty into predictions, and 4) gathering new data for future model refinement. These tasks take the form of Bayesian inference and optimal experimental design. However, traditional methods used to solve UQ problems are no longer possible for SciML. This necessitates approximation methods adapted to ML such as variational inference. We will discuss two aspects of these approximations. First, we will consider methods to quantify the effects of approximation using information theoretic metrics, and designing algorithms that are robust to the approximation. This is particularly important for streaming data where errors can accumulate leading to incorrect inference. Secondly, we will consider goal-oriented methods that seek approximations that prioritize representing, updating, and refining uncertainty about key quantities of interest. This contrasts with standard information theoretic measures of uncertainty for variational inference and optimal experimental design that consider a general notion of uncertainty over all model parameters. Finally, we will discuss these aspects of UQ in the context of building surrogate models for climate simulation.
(Received September 11, 2023)

Sam Zhang*, University of Colorado Boulder. Quantifying the impact of Indigenous land rights and communal land ownership on de/reforestation in Brazil. Preliminary report.
A growing body of research demonstrates that tenure of Indigenous lands improves livelihoods and protects forests in addition to inherently recognizing human rights. However, the effect of tenure on environmental outcomes has scarcely been tested in regions with high development pressure, such as those with persisting forest-agriculture conflicts. I will present joint work with Rayna Benzeev (UC Berkeley) on statistical methods to estimate the average treatment effect of land tenure on forest cover change for 129 Indigenous lands in the Atlantic Forest of Brazil from 1985 to 2019. I will also present ongoing work studying the impacto f communal land ownership on reforestation outcomes in the Atlantic Forest of Brazil.
(Received September 12, 2023)

1192-62-31792
Sophie Farr*, Vassar College, Erin O'Neil*, University of California, Los Angeles, Nicholas Simafranca*, University of Minnesota, Twin Cities, Bryant Willoughby, North Carolina State University. Predicting Post-Wildland Fire Burn Severity in California; A Super Learner Approach. Preliminary report.
Given the increasing prevalence of wildland fires in the Western US, there is a critical need to develop tools to accurately predict burn severity. We developed a novel machine learning model to predict post-fire burn severity using pre-fire remotely sensed data. Hydrological, ecological, and topographical variables collected from four regions of California - the site of the Kincade fire (2019), the CZU Lightning Complex fire (2020), the Windy fire (2021), and the KNP Fire (2021) - were used as predictors of the differenced normalized burn ratio (dNBR). We hypothesized that a Super Learner algorithm that accounts for spatial autocorrelation using Vecchia's Gaussian approximation will accurately predict burn severity. In all combinations of test and training sets explored, the results of our model showed the Super Learner algorithm outperforming each base model i.e. Random Forests, Extra Trees Regression etc.. An increase in predictability after accounting for spatial autocorrelation is also reported. These findings provide actionable insights that enable communities to strategize interventions, such as early fire detection systems and resource allocation during emergency responses. When implemented, this model has the potential to minimize the loss of human life, property, resources, and ecosystems in California.
(Received September 12, 2023)

## 1192-62-31807

Jiazhang Cai, University of Georgia, Huimin Cheng, Boston University, Ping Ma, University of Georgia, Shushan Wu*, University of Georgia, Wenxuan Zhong, University of Georgia. Subsampling in Large Graphs Using Ricci Curvature. In the past decades, many large graphs with millions of nodes have been collected/constructed. The high computational cost and significant visualization difficulty hinder the analysis of large graphs. To overcome the difficulties, researchers have developed many graph subsampling approaches to provide a rough sketch that preserves global properties. By selecting representative nodes, these graph subsampling methods can help researchers estimate the graph statistics, e.g., the number of communities, of the large graph from the subsample. However, the available subsampling methods, e.g., degree node sampler and random walk sampler, tend to leave out minority communities because nodes with high degrees are more likely to be sampled. To overcome the shortcomings of the existing methods, we are motivated to apply the community information hidden in the graph to the subsampling method. Though the community structure is unavailable, community structure information can be obtained by applying geometric methods to a graph. An analog of Ricci curvature in the manifold is defined for the graph, i.e., Ollivier Ricci curvature. Based on the asymptotic results about the within-community edge and betweencommunity edge's OR curvature, we propose a subsampling algorithm based on our theoretical results, the Ollivier-Ricci curvature Gradient-based subsampling (ORG-sub) algorithm. The proposed ORG-sub algorithm has two main contributions: First, ORG-sub provides a rigorous theoretical guarantee that the probability of ORG-sub taking all communities into the final subgraph converges to one. Second, extensive experiments on synthetic and benchmark datasets demonstrate the advantages of our algorithm.
(Received September 12, 2023)

## 1192-62-31845

Julian Bennett*, Research Partner, Lauren Eriksen*, Research Partner. Finding Correlations between External Factors and Covid-19 Rates in Florida. Preliminary report.
Covid-19 swept through the United States very quickly and went on for a very long period of time. In order to study how the disease spread and its growth rate, we developed a series of Susceptible-Infected-Removed (SIR) models that consider various factors such as the time-delay for Covid-19 symptoms, reinfection, or a changing susceptible population. Based on the data analyzed by our new SIR models, we then studied the correlation and causal inference between Covid cases and outside factors. These outside factors include age, gender, education, and weather, and the data are obtained from open source websites. Through multivariate regression, we could pinpoint the particular factors that were more or less correlated to the growth of Covid-19 cases. With this information, we can help suggest new government protocol in order to slow down the pandemic spread.
(Received September 12, 2023)

1192-62-31884
Robert J. Ravier*, Sarcos Technology and Robotics Corporation. Hypothesis Testing on Patch Spaces via Manifold Moving Least Squares with Application to Evolutionary Anthropology.
Methods for performing hypothesis testing on anatomical surfaces, such as multivariate T-tests on corresponding vertices, tend to produce nonmeaningful results when the collection of surfaces vary over multiple species, as is often the case in evolutionary anthropology; it is difficult to determine which areas on which surfaces differ in a statistically significant sense Furthermore, anthropological settings often suffer from low sample sizes, calling the reliability of parametric methods into question. Motivated by these problems, we propose a novel nonparametric statistical test based on apply manifold moving least-squares (MMLS), a technique for computing approximating manifolds for high-dimensional point clouds, to spaces of corresponding patches on a given collection of surfaces of interest. Specifically, we use the Riemannian metric of the learned manifold to define a notion of distance between any two patches, for which we have theoretical guarantees of approximation accuracy under mild assumptions, and propose a test based on bootstrapped distributions of distances on the manifold. After
going over details of the registration and statistical methodology, we apply our proposed methodology to an anthropological case study, illustrating the potential our methodology has to shed light on evolutionary timelines. Joint work with Barak Sober, Doug Boyer, and Ingrid Daubechies.
(Received September 12, 2023)

Guangliang Chen*, Hope College. Fast, memory-efficient spectral clustering with cosine similarity.
Spectral clustering is a popular and effective clustering method but known to face two significant challenges: scalability and out-of-sample extension. In this talk, we address these two issues of spectral clustering in the special setting of cosine similarity in order to deal with massive or online data that are too large to be fully loaded into computer memory. We start by assuming a small batch of data drawn from the full set and develop an efficient procedure that learns both the nonlinear embedding and clustering map from the sample and extends them easily to the rest of the data as they are gradually loaded. We then introduce an automatic approach to selecting the optimal sample size based on the Grassmannian metric. The combination of the two steps leads to a streamlined memory-efficient algorithm that only uses a small number of batches of data (as they become available), with memory and computational costs that are independent of the size of the data. Experiments are conducted on benchmark data to demonstrate the fast speed and excellent accuracy of the proposed algorithm. We conclude the talk by pointing out several future research directions
(Received September 12, 2023)

1192-62-32024
Mark Curiel, University of Hawai'i at Manoa, Elizabeth Gross*, University of Hawai`i at Manoa, Nicolette Meshkat, Santa Clara University. The singular locus of directed graphical models. Preliminary report.
A structural equation model (SEM) is a multivariate statistical model having a parametrization induced by a mixed graph. Because of their flexibility and ability to model the effect of latent random variables, these directed graphical models have applications to a variety of fields including ecology, epidemiology, and microbiology. In this talk, we look at parameter identifiability, the question of which parameters of a model can be determined from known data. For structural equation models, this question amounts to testing if the parameters of the model can be determined from the covariance matrix, or more precisely, if the mapping from the parameters to the covariance matrix entries is generically finite-to-one. This amounts to checking the rank of the Jacobian of this mapping evaluated at a generic point. However, an interesting question arises: for what parameter values does this Jacobian matrix drop in rank? This can be answered by examining the singular locus, which is given, e.g., by the determinant of the Jacobian matrix when it is square, or more generally by its minors. We consider some families of graphs and find the corresponding formulas for the singular locus equations.
(Received September 12, 2023)

1192-62-32094
Lake Bookman*, Monash University. Mixtures of Nonlinear Regressions: Experiments with Expectation-Maximization. Preliminary report.
Gaussian mixture models provide a versatile framework widely used for data clustering. The fundamental structure of these models is to posit that each sample from the data set is drawn from one of a collection of distributions, but which distribution and its parameters remain unknown. While finding optimal algoirthms for fitting such models to large data sets is a rich area of research, the classical expectation-maximization scheme provides an efficient, easy to program method suitable for a wide range of problems. While mixture models enjoy popularity for data clustering, they are less widely utilised for regression. Nevertheless, in a many applications the observed data are drawn mutliple underlying models. In order to accurately infer the parameters of these underlying models, the latent variable assignment of each data point to a model must also be solved. While existing work establishes the convergence of expectation-maximization for mixtures of linear regressors, little work has been done to establish the utilty for method mixtures of nonlinear models. This talk discusses the generalization of expectation-maximization methods to fully nonlinear models and variations on the underlying noise model of the data. The utility of these algorithms is discussed in the context of data drawn from multiple applications: specifically multitarget tracking and image processing.
(Received September 12, 2023)

1192-62-32104
Lorin Crawford, Brown University, Cassie Ding*, Brown University, Ani Eloyan, Brown University School of Public Health, Mattie Ji, Brown University, Henry Kirveslahti, Laboratory for Topology and Neuroscience, EPFL, Kun Meng, Brown University, Jinyu Wang, Brown University. Statistical Inference on Grayscale Images via the Euler-Radon Transform. Tools from topological data analysis have been widely used to represent binary images in many biological and medical applications, like magnetic resonance imaging (MRI), computed tomography (CT), and positron emission tomography (PET.) Methods that aim to represent grayscale images (i.e., where pixel intensities instead take on continuous values) have been relatively underdeveloped. In this talk, we introduce the Euler-Radon transform (ERT,) which generalizes the Euler characteristic transform (ECT) to grayscale images by using o-minimal structures and Euler integration over definable functions. Specifically, we extend the invertibility findings of the ECT and smooth Euler characteristic transform (SECT) by Ghrist et al (2018) to ERT and smooth Euler-Radon transform (SERT.) Applying these results, we characterize the relationship between ERT and SERT. Coupling the Karhunen-Lo'eve expansion with our proposed topological representation, we offer hypothesis-testing algorithms based on the $\chi^{2}$ distribution for detecting significant differences between two groups of grayscale images.
(Received September 12, 2023)

1192-62-32338
Md Sazib Hasan*, Utah Tech University. Fostering Inner Creativity: The Benefits and Challenges of Encouraging Student-led Projects Based on Class Curriculum. Preliminary report.
The purpose of the talk is to present the significance of projects that involve students utilizing their own ideas following exposure to the class curriculum. The students' ideas are indicative of their creativity, authenticity, diversity, and the quality of
their cognitive processes. This pedagogical approach was applied in various classes throughout the semester, resulting in the development of students' inner creativity. The talk will discuss the benefits, challenges, and rationale behind this approach, as well as its impact on the students' learning process in an authentic learning environment. Additionally, the talk will showcase some exemplary projects that were created by the students as a final group project, emphasizing the need for building a collaborative environment within the classroom.
(Received September 12, 2023 )

1192-62-32364
Myrine Barreiro-Arevalo*, The University of Texas Rio Grande Valley, Hansapani Rodrigo, The University of Texas Rio Grande Valley. Significant Gene Array Analysis and Cluster-Based Machine Learning Modeling for Breast Cancer Relapse Prediction.
Gene expression analysis has been of major interest to biostatisticians for many decades. Such studies are necessary for the understanding of disease risk assessment and prediction, so that medical professionals and scientists alike may learn how to better create treatment plans to lessen symptoms and perhaps even find cures. In this study, we will investigate various gene expression analyses and machine learning techniques for disease class prediction, as well as assess predictive validity of these models and uncover differentially expressed (DE) genes for their relevant pathology datasets. Multiple gene expression datasets will be used to test model accuracies and will be obtained using the Affymetrix U133A platform (GPL96). Significant Analysis of Microarrays (SAM) had been used to identify potential disease biomarkers, followed by these predictive models: (a) random forest, (b) random forest with Gene eXpression Network Analysis (GXNA), (c) RF++, (d) LASSO, and (e) Bayesian Neural Networks. One of the intended goals for this study is to find clusters of co-expressed genes and identify the effect of clustering classification based on knowledge in gene expression data/microarray data. The other goal is to determine the usefulness of Automatic Relevancy Determination in Bayesian neural networks.
(Received September 12, 2023)

## 1192-62-32718

Mayleen Cortez-Rodriguez*, Cornell University. Causal Inference Under Interference: Estimating Effects When the Network is Known or Unknown.
Randomized experiments are widely used to estimate causal effects across a variety of domains. Network interference, where the outcome of an individual is affected by the treatment assignment of those in their social network, poses a challenge to estimating causal effects since it violates standard independence assumptions in the causal inference literature. In this work, we focus on two approaches to estimating the total treatment effect (TTE), or the difference between the average outcomes of the population when everyone is treated versus when no one is, under network interference. In the first approach, we consider a simple Bernoulli randomized design and provide an unbiased estimator for the TTE along with variance bounds when the network is fully known. We also prove that our estimator is asymptotically normal under boundedness conditions. In the second approach, we leverage a staggered rollout design, in which treatment is incrementally given to random subsets of individuals, to derive unbiased estimators for the TTE and variance bounds when the underlying network is unknown, so long as the network interference effects are constrained to low-order interactions among neighbors of an individual. Central to both approaches is a new framework for balancing model flexibility and statistical complexity as captured by this low-order interactions structure. We also consider extensions to settings where we might have some, but not complete, knowledge of the network.
(Received September 12, 2023)

## 1192-62-32871

Khang Bui*, Villanova University, Jesse Frey, Professor. Bootstrap Confidence Intervals in Ranked-Set Sampling Across Distributions.
In this study, we applied bootstrap techniques, specifically the pivotal and percentile methods, to compute confidence intervals for the population variance based on a ranked-set sample. Drawing from five different distributions-normal, exponential, lognormal, uniform, and $t(3)$-we observed that the coverage probability converges to the nominal level as the set size increases. Our findings underscore the efficacy of bootstrap methods in variance estimation within the context of ranked-set sampling, especially as the sample set size grows.
(Received September 12, 2023)

## 1192-62-33076

Ashley Birnesser, Lake Forest College, Patrick Breheny, University of Iowa, Zoe Chafouleas, Dartmouth College, Logan Harris, University of Iowa, Bryan Salcedo*, Kean University. Ititle \{Predicting Renal Failure Time in Newly Diagnosed C3G Patients\} lauthor\{Ashley B.|textsuperscript \{1\}, Zoe C.|textsuperscript \{2\}, Bryan S.|textsuperscript\{3\}\} |date\{\}. \abstract Complement-c3 glomerulopathy (C3G) is a group of rare conditions that cause kidney disease through abnormal activation of the autoimmune system. The immune system of C3G patients lacks control of the protein C3 - a protein on the surface of red blood cells - resulting in a constitutive inflammatory response that causes damage to patient kidneys over time. The focus of this research is to develop a model to predict time until renal failure in newly diagnosed C3G patients using various measures of kidney function at the time of diagnosis. Our methods of modeling include Cox proportional hazards, Kaplan-Meier curves, and decision trees. These models identify predictor variables for the risk of reaching ESRD using data collected from a cohort of C3G patients by the University of Iowa. Methodology for predicting time until renal failure will be accomplished using a statistical survivorship function and decision tree based on the initial measurements of these variables in C3G patients. \endabstract
(Received September 13, 2023)

1192-62-33100
Michael J Orr*, Utah Tech University. Rock Column Displacement in Zion National Park: A Statistical Analysis. Preliminary report.
Every year Landslides claim lives and cause billions in property damage. Zion National Park in Southwestern Utah has been monitoring a precarious rock column contained within the parks borders since 2019. The results of this rockfall will be damage
to the land and historical structures located in the fall area. The data are collected by two instruments called vibrating-wire crackmeters that are connected from the cliff face to the pillar-these are electronic and provide a continuous feed of distance measurements. The project will be focused on analyzing a 4 -year record of displacement of a precarious rock pillar from the adjacent cliff face from which it will eventually separate and topple onto a slope below. We will also be analyzing the environmental factors causing the movement of the rock column. Results of this modeling will help us better understand the driving forces behind rockfalls, possibly helping to mitigate future damage.
(Received September 13, 2023)

## 1192-62-33110

Adam Ahmed, Metron, Inc., Jim Ferry*, Metron, Inc.. Optimal Bayesian Decisions for Adaptive System Testing. Typical acquisition processes rely on simple experimental design protocols to test and evaluate complex systems in order to make purchasing decisions. We present results from a more sophisticated, adaptive framework based on Sequential Bayesian Decision Theory. This framework uses a stochastic model for system behavior as a function of its operating environment $x$ and its unknown parameters $\theta$. When the parameters $\theta$ can be modeled via a conjugate family of distributions $P(\theta \mid \kappa)$, one can define a utility $U(\kappa, x)$ which defines the value of the system in environment $x$ given the current state of knowledge $\kappa$ about its parameters. We demonstrate this framework using a utility $U(\kappa, x)$ formulated in the same units as testing cost. This produces an optimal decision procedure (for which tests to conduct and when to stop testing) that is sensitive to the costs of the various testing options. The framework is illustrated in two scenarios. The first assesses a simple Bernoulli process $B(p)$ and compares the utility of retaining real-valued error measurements to retaining only hit/miss data. It is shown, for example, that each real measurement from an isotropic 2 -d Gaussian is asymptotically worth $p /\left((1-p) \log ^{2}(1-p)\right)$ hit/miss measurements from the corresponding process $B(p)$. The second demonstrates the framework on a normal-inverse-gamma model of a system operating in a mixed discrete-continuous space of environments. In each case, the adaptive framework improves on existing, static experimental designs by providing optimal testing decisions on the fly as results arrive. (Received September 13, 2023)

## 1192-62-33260

Iramir Da Silva Barros Neto*, Kean University, Hanna Mullins, Wheaton College, Andrés Vazquez, University of Puerto Rico-Mayagüez, Rachel Watson, Bradley University, Gideon K. D. Zamba, University of IOWA. Why is learning so often difficult to achieve?.
A typical college student today has access to learning resources, but they may also face adverse learning experiences that lead to discomfort during learning, through challenged beliefs or failures. This discomfort causes some students to ruminate on negative thoughts and emotions which may lead to distal behaviors, such as dropping classes or accepting defeat. Researchers from the Department of Neuropsychology at the University of Iowa conducted a pilot study to determine the role that biofeedback plays in sustaining students' efforts to engage with ideas that induce cognitive and emotional discomfort. Our methods for identifying significant electrodermal activity (EDA) were based on change point models. The change-point model was used to identify structural changes in the EDA data, while video evidence was deployed to validate the changes and ensure the activities engaged in have appropriately led to a surge, a decrease, or stabilization of EDA profile. The findings from the pilot study suggest that EDA may be used as a means for detecting learners' discomfort, particularly while facing failure. A correlation such as this will address a critical need neglected by prior research that uses self-report data and learners' overt talk as the sole metrics to construct claims regarding learners' cognitive and emotional discomfort. EDA technologies can augment people's capacity to monitor and reflect upon the emergence of their discomfort and thus deliver to learners a psychological buffer that supports their continued learning-transforming a liability into an asset.
(Received September 13, 2023)

1192-62-33346
Hannah Friedman, University of California, Berkeley, Bernd Sturmfels, University of California, Berkeley, Maksym Zubkov*, University of California, Berkeley. Likelihood Geometry of Determinantal Point Processes. We study determinantal point processes (DPP) through the lens of algebraic statistics. We count the critical points of the loglikelihood function, and we compute them for small models, thereby disproving a conjecture of Brunel, Moitra, Rigollet and Urschel.
(Received September 13, 2023)

1192-62-33353
Raphiel J Murden*, Rollins School of Public Health, Emory University. Analyzing Data from Ambulatory Blood Pressure Monitoring.
Increasing rates of obesity, specifically class 3 obesity, and elevated blood pressure (BP) levels are worldwide public health problems that have been especially pernicious for Black women in the United States (US). Simultaneously, elevated BP and obesity are risk factors for cardiovascular disease (CVD), which disproportionately impacts Black women. Studies that collect ambulatory blood pressure (ABP), the reference standard for non-invasive measuring of BP, often use mean ABP in statistical analyses. However, methods based on functional data analysis (FDA) allow the assessment of time-specific ABP levels. An FDAbased approach assessed whether BMI classification modified associations between time of day and ABP in a cohort of Black women living in and around Atlanta, GA, and ages 30-45 years old between 2016 and 2019. Analyses revealed that the magnitudes of differences in covariate-adjusted systolic ABP (SBP) between women with class 3 obesity and those with normal/overweight were time-dependent, around 6 mmHg during the day and up to 11 mmHg overnight. Moreover, among women with class 3 obesity, SBP declined more slowly and reached its minimum level later at night than SBP of other women. Developing time-specific models of ABP may inform interventions to prevent high levels of nighttime BP, a risk factor for CVD. (Received September 13, 2023)

Preliminary report.
Many sociology studies have been published regarding the experiences and development of medical school students, but there is a gap of research observing undergraduate students on pre-health professions tracks. Specifically, studies have been published noting a significant decrease in the empathy of medical school students during their third year, but no research has been conducted to identify development patterns of these students during their undergraduate years. This study aims to identify groups of undergraduate students on pre-health professions tracks based on typologies formed from longitudinal survey responses, and also to identify any significant transitions between these groups over time. Because the survey responses were collected between Spring 2019 and Spring 2022, the experience of the COVID-19 pandemic added an additional dimension to this study making it a natural experiment. Using our longitudinal models to compare the transitions between groups across different cohorts, we address whether the pandemic had a significant impact on the identity and empathy development of undergraduate pre-health students. To group our sample based on their survey responses we used latent class analysis, which categorized individuals based on their response pattern to questions regarding religion, political views, moral foundation, empathy, and demographics. Upon completing this analysis and comparing transitions across the discovered groups, we found significant differences in transitions between identity groups across the cohorts, which has implications for the COVID-19 pandemic having a significant impact on undergraduate pre-health and nursing students' identity and empathy development. Further research can be done to see if these potential variations in normal development have a significant impact on the identities and empathy levels of these individuals in the future.
(Received September 14, 2023)

1192-62-33381
Noah Couch*, UNC Greensboro, Geoffrey Kleinberg*, Moravian University. Evaluating the EM algorithm in recombination model for constructing phylogenetic trees. Preliminary report.
The study of phylogenetic trees is integral to the field of biology, especially in its application to the study of evolution and research into genetic markers for diseases. To this end, statistical methods have been applied to DNA sequences to assist in construction of such trees. Past research has studied the effect of genetic mutations over generations, creating an ancestral mixture model to estimate ancestral DNA proportions. More recent research has taken genetic recombination into account, proposing a hierarchical estimator based on Markov Chain Composite Likelihood. However, there has not yet been discussion on the accuracy of this estimator. The purpose of this paper is to compare the hierarchical estimator to the EM algorithm in terms of accuracy and computation time, for small lengths $L$ of DNA sequences. We find that, in general, the EM algorithm is more accurate while the hierarchical estimator is significantly more efficient. In cases where the distribution can be represented by a Markov Chain, the hierarchical estimator performs about as well as the EM algorithm. (Received September 14, 2023)

1192-62-33457
Hai Van Le*, University of Washington. Firms' financial misconduct impacts on rival companies. Preliminary report.
The purpose of this study is to evaluate the financial performance as measured by return on assets, return on equity, Altman's Z-score, and gross profit margin of publicly-listed companies in Northern America following a major financial scandal committed by their rival companies. We examine cases collected from the Accounting and Auditing Enforcement Releases of the U.S. Securities and Exchange Commission that occurred between 1999 and 2020. The firms' financial data were sourced from Compustat databases. Our final analysis includes 160 accused companies and 16,297 industry competitors. This paper relies on an ordinary least squares regression model controlling for the Sarbanes-Oxley Act policy and firm size and age, fitted to the dataset according to the ordinary least squares approach. The results present a statistically significant difference in financial performance of rival companies between the pre- and post- period after the scandal was committed by their industry peers. We find that about two-thirds of the accused companies adversely impacted their competitors in terms of core financial metrics. The remaining cases show conflicting patterns: some rivals recorded better financial performance while others remained stable or experienced gentle fluctuations in their financial performance a couple of years after a major scandal. (Received September 17, 2023)

## 1192-62-33500

Erin Malloy Cooper*, Butler University. Modeling Tornado Occurrence and Uncertainty Quantification in the USA Using the Right Extremes Data. Preliminary report.
Tornadoes are one of the most destructive and unpredictable meteorological phenomena, posing significant risks to life, property, and the financial sector in the United States. Understanding their occurrence and intensity is essential for risk assessment and disaster preparedness. This study investigates the risk and uncertainty quantification by modeling monthly tornado occurrences in the USA from 1950 to 2022. Through the utilization of different thresholding methods - local mean, local median, global mean, and global median - initial exploratory data analysis reveals the presence of extreme events with heavy-tailed distributions, indicating the need for extreme value modeling. We apply four extreme value distributions: exponential distribution, Weibull distribution, Gumbel distribution, and generalized Pareto distribution, to model monthly tornado occurrence and intensity. The Generalized Pareto Distribution offers the best fit for modeling extreme tornado intensities. This distribution provides valuable insights into the tails of the intensity distribution, helping identify seasons prone to exceptionally severe tornadoes. We have employed parametric bootstrap method for uncertainty quantification. These findings contribute to improved risk assessment, disaster mitigation, and the development of more robust tornado prediction models, ultimately enhancing the safety and resilience of the nation at large.
(Received September 19, 2023)

1192-62-33561
Eli Edwards-Parker*, Hope College, Colin Kalkman*, Hope College. The impact of non-pharmaceutical interventions and environmental factors on dengue fever incidence in Singapore. Preliminary report.
Dengue fever is a serious and potentially fatal disease, endemic in many tropical countries around the world. Due to the covid19 pandemic, unprecedented non-pharmaceutical mitigation measures were implemented in Singapore. This study attempts to identify the statistical significance of the effect of those measures on dengue fever incidence. We will also propose prediction models for dengue fever incidence which incorporate the implementation of measures for covid-19 mitigation as well as environmental factors like total rainfall.
(Received September 20, 2023)

## 1192-62-33662

Jackson Christopher Krebsbach*, Hope College, Brian Yurk, Hope College. Mapping Plant Populations Using Drones and Machine Learning.
Active coastal sand dunes are dynamic environments impacted by interactions between plant populations, topography changes, and physical processes. Using machine learning and photography both from the ground and from the air (using a drone), our group developed a method to map vegetation density at high resolution across an entire dune complex. By comparing maps across time we hope to better understand the interactions between sand dune activity and marram grass population growth and spread. First, an XGBoost model was trained to classify pixels in ground-based images as dead vegetation, live grass, or sand. We used the resulting classifier, which achieved an overall accuracy of $96 \%$ to create 200 coverage estimates for square regions on the ground ( 45 cm by 45 cm ) at various locations within the dune complex. Each of these regions, which included thousands of pixels in the ground-based images, were covered by 50 pixels in the coarser multispectral drone imagery. The multispectral photographs included red, green, blue, near infrared, and red edge bands. Additional features were constructed from these bands, including the Normalized Difference Vegetation Index (NDVI), which can contrast healthy live vegetation from the ground. We used the feature images and original coverage estimates from the ground based images to train a regularized multinomial regression model. The trained model, which predicts surface composition using drone imagery, was used to create an orthomosaic map of vegetation coverage spanning the entire dune complex.
(Received September 22, 2023)

## 1192-62-33766

Callie Neal Reid*, Valdosta State University. A Machine Learning analysis on the unemployment rates in the U.S.. Preliminary report.
We use unsupervised tools from Machine Learning such as principal components and hierarchical clustering to analyze data obtained from the U.S. Bureau of Labor Statistics concerning unemployment rates in all the states in the United States. We use the statistical computing software R to obtain our conclusions. We are also interested in comparing our results with those previously obtained by using tools from topological data analysis on the same data set.
(Received September 25, 2023)

## 1192-62-33860

Anna Rittenhouse*, Clark Atlanta University, Grace Rojo*, Massachusetts, Alia Valentine*, Michigan State University. Assessing the Robustness of VBMC for Extracting Parameters in Differential Equations.
Many computational models rely on parameter estimation based on physical measurements. Variational Bayesian Monte Carlo (VMBC) is a novel algorithm that is well-suited to handle models with computationally expensive likelihoods and multiple parameters. In our research, we used synthetic data to assess the performance of VBMC for parameter estimation, for a variety of ordinary differential equations. To measure its robustness, we created three metrics to evaluate the difference between a variational posterior distribution from VBMC and the true values of parameters: a Euclidean metric, a confidence interval metric via a concave hull, and a confidence interval metric via a multi-dimensional Delaunay integration. With these metrics we found that VBMC is able to extract simple relationships between parameters and is able to handle noisy datasets effectively. We also found that the algorithm is sensitive to randomness, and that it is not able to find distinct parameters that belong to different manifolds. (Received September 26, 2023)

1192-62-33881
Liora Mayats-Alpay*, Chapman University. Parkinson Patients and Hospital related - Length of Stay in the hospitals analysis: A Bayesian Hierarchical Model Approach. Preliminary report.
Parkinson's Disease (PD) is an age-related neurological disease that impacts the motor system. Several factors influence the development of the disease such as genetic predisposition and environmental conditions. Progressive PD disease requires clinical care. Since the disease is associated with a motor disorder, PD patients are admitted to hospitals with common problems related to falls, medical management, injuries, respiratory problems, as well as mental and psychiatric emergencies. Aim: In this study, we present an analysis of the length of hospital stay of PD patients over 16 years and 107 hospitals. We queried one of the largest, high quality medical databases (Cerner) and obtained data on 51,204 PD patients. We performed Bayesian hierarchical modeling to investigate the Hospital-related - Length of Stay for PD patients. The result of this study shows that the average PD stay in the hospitals was 3.0 days with a maximum of 6.8 days. The average stay of the 2.5 -th and 97.5 th percentiles were 0.5 days and 5.4 days respectively. Our findings provide insight into the distribution of hospital lengths of stay of PD patients and the associated medical cost of treatment.
(Received September 27, 2023)

## 65 Numerical analysis

## 1192-65-25469

Thu Thi Anh Le*, Kansas State University. A sampling-type method combined with deep learning for inverse scattering with one incident.
We consider the inhomogeneous acoustic inverse problem of determining the geometry of penetrable objects from scattering data generated by one incident wave at a fixed frequency. We first study an Orthogonality Sampling-type method which is fast, simple to implement, regularization-free and robust against noise in the data. This sampling method has a new imaging functional that is applicable to data measured in near-field or far-field regions. The resolution analysis of the imaging functional is analyzed where the explicit decay rate of the functional is established. The sampling method is then combined with a Deep Neural Network to solve the inverse scattering problem. This combined method can be understood as a network using the image computed by the sampling method for the first layer, followed by the U-net Xception architecture for the rest of the layers. The fast computation and the knowledge from the results of the sampling method help speed up the training of the network. The combination leads to a significant improvement in the reconstruction results initially obtained by the
sampling method. The combined method is also able to invert some limited aperture experimental data without any additional transfer training. Numerical results testing against simulated data and experimental data will be presented. This talk is based on joint work with Dinh-Liem Nguyen, Vu Nguyen, and Trung Truong.
(Received June 13, 2023)

## 1192-65-26131

Irene A Caracioni, Illinois Institute of Technology, Fred Hickernell, Illinois Institute of Technology, Guangyu Qiu, Illinois Institute of Technology, Dylan Xu*, University of California, Berkeley, Anita Ye, Northern Illinois University. Performing the Euler-Bernoulli Beam Uncertainty Quantification Using QMCPy.
Quasi-Monte Carlo (QMC) methods often achieve greater computational efficiency than classic Monte Carlo methods by using low-discrepancy, instead of IID, sequences. We tested QMC through implementing and experimenting with the Python package QMCPy, combined with UM-Bridge, on an uncertainty quantification simulation of an Euler-Bernoulli beam. While QMC was generally faster than, or as fast as, Monte Carlo, it sometimes had trouble with high dimensionality and peaky functions. We found various mechanisms for speeding up the simulation by orders of magnitude through choosing which types of lowdiscrepancy sequences, periodization transforms (when applicable), and stopping criteria to use. We also explored using higher-order nets in our simulations. Finally, we found and implemented algorithms that made our beam model more realistic and complex, and thus a more difficult test of QMC, through replacing the segmented beam by a Gaussian process for the stiffness. Our work can help elucidate the versatility and limitations of QMC in complex simulations.
(Received July 18, 2023)

## 1192-65-26675

Thu Thi Anh Le*, Kansas State University. A sampling type method combined with deep learning for inverse scattering with one incident.
We consider the inhomogeneous acoustic inverse problem of determining the geometry of penetrable objects from scattering data generated by one incident wave at a fixed frequency. We first study an Orthogonality Sampling-type method which is fast, simple to implement, regularization-free and robust against noise in the data. This sampling method has a new imaging functional that is applicable to data measured in near-field or far-field regions. The resolution analysis of the imaging functional is analyzed where the explicit decay rate of the functional is established. The sampling method is then combined with a Deep Neural Network to solve the inverse scattering problem. This combined method can be understood as a network using the image computed by the sampling method for the first layer, followed by the U-net Xception architecture for the rest of the layers. The fast computation and the knowledge from the results of the sampling method help speed up the training of the network. The combination leads to a significant improvement in the reconstruction results initially obtained by the sampling method. The combined method is also able to invert some limited aperture experimental data without any additional transfer training. Numerical results testing against simulated data and experimental data will be presented. This talk is based on joint work with Dinh-Liem Nguyen, Vu Nguyen, and Trung Truong.
(Received August 03, 2023)

## 1192-65-26849

Alan Demlow, Texas A\&M University, Michael Neilan*, University of Pittsburgh. A tangential and penalty-free finite element method for the surface Stokes problem.
Surface Stokes equations have attracted significant recent attention in numerical analysis because approximation of their solutions poses significant obstacles not encountered in the Euclidean context. One of these challenges involves simultaneously enforcing tangentiality and $H^{1}$ conformity of discrete velocity approximations. Existing methods all enforce one of these two constraints weakly either by penalization or by use of Lagrange multipliers. However, a robust and systematic construction of surface Stokes finite element spaces with nodal degrees of freedom is still missing. In this talk, we introduce a novel approach addressing these challenges by constructing surface MINI spaces with tangential velocity fields. They are not $H^{1}$-conforming, but do lie in $H$ (div) and do not require penalization. We prove stability and optimal-order energy-norm convergence of the method. The core advancement is the construction of nodal degrees of freedom for the velocity field. This technique also may be used to construct surface counterparts to many other standard Euclidean Stokes spaces. (Received August 07, 2023)

1192-65-27138
Qin Sheng*, Baylor University. A review of splitting, adaptive splitting, operator splitting, and exponential splitting methods. Preliminary report.
Splitting methods have been playing an important role in approximating solutions of numerous modeling problems including those in the ice sheet dynamics. Of course, these methods are not limited for solutions of differential equations. They are also used in statistical computations and optimization. They may also preserve important physical features. The modern ideas of splitting can be traced back to Henry F. Baker (1866-1956), John E. Campbell (1862-1924) and Felix Hausdorff (1868-1942). The strategies have been booming since the arrival of the first electronic computer. A splitting method separates a sophisticated mathematical model into several subproblems, separately computes the solution to each of them, and then combines all sub-solutions to form an approximation of the solution to the original problem. A canonical example is splitting of waves of different frequencies in a general wave partial differential equation. The splitting idea generalizes in a natural way to problems with multiple operators too. In all cases, the computational advantage is that it is faster to compute the solution of the split components separately, than to compute the solution directly when they are treated together. However, this comes at the cost of an error introduced by the splitting, so strategies must be devised for controlling the error. This presentation recalls the phenomenal work done by the pioneers and introduces different splitting ideas via modern methodologies. A short conversation will be given in global error analysis of popular exponential splitting formulations. Graduate and undergraduate students are very welcome to this talk.
(Received August 10, 2023)

Praveeni Mathangadeera*, Oregon State University. Sensitivity to different assumptions in a permafrost model responding to surface temperature variations. Preliminary report
We start with a one-dimensional finite difference model of phase change in permafrost soil defined in [Bigler, Peszynska, Vohra, 2022]. This computational model requires careful handling of nonlinear relationships. We also use data and constitutive parameters from [Ling, Zhang'2003] which require extensions of this model; we also use daily temperature data from databases in the Arctic. To introduce these elements, we add to our computational model a new possibility of sequential rather than implicit treatment of nonlinearities. Next, we evaluate the response of the model depending on which assumptions are made. In particular, we allow presence of a snow layer which influences the top boundary condition for the model; and we study the effects of snow thermal conductivity, volumetric heat capacity, and albedo.
(Received August 14, 2023)

## 1192-65-27887

Maria Van Der Walt*, Westmont College. A function approximation approach to the prediction of blood glucose levels. The problem of real time prediction of blood glucose (BG) levels based on the readings from a continuous glucose monitoring device is a problem of great importance in diabetes care, and therefore, has attracted a lot of research in recent years, especially based on machine learning. An accurate prediction with a 30, 60, or 90 minute prediction horizon has the potential of saving millions of dollars in emergency care costs. In this paper, we treat the problem as one of function approximation, where the value of the BG level at time $t+h$ (where $h$ the prediction horizon) is considered to be an unknown function of $d$ readings prior to the time $t$. This unknown function may be supported in particular on some unknown submanifold of the $d$ dimensional Euclidean space. While manifold learning is classically done in a semi-supervised setting, where the entire data has to be known in advance, we use recent ideas to achieve an accurate function approximation in a supervised setting; i.e., construct a model for the target function. We use the state-of-the-art clinically relevant PRED-EGA grid to evaluate our results, and demonstrate that for a real life dataset, our method performs better than a standard deep network, especially in hypoglycemic and hyperglycemic regimes. One noteworthy aspect of this work is that the training data and test data may come from different distributions. This is joint work with Hrushikesh Mhaskar and Sergei Pereverzyev. (Received August 21, 2023)

## 1192-65-28007

Zhen Chao*, University of Michigan-Ann Arbor, Dexuan Xie, University of Wisconsin-Milwaukee. A Poisson-Nernst-Planck Single Ion Channel Model and Its Effective Finite Element Solver.
A single ion channel is a membrane protein with an ion selectivity filter that allows only a single species of ions (such as potassium ions) to pass through in the "open" state. Its selectivity filter also naturally separates a solvent domain into an intracellular domain and an extracellular domain. Such biological and geometrical characteristics of a single ion channel are novelly adopted in the construction of a new kind of dielectric continuum ion channel model, called the Poisson-Nernst-Planck single ion channel (PNPSIC) model. In this talk, I introduce an effective PNPSIC finite element solver for a single ion channel with a three-dimensional X-ray crystallographic molecular structure and a mixture of multiple ionic species. Numerical results for a potassium channel confirm the convergence and efficiency of the PNPSIC finite element solver and demonstrate the high performance of the software package. Moreover, the PNPSIC model is applied to the calculation of electric current and validated by biophysical experimental data. (Received August 24, 2023)

## 1192-65-28049

John Peca-Medlin*, University of Arizona. Growth factors of orthogonal matrices and local behavior of Gaussian elimination with partial and complete pivoting.
Gaussian elimination (GE) is the most used dense linear solver. Error analysis of GE with selected pivoting strategies on wellconditioned systems can focus on studying the behavior of growth factors. Although exponential growth is possible with GE with partial pivoting (GEPP), growth tends to stay much smaller in practice. Support for this behavior was provided last year by Huang and Tikhomirov's average-case analysis of GEPP, which showed GEPP growth factors stay at most polynomial with very high probability when using small Gaussian perturbations. GE with complete pivoting (GECP) has also seen a lot of recent interest, with recent improvements to lower bounds on worst-case GECP growth provided by Edelman and Urschel earlier this year. We are interested in studying how GEPP and GECP behave on the same linear systems as well as studying large growth on particular subclasses of matrices, including orthogonal matrices. We will also study systems when GECP leads to larger growth than GEPP, which will lead to new empirical lower bounds on how much worse GECP can behave compared to GEPP in terms of growth. We also present an empirical study on a family of exponential GEPP growth matrices whose polynomial behavior in small neighborhoods limits to the initial GECP growth factor.
(Received August 24, 2023)

## 1192-65-28089

Harish Bhatt*, Utah Valley University. Second-order time integrators with the Fourier spectral method in application to multidimensional space-fractional FitzHugh-Nagumo model. Preliminary report.
Fractional-in-space reaction-diffusion models have emerged as powerful tools for understanding intricate patterns and dynamics in various scientific domains. In this talk, we investigate the propagation and interaction behavior of the fractional-in-space multidimensional FitzHugh-Nagumo model using second-order time integrators in combination with the Fourier spectral method. Furthermore, we analyze the accuracy, efficiency, and stability of second-order time integrators through a comparative study of numerical outcomes. The experimental findings highlight the straightforward implementation and suitability of the methods for long-time simulations. Finally, we discuss the methods' capability in capturing the influence of the fractional operator on the equation's dynamics, leading to the observation of novel propagation behaviors in the threedimensional (3D) model.
(Received August 25, 2023)

1192-65-28152
Sophie Boileaus*, Carleton College, Atmik Das*, University of California San Diego, Kellen Arnold Kanarios*, University
of Michigan, Lucia Krajcoviechova*, University of Cambridge. Parallel Algebraic Multigrid for Fusion and Higher-Order PDEs. Preliminary report.
Multigrid methods play a key role in large-scale scientific simulation because they are among the fastest and most scalable approaches for solving the underlying sparse linear systems of equations that arise from a wide array of PDE discretizations. Algebraic multigrid (AMG) is a special type of multigrid method that depends only on the description of the linear system, giving it better portability and broader applicability than geometric multigrid, as it requires no explicit knowledge of the problem geometry. Though these methods are widely used today, there are still applications where further development is needed. For instance, PDEs with higher order terms violate assumptions of existing AMG methods, causing an unbounded number of $V$-cycles for convergence. In our case, we are interested in a PDE that arises in kinetic-edge plasma simulation. This PDE contains an isotropic fourth-order term, making existing methods infeasible. To remedy this, we propose a $O(n)$ highly parallelizable exact method to solve the system solely containing the isotropic fourth-order term. We then extend this algorithm to solve the original system, obtaining drastic improvement over existing methods.
(Received August 27, 2023)

1192-65-28154
Sophie Boileaus, Carleton College, Atmik Das, University of California San Diego, Kellen Arnold Kanarios*, University of Michigan, Lucia Krajcoviechova, University of Cambridge. Parallel Algebraic Multigrid for Higher-Order PDEs. Preliminary report.
Existing algebraic multigrid (AMG) methods rely on assumptions about the near-kernel components of a given linear system. Namely, that these components are "smooth" in the sense that they can be sufficiently approximated by few degrees of freedom. PDEs with higher order terms violate these assumptions, causing an unbounded number of $V$-cycles for convergence. As an example, we introduce a PDE that arises in kinetic-edge plasma simulation. This PDE contains an isotropic fourth-order term, making existing methods infeasible. In this work, we propose an $O(n)$ highly-parallelizable exact method to solve the system solely containing the isotropic fourth-order term. We then extend this algorithm to solve the original system, including periodic boundary conditions. Our algorithm obtains drastic improvement over existing methods.
(Received August 27, 2023)

1192-65-28241
Bo Li*, University of California, San Diego. Variational Implicit Solvation and Fast Algorithms for Molecular Binding and Unbinding.
Ligand-receptor binding and unbinding are fundamental molecular processes, whereas water fluctuations impact strongly their thermodynamics and kinetics. We develop a variational implicit-solvent model (VISM) and a fast binary level-set method to calculate the potential of mean force and the molecule-water interfacial structures for dry and wet states. Monte Carlo simulations with our model and method provide initial configurations for efficient molecular dynamics simulations. Moreover, combined with the string method and stochastic simulations of ligand molecules, our hybrid approach enables the prediction of the transition paths and rates for the dry-wet transitions and the mean first-passage times for the ligand-pocket binding and unbinding. Without any explicit description of individual water molecules, our predictions are in a very good, qualitative and semi-quantitative, agreement with existing explicit-water molecular dynamics simulations.
(Received August 28, 2023)

## 1192-65-28491

Owen Deen*, University of North Carolina Wilmington. The Generalized Matrix Separation Problem. Preliminary report. We will present an approach for solving the generalized matrix separation problem, focusing on recoverability of sparse and low-rank matrices under specific constraints. We explore the problem's solvability conditions while offering a robust numerical algorithm and fine-tuned parameters. The algorithm has foundations in convex optimization principles and presents a solution to filter noise in various data analysis tasks. We provide a Python-based implementation with emphasis on efficiency and scalability, as well a preliminary investigation of the mathematical framework. In particular, we will look at practical applications in video cleanup and image deblurring.
(Received August 31, 2023)

## 1192-65-28656

Christian Alexander Glusa, Sandia National Laboratories, Shuai Jiang*, Sandia National Laboratories. Optimization-based coupling without the optimization. Preliminary report.
In this talk, we propose a non-intrusive and simple "splicing" method for local-to-nonlocal (LtN) coupling. While previous splicing approaches generally considered particle methods, we extend it to coupling variational solutions of weak forms using potentially different polynomial orders. We show that our approach is in fact equivalent to a specific optimization-based LtN coupling method, and, as a result, inherits many valuable theoretical properties such as well-posedness and convergence. However, our coupling method does not need to perform expensive optimization, and only a single matrix, albeit nonsymmetric, solve is needed.
(Received September 01, 2023)

## 1192-65-28657

Owen Deen*, University of North Carolina Wilmington, Colton Waller, North Carolina Agricultural and Technical State University, John Paul Ward, Department of Mathematics, North Carolina Agricultural and Technical State University. Fast and Accurate Log-Determinant Approximations. Preliminary report.
We consider the problem of estimating log-determinants of large, sparse, positive definite matrices. We will present an algorithm for approximating log-determinants based on sparse approximate inverses. The algorithm is adaptive and uses graph spline approximations to improve accuracy. We illustrate our approach on classes of large sparse matrices. In our experiments, the spline-based approximation uses the smooth decay of the data to predict the results more accurately, reducing computational demands. Our choice to employ graph-based splines is rooted in the discrete nature of sparsity patterns, where we currently focus on a specific path subgraph within essentially one-dimensional data. We have also provided a new proof of
the monotonicity of the approximations with respect to sparsity pattern inclusion. This work was part of the Summer 2023 Research Experiences for Undergraduates in Data Science and Analytics at North Carolina Agricultural and Technical State University.
(Received September 01, 2023)

## 1192-65-28701

Christian Alexander Glusa*, Sandia National Laboratories. Optimal control for fractional order equations.
We consider adjoint-based optimization for control problems involving fractional-order state equations, applied to the inference of kernel parameters. We will discuss optimality conditions, error estimates and techniques to efficiently explore the parameter space and approximate gradients
(Received September 01, 2023)

1192-65-28707
Mikheil Tutberidze*, San Diego State University. Investigation of the Difference Scheme for the Initial-Boundary Value Problem to One Nonlinear Parabolic Equation.
Investigation of some biological models bring us to the following initial-boundary value problem to nonlinear parabolic equation:

$$
\begin{gathered}
U_{t}=\left(k\left(x, t, U_{x}\right) U_{x}\right)_{x}+f\left(x, t, U, U_{x}\right),(x, t) \in \Omega \times(0, T], \\
U(x, 0)=\varphi(x), x \in \Omega, \\
U(0, t)=\phi_{0}(t), U(1, t)=\phi_{1}(t), t \in(0, T],
\end{gathered}
$$

where $U=U(x, t)$ is unknown function, $k, f, \varphi, \phi_{0}$ and $\phi_{1}$ are given functions, $T=$ const $>0, \Omega=(0,1)$. For this problem we construct the difference scheme and prove existence of its solution under some conditions on functions $k$, $f$. For the mentioned scheme we also prove the theorems of comparison and uniqueness of its solution. Also, for the solution of the mentioned difference scheme we construct the iteration scheme and prove convergence of the iteration scheme to the solution of the difference scheme. If solution $U$ of the source problem is smooth enough, we also prove the convergence of the solution of the difference scheme to the solution of the source problem.
(Received September 01, 2023)

## 1192-65-28726

Noufe Aljahdaly*, Department of Mathematics, Faculty of Sciences and Arts, King Abdulaziz University, Rabigh, Saudi Arabia. The Improved Laplace homotopy perturbation method for solving non-integrable PDEs.
The Laplace homotopy perturbation method (LHPM) is an approximate method that help to compute the approximate solution for partial differential equations. The method has been used for solving several problems in science. It requires the initial condition, so it solves the initial value problem. In physics, when some important terms are taken in account, we may obtain non-integrable partial differential equations that do not have analytical integrals. This type of PDEs do not have exact solution therefore, we need to compute the solution without initial condition. In this work, we improved the LHPM to be able to solve non-integrable problem, especially the damped PDEs, which are the PDEs that include a damping term which makes the PDEs non-integrable. We improved the LHPM by setting a perturbation parameter and an embedding parameter as the damping parameter and using the initial condition for damped PDE as the initial condition for non-damped PDE.
(Received September 02, 2023)

Feng Bao*, Florida State University, Guannan Zhang, ORNL, Zezhong Zhang, Florida State University. AN ENSEMBLE SCORE FILTER FOR TRACKING HIGH-DIMENSIONAL NONLINEAR DYNAMICAL SYSTEMS. Preliminary report. We propose an ensemble score filter (EnSF) for solving high-dimensional nonlinear filtering problems with superior accuracy. A major drawback of existing filtering methods, e.g., particle filters or ensemble Kalman filters, is the low accuracy in handling high-dimensional and highly nonlinear problems. EnSF attacks this challenge by exploiting the score-based diffusion model, defined in a pseudo-temporal domain, to characterizing the evolution of the filtering density. EnSF stores the information of the recursively updated filtering density function in the score function, in stead of storing the information in a set of finite Monte Carlo samples (used in particle filters and ensemble Kalman filters). Unlike existing diffusion models that train neural networks to approximate the score function, we develop a training-free score estimation that uses mini-batch-based Monte Carlo estimator to directly approximate the score function at any pseudo-spatial-temporal location, which provides sufficient accuracy in solving high-dimensional nonlinear problems as well as saves tremendous amount of time spent on training neural networks. Another essential aspect of EnSF is its analytical update step, gradually incorporating data information into the score function, which is crucial in mitigating the degeneracy issue faced when dealing with very high-dimensional nonlinear filtering problems. High-dimensional Lorenz systems are used to demonstrate the performance of our method. EnSF provides surprising performance in reliably and efficiently tracking extremely high-dimensional Lorenz systems (up to 1,000,000 dimension) with highly nonlinear observation processes. (Received September 03, 2023)

Jehanzeb H Chaudhary*, University of New Mexico. Robust Numerical Solvers for the Poisson-Bolztmann and the Poisson-Nernst-Planck Equations.
This talk traces the evolution of solvers for the Poisson Bolztman-Equation (PB) and the Poisson-Nernst-Planck Equations (PNPE). In addition to discussing the history and modern day PBE solvers, we focus on approximating first order flux variables and adjoint based a posteriori error estimation in the finite element and boundary element method contexts. Another topic highlighted in this talk is the application of the PNPE to emerging challenges in carbon sequestration.

Long Chen, UC Irvine, Ruchi Guo, University of California, Irvine, Jingrong Wei*, University of California, Irvine.
Transformed Primal-Dual Methods for Nonlinear Partial Differential Equations.
A transformed primal-dual (TPD) flow is developed for a class of nonlinear saddle point system, including convex optimization problem with affine constraints. The flow for the dual variable contains a Schur complement which is strongly convex.
Exponential stability of the saddle point is obtained by showing the strong Lyapunov property. A TPD iteration is derived by time discretization of the TPD flow. Under mild assumption, the algorithm is linear convergent for smooth problems. The convergence rate depends on the relative condition number of the objective function and the Schur complement under variant metric as preconditioners. The developed algorithm is then applied to nonlinear partial differential equations: DarcyForchheimer model and a nonlinear electromagnetic model. Numerical results demonstrate the efficiency of the method. This is joint work with Long Chen and Ruchi Guo.
(Received September 04, 2023)

## 1192-65-28879

Ruchi Guo*, University of California, Irvine. Solve Electromagnetic Interface Problems on Unfitted Meshes.
Electromagnetic interface problems widely appear in a lot of engineering applications, such as electric actuators, invasive detection techniques and integrated circuit, which are typically described by Maxwell equations with discontinuous coefficients. Conventional finite element methods require a body-fitted mesh to solve interface problems, but generating a high-quality mesh for complex interface geometry is usually very expensive. Instead, using unfitted mesh finite element methods can circumvent mesh generation procedure, which greatly improves the computational efficiency. However, the low regularity of Maxwell equations makes its computation very sensitive to the conformity of the approximation spaces. This very property poses challenges on unfitted mesh finite element methods, as most of them resort to non-conforming spaces. In this talk, we will present our recent progress including several methods for this topic.
(Received September 04, 2023)

## 1192-65-28884

Xiaobing Feng*, The University of Tennessee. Recent Advances in Fractional Calculus of Variations and Their Numerical Methods.
In this talk, I shall first briefly discuss some ecent advances in weak fractional calculus and fractional Sobolev spaces based on a new weak fractional derivative concept which is a natural generalization of integer order weak derivatives and helps to unify multiple existing fractional derivative concepts. I shall then introduce a class of fractional calculus of variations problems and their associated Euler-Lagrange (fractional differential) equations. This new framework/theory is based on the aforementioned theory of weak fractional derivatives and their associated fractional order Sobolev spaces. It leads to new types of fractiona differential equations, including new one-side fractional Laplace operators and future value problems. Finally, I shall also introduce some new finite element (and DG) methods for approximating the weak fractional derivatives and the solutions of fractional calculus of variations problems and their associated fractional differential equations, which requires developing some novel technique to estimate the finite element interpolation errors in the new fractional Sobolev spaces. (Received September 04, 2023)

## 1192-65-29001

Sylvia Amihere*, University of Alabama, Weihua Geng, Southern Methodist University, Shan Zhao, University of Alabama. Benchmarking Electrostatic Free Energy of the Nonlinear Poisson-Boltzmann Model for the Kirkwood Sphere.
Various numerical packages have been developed to solve the Poisson-Boltzmann equation (PBE) for the electrostatic analysis of solvated bio-molecules. A common benchmark test for the PBE solvers is the Kirkwood sphere, for which analytical potential and free energy are available for the linearized PBE. However, the Kirkwood sphere does not admit an analytical solution for the nonlinear PBE involving a hyperbolic sine term. In this talk, we will propose a simple numerical approach, so that the energy of the Kirkwood sphere for the nonlinear PBE can be calculated at a very high precision. Thus, providing a new benchmark test for the future developments of nonlinear PBE solvers. In addition, we will introduce a novel boundary treatment that is valid for both linearized and nonlinear PBE and can be employed in 3D PBE implementations. (Received September 05, 2023)

## 1192-65-29086

Elaine Gorom*, University of North Carolina at Charlotte, Xingjie Helen Li, University of North Carolina Charlotte Analytical Study on the Dynamics of a Bi-Material System Utilizing Bond-Based Peridynamics. Preliminary report. We couple Peridynamics to Peridynamics at an interface with two materials. When two different materials meet, the resulting system is more susceptible to fracture near their interface. While Peridynamics can be more computationally expensive than other models, it gives high accuracy and naturally allows the simulation of crack propagation in its model due to its use of integro-differentials and time derivatives instead of the spatial derivatives typical of classical models. We prove conservation of momentum and conservation of energy for this Peridynamics-to-Peridynamics system. Utilizing the conservation properties, we calculate the reflection coefficients of a plane wave near the interface between the two materials. We use this coefficient of reflection to calculate the error between the Peridynamics framework and classical models. We then design a novel numerical scheme that reduces numerical artifacts and preserves the conservation laws of the system. A comprehensive stability analysis and error estimate is conducted for the numerical scheme. Finally, we conduct several numerical experiments to confirm the theoretical findings.
(Received September 05, 2023)

In this talk, we will discuss the use of neural networks to solve high-dimensional partial differential equations (PDEs) without being affected by the curse of dimensionality. We will explore three key questions: (1) How to formulate PDE problems as optimization problems for deep learning techniques, (2) The accuracy of neural network approximations, and (3) Systematic training for global minimum convergence. Specifically, We will present various optimization formulations for the highdimensional quadratic porous medium equation, analyze generalization and approximation errors for Ritz methods, and propose an adaptive optimization strategy for training residual neural networks. Numerical results will be provided to demonstrate the effectiveness of the proposed methods.
(Received September 05, 2023)

## 1192-65-29347

Rajan Adhikari*, Department of Mathematics, Oklahoma State University. Reduced Mixed Finite Element Method The traditional mixed finite element method (FEM) approximates the flux and the primary functions simultaneously. We propose a simple, accurate, and efficient FEM to compute the flux independently of the primary function and present results to establish the convergence and accuracy of our approximation. We also propose a method to obtain an accurate approximation of the primary function by utilizing the flux approximated via our reduced mixed FEM and introduce a local post-processing scheme to increase its accuracy. We present numerical examples confirming the theoretical analysis. (Received September 06, 2023)

1192-65-29469
Madeline Gorman, Embry Riddle Aeronautical University, Thomas Pasfield, Embry Riddle Aeronautical University, Eleanor Sigel*, University of Southern California. Uncertainty Propagation in Image Deblurring: Comparing Tikhonov Regularization and Total Variation Denoising.
In image processing, noise resulting from motion, features of the detector, and the radiographic source itself all contribute to "blur", presenting a challenge for researchers attempting to derive conclusions from radiographic data. Various regularization methods, particularly Tikhonov (L2) Regularization, while Total Variation (TV or ROF) Regularization may be applied to remove blur from smooth or "blocky" signals, respectively, depending on knowledge of the blurring parameters and the structure of the signal. These methods of regularization reframe deblurring as a linear inverse problem, simplifying various blurring factors into a single linear operator in order to estimate the degree of blur mapped onto an original signal as a Point Spread Function. However, estimates of the PSF are necessarily uncertain, resulting in the compounding of uncertainty in the process of deblurring by regularization. As such, we have partnered with the Nevada National Security Site, whose research relies upon accurate models of radiographic blur, to model the uncertainty associated with Total Variation and Tikhonov Regularization. We will begin by applying the listed regularization approaches in order to estimate blur, acknowledging uncertainty by estimating the error bounds for each respective method. Then, we quantify the uncertainty propagated by the blur estimates yielded by each technique, beginning with the case of 1-dimensional signals and extending to 2-dimensional images. These results will clarify the error associated with radiographic deblurring techniques and inform corresponding expectations of accuracy, improving predictions of the true signal with radiographic analysis more broadly. (Received September 06, 2023)

1192-65-29609
Zhu Wang*, university of South Carolina. Time Stepping Schemes for the Baroclinic-Barotropic Split Dynamics in Primitive Equations.
To treat the multiple time scales of ocean dynamics in an efficient manner, the baroclinic-barotropic splitting technique has been widely used for solving the primitive equations for ocean modeling. In this talk, we will discuss several recently developed time stepping algorithms for efficiently simulating such a split dynamics, together with the corresponding temporal truncation error analyses. Two benchmark tests drawn from the "MPAS-Ocean" platform will be presented to illustrate the accuracy and parallel performance of the schemes.
(Received September 07, 2023)

1192-65-29666
Danielle Vogt*, Roger Williams University, Yajni Warnapala, Roger Williams University. An Integral Equation Approach for Long Covid -19 (LC Model)-Preliminary Report. Preliminary report.
This research is based on creating a Volterra- Fredholm integral Equation Model of the second kind for determining the variables associated with recovery $\mathrm{R}(\mathrm{T})$ of long Covid symptoms. Long Covid was determined to be having post Covid symptoms longer than 3 months. The analysis was done using Multiple Regression techniques and incorporating the Neural Network model and the Gaussian Quadrature method. (Received September 07, 2023)

1192-65-29689
Sam Bielawa*, Roger Williams University, Yajni Warnapala, Roger Williams University. Integral Equation Method for Maxwell Integral Equation: MRI Scan. Preliminary report.
The model uses the Maxwell integral equation to solve the electromagnetic field in an MRI scanner. The Gallekin method will be used to numerically approximate the integral to obtain good conversion results. The model uses electric permittivity, magnetic permittivity, electric conductivity, the frequency of the electromagnetic wave, and vacuum permeability. The assumption is that the patient is placed in a strong constant magnetic field and the patients body (head) is assumed to be a sphere or an ellipsoid.
(Received September 07, 2023)

1192-65-29732
Shanya Sanjay Verma*, University of Wisconsin. The Zeros of a Continuous Cubic Spline Functions. Preliminary report. The Zeros of a Continuous Cubic Spline Function In Numerical Analysis, to find the zeros of a Continuous Cubic Spline

Function $F(x)=A x \wedge\{3\}-B x+C=0$, the function is reduced to $f(x)=x \wedge\{3\}-b x+c=0$. For a given value of $b$ and $c$; if 【 $4 b \rrbracket \wedge\{3\}-$ $27 c^{\wedge}\{2\} \geq 0$, then the zeros of the function $f(x)=x^{\wedge}\{3\}-b x+c=0$ are $-2 \rrbracket(r) \rrbracket \wedge\{( \} 1 / 3) \cos ((2 n \pi+\theta) / 3)$; for $n=0$, 1 , and 2 ; where $r=\llbracket\left[\llbracket \mathrm{c} \rrbracket \wedge\{2\} / 4+1 / 4\left(\left(4 b^{\wedge}\{3\}-27 c^{\wedge}\{2\}\right) / 27\right)\right] \rrbracket \wedge\{( \} 1 / 2)$; and $\theta=\llbracket \tan \rrbracket \wedge\{( \}-1)\left[-\llbracket 1 / c\left(\left(4 b^{\wedge}\{3\}-27 c^{\wedge}\{2\}\right) / 27\right) \rrbracket\right.$ ^\{(\}1/2)]
(Received September 07, 2023)

## 1192-65-29833

Yakov Berchenko-Kogan, Florida Institute of Technology, Evan Gawlik*, University of Hawaii at Manoa, Michael Neunteufel, TU Wien. Finite element discretizations of curvature tensors. Preliminary report.
The finite element method is used ubiquitously to solve partial differential equations posed in Euclidean space, but it sees relatively limited use in Riemannian geometry. Recently this has changed with the debut of the Regge finite element space, which is a finite element space that is well-suited for discretizing Riemannian metric tensors on simplicial triangulations. It consists of piecewise polynomial symmetric ( 0,2 )-tensor fields possessing single-valued tangential-tangential components along simplex interfaces. Riemannian metrics belonging to the Regge finite element space are not classically differentiable, but it turns out that one can still make sense of various notions of curvature for such metrics, like the scalar curvature and the Einstein curvature tensor, in a distributional sense. We study these distributional curvatures and show that they converge in certain dual Sobolev norms to their smooth counterparts under refinement of the triangulation. The rates of convergence depend on the polynomial degree of the approximate metric tensor, with higher polynomial degrees yielding higher rates of convergence.
(Received September 08, 2023)

1192-65-29875
Talon Johnson*, University of Texas Southwestern Medical Center. A Novel Compressive Deconvolution Method with MRI Imaging Application.
The rapid evolution of technology has significantly impacted the field of medical imaging, particularly magnetic resonance imaging (MRI). In contrast to other imaging modalities, MRI offers distinct advantages, such as the ability to differentiate between various tissues and fat, a radiation-free imaging process, and the provision of detailed anatomical and molecular information about the examined tissue. However, one significant drawback lies in the prolonged data acquisition times required to generate high-quality MRI images, which necessitates patients to remain motionless for extended periods. This often results in patient discomfort and may lead to voluntary or involuntary movements, further exacerbating the inherent challenges of MRI, including image blurring and the presence of artifacts. Mathematically, these issues can be described as the convolution between the acquired MRI data and undesirable Point Spread Functions (PSF). In this presentation, we introduce a novel approach to expedite MR data acquisition. This approach employs sparse signal reconstruction and simultaneous deconvolution of the unwanted convolution using $\ell 1-\ell 2$ regularization via $\ell 1$-magic: TV2 and Tikhonov regularization.
(Received September 08, 2023)

## 1192-65-29883

Thir Raj Dangal*, Augusta University, Balaram Khatri Ghimire, Alabama State University, Anup R Lamichhane, Ohio Northern University. Localized oscillatory radial basis functions collocation method using augmented polynomials for solving elliptic partial differential equations. Preliminary report.
We have proposed a numerical scheme in which we added polynomial terms with localized oscillatory radial basis functions (ORBFs) collocation method to address the computational issues of ORBFs collocation method for solving elliptic partial differential equations in 2D. The accuracy of our proposed numerical scheme is very high and clearly outperformed the result of localized ORBFs collocation method. (Received September 08, 2023)

Ariana Brown*, Emory University, James Nagy, Emory University, Malena Sabaté Landman, Emory University. Hybrid Iterative Solver for Inverse Problems.
Inverse problems arise in a variety of applications: machine learning, image processing, finance, mathematical biology, and more. Solution schemes are formulated by applying algorithms that incorporate regularization techniques and/or statistical approaches. In most cases these solution schemes involve the need to solve large-scale ill-conditioned linear systems that are corrupted by noise and other errors. In this talk we consider new hybrid Krylov subspace methods to solve these linear systems, including how to choose regularization parameters.
(Received September 08, 2023)

1192-65-30096
Treena Basu*, Occidental College, Ron Buckmire, Occidental College. NSFD Unity Approximations]Using Unity Approximations to Construct Nonstandard Finite Difference Schemes for Bernoulli Differential Equations. We explore the use of nonstandard finite difference (NSFD) methods in numerically approximating solutions to Bernoulli ordinary differential equations (BDEs) with constant coefficients. Specifically, we investigate the numerical performance of NSFD schemes that deploy unity approximations. A unity approximation can be created by introducing and replacing the number 1 with an approximation using non-local terms of the form $\backslash \mathrm{ds} \frac{2 x_{k+1}}{x_{k+1}+x_{k}}$ where $x_{k} \approx x_{k+1}$ on a discrete grid. Our NSFD schemes derived from unity approximations produce solutions that compare favorably with numerical solutions obtained using standard finite difference (SFD) schemes. The NSFD schemes presented here demonstrate their utility by possessing one or more of the following properties: enhanced accuracy, preserving positivity, and maintaining dynamic consistency (i.e., other essential qualities of the ODE pertaining to its domain and the asymptotic properties of the equilibria such as stability). (Received September 08, 2023)

Jorge E. Macias-Diaz*, Autonomous University of Aguascalientes. Numerically efficient methods for partial differential equations which preserve physical quantities.
There are many systems in mathematical physics which preserve meaningful quantities. Indeed, it is easy to find examples of mathematical models which conserve mass, energy, linear momentum, etc. When these systems are discretized, numerical analysts strive to provide models which are also capable of preserving those quantities. From the physical point of view, such discretizations are obviously preferred in view that they reflect the physics of the continuous problem. However, numerical models which preserve discrete quantities are easier to analyze from a theoretical perspective. To start with, the proofs for the properties of stability and convergence are easier to establish when the discrete mass or the discrete energy are conserved. In those cases, a discrete form of Gronwall's inequality is employed together with the conserved properties. In this talk, we will consider a particular model from mathematical physics which preserves various quantities. We will provide a discretization of the mathematical model together with discrete forms of the conserved quantities, and we will prove that they are also preserved in the discrete domain. The proof for the existence of discrete solutions will be carried out using a suitable fixedpoint theorem, and the consistency will be proved using Taylor's theorem. In turn, the properties of stability and convergence will be proved using the conservation rules and a discrete form of Gronwall's inequality. If time allows, we will talk about the computer implementation of the method, ad we will provide some simulations which confirm the convergent character of the scheme.
(Received September 08, 2023)

## 1192-65-30103

Gunay Dogan*, National Institute of Standards and Technology, Gaithersburg. Variational Models and Algorithms for Image Segmentation.
Image segmentation is the task of identifying distinct regions, objects or their boundaries in given images. It is a fundamental task required for many higher level image analysis or computer vision tasks. In this talk, I will review some powerful variational models that are used for image segmentation. These models are formulated as energies whose free variables are the geometric entities representing possible segmentations/partitions of the image. The correct segmentations are obtained by minimizing these energies, and computing the optimal geometries. I will describe a few alternative algorithms to carry out the minimization and to compute the optimal solutions. I will explain the pros and cons of each algorithm, and demonstrate them on real images.
(Received September 08, 2023)

## 1192-65-30116

Shraddha Rajpal*, George Mason University. Data Assimilation For Quantum NV Diamond Spectroscopy. Preliminary report. Nitrogen-vacancy (NV) defect centers in diamond have generated much interest for their uses in quantum information and sensing. Negatively charged centers (NV-) are used for high spatial-resolution sensing (magnetometry, electrometry, thermometry, and barometry) and for quantum information (with a scalable quantum computer as the eventual goal). Despite the rapid NV applications development, our grasp of basic NV properties is incomplete, which is important to understand to fully exploit potential uses. In this work we construct a statistical model for NV spectroscopy and use it in synthetic experiments to solve inverse problems. Our principal application is to develop a primary sensor based on the NV diamond quantum optical properties. This is a significant challenge because the NV diamond structure is sensitive to temperature and pressure as well as magnetic and electric fields, including electromagnetic fields of nearby atoms and molecules. First, using the Hamiltonian for the effects of local strain and the environmental variables, we identify the observable components based on the invertibility of various observation systems. Next, we observe the influence of temperature and pressure on the NV center by solving the Schrödinger Equation and computing the theoretical spectroscopy curve. We assume that the observed photon counts are Poisson random variables with rates proportional to the theoretical spectroscopy. Then, using the Maximum Likelihood Estimation we find the parameter values that maximize the likelihood. Last but not the least we determine the robustness of the model using sensitivity analysis. (Received September 08, 2023)

1192-65-30142
Nadun L Kulasekera Mudiyanselage*, Mount St. Mary's University, Cecile M Piret, Michigan Technological University. New Numerical Approach to Solve PDEs with Interfaces in Heterogeneous Systems Using Radial Basis Functions. The talk will focus on using the Radial Basis Functions (RBF) method to solve the groundwater flow models in fractured porous media. An interface represents changes, such as between layers or abrupt exposures. Our new methodology was first implemented for solving heat equilibrium problems with discontinuous thermal diffusive coefficients across an interface. This work can also be adapted to solve similar groundwater flow equations in the presence of fractures. Furthermore, the methodology was also used to solve the heat conduction equation for a two-material system (snowpack overlying soil) in 1D. In order to develop the numerical method, we used a local RBF approach called the RBF-FD (Radial Basis Functions Finite Difference) method. We observed that the convergence of the new method is impacted by the number of nodes placed on the interface. Therefore, we proposed a coordinate transformation for curvilinear interfaces to fix this issue. We validate our approach by implementing it on PDEs with linear, quadratic, and cubic interfaces.
(Received September 08, 2023)

## 1192-65-30235

Gary Choi*, The Chinese University of Hong Kong. Quantifying shape variation using quasi-conformal geometry.
Quasi-conformal geometry has recently emerged as a useful tool in imaging science. In this talk, we will discuss how quasiconformal theory can be applied for quantifying biological shape variation. More specifically, quasi-conformal mappings can be used for establishing a 1-1 correspondence between two biological shapes with prescribed feature landmarks exactly matched. Moreover, the quasi-conformal distortion encodes important information about the local geometric difference between two shapes. Examples across biological scales are presented to demonstrate the effectiveness of the method.
(Received September 09, 2023)
C.-S. Chien, National Chung Hsing University, Y.-T. Lin, Academia Sinica, Qin Sheng, Baylor University, Yintzer Shih*, National Chung Hsing University. A note on stochastic polynomial chaos expansions for uncertain volatility and Asian option pricing.
This study concerns accurate and efficient polynomial chaos expansions (PCEs) for Asian option pricing with uncertain volatilities. While arbitrary distributions of the volatility parameter are applied for estimating real world option prices, arbitrary polynomial chaos (aPC) are incorporated for approximating raw data of the historical volatility distributions. Rigorous analysis is carried out to ensure the numerical stability of the compact aPC Crank-Nicolson finite difference method accomplished. Numerical results acquired are compared with solutions via standard Monte Carlo schemes (MCSs) and generalized polynomial chaos (gPC) with different random volatilities. Stock data from Asian financial industry are used. It is evident that the novel schemes derived are highly accurate and efficient for evaluating means and variances of uncertain volatility and stochastic Asian option pricing. (Received September 10, 2023)

1192-65-30529
Huan Lei*, Michigan State University, Liyao Lyu, Michigan State University. Construction of coarse-grained molecular dynamics with many-body non-Markovian memory.
We introduce a machine-learning-based coarse-grained molecular dynamics (CGMD) model that faithfully retains the manybody nature of the inter-molecular dissipative interactions. Unlike the common empirical CG models, the present model is constructed based on the Mori-Zwanzig formalism and naturally inherits the heterogeneous state-dependent memory term rather than matching the mean-field metrics such as the velocity auto-correlation function. Numerical results show that preserving the many-body nature of the memory term is crucial for predicting the collective transport and diffusion processes, where empirical forms generally show limitations.
(Received September 10, 2023)

## 1192-65-30639

Caroline Hills*, University of Notre Dame. Robotic Kinematic Optimization Applications.
Task requirements and analyses of robotic kinematic systems can be represented by a system of polynomial equations. As the requirements become more complex, so do the resulting systems, ultimately leading one to consider an optimization formulation using approximations. Homotopy continuation methods can be employed to these polynomial formulations to yield a nearly, if not totally, complete solution set of the critical points of the optimization problem or to describe the characteristics of the mechanism within its workspace. This talk will describe the use of homotopy continuation methods in robotic kinematic applications and analyses with application examples.
(Received September 10, 2023)

## 1192-65-30672

Martin W. Licht*, EPFL. Computable reliable bounds for Poincaré-Friedrichs constants via Čech-de-Rham complexes. We derive computable and reliable upper bounds for Poincaré-Friedrichs constants of classical Sobolev spaces and, more generally, $L^{2}$ de-Rham complexes. The upper bounds are in terms of local Poincaré-Friedrichs constants over subdomains and the smallest singular value of a finite-dimensional operator that is easily assembled from the geometric setting. Thus we reduce the computational effort when computing the Poincaré-Friedrichs constant of finite de-Rham complexes, and we provide computable reliable bounds even for the original $L^{2}$ de-Rham complex. The reduction to a finite-dimensional system uses diagram chasing within a Čech-de-Rham complex.
(Received September 10, 2023)

## 1192-65-30885

Li Wang*, University of Minnesota. Deep JKO: time-implicit particle methods for general nonlinear gradient flows. Preliminary report.
We develop novel neural network-based implicit particle methods to compute high-dimensional Wasserstein-type gradient flows. The main idea is to use the Lagrangian formulation in the Jordan-Kinderlehrer-Otto framework, where the velocity field is approximated using a neural network. Our methodology demonstrates versatility in handling a wide range of gradient flows, accommodating various potential functions and nonlinear mobility scenarios.
(Received September 11, 2023)

## 1192-65-30936

Divya Jaganathan*, International Centre for Theoretical Sciences (ICTS-TIFR), Vishal Vasan, International Centre for Theoretical Sciences (ICTS-TIFR). Markovian embedding of nonlocal evolution equations using spectral representations. We consider physical systems whose dynamics are modelled by nonlocal evolution equations. The governing integrodifferential equations in such systems are characterised by persistent memory effects which make numerical computations increasingly prohibitive with the simulation time. The non-Markovian nature of the equations, due to the memory effects, precludes the use of standard time-integrators which are based on local rules. Our approach involves embedding the nonlocal (non-Markovian) equation into a Markovian system in an inflated state space. The inflated state space is constructed by introducing co-evolving variables, along with their dynamical equations, which are suitably derived from a spectral representation relevant to the governing equation. We derive iterative schemes for the resultant Markovian system. The local reformulation, as a result of the Markovian embedding procedure, exhibits benefits typical to numerical integration of canonical dynamical systems namely, a constant memory storage cost, a linear growth in operational effort with simulation time, and the ability to self-start. As concrete examples, we discuss the Maxey-Riley equation for an inertial particle in viscous flow, and the evolution of movable boundary in the classical Stefan problem for phase change.
(Received September 11, 2023)

John Urschel*, MIT. From Moments to Matrices.
More than a hundred years ago, Chebyshev posed the following problem: "Given the length, weight, position of mass centre and moment of inertia of a material straight line with an unknown density... find the narrowest possible limits for the weight of any segment of the line." This is one of the earliest examples of a moment problem, the task of obtaining information about a measure from some sequence of its moments. In this talk, we will explore the classical moment problems of the late 19 th and early 20th centuries and how they laid the groundwork for fundamental concepts and techniques in numerical analysis and numerical linear algebra.
(Received September 11, 2023)

1192-65-31023
Gabriela Jaramillo, University of Houston, Sameel Imran Siddiqi*, University of Central Florida, Kevin Sony*, University of Houston. Localized Target Patterns in Oscillatory Media. Preliminary report.
Oscillatory systems that permit interactions between different elements within the system are said to be coupled. It is well known that in the presence of a defect, these systems generate target patterns, a series of concentric circles emanating from a center. Past experiments involving CO oxidation on Pt found never before seen localized target patterns, and numerical simulations with the assumption of global coupling were able to explain these structures. In this project, we investigate the role of more general nonlocal forms of coupling in the formation of these localized patterns. To do this, we consider a nonlocal FitzHugh-Nagumo system undergoing a Hopf bifurcation as an abstract model for oscillatory media, and using multiple scale analysis, derive a complex Ginzburg-Landau (CGL) equation as a reduced model. To find the conditions needed for the formation of localized target patterns, we numerically simulated the CGL equation. These conditions are then extrapolated to the FitzHugh-Nagumo system. Our results confirm that localized target patterns can be found in reaction-diffusion equations that are near a Hopf bifurcation, but also show that not all types of nonlocal coupling give rise to these patterns. This works is well supported by NSF grant DMS-1911742
(Received September 11, 2023)

## 1192-65-31073

Anup R Lamichhane*, Ohio Northern University. Fast oscillatory radial basis functions collocation method. Preliminary report.
Fast oscillatory radial basis functions (F-ORBFs) collocation method is a numerical method to solve partial differential equations. F-ORBFs collocation method couples oscillatory radial basis function collocation method with fast summation method. In this talk, we present F-ORBFs numerical scheme to solve an elliptic partial differential equations. (Received September 11, 2023)

1192-65-31245
Alen Alexanderian, North Carolina State University, Ethan Ebbighausen*, University of North Carolina Chapel Hill, Joseph Hart, Sandia National Laboratories, Janet Jiang*, Trinity University, Riley Link*, Creighton University, Julia Martello*, Liberty University, Paul Spears, North Carolina State University. On Approximate Solutions for Perturbed Optimal Control Problems.
We consider the optimal control of a space shuttle trajectory where we seek to maximize longitudinal range as the shuttle descends into the atmosphere. The optimal trajectory depends on parameters, such as aircraft shape, whose importance we determine by derivative-based global sensitivity measures. Parameter uncertainty limits the applicability of control trajectories computed prior to flight and often begets a need for new solutions mid-flight. However, it is often prohibitive or impossible to recalculate the optimal control mid-flight due to strict time constraints. Instead, we approximate new trajectories using integration methods on post-optimality sensitivity information. We test the accuracy of these approximations for computing entire trajectories along with mid-flight changes to determine the best method of approximating perturbed solutions of optimal control problems.
(Received September 11, 2023)

1192-65-31262
Alexander Cloninger*, UCSD, Scott Mahan, University of California, San Diego. Linearized Optimal Transport with Input Convex Neural Networks for Point Cloud Classification.
We focus on the problem of classifying distributions and point clouds via the linear optimal transport embedding. This is a method for embedding distributions into a Hilbert space with guarantees on when this embedding is nearly isometric to Wasserstein-2 distance. However, learning these embeddings is expensive with traditional optimal transport solvers. In this talk, we focus on two tasks: learning optimal transport maps with neural networks, and training a classifier on the learned maps. For the first task, we prove that our approach linearly embeds certain classes of probability distributions into Euclidean space nearly isometrically with high probability. Consequently, we can correctly classify these families of distributions using neural networks with finite input dimension, provided that the classes are separable in the space of distributions. This result establishes theoretical guarantees for neural networks as a method for classifying distributions and point clouds. It can similarly be viewed as incorporating geometric priors into the Deep Sets family of algorithms. We demonstrate the benefits of this geometric perspective in small data problems where there are only a handful of point clouds in the training data. (Received September 11, 2023)

## 1192-65-31272

Alexander Panchenko*, Washington State University. Transportation matchings with bounded distances. Preliminary report. Consider a set $S$ of $N$ points in $\mathbf{x}_{n} \in \mathbf{R}^{d}$ contained in a unit cube. We give sufficient condition for existence of a uniformly distance-bounded perfect matching between $S$ and points $\mathbf{z}_{n}, n=1,2, \ldots, N$ of a lattice (restricted to the unit cube). An interesting special case is when the distance bound depends on $N$. Such bounds may be used to obtain estimates for scattered data quasi-interpolation and integral quadratures.
(Received September 11, 2023)

## 1192-65-31298

Emma R Cobian*, University of Notre Dame. Optimization Techniques in Variational Inference.
Approximating probability distributions is an important task in machine learning and statistics. Recently, optimization-based methods in variational inference have gained popularity, such as normalizing flows, to provide approximations which allow both sampling and density estimation. Normalizing flows are invertible mappings used to transform simpler distributions into ones that are more complex through optimizing parameters associated with these mappings. With computationally expensive model evaluations or complicated geometrical structures underlying a distribution, this can be a challenging task providing an accurate approximation. I will be presenting optimization techniques which facilitate computational efficiency and accurate convergence to the desired distribution.
(Received September 11, 2023)

## 1192-65-31306

Emily Ellen Diegel*, Embry Riddle Aeronautical University. Gamma Emission Tomography for the Verification of Nuclear Spent Fuel.
Radioactive waste from nuclear-energy facilities poses challenges for the world both in safe, long-term disposal of material and ensuring nonproliferation of weapons of mass destruction. Spent nuclear fuel assemblies contain plutonium, an element that can be used to make nuclear weapons, and a mission of the International Atomic Energy Agency (IAEA) is safeguarding against its illicit use. Verifying that the nuclear material contained in the spent fuel is not being diverted for non-peaceful activities and determining whether countries are abiding by their non-proliferation responsibilities - is an essential component of providing assurance to the international community. Verifying the declaration of the geometry of a spent-fuel assembly can be accomplished by determining the distribution of radioactive material. Ideally, one would measure the distribution of plutonium, but the distribution of any fission product can be used for this metric of integrity. The IAEA uses the Passive Gamma Emission Tomography (PGET) device to perform a measurement and reconstruction of cross-sectional activity from gamma emitters in spent-fuel assemblies. Reconstructed images are analyzed to determine the number of pins present in the assembly. Unfortunately, image artifacts and variations in pin-to-pin burnup complicate unambiguous counting of pins. Figures of merit have been developed where individual pin intensities of the assemblies are compared to a neighborhood of average pin intensities to determine missing rods based on a statistical deviation. This study describes a new metric for discriminating missing pins called radial neighbors, where pins are compared at similar radii for verification of nuclear material. PNNL-SA189972
(Received September 11, 2023)

## 1192-65-31324

Iván Ojeda Ruiz*, Texas State University. Spectral Clustering with Fairness Constraints/Lessons from Algorithms, Fairness and Equity.
In this session, I will discuss a working definition of Fairness. In various recent works, the inclusion of fairness constraints in clustering methods has been studied. The notions of disparate impact, community cohesion, and cost per population have all been implemented as constraints previously. We shall consider the notion of balance as a metric of fairness. There have been scarce results in the implementation of fairness constraints in Spectral Clustering, so we study the notion of balance as a constraint for Spectral Clustering. The resulting problem has been studied with a variation of the Stochastic Block Model and we shall present guarantees for the distance of the result from a fair ground-truth. We implement the so-called Constrained Normalized Cut model in order to obtain results comparable to the fair ground truth, which can achieve a similar balance and can improve the speed of the computation. We show the results obtained using the FriendshipNET and FacebookNET databases.
(Received September 12, 2023)

1192-65-31337
Shiying Li*, University of North Carolina, Chapel Hill, Caroline Moosmueller, University of North Carolina - Chapel Hill. Exploring Iterative Slice-Matching for Measure Transport.
In the domain of measure transport, we explore (stochastic) iterative slice-matching techniques. These methods come into play when dealing with shape deformation, especially when shapes are modeled as probability distributions. Although these techniques have proven effective in various data science applications, a comprehensive understanding of their convergence properties remains elusive. Our primary focus is on establishing an almost sure convergence proof, shedding light on intriguing connections with stochastic gradient descent in the context of the sliced-Wasserstein distance. Additionally, we delve into recovery and stability considerations under specific structural assumptions about the measures (shapes). The talk is based on joint work with Caroline Moosmueller.
(Received September 12, 2023)

1192-65-31371
Satyajith Bommana Boyana*, University of North Carolina at Greensboro, Thomas Lee Lewis, University of North Carolina at Greensboro, Aaron Frost Rapp, University of the Virgin Islands, Yi Zhang, University of North Carolina at Greensboro. Novel Discontinuous Galerkin Methods for Optimization Problems with Inequality Constraints.
A symmetric DWDG method is used to discretize a control-constrained elliptic optimal control problem with the PDE constraint as Poisson's equation. We develop a scheme to obtain a finite-dimensional optimization problem which is then solved with a primal-dual active set strategy. The convergence of the numerical solution pair ( $\bar{y}_{h}, \bar{u}_{h}$ ) is proved. The rates of convergence are established in $L^{2}$ and energy norms. Next, to solve the parabolic variational inequality with a general obstacle function, a fully discrete scheme that uses a symmetric dual-wind discontinuous Galerkin discretization in space and a backward Euler discretization in time is proposed and analyzed. The convergence of numerical solutions in $L^{\infty}\left(L^{2}\right)$ and $L^{2}\left(H^{1}\right)$ like energy norms established and the rates are computed. Several numerical tests are provided to demonstrate the robustness and effectiveness of the proposed methods. This is joint work with Tom Lewis, Aaron Rapp, and Yi Zhang. (Received September 11, 2023)

## 1192-65-31490

Junping Wang*, National Science Foundation. Weak Galerkin Finite Element Methods.
The speaker will first provide an overview on the weak Galerkin finite element methods for partial differential equations. This part of the talk should be accessible to graduate students who are interested in numerical methods for differential equations. The second part of the talk will focus on recent developments in weak Galerkin finite element methods.
(Received September 11, 2023)

1192-65-31760
Qingshan Chen*, Clemson University. A conservative scheme for the multilayer shallow water equations by the Hamiltonian principle.
The system of multilayer shallow water equations is an important model for large-scale geophysical flows, namely the ocean and atmosphere. Without frictional forces, the model is reversible, and can be reformulated as a Hamiltonian system. Compared with traditional formulations, the Hamiltonian formulation of the system exposes its inherent geometric structure. In this work, we exploit the skew-symmetry of the system, which is made apparent by the Hamiltonian formulation, to derive an exactly energy conserving scheme for the system. Recent numerical results from the scheme will be presented, and discussions will also be given regarding the challenges in simulating large-scale geophysical flows.
(Received September 12, 2023)

1192-65-31768
Thomas Lee Lewis, University of North Carolina at Greensboro, Xiaohuan Xue*, University of North Carolina at Greensboro. Numerical moment stabilization of central difference approximations for linear stationary reaction-convection-diffusion equations.
This talk will explore the impact of adding a high-order stabilization term called a numerical moment when approximating linear stationary reaction-convection-diffusion equations with Dirichlet boundary conditions using central difference approximations for the gradient. The focus will be on convection-dominated equations, and the formulation for the method is motivated by various results for approximating viscosity solutions to fully nonlinear degenerate elliptic equations. The methods are shown to be uniformly stable in the $\ell^{\infty}$ norm independent of the scaling of the diffusive term while featuring higher-order local truncation errors than monotone methods consistent with the use of the central difference approximation for the gradient. Stability and rates of convergence are also derived in the $\ell^{2}$ norm for the constant-coefficient case. Numerical tests are provided to compare the new methods to monotone methods. The methods are also tested for stationary Hamilton-Jacobi equations where they demonstrate higher rates of convergence than the Lax-Friedrich's method when the underlying viscosity solution is smooth and comparable performance when the underlying viscosity solution is not smooth. (Received September 12, 2023)

## 1192-65-31859

Jacob Blazejewski*, Michigan Technological University, Nadun L Kulasekera Mudiyanselage, Mount St. Mary's University, Benjamin Ong, Michigan Technological University, Cecile M Piret, Michigan Technological University. Coupling RBF-FD with the Parareal Framework to Solve Time-Dependent PDEs. Preliminary report.
The meshless Radial Basis Functions-Finite Difference (RBF-FD) method discretizes differential operators in both ODEs and PDEs. It is commonly used to discretize the spatial operator of a time-dependent PDE to convert the problem to a system of ODEs via the Method of Lines (MOL) approach. The resulting system of ODEs is solved by selecting an appropriate numerical technique that is traditionally viewed as a sequential process. Alternative ODE solvers include the Parallel in Time (PinT) integrators, such as the Parareal Framework. As their name implies, PinT integrators operate along segments of the time domain in parallel rather than sequentially. In this presentation, we summarize options from our recent paper on how to couple the RBF-FD method with the Paraeal Framework. Additionally, we will demonstrate the coupling's potential for mitigating the impact of spurious eigenvalues present in the spectrum of certain RBF-FD differentiation matrices. (Received September 12, 2023)

## 1192-65-31998

Zhaolong Han, UC San Diego, Xiaochuan Tian*, University of California, San Diego. Asymptotically compatible scheme for nonlocal saddle point problems.
In this talk, we present an abstract framework for studying asymptotically compatible schemes for parameterized saddle point problems. This framework is then applied to the study of robust numerical discretization for several nonlocal problems, including nonlocal Poisson's problem, nonlocal convection-diffusion equations, nonlocal Stokes equation, as well as nonlocal biharmonic problems.
(Received September 12, 2023)

## 1192-65-32022

Xiaochuan Tian*, University of California, San Diego, Qihao Ye, University of California, San Diego. Monotone meshfree methods for linear elliptic equations in non-divergence form via nonlocal relaxation.
We design a maximum principle preserving meshfree finite difference method for linear elliptic PDEs on point clouds via a nonlocal relaxation method. The key idea is a novel combination of a nonlocal integral relaxation of the PDE problem with a robust meshfree discretization on point clouds. Minimal positive stencils are obtained through a linear optimization procedure that automatically guarantees the stability and, therefore, the convergence of the meshfree discretization. A major theoretical contribution is the existence of consistent and positive stencils for a given point cloud geometry. We provide sufficient conditions for the existence of positive stencils by finding neighbors within an ellipse (2d) or ellipsoid (3d) surrounding each interior point. Our result represents a significant improvement in the stencil width estimate for positive-type finite difference methods for linear elliptic equations in the near-degenerate regime (modeling high-contrast media), compared to previously known works in this area. Numerical results are conducted in 2d and 3d, examining a range of ellipticity constants including the near-degenerate regime. Some new experiments on degenerate elliptic equations will also be discussed.

Mario Javier Bencomo*, University of California Fresno. Finite difference discretizations of singular anisotropic seismic sources.
Seismic sources are commonly idealized as point-sources due to their small spatial extent relative to seismic wavelengths. To account for anisotropic radiation patterns, seismic sources can be modeled as multipoles, that is, a time-dependent linear combination of spatial derivatives of the spatial delta function. Since the solutions of linear hyperbolic systems with pointsources are necessarily singular, standard finite difference (FD) convergence results are no longer applicable for these problems. We present a discretization of these singular sources compatible with FD methods, an extension of the moment matching conditions developed for the Dirac delta function. We also develop a weak convergence theory applicable to a large family of wave propagation problems with singular source terms, solved via staggered-grid finite difference methods: we show that grid-independent space-time averages of the numerical solutions converge to the same averages of the continuum solution, and provide an estimate for the error in terms of moment matching and truncation error conditions. Numerical experiments confirm this result, but also suggest a stronger one: optimal convergence rates appear to be achieved point-wise in space away from the source.
(Received September 12, 2023)

1192-65-32050
Emi Cervantes*, University of California, Irvine, Jenny Tran*, University of California, Irvine. A Greedy Approach to Kaczmarz Method with Oblique Projections. Preliminary report.
Randomized iterative algorithms are commonly used to solve large-scale linear systems of the form $A x=b$. The Kaczmarz method is one such algorithm that utilizes only one row of $A$ in each iteration to perform orthogonal projections to solve linear systems. There are many ways to choose the next row. One such approach is a greedy method, also known as Motzkin's Method. A variation of the Kaczmarz algorithm called Kaczmarz with oblique projections (KO), was proposed recently and utilizes oblique projections to improve convergence speed. To optimize computation, we propose the Greedy Kaczmarz with oblique projections (GKO) which chooses the row associated with the largest step length when allowing for oblique projections. Our theoretical results and numerical experiments show that in some settings, GKO performs better than alternative greedy oblique projection methods. In this talk, we present our proposed method, our theoretical results, and discuss our numerical experiments.
(Received September 12, 2023)

1192-65-32090
Elizabeth Newman*, Emory University, Lars Ruthotto, Emory University, Deepanshu Verma, Emory University, Samy Wu Fung, Colorado School of Mines. Efficient Training of Deep Neural Networks with Gauss-Newton. Preliminary report. Deep neural networks (DNNs) have achieved inarguable success as high-dimensional function approximators in countless fields, including numerous scientific applications such as surrogate modeling, operator learning, and model discovery. However, this success comes at significant hidden costs, notably a long training time. Typically, training is posed as a stochastic optimization problem to learn the DNN weights. However, we can interpret training as solving a high-dimensional, nonlinear data fitting problem for which Gauss-Newton has traditionally been the go-to optimization strategy. Despite this tradition, Gauss-Newton has not gained widespread use for DNN training, often due to computational bottlenecks. In this talk, we propose a memory and computationally efficient Gauss-Newton implementation that can solve the training problem well and learn weights that generalize. We will describe how we obtain this efficiency by approximating the Jacobian with only a few additional passes through the network. We will outline how our optimizer reliably solves the training problem using a trust-region method. We will demonstrate the computational advantages of our approach over traditional stochastic optimizers on several benchmark deep learning tasks.
(Received September 12, 2023)

1192-65-32091
Olena Burkovska*, Oak Ridge National Laboratory. Efficient approximation of nonlocal variational inequalities. In this talk, we present an efficient discretization method for time-dependent nonlocal variational inequalities with integrable kernels. The method is based on a Fourier collocation approximation in space together with a semi-implicit time discretization. We show that a special structure of the model and nonlocal kernel allows for the characterization of the solution in terms of a projection formula. This, together with the FFT-based implementation, allows for a fast evaluation of the solution that can bypass a solution of both a nonlinear and nonlocal system. We also discuss higher-order time-stepping schemes and present several numerical experiments, including application to the nonlocal Allen-Cahn phase-field system.
(Received September 12, 2023)

1192-65-32332
Tom Chou, UCLA, Sihong Shao, Peking University, Mingtao Xia*, Courant Institute of Mathematical Sciences. Adaptive spectral methods in unbounded domains for solving spatiotemporal equations.
We shall introduce a novel adaptive spectral method to numerically solve partial differential equations (PDEs) in unbounded domains. To achieve accuracy and improve efficiency, the method relies on the dynamic adjustment of three key tunable parameters: the scaling factor, a displacement of the basis functions, and the spectral expansion order. Numerical examples will be provided to show efficiency of our method. We shall also then introduce how the implementation of the adaptive spectral methods affects numerical results, thereby providing guidelines for the proper tuning of parameters. (Received September 12, 2023)
using neural networks to predict dynamics of epidemiological models incorporating human behavior.
One of the popular ways to model the dynamics of infectious diseases is using compartmental models that can be described by a coupled system of non-linear differential equations. This governing system of equations often includes several parameters such as transmission and recovery rates, which are usually hard to estimate efficiently from an available dataset for a certain disease. In this work, we present modeling, analysis and simulation of a mathematical epidemiological model which incorporates human social, behavioral, and economic interactions. We discuss an approach based in Physics-Informed Neural Network, which is capable of predicting the dynamics of a disease described by modified compartmental models that include parameters, and variables associated with the governing differential equations. (Received September 12, 2023)

1192-65-32456
Alexander Cloninger, UCSD, Dhruv Kohli, University of California San Diego, Gal Mishne*, University of California San Diego. Low distortion embeddings with bottom-up manifold learning. Preliminary report.
Manifold learning algorithms aim to map high-dimensional data into lower dimensions while preserving local and global structure. In this talk, I present Low Distortion Local Eigenmaps (LDLE), a bottom-up manifold learning framework that constructs low-distortion local views of a dataset in lower dimensions and registers them to obtain a global embedding. Motivated by Jones, Maggioni, and Schul (2008), LDLE constructs local views by selecting subsets of the global eigenvectors of the graph Laplacian such that they are locally orthogonal. The global embedding is obtained by rigidly aligning these local views, which is solved iteratively. Our global alignment formulation enables tearing manifolds so as to embed them into their intrinsic dimension, including manifolds without boundary and non-orientable manifolds. We define a strong and weak notion of global distortion to evaluate embeddings in low dimensions. We show that Riemannian Gradient Descent (RGD) converges to an embedding with guaranteed low global distortion. Compared to competing manifold learning and data visualization approaches, we demonstrate that LDLE achieves lowest local and global distortion on real and synthetic datasets.
(Received September 13, 2023)

1192-65-32457
Thomas Lee Lewis, University of North Carolina at Greensboro, Quinn Alexander Morris*, Appalachian State University, Yi Zhang, University of North Carolina at Greensboro. Approximating sublinear positone and semipositone boundary value problems using finite difference methods.
In this talk, we will discuss a method for approximating solutions to sublinear positone and semipositone boundary value problems using a finite difference method based on the method of sub- and supersolutions. We prove convergence and existence results for our method, and show a correspondence between solutions to the discretized boundary value problem and its continuous analogue.
(Received September 12, 2023)

## 1192-65-32533

Lander Besabe*, University of Houston, Michele Girfoglio, SISSA, Annalisa Quaini, University of Houston, Gianluigi Rozza, SISSA. Large Eddy Simulation for the quasi-geostrophic equations. Preliminary report.
In this talk, we will explore the utilization of various nonlinear differential low-pass filters for the Large Eddy Simulation (LES) of the quasi-geostrophic equations. Our goal is to assess the accuracy of the filters by analyzing time-averaged fields of interest within the framework of the classical double-gyre wind forcing experiment. Moreover, we will introduce some preliminary results of its extension for the stratified two-layer quasi-geostrophic equations (2LQGE). We will then propose an efficient segregated algorithm using Finite Volume methods for 2LQGE. Lastly, we will also present manufactured numerical tests to show the rate of convergence of the method for solving steady state solutions of the said model. These results could enhance our understanding of large-scale geophysical fluids and contribute to the development of more robust numerical models for ocean flows.
(Received September 12, 2023)

## 1192-65-32555

Ram Sharan Adhikari*, Rogers State University. A weak Simpson method and its mean square stability analysis for a system of stochastic differential equations.
We present a weak numerical algorithm to approximate the solutions for a system of stochastic differential equations. The proposed method constructs paths that are accurate in third order under certain additional assumptions. This work aims to analyze the mean-square stability region of the proposed method for stochastic ordinary differential equations with multiplicative noises. In this work, we will provide step-sizes for our scheme where errors propagation is under control in a well-defined sense are given. The main results are illustrated with numerical examples.
(Received September 12, 2023)

1192-65-32561
David P. Nicholls*, University of Illinois at Chicago, Jon Wilkening, University of California, Berkeley, Xinyu Zhao, McMaster University. High-Order Spectral Methods for the Computation of Dirichlet-Neumann Operators for Laplace's Equation with Quasiperiodic Boundary Conditions.
In this talk we describe stable High-Order Spectral algorithms for the numerical simulation of Dirichlet-Neumann operators (DNOs) which arise in boundary value and free boundary problems from a wide variety of applications (e.g., fluid and solid mechanics, electromagnetic and acoustic scattering). More specifically, we consider DNO defined on domains inspired by the simulation of ocean waves subject to quasi-periodic boundary conditions. We have recently shown that the DNO, when perturbed from a flat interface configuration, is parametrically analytic (as a function of deformation height/slope) for profiles of finite smoothness. The method of proof suggests a stable and high-order method of numerical simulation that we now present.
(Received September 12, 2023)

Bruce A Wade*, University of Louisiana at Lafayette. Exponential Time Differencing Schemes with Dimensional Splitting for Non-linear Reaction-Diffusion Systems.
Second and fourth order Exponential Time Differencing (ETD) schemes with dimensional splitting are developed to solve nonlinear systems of reaction-diffusion equations. The splitting methods are shown to significantly improve computational efficiency over existing ETD schemes which do not employ any splitting strategy.
(Received September 12, 2023)

## 1192-65-32636

Joe Kileel*, University of Texas at Austin. Mathematics of cryo-EM.
In this talk I'll discuss the cryo-electron microscopy problem of recovering the 3D shape of biomolecules from datasets of noisy 2D images. In important cases, the problem is governed by invariant theory and may be reduced to solving an overdetermined polynomial system. In other cases, spectral embedding methods give useful qualitative information.
(Received September 12, 2023)

Brian E Moore, University of Central Florida, Poroshat Yazdanbakhsh*, Rollins College. Linear Stability of StructurePreserving Exponential Time Differencing Schemes for Damped Hamiltonian Systems. Preliminary report.
This talk considers structure-preserving exponential time differencing methods for solving conservative Hamiltonian systems that are perturbed by a linear, time dependent damping term. Analysis of the linear stability of the methods, which provides a relationship between the damping parameter and the step size, reveals when the schemes give the greater stability region. These methods are further explored through applications to various damped-driven ODEs and PDEs. Finally, to demonstrate accuracy of the methods and the advantages of structure-preservation, the methods are numerically compared to standard methods.
(Received September 12, 2023)

## 1192-65-32791

Muhammad Adnan Anwar*, Instituto Superior Técnico, Jorge Tiago, Instituto Superior Técnico. Numerical approximation techniques and error analysis with FEniCS.
This study explores the realm of numerical approximation techniques and error analysis within the context of the FEniCS computational framework. FEniCS is a powerful tool for solving partial differential equations (PDEs) numerically, making it essential for a wide range of scientific and engineering applications. Understanding the accuracy and reliability of numerical solutions is crucial in ensuring the fidelity of computational simulations. This research delves into various numerical approximation methods, such as finite element methods, finite difference methods, as implemented in FEniCS. Through rigorous error analysis, the study investigates the sources of errors inherent in these techniques, including discretization errors, round-off errors, and convergence behavior. By systematically examining error sources and quantifying their impact, this research provides insights into optimizing numerical simulations, choosing appropriate discretization schemes, and estimating the accuracy of results. Furthermore, it sheds light on the trade-offs between computational efficiency and solution accuracy, enabling practitioners to make informed decisions when utilizing FEniCS for solving PDEs. Ultimately, this exploration of numerical approximation techniques and error analysis with FEniCS contributes to the broader understanding of computational modeling and its applicability to complex real-world problems across various scientific disciplines. (Received September 12, 2023)

## 1192-65-32825

Mona Baroonian, Georgia State University, Alexandra B. Smirnova*, Georgia State University. Reconstruction of Incidence Reporting Rate for SARS-CoV-2 Delta Variant of COVID-19 Pandemic in the US.
In recent years, advanced regularization techniques have emerged as a powerful tool aimed at stable estimation of infectious disease parameters that are crucial for future projections, prevention, and control. Unlike other system parameters, i.e., incubation and recovery rates, the case reporting rate, $\Psi$, and the time-dependent effective reproduction number, $\mathcal{R}(t)$, are directly influenced by a large number of factors making it impossible to pre-estimate these parameters in any meaningful way. In this study, we propose a novel iteratively-regularized trust-region optimization algorithm, combined with $S_{u} S_{v} I_{u} I_{v} R D$ compartmental model, for stable reconstruction of $\Psi$ and $\mathcal{R}(t)$ from reported epidemic data on vaccination percentages, incidence cases, and daily deaths. The innovative regularization procedure exploits (and takes full advantage of) a unique structure of the Jacobian and Hessian approximation for the nonlinear observation operator. The proposed inversion method is thoroughly tested with synthetic and real SARS-CoV-2 Delta variant data for different regions in the United States of America from July 9, 2021, to November 25, 2021. Our study shows that case reporting rate during the Delta wave of COVID-19 pandemic in the US is between $12 \%$ and $37 \%$, with most states being in the range from $15 \%$ to $25 \%$. This confirms earlier accounts on considerable under-reporting of COVID-19 cases due to the impact of "silent spreaders" and the limitations of testing.
(Received September 12, 2023)

## 1192-65-32862

Margaret Regan*, College of the Holy Cross. Using monodromy to recover symmetries of polynomial systems. Preliminary report.
Many problems in computer vision are represented using a parameterized polynomial system, where the solution set is critical for 3D reconstruction. Minimal problems can be of special interest as these polynomial systems are well-constrained and generically have finitely many solutions. Computing the Galois/monodromy group of the associated branch cover can yield information and understanding about the underlying structure of the minimal problem. One potential outcome of this computation is identifying the existence of possible decompositions or symmetries. Beyond the question of existence, one would like to compute formulas for these symmetries, towards the eventual goal of solving the systems more efficiently. These Galois/monodromy groups can be computed using numerical homotopy continuation via a multitude of softwares. The
equations for the symmetries can be found with the additional technique of multivariate rational function interpolation. This talk will discuss these methods in theory, as well as illustrate the approach on practical examples of minimal problems in computer vision. This is joint work with Timothy Duff, Viktor Korotynskiy, and Tomas Pajdla.
(Received September 12, 2023)

Jacob Blazejewski*, Michigan Technological University, Cecile M Piret, Michigan Technological University. Restricting Differential Operators to Curves and Surfaces using Fast Orthogonal Gradients with the RBF-FD Method. Preliminary report. Partial Differential Equations (PDEs) on arbitrary surfaces arise in many applied and natural science models. A notable example of solving PDEs on static surfaces is image processing. Applications of PDEs on evolving surfaces occur in material science and fluid dynamics. Additionally, the fields of biology and computer graphics have applications for PDEs on both static and evolving surfaces. There are three main categories of methods for solving PDEs on arbitrary surfaces: the methods that rely (i) on parametrization, (ii) on an embedding, and (iii) on triangulation. Embedding-type methods are quite simple in that they are based on the discretization of standard $\mathbb{R}^{3}$ operators rather than curve or surface-specific operators. One of the most common embedding methods is the closest point method (CPM). The surface is enclosed inside a thick layer of nodes that belong to a dense three-dimensional grid. Each one of these nodes takes the function value of the one associated with their closest point to the surface, implicitly imposing that the normal derivatives at each node is null. Under that constraint, the surface Laplacian is equivalent to its $\mathbb{R}^{3}$ analog. The Radial Basis Functions Orthogonal Gradients method (RBF-OGr) is another embedding method. It benefits from the meshfree character of RBFs, which gives the flexibility to represent complex geometries in any spatial dimension while providing a high order of accuracy. This method is different from the CPM in that its computational domain is the point cloud that defines the manifold, instead of being being a thick layer of nodes around the surface. Every computation is performed on the surface, including the constraints of having null normal derivatives. The fast RBF-OGr method uses the finite-difference based RBF method, instead of the global standard RBFs which gave rise to dense differentiation matrices, a limiting factor on the size of the point cloud representing the surface. However, going from the global to the local RBF method have introduced a few sources of instabilities in the process. In this presentation, we will address the different stability issues and provide solutions. We will illustrate the procedure with a number of interesting examples.
(Received September 12, 2023)

## 1192-65-32928

Kevin Wan, Homestead High School, George Zhu*, Homestead High School. On mathematical modeling, simulation, and analyses of opioid use disorder with treatment. Preliminary report.
In this paper, the dynamics of an SITR model are examined in which the incidence rate is Beddington-DeAngelis type and the treatment rate is Holling type II (saturated). Analysis shows that the disease free equilibrium (DFE) point is locally asymptotically stable when the reproduction number is below one. Analysis is also done on the existence of the endemic equilibrium (EE). Analysis is also conducted on a stochastic version of the model with a continuous time Markov chain. Numerical analyses are also conducted to further understanding.
(Received September 13, 2023)

1192-65-33015
Balram Singh Aulakh*, Self. A Generalized Relationship Of The Zeros Of The Two Functions. Preliminary report. A Generalized Relationship Of The Zeros Of Two Functions If $x_{1}$ and $x_{2}$ are the zeros of the function
$f(x)=x^{2}-(a+d) x+(a d-b c)$, then for $p=2,3 ; x_{1}^{p}$ and $x_{2}^{p}$ will be the zeros of the function $F(y)=y^{2}$ $-\left[a^{p}+d^{p}+p b c(a+d)^{p-2}\right]+(a d-b c)^{p}$. \par \end \{document\} \par }
(Received September 13, 2023)

## 1192-65-33053

Stefano Colafranceschi*, Eastern Mennonite University, Roger Thelwell, James Madison University. Title: n-Body problem via the Power Series Method.
We apply the Power Series Method (PSM) to the Hamiltonian $n$-body problem of celestial mechanics implemented on heterogeneous computing. Combining different types of processing units such as FPGAs (Field-Programmable Gate Arrays) and GPUs (Graphics Processing Units) the execution of the algorithm is efficient and accelerated. This is a highly parallelizable problem, where the proposed computing platforms can dynamically adjust the allocation of tasks to different hardware components based on workload characteristics. The presented program for solving the $n$-body problem using PSM benefits from a heterogeneous computing approach by leveraging FPGAs and GPUs in conjunction with CPUs. The solver achieves higher performance, faster simulations, and better resource utilization for large-scale $n$-body simulations, while also offering adaptability to various simulation scenarios.If there is time, we will consider error estimate control using angular momentum. (Received September 13, 2023)

1192-65-33095
Peter Call, Brigham Young University, Xander de la Bruere, Brigham Young University, Samuel Goldrup, Chicago Booth School of Business, Tyler J. Jarvis, Brigham Young University, Timothy Jay Keith, Brigham Young University, Erik Parkinson, Emergent Trading, Jane Slagle, Tufts University, Daniel Ryan Treuhaft*, Brigham Young University, Kate Wall, Brigham Young University. Chebyshev Subdivision and Reduction Methods for Solving Multivariable Systems of Equations.
We use Chebyshev approximations and their properties to efficiently find common roots of multivariate systems of smooth functions. Our algorithm employs subdivision and interval reduction techniques with R-quadratic convergence. Comparison of speed and accuracy with other methods, including algebraic, homotopy, subdivision, resultant, eigenvalue, and Chebyshev proxy methods. Tests used consist of dense polynomial systems and transcendental systems on $[-1,1]^{n}$.
(Received September 13, 2023)

Peter Call, Brigham Young University, Xander de la Bruere, Brigham Young University, Samuel Goldrup, Chicago Booth School of Business, Tyler J. Jarvis, Brigham Young University, Timothy Jay Keith, Brigham Young University, Erik Parkinson, Emergent Trading, Jane Slagle, Tufts University, Daniel Ryan Treuhaft, Brigham Young University, Kate
Wall*, Brigham Young University. A Chebyshev Subdivision and Reduction Method for Multivariate Root-finding with Quadratic convergence.
We use Chebyshev approximations and their properties to efficiently find common roots of multivariate systems of smooth functions. Our algorithm takes advantage of both the orthogonality and bounds of Chebyshev monomials to achieve Rquadratic convergence. We also employ subdivision techniques that avoid function re-approximation to speed up our method. Numerical tests show comparative advantages to spectral methods and other root-finding solvers.
(Received September 13, 2023)

## 1192-65-33443

Janak Joshi*, Dallas College. Fourier Spectral High-Order Time-Stepping Method for Numerical Simulation of the MultiDimensional Allen-Cahn Equations.
In this talk, I will present the Fourier spectral method combined with the strongly stable exponential time difference method as an attractive and easy-to-implement alternative for integrating the multi-dimensional Allen-Cahn equation with no-flux boundary conditions. The main advantages of the proposed method are that it utilizes the discrete fast Fourier transform, which ensures efficiency, allows an extension to two and three spatial dimensions in a similar fashion as one-dimensional problems, and deals with various boundary conditions. I will show some numerical results demonstrating that the proposed method is fourth-order accurate in the time direction and can satisfy the discrete energy law.
(Received September 16, 2023)

## 1192-65-33475

Andrea Bertozzi*, University of California Los Angeles. Active learning with graphs for semi-supervised remote sensing applications.
Graphical models are well-known in machine learning in which information is assigned to nodes on a graph and comparisons between pieces of data are measured as graph weights. This talk features new research that incorporates active learning into this process. Active learning is an iterative method in which specific pieces of data are shown to a "human in the loop" or ground truth oracle to assign labels to those data. Part of the algorithm is the automation of the data chosen to label. We combine this with semi-supervised Laplace learning on graphs and apply it to a number of remote sensing problems such as hyperspectral pixel classification, nearly blind hyperspectral unmixing, multispectral surface water detection, and classification of synthetic aperture radar images.
(Received September 18, 2023)

## 1192-65-33499

Joseph Hunter Kee Nakao*, Swarthmore College. Implicit and implicit-explicit low-rank integrators for solving timedependent problems.
Dynamical low-rank (DLR) methods have become a popular way to efficiently solve time-dependent problems that have lowrank structures. Some of the equations of interest contain stiff operators that require implicit time integrators for reasonable computational efficiency. However, achieving high-order accuracy using implicit time integrators in the low-rank framework remains a challenge. We propose a new low-rank method, similar in spirit to the DLR framework, that incorporates high-order implicit time integrators
(Received September 19, 2023)

## 1192-65-33504

Allen J Minch*, Co-Author, Hung Anh Dinh Vu*, Co-Author, Anne Marie Warren*, Co-Author. Fast \& Fair: Efficient Second-Order Robust Optimization for Fairness in ML.
This project explores adversarial training techniques to develop fairer Deep Neural Networks (DNNs) to mitigate the inherent bias they are known to exhibit. DNNs are susceptible to inheriting bias with respect to sensitive attributes such as race and gender, which can lead to life-altering outcomes (e.g., demographic bias in facial recognition software used to arrest a suspect). We propose a robust optimization problem to improve fairness in DNNs, and leveraging second-order information, we are able to efficiently find a solution.
(Received September 19, 2023)

## 68 Computer science

## 1192-68-25482

Janani Lakshmanan*, University of Hawaii at Manoa. Automatic Complexity and Quantum Logic over Finite Fields. Preliminary report.
The automatic complexity of finite words was introduced by Shallit and Wang (2001). It measures the complexity of a word $x$ as the minimum number of states of a finite automaton that uniquely accepts $x$. Here, an automaton $M$ uniquely accepts a word $x$ if $x$ is the only word of length $|x|$ accepted by $M$. Via the digraph representation of automata we can view the computation of this number of states as a problem of extremal graph theory. A quantum version of automatic complexity was studied by Kjos-Hanssen (2017). We explore a finite field analogue of quantum automatic complexity, with particular attention to the subspace structure of the automata and the associated quantum logic.
(Received June 16, 2023)

Gianluca Barone*, Rowan University, Aashrit Cunchala*, University of Pittsburgh, Rudy A Nunez*, Emory University. Improving Accuracy When Classifying Out of Distribution Data in Facial Recognition Models.
Standard classification theory assumes that the distribution of images in the test and training sets are identical. Unfortunately, real-life scenarios typically feature unseen data ("out-of-distribution data") which is different from data in the training distribution("in-distribution"). This issue is most prevalent in social justice problems where data from under-represented groups may appear in the test data without representing an equal proportion of the training data. This may result in a model returning confidently wrong decisions and predictions. We are interested in the following question: Can the performance of a neural network improve on facial images of out-of-distribution data when it is trained simultaneously on multiple datasets of indistribution data? We approach this problem by incorporating the Outlier Exposure model and investigate how the model's performance changes when other datasets of facial images were implemented. We observe that the accuracy and other metrics of the model can be increased by applying Outlier Exposure, incorporating a trainable weight parameter to increase the machine's emphasis on outlier images, and by re-weighting the importance of different class labels.
(Received July 20, 2023)

## 1192-68-26375

Keira Behal*, Emory University, Jiayi Chen*, Emory University, Caleb J. Fikes*, Rice University, Sophia Xiao*, Emory University. Synthetically Rebalancing Healthcare Datasets via Conditional DDPM.
In the field of healthcare, electronic health records (EHR) serve as crucial training data for developing machine learning models for diagnosis, treatment, and management of healthcare resources. However, medical datasets are often imbalanced in terms of sensitive attributes like race/ethnicity, gender, and age. Machine learning models trained on class-imbalanced EHR datasets perform significantly worse in deployment for individuals of the minority classes compared to samples from majority classes, which may lead to inequitable healthcare outcomes for minority groups. To address this challenge, we propose Minority Class Rebalancing through Augmented data Generation (MCRAGE), a novel approach to augment imbalanced datasets using samples generated by a deep generative model. The MCRAGE process involves training a Conditional Denoising Diffusion Probabilistic Model (CDDPM) capable of generating high-quality synthetic EHR samples from underrepresented classes. We use this synthetic data to augment the existing imbalanced dataset, thereby achieving a more balanced distribution across all classes which can be used to train an unbiased machine learning model. We measure the performance of MCRAGE versus alternative approaches using several metrics including Precision, Recall, F1 score, and AUC. We provide theoretical justification for our method in terms of recent convergence results for DDPM's with minimal assumptions. (Received July 26, 2023)

1192-68-26643
Kason Ancelin*, UCLA, Ana Paola Arias Martinez, Universidad Nacional Autonoma de Mexico, Edward T Gilman*, Rice University, Luca Ponzoni, Relay Therapeutics, Joia Zhang, Cornell University, Yanqiao Zhu, UCLA. Evaluation and Interpretation of Chemical Large Language Models.
In this project, we evaluated IBM's transformer-based masked language model MoLFormer on textual molecular datasets of interest in the drug discovery process. We detailed the performance of MoLFormer on activity cliff datasets, which contain molecules with small differences in structure but large differences in function. Further, we studied the use of symmetries in molecular representations to improve the performance of IBM's model. Using conclusions suggested by these experiments, we explored the model's learning process, allowing us to better interpret results and propose further modifications to fix the model's common mistakes. Applications of our results could enable the drug discovery process to be less labor intensive and more cost effective.
(Received August 02, 2023)

1192-68-26681
Donald E Knuth*, stanford university. Recreational computer programming. Preliminary report.
Topics related to "recreational mathematics" have always been a great resource for teaching concepts of computer science. The speaker will discuss those that have given him the greatest pleasure, during the sixty years that he has been writing about The Art of Computer Programming. Besides the classic subjects, such as aspects of graph theory that were inspired by chess, as well as fractals, packing, and Conway's Game of Life, there's great math to be found also in more recent recreations such as slitherlink and masyu. Intriguing open problems will be highlighted.
(Received August 03, 2023)

1192-68-27169
Jean Ponce*, Ecole normale supérieure. Physical models and machine learning for photography and astronomy. We live in an era of data-driven approaches to image analysis, where modeling is sometimes considered obsolete. I will propose in this talk giving back to accurate physical models of image formation their rightful place next to machine learning in the overall processing and interpretation pipeline, and discuss two applications: super-resolution and high-dynamic range imaging from raw photographic bursts, and exoplanet detection and characterization in direct imaging at high contrast. This is joint work with Theo Bodrito, Yann Dubois de Mont-Marin, Thomas Eboli, Olivier Flasseur, Anne-Marie Lagrange, Maud Langlois, Bruno Lecouat and Julien Mairal. (Received August 11, 2023)

1192-68-27600
Kyle Hogan, Massachusetts Institute of Technology, Sophia Lichterfeld*, MIT PRIMES, Garima Rastogi, Virtual Learning Academy Charter School. Leveraging the Escrow-Holding Abilities of Ethereum Smart Contracts to Incentivize Account Creation for the Widespread Adoption of Web Monetization Schemes.
Web Monetization (WM) aims to provide users an alternative method (other than ads or subscription models) of compensating creators by streaming micropayments throughout the time spent on the website directly to website owners. Traditional WM schemes have faced significant challenges in acquiring the widespread adoption of their programs because they require full website participation to be implemented. However, establishing the necessary elements may take time and expertise that is
not immediately available to small content creators. Therefore, the obligation for website owners to have already set up a completely functional system before any user can begin WM poses a major hindrance to WM's expansion. In this paper, we develop a scheme that allows users to start WM even before website owners have had the occasion to establish all required components. Specifically, we focused on cases where, in the website's HTML head, a payment pointer tag, which indicates the transaction destination endpoint and resolves to the website owner's wallet address, is absent or illegible to the browser. Our novel approach leverages the ability of Solidity smart contracts on the Ethereum blockchain to hold money in escrow; users employing WM stream micropayments into a smart contract where it will be stored securely until owners have correctly implemented WM on their site. The money accumulated from user sessions for all web pages is kept in one single, general smart contract. Within this smart contract, we create distinct escrow folders for each particular website. We map the website's URL to a website ID based on a one-way hash of the URL's root domain. By routing payments into one aggregated smart contract rather than directly to separate payment locations for individual websites, we ensure that a user's transaction history does not reveal their browsing history and, thereby, their privacy is preserved. Owners wanting to retrieve this revenue must adopt W3C's WM API payment pointer standard for future use; thus, our approach ultimately aims to encourage the propagation of WM as a viable alternative to ads or subscriptions, especially for small websites. We believe that our proposal acts as an important stepping stone on the pathway toward a decentralized financial system centered around WM. (Received August 16, 2023)

1192-68-27700
Anne Deyu Shuai*, Oviedo High School, Florida. Using Machine Learning to Predict Terms in Number Sequences. Preliminary report.
The On-Line Encyclopedia of Integer Sequences (OEIS) is a tool that many mathematicians and other scientists use to find sequences that have been discovered. OEIS contains over 300,000 number sequences. The goals of this project was to predict the next term of any given number sequence by using data from OEIS and to visualize the sequences. This was done using a combination of machine learning and graphing methods to create functions that would inform what the next term is most likely going to be.
(Received August 18, 2023)

## 1192-68-27710

Dirk Jäger, Department of Medical Oncology, National Center for Tumor Diseases (NCT), University Hospital Heidelberg (UKHD), Im Neuenheimer Feld 460, 69120, Heidelberg, Germany, Bénédicte Lenoir, Center for Quantitative Analysis of Molecular and Cellular Biosystems (Bioquant), Heidelberg University, Im Neuenheimer Feld 267, 69120, Heidelberg, Germany, Ferdinand Popp, Applied Tumor Immunity Clinical Cooperation Unit, National Center for Tumor Diseases (NCT), German Cancer Research Center (DKFZ), Im Neuenheimer Feld 460, 69120, Heidelberg, Germany, Nektarios A. Valous*, Center for Quantitative Analysis of Molecular and Cellular Biosystems (Bioquant), Heidelberg University, Im Neuenheimer Feld 267, 69120, Heidelberg, Germany, Inka Zörnig, Applied Tumor Immunity Clinical Cooperation Unit, National Center for Tumor Diseases (NCT), German Cancer Research Center (DKFZ), Im Neuenheimer Feld 460, 69120, Heidelberg, Germany. Image analysis workflows rooted in quaternion algebra.
The proposed methodology relies on a straightforward and adaptable technique, leveraging quaternions, for working with natural and biomedical images across a spectrum of computational workflows. These workflows include: computational recolorization and de-colorization, computational re-staining, computational contrast enhancement, and computational stain separation. Additionally, this method holds potential for enhancing the effectiveness of machine learning approaches, including conventional and deep learning pipelines, as measured by various performance metrics. The utility of the proposed approach renders it suitable for standalone applications but also integrated into other image processing systems. Ultimately, the proposed technique underscores the capacity of sophisticated mathematical frameworks to yield inventive and efficient resolutions for natural and biomedical image processing. (Received August 18, 2023)

1192-68-27801
Ray Li*, Santa Clara University. Recent developments in list-decoding.
A list-decodable code is a subset $C \subset \mathbb{F}_{q}^{n}$ that, roughly, is "not too clustery." Formally, a code is $(p, L)$-list-decodable if every Hamming ball of radius $p n$ contains at most $L$ codewords (elements of $C$ ). The "gold standard" in list-decoding is uniformly random codes, where each codeword is sampled independently at random from $\mathbb{F}_{q}^{n}$. I will discuss recent progress on whether much more structured random ensembles of codes - such as random linear codes (random subspaces of $\mathbb{F}_{q}^{n}$ ) and randomly punctured Reed-Solomon codes - enjoy list-decoding properties comparable to those of uniformly random codes. (Received August 20, 2023)

1192-68-27810
Filippo Posta*, Phoenix College. Mathematical Challenges deploying Patient-Matching Algorithms.
The "Patient Matching Problem" is the issue raised whenever different Institutions/Companies are trying to merge or clean a database of patients/clients. In that scenario there is often the risk of duplication due to misspelled names, ID, addresses and the likes. One of the best (but not perfect) ways to solve this problem is to use computers and artificial intelligence approaches. We have been profiling the most used algorithms (i.e., Fellegi-Sunter algorithm, and similar) to solve the patient matching problem and their implementation in Python. The algorithms consists of many different parts and each of them can be tweaked toward a certain improvement including machine learning extensions such as neural networks. We present the problem and some of the ways to approach together it with their matching outcomes.
(Received August 21, 2023)

1192-68-27914
Haizhao Yang*, University of Maryland College Park. Finite Expression Method: A Symbolic Approach for Scientific Machine Learning.
Machine learning has revolutionized computational science and engineering with impressive breakthroughs, e.g., making the
efficient solution of high-dimensional computational tasks feasible and advancing domain knowledge via scientific data mining. This leads to an emerging field called scientific machine learning. In this talk, we introduce a new method for a symbolic approach to solving scientific machine learning problems. This method seeks interpretable learning outcomes in the space of functions with finitely many analytic expressions and, hence, this methodology is named the finite expression method (FEX). It is proved in approximation theory that FEX can avoid the curse of dimensionality in discovering high-dimensional complex systems. As a proof of concept, a deep reinforcement learning method is proposed to implement FEX for learning the solution of high-dimensional PDEs and learning the governing equations of raw data.
(Received August 22, 2023)

1192-68-28021
Zahra Askarzadeh, SAP, Camilla de Oliveira Fonseca*, Institute for Pure and Applied Mathematics, UCLA, Nour Kawni*, An Najah National University, Maria Nicos Alain Pasaylo*, Institute for Pure and Applied Mathematics, UCLA, Justin Sunu, Institute for Pure and Applied Mathematics, UCLA, Bethlehem Tassew*, SAP. Applying Computer Vision for Out-of-stock Detection in Retail Stores. Preliminary report.
Computer vision has emerged as a powerful technology that enables computers to perceive and understand visual information. Retail stores today have difficulty keeping track of the product inventory on shelves due to the current inventory management system that involves manual scanning and counting of products which is time-consuming and error prone. Computer vision enables us to automate inventory management by using cameras to capture real-time data on product availability. Specifically, we use deep learning to detect out-of-stock (OOS) items and price tags on the shelves. We also implement an algorithm to match the OOS item to the appropriate price tag and extract its barcode and textual information to successfully identify OOS items.
(Received August 24, 2023)

1192-68-28022
Zahra Askarzadeh, SAP, Camilla de Oliveira Fonseca, Institute for Pure and Applied Mathematics, UCLA, Nour Kawni, An Najah National University, Maria Nicos Alain Pasaylo*, Institute for Pure and Applied Mathematics, UCLA, Justin Sunu, Institute for Pure and Applied Mathematics, UCLA, Bethlehem Tassew, SAP. Detection of Out-of-Stock Items at Retail Stores using Computer Vision. Preliminary report.
Computer vision has emerged as a powerful technology that enables computers to perceive and understand visual information. Retail stores today have difficulty keeping track of the product inventory on shelves due to the current inventory management system that involves manual scanning and counting of products which is time-consuming and error prone. Computer vision enables us to automate inventory management by using cameras to capture real-time data on product availability. Specifically, we use deep learning to detect out-of-stock (OOS) items and price tags on the shelves. We also implement an algorithm to match the OOS item to the appropriate price tag and extract its barcode and textual information to successfully identify OOS items.
(Received August 24, 2023)

1192-68-28197
Jie Gao*, Rutgers University. Graph Ricci Flow and Applications in Network Analysis and Learning.
The notion of curvature describes how spaces are bent at each point and Ricci flow deforms the space such that curvature changes in a way analogous to the diffusion of heat. In this talk I will discuss some of our recent work on discrete Ollivier Ricci curvature defined on graphs. Discrete curvature defined on an edge captures the local connectivity in the neighborhood. In general edges within a densely connected community have positive curvature while edges connecting different communities have negative curvature. By deforming edge weights with respect to curvature one can derive a Ricci flow metric which is robust to edge insertion/deletion. I will present applications of graph Ricci flow in graph analysis and learning, including network alignment, community detection and graph neural networks.
(Received August 28, 2023)

## 1192-68-28350

Samuel Lanthaler*, California Institute of Technology. Learning operators with neural networks. Neural networks have proven to be effective approximators of high dimensional functions in a wide variety of applications. In scientific computing the goal is often to approximate an underlying operator, which defines a mapping between infinitedimensional spaces of input and output functions. Extensions of neural networks to this infinite-dimensional setting have been proposed in recent years, giving rise to the emerging field of operator learning. Despite their practical success, our theoretical understanding of these approaches is still in its infancy; Why are neural networks so effective in these applications? In this talk, I will discuss recent theoretical work aiming to address what neural networks can and cannot achieve in this context. (Received August 29, 2023)

## 1192-68-28455

Kelli Lynn Galbraith*, Slippery Rock University of Pennsylvania. Fractal dimension applied to human iris recognition: All eyes on machine learning. Preliminary report.
This project develops a new machine learning (ML) model for human iris identification by utilizing the fractal dimension of image components. Though iris identification often incorporates infrared scanning, this project is instead an exploration of ML classification on visible light, colored photographs with a nuanced feature extraction method. After creating a database of 2,012 colored eye images, the authors used the programing language Processing to normalize the iris images and compute the fractal dimension of a range of RGB codes. With this feature, ML classification using the K-Nearest Neighbors algorithm was performed in Python and the model was subsequently evaluated.
(Received August 30, 2023)

Constantin-Daniel Nicolae*, University of Cambridge, Sara Sameer*, National University of Computer and Emerging Sciences, Pakistan, Nathan Sun*, Harvard University, Karena Yan*, Harvard University. Optimizing Cycle Life Prediction of Lithium-ion Batteries via a Physics-Informed Model. Preliminary report.
As a battery cell is repeatedly charged and discharged, various degradation mechanisms lead to an increasing loss of the total electric charge it can store. In particular, predicting the cycle life of commercial lithium-ion batteries remains challenging due to the complex chemical processes that alter the state of the battery. Our research improves upon prior work by introducing a hybrid prediction model that combines data-driven methods with a physical model of capacity loss. The former uses statistical summaries of various physical quantities and is based on the mathematical equivalence of self-attention to a support vector regression problem. The latter follows an Arrhenius law and is motivated by known descriptions of certain chemical processes, like solid electrolyte interphase formation, occurring throughout the lifetime of lithium-ion batteries. To optimize our model, we further introduce a multi-stage training scheme to ensure numerical stability while preserving accuracy in a complex loss landscape. A significant advantage of our model is the ability to forecast the evolution of electric charge capacity over the entire lifetime of a battery, as opposed to the cycle life only, allowing us to optimize cycle life prediction in complex battery systems.
(Received August 31, 2023)

## 1192-68-28781

Thomas C. Hull*, Franklin \& Marshall College, Inna Zakharevich, Cornell. Flat origami is Turing complete.
Flat origami refers to the folding of flat, zero-curvature paper such that the finished object lies in a plane. Mathematically, flat origami consists of a continuous, piecewise isometric map $f: P \subseteq \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ along with a layer ordering $\lambda_{f}: A \subset P \times P \rightarrow\{-1,1\}$ that tracks which points of $P$ are above/below others when folded. The set of crease lines that a flat origami makes (i.e., the set on which the mapping $f$ is non-differentiable) is called its crease pattern. Flat origami mappings and their layer orderings can possess surprisingly intricate structure. For instance, determining whether or not a given straight-line planar graph drawn on $P$ is the crease pattern for some flat origami has been shown to be an NP-complete problem, and this result from 1996 led to numerous explorations in computational aspects of flat origami. In this presentation we prove that flat origami, when viewed as a computational device, is Turing complete. We do this by showing that flat origami crease patterns with optional creases (creases that might be folded or remain unfolded depending on constraints imposed by other creases or inputs) can be constructed to simulate Rule 110, a one-dimensional cellular automaton that was proven to be Turing complete by Matthew Cook in 2004.
(Received September 03, 2023)

1192-68-29081
Hum Nath Bhandari, Rogers William University, Binod Rimal*, The University of Tampa, Ramchandra Rimal, Middle Tennessee State University. Transformer-based stock market prediction. Preliminary report.
The transformer model, originally designed for language modeling, has recently gained popularity in various other domain. However, its application in stock market prediction presents a significant challenge due to the distinct characteristics of financial data, which is often multifaceted and volatile time series. To tackle this challenge, we've developed a computational framework based on transformers for forecasting major stock market indices, incorporating a diverse set of input features. The goal is to utilize the predictive power of the transformer architecture in the context of multivariate time series. Rigorous hyperparameter tuning and model selection strategies are implemented to identify the best performing model. (Received September 05, 2023)

## 1192-68-29119

Haewon Jeong*, UCSB. Coded Computing for Fault-Tolerant Parallel QR Decomposition.
QR decomposition of a matrix is one of the essential operations that is used for solving linear equations and finding leastsquares solutions. We propose a coded computing strategy for parallel QR decomposition with applications to solving a fullrank square system of linear equations in a high-performance computing system. Our strategy is applied to the parallel GramSchmidt algorithm, which is one of the three commonly used algorithms for QR decomposition. While conventional coding strategies cannot preserve the orthogonality of $Q$, recent work has proven a post-orthogonalization condition that allows lowcost restoration of the degraded orthogonality. In this paper, we construct a checksum-generator matrix for multiple-node failures that satisfies the post-orthogonalization condition and prove that our code satisfies the maximum-distance separable (MDS) property with high probability. Furthermore, we consider in-node checksum storage setting where checksums are stored in original nodes. We obtain the minimal number of checksums required to be resilient to any $f$ failures under the innode checksum storage, and also propose a in-node systematic MDS coding strategy that achieves the lower bound. (Received September 05, 2023)

1192-68-29147
Robert Mepham, Middle Tennessee State University, Ramchandra Rimal*, Middle Tennessee State University. Machine Learning Algorithms for Predicting Game Outcomes in Division 1 Men's College Basketball.
This study aims to predict the probability of winning each game in Division 1 Men's College Basketball utilizing both basic and advanced basketball statistics. Men's NCAA basketball data from 2010-11 to 2021-22 seasons were collected and prepared. Multiple machine learning algorithms were employed, and their performance was evaluated using various metrics. Furthermore, a comparative analysis was conducted between these optimized models and the standard models used by sportsbooks, revealing a $7.48 \%$ advantage in favor of the top-performing models developed in this research. Finally, statistical analysis was carried out to further validate the model's robustness.
(Received September 05, 2023)

Jake Slater Rhodes*, Brigham Young University. Leveraging GitHub Organizations for Research Mentorship.
Mentoring undergraduate students in data-driven research projects presents a unique set of opportunities and challenges. This talk explores the utilization of Git, GitHub, and GitHub Organizations as powerful tools for navigating research projects.

GitHub provides a collaborative platform that enhances transparency, communication, and project management throughout the research journey. GitHub offers a structured environment for students to engage in version control, facilitating the tracking of project progress and the management of codebase evolution. Moreover, GitHub Organizations empower mentors to oversee multiple projects simultaneously while ensuring that students can collaborate within their teams. However, navigating these tools comes with its own set of challenges. For novice students, GitHub may present a steep learning curve, necessitating mentorship in version control concepts and best practices. Ensuring students adopt a standardized workflow and adhere to coding conventions is important for maintaining project cohesion. This talk discusses the experiences (past, present, and future) of mentors who leverage GitHub and GitHub Organizations in guiding students through research projects. It explores strategies to overcome challenges and enjoy the benefits of these platforms to foster a collaborative and productive research environment. By using these tools, educators and mentors can empower future data scientists and researchers to thrive in an increasingly data-centric world.
(Received September 06, 2023)

## 1192-68-29673

Matthew Patitz*, University of Arkansas. Designing DNA That Does Math On Its Own.
In biological systems DNA is used primarily as a medium for storing information. That information guides the processes that build the components of living systems out of building blocks based on amino acids. Alternatively, the field of DNA nanotechnology uses DNA as the actual building blocks of structures. However, the information storage potential of DNA is not wasted - information is instead embedded within these building blocks (in the form of their nucleotide sequences) to guide their self-assembly, following the basic base-pairing rules, into complex structures. Careful design of the nucleotide sequences of sets of DNA strands, combined with detailed understanding of their structural properties and binding affinities, can yield systems that autonomously self-assemble into structures while following the steps of arbitrary algorithms. In this talk, I will present designs of DNA-based systems that can be "programmed" to compute via self-assembly, showing high-level design motifs as well as abstract mathematical models in which such systems can be designed and tested. The goal will be to introduce the notion of algorithmic self-assembly and to show how, in theory, DNA-based systems can compute anything that can be computed.
(Received September 07, 2023)

## 1192-68-29846

David Doty, UC Davis, Joshua Petrack*, UC Davis, David Soloveichik, UT Austin. Thermodynamically Driven Signal Amplification
The field of chemical computation attempts to model computational behavior that arises when molecules, typically nucleic acids, are mixed together. Thermodynamic binding networks (TBNs) is a highly abstracted model that focuses on which molecules are bound to each other in a "thermodynamically stable" sense. TBNs attempt to naturally model the long-term behavior of a mixture (i.e., its thermodynamic equilibrium). We study the problem of signal amplification: detecting a small quantity of some molecule and amplifying its signal to something more easily detectable. This problem has natural applications such as disease diagnosis. By focusing on thermodynamically favored outcomes, we seek to design chemical systems that perform the task of signal amplification robustly without relying on kinetic pathways that can be error prone and require highly controlled conditions (e.g., PCR amplification). It might appear that a small change in concentrations can result in only small changes to the thermodynamic equilibrium of a molecular system. However, we show that it is possible to design a TBN that can "exponentially amplify" a signal represented by a single copy of a monomer. The system can be programmed to any desired level of resilience to false positives and false negatives. We also show a corresponding negative result: a doubly exponential upper bound, meaning that there is no TBN that can amplify a signal by an amount more than doubly exponential in the number and sizes of different molecules that comprise it. We leave as an open question to close this gap. Our work informs the fundamental question of how a thermodynamic equilibrium can change as a result of a small change to the system (adding a single molecule copy). While exponential amplification is traditionally viewed as inherently a non-equilibrium phenomenon, we find that in a strong sense exponential amplification can occur at thermodynamic equilibrium as well-where the "effect" (e.g., fluorescence) is exponential in types and complexity of the chemical components. (Received September 08, 2023)

## 1192-68-29946

Benjamin McFarland*, Ripon College, Erin Munro Krull, Ripon College. Predicting Results of ICA on Overcomplete Data. Preliminary report.
The Independent Component Analysis (ICA) algorithm takes a recording with multiple channels and decomposes the recording into components that are as independent as possible. For example, ICA could be used on a two-microphone recording of two people talking over each other. ICA returns an unmixing matrix which, when multiplied to the recordings, gives us two components estimating the two individual voices. Because ICA returns a matrix, it always returns the same number of components as channels. If there are more voices than components then there aren't enough components for each voice. So ICA cannot fully separate the voices. We call data with an equal number of channels and sources complete, and data with more channels than sources overcomplete. ICA is currently used in many applications where there are likely more channels than sources, such as EEG and fMRI. Likewise, if there are fewer channels than sources, such as ECOG, ICA is not readily used. Our aim is to predict how ICA decomposes overcomplete data. We have found that ICA groups sources together based on relative amplitude over the channels. We hypothesize that this grouping of sources can be modeled by the Least Square Mean (LSM) solution for the unmixing matrix, using a binning matrix to identify the groupings. Our findings show a pattern in the way ICA groups sources into components. This may allow researchers to better interpret components from overcomplete data. (Received September 08, 2023)

1192-68-30064
Kartik Lakhotia*, Intel. Extreme-scale Graphs in HPC Networks: Connecting the Dots. Preliminary report.
The degree-diameter problem aims to find the largest possible graphs under given degree and diameter constraints. It has profound applications in design of High-Performance Network topologies where degree is often restricted by the technology constraints and low diameter is highly preferred for performance reasons. The emergence of new technologies such as copackaged photonics further emphasizes the importance of scale and diameter of the network topologies. In this talk, we bridge the gap between mathematical graph constructions derived from degree-diameter problem and the design of High-

Performance Networks. Pertaining to low-diameter networks, we discuss graph theoretical methodologies to construct largescale diameter-2 and diameter-3 graphs. We present a new family called PolarStar, which provides the largest diameter-3 graphs known till date for several degrees and are near-optimal for a category of product graphs. To place the theoretical properties of these graphs in the context of networking applications, we further discuss practical challenges that include routing, performance, and in-network computing.
(Received September 11, 2023)

## 1192-68-30083

Jeffrey W Calder*, University of Minnesota, Twin Cities. Using machine learning to classify broken animal bone fragments. Distinguishing agents of bone modification at paleoanthropological sites is at the root of much of the research directed at understanding early hominin exploitation of large animal resources and the effects those subsistence behaviors had on early hominin evolution. However, current methods, particularly in the area of fracture pattern analysis as a signal of marrow exploitation, have issues with replicability and validity for analyzing bone modifications. Here we present a new approach to fracture pattern analysis aimed at distinguishing bone fragments resulting from hominin bone breakage and those produced by carnivores. Our method uses supervised machine learning algorithms applied to 3D models of fragmentary bone and shape statistics thereof. This talk will overview this field and present the results of our work, as well as outline future directions of research.
(Received September 08, 2023)

## 1192-68-30109

Ricardo Reyna*, University of Texas Rio Grande Valley. Bayesian Inference for Deep Learning. Preliminary report. Convolutional neural networks [CNN] have been used widely for image classification. However, CNN usually needs large amounts of data. Small amounts make CNNs prone to over fitting [Blei D. M., Kucukelbir A., and McAuliffe J. D. (2018)]. This makes it unable to properly weigh what inputs properly correspond to the outputs of a data analysis. Bayesian Inference studies has been a popular topic in machine learning, where it helps to reduce the problem network overfitting associated with neural networks. We have seen several integrations of Bayesian ideas into several types of neural networks [Yu, H., et al. (2021)][Rodrigo, H. and Tsokos, C. (2020)]. By incorporating Bayesian analysis in to CNN, we could possibly make them more efficient, even with a small amount of data. In this project, we will introduce three different Bayesian Inference methods, particularly the evidence approach [MacKay, D. J. C. (1992)], Hybrid Monte Carlo, and the combination of both [Rodrigo, H. and Tsokos, C. (2020)]. Specifically the Hybrid Monte Carlo has with the intention increasing the efficacy of the convolutional neural networks with the Bayesian approaches with the image analysis.
(Received September 08, 2023)

## 1192-68-30175

Ankur Moitra*, Massachusetts Institute of Technology. Learning From Dynamics.
Linear dynamical systems are the canonical model for time series data. They have wide-ranging applications and there is a vast literature on learning their parameters from input-output sequences. Moreover they have received renewed interest because of their connections to recurrent neural networks. But there are wide gaps in our understanding. Existing works have only asymptotic guarantees or else make restrictive assumptions, e.g. that preclude having any long-range correlations. In this work, we give a new algorithm based on the method of moments that is computationally efficient and works under essentially minimal assumptions. Our work points to several missed connections, whereby tools from theoretical machine learning including tensor methods, can be used in non-stationary settings.
(Received September 09, 2023)

1192-68-30266
Aliah Sow*, Iowa State University (REU at Florida International University). Transformers 8: BERT vs. Electra.
In times of crisis, Twitter has become a vital means of communication. The prevalence of smartphones enables people to announce emergencies in real-time. Sometimes, this information is useful, while other times, it can be the opposite. For this reason, news agencies and relief organizations aim to utilize real-time tweet monitoring to respond swiftly during emergencies using natural language processing. Natural language processing (NLP) uses mathematics and computational techniques to enable computers to comprehend written and spoken language like humans. The complexities of human language present significant challenges in developing software that can accurately decipher the intended message from textual or spoken data. For our project during the REU, we gathered a data set of over 10,000 tweets that potentially suggest an emergency. Then, we built machine learning models using transfer learning that predict whether a tweet indicates a real emergency. Lastly, we compared the accuracy of two pre-trained NLP models, BERT and Electra, in determining the legitimacy of a threat in a tweet. (Received September 09, 2023)

## 1192-68-30278

Andrej Risteski*, Carnegie Mellon University. The statistical cost of score matching.
Score matching is a recent approach to learning probabilistic generative models parameterized up to a normalizing constant (energy-based models). Fitting such models using maximum likelihood (that is, finding the parameters which maximize the probability of the observed data) is computationally challenging, as evaluating the normalizing constant involves a high dimensional integral. Score matching instead fits the score (which is the gradient with respect to the input) of the data distribution. What's gained with these approaches is a tractable gradient-based algorithm. What's lost is less clear: in particular, since maximum likelihood is asymptotically optimal in terms of statistical efficiency, how suboptimal are losses like score matching? We will provide partial answers to this question - and in the process uncover connections between geometric properties of the distribution (Poincaré, log-Sobolev, isoperimetric constants) and the statistical efficiency of score matching. The talk will survey several recent papers in this developing line of work, as well as highlight open directions for further study. (Received September 09, 2023)

Aras Asaad, University of Buckingham, Vidit Nanda, University of Oxford, Alexander M Tanaka*, University of Oxford. Combining Computational Topology and Machine Learning for Drug Discovery. Preliminary report.
We devise a novel shape-based method for ligand-based virtual screening. Taking inspiration from computational topology, we create a geometric feature vector for identifying active molecules for given drug targets. The feature vector is calculated by mapping a simplicial complex approximation of the molecular van der Waals surface to a one-dimensional vector. We then use this feature vector as an input to a binary classification task for distinguishing whether or not a small molecule would bind to a target protein. We train a shallow neural network on our features generated from molecular points clouds in the DUD-E database. We take advantage of the symmetries present in our feature vectors to make our neural network approximately invariant to 3-D rotations of the molecular points clouds.
(Received September 09, 2023)

## 1192-68-30332

John Cobb*, University of Wisconsin-Madison, Thomas Gebhart, University of Minnesota. Sheaf Laplacians and Missing Data.
The Laplacian of a graph is a matrix that encodes the local geometry of the graph. It is a fundamental object in spectral graph theory, and has been used to study a variety of problems in machine learning. In this talk, I will discuss a generalization called the sheaf Laplacian and how this can be used to study problems with missing data.
(Received September 09, 2023)

## 1192-68-30395

Qixing Huang*, UT Austin. Enhancing Implicit Shape Generators Using Topological Regularizations.
A fundamental problem in learning generative models of 3D shapes is that when merely fitting the generative model to the training data, the resulting synthetic 3D models may present various artifacts. Many those artifacts are topological in nature, e.g., broken legs, unrealistic thin structures, and small holes. In this paper, we introduce a principled approach that utilizes topological regularization losses on an implicit shape generator to rectify topological artifacts among synthetic models. The objectives are two-fold. The first is to align the persistent diagram (PD) distribution of training shapes with that of synthetic shapes. The second ensures that PDs are smooth among adjacent synthetic shapes. We show how to achieve these two objectives using two simple yet effective formulations. Specifically, distribution is achieved for learning a generative model of PDs. We show how to handle discrete and continuous variabilities of PDs. Based on the stability theorem of PDs, smoothness of PDs is enforced using the smoothness of the implicit generator under a suitable norm. Experimental results on ShapeNet and DFaust show that our approach leads to much better generalization behavior than state-of-the-art implicit shape generators. Our approach can be combined with any implicit shape generators, making it an universal approach for enhancing topological generalization.
(Received September 09, 2023)

## 1192-68-30645

Hum Nath Bhandari, Roger Williams University, Hasala Gallolu Kankanamalage, Roger Williams University, Sarita Poudyal Bhandari*, Roger Williams University. Medical Image Classification Using Deep Learning Models. Preliminary report.
Machine learning has recently become a widely popular and useful technique in various fields, ranging from everyday households to security, banking, healthcare, insurance, and other scientific advancements. Due to the widespread use of IoT devices and the digitalization of almost every sector of human society, data has grown enormously, creating both challenges and opportunities. Researchers are trying to capitalize on this massive opportunity by building several data-driven AI and machine learning solutions to mitigate fraud and enhance our human lives. Interestingly, the Healthcare, Bio-Tech, and pharmaceutical industries are seen benefiting significantly from these emerging technologies as machine learning can play vital roles in analyzing healthcare data, early and anomaly detection of diseases from medical imaging, predicting the disease spreads, and inventing new drugs and vaccines. This study aims to solve a medical imaging problem by implementing deep learning models based on Convolutional Neural Networks (CNN) architecture. Input data consists of Magnetic Resonance Imaging (MRI) images collected independently from several patients. The goal is to train the CNN-based model to detect the tumor and its types. Several model variants are constructed with different configurations of the CNN architecture, and their performances are evaluated to identify the best model. Model parameters are optimized by implementing effective training, hyperparameter tuning, and model selection strategies. Preliminary experimental results suggest the impressive predictive power of deep learning in this particular computer vision problem domain.
(Received September 10, 2023)

## 1192-68-30649

Erik Demaine*, Massachusetts Institute of Technology. Energy-Efficient Algorithms.
The field of energy-efficient algorithms aims to develop new techniques for solving computational problems with vastly reduced energy consumption - for some problems, by several orders of magnitude - in exchange for a small increase in time and memory requirements. Specifically, we explore how to algorithmically exploit reversible computation, an idea that has been around since the 1970s and has just started to become a practical reality, but for which we have only just begun understanding how to design efficient algorithms. Our preliminary investigations indicate that some basic problems (but not others) can have their energy cost substantially reduced, far less than that spent by a traditional algorithm on traditional hardware. This theoretical ground work will become especially important over the next decade, when Landauer's Principle predicts that the energy efficiency of traditional chips (kilowatt hours spent per nonreversible computation) must stop improving, so the only alternative is to do more reversible computation. Reducing the energy consumed by computation will help save the planet from Global Warming, will save money, will help portable devices run longer on the same battery, and will help chips run faster (as cooling from waste heat is a major bottleneck in chip speeds).
(Received September 10, 2023)

Hum Nath Bhandari*, Roger Williams University (RWU). A Computational Framework for Implementing Deep Learning Model Architectures in Solving Computer Vision Problems. Preliminary report.
Computer vision is a field of artificial intelligence(AI) that enables machines to extract meaningful information from digital images, videos, and other visual objects. Solving computer vision problems, such as image detection and video classification, has become an important application area for emerging AI and deep learning technologies. Real-world implications are enormous, benefiting several industries, ranging from security, energy and utilities, and manufacturing to medical imaging, DNA sequencing, and automotive. This study aims to provide an integrated computational framework for implementing state-of-the-art deep learning models, such as CNN and CNN-LSTM, for solving different computer vision problems. Theoretical aspects of these model architectures are discussed, and their implementation frameworks are illustrated using multiple case studies drawn from various fields, including Biology and Medical Imaging. The entire deep learning models implementation pipeline is presented, which includes data extraction and visualization, resizing and normalization, input preparation, model construction and implementation, performance evaluation and hyperparameter tuning, and model selection and validation. Finally, the best-performing model configuration is identified using empirical results and robust theoretical analysis from several constructed models. The preliminary experimental results demonstrate the effectiveness of deep learning model architectures in this particular problem domain.
(Received September 10, 2023)

1192-68-30801
Keller Blackwell, Stanford University, Mary Wootters*, Stanford. Coding-theoretic techniques in Homomorphic Secret Sharing.
Homomorphic Secret Sharing (HSS) is a variant on secret sharing where a function $f$ can be computed using only a small amount of information from each party. The value of "small" in the previous sentence can be quantified by the download rate of an HSS scheme. We establish bounds on the download rate for information-theoretic HSS schemes, complemented by (sometimes matching) constructions. Both our impossibility results and our constructions exploit coding-theoretic connections, which we explain in this talk. This talk is based on two works, joint with Ingerid Fosli, Yuval Ishai, and Victor Kolobov; and with Keller Blackwell, respectively.
(Received September 11, 2023)

## 1192-68-30833

Ahmed Abdelsalam*, The City University of New York-BMCC, David Allen, The City University of New York-BMCC. A Comparative Analysis of K-means using the Pythagorean Means. Preliminary report.
The project, "A Comparative Analysis of K-Means using Pythagorean Means," explores various types of clustering techniques using different distance metrics: Euclidean, Geometric, and Harmonic means. The study employs the "Gas Turbine CO and NOx Emission Data Set" from the UC Irvine archive, focusing on one year of data (2011) for applying all three k-means clustering algorithms (Euclidean, Geometric, and Harmonic). The main objective is to compare the clustering performance of each distance metric by analyzing the clusters.
(Received September 11, 2023)

## 1192-68-30879

Mohit Gupta*, University of Wisconsin-Madison. A Coding Theory of 3D Cameras.
The advent (and commoditization) of low-cost 3D cameras is revolutionizing many application domains, including robotics, autonomous navigation, human computer interfaces, and recently even consumer devices such as cell-phones. Most modern 3D cameras (e.g., LiDAR, Microsoft Kinect and Azure) are active; they consist of a light source that emits spatio-temporal coded light into the scene, i.e., its intensity is modulated over space, and/or time. The performance of these cameras is determined by their illumination coding functions. This work develops a coding theory of active 3D cameras. This theory, for the first time, abstracts several seemingly different 3D camera designs into a common, geometrically intuitive space. Using this framework, we design families of novel coding functions that are based on Hamiltonian cycles on hypercube graphs. Given a fixed power and acquisition time, the new Hamiltonian coding scheme can achieve up to an order of magnitude higher resolution as compared to the current state-of-the-art methods, especially in low signal-to-noise ratio (SNR) settings. These results could be useful toward developing 'All-Weather' 3D cameras that can operate in extreme real-world conditions, including outdoors (e.g., a robot navigating outdoors in bright sunlight and poor weather), under multi-camera interference, and handle optically challenging objects such as shiny metal (e.g., for an industrial robot sorting machine parts). (Received September 11, 2023)

## 1192-68-30922

Ryuhei Uehara*, Japan Advanced Institute of Science and Technology. Research on Common Shape Puzzles. In this study, we investigate the computational complexity of some variants of generalized puzzles. We are provided with two sets $S_{1}$ and $S_{2}$ of polyominoes. The first puzzle asks us to form the same shape using polyominoes in $S_{1}$ and $S_{2}$. We demonstrate that this is polynomial-time solvable if $S_{1}$ and $S_{2}$ have constant numbers of polyominoes, and it is strongly NPcomplete in general. The second puzzle allows us to make copies of the pieces in $S_{1}$ and $S_{2}$. That is, a polyomino in $S_{1}$ can be used multiple times to form a shape. This is a generalized version of the classical puzzle known as the common multiple shape puzzle. For two polyominoes $P$ and $Q$, the common multiple shape is a shape that can be formed by many copies of $P$ and many copies of $Q$. We show that the second puzzle is undecidable in general. The undecidability is demonstrated by a reduction from a new type of undecidable puzzle based on tiling. Nevertheless, certain concrete instances of the common multiple shape can be solved in a practical time. We present a method for determining the common multiple shape for provided tuples of polyominoes and outline concrete results, which improve on the previously known results in puzzle community. (Received September 11, 2023)

1192-68-31235
Liljana Babinkostova*, Boise State University. Analysis of Deep Learning Side Channel Attacks on Lightweight
Cryptographic Systems. Preliminary report.
Lightweight Cryptography allows for the encryption of sensitive information from devices with constraints on power usage,
processing power, storage, and more. Side-channel resistance is widely accepted as a crucial factor in deciding the security assurance level of cryptographic implementations. Deep learning side-channel attacks (DL-SCA) are new methods that utilize the high identifying ability of the neural network to try and unveil a secret key of the cryptographic module. The DL-SCA methods are considered as a powerful alternative to the conventional side-channel attacks (SCA) and have received much attention because they bypass the need for signal processing and statistical processing. This talk focuses on the side channel resistance of the GIFT-COFB hardware implementation and the mathematical techniques in exploiting deep learning in cryptanalysis. In particular we focus on DL-SCA methods in non-profiled scenarios and present experimental results performed on simulated data and real traces collected from the ChipWhisperer board that show the vulnerabilities of GIFT-COFB against the recently proposed DL-SCA attacks.
(Received September 11, 2023)

## 1192-68-31303

Dominic Cugliari*, Hope College, David Slakes*, Hope College. Reconstructing Tomographic Images with Artificial Neural Networks. Preliminary report.
In this project we explore artificial neural networks' (ANN) functionality in tomographic image reconstruction, a type of inverse problem. We are interested in using this technology to aid in image reconstruction, specifically in Compton scatter tomography, which is used in nondestructive inspection. Compton scatter tomography instruments produce a limited amount of data compared to computed tomography. We extended an existing technique using ANN's to compute an optimal filter in reconstructing tomographs with filtered back projection using very limited data. We developed an image reconstruction algorithm incorporating existing packages for ANN implementation and image reconstruction within the R programming language. This algorithm was used to create weighted filters to reconstruct tomographs. (Received September 11, 2023)

1192-68-31310
Alperen Ergur*, University of Texas at San Antonio, Grigorios Paouris, Texas A\&M University, Petros Valettas, University of Missouri, Department of Mathematics. A Metric Geometry Approach to Semidefinite Extension Complexity. Preliminary report.
A basic question in convex optimization is to find optimal (re)formulation of a problem by allowing additional variables. This corresponds to lifting a geometric object (optimization domain) to higher dimensions for obtaining a simpler description. For a given convex body, or a family of convex bodies, we ask for the best possible representation using semidefinite programming, this is called semidefinite extension complexity. In this talk, we present a general framework to reason about semidefinite extension complexity lower bounds. We apply the framework on families of polytopes and some specific convex bodies with algebraic structure. (Received September 11, 2023)

## 1192-68-31539

Ellie Kienast, Georgia Technical University, Ioannis Paraschos*, Embry-Riddle Aeronautical University, Nathaniel Reimer, Macalester College, Hadley Santos-Del Villar, State University of New York at Albany. Uncertainty Quantification in Segmentation of Computed Tomography. Preliminary report.
Image processing and analysis play pivotal roles in materials science, manufacturing, and non-destructive testing. By processing computed tomography (CT) images, researchers can derive physical characteristics of an object, such as its boundary, surface, and internal features. Identifying different areas of an image and the edges between them is known as segmentation. Pacific Northwest National Laboratory (PNNL) utilizes CT scans to study objects and materials in an efficient and nondestructive manner. Partnering with PNNL, this Research Experience for Undergraduates project at Embry-Riddle Aeronautical University aims to quantify the uncertainty of edge detection methods applied to CT scans of machined objects. To ensure the reliability and effectiveness of CT, the associated uncertainty in segmentation must be addressed. The first phase in our research applies gradient, statistical, and area-based edge detection methods to a dataset of 176 CT images comprising a single test object provided by PNNL. Evaluation metrics are employed on each edge detection method, and a ground truth reference is generated to determine the accuracy of the algorithms. After selecting the most effective method, frameworks for quantifying error in edge detection are developed, tested, and evaluated. Using the developed error models as a basis, the uncertainty of the CT scan segmentation can be quantified to generate a more robust and productive process to study materials in industry application.
(Received September 11, 2023)

## 1192-68-31585

Sara Fridovich-Keil*, Stanford University. Gradient Descent Provably Solves Nonlinear Tomographic Reconstruction. Computer vision has recently made great progress in recovering photorealistic 3D scenes from 2D photographs, using a forward model based on a discretization of the Beer-Lambert Law in chemistry. Omitting color, the same forward model describes X-ray measurements in computed tomography. Both of these forward models are nonlinear due to the exponential attenuation of light as it passes through absorbing media, resulting in a nonconvex least squares objective. We show that gradient-based optimization nonetheless converges to global optimality in these cases, and give reconstruction error bounds in terms of the number of measurements and optimization steps. (Received September 12, 2023)

1192-68-31597
Alex Wein*, UC Davis. Optimality of Approximate Message Passing.
The approximate message passing (AMP) framework has been widely successful at producing algorithms with provable guarantees for a variety of high-dimensional statistical inference tasks. In some settings, AMP is conjectured to be optimal in the sense that no computationally efficient estimator can achieve a better mean squared error (MSE). In a simple "signal plus noise" model (spiked Wigner), we prove a variant of this conjecture by showing that AMP has the best possible MSE within a larger class of algorithms, namely those that can be described as constant-degree polynomials. This is joint work with Andrea Montanari, available at: https://arxiv.org/abs/2212.06996

Holden Lee*, Johns Hopkins University. Parallelising Glauber Dynamics.
For distributions over discrete product spaces $\prod_{i=1}^{n} \Omega_{i}^{\prime}$, Glauber dynamics is a Markov chain that at each step, resamples a random coordinate conditioned on the other coordinates. We show that $k$-Glauber dynamics, which resamples a random subset of $k$ coordinates, mixes $k$ times faster in $\chi^{2}$-divergence, and assuming approximate tensorization of entropy, mixes $k$ times faster in KL-divergence. We apply this to Ising models $\mu_{J, h}(x) \propto \exp \left(\frac{1}{2}\langle x, J x\rangle+\langle h, x\rangle\right)$ with $\|J\|<1-c$ (the regime where fast mixing is known), where we show that we can implement each step of $\widetilde{O}\left(n /\|J\|_{F}\right)$-Glauber dynamics efficiently with a parallel algorithm, resulting in a parallel algorithm with running time $\widetilde{O}\left(\|J\|_{F}\right)=\widetilde{O}(\sqrt{n})$.
(Received September 12, 2023)

1192-68-31749
Aleyah Dawkins*, George Mason University, Kelly Isham, Colgate University, Laura Monroe, Los Alamos National Laboratory. Constructing Spanning Trees in Post-Exascale Networks. Preliminary report.
The Star product, a generalization of the Cartesian product, is emerging in post-exascale network topologies such as SlimFly and PolarStar. For a network topology to be useful, it must support collectives such as Allreduce to increase bandwidth. This problem maps immediately to the problem of finding a large number of edge disjoint spanning trees in the corresponding graph. In this talk, we discuss a construction of a near optimal set of edge disjoint spanning trees in Star product networks. We then discuss conditions under which the theoretical optimal number of edge disjoint spanning trees can be found. This talk is based on preliminary joint work with Laura Monroe and Kelly Isham, along with researchers from Intel. (Received September 12, 2023)

## 1192-68-31798

Irina A Kogan*, North Carolina State University. Signatures of Smooth and Algebraic Curves. Deciding whether or not two closed curves are congruent under a group of transformations is a fundamental problem in differential and algebraic geometry, and has numerous applications in computer vision and image processing, face recognition, medical imaging, automated assembly, and more. To address this problem, the signature curve parameterized by differential invariants was introduced by Calabi, Olver, Shakiban, Tannenbaum, and Haker (1998). In this talk, we will discuss the subsequent developments and applications of signature construction in smooth and algebraic settings. (Received September 12, 2023)

1192-68-31810
Andrii Iarmolenko*, Borough of Manhattan Community College, CUNY, Marc Legaspi, Borough of Manhattan Community College, CUNY, Andrew R Stout, Borough of Manhattan Community College, CUNY, Peter Vaiciulis, Borough of Manhattan Community College, CUNY. Using the Gini Coefficient to rank AI systems: a case study using Rubik's cube AI solvers. We propose a new way to rank the performance of AI systems by adapting the concept of the Gini coefficient. The main idea is that by adjusting the number of solvable or goal states within an algorithm, we can have a finer measure of performance and can compare AI systems which are designed to solve different problems. To gauge the strength of this approach, we focus on AIs which solve rubik's cubes by adjusting these algorithms to increase the number of goal states. (Received September 12, 2023)

1192-68-31865
Kuikui Liu*, Massachusetts Institute of Technology. Spectral Independence: A New Tool to Analyze Markov Chains. Sampling from high-dimensional probability distributions is a fundamental and challenging problem encountered throughout science and engineering. One of the most popular approaches to tackle such problems is the Markov chain Monte Carlo (MCMC) paradigm. While MCMC algorithms are often simple to implement and widely used in practice, analyzing the rate of convergence to stationarity, i.e. the "mixing time", remains a difficult problem in many settings of interest. This talk will describe a recent technique called "spectral independence", which has been used to break long-standing barriers and resolve several decades-old open problems in the theory of sampling algorithms. Through this technique, we've further established new connections with other areas such as statistical physics, geometry of polynomials, the emerging study of high-dimensional expanders, and more. Applications include discrete log-concave distributions, graphical models, and concentration inequalities. Based on several joint works with Dorna Abdolazimi, Nima Anari, Zongchen Chen, Shayan Oveis Gharan, Nitya Mani, Ankur Moitra, Eric Vigoda, Cynthia Vinzant, and June Vuong.
(Received September 12, 2023)

## 1192-68-31870

Alejandro Antonio Mayorga*, Author, Letian Wang*, Author. Prediction of Credit Defaults based on Weight Dimensionality Reduction Neural Network and M-Band Discrete Wavelet Transform.
Credit Defaults of Companies are of utmost importance for many people such as investors that may include the general public, entrepreneurs looking to fund the next Apple or companies looking to support small companies. These Defaults can be modeled using financial information ,however there are too many companies in the world to perform a careful analysis on each one. A system which is already trained and takes financial information and returns a credit Default is the best for this task. Firstly, We implemented M-band discrete wavelet transform (MDWT) to decompose our dataset into M different frequency components to discover some hidden information we would not get otherwise In this paper we propose a novel weight dimensionality reduction neural networks (WDR-NN) that uses dimensionality reduction techniques such as UMAP Wavelets, PCA, convolutions and Max Pooling to generate a new neural network and then pass relevant information that is in a reduced dimension but preserves the overall structure of the networks weights.. In our research, two Datasets were used:

England Companies Binary Classification of whether the company went bankrupt at some point and Moody's and Fitch Credit Defaults using binary classification to see if their rating by the agency was higher than a given threshold. The results have shown that our WDR-NN model outperformed a regular neural network by yielding a 13
(Received September 12, 2023)

## 1192-68-31897

Sitan Chen*, Harvard University. Theory for diffusion models.
This talk will survey recent progress on providing rigorous convergence guarantees for score-based generative models (SGMs) such as denoising diffusion probabilistic models (DDPMs), which constitute the backbone of large-scale real-world generative models such as DALL•E 2. The first part of the talk will present a simple proof that such SGMs can efficiently sample from essentially any realistic data distribution, even ones which are highly non-log-concave. In the second part of the talk, we will see how to extend these guarantees to deterministic samplers based on discretizing the so-called probability flow ODE, which ultimately leads to better dependence on the dimension. All of these results assume access to an oracle for score estimation; time permitting, at the end we will briefly touch upon how to provably implement this oracle for interesting classes of distributions like Gaussian mixtures.
(Received September 12, 2023)

## 1192-68-31908

Emma Chow*, Baycodingclub, Yinbo Liu*, Baycodingclub, Jiayu Su*, Baycodingclub, Xinrui Wang*, Baycodingclub, Linqi
Zhang*, Baycodingclub. Enhancing Data Capacity in Color QR Codes with Wavelet based Steganography and Denoise Algorithms. Preliminary report.
As the barcode becomes more widely used, its applications and data capacity demands grow, increasing the need for color barcodes with greater data density. Moreover, the necessity of increasing information capacity motivates the use of more colors to create color barcodes. In this research, we utilize the quick response ( QR ) code-one of the many types of barcodeswe develop an algorithm to create a color QR code with n color channels ( (Received September 12, 2023)

## 1192-68-31944

Ankur Moitra*, Massachusetts Institute of Technology. Finding a giant component in random k-SAT In this work we study the problem of approximately counting and sampling from the set of satisfying assignments to a random $\mathrm{k}-\mathrm{CNF}$ on n variables for large k . Along the way, we show a new structural result that at clause density $2 \wedge\{\mathrm{k} / 6\}$ with high probability there is a giant component in the graph where we connect all satisfying assignments that are within Hamming distance $\mathrm{O}(\operatorname{logn})$. We show similar results for bounded degree k-CNFs at similar density. Based on joint work with Zongchen Chen and Nitya Mani
(Received September 12, 2023)

## 1192-68-32003

Pelin Ersin, Izmir University of Economics, Emma Hayes*, Carnegie Mellon University, Peter Matthews, University of Warwick, Paramjyoti Mohapatra*, Case Western Reserve University. Neural Networks for solving PDEs and Applications to Inverse Problems. Preliminary report.
Neural networks have become a powerful tool to provide numerical solutions for scientific problems with increased computational efficiency. This efficiency can be advantageous for numerically challenging problems where time to solution is important or when evaluation of many similar analysis scenarios is required. In this work, we considered solutions of the 2D acoustic wave equation where pressure and velocity vary over space and time under the influence of a driving force at a source point. We trained neural networks on data generated with a single driving force at one source point, as well as on data with multiple source locations. Our ground truth data was generated on a coarse grid using a Discontinuous Galerkin method. We show that our neural networks are able to produce an accurate solution over a square domain over a fine grid roughly 300 times faster than the Discontinuous Galerkin method on a single source. Moreover, we study the performance of the network with physics-based regularization terms added to the loss function using a variety of schemes. We outline some of the potential failure modes of such methods, particularly in the presence of a forcing term. Finally, due to the computational efficiency and accuracy achieved via neural networks, we show that the inverse problem of detecting source location from the pressure signals at given receiver points can be solved in a Bayesian setting by using MCMC methods. This would not be feasible using Discontinuous Galerkin methods due to the computational cost.
(Received September 12, 2023)

## 1192-68-32056

Aydin Buluc*, Lawrence Berkeley National Laboratory. The Ubiquitous Sparse Matrix-Matrix Products. Preliminary report. Multiplication of a sparse matrix with another matrix is a fundamental operation that captures the computational patterns of many data science applications, including but not limited to graph algorithms, sparsely connected neural networks, graph neural networks, clustering, and many-to- many comparisons of biological sequencing data. In the majority of these application scenarios, the matrix multiplication takes places on an arbitrary algebraic semiring where the scalar operations are overloaded with user-defined functions with certain properties or a more general heterogenous algebra where even the domains of the input matrices can be different. Here we provide unifying treatment of the sparse matrix-matrix operation and its rich application space.
(Received September 12, 2023)

1192-68-32111
Marta D'Elia, Pasteur Labs, Patrick Diehl, Louisiana State University, Christian Alexander Glusa, Sandia National Laboratories, Noujoud Nader*, Louisiana State University, Serge Prudhomme, Polytechnique Montreal. Automated Identification of Regions for Coupling Local and Nonlocal Models in Material Modeling Using Machine Learning. Preliminary
report.
Recent research has shown a growing interest in coupling local and nonlocal models within the field of mechanics to address complex material behavior challenges. Local models, rooted in continuum mechanics, traditionally assume that local properties solely determine material behavior, yet they often prove inadequate in predicting intricate behaviors related to phenomena such as cracks, voids, or material heterogeneity. In response, nonlocal models have been developed, considering the influence of surrounding material to improve predictions. However, their accuracy comes at the cost of computational intensity. Various coupling approaches have been proposed to mitigate these limitations, including methods like the Variable Horizon Method (VHM) and matching displacements or stresses in overlapping regions. In this work, we propose an innovative approach that utilizes machine learning techniques to automatically identify local and nonlocal regions in a coupling process. The identification process takes into consideration various factors, including load functions, boundary conditions, and material geometry. We initially employed load functions with a single discontinuity region. Subsequently, we expanded our research scope by incorporating load functions with multiple discontinuities. This deliberate extension serves the purpose of evaluating the robustness and adaptability of our machine learning-driven approach in identifying multiple nonlocal regions within a coupling process. The machine learning algorithms were trained on combined datasets, encompassing more than one discontinuity, thus spanning both local and nonlocal information. Furthermore, we also investigate the adaptability of our model to discretization parameters. Hence, we explore if we can train our model with a coarse discretization to save computational time and make prediction on a much finer scale. This exploration further enhances the versatility and practicality of our methodology, enabling it to address a broader range of real-world challenges.
(Received September 12, 2023)

1192-68-32127
Megan Chen*, Boston University. Proof-Carrying Data From Arithmetized Random Oracles.
Proof-carrying data (PCD) is a powerful cryptographic primitive that allows mutually distrustful parties to perform distributed computation in an efficiently verifiable manner. Known constructions of PCD are obtained by recursively-composing SNARKS, a cryptographic protocol that enables verifying whether a computation execution is correct. One way to construct SNARKs is in the random oracle model, which assumes that all parties have query-access to the same random function. However, using such SNARKs to construct PCD requires heuristically instantiating the random oracle with a hash function circuit and using it in a non-black-box way. As a result, the security proof requires an additional assumption: that replacing the random oracle with a hash function preserves security. \cite\{ChenCS22\} constructed SNARKs in the low-degree random oracle model, circumventing this issue, but instantiating their model in the real world appears difficult. In this talk, I will introduce a new 'arithmetized' random oracle model (AROM), which has a plausible standard-model (software-only) instantiation. We can construct PCD in the AROM, given only a standard-model collision-resistant hash function. We obtain our PCD construction by showing how to construct SNARKs in the AROM for computations that query the oracle, given an accumulation scheme in the AROM that batch-verifies oracle queries. This talk is based on joint work with Alessandro Chiesa, Tom Gur, Jack O'Connor, and Nicholas Spooner.
(Received September 12, 2023)

## 1192-68-32453

Rushil Anirudh*, Lawrence Livermore National Laboratory, Yamen Mubarka, Lawrence Livermore National Laboratory, Vivek Sivaraman Narayanaswamy, Lawrence Livermore National Laboratory, Matthew Olson, Lawrence Livermore National Laboratory, Ankita Shukla, Arizona State University, Luning Sun, Lawrence Livermore National Laboratory, Jayaraman J. Thiagarajan, Lawrence Livermore National Laboratory, Kowshik Thopalli, Lawrence Livermore National Laboratory. SciML under distribution shifts: Achieving reliable and accurate performance via geometric and probablistic priors. Preliminary report.
The computational modeling problems in most SciML applications tend to present unique set of challenges such as: (a) the data is typically multimodal (often consisting of more than 2 modalities); (b) there are known scientific constraints or equations that must be considered in training; (c) large distribution shifts are expected by design when training with simulation data; (d) Understanding failure of SciML models is non-trivial; (e) data can come from varying (related) fidelities. We present our progress on these problems, with the use and design of novel priors for SciML. These include, modern architectural priors such as Transformers-based surrogates that show improved modeling and generalization capabilities; geometric priors such as enforcing hierarchical priors in the latent space for improved generative modeling and multi-fidelity training; and finally probabilistic priors that allow us to build frameworks for accurately assessing failures in deep learning models. Finally, using applications from high energy density physics, and inertial confinement fusion we will also show how the resulting ML models are better suited to generalize across distribution shifts, both in zero-shot (extrapolation) as well as few-shot training settings (transfer learning).
(Received September 12, 2023)

1192-68-32600
Maciej Besta, ETH Zurich, Torsten Hoefler, ETH Zurich, Kelly Isham, Colgate University, Kartik Lakhotia, Intel, Laura Monroe*, Los Alamos National Laboratory, Fabrizio Petrini, Intel. In-network Allreduce with Multiple Spanning Trees. Allreduce is a fundamental collective used in parallel computing and distributed training of machine learning models. This collective maps directly to the graph-theoretic problem of finding a large set of spanning trees with a small amount of edgeoverlap between trees. In this talk, we develop, compare and contrast two different sets of Allreduce spanning trees embedded into PolarFly, a high-performance diameter-2 network topology based on the Erdos-Renyi family of polarity graphs. One is a maximal set of edge-disjoint spanning trees (EDSTs) composed of Hamiltonian paths. The other is not edge-disjoint, but has minimal edge overlap and has the benefit of constant low depth 3 on ER polarity graphs of any size. We will present the construction of these spanning trees, and discuss benefits to computations involving Allreduce, including theoretically guaranteed near-optimal performance, performance boosts that are a factor of half the network router radix, low latency in the low-depth case, and congestion-free routing in the case of the EDSTs. We also distinguish PolarFly as a highly suitable network for distributed deep learning and other applications that employ throughput-bound large Allreductions.
(Received September 12, 2023)

Helena K Chaine*, Western Connecticut State University, Ellen Wang, Western Connecticut State University, Xiaodi Wang, Western Connecticut State University. Defending Quantum Neural Networks against Adversarial Attacks using Lattice-Based Homomorphic Data Encryption. Preliminary report.
Several studies have shown that quantum neural networks are susceptible to adversarial attacks, which pose a significant threat to the accuracy of quantum neural networks. Adversarial attacks may exploit the input data of a network by feeding incorrect data to a model while manipulating one that has already been trained, which may cause erroneous results. In this research, we propose a novel defense model to protect quantum neural networks against adversarial attacks by using homomorphic data encryption. Homomorphic encryption allows computations to be processed on encrypted data that has not yet been decrypted by converting the data into ciphertext, which allows the data to be handled in its original form without any risks of privacy breach. By incorporating homomorphic data encryption into quantum neural networks, our proposed model tries to reduce the threats posed by adversarial attacks that may perturb network outputs. Taken together, our approach provides insight on the future of quantum computing and the preservation of sensitive information.
(Received September 12, 2023)

1192-68-32826
Mahmud Hasan*, Graduate Student, Hailin Sang, Associate Professor. Estimation bounds of various f-divergence measures for Generative Adversarial Networks.. Preliminary report.
Generative Adversarial Networks (GANs) constitute a minimax problem involving two neural network components: a discriminator and a generator. Estimating this minimax problem, using various $f$-divergence metrics, is crucial for analyzing errors in GANs. This research article establishes upper bounds for several $f$-divergences, including total variation, Hellinger divergence, Pearson $\mathcal{X}^{2}$ divergence, and Jensen-Shannon divergence, within the context of GANs minimax estimation. These upper bounds are contingent on factors such as the discriminator and generator's sample sizes and the VC dimension of a new function class. Notably, as the sample size approaches infinity, the bounds become dependent solely on the discriminator class, and the existing bounds can be seen as a specific case of the results presented in this paper. Furthermore, this estimation bound is extended to the error analysis of GANs concerning the mentioned $f$-divergence measures. As a part of the proof, the study introduces an oracle inequality bounded by three empirical objective functions. These objective functions serve as the foundation for bounding various $f$-divergence metrics, including total variation, Hellinger divergence, Pearson $\mathcal{X}^{2}$ divergence, and Jensen-Shannon divergence. The computation of these empirical objectives is bounded by the Rademacher complexity of the function class formed by the discriminator and generator. We consider the neural network architecture of the discriminator and generator networks with weight parameters and using Lipschitz activation functions. The Rademacher complexity is bounded by the VC dimension and the sample sizes of the function class formed by the neural network structure. Finally, the relationship with Rademacher bound and Oracle inequality is used to prove the GANs estimation bound and extend to error analysis for $f$-divergences, including total variation, Hellinger divergence, Pearson $\mathcal{X}^{2}$ divergence, and JensenShannon divergence. (Received September 12, 2023)

Sophia Gu*, CosmosQuotient. A Study on LLMs' Collaborative Brainstorming Prowess in Math and Science Problems. Preliminary report.
Recently, there has been a growing interest in examining the capabilities of state-of-the-art deep learning models to collaborate on mathematical and scientific problems that require creative problem-solving and brainstorming. Traditional ML/DL methods often manifest as inscrutable black boxes, making their decision-making processes opaque. The rise of Large Language Models (LLMs) this year offers a solution by facilitating transparent conversations, elucidating Chain-of-Thoughts (CoT) processes, and providing greater flexibility such as iterative idea exchange and development. In this work, we present detailed case studies exploring the capabilities and limitations of LLMs with a specific focus on GPT-4, in addressing both welldefined and open-ended mathematical questions, such as the n-body problem. We find that the unique attributes of LLMs that complement human abilities, including higher-dimensional thinking and extensive world knowledge, have the potential to reshape the future of mathematical learning and research. Our evaluation of GPT-4 also covers its ability to comprehend complex queries, recommend appropriate methodologies, clarify vague problems, and more. Furthermore, we provide a blueprint and several successful strategies for optimizing communication with LLMs in this context.
(Received September 12, 2023)

1192-68-32942
Helena K Chaine*, Western Connecticut State University, Ellen Wang*, Western Connecticut State University, Xiaodi Wang, Western Connecticut State University. Defending Quantum Neural Networks against Adversarial Attacks with LatticeBased Homomorphic Data Encryption. Preliminary report.
Several studies have shown that quantum neural networks are susceptible to adversarial attacks, which pose a significant threat to the accuracy of quantum neural networks. Adversarial attacks may exploit the input data of a network by feeding incorrect data to a model while manipulating one that has already been trained, which may cause erroneous results [19],[20]. In this research, we propose a novel defense model to protect quantum neural networks against adversarial attacks by using homomorphic data encryption. Homomorphic encryption allows computations to be processed on encrypted data that has not yet been decrypted by converting the data into ciphertext, which allows the data to be handled in its original form without any risks of privacy breach [8],[9],[10]. By incorporating homomorphic data encryption into quantum neural networks, our proposed model tries to reduce adversarial attacks that may perturb network outputs. Taken together, our approach sheds light on the future of quantum computing and the preservation of sensitive information.
(Received September 13, 2023)

1192-68-33017
Andrew Li*, Western Connecticut State University. Mid-term Seasonal Arctic Sea Ice Concentration Forecasts by M-Band Wavelet Based CNN and ConvLSTM algorithms.
Mid-term Seasonal Arctic Sea Ice Concentration Forecasts are vital for navigation routing, policy-making, and predicting cetacean migrations for safety reasons. The accuracy of such predictions typically declines with extended lead times. Notably, forecasting during the melting season (June-October) presents more challenges than the freezing season (November-May),
largely due to sea ice thinning. In this research, we try to address these challenges by employing convolutional neural networks (CNN) and Convolutional long short-term memory networks (ConvLSTM) in M-Band Wavelet domain and integrating three-dimensional spatio-temporal climate and sea ice data for enhancing mid-term sea ice concentration (SIC) forecast. Our experimental results revealed that ConvLSTM consistently surpassed CNN. Moreover, the inclusion of climate variables, expanded historical data, and pre-processing our spatial data with M-Band wavelet transform further enhanced model efficacy. (Received September 13, 2023)

## 1192-68-33118

Shankar Balasubramanian*, Massachusetts Institute of Technology, Aram Harrow, Massachusetts Institute of Technology, Tongyang Li, Peking University. Exponential speedups for quantum walks in random hierarchical graphs.
There are few known exponential speedups for quantum algorithms and these tend to fall into even fewer families. One speedup that has mostly resisted generalization is the use of quantum walks to traverse the welded-tree graph, due to Childs, Cleve, Deotto, Farhi, Gutmann, and Spielman. We show how to generalize this to a large class of hierarchical graphs in which the vertices are grouped into "supervertices" which are arranged according to a $d$-dimensional lattice. Supervertices can have different sizes, and edges between supervertices correspond to random connections between their constituent vertices. The hitting times of quantum walks on these graphs are related to the localization properties of zero modes in certain disordered tight binding Hamiltonians. The speedups range from superpolynomial to exponential, depending on the underlying dimension and the random graph model. We also provide concrete realizations of these hierarchical graphs, and introduce a general method for constructing graphs with efficient quantum traversal times using graph sparsification.
(Received September 13, 2023)

## 1192-68-33145

Charlotte Aten*, University of Denver. Discrete neural nets and graph polymorphisms for learning.
Classical neural network learning techniques have primarily been focused on optimization in a continuous setting. Early results in the area showed that many activation functions could be used to build neural nets that represent any function, but of course this also allows for overfitting. In an effort to ameliorate this deficiency, one seeks to reduce the search space of possible functions to a special class which preserves some relevant structure. I will propose a solution to this problem of a quite general nature, which is to use polymorphisms of a relevant family of grpahs. I will give some concrete examples of this, then hint that this specific case is actually of broader applicability than one might guess.
(Received September 25, 2023)

1192-68-33154
Andrew Li*, Western Connecticut State University. Mid-Term Arctic Sea Ice Concentration Forecast Based on CNN, ConvLSTM and Wavelet Multi-Scale Deep Learning Algorithms.
Mid-term forecasts of Arctic Sea Ice Concentration (SIC) are crucial for safe navigation, informed policy decisions, and anticipating cetacean migrations. The longer the prediction lead time, the more accuracy tends to diminish. Especially challenging is the melting season (June-October), attributed largely to the thinning of sea ice. This study tackles these challenges by utilizing convolutional neural networks (CNN) and Convolutional long short-term memory networks (ConvLSTM) within the M-Band Wavelet domain. We enhanced predictions by integrating three-dimensional spatio-temporal data on both climate and sea ice. Experimental outcomes demonstrated that ConvLSTM consistently outperformed CNN. Additionally, integrating climate variables, broadening the historical data scope, and processing our spatial data with M-Band wavelet transform significantly improved model performance.
(Received September 13, 2023)

1192-68-33679
Stellina Xingxing Ao*, California State University, Los Angeles, Deborah Won, California State University, Los Angeles, Jie Zhong, California State University Los Angeles. A Beta Regression Model Using Fractal Dimensionality to Predict Grip Force from EEG for Applications in Brain-Computer Interfaces. Preliminary report.
Brain-computer interfaces (BCI) are neuroprosthetics that help tetraplegic individuals regain mobility by interpreting motor intent from neural signals and commanding a prosthetic limb to perform the task. While decoders have achieved high accuracy in decoding grip force from neural signals collected with electroencephalography (EEG), most of the literature has implemented deep learning algorithms. However, beta regression models can utilize the linear relationship between the fractal dimensionality (FD) of EEG signals and the degree of force exerted. FD measures the complexity of the EEG, which increases as more motor neurons are recruited to execute a stronger grip force. Beta regression models are advantageous in their simplicity, efficiency, and interpretability. To train the model, EEG data from C3, C4, F7, and F8 (10-20 system) were collected from subjects clenching a pipe with force-sensing resistors to four degrees of their maximum force, guided by a virtual interface. EEG data was bandpass-filtered from 0.5 Hz to 35 Hz , and eyeblink artifacts recorded with electrooculography were filtered out. Kat'z algorithm for FD calculation was applied to one second of EEG from each channel correlated with each force datum. The four-tuple of fractal dimensionality was then used to train a beta regression model. This model provides a resource-efficient method of decoding force on a continuous scale from EEG data, contributing to sensitive BCIs restoring the full range of motion to tetraplegic individuals.
(Received September 22, 2023)

Simon Langowski, MIT, Dongchen Zou*, Phillips Exeter Academy. Intersection Attacks in Non-Uniform Setting. Recently consumer demand for privacy has spurred growth in private messaging systems. However, formally, privacy degrades in such systems when users log on and off and this change of status exposes the ongoing conversations. Intersection attacks (also known as statistical disclosure attacks) use messaging patterns or liveness information to reconstruct re- lationships, deanonymize users, and track user behaviors. Prior attacks assume users have an underlying uniform communication pattern for sim- plicity, leaving the question open of how effective such attacks would be in a non-uniform real world. We observe that effects like clustering in real social graphs, and correlation between repeated conversations change the behavior and potential of such attacks. This paper provides a new approach that can take into account some of these additional factors, by
constructing a polynomial that can be used to determine the social graph. We analyze the performance, accuracy, and convergence rate of our attack, and compare with prior work. Our attack is applicable to many existing anonymous communication systems, and our technique can be extended to incorporate additional factors.
(Received September 23, 2023)

Omar El Nesr*, Massachusetts Institute of Technology, Axel Feldmann, Massachusetts Institute of Technology. Fast GPU Accelerated Ising Models for Practical Combinatorial Optimization. Preliminary report.
Combinatorial optimization (CO) problems exist in every scientific field, spanning disciplines such as biology, chemistry, finance and mathematics. These problems are NP-hard, and difficulty explodes with size. Methods for quickly finding high quality solutions to many problems enable breakthroughs in research. The Ising model is a flexible and general system from statistical mechanics that optimization problems can be mapped onto. Solving the Ising model then becomes equivalent to solving the original problem. However, current solvers do not scale well to large problem sizes. In this paper, we take advantage of the sparsity of large graphs and present an optimized GPU Ising solver for both MAXCUT and the Traveling Salesman Problem (TSP). Benchmarking shows the solver outperforms state-of-the-art tools in both solution quality and runtime, achieving a $3 x$ geometric mean speedup and a maximum speed up of over 2,000x over the prior leading implementation. Our method scales well to problems of over 40,000 binary variables and runs thousands of simulations simultaneously, giving near-optimal solutions in seconds. To the best of our knowledge, this is the fastest open-source tool for Ising optimization and will enable researchers to solve these computational problems with ease.
(Received September 25, 2023)

## 1192-68-33786

Michelle Wei*, MIT PRIMES-USA. Solving Second-Order Cone Programs in Matrix Multiplication Time. We propose a deterministic algorithm for solving second-order cone programs of the form

$$
\min _{A x=b, x \in \mathcal{L}_{1} \times \ldots \times \mathcal{L}_{r}} c^{\top} x
$$

which optimize a linear objective function over the set of $x \in \mathbb{R}^{n}$ contained in the intersection of an affine set and the product of $r$ second-order cones. Our algorithm achieves a runtime of $\widetilde{O}\left(\left(n^{\omega}+n^{2+o(1)} r^{1 / 6}+n^{2.5-\alpha / 2+o(1)}\right) \log (1 / \epsilon)\right)$, where $\omega$ and $\alpha$ are the exponents of matrix multiplication, and $\epsilon$ is the relative accuracy. For the current values of $\omega \sim 2.37$ and $\alpha \sim 0.32$, our algorithm takes $\widetilde{O}\left(n^{\omega} \log (1 / \epsilon)\right)$ time. This nearly matches the runtime for solving the sub-problem $A x=b$. To the best of our knowledge, this is the first improvement on the computational complexity of solving second-order cone programs after the seminal work of Nesterov and Nemirovskii on general convex programs. For $\omega=2$, our algorithm takes
$\widetilde{O}\left(n^{2+o(1)} r^{1 / 6} \log (1 / \epsilon)\right)$ time. To obtain this result, we utilize several new concepts that we believe may be of independent interest: - ize
- We introduce a novel reduction for splitting \(\ell_{p}\)-cones.
- We propose a deterministic data structure to efficiently maintain the central path of interior point methods for general convex programs. \enditemize
(Received September 26, 2023)


## 1192-68-33811

Nick David Wharff*, Drake University. A deep neural network for a hemiarray EIT system.
Electrical Impedance Tomography (EIT) can map electrical property distributions within the body using a surface electrode array. EIT systems using a circumferential array applied to the abdomen can be used to monitor acute intra-abdominal hemorrhages in trauma patients. Nevertheless, these patients may also have suffered spinal injuries that might be exacerbated by lifting the patient to place the array. Thus, a half array ('hemiarray') applied only to the anterior abdomen may be more practical. However, severe reconstruction artifacts result in posterior regions using standard EIT reconstruction methods. This study proposes a novel machine learning-based approach for standard full and hemiarray EIT reconstructions, demonstrating superior reconstruction characteristics compared to conventional methods. Notably, our method mitigates the challenges of reconstructing anomalies in posterior regions. This performance advantage was consistently observed across reconstructions from simulated and real tank data. Based on our findings, we conclude that the machine learning-based hemiarray reconstruction method holds significant promise for challenging imaging scenarios, particularly when access to the anterior or posterior abdomen is restricted.
(Received September 26, 2023)

## 1192-68-33841

Eric A. Autry, Grinnell College, Mai A Hoang*, Grinnell College, Guochen Liao*, Grinnell College, Chong Zhao*, Grinnell College. Redistricting Score for Community Preservation
In this work, we evaluate the preservation of municipalities and communities of interest in the redistricting process Preserving these communities protects the residents' right to choose their local representatives. The Supreme Court of the United States emphasized the importance of communities of interest in their recent ruling on the Allen v. Milligan case from Alabama (June 2023). To quantitatively measure the preservation of a community, we introduce the novel idea of an 'ousted population' that has been separated from their community. We define the ousted population as the number of residents not included in the community's core voting districts. We analyze some small-scale artificial models and real-world examples from North Carolina and Michigan to examine the impact of utilizing this preservation score.
(Received September 26, 2023)

## 1192-68-33850

Abdurrahim Birik*, Emory University, Christopher Cornwell, Towson University, Conor Meise*, Geneva College, Zhang Na, Towson University, Jilian Thomas, Elon University. Probabilistic Analysis of ReLU Neural Network Collapse. Preliminary report. In a feedforward ReLU neural network, upon initialization of the weights and biases, the function form of the neural network is
necessarily a continuous piecewise linear function. The expressiveness of this function can be limited by a significant number of constant regions due to the 'dying ReLU problem' which occurs when neurons only output zero values. Consequently, as the depth of the network approaches infinity, it has been shown that the network will eventually die, in probability (i.e., the function is constant with probability 1). In the current era of big data, avoiding an excessive number of parameters and the need to repeatedly re-initialize collapsed networks during training can lead to substantial savings in both space and time. In this work, we develop theoretical results on the probability of collapse for a given architecture. Specifically, we seek to understand the probability of collapse at each layer as inputs propagate through the network. To this end, we investigate the shrinking nature of the range size of the truncated network function and the polyhedral complexes associated with the bent hyperplane arrangement within neural networks. Numerical experiments and empirical data are provided to illustrate and support our various findings.
(Received September 26, 2023)

1192-68-33865
Simon Langowski, MIT, Boyan Litchev*, Lexington High School. Updatable Private Information Retrieval Without Noise Reduction. Preliminary report.
In traditional fully homomorphic encryption (FHE), number-theoretic transforms (NTTs) are utilized to speed up the process of multiplication. After multiplication, the ciphertext noise increases multiplicatively, meaning that few multiplications can be applied successively. To reduce this noise, schemes apply modulus and key-switching after multiplication. However, these operations cannot be applied to the NTT forms of ciphertexts, so ciphertexts have to be converted out of NTT form, using a significant amount of processing time and preventing parallelization. In the setting of private information retrieval (PIR) - an important building block of recent systems in anonymous messaging and private streaming - small ciphertext values, low multiplicative depth, and the usage of fresh ciphertexts in multiplications mitigate noise even without key and modulusswitching. We explore the efficiency of removing key and modulus-switching from the computation process for PIR, eliminating the need for intermediate number-theoretic transforms. This also aids in updating the result of a query when the database is modified.
(Received September 27, 2023)

## 1192-68-33888

Zaz Brown*, Kent State University, Mikhail Nesterenko, Kent State University. Using Braids for Byzantine-Resistant Geographic Routing on Polyhedral Networks. Preliminary report.
Geographic routing protocols enable routing in networks where the nodes have very little state, but know their position. This finds use, for example, in embedded devices that have GPS position information, but cannot maintain routing tables either due to lack of resources or lack of a constant position within the network (such as inter-vehicular networks). Current geographic routing protocols are close to optimal in terms of path stretch and throughput, but cannot tolerate a single malfunctioning (Byzantine) node. Byzantine-resistance requires $2 f+1$ connectivity, where $f$ is the number of Byzantine nodes. In particular, to tolerate 1 Byzantine node, the network must be 3-connected. We assume that the network is also planar, as there are techniques to planarize a network. Planar, 3-connected networks are called polyhedral networks because nodes can be represented as the vertices of a polyhedron. Polyhedral networks have nice properties that allow Byzantine resistant routing while only knowing local network topology and while using very few properties of the metric space which we assume nodes know their position in. The most basic technique to achieve Byzantine-resistance is to find 3 disjoint paths between the source and target node, but this proves challenging because nodes only know local topology; this makes it very hard to correctly route packets that are not traversing a face intersected by the line segment between source and target. A second technique we can use is braiding. The combination of these two techniques is what we explore in this work. (Received September 27, 2023)

## 70 Mechanics of particles and systems

## 1192-70-25452

Carlos Rodolfo Barrera-Anzaldo*, IIMAS-UNAM. Uniform bifurcation: Finding an infinitely many periodic solutions in a nonNewtonian restricted problem.
In this talk we treat the restricted $(n+1)$-body problem with a non-Newtonian homogeneous potential where the $n$ primaries move on an arbitrary $2 \pi$-periodic orbit. We prove that the satellite equation has infinitely many periodic solutions. These solutions are obtained as critical solutions of a family of time-dependent perturbed Lagrangian systems, bifurcating uniformly from a compact set of periodic solutions of the unperturbed Lagrangian system.
(Received June 09, 2023)

1192-70-26264
Emily Helene Almgren*, Haverford College, Rachel Kuske, Georgia Institute of Technology, Hung Nguyen*, University of Pennsylvania, Nilay J. Patel*, Cornell University. Dynamics of a Vibro-Impact Energy Harvester Under Non-Smooth Forcing. We study the dynamics of a ball-and-capsule vibro-impact energy harvester (VI-EH) under triangle wave forcing to contrast with ideal models based on harmonic forcing. We obtain a comprehensive bifurcation structure of our model via simulations, a semi-analytical approach based on nonlinear maps, and linear stability analysis under both the triangle wave forcing and its smooth Fourier approximations. Across a range of relevant parameters, we observe and characterize general shifts of periodic solutions and the bifurcations to smaller capsule lengths. Further analysis of these bifurcation structures also reveals novel phenomena not seen under harmonic forcing. Energy-harvesting analysis shows that low-order Fourier approximations provide an accurate estimate of the energy harvested under the non-smooth triangle wave. We find that the VI-EH is more efficient under the harmonic forcing than the triangle wave forcing in the same regime of motion, while the energy harvester generally remains in the more efficient regime of alternating periodic motion for a larger range of capsule lengths under the triangle wave forcing. This bridges the gap between previous work and experimental conditions.
(Received July 23, 2023)

Shangzhong Sun, Capital Normal University, Zhifu Xie*, The University of Southern Mississippi, Peng You, Hebei
University of Economics and Business. On the Uniqueness of Convex Central Configurations in the Planar 4-body Problem. A central configuration is a specific arrangement of masses, and a planar central configuration can lead to a homographic periodic solution. It is crucial for understanding the dynamic behavior of the N-body problem, and the question of its finiteness has been a challenge for mathematicians in the 21st century. For the planar four-body problem, its finiteness has been proven by computer-assisted proof in 2006 by Hampton and Moeckel, but there is still much to understand. One conjecture is that there exists a unique convex central configuration for any four positive masses in a given order. Many research paper has attempted this question by assuming either having some equal masses or having restrictions of the geometric shape such as a trapezoid or co-circular shape. In this talk, we provide a rigorous computer-assisted proof (CAP) of the conjecture for four masses belonging to a closed domain in the mass space. The proof employs the Krawczyk operator and the implicit function theorem. Notably, we demonstrate that the implicit function theorem can be combined with interval analysis, enabling us to estimate the size of the region where the implicit function exists and extend our findings from one mass point to its surrounding neighborhood. Such methods may be applied to general nonlinear equations.
(Received July 27, 2023)

## 1192-70-26819

Jodin Christopher Morey*, University of Minnesota. Approximating the Full Two-Body Problem.
You've sent a satellite to a far off planet or asteroid, and want to find a stable orbit. Newton found stable orbits when these two bodies are assumed to be infinitely small. We call this the two-body problem. However, when the bodies take on various shapes and sizes, we call this the full two-body problem, and there is still much work to be done! My research finds stable orbits when the orbiting system's motion is restricted to a plane, and each body is modeled as two point-masses connected by a massless rod, a dumbbell. In particular, I find stability when the orbits are symmetric; the bodies being oriented colinearly, perpendicularly, or trapezoidally.
(Received August 06, 2023)

## 1192-70-27423

Toshiaki Fujiwara, Kitasato University, Ernesto Perez-Chavela*, ITAM. Euler and Lagrange relative equilibria on the sphere. Preliminary report.
For $N=3$, it is well known that on the Euclidean space there are exactly five relative equilibria: three collinear (Euler relative equilibria) and two planar relative equilibria forming an equilateral triangle (Lagrange relative equilibria). The big difficulty to study relative equilibria on the sphere $\mathbb{S}^{2}$, that we call $R E$ by short, is the absence of the center of mass as a first integral, since many of the standard methods used in the classical case don't apply any more. Without the center of mass we do not know how to determine the rotation axis. In this talk I will show a geometrical method to study $R E$ on the sphere, when the masses are moving under the influence of a general potential which only depends on the mutual distances among the masses. First we prove the existence of two new integrals of motion, which can be seen as an extension of the center of mass. These two new integrals allow us determine the rotation axis. For the case $N=3$, we give some new families of Euler and Lagrange $R E$ on the sphere for the cotangent potential (the natural extension of the Newtonian potential to the sphere). (Received August 14, 2023)

## 1192-70-27967

Abimael Javier Bengochea Cruz*, ITAM. Periodic orbits near collision in a restricted four-body problem for the figure-eight choreography. Preliminary report.
Consider the following time-dependent planar restricted four-body problem: a massless particle moves under the gravitational influence due to three bodies following the figure-eight choreography. We use reversing symmetries and a regularizing technique, introdyced by D. G. Bettis and V. Szebehely, to study both theoretically and numerically symmetric periodic orbits near collision.
(Received August 23, 2023)

## 1192-70-28262

Jaime Burgos-Garcia*, Faculty of Physics and Mathematics.Autonomous University of Coahuila.. Periodic orbits in some gravitational perturbed environments. Preliminary report.
In this talk, some periodic orbits will be shown for the three-dimensional problem where a binary system, called asteroidmoonlet, is perturbed by the gravitational influence of two distant bodies and an irregular shape of the asteroid. These orbits were computed with analytic techniques such as symmetries, quadrature schemes, special functions, and rigorous numerics. We will discuss some properties of the orbits related to their existence and stability. This is joint work with A. Bengochea, L. Franco, M. Gidea, Wai-Ting Lam, C. Gales and A. Celletti. (Received August 28, 2023)

## 1192-70-31954

Gareth E Roberts*, College of the Holy Cross. On Kite Central Configurations. Preliminary report.
Central configurations play a key role in understanding solutions of the Newtonian $n$-body problem. From rest, a central configuration (c.c.) will collapse in on itself homothetically; given the correct initial velocity, a planar c.c. rotates rigidly about it's center of mass. Any group of bodies heading toward a collision must asymptotically approach a c.c. We focus on four-body kite c.c.'s, where two of the bodies lie on an axis of symmetry and the other two bodies are positioned equidistant from that axis. Kites may either form a convex or concave quadrilateral. The masses of the bodies not on the axis of symmetry must be equal. Following the approach used by Santoprete for co-circular c.c.'s, we explore the question of uniqueness, that is, for a given choice of masses and a particular ordering of the bodies, does there exist a unique convex kite c.c.? The idea is to replace the complicated Cayley-Menger determinant with a simpler constraint function, and then apply a topological argument. We contrast the convex and concave cases, and discuss how this approach may apply to the broader question of uniqueness for four-body convex central configurations.
(Received September 12, 2023)

## 1192-70-32103

Lora Bailey, Grand Valley State University, Weiqi Chu, University of Massachusetts Amherst, Daniel Brendan Cooney, University of Pennsylvania, Casey Johnson, University of California, Los Angeles, Edith Jin Zhang*, Columbia University. Higher Dimension Opinion Dynamics. Preliminary report.
Mathematical models of opinion dynamics explore the long-term behavior of a group of individuals, such as whether or not the group will reach consensus over time. Bounded confidence models (BCM) such as the Hegselmann-Krause (HK) and DeffuantWeisbuch (DW) models are used in opinion dynamics to model how individuals are influenced by the opinions of others. When considering a single topic, a BCM has individuals update their opinion by considering the opinions of other individuals whose opinions are within a fixed distance of their own. An important question to address is how our opinions change based on the opinions of others when opinions on multiple issues are relevant. We will build off the HK and DW models to expand to $n$ issues, or $n$ dimensions. In the presence of multiple issues, how do individuals use differences on each issue to determine how to update their own opinions? In higher dimensions, what constitutes consensus? We can explore higher-dimensional opinion dynamics from various perspectives. From a numerical perspective, a stochastic differential equation approach can be used to investigate long-term behavior. Using linear and weakly nonlinear stability analysis for the PDE obtained in the mean-field limit, we can explore the emergence of opinion clusters as an instability of a uniform distribution of opinions and study the patterns of opinion clusters arising under the interaction of opinions on multiple issues.
(Received September 12, 2023)

1192-70-32144
Jaime Eduardo Andrade*, Universidad del Bío-Bío. On the Restricted ( $N+1$ )-Body Problem on Surfaces of Constant Curvature.
We consider a restricted $(N+1)$-body problem on a constant curvature surface. This problem consists in the study of the dynamics of an infinitesimal mass particle attracted by $N$ primaries of identical masses describing elliptic relative equilibria of the $N$-body problem i.e., a solution where the primaries are rotating uniformly with constant angular velocity $\omega$ on a fixed parallel of $\mathbb{S}^{2}$ or $\mathbb{H}^{2}$ and placed at the vertices of a regular polygon. In a rotating frame, this problem yields a two degrees of freedom Hamiltonian system. In is ferified that the poles of $\mathbb{S}^{2}$ and vertex of $\mathbb{H}^{2}$ represent equilibrium points for any value of the parameters. Thus, analysis of the nonlinear stability of these equilibria is carried out, as well as two types of bifurcations are detected: Hamiltonian-Hopf and $N$-bifurcation. Additionally, we prove the existence of a family of Hill's orbits and a family of periodic orbits when the primaries are located near the poles of $\mathbb{S}^{2}$ or the vertex of $\mathbb{H}^{2}$. Finally we prove the existence of KAM 2-tori related to these periodic orbits.
(Received September 12, 2023)

## 1192-70-32161

Anju Saini*, Graphic Era Deemed to be University Dehradun , India. Mathematical Modeling of Unsteady Magneto hydrodynamic flow of nanoparticle deposition in the porous alveolar ducts. Preliminary report.
This study aims to investigate a numerical model of dust particle deposition in the human lung. There were significant discrepancies in deposition inside each alveolated duct and between ducts of a given generation, indicating that the mean acinar concentration can be greatly exceeded by limited particle concentrations. During expiration, the huge particles try to exit the construction but are failed. The similarity transformation is used to describe the differential equations controlling the flow. The various systematic quantities, such as the fluid velocity, dust particle velocity are determined. In this research, nonlinear governing coupled partial differential equations are explained by developing a finite-difference method established on the Crank-Nicolson model that is accurate, precise, widely validated, and unconditionally stable. The technique's precision and efficacy are proven. To show how various physical factors affect the velocity and temperature profiles, various numerical data are collected and visually displayed. For each of the characteristics tested, the results show that the nano particles velocity is more in compare of the fluid particles.
(Received September 12, 2023)

## 1192-70-32239

Gabriel Martins*, CSU Sacramento. Topological Study of Magnetic Confinement and Magnetic Quantum Tunneling. We will study the confinement of a charged particle to a bounded region in space by the action of a magnetic field. We describe a few topological tools that can be used and a very simple example where this idea can be clearly understood. Then we analyze how the shape of the region might help or obstruct the confinement of particles to its interior and mention a number of open questions. Lastly, we study a magnetic quantum tunneling effect, and consider choices of magnetic fields which are able to confine classical particles but are unable to confine quantum particles. We mention a number of open questions in that direction as well.
(Received September 12, 2023)

## 1192-70-32682

Sarah Post*, University of Hawaii. Superintegrable systems and exact solvability.
In this talk, I will give a brief review of the study of superintegrable systems-Hamiltonian systems with the maximum number of algebraically independent integrals of motion. We will discuss how this concept is related to exact-solvability and the role of parameters in the classification and control of these systems. We will end with a brief discussion of the quantum analog of these systems and control of quantum spin systems.
(Received September 12, 2023)
integral of motion in galactic dynamics. In order to do this, they reduced the system down to a 2-dimensional axisymmetric Hamiltonian system. Now this is known as the Hénon-Heiles (HH) System. Due to its apparent simplicity but extremely complicated dynamical behavior, this system is currently a paradigm in dynamical systems. In this paper, we perform a series expansion up to the seventh order of a potential with axial and reflection symmetries. After some transformations, this turns into the generalized Hénon-Heiles (GHH) system where we separate the fifth and seventh-order terms. We qualitatively analyze this system for energies near the threshold between bounded and unbounded motion with Poincaré sections and quantitatively analyze with Lyapunov Exponents. We find that particles far from the critical energy demonstrate less chaos. Additionally, the fifth-order term creates more regularity while the seventh-order term does the opposite. (Received September 19, 2023)

## 74 Mechanics of deformable solids

## 1192-74-28655

Siavash Jafarzadeh, Lehigh University, Stewart A Silling*, Sandia National Laboratories, Yue Yu, Lehigh University. Nonlocal properties of random media.
In modeling heterogeneous solids, it is often necessary to use smoothed, upscaled variables rather than attempting to resolve all the small-scale features in full detail. The evolution laws followed by these upscaled variables generally involve nonlocal interactions. A coarse graining method to be described in this talk maps the small-scale interactions in the original heterogeneous system into peridynamic bond forces at the larger length scale. These coarse-grained forces can be used to calibrate a peridynamic material model suitable for use with the upscaled equations of motion. With this technique, upscaled peridynamic descriptions of random media are found to accurately reproduce the response of the original system. Because the upscaled model is heterogeneous, it accounts for mesoscale variations in the statistics of the small-scale model. This provides an advantage over assigning a single set of effective properties that do not account for such variations. These mesoscale variations are important in additively manufactured materials, in which the manufacturing process imposes a length scale larger than the grain sizes but smaller than the overall dimensions of the body. This intermediate length scale can influence the fracture properties of the material. Damage thresholds for the upscaled model can also be obtained using coarse graining. This is accomplished by mapping the upscaled displacements to critical values of variables leading to damage in the original system. The resulting model, including both deformation and damage response, is found to reproduce fracture behavior in test problems on the small-scale system.
(Received September 01, 2023)

## 1192-74-29392

Siavash Jafarzadeh*, Lehigh University, Ning Liu, Global Engineering and Materials, Stewart A Silling, Sandia National Laboratories, Yue Yu, Lehigh University. Neural Peridynamic Operators: data-driven nonlocal constitutive models. Preliminary report.
In this work, we introduce an innovative integral neural operator architecture, designed to acquire nonlocal constitutive laws from data for materials with complex responses, including nonlinear large deformation, anisotropy, and heterogeneity. While most existing neural operator frameworks have primarily relied on data-driven approaches, overlooking the inherent preservation of fundamental physical principles within the data, our proposed neural peridynamic operator establishes a forward model following the principles of state-based peridynamics, thereby automatically ensuring the preservation of essential conservation and momentum balance laws. In this study, we showcase the versatility and effectiveness of our model by applying it to synthetic and experimental datasets, demonstrating its capacity to learn complex material responses. By employing neural networks to capture intricate behaviors while simultaneously adhering to fundamental physical laws, our learned neural operator outperforms baseline peridynamic models in terms of accuracy and efficiency. (Received September 06, 2023)

## 1192-74-29536

Ornella Mattei*, San Francisco State University, Charlie McMenomy, University of Colorado, Boulder. Bounds on the response of lossy 3d-printed composites in the time domain.
In this talk, we will present bounds on the quasi-static response of those 3d-printed composite materials composed by three phases: void, an elastic phase and a viscoelastic phase. As an example, consider the case of a 3d-printed polymer-based composite reinforced by carbon fibers. The presence of voids in the microstructure of the composite is either by design or a result of the printing process. The goal is to determine bounds on the antiplane shear response of the composite in the time domain, when the material is subject to a relaxation experiment. By exploiting the analytic properties of the effective tensor as a function of the shear moduli of the phases, we determine the minimum and maximum values of the shear response in time of the composite in the following two scenarios: (i) no information is provided regarding the microstructure of the composite; (ii) the volume fraction of the three phases is provided. Very interestingly, the bounds for the second scenario are incredibly tight at certain moments of time, allowing us to use them in an inverse fashion to determine the volume fraction of voids. Furthermore, by comparing such bounds to the ones obtained for the case when there are no voids, one can quantify the effects of the presence of voids in the shear response of the composite.
(Received September 07, 2023)

1192-74-33197
Natalie Frey, Worcester Polytechnic Institute, Dashiell Lipsey, Pomona College, Anthony Charles Miller*, University of Maryland Baltimore County. Terahertz Time Domain Spectroscopy for Characterizing Properties of Carbon Nanotube Yarns.. Carbon Nanotube yarn is a material with a vast array of potential applications. Among the most significant properties of these materials is their high tensile strength and conductivity. This research studied two areas. First, we sought to use a polarized terahertz laser to capture insight into the structure, particularly the alignment of the nanotubes in the yarn. The second aspect of our research involved in-depth data analysis of available measurements on finished nanotube yarn during the production process. The motivation behind this was the understanding that Van der Waals forces hold the yarn together, so a more aligned yarn might be stronger, meaning a way to measure the alignment of a yarn would be of value. The experiment had us shine terahertz, a laser perpendicular to a length of wire, measure absorption, and then use those measures to calculate the polarization of the laser as it passed through the wire, giving insight into the alignment of the yarn. Our most significant
finding in this part of the research was that while more aligned yarn did not correspond to the stronger yarn we tested, the variance of the alignment measures was lower for the stronger yarn. lower variance in the alignment of the yarn corresponded to higher strength. Our data analysis research focused on two areas, Fourier analysis and time series regression. One common problem we faced in data analysis was the highly variate and noisy nature of the data. In the Fourier analysis we took Fourier transformation and extracted high amplitude Fourier modes, then applied reverse transformations to identify and extract signal amidst noise. In our time series regressions, we sought to make accurate predictions of resistance (ohms) measurements in the yarn down the length of the yarn. We applied and compared various data structures and models to see what models could perform well and how far down the length of a yarn we could make predictions. (Received September 13, 2023)

## 76 Fluid mechanics

## 1192-76-26396

Keith Daniel Brauss*, . Approximating the Induced Magnetic Field of Velocity-Current Magnetohydrodynamics Equations. (Received July 27, 2023)

## 1192-76-28356

Ezra Baker, Institute for Pure and Applied Mathematics, Lila Fowler*, Institute for Pure and Applied Mathematics, Teagan Johnson*, Institute for Pure and Applied Mathematics, Isabel Renteria*, Institute for Pure and Applied Mathematics. Multifidelity Modeling for Rarefied Gas Kinetics Using POD and CFD. Preliminary report.
Developing efficient and accurate methods to model the Hall Effect Thruster (HET) is essential to improving satellite propulsion. Conventional high-fidelity models of the HET such as Direct Simulation Monte Carlo (DSMC) achieve acceptable accuracy, but only at high computational cost. This project seeks to develop more computationally efficient modeling techniques to model HET devices. To do so, we consider the simplified scenario of supersonic air experiencing a shock as it travels over a wedge, aiming to model the density of this air across space and time given varied initial densities and velocities. We combine low and medium fidelity models into a single multifidelity model that will produce a solution with comparable accuracy to DSMC in significantly less run time. Our low fidelity approach uses Proper Orthogonal Decomposition (POD) and least squares regression in a data-fit model based on limited DSMC simulation data. Our medium fidelity approach implements a Computational Fluid Dynamics (CFD) based model which solves the Euler equations. By combining the CFD with POD as an error term into a single multifidelity framework, we are able to produce a model with similar accuracy to DSMC that runs orders of magnitude faster.
(Received August 29, 2023)

## 1192-76-28479

John D Carter*, Seattle University. Stability of Near-Extreme Solutions of the Whitham Equation.
The Whitham equation is a model for the evolution of small-amplitude, unidirectional waves of all wavelengths on shallow water. It has been shown to accurately model the evolution of waves in laboratory experiments. We compute $2 \pi$-periodic traveling-wave solutions of the Whitham equation and numerically study their stability with a focus on solutions with large steepness. We show that the Hamiltonian oscillates as a function of wave steepness when the solutions are sufficiently steep. We show that a superharmonic instability is created at each extremum of the Hamiltonian and that between each extremum the stability spectra undergo similar bifurcations. Finally, we compare these results with those from the Euler equations. (Received August 31, 2023)

## 1192-76-28567

Andre Nachbin*, Worcester Polytcehnic Institute. Solitary water waves on graphs.
We have deduced a weakly nonlinear, weakly dispersive Boussinesq system for water waves on a 1D branching channel, namely on a graph. The model required a new compatibility condition at the graph's node, where the main reach bifurcates into two reaches. The new nonlinear compatibility condition arises from a stationary shock condition. We present numerical simulations comparing solitary waves on the 1D graph model with results of the (parent) 2D model, where a compatibility condition is not needed. We will comment on some new problems and applications that arise.
(Received August 31, 2023)

## 1192-76-29265

Bernard Deconinck, University of Washington, Sergey A Dyachenko, University at Buffalo, Pavel M Lushnikov, University of New Mexico, Anastassiya Semenova*, University of Washington. Instability of Stokes Waves. We study periodic traveling waves on the free surface of an ideal 2-dimensional fluid with infinite depth. Stability of such waves is examined by linearizing nonlinear equations of motion around Stokes waves and by studying the resulting eigenvalue problem numerically. In this talk, we demonstrate the spectrum of Stokes waves with various steepness. Additionally, we present the growth rate of the dominant instability and discuss the Benjamin-Feir, high-frequency and superharmonic instabilities associated with these waves.
(Received September 06, 2023)

1192-76-29412
Nazish Iftikhar*, Government College University Lahore. Triple Fractional Analysis of MHD Oldroyd-B Fluid Under Newtonian Heating and Radiation Effect.
This paper is an analysis of flow of a non-Newtonian fluid, Oldroyd-B fluid (OBF), with magnetohydrodynamic (MHD), Newtonian heating, radiation effect and non-uniform velocity conditions. The constitutive equations for heat, mass and momentum are established in the terms of Caputo (C), Caputo-Fabrizio (CF) and the Atangana-Baleanu in Caputo sense (ABC) fractional derivatives. The solutions are evaluated by employing Laplace transform and inversion algorithm. The flow in
momentum profile due to variability in the values of parameters are graphically illustrated among C, CF and ABC models. It is concluded that Atangana Baleanu fractional operator presents larger memory effects than Caputo and Caputo-Fabrizio
fractional operators.
(Received September 06, 2023)

## 1192-76-29561

Shuwang Li*, Illinois Institute of Technology, John Lowengrub, uc irvine, Steven Wise, University of Tennessee. PDE modeling and computation of fluid-structure interaction problems.
In this talk, I will use vesicle problem as an example to demonstrate two modeling approaches and discuss challenges in computation. The first approach is based on a sharp interface description. I will show the derivation of the model and corresponding interface (boundary) conditions and discuss vesicle dynamics in a viscous fluid. In the second approach, I will present a phase field model for vesicle expanding or shrinking, induced by an osmotic pressure that arises due to a chemical potential gradient. The model consists of an Allen-Cahn equation, which describes the phase field evolution; a Cahn-Hilliard equation, which simulates the fluid concentration; and a Stokes equation, which describe the motion of the vesicle. Conditions for vesicle growth or shrinkage are analyzed via the common tangent construction in phase energy. Finally, I will show some numerical results.
(Received September 07, 2023)

## 1192-76-29806

Katie L Oliveras*, Seattle University. Stability of Waves with Shear Currents. Preliminary report.
In this talk, we discuss the stability of periodic traveling wave solutions to the Euler equations under the influence of a linear shear and gravity. We explore the regime where the flow is dominated by the strength of the shear relative to the strength of gravity. In this regime, we show that the growth rates associated with the spectral stabilty of non-trivial traveling wave solutions grows without bound - precisely when the Taylor sign condition is violated. We compare these results with the results due to Blythe \& Parau based on the work of Hur \& Wheeler in the case of zero gravity. Specifically, we introduce gravity as a free parameter and use a continuation method to show that the results in the limiting zero gravity case are in agreement. (Received September 07, 2023)

## 1192-76-30148

Thomas Barker, Cardiff University, Yuhao Hu*, Shanghai Jiao Tong University, David G Schaeffer, Duke University. Onsager-symmetric constitutive laws for granular flow.
Assuming material frame-indifference, Onsager symmetry, and a yield condition, the search for a constitutive law for 3D granular flow is reduced to a geometric problem of finding a suitable Lagrangian submanifold of $\mathbb{R}^{6}$. Building on this idea, we devise a new constitutive theory for 3D granular flow in the inertial regime. The theory can be viewed as an extension of the Compressible I-Dependent Rheology (CIDR) of Barker et al. A link to Edelen's theory of irreversibility will be demonstrated. (Received September 08, 2023)

## 1192-76-30602

Jon Wilkening, University of California, Berkeley, Xinyu Zhao*, McMaster University. Spatially quasi-periodic water waves of finite depth.
We present a framework for computing and studying two-dimensional spatially quasi-periodic gravity-capillary water waves of finite depth. Specifically, we adopt a conformal mapping formulation of the water wave equation and represent quasi-periodic water waves by periodic functions on a higher-dimensional torus. We will present numerical examples of traveling quasiperiodic water waves and the time evolution of water waves over quasi-periodic bathymetry. We will also discuss the occurrence of small divisor phenomenon in this quasi-periodic setting.
(Received September 10, 2023)

1192-76-30643
Yassaya Batugedara*, The University of Virginia's College at Wise, Alexander E Labovsky, Michigan Technological University. Approximate Deconvolution with Correction - A High Fidelity Model for Magnetohydrodynamic Flows at High Reynolds and Magnetic Reynolds Numbers. Preliminary report.
Large Eddy Simulations (LES) are widely used in modeling turbulent flows including Magnetohydrodynamic(MHD) turbulence. In Labovsky(2020), a method called Large Eddy Simulation with Correction (LES-C) was proposed to reduce the modeling error using a "predictor-corrector" approach and the Approximate Deconvolution with Correction (ADC) model was introduced. In Batugedara et al. (2021), the time discretization error of the LES-C model was reduced using a scheme called Deferred correction. Using the above approach here we propose a model (ADC-MHD) to improve the accuracy of MHD. Here we used the decoupled MHD with Elsässer variables (Trenchea, 2014) and improve the accuracy by adding several terms to the righthand side of the correction step. We perform full numerical analysis and provide several computational tests that demonstrate the effectiveness of the new method.
(Received September 10, 2023)

## 1192-76-30920

Jon A Wilkening*, UC Berkeley. Long-time nonlinear dynamics of perturbed traveling and standing water waves.
We present a unified method of computing the spectral stability of traveling and standing water waves to harmonic or subharmonic perturbations. We explore the long-time dynamics of these perturbations as they grow in amplitude beyond the realm of linear theory, which we find can lead to Fermi-Pasta-Ulam recurrence. To track the growth of subharmonic perturbations, we develop a framework to compute and study fully nonlinear spatially quasi-periodic water waves, which are represented as periodic functions on a higher-dimensional torus evaluated along irrational directions. (Received September 11, 2023)

Christopher W Curtis*, San Diego State University. Evolution of Spectral Distributions in Deep-Water Constant Vorticity Flows.
A central question in ocean-state modeling is the role that various physical effects have on the evolution of the statistical properties of random sea states. This becomes a critical issue when one is concerned with the likelihood of rare events such as rogue, or freak, waves which can have significant destructive potential on deep sea ships and other offshore structures. In this talk then, building from a recently derived higher-order model of deep-water nonlinear waves, we examine the impact of constant vorticity currents on the statistical properties of nonlinearly evolving random sea states. As we show, these currents can both decrease and increase the kurtosis of the affiliated distributions of the sea states, thereby diminishing or enhancing the likelihood of rare events. We likewise numerically study the relationship between the kurtosis and a non-dimensional parameter, the Benjamin-Feir Index, which has proven to be a useful measure of when rare events are likely in oceanographic application.
(Received September 11, 2023)

1192-76-31386
Sergey A Dyachenko*, University at Buffalo. Singularities in 2D flows: The Tale of Two Branch Points. Preliminary report. We consider the classical problem of 2D fluid flow with a free boundary. Recent works strongly suggest that square-root type branch points appear naturally in 2D hydrodynamics. We illustrate how the fluid domain can be complemented by a "virtual" fluid, and the equations of motion are transplanted to a branch cut (a vortex sheet) in the conformal domain. A numerical and theoretical study of the motion of complex singularities in multiple Riemann sheets is suggested. Unlike preceding work for dynamics of singularities: the short branch cut approximation, and the study of viability of meromorphic solutions in fluid dynamics, the present approach neither simplifies the equations of fluid flow, nor uses local Laurent expansions. Instead the new approach is based on analytic functions and allows construction of global solutions in 2D hydrodynamics. A natural extension of the approach considers fluid flows described by many pairs of square-root branch points. (Received September 11, 2023)

1192-76-31528
Bernard Deconinck*, University of Washington. The instabilities of Stokes Waves: an overview.
A few different talks in this session will present new results on the stability of so-called Stokes waves (surface water waves of permanent form, traveling at constant speed). I will set the stage for these presentations by providing an overview of relevant results (analytical, asymptotic, computational) in the literature, both classical and more recent.
(Received September 11, 2023)

1192-76-31933
Souradip Chattopadhyay, North Carolina State University, Jessie Chen, North Carolina State University, Karen Daniels, North Carolina State University, Keith Hillaire, North Carolina State University, Hangjie Ji, North Carolina State University, Shawn Koohy, University of Massachusetts Dartmouth, Carmen Lee, North Carolina State University, Kathryn Massey*, Marist College, Luis Schneegans, University of Missouri - St. Louis, Megan Vezzetti, North Carolina State University. Mathematical Modeling of EGaIn Droplets Sliding Down an Inclined Plane. Preliminary report.
Eutectic Gallium-Indium (EGaIn) is a room-temperature liquid metal alloy that dramatically changes its surface tension and dynamics under an applied electric field. EGaIn has been used heavily in soft electronics engineering due to its high conductivity, malleability, and safety. However, the absence of mathematical modeling in the current literature makes its behavior difficult to understand and predict. In this study, we present a one-dimensional lubrication model for the dynamics of an EGaIn droplet moving along an inclined plane. Our model incorporates essential physical effects and parameters including oxidation, capillary action, diffusion, gravity, and Marangoni effects. In particular, we incorporate effects of the electric field, both through electric forces and changes in oxidation flux. We model the thin oxide skin of the droplet, which modulates the interfacial surface tension, as an insoluble surfactant at the surface. Oxidation, while observable in the physical setting, cannot be well measured, calling for an alternative method to quantify oxidation flux. Utilizing experimental data, we calibrate system parameters and qualitatively obtain numerical simulation results comparable to experimental observations. Stability analysis was conducted to understand the impacts of physical effects on our model. We find azimuthal curvature to be the main contributor in the process of threading that also promotes the formation of satellite droplets. Our model has demonstrated success in reproducing the observed dynamics of an EGaIn droplet and provides a valuable resource for further investigation and uses of EGaIn.
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## 1192-76-31962

Souradip Chattopadhyay, North Carolina State University, Jessie Chen, North Carolina State University, Karen Daniels, North Carolina State University, Keith Hillaire, North Carolina State University, Hangjie Ji, North Carolina State University, Shawn Koohy*, University of Massachusetts Dartmouth, Carmen Lee, North Carolina State University, Kathryn Massey*, Marist College, Luis Schneegans*, University of Missouri - St. Louis, Megan Vezzetti*, North Carolina State University. Mathematical modeling of liquid metal dynamics. Preliminary report.
Eutectic Gallium-Indium (EGaIn) is a room-temperature liquid metal alloy that dramatically changes its surface tension and dynamics under an applied electric field. EGaIn has been used heavily in soft electronics engineering due to its high conductivity, malleability, and safety. However, the absence of mathematical modeling in the current literature makes its behavior difficult to understand and predict. In this study, we present a one-dimensional lubrication model for the dynamics of an EGaIn droplet moving along an inclined plane. Our model incorporates essential physical effects and parameters including oxidation, capillary action, diffusion, gravity, and Marangoni effects. In particular, we incorporate effects of the electric field, both through electric forces and changes in oxidation flux. We model the thin oxide skin of the droplet, which modulates the interfacial surface tension, as an insoluble surfactant at the surface. Oxidation, while observable in the physical setting, cannot be well measured, calling for an alternative method to quantify oxidation flux. Utilizing experimental data, we calibrate system parameters and qualitatively obtain numerical simulation results comparable to experimental observations. Stability analysis
was conducted to understand the impacts of physical effects on our model. We find azimuthal curvature to be the main contributor in the process of threading that also promotes the formation of satellite droplets. Our model has demonstrated success in reproducing the observed dynamics of an EGaIn droplet and provides a valuable resource for further investigation and uses of EGaIn.
(Received September 12, 2023)

1192-76-32081
Denis Silantyev*, UCCS. Obtaining Stokes waves with high-precision using conformal maps and spectral methods on nonuniform grids.
Two-dimensional potential flow of the ideal incompressible fluid with free surface and infinite depth has a class of solutions called Stokes waves which are fully nonlinear periodic gravity waves propagating with the constant velocity. We developed a new highly efficient method for computation of Stokes waves. The convergence rate of the numerical approximation by a Fourier series is determined by the complex singularity of the travelling wave in the complex plane above the free surface. We study this singularity and use an auxiliary conformal mapping which moves it away from the free surface thus dramatically speeding up Fourier series convergence of the solution. Three options for the auxiliary conformal map are described with their advantages and disadvantages for numerics. Their efficiency is demonstrated for computing Stokes waves near the limiting Stokes wave (the wave of the greatest height) with 100 -digit precision. Drastically improved convergence rate significantly expands the family of numerically accessible solutions and allows us to study the oscillatory approach of these solutions to the limiting wave in great detail.
(Received September 12, 2023)

## 1192-76-32187

Monika Nitsche, University of New Mexico, Anand Oza, NJIT, Michael Siegel*, NJIT. Vortex sheet interactions with flapping swimmers.
The effect of hydrodynamic interactions on schooling behavior of animal collectives is studied via a simple model of multiple flapping plates (or swimmers) in an inviscid, incompressible fluid. The plates move vertically with a prescribed oscillatory motion, and horizontally via a self-induced thrust. Imposition of the Kutta condition at the trailing edge of each plate generates a shed vortex sheet, which then interacts with the other swimmers. A novel corrected quadrature is used to resolve the fluid motion near the plates. Results are obtained on the equilibrium swimming speeds of 2,3 and 4 in-line plates as a function of the flapping amplitude, and the stability of the equilibria is investigated. A simple control mechanism is introduced and shown to stabilize the equilibrium configurations.
(Received September 12, 2023)

## 1192-76-32290

Dipa Ghosh*, IIIT Delhi, India. Non-Newtonian Saffman-Taylor fingers: A topologically singular problem through WKB approximation. Preliminary report.
We present an analytical approach to the problem of predicting the finger width of a simple fluid driving a non-Newtonian (power-law) fluid. Our analysis is based on the Wentzel-Kramers-Brillouin approximation, by representing the deviation from the Newtonian viscosity as a singular perturbation in a parameter, leading to a solvability condition at the finger tip, which selects a unique finger width from the family of solutions. We find that the relation between the dimensionless finger width, $\Lambda$, and the dimensionless group of parameters containing the viscosity and surface tension, $\nu$, has the form $\Lambda \sim \frac{1}{2}-\mathcal{O}\left(\nu^{-1 / 2}\right)$ for the shear thinning case and $\Lambda \sim \frac{1}{2}+\mathcal{O}\left(\nu^{2 /(4-n)}\right)$ for the shear thickening case, in the limit of small $\nu$. This theoretical estimate is compared with the existing experimental, finger width data as well as the one computed with the linearized model, and a good agreement is found near the power-law exponent, $n=1$.
(Received September 12, 2023)

## 1192-76-32333

Roberto Camassa, The University of North Carolina at Chapel Hill, Daniel M Harris, Brown University, Robert Hunt*, Brown University, Richard M McLaughlin, The University of North Carolina at Chapel Hill, Rebecca Rosen, John Hopkins University. Diffusion-induced aggregation and diffusion-limited settling. Preliminary report.
Particles in density-stratified fluids are ubiquitous and relevant to the carbon cycle and the dispersion of microplastics. This talk will present a mechanism for particle aggregation in stratified fluids, in the absence of adhesion, which is due to the interplay of solute diffusion, boundary conditions, and gravity. A neutrally-buoyant particle immersed in a quiescent stratified fluid self-induces a flow which mediates an effective force between other immersed particles. The influence of particle size, shape, and material parameters on this induced flow and the consequent aggregation will be investigated theoretically, numerically, and experimentally with strong agreement. Porous particles carrying excess solid density will also be discussed; these particles can reach a terminal settling velocity dominated by mass exchange, and the size dependence of this terminal velocity behaves inversely to that of the Stokes settling velocity. A model for the particle dynamics as a function of the fluid and solid parameters and particle geometry will be presented which demonstrates quantitative agreement with experiments. (Received September 12, 2023)

1192-76-32439
Jose Claudio Vidal Diaz*, University of Bio-Bío. Crown relative equilibria for the vortex problem on the plane. Preliminary report.
We consider planar central configurations of the $2 n$-vortices problem consisting in 2 groups of regular $n$-gons of equal vorticities, called $(2, n)$-crown, or equivalently, we study the existence of periodic solutions, called relative equilibrium, for which the vortices rigidly rotate about the center of vorticity, with angular velocity $\lambda \neq 0$. We derive the equations of central configurations for the general $(2, n)$-crown. We prove that vorticities of each $n$-gon must be equal. Next, we give a necessary condition for a $(2, n)$-crown: either the rings are nested (the vertices of the two gons are aligned) or they must be rotated at an angle $\pi / n$ (twisted case). After that, we are able to give the exact number of central configurations as a function of the ratio of
vorticities. More precisely, we show that in the nested case there are two central configurations when the ratio of vorticity is positive, while for negative ratio of vorticity there exists a unique central configuration for appropriate radius. For the twisted case, it is observed that the study depends on the number of vortices in each $n$-gon and the admissible ratio of vorticities must be in an appropriate interval. Our analysis is analytic and it presents some important differences when compared with the Newtonian case.
(Received September 12, 2023 )

## 1192-76-32510

Christel Hohenegger*, University of Utah. Particle diffusion in complex fluids.
Complex fluids, like polymer melts or cake batter, have prominent viscoelastic properties, which lead to the subdiffusive behavior of immersed particles. Passive microrheology records displacement of such particles and extract mechanical properties of the bulk fluid. It is premised on the idea that statistics of particles trajectories can reveal fundamental information about the fluid environment. We discuss two different modeling and simulation approaches: a viscoelastic generalization of the Landau-Lifschitz Navier-Stokes fluid model for passive particles and generalized Langevin dynamics for particles. Ultimately, simulated data are used to address the uncertainty in reconstructing loss and storage moduli, which characterize the elastic and viscous properties of the fluid.
(Received September 12, 2023)

## 1192-76-32568

David M. Ambrose, Drexel University, Pavel M. Lushnikov*, Department of Mathematics and Statistics, Michael Siegel, NJIT, Denis Silantyev, University of Colorado. Nonlinear eigenvalue problem for collapse in the Generalized Constantin-LaxMajda Equation with and without dissipation.
We analyze the dynamics of singularities and finite time blowup of generalized Constantin-Lax-Majda equation which corresponds to non-potential effective motion of fluid with competing convection and vorticity stretching terms. Both nonviscous fluid and fluid with various types of dissipation including usual viscosity are considered. An infinite families of exact solutions are found together with the different types of complex singularities approaching the real line in finite times. A nonlinear eigenvalue problem is formulated and solved to determine the rate of blow up and the corresponding self-similar solutions. Both solutions on the real line and periodic solutions are considered. In the periodic geometry, a global-in-time existence of solutions is proven when the data is small and dissipation is strong enough. The found analytical solutions on the real line allow finite-time singularity formation for arbitrarily small data, even for various form of dissipation, thereby illustrating a critical difference between the problems on the real line and the circle. The analysis is complemented by accurate numerical simulations, which are able to track the formation and motion singularities in the complex plane. The computations validate and extend the analytical theory (Received September 12, 2023)

1192-76-32991
Dr. Caroline Lubert, James Madison University, Valentina Paz Soldan Viscarra*, James Madison University, Joseph Ungerleider, James Madison University. Prediction of Acoustic Loads by Empirical Analysis. Preliminary report. Rockets are powerful vehicles that play a critical role in space exploration, satellite deployment, and scientific research, and their importance lies in their ability to overcome the challenges of Earth's gravity and enable human exploration of space. Predicting acoustic loads on rockets is crucial because excessive noise and vibration can cause damage to the vehicle's structure, payload, and instrumentation. A model, NASA SP-8072, was developed in 1971 using old rocket data to predict the acoustic power generated by a supersonic rocket exhaust. This model presents two different distribution source methods (DSM): DSM-1 and DSM-2. DSM-1 is a straightforward approach that employs a source-distribution technique, while DSM-2 is an improved source-distribution method that considers exhaust shielding and reflections, providing more accurate prediction. The team implemented a MatLab code of the NASA-SP8072 rocket noise model to determine the sound pressure levels in each frequency band contributed by each slice at any observer point.
(Received September 13, 2023)

## 1192-76-33056

Kausik Das*, Harvey Mudd College. Effect of substrate-product competition on the swimming velocity of catalytic Janus spheres.
We explore spherical Janus particles in which a chemical reaction occurs on one face, depleting a substrate in the suspending fluid and synthesizing a product, while no reaction occurs on the other face. The steady state substrate concentration field is governed by Laplace's equation with mixed boundary conditions. We use the collocation method to obtain numerical solutions to the equation in spherical coordinates and we show that the product concentration field can be expressed in terms of the substrate concentration field. The asymmetry of the reaction gives rise to a slip velocity that causes the particle to move spontaneously in the fluid through a process known as self-diffusiophoresis. Using the Lorentz reciprocal theorem, we obtain the swimming velocity of the particle. The magnitude and direction of propulsion depends on competition between the substrate and product mobilities and diffusivities. We extend the results to Janus particles with arbitrary surface coverage, maximizing the swimming velocity as a function of the size of the reaction site.
(Received September 13, 2023)

## 1192-76-33131

Eleanor Devin Byrnes*, University of Washington. The Instabilities of Finite-Depth Stokes Waves. Preliminary report. I will present a progress report on numerical results for the stability spectrum of Stokes wave solutions to the full water wave problem in finite depth. Following the work of Dyachenko and Semenova, we apply the Fourier-Floquet-Hill Method to a conformal mapping reformulation of the water wave problem to compute both sub- and co-periodic instabilities. (Received September 13, 2023)

Michelle Bartolo*, North Carolina State University. Development of a multiscale pulse wave propagation model of the pulmonary arteries, veins, and capillaries.
Patient-specific computational modeling offers an efficient, non-invasive method to better understand complex, multiscale pathologies. Pulmonary hypertension (PH) is an understudied, yet severe and heterogeneous disease that afflicts the pulmonary vasculature. PH initiates in the smallest vessels, such as the arterioles, venules, and capillaries, and progressively reaches the main pulmonary artery, leading to increases in pulmonary vascular resistance and pressure. To study the pathogenesis of PH , we develop a novel pulse wave propagation model of blood flow through the complete pulmonary vasculature. Our model predicts hemodynamics in: (1) proximal vessels by solving the 1D Navier-Stokes equations in a reconstructed geometry from imaging data, (2) arterioles and venules by solving a linearized model in symmetry-changing fractal trees, and (3) capillaries by connecting the microcirculation in a ladder-like manner. We predict blood pressure, shear stress, and cyclic stretch in a wide range of spatial scales, allowing us to explore mechanobiological relationships related to PH progression. Our computational analyses reveal that rarefaction and vasoconstriction in the capillaries propagates to proximal arteries, elevating pressure and pulsatility.
(Received September 13, 2023)

## 1192-76-33289

Yanyan He, University of North Texas, Shilpa Khatri*, University of California, Merced, Lindsay Waldrop, Chapman University. The performance of crab odor-capture organs.
Olfaction, gathering information from dissolved chemicals in the ambient environment, is important for animals during reproduction, when searching for food, and when avoiding predation. Odor capture, when dissolved chemical cues or odorant molecules come into contact with chemosensory structures, is an important step in the olfaction process. We develop a mathematical model and computational tools to simulate a chemical concentration coupled with flow fields to study the antennule flicking of crabs in air and water. We will present simulations of odors as antennae flick and return. Furthermore, we will present results of the role of various nondimensional parameters, for example the Reynolds number or spacing between chemosensory structures, in odor-capture performance and the sensitivity of this performance to these parameters. (Received September 13, 2023)

## 1192-76-33536

Samuel Armstrong*, Buena Vista University, Dante Buhl*, University of California Santa Cruz, Garrett Hauser*, University of Rhode Island, Kristin Lloyd*, Towson University, Jazmin Sharp*, Towson University. Swimming with Deep Learning. Preliminary report.
The study of micro-organisms' propulsion has intrinsic relevance for the development of micro-robots designed for targeted drug delivery and as a foundation for further studies on hydrodynamic interactions between micro-organisms in complex environments. Numerical simulations have been used extensively to investigate micro-organisms' locomotion. Recently, physics-informed neural networks (PINNs) have shown promise for approximating solutions to differential equations that govern various physical problems. In this poster, we evaluate the effectiveness of using PINNs to predict the low Reynolds dynamics that characterize the propulsion of micro-organisms.
(Received September 20, 2023)

## 1192-76-33772

Halle Theriault*, University of North Carolina at Asheville. Reduced wave speed in $3 D$ printed corrugated pipes. Preliminary report.
Waves in periodic media travel at a group velocity that can be reduced compared to the speed in unperturbed media. This important phenomenon occurs in electronics, solid-state physics, optics, and-as is the focus of this study-acoustic waves in corrugated pipes. The speed reduction coefficient's dependence on the depth, width, and spacing of the corrugations is predicted by two models: the Bloch periodic wave theory and the Cummings suppressed mass model. To test these two models, the speed of sound was measured using 3D modeled and printed corrugated tubes which allowed for fine control of the corrugation parameters. The speed of sound was measured by tracking the change in the time it takes a pulse to reach the microphone as it is moved down the tube. Computational finite element analysis supported our experimental results, which suggests that the Bloch theory underpredicts and the Cummings model overpredicts the speed reduction. A new model taking elements from each established theory and our experimental results is developed.
(Received September 26, 2023)

1192-76-33954
Alexander Hoover*, Cleveland State University. Modeling the Mechanospace of Larvacean Pumping and Swimming. Far from the surface, the ocean's midwater present a rich frontier of biodiversity that is not well understood. Part of this gap in our knowledge is the great expense involved in collecting data with remotely operated vehicles. In this presentation, we will discuss the pipeline of developing in-silico computational experiments in concert with in-situ experimental data. Using a combination of particle image velocimetry data, optical scans, and confocal microscopy, we will discuss the creation of fluidstructure interaction models for organismal pumping and fluid transport, with the goal of developing an intuition on the physical mechanisms that drive their success, with a focus on understand the role that passive flexibility of oscillating flexible panels, and their associated beam modes, can in help in determining the success of active undulation. Using a combination of simplified geometries and scanned body meshes, we will employ the immersed boundary/finite element (IB/FE) method to simulate the swimming and chambered, valveless pumping mechanism of a giant larvacean. All motion described in this model will emerge from the interaction of active muscular tension, passive elastic recoil, and the local fluid environment (Received October 24, 2023)

## 78 Optics, electromagnetic theory

Scott Ziegler*, U.S. Naval Research Laboratory. A Topological Regularization Term for Inverse Scattering Problems. Quantitative inverse scattering (QIS) algorithms seek to reconstruct the support and electrical properties of an object through knowledge of transmitted signals and measured scattered waves. In qualitative inverse scattering, only the support of the object being imaged is considered. There is valuable information within a qualitative reconstruction concerning the size and location of the object under consideration, and thus techniques to leverage this information within the framework of QIS are of interest. Further, there are a myriad of techniques for quantitative reconstructions that provide advantages in different imaging environments. It would thus be beneficial to use topological knowledge from an inverse scattering reconstruction to improve imaging performance using a different technique. We propose incorporating prior information concerning the topology of the object being imaged into a regularization term constructed using persistent homology. The topological regularization term penalizes reconstructions which have an incorrect number of 0 and 1-dimensional Betti numbers and updates the image to better match previous information. A topological prior constructed using persistent homology has been explored for deep-learning based image segmentation (Clough et al., IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. 44, 2019), but has yet to be explored for inverse scattering algorithms. We aim to improve the quality of quantitative reconstructions obtained using the well-known distorted Born iterative method (DBIM) through the use of this topological regularization term. In this talk, we will outline the steps of the standard DBIM algorithm and give an overview of our use of persistent homology. We will then explain how prior topological information can be incorporated into a regularization function and describe a technique for updating the reconstructed image to decrease the value of that term. Both the standard approach and topologically-enhanced methods will be applied to a series of simulated datasets in order to evaluate performance. The success of the algorithm will be evaluated for higher-contrast targets, denser targets, and data sets corrupted by noise.
(Received August 29, 2023)

## 1192-78-29267

Miguel Moscoso, Universidad Carlos III de Madrid, Alexei Novikov, Penn State University, George Papanicolaou, Stanford University, Chrysoula Tsogka*, University of California Merced, Merced. Correlation-informed ordered dictionary learning for imaging in complex media.
We propose an approach for imaging in scattering media when large and diverse data sets are available. We use algorithms that come from computer science and statistics rather than traditional imaging techniques. Our approach has two steps. Using a dictionary learning algorithm, the first step estimates the true Green's function vectors as columns in an unordered sensing matrix. The array data comes from many sparse sets of sources whose location and strength are not known to us. In the second step the columns of the estimated sensing matrix are ordered for imaging using Multi-Dimensional Scaling with connectivity information derived from cross correlations of its columns, as in time reversal. For these two steps to work together we need data from large arrays of receivers, so that the columns of the sensing matrix are incoherent for the first step, as well as from sub-arrays, so that they are coherent enough to obtain the connectivity needed in the second step. Through simulation experiments, we show that the proposed approach is able to provide images in complex media whose resolution is that of a homogeneous medium.
(Received September 06, 2023)

## 1192-78-33018

Henok Mawi*, Howard University (Washington, DC, US). Optimal Transport in the Design of Refractors in Anisotropic Media. The use of freeform optical surface allows generation of complex, compact and highly efficient imaging systems. A freeform optical surface refers to a lens or mirror whose shape lacks rotational symmetry. One of the techniques used in the design problem of such surfaces is optimal transport technique. In this talk, we formulate a law of refraction for anisotropic media by using Fermat's principle of least time and exhibit the existence of a far field refracting lens by using optimal transport method. This is joint work with Cristian Gutiérrez. (Received September 13, 2023)

## 1192-78-33732

Cade Boggan*, Howard University, Nora Fischer, Howard University. Implementation of the Near Field Refractor Problem Solver Algorithm.
A near field refractor is an interface between two media of propagation that refracts light to a desired target in accordance with Snell's Law and satisfying a given energy conservation constraint. The study of the near field refractor is primarily of interest in the design of compact optical systems used in street lighting, automotive headlights, medical lighting, and fiber optics. This project is an implementation of a computational algorithm that is recently developed in the work of Gutierrez and Mawi, On the Numerical Solution of the Near Field Refractor Problem, Applied Mathematics and Optimization, Vol. 84 Issue (2021) to obtain a near field refractor. By using MATLAB, a plot for the desired near field refractor model was constructed. (Received September 24, 2023)

## 81 Quantum theory

## 1192-81-25419

Martin Weigt*, Nelson Mandela University, Port Elizabeth, South Africa. Local nets of unbounded operator algebras. One approach to a rigorous mathematical foundation to relativistic quantum field theory is to make use of local nets of von Neumann algebras, where, for a given region of spacetime, self-adjoint bounded linear operators in a von Neumann algebra are used as observables in the given region of spacetime (we note that the von Neumann algebra of observables generally depends on the region of spacetime under consideration). In reality, the observables are unbounded linear operators on a Hilbert space, so that one would have to consider local nets of unbounded operator algebras in which the closure of the operators are affiliated to a von Neumann algebra (in general, the von Neumann algebra depends on the unbounded operator algebra within the local net under consideration). The study of unbounded operator algebras started in around the 1960s and is not as well developed as the theory of operator algebras, where only bounded linear operators come into play. There are various types of unbounded operator algebras, two of which are the generalized B*-algebras due to G. R. Allan in 1967, and the $\mathrm{GW}^{*}$-algebras (short for generalized $\mathrm{W}^{*}$-algebras) due to A. Inoue in 1978. The purpose of this talk is to define local nets of GW*-algebras in a suitable manner, and to give some of their basic properties. For this, one must take into account that there
are various ways in which to extend the notion of a commutant of a set of bounded linear operators, to those of unbounded linear operators, and then finding the suitable extension of the concept to physical applications. Connections to Wightman theory will also be discussed, which forms a significant component of the talk.
(Received June 07, 2023)

## 1192-81-27099

Martin Fraas*, UC Davis. DERIVATION OF KUBO'S FORMULA FOR DISORDERED SYSTEMS AT ZERO TEMPERATURE.
This work justifies the linear response formula for the Hall conductance of a two- dimensional disordered system. The proof rests on controlling the dynamics associated with a random time-dependent Hamiltonian. The principal challenge is related to the fact that spectral and dynamical localization are intrinsically unstable under perturbation, and the exact spectral flow - the tool used previously to control the dynamics in this context - does not exist. We resolve this problem by proving a local adiabatic theorem: With high probability, the physical evolution of a localized eigenstate $\psi$ associated with a random system remains close to the spectral flow for a restriction of the instantaneous Hamiltonian to a region R where the bulk of $\psi$ is supported. Allowing $R$ to grow at most logarithmically in time ensures that the deviation of the physical evolution from this spectral flow is small. To substantiate our claim on the failure of the global spectral flow in disordered systems, we prove eigenvector hybridization in a one-dimensional Anderson model at all scales. (Received August 10, 2023)

1192-81-27250
Davide Gaiotto, Perimeter Institute for Theoretical Physics, Theo Johnson-Freyd*, Perimeter Institute For Theoretical Physics. Mock modularity and a secondary elliptic genus.
The theory of Topological Modular Forms suggests the existence of deformation invariants for two-dimensional supersymmetric field theories that are more refined than the standard elliptic genus. In this note we give a physical definition of some of these invariants. The theory of mock modular forms makes a surprise appearance, shedding light on the integrality properties of some well-known examples.
(Received August 12, 2023)

## 1192-81-27877

Alexis Drouot*, University of Washington. Topological edge spectrum for curved boundaries.
Topological insulators (TI) are a class of 2D materials which behave like insulators in their bulk but support robust states along their edges. This property has a rigorous explanation rooted in an index theorem when the TI fills a half-space. In this talk, we show that edge states emerge more generally when the boundary of the TI divides the plane in two regions with infinite filling radii. We will then give counter-examples when this condition fails. Joint work with Xiaowen Zhu.
(Received August 21, 2023)

## 1192-81-27959

Jan Manschot*, Trinity College Dublin. Mock Modular Forms and Instanton Partition Functions
Instanton partition functions are generating functions of topological invariants of moduli spaces of instants on four-manifolds. They are of interest in theoretical physics and geometry. Moreover, they are of interest for analytic number theory, since such partition functions can often be expressed in terms of modular forms and mock modular forms and generalisations. I will review this connection between physics, geometry \& mock modular forms, and discuss recent results. (Received August 23, 2023)

## 1192-81-28449

Ellen Chlachidze*, University of Michigan, Triet Ha*, Rhodes College, Sudatta Hor*, Brown University, Yuan Qiu*,
Williams College, Scott Smart, University of California, Los Angeles. Expressibility of Quantum Circuits for Chemistry Applications. Preliminary report.
Abstract: The expressibility of a parameterized quantum circuit (PQC) quantifies how well a PQC spans the Hilbert space by using the Kullback-Leibler divergence to compare the distribution of states obtained from the PQC to the uniform distribution of Haar-random states. The expressibility of a PQC has been shown to have a relationship with the performance in quantum machine learning. However, the relationship to quantum chemistry applications has been unexplored. In our work, we explore this relationship and devise a new strategy of computing expressibility that is better suited for chemistry applications. (Received August 30, 2023)

## 1192-81-28485

Eleanor Rieffel*, NASA Ames Research Center. Problem Session $a$.
A discussion of open mathematical problems relating to quantum theory
(Received August 31, 2023)

1192-81-28602
Anna Vershynina*, University of Houston. Quantum resource theory of coherence. Preliminary report
We will discuss both static and dynamic quantum resource theory of coherence. Coherence describes the existence of quantum interference. Quantum resource theory considers some resources (some states in the static theory and channels in the dynamic one) as free and the rest as a non-free resource. Accompanying these free resources are free quantum operations on them (quantum channel in the static theory and super-channels in the dynamic one), which leave the set of free resources invariant. The resource theory then studies information processing tasks that become possible using these operations. In this talk I will review the theory of coherence, as well as a relatively new dynamic resource theory of coherence. I will talk about the often used coherence measures of states, and will present new measures of the coherence of channels.
(Received August 31, 2023)

1192-81-28712
Simone Warzel, Technical University of Munich, Amanda Young*, Technical University Munich. A bulk gap in the presence of edge states for a truncated Haldane pseudopotential.
In this talk, we discuss a proof of a bulk gap for a truncated $1 / 3$-filled Haldane pseudopotential for the fractional quantum Hall effect in the cylinder geometry. In the case of open boundary conditions, a lower bound on the spectral gap (which is uniform in the volume and particle number) accurately reflects the presence of edge states. However, these edge states do not persist into the bulk, and so their low-lying energies do not accurate reflect the behavior of the bulk gap. A uniform lower bound for the Hamiltonian with periodic boundary conditions is also obtain. Both bounds are proved by identifying invariant subspaces to which spectral gap and ground state energy estimating methods originally developed for quantum spin Hamiltonians are applied. However, by customizing the gap technique to the invariant subspace, however, we are able to avoid the edge states and establish a more precise estimate on the bulk gap in the case of periodic boundary conditions. This is based off joint work with S. Warzel.
(Received September 01, 2023)

## 1192-81-29061

Michael H Freedman*, retired. Anyons vs. Electrons for Quantum Computation. Preliminary report.
Historically, the idea of doing quantum computation by braiding anyons largely gave way to measurement-only proposals in which the topological charge of a pair of anyons is measured. Strikingly, projective measurement of the total spin of pairs of ordinary spin $1 / 2$ particles is also BQP-complete. So, the virtue of topological physics in computation now rests only on protection of quantum information against decoherence rather than the universality of the Jones representations. (Received September 05, 2023)

1192-81-29140
Nathan Geer, Utah State University, Aaron D Lauda*, University of Southern California, Bertrand Patureau, University of South Brittany, Joshua Sussan, CUNY. An introduction to nonsemisimple topological quantum compiling.
Since the work of Freedman, Kitaev, Larsen, and Wang, it has been known that 2+1-dimensional TQFTs support universal quantum computation for certain unitary modular tensor categories (UMTC). This form of quantum computation is naturally robust against decoherence errors, as the quantum state of the system is encoded into topological degrees of freedom. In topological quantum computation, quantum states are encoded into subspaces of a larger Hilbert space associated with several quasiparticles corresponding to simple objects of the UMTC. The evolution of these quantum states is achieved via the braiding of quasiparticles. Topological quantum compiling is the process of constructing braids that achieve the desired computation on the encoded computational space. In this talk, we will review this theory in the context of Fibonacci anyons associated with $S U(2)_{3}$. We will then show how recent advances in nonsemisimple TQFT support a universal theory of topological quantum computation with many advantages over the semisimple counterpart.
(Received September 05, 2023)

1192-81-29280
Omar Hurtado*, University of California, Irvine. Unique continuation, Wegner lemma, and localization for non-stationary random Schrödinger operators on $\mathbb{Z}^{2}$.
Breakthrough work of Ding and Smart showed Anderson localization for certain random Schrödinger operators on $\ell^{2}\left(\mathbb{Z}^{2}\right)$ via a quantitative unique continuation principle and Wegner estimate. We extend their methods, relaxing the requirement of identical distribution with the requirement of a uniform bound on the essential range of potential and a uniform positive lower bound on the variance of the variables giving the potential. Under those assumptions, we recover the unique continuation and Wegner lemma results, using Bernoulli decompositions and modifications of the arguments therein. Under additional assumptions, we obtain a localization result at the bottom of the spectrum. (Received September 06, 2023)

## 1192-81-29333

Matthew B. Young*, Utah State University. Abelian supergroup Chern-Simons theories and their variants. Preliminary report.
Blanchet, Costantino, Geer and Patureau-Mirand discovered a non-semisimple generalization of the construction of Reshetikhin and Turaev of a three dimensional topological quantum field theory from a modular tensor category. The goal of this talk is to use this generalization to construct new examples of non-semisimple theories which appear as topological twists of certain supersymmetric quantum field theories, namely, abelian supergroup Chern-Simons theories and their variants. Based on joint works with Niklas Garner and Nathan Geer.
(Received September 06, 2023)

## 1192-81-29491

Radhakrishnan Balu*, University of Maryland. 1D self-similar fractals with centro-symmetric Jacobians: asymptotics and modular data. Preliminary report.
We establish asymptotics of growing one dimensional self-similar fractal graphs, that are relevant in confining fractons, with weighted edges in terms of quantum central limit theorems, with applications to quantum query complexity, for algebraic probability spaces in pure state. An additional structure is endowed with the repeating units of centro-symmetric Jacobians in the adjacency of a linear graph creating a self-similar fractal. The family of fractals induced by centro-symmetric Jacobians formulated as orthogonal polynomials satisfying three term recurrence relations support such limits. The construction proceeds with the interacting Fock spaces, T-algebras endowed with a quantum probability space, corresponding to the Jacobi coefficients of the recurrence relations and when some elements of the centro-symmetric matrix are constrained in a specific
way we obtain, as the same Jacobian structure is repeated, the central limits. The generic formulation of Leonard pairs and probablistic laplacians used in physics provide choice of centro-symmetric Jacobians widening the applicability of the result. We establish that the T-algebras of these 1D fractals, as they form a special class of distance-regular graphs, are thin and the induced association schemes are self-duals that lead to anyon systems with modular invariance.
(Received September 07, 2023)

1192-81-29570
Robert Calderbank, Duke University, Siyi Yang*, Duke University. Spatially-Coupled QLDPC Codes
Spatially-coupled (SC) codes is a class of convolutional LDPC codes that has been well investigated in classical coding theory thanks to their high performance and compatibility with low-latency decoders. We describe toric codes as quantum
counterparts of classical two-dimensional spatially-coupled (2D-SC) codes, and introduce spatially-coupled quantum LDPC (SCQLDPC) codes as a generalization. We use the convolutional structure to represent the parity check matrix of a $2 \mathrm{D}-\mathrm{SC}$ code as a polynomial in two indeterminates, and derive an algebraic condition that is both necessary and sufficient for a 2D-SC code to be a stabilizer code. This algebraic framework facilitates the construction of new code families. While not the focus of this paper, we note that small memory facilitates physical connectivity of qubits, and it enables local encoding and low-latency windowed decoding. In this paper, we use the algebraic framework to optimize short cycles in the Tanner graph of 2D-SC hypergraph product (HGP) codes that arise from short cycles in either component code. While prior work focuses on QLDPC codes with rate less than $1 / 10$, we construct 2D-SC HGP codes with small memories, higher rates (about $1 / 3$ ), and superior thresholds.
(Received September 07, 2023)

1192-81-29845
Sergei Korotkikh*, University of California, Berkeley. Integrability of $q$-Hahn vertex models explained via representation theory of quantum loop algebras, and its applications.
I will talk about vertex models with weights given by the orthogonality weights of $q$-Hahn polynomials. The main feature of these models is that their integrability cannot be explained by the classical Yang-Baxter equations used for other integrable vertex models. Instead, we need to consider families of isomorphisms of $U_{q}(L \mathfrak{g})$-modules which are different from $R$-matrices and then use the resulting versions of the Yang-Baxter equations to get integrability. In my talk I will present these families of isomorphisms and will show how they are used to study certain random polymer models and families of combinatorial special functions.
(Received September 08, 2023)

## 1192-81-29893

Saibal De, Sandia National Laboratories, Oliver Knitter*, University of Michigan, Rohan Kodati, University of Michigan, James Stokes, University of Michigan, Shravan Veerapaneni, University of Michigan. Applying Variational Quantum and Quantum-Inspired Algorithms to the Linear Complementarity Problem. Preliminary report.
Variational quantum algorithms (VQAs) are hybrid quantum-classical algorithms that seek to harness the advantage of quantum computers while simultaneously mitigating the drawbacks of the noisy, intermediate-scale (NISQ) quantum hardware existing today. VQAs have an established theoretical potential, but their ability to effectively solve problems arising from practical applications, and whether this utility can be wholly replicated by quantum-inspired classical algorithms, remains an active area of interest. We present a novel application of both the Variational Quantum Linear Solver (VQLS) and the Variational Neural Linear Solver (VNLS)—an existing VQA for solving systems of linear equations, and its quantum-inspired fully classical counterpart-as the key component within a larger minimum map Newton solver for a complementarity-based rigid body contact model. Using each algorithm, we demonstrate that this solver accurately depicts the dynamics of the model system's rigid spherical bodies as they collide. These results indicate that quantum and quantum-inspired linear algebra algorithms may provide a satisfactory alternative to standard linear algebra solvers for modeling certain physical systems. (Received September 08, 2023)

## 1192-81-30280

Zhenghan Wang*, Microsoft Station Q, UC Santa Barbara. Problem session B.
A discussion of open problems mathematical problems relating to quantum theory.
(Received September 09, 2023)

## 1192-81-30553

Wayne Polyzou, University of Iowa, Shaikh Gohin Samad*, University of Iowa. Construction of reflection positive kernels for $n$ - particle relativistic quantum system.. Preliminary report.
We investigate a system of relativistic particles with a finite number of degrees of freedom. To incorporate quantum theory with special relativity in Euclidean representation, the system must satisfy a condition known as reflection positivity. The purpose is to construct a class of reflection positive kernels that define the dynamics in Euclidean formulations of relativistic quantum mechanics and to apply it to a system to compare computational efficiency. Relativistic treatments of quantum theory are needed to study hadronic systems at sub-atomic distance scales. The direct interaction approaches to relativistic quantum mechanics have two disadvantages: a) cluster properties are difficult to realize for systems of more than two particles. b) the relation to quantum field theories is indirect. Euclidean formulation provides an alternative representation that does not have these difficulties and can be formulated without analytic continuation. An appealing feature of the Euclidean axioms is that the locality axiom is logically independent of the other axioms, so it can be relaxed (which is necessary for models of a finite number of degrees of freedom) without violating relativistic invariance, the spectral condition, cluster properties, and the Hilbert space representation of the theory. The Euclidean axioms of Osterwalder and Schrader lead to a Hamiltonian formulation of field theory that gives the physical Hilbert space inner product and a unitary representation of the Poincaré group directly in the Euclidean representation without analytic continuations which could provide a path to direct treatments of scattering, time-like form factors, Lorentz transformations, and light-front dynamics.
(Received September 10, 2023)

Anastasiia Minenkova*, University of Hartford, Gamal Mograby, University of Maryland. Linear Algebra in Quantum Computing. Preliminary report.
In this talk we discuss the relation between linear algebra and problems in quantum computing: from the perfect quantum state transfer to quantum walks on graphs. Using this relation we give a quick exposition of the mathematical aspects of these concepts.
(Received September 11, 2023)

## 1192-81-30648

Chi-Kwong Li*, College of William and Mary. Adiabatic quantum computing and graph theory problems. Preliminary report. We discuss some new methods in formulating some graph theory problems such as the traveling salesman problem in adiabatic quantum computing setting. The theoretical background and implementation issues will be discussed. (Received September 10, 2023)

## 1192-81-30651

Stan Srednyak*, Duke University. Functional differential equations in Quantum Field Theory.
There are two complementary approaches to second quantization of fields - canonical operator quantization and functional quantization. Both point to a unified object - theory of bundles on infinite dimensional submanifolds in spaces of sections of classical fields with varying regularity properties. This point of view sheds new light on classical objects such as renormalization theory, operator product expansions, theory of bound states, chaos in quantum systems etc. In particular, it highlights the search for bound state equations among families of functional differential equations. We will discuss our recent work on the development of this theory, focusing mainly on perturbative aspects. We will show relation to the theory of singular integral equations with holomorphic kernels and the theory of special functions that solve them. We will discuss aspects of theory of flat connections on complex analytic manifolds and multidimensional analogies of isomonodromic deformations and Riemann Hilbert problems that arise in this context. We will show how these finite dimensional approximations integrate to perturbative solutions of functional differential equations and discuss the structure of function spaces in which these solutions naturally belong. This will lead us to the definition of a natural class of infinite dimensional manifolds that arise in quantum field theory, as well as a class of holomorphic functions which can be used to construct approximate solutions. We will discuss implications for Yang Mills mass gap problem.
(Received September 10, 2023)

1192-81-30675
Natanael Alpay*, University California Irvine, Tiju Cherian John, The University of Arizona. Thermal States on Mittag Leffler Fock Space of the Slitted Plane. Preliminary report.
Number states and thermal states form an important class of physical states in quantum theory. A mathematical framework for studying these states is that of a Fock space over an appropriate Hilbert space. Several generalizations of the usual Bosonic Fock space have appeared recently due to their importance in many areas of mathematics and other scientific domains. One of the most prominent generalization of Fock spaces is the Mittag-Leffler (ML) Fock space of the slitted plane. Natural generalizations of the basic operators of quantum theory can be obtained on ML Fock spaces. Following the construction of the creation and annihilation operators in the Mittag-Leffler Fock space of the slitted plane by Rosenfeld, Russo, and Dixon, (J. Math. Anal. Appl. 463, 2, 2018). We construct and study the number states and thermal states on the ML Fock space of the slitted plane. Thermal states on usual Fock space form an important subclass of the so called quantum gaussian states, an analogous theory of more general quantum states (like squeezed states and Bell states) on ML Fock spaces is an area open for further exploration
(Received September 10, 2023)

## 1192-81-30747

Derek Zhang*, Amherst College. Quantum Network Routing and Graph Foliage. Preliminary report.
Future quantum networks can facilitate communication of quantum information between various nodes. We are particularly interested in whether multiple pairs can communicate simultaneously across a network. Quantum networks can be represented with graph states, and producing communication links amounts to performing certain quantum operations on graph states. This problem can be formulated in a graph-theoretic sense with the (Bell) vertex-minor problem. We discuss the recently introduced foliage partition and provide a generalization. This generalization leads us to a useful result for approaching the vertex-minor problem. We apply this result to identify the exact solution for the Bell vertex-minor problem on line and tree graphs.
(Received September 10, 2023)

1192-81-30800
Hanmeng (Harmony) Zhan*, Worcester Polytechnic Institute. Quantum search: an averaging perspective. Many quantum search algorithms use discrete quantum walks as subroutines. Naturally, the performance of the search depends on the structure of the undelying graph. In this talk, I will discuss quantum search from an averaging perspective instead of looking for the optimal stopping time, I will look at the average probability, over any period of time, that the quantum walk finds the marked vertex. For distance regular graphs satisfying a valency-size condition, this average success probability approaches $1 / 4$ as the valency goes to infinity. (Received September 11, 2023)
report.
Quantum groups at roots of unity provide invariants of links with flat connections in the complement. In this talk I will report some of the recent developments in this direction.
(Received September 11, 2023)

## 1192-81-30913

Nicolai Reshetikhin*, YMSC, Tsinghua University. On the semiclassical limit of solvable lattice models. Preliminary report. A quantum integrable system in the semiclassical limit gives a hybrid system: a quantum system that "rides on the top" of the corresponding classical system. In the talk I will focus on such phenomenon for integrable lattice systems.
(Received September 11, 2023)

## 1192-81-30927

Mihai Stoiciu*, Williams College. The Eigenvalue Distribution for Random Unitary Matrices: An Approach Using Entropy. Preliminary report.
CMV matrices are the unitary analogues of one-dimensional discrete Schrödinger operators. Depending on the distribution of their coefficients, random CMV matrices exhibit a transition in their eigenvalue distribution from a Poisson process (no eigenvalue correlation) to "picket fence" (strong eigenvalue repulsion). We investigate this transition from the perspective of the joint entropy of the coefficients of the random CMV matrix.
(Received September 11, 2023)

## 1192-81-31121

Katrin A. M. Wendland*, Trinity College Dublin. A fresh view on Gepner models.
In conformal field theory, Gepner models provide particularly nice examples of „exactly solvable" models. Their construction, which has been well established since the late 1980s, can be described purely combinatorially. Gepner models have fascinated ever since, thanks to their beautiful properties linking them both to modular (sometimes also Mock modular) forms and to geometry. We attempt a fresh view on Gepner models, addressing some old and some new mysteries around these conformal field theories and their representation content.
(Received September 11, 2023)

## 1192-81-31246

Jeffrey H Schenker*, Michigan State University. Theory of Ergodic Quantum Processes.
Recent results on the evolution of open quantum systems will be discussed. An open quantum system has non-trivial interactions with its environment, or with a measurement apparatus. In this talk, we will first review the background of open quantum systems and their description and then introduce the notion of a quantum process, as a sequence of quantum channels (trace preserving, completely positive maps on the space of density matrices). We will present results about the longtime behavior of ergodic quantum processes, composed of random channels sampled along the trajectory of an ergodic map, and present applications of these results to the theory of noisy repeated measurement processes. (Joint work with Ramis Movassagh, Lubashan Pathirana, Renaud Raquépas, Owen Ekblad, and Eloy Nadales.) (Received September 11, 2023)

## 1192-81-31339

Carlos M Ortiz-Marrero*, Pacific Northwest National Laboratory. Computing Quantum Strategies for Non-Local Games. Preliminary report.
In a non-local game, two non-communicating players cooperate to convince a referee about a strategy that does not violate the rules of the game. A quantum strategy for such a game enables players to determine their answers by performing joint measurements on a shared entangled state. In this talk, we will discuss a new variational quantum algorithm for computing these quantum strategies. In particular, we will showcase how we can use these quantum strategies to benchmark near-term quantum computing devices utilizing non-local games that come from problems in graph theory.
(Received September 11, 2023)

## 1192-81-31659

Jessalyn Bolkema*, California State University, Dominguez Hills. Combinatorics and Quantum Fault Tolerance.
Quantum error correction and quantum fault tolerance are critical to the development of practical quantum computers. Unfortunately, not every quantum error-correcting code can be used for fault-tolerant computation. Rengeswamy et. al. define CSS-T codes, which are CSS codes that admit a physical transversal T-gate. In this talk, we give an overview of the combinatorics of quantum error correcting codes and a comprehensive study specifically of CSS-T codes from Reed-Muller codes. These codes allow for the construction of CSS-T code families with desirable properties for fault-tolerant quantum computation. This work was partially supported by the National Science Foundation under grant DMS-1547399.
(Received September 11, 2023)

Daniel David Spiegel*, University of California, Davis. A $C^{*}$-Algebraic Approach to Parametrized Spin Chains and Their Phases.
The study of parametrized many-body gapped Hamiltonians and their phases is a fast-growing frontier at the intersection of condensed matter physics, functional analysis, and algebraic topology. In one spatial dimension, spin systems parametrized by a topological space $X$ possess a quantized phase invariant valued in the third cohomology group $H^{3}(X ; \mathbb{Z})$ of the parameter space; this invariant was originally constructed by Kapustin and Spodyneiko in 2020 as the de Rham class of a differential form generalizing the Berry curvature for higher dimensional systems. I will present an alternative derivation of an $H^{3}(X ; \mathbb{Z})$ phase
invariant for infinite spin chains using $C^{*}$-algebra theory and Cech cohomology, in which the phase invariant first appears as an element of $H^{1}(X ; \mathbb{P} \mathrm{U}(\mathcal{H}))$-where $\mathbb{P} \mathrm{U}(\mathcal{H})$ is the projective unitary group of an infinite-dimensional separable Hilbert space $\mathcal{H}$-and is mapped to $H^{3}(X ; \mathbb{Z})$ using Dixmier-Douady theory. This construction will be exhibited for an exactly solvable model in a nontrivial phase parametrized by the 3-sphere $X=S^{3}$, introduced in a paper by Xueda Wen, et al. (Received September 11, 2023)

## 1192-81-31682

Hermie Monterde*, University of Manitoba. Sedentariness in twin sets of size two.
Let $A$ be the adjacency matrix of a graph $X$. The transmission of quantum states within a quantum spin network represented by $X$ is determined by the unitary matrix $U(t)=\exp (i t A)$, where $i^{2}=-1$ and $t \in \mathbb{R}$. Here, $\left|U(t)_{u, v}\right|^{2}$ is interpreted as the probability of quantum state transfer between vertices $u$ and $v$ in $X$. In particular, we say that vertex $u$ is sedentary in $X$ if

$$
\operatorname{in} f_{t \in \mathbb{R}}\left|U(t)_{u, u}\right| \geq C
$$

for some constant $C>0$. Two vertices in $X$ are said to be twins if they share the same neighbours, and a twin set in $X$ is a maximal subset of the vertex set of $X$ whose elements are pairwise twins. Recently, it was shown that vertices in a twin set of size at least three are all sedentary [Quantum Inf. Process. 22:273, 2022]. In this talk, we investigate sedentariness of vertices in a twin set of size two.
(Received September 11, 2023)

## 1192-81-31777

Roy J Garcia*, Harvard University. Resource theory of quantum scrambling.
Quantum computers are devices which one can use to perform computations by exploiting the principles of quantum mechanics. One such principle is scrambling, which describes the spread of local information throughout a quantum system. We introduce a framework, known as a resource theory, to mathematically define and measure scrambling. We show its connection to several topics in quantum information. Among them, we show that scrambling results in training obstacles in an area of quantum computation known as quantum machine learning. We use our framework to explain recent experimental results, in which scientists at Google used scrambling to measure the source of computational speed-ups using their quantum processor. Our theory provides an approach to study quantum chaos and its applications to quantum computation. (Received September 12, 2023)

## 1192-81-31967

Tepper L. Gill*, Howard University. Isotopes in Physics and in Mathematics. Preliminary report
It is well known in physics that there are many Hamiltonians (or Lagrangians) for a given set of dynamical equations of motion. Santilli [1] observed that many of these Hamiltonians can be obtained from a fixed one via a change in the definition of the Lie algebra bracket, in both particle and field theory. He called these algebras Lie-isotopes, by analogy with a similar phenomenon in nuclear physics where the same atom can have a varying number of neutrons. In this talk, we introduce a few simple examples to show how natural this concept is (even in arithmetic). We then use it to provide an improvement in the big bang model of the universe (physics) and to add another degree of freedom to cryptographic schemes (mathematics). [1] R. M Santilli, Isonumbers and genonumbers of dimension 1, 2, 4, 8, their isoduals, and pseudoduals, and "hidden numbers" of dimension 3, 5, 6, 7, Algebras, Groups and Geometries, Vol. 10, 273-322, 1993.
(Received September 12, 2023)

## 1192-81-31996

Victor Galitski, University of Maryland, Abu Musa Patoary, University of Maryland, Laura Shou*, University of Maryland, Amit Vikram, University of Maryland. Quantum baker maps. Preliminary report. We study the spectral properties of the quantum baker's map and generalized quantum baker maps on the torus. We numerically explore anomalies in the spectral statistics, and investigate their relationship to the preservation or breaking of symmetries in the quantized maps.
(Received September 12, 2023)

## 1192-81-32594

Adrian I. Vajiac*, Chapman University. Hypertwined Quantum Field Theory. Preliminary report
Hypertwined analysis is a refinement of general hypercomplex theories of differential operators. In the assumption that the superspace has a hypertwined structure, I will discuss several (re)interpretations of notions and results in Quantum Field Theory (QFT), such as supersymmetry, path-integral quantization, cancellation of anomalies, et al. Particular QFTs of interest in this study are Topological QFTs such as supersymmetric Yang-Mills theory on a four-manifold, which are deeply related with Donaldson and Seiberg-Witten invariants. (Received September 12, 2023)

1192-81-32640
Adam Bene Watts*, Institute for Quantum Computing, University of Waterloo, Anirban Chowdhury, Institute for Quantum Computing, University of Waterloo, Aidan Epperly, UC Davis, Bill Helton, UC San Diego, Igor Klep, University of Ljubljana. Relaxations and Exact Solutions to Quantum Max Cut via the Algebraic Structure of Swap Operators.
The Quantum Max Cut (QMC) problem has emerged as a test-problem for designing approximation algorithms for local Hamiltonian problems. In this talk I'll describe an approach to solving this problem using the algebraic structure of QMC, in particular the relationship between the quantum max cut Hamiltonian and the representation theory of the symmetric group. The first major result I'll describe is an extension of non-commutative Sum of Squares (ncSoS) optimization techniques to give a new hierarchy of relaxations to Quantum Max Cut. This hierarchy is based on optimizations over polynomials in the qubit
swap operators. This is in contrast to the "standard" quantum Lasserre Hierarchy, which is based on polynomials expressed in terms of the Pauli matrices. The second major result covered will be a polynomial-time algorithm that exactly computes the maximum eigenvalue of the QMC Hamiltonian for certain graphs, including graphs that can be "decomposed" as a signed combination of cliques. A special case of the latter are complete bipartite graphs with uniform edge-weights, for which exact solutions are known from the work of Lieb and Mattis [LM62]. Our methods, which use representation theory of the symmetric group, can be seen as a generalization of the Lieb-Mattis result. This talk is based on arxiv:2307.15661
(Received September 12, 2023)

## 1192-81-32874

Alexis Drouot, University of Washington, Xiaowen Zhu*, University of Washington. Closure of bulk spectral gaps for topological insulator with general edges.
Topological insulators are materials that have unique physical properties due to their non-trivial topological invariants (the Chern number). One of the most notable consequences of a topological insulator is that they behave like insulators in the bulk and conductors at the edge. Or mathematically speaking, the bulk spectral gaps close and the protected edge states emerge, which is known as the bulk-edge correspondence. In this talk, I will start with the definition of chern number and discuss closure of bulk spectral gaps for topological insulators with general edges. The talk is based on a joint work with Alexis Drouot. (Received September 12, 2023)

## 1192-81-32902

Nicolai Reshetikhin*, YMSC, Tsinghua University. Poisson sigma model and quantum mechanics. n this talk I will focus on the path integral quantization of the Poisson sigma model with boundary conditions given by Lagrangian fibrations.
(Received September 12, 2023)

1192-81-33367
Elsa Marie Church*, Macalester College. Matrix Product States, Markov Chains, and Local Probabilities.
We investigate a crucial connection between matrix product states (MPS), tensor networks, and the measurement of a local walk, a finite sequence of states within an infinitely transitioning Markov Chain. Markov Chains are stochastic models tracking the probability of some object transitioning from one state to the next. Matrix product states (MPS) are a fundamental tool used in quantum theory, defining quantum states and their corresponding probabilistic nature. We unify the concepts of the local walk through a Markov chain with MPS by encoding the transition matrix of the Markov Chain into the MPS matrices. This exploration naturally leads to the emergence of tensor networks. Subsequently, we introduce the transfer matrix, seen in many MPS applications, and its relevance in this context. Finally, the results culminate in the validation of a MPS equation, offering a reliable means to determine the probability of observing the targeted local walk within a perpetually transitioning Markov Chain.
(Received September 14, 2023)

1192-81-33733
Peter Burton, Iowa State University, Phillip Diamond*, Kenyon College, Rachel Hill*, Iowa State University, Sam Sunday Trombone*, Hamilton College, Philip Yang, Iowa State University. A Statistical Strategy for The CHSH Quantum Game. Nonlocal games are a mathematical construction which highlight the utility of quantum entanglement. In these games, a referee sends questions to players Alice and Bob, who use entanglement to correctly answer the questions more often than classically possible. Standardly, the players' quantum machinery is represented by a collection of projection operators on the tensor product of their local spaces. By applying these operators to an entangled state, they generate a table of probabilities for various answer options. The nonlocal game itself is formalized as a linear functional of these probabilities which the players attempt to maximize. Our approach is to give the entangled state as a square grid of values. Alice's projections are implemented as component-wise averages on rows while Bob's average columns. This has the advantage that even large grids can be visualized in a two-dimensional image, while networks of projections in high-dimensional tensor product spaces are far more difficult to picture. Using this perspective we have reproduced optimal winning strategies for the simplest nontrivial game, the CHSH game. We have also applied our methods to generate a sequence of strategies converging to the optimal win probability for certain more complicated three-question, two-answer games.
(Received September 24, 2023)

1192-81-33844
James Cook, Liberty University, Ernesto Jose Ugona Santana*, Liberty University. Examples of Supermanifolds with Nontrivial Toplogies in the Odd Coordinate Directions.
In the present research project, we study supermanifolds, which are based on supernumbers generated with infinitely many Grassman generators. In particular, we construct examples of supermanifolds with non-trivial topology in the odd coordinate directions. These examples illustrate that it is possible to have non-trivial topology outside the body of the supermanifold. Finally, we show how to construct a super-version of the Heisenberg 3-manifold. This presentation will consist in summarizing the results of an undergraduate, summer-long research project alongside Dr. James Cook, which consisted of investigating the nature of supernumbers, exploring analysis on vector spaces over the supernumbers, and ultimately understanding supermanifolds with the intention of developing the aforementioned examples.
(Received September 26, 2023)

Chi-Kwong Li, College of William and Mary, Kevin Yipu Wu*, The college of William and Mary, Zherui Zhang*, The college of William and Mary. Efficient Circuit-Based Quantum State Tomography via Sparse Entry Optimization. Preliminary report. Quantum state tomography (QST) is the procedure for reconstructing unknown quantum states from a series of measurements of different observables. We propose an efficient circuit-based QST scheme demonstrating that for an n-qubit state $|\psi\rangle$, only
$2 n+1$ measurements are needed for determination. A unitary operator consisting of a combination of single qubit gates and up to $n-1$ CNOT gates is applied before each measurement. In addition, we establish that the measurement complexity of state determination is influenced by the sparsity of nonzero entries in $|\psi\rangle$, allowing us to accordingly reduce the number of measurements. Our scheme incorporates a method to minimize the count of CNOT gates used by leveraging insights from the distribution of $|\psi\rangle$ 's nonzero entries in Hamming space. We present the scheme's details, outcomes, and conclude with numerical validation using IBM quantum computers.
(Received September 27, 2023)

## 82 Statistical mechanics, structure of matter

## 1192-82-28127

Jimmy He*, MIT. Boundary current fluctuations for the half space ASEP and six vertex model.
The half space asymmetric simple exclusion process (ASEP) is an interacting particle system on the half line, with particles allowed to enter/exit at the boundary. I will discuss recent work on understanding fluctuations for the number of particles in the half space ASEP started with no particles, which exhibits the Baik-Rains phase transition between GSE, GOE, and Gaussian fluctuations as the boundary rates vary. As part of the proof, we must work at the level of the six vertex model, and find new distributional identities relating this lattice model to two other models, the half space Hall-Littlewood process, and the free boundary Schur process, which allows exact formulas to be computed.
(Received August 26, 2023)

1192-82-28189
Surl-Hee Ahn*, University of California, Davis. Bridging timescales between simulations and catalytic and biological processes. Preliminary report.
To investigate the structures and dynamics of catalytic and biological processes at an atomic level, computer simulations can be useful since they can serve as a computational microscope. However, electronic structure calculations that are often used to investigate catalytic processes are computationally expensive and cannot reach long timescales as a result. Hence, we have recently developed machine-learned potentials to model gold nanoclusters in zeolite pores or catalysts using molecular dynamics (MD) simulations with density functional theory (DFT) accuracy. Although MD simulations can reach longer timescales compared to DFT, MD simulations also suffer from the timescale barrier between simulations (run using femtosecond time steps) and biological processes (usually milliseconds or longer) by being limited to the slowest motions in the system (e.g., the vibration of bonds). Thus, we have been developing enhanced sampling methods for MD simulations to bridge the timescales and make MD simulations closer to being a true computational microscope that uncovers the fundamental mechanisms of biological processes. In this talk, I will present and discuss both efforts. (Received August 27, 2023)

## 1192-82-28639

Richard W Kenyon*, Yale University. Higher-rank dimers.
Given a planar bipartite graph of vector spaces and linear maps, we associate an operator, the Kasteleyn matrix, and show that its determinant counts "traces of multiwebs", where multiwebs are certain subgraphs with degree restrictions, and the trace of a multiweb has a tensorial definition. We characterize connections with positive traces, and define and associated probability measure on multiwebs. By careful choice of connection we can encode the "free-fermionic" subvarieties for vertex models such as the six-vertex model and 20-vertex model. This is joint work with Nicholas Ovenhouse (Yale).
(Received September 01, 2023)

1192-82-28676
Bruno Nachtergaele*, UC Davis. Quantum Lattice Systems in the GNS representation
Quantum many-body systems describe a wide variety of fascinating phenomena in the physical world. The last decade has seen impressive progress in our understanding of these systems. Notable topics are the classification of symmetry protected and symmetry enhanced topological phases, the fractional quantum Hall effect, and the stability of spectral gaps for many-body quantum systems. As a rule, these topics call for the analysis of arbitrarily large systems or infinite systems. While many works restrict themselves to obtain results for finite systems that can be shown to be independent of system size, studies of infinite systems are increasingly common and are often preferred. Analysis of infinite many-body systems leads one to transition from the standard algebraic setting to the Hilbert space description provided by the GNS representation. In this talk I will aim to dispel the myth that the GNS representation is arcane or unwieldy and show that it offers in some aspects an attractive simplification.
(Received September 01, 2023)

1192-82-28850
Istvan Prause*, University of Eastern Finland. Random Young tableaux and harmonic envelopes. Preliminary report. Young tableaux are combinatorial objects obtained by filling in the cells of a given Young diagram in a monotonic manner. A large uniformly random Young tableau develops a deterministic limiting shape after a suitable rescaling. These shapes when considered surfaces in three dimensions resemble crystals with formation of facets and intriguing "arctic boundaries". I'll introduce an intrinsic conformal structure on these limit shapes and describe how harmonic functions naturally arise in their study. I'll use a "harmonic envelope" method to solve the Young tableaux limit shape problem on an arbitrary diagram. Specific examples and connections to free probability will be mentioned.
(Received September 04, 2023)
temperature.
The spherical Sherrington-Kirkpatrick (SSK) model and its bipartite analog both exhibit the phenomenon that their free energy fluctuations are asymptotically Gaussian at high temperature but asymptotically Tracy-Widom at low temperature. This was proved in two papers by Baik and Lee, for all non-critical temperatures. The case of critical temperature was recently computed for the SSK model in two separate papers, one by Landon and the other by Johnstone, Klochkov, Onatski, Pavlyshyn. In the current work, we derive the critical temperature result for the bipartite SSK model. In particular, we find that the free energy fluctuations exhibit a transition when the temperature is in a window of size $n^{-1 / 3} \sqrt{\log n}$ around the critical temperature, the same window for the SSK model. Within this transitional window, the asymptotic fluctuations of the free energy are the sum of independent Gaussian and Tracy-Widom random variables. (Received September 07, 2023)

1192-82-29730
Zhongyang Li*, University of Connecticut. Perfect matchings on Rail-Yard graphs: limit shapes and height fluctuations. I will discuss random perfect matchings on a large class of graphs called rail yard graphs, such that the perfect matchings form Schur processes. I will then talk about the limit shape (Law of Large Numbers) and convergence of height fluctuations to Gaussian Free Field (Central Limit Theorem) for random perfect matchings on these graphs in the scaling limit. The techniques used include analyzing a large class of Macdonald processes which contain dual partitions as well. Part of this talk is based on joint work with Mirjana Vuletic.
(Received September 07, 2023)

## 1192-82-30183

Michael P Brenner*, Harvard University. Scientific Applications of Automatic Differentiation
There is much excitement (some of it legitimate) about applications of machine learning to the sciences. Here I'm going to argue that a primary opportunity is not machine learning per se, but instead that the tools underlying the ML revolution yield significant opportunities for scientific discovery. Primary among these tools is automatic differentiation and the scalability of codes. Neural network architectures are similar to time rollouts in dynamical systems, and therefore the technical advances underlying the ML have the potential to directly translate into the ability to solve important optimization problems in the sciences that have heretofore not been tackled. I will describe a number of different directions we have been undertaking using automatic differentiation and large scale optimization to solve science problems, including developing new algorithms for solving partial differential equations, the design of energy landscapes and kinetic pathways for self assembly, the design of fluids with designer rheologies, "optimal porous media", and learning the division rules for models of tissue development. (Received September 09, 2023)

## 1192-82-32256

Christoph Fischbacher*, Baylor University. Droplet States and Scaling of Entanglement Entropy in the Infinite XXZ Chain. Preliminary report.
In this preliminary report, I will talk about recent progress made when considering the entanglement entropy of droplet states in the infinite XXZ chain. Unlike in previous work, droplet states in the infinite model are part of the absolutely continuous subspace. I will give a full description of these states and point out suitable candidates whose entanglement entropy exhibits a so-called logrithmically enhanced area.
(Received September 12, 2023)

1192-82-32729
Yue Zhao*, Department of Computational Mathematics, Science, and Engineering, Michigan State University. The Random Batch Ewald Method for Molecular Dynamics Simulations.
Molecular dynamics simulation is one of the most popular numerical methods to understand dynamical and equilibrium properties of many-body particle systems in many areas such as chemical physics, soft materials and biophysics. The longrange interactions such as electrostatic Coulomb interactions pose a major challenge to particle simulations. A lot of effort in literature has been devoted to computing the long-range interactions efficiently, and widely studied methods include lattice summation methods such as particle mesh Ewald (PME) and particle-particle particle mesh Ewald (PPPM), and multipole type methods such as tree-code and fast multipole methods (FMM). These methods can reduce the operations per step to O(N logN) or $\mathrm{O}(\mathrm{N})$, and have gained big success in practice, but many problems remain to be solved as the prefactor in the linear scaling can be large, or the scalability for parallel computing is not high. We develop a random batch Ewald (RBE) method, which achieves an $\mathrm{O}(\mathrm{N})$ complexity in each timestep, based on the Ewald splitting for the Coulomb kernel with a random "minibatch" type technique introduced to speed up the summation of the Fourier series for the long-range part of the splitting. Simulations are presented to illustrate the accuracy, efficiency and parallel efficiency of the RBE method (Received September 12, 2023)

## 83 Relativity and gravitational theory

## 1192-83-30605

Mason Huffman*, n/a. Data Analysis of Gravitational Wave Signals GW150914 and GW1 70817 Using Python. Preliminary report
Gravitational waves are ripples in spacetime caused by violent and energetic events such as the merging of black holes, neutron stars, or normal stars. Gravitational waves have allowed scientists to observe these events which were once invisible to electromagnetic wave detectors. The project focuses on the analysis of GW150914 and GW170817 signals that were detected by the Advanced Laser Interferometer Gravitational Wave Observatory, in 2014 and 2017 respectively. The objective is to determine the source of a GW150914 and GW170817 gravitational waves given only their signal data. To do this, the period/frequency of the gravitational wave must be measured as it changes over time. The frequency of the gravitational waves was estimated using Python program. The frequency was used to calculate the chirp mass, component masses, orbital separation, and tangential velocity. The analysis of the data from both signals led to conclude that the GW150914 gravitational
wave signal source was from a binary black hole system and the GW170817 gravitational wave signal source was from a binary neutron star system.
(Received September 13, 2023)

## 1192-83-32532

Arthur E Fischer*, Department of Mathematics, University of California, Santa Cruz, Vincent Moncrief, Departments of Mathematics and Physics, Yale University. Hamiltonian Reduction of Einstein's Equations.
We review the mathematical machinery for symplectically reducing the Einstein field equations to an unconstrained gaugefixed Hamiltonian system derived from a reduced Hamiltonian that is topologically significant for spatially closed universes. We then describe a surprising dynamical mechanism at work within the Einstein flow that strongly suggests that many closed 3-manifolds that do not even admit a locally homogeneous and isotropic metric will nevertheless evolve in such a way as to be asymptotically compatible with the observed approximate spatial homogeniety and isotropy of the universe. A key step in the argument is the proof that the reduced Hamiltonian is monotonically decreasing in the temporal direction of cosmological expansion. Moreover, the reduced Hamiltonian is a weak Lyapunov function for the dynamics whose infimum is related to the $\sigma$-constant of the closed spatial hypersurface, which in turn has a fundamental topological significance in light of the recent completion of the Perelman-Thurston geometrization program for closed 3-manifold topology. Our main results suggest that the universe's approximate spatial homogeniety and isotropy is a dramatic natural consequence of a combination of the topological structure of closed 3-manifolds, which is independent of the physics, and the Einsteinian dynamics of the universe as generated by our reduced Hamiltonian.
(Received September 12, 2023)

1192-83-33012
Arthur E Fischer*, Department of Mathematics, University of California, Santa Cruz. Introduction of the Microverse $\mathcal{U}_{\text {micro }}\left(\mathbf{S}^{3}\right)$ and a Mathematical Model for the Big Bang, Inflation, and our Spatially Flat Universe.
We construct the microverse $\mathcal{U}\left(\mathbf{S}^{3}\right)$, a micro-cosmological model that existed before the big bang, that is the progenitor of our current observable universe, and that provides a viable mathematical model for the big bang, inflation, and the spatially flat universe that we have today. The microverse is an all-radiation static metastable Friedmann micro-cosmology, about the size of a proton, with a large positive cosmological constant $\Lambda_{\text {micro }}$, the geometry of a round 3-sphere, and whose photons packed together have the same total energy as our observable universe. The microverse is unstable with respect to radiation converting to matter inasmuch as because of radiation pressure, radiation density "pound for pound" is twice as effective in decelerating the universe as an equivalent amount of matter density. Thus as matter condenses out of radiation, along the lines of colliding photons to give electrons and positrons, and then proceeding to the creation of quarks, gluons, and nucleons, at a critical point the microverse destabilizes into an exponential expansion which initiates the big bang. Briefly following this initial exponential expansion, there is a topology change modelled by the 3 -sphere $\mathbf{S}^{3}$ rupturing at a point $P$ to give rise to a solid 3-ball $\mathbf{B}^{3}$, whereupon round neighborhoods of the rupture point $P$ get exponentially mapped to round neighborhoods (or collars) of the boundary $\partial \mathbf{B}^{3}=\mathbf{S}^{2}$ of $\mathbf{B}^{3}$, thereby providing a mathematical model of inflation through a change of topology of the microverse from $\mathbf{S}^{3}$ to $\mathbf{B}^{3}$. This inflation map $\Phi$ can be made explict by the diffeomorphism

$$
\Phi: \mathbf{S}^{3}-P \longrightarrow \mathbf{B}^{3}, x \longmapsto\left(E x p_{P^{\prime}} \mid \mathbf{B}^{3}\right)^{-1}(x)
$$

where $P^{\prime}=-P$ is the antipodal point of $P$ and $\operatorname{Exp}_{P^{\prime}}: T_{P^{\prime}} \mathbf{S}^{3} \approx \mathbf{R}^{3} \rightarrow \mathbf{S}^{3}$ is the Riemannian exponential map. Lastly, inflation ceases when the punctured round sphere $\mathbf{S}^{3}-\{P\}$ flattens out to the flat Euclidean ball $\mathbf{B}^{3}$ and this change of topology explanation of inflation thus also gives rise to an explanation of the spatially flat universe that we observe today. (Received September 13, 2023)

## 86 Geophysics

## 1192-86-32968

Daniela Beckelhymer*, University of Minnesota, Twin Cities. Using An Energy Balance Model to Predict When the Arctic Ocean Will Have Its First Ice-Free Summer. Preliminary report.
Arctic sea ice is an important indicator of climate change. The rate at which its extent decreases or increases is crucial to monitor closely because Arctic sea ice affects the entire planet's physical landscape, through weather patterns, local and global ecosystems, and ocean circulations. Recently, the annual average Arctic sea ice area reached its lowest level since at least 1850 and late summer Arctic sea ice area was smaller than at any time in at least the past 1000 years. The most current report by the Intergovernmental Panel on Climate Change (IPCC) noted the loss of sea ice is mostly driven by the increase of atmospheric greenhouse gasses, unequivocally caused by human activity. Similar studies resulted in strong correlations between the loss in Arctic sea ice and the increase in global mean temperature, $\mathrm{CO}_{2}$ concentration in the atmosphere, and cumulative anthropogenic $\mathrm{CO}_{2}$ emissions. In this presentation, I will discuss the development of a model of the Arctic sea ice using an Energy Balance Model (EBM) and expanding on the work of Dr. Somyi Baek, with the goal of refining the prediction for the first ice-free Arctic summer. The most current complex models used by the IPCC predict the Arctic ocean will have its first ice free summer by September 2050. The goal is to discuss how a prominent minimal-complexity model can produce similar results while maintaining its analyzablility.
(Received September 13, 2023)

François Blanchette, University of California, Merced, Conor Edward Olive*, University of California, Merced, Nathan Willis, University of California, Merced. Augmented Box Model for Colliding Turbidity Currents. A modified box model for colliding turbidity currents-motivated by sediment plumes generated during deep-sea mining-is developed to investigate collision dynamics that are not captured in the standard box model. The SINDy algorithm is used to determine coefficients in a system of ordinary differential equations that describe the evolution of the turbidity currents post-
collision. Training data for this equation discovery method is generated from numerical simulations of the shallow-water equations that are post-processed to track the fronts of the current and the horizontally-averaged values of relevant quantities. (Received September 25, 2023)

## 1192-86-33799

Adam Dorsky, University of Utah, Kenneth M Golden, University of Utah, Daniel Hallman, University of Utah, Nash Ward*, University of Utah. Fractal Seas; Measuring sea ice geometry from millimeters to kilometers.
Fractal structures appear throughout nature. They are objects that display self-similarity and possess a non-integer dimension. For over 40 years researchers have suspected that brine inclusions and channels possess fractal characteristics.
Understanding the geometries of these brine structures is critical as the fluid which flows through them mediates key processes such as melt pond evolution and algal growth. Using X-ray tomography data, we have conducted the first comprehensive quantitative study of the fractal dimension of brine in sea ice and its strong dependence on temperature and porosity. Furthermore, we took our analysis to a much larger scale. Utilizing aerial images, we explored the fractal geometry of the ice pack, which determines surface wave dynamics, and rigorously investigated widely used relationships in geophysics literature.
(Received September 26, 2023)

## 90 Economics, operations research, programming, games

## 1192-90-26613

Rachael May Alfant*, Rice University. Periodicity of Mixed Integer Programming Gap Functions.
A critical measure of the quality of a mixed integer programming (MIP) model with fixed data is the difference, or gap, between the optimal objective value of the linear programming relaxation and that of the corresponding MIP. In many contexts, only an approximation of the right-hand side may be available, or there may be multiple right-hand sides of interest. Yet, there is currently no consensus on appropriate measures for MIP model quality over a range of right-hand sides. We provide conditions under which absolute MIP gap functions are periodic, as well as formulations of optimization problems that represent the expectation and extrema of absolute and relative MIP gap functions over finite discrete sets. Thus, we provide a framework by which to evaluate a MIP model's quality over multiple right-hand sides.
(Received August 01, 2023)

## 1192-90-26997

Todd Munson*, Argonne National Laboratory. Complementarity Problems in the Power Grid. Preliminary report
During this talk, I will provide a brief overview of the session and then present the complementarity formulation of some problems in the power grid, discuss solution methods for solving these problems, and provide some numerical results. The solution methods will include PATH, a Josephy-Newton method for solving complementarity problems, that solves a linear complementarity problem at each step to determine the direction and a projected line search for stabilization. Techniques used to improve the method, including preprocessing techniques and crashing methods, will be discussed.
(Received August 08, 2023)

1192-90-27301
Sven Leyffer*, Argonne National Laboratory. Nonlinear and Combinatorial Optimization. Preliminary report.
Many applications of interest to the US Department of Energy can be modeled as optimization problems that involve both nonlinear physical laws and combinatorial constraints. We review several such applications including optimal topology design, the analysis of gamma-ray, the placement of blocking devices to mitigate the effect of solar storms on the power grid, the optimal design of experiments, and disaster recovery planning. These problems have in common that they cannot be handled with existing commercial solvers. We discuss the challenges of each application, and review solution techniques based on rounding and improvement heuristics.
(Received August 13, 2023)

## 1192-90-27583

Zichao Wendy Di*, Argonne National Lab. Multilevel Optimization in Image Reconstruction. Preliminary report.
The challenge of solving image reconstruction problems is complicated by the expansion of both parameter space dimensionality and data volume, particularly when dealing with intricate phenomena. In this talk, I will introduce a multilevel optimization framework that addresses this issue. This method involves tackling a spectrum of scales within both the data and parameter spaces during the reconstruction process. Leveraging the inherent hierarchical organization not only within discretized image/sample grids but also across diverse data and experiment fidelities, these integrated domain-, data-, and approximation-based hierarchies can potentially reduce the computational cost and scale efficiently to the ever-increasing data volume without sacrificing the accuracy of the discovery.
(Received August 16, 2023)

Kibaek Kim*, Argonne National Laboratory, Youngdae Kim, ExxonMobil Technology and Engineering Company, Weiqi Zhang, University of Wisconsin-Madison. On Solving Unit Commitment with Alternating Current Optimal Power Flow on GPU. Unit commitment (UC) is one of the important problems solved to schedule generation units in U.S. electricity markets. Most UC formulations are based on a direct current (DC) approximation of the transmission network constraints, enabling to use off-the-shelf mixed-integer linear programming solvers such as CPLEX and GUROBI. However, the DC approximation fails to represent the transmission losses, reactive power, and voltage constraints. To address these, the UC problem can be formulated by directly employing the alternating current optimal power flow (ACOPF) constraints. We call such problem UCACOPF. The resulting formulation is of a mixed-integer nonlinear programming problem and thus challenging to solve in
practice. In this paper, we develop a new algorithm that quickly finds a feasible solution of the UC-ACOPF problem by employing the alternating direction method of multiplier (ADMM). Specifically, our algorithm decomposes the UC-ACOPF problem into the UC and the ACOPF subproblems first. The ACOPF subproblem is solved by the component-based decomposition based on the Lagrangian relaxation. The UC subproblem schedules generating units subject to operating constraints (e.g., minimum up/down limits). As a result, our ADMM algorithm solves a large number of independent small subproblems, each of which represents an individual grid component (e.g., generator, bus, and transmission line), as well as the UC subproblem. This enables running our algorithm on parallel computing architecture. Furthermore, we develop dynamic programming (DP) to solve the UC subproblem for each unit, which is key to enabling the use of GPU parallel computing as integer programming solver is not needed. We have implemented the algorithm in Julia, as part of the existing package ExaAdmm.jl, which can efficiently run on GPU as well as on CPU. We will present extensive numerical results of the algorithm performance on GPU, as well as CPU.
(Received September 05, 2023)

1192-90-29599
Yun Lu*, Kutztown University, Emre Shively-Ertas, Kutztown University, Myung Soon Song, Kutztown University, Francis Vasko, Kutztown University. Generating Bounded Solutions for Multiple Knapsack Assignment Problems.
The multiple knapsack assignment problem (MKAP) is an interesting generalization of the multiple knapsack problem which has logistical applications in transportation and shipping. In this talk, we will present a simple methodology that iteratively solves MKAPs using commercial software to generate solutions that are within a tight tolerance of the optimums, using a reasonable amount of computer time. Furthermore, for very large MKAPs, the chosen commercial software's performance can be significantly improved when an initial feasible solution is provided.
(Received September 07, 2023)

## 1192-90-29819

Gurcharan Singh Buttar*, Department of Mathematics, Chandigarh University, Mohali. The Applications of Trigonometric Fuzzy Entropic Models to the Maximum Entropy Principle.
There are two kinds of uncertainties: Although probabilistic and fuzzy uncertainties are opposites, they both contribute significantly to the reduction of uncertainties and, as a result, to the improvement of the system under study. In addition, it has been realised that the maximum entropy principle plays a crucial role in the investigation of optimization issues related to theoretical information measures. For the purpose of applying two brand-new trigonometric entropic models to discrete fuzzy distributions for the knowledge of the maximum entropy principle within a set of fuzzy constraints, we created the models. (Received September 08, 2023)

## 1192-90-31038

Annika King*, Brigham Young University, Dallas Smith, Utah Valley University, Ben Webb, Brigham Young University. Integrated Minimal Specialization: Coevolution of Network Structure and Dynamics.
Networks are used to study systems in a variety of settings, including in the biological, social, and technological sciences. These systems are inherently dynamic in two ways: the dynamics on the network and the dynamics of the network, i.e. the changing state of the network's components and the evolution of the network's structure, respectively. Typically, these dynamics are studied separately. Here, we study the interplay of the two by using the dynamics on the network to determine where to evolve the structure, which in turn alters the dynamics on the network. We model the dynamics on the network using Jackson network dynamics and the dynamics of the network using the newly proposed model of network specialization called Minimal Specialization, which mimics the specialization most likely to appear in Jackson-type networks. We then use the dynamics on the Jackson network to determine where to specialize, calling this Integrated Minimal Specialization and show that this model exhibits real-world properties, such as right-skewed degree distributions, sparsity, small-world property, and non-trivial equitable partitions. (Received November 13, 2023)

## 1192-90-31398

Matthias Koeppe, UC Davis, Moises Reyes Rivas*, Andrews University, Luze Xu, UC Davis. Determining sharp proximity bounds for low row rank and Delta-modularity. Preliminary report.
A question of much interest in integer programming is that of proximity bounding: how big can the distance be between a vertex optimal solution of the corresponding linear program relaxation and the closest integer optimal solution in the integer linear program? The proximity bound has many applications, for example, it can be used to bound the integrality gap and estimate nearby integer solutions in a dynamic programming framework. The maximum absolute subdeterminant of the coefficient matrix is known to be an important parameter to measure the proximity bound. Here, we consider the integer program in the standard form $\max \left\{c^{\top} x: A x=b, x \geq 0\right\}$ with a bounded feasible region, where the coefficient matrix $A$ is of full row rank $m$ and has a maximum order- $m$ absolute subdeterminant of $\Delta$ (where we call the rank- $m$ matrix $A \Delta$-modular). Using Wolsey's $b$-hull results and the polyhedral fan theory, we propose a method to determine the proximity bound for a fixed $A$ and parametric $b, c$. Furthermore, applying a modified algorithm by Averkov and Schymura for classifying such rank-m $\Delta$ modular matrices up to unimodular transformation, we calculate sharp proximity bounds for small $m$ and $\Delta$ using SageMath. (Received September 11, 2023)

1192-90-31568
Nicholas Grabill*, University of Michigan, Stephanie Wang*, University of Rochester. Enhanced Reliability Predictions: An AI-based Framework for Performing Failure Modes, Effects, and Criticality Analysis in Industrial Environments. Preliminary report.
Reliability engineering grapples with the imperative task of predicting and comprehending product failures across an array of sectors. Although traditional approaches like Failure Mode Effects and Analysis (FMEA) and Failure Mode, Effects, and Criticality Analysis (FMECA) have substantially bolstered product safety and quality, their inherent manual intensity and reliance on expert insights introduce limitations. As today's systems surge in complexity, a more holistic approach becomes
paramount. In response, we introduce an AI-driven risk assessment tool that guides the user to a host of failure modes and their effects for each component contained in a bigger system. Through a user-friendly graphical interface and a robust statistical modeling backend, the AI-driven tool streamlines the risk assessment process by prompting users to input a system's name and subsequently generating an extensive array of failure modes and associated effects for each constituent component, including Weibull, Rayleigh, and Bathtub distribution curves. By automating this aspect of FMEA/FMECA, the AIbased solution seeks to not only enhance reliability analyses but also optimize development timelines, improve resource allocation, and provide valuable educational avenues for junior across sectors, including chemical, automotive, aerospace, and beyond.
(Received September 11, 2023)

## 1192-90-31825

Clara R Chaplin*, Bucknell University, Stanley Gai, Bucknell University, Tsugunobu Miyake*, Bucknell University. Optimizing Final Exam Schedules at Bucknell University.
Final exam scheduling at Bucknell University is a complex problem: scheduling rules make a one-to-one mapping between lecture times and final exam timeslots impossible. It is challenging to develop a final exam schedule that avoids unpleasant student experiences, including students having overlapping exams or too many exams within the same day. Historically, final exams at Bucknell have been scheduled by the registrar's office. Even with their substantial scheduling experience, this process takes months of manual work, and it is difficult to detect and minimize all of the inconveniences for students. To assist the registrar's office with finding better schedules, we developed a mixed-integer programming model that optimizes final exam schedules by reducing the number of students and instructors with inconveniences. We employed a novel, multiphase strategy that emulates the registrar's approach to scheduling, first scheduling exams for courses with large enrollments. This breaks some of the innate symmetry in an integer programming model, allowing us to efficiently and optimally schedule the remaining exams. The student inconveniences in our generated schedules were reduced by a factor of two or more, as compared to the final exam schedules used in the past.
(Received September 12, 2023)

## 1192-90-32346

Victoria Denise Robinson*, EDGE 2019 Cohort. Long Short-Term Memory (LSTM) Model for Predicting Groundwater Level in the State of $A L$.
A predictive tool to forecast the groundwater response time to precipitation events and other major climatic parameters can be beneficial to evaluate the short- and long-term impacts of natural and anthropogenic stresses on groundwater resources. It will also be beneficial for stakeholders and decision makers to effectively estimate the key factors controlling groundwater availability/quality in the area. We investigated the applicability and ability of a Long Short- Term Memory (LSTM) neural network model for predicting fluctuations of groundwater levels by using widely available and easy to measure input features such as maximum/minimum air temperature, daily precipitation, and groundwater level data. Continuous time-series data collected over eight years, from January 2012 to May 2020, from 8 monitoring wells located across the state of Alabama have been used in the model. The wells are located outside of the known area of influence of any other groundwater sources and are not affected by usage stress. The LSTM model parameters were selected and initialized according to the insights we gained from a preliminary statistical analysis prior to the application of LSTM. A continuous wavelet transform (CWT) analysis was performed to estimate the lag in response time of groundwater levels to precipitation events and was used further to select the dropout for the model. Based on the CWT analysis outcome the short-term response time of groundwater levels can be expected within 15-30 days after a precipitation event, depending on the well locations. This outcome was used to construct the 30-day sequences for short term memory training. The performance of our prediction was compared with different prediction dates and prediction training datasets. Our results suggest that the proposed LSTM network can be an efficient tool for forecasting the groundwater level fluctuations in the corresponding monitoring wells. The predicted data offered a Nash Sutcliffe efficiency (NSE) that ranged from 0.85 to 0.96 . Our proposed model is able to provide predictions within a 95 percent confidence interval of the actual measured groundwater levels.
(Received September 12, 2023)

1192-90-32611
Annabelle Piotrowski*, University of Wisconsin - Eau Claire. Math Department Scheduling Using Linear Programming. Preliminary report.
Scheduling classes is a challenging and time-consuming task. The mathematical technique of linear programming has the potential to simplify this challenge, by building a model of linear constraints to find the most optimal solution that satisfies all the constraints. In this project, we are implementing a linear programming model using the Cplex library in Python. The objective function represents instructor satisfaction with different courses and the constraints represent limitations such as the fact that one instructor cannot teach two courses at the same time. These constraints allow many ways to build a schedule. The goal of our program is to identify the most optimal solution, that maximizes the professor's satisfaction and class availability. Our model can successfully use binary variables to assign professors to classes at specific times during the week on a small scale. We will present a system for encoding the preferences about class times and preps for a professor's weekly schedule, and the constraints about maximum and minimum numbers of credits and classes, as well as the results of applying our code to scheduling math courses at UW-Eau Claire. (Received September 12, 2023)

1192-90-32870
D.W. Hall*, Lee University, Jason Schmurr, Lee University. Vacation for All: Optimizing Scheduling Coverage for Coptic Orthodox Priests in the Southern Diocese of the United States. Preliminary report.
The Coptic Orthodox Diocese of the Southern United States has considerably grown since its establishment in 1993. Congregational growth has tended to outpace clergy growth, and it is becoming increasingly difficult to provide adequate staffing for weekly liturgies. These growing pains are especially evident in the instances of priests taking paid time off, with many clergy being unable to take accrued yearly time off. A discussion of current scheduling procedures and literature review precedes the proposed assumptions and criteria sought to be optimized. The multifaceted mathematical modeling approach considers a data- analytic and computer-simulated approach before suggesting best practices for scheduling.
(Received September 12, 2023)

## 91 Game theory, economics, social and behavioral sciences

## 1192-91-26227

Grace A Brophy, Hamilton College, Audrey Ruth Rips-Goodwin*, The University of Kansas, Lucy A Wilson, Bryn Mawr College. Using agent-based modeling to understand the impact of community interactions on voter apathy and election outcome. Preliminary report.
Voter turnout in U.S. elections tends to fluctuate, with multiple factors influencing a prospective voter's participation. In this investigation, we focus on how a voter might weigh their inclination to vote based on their individual demographics and their perception of others' political views within their community. We present an agent-based model (ABM) to explore how voter turnout and election results change as we vary the model settings. Through sensitivity analysis, we examine which of our input parameters-including such factors as party breakdown and the degree to which voters tend to communicate in echo chambers-have the greatest impact on voter turnout and the extent to which election results represent the electorate. We find that turnout rates depend heavily on party affiliation heterogeneity, as well as how strongly a voter is influenced to vote or abstain based on their political interactions within their community. Additionally, we see that it is possible for an unexpected winner to emerge due to an increase in mobilization among minority party members.
(Received July 21, 2023)

1192-91-26245
Grace A Brophy*, Hamilton College, Audrey Ruth Rips-Goodwin*, The University of Kansas, Lucy A Wilson*, Bryn Mawr College. Using agent-based modeling to understand the impact of community interactions on voter apathy. Preliminary report. In recent decades, voter turnout in U.S. elections has fluctuated. Here we present an agent-based model (ABM) to study several factors contributing to voter apathy; specifically, we ask how voters weigh their individual demographic characteristics against their community's political opinions to inform their likelihood to participate in an election. We examine how varying initial settings-such as party breakdown and the degree to which voters tend to communicate in echo chambers-impacts election results and voter turnout. Using sensitivity analysis, we deduce which of our input parameters have the greatest effect on turnout, as well as the extent to which the electorate is represented in the election results. Our model shows that turnout rates are tied to the level of political similarity in an agent's surroundings, as well as the degree to which individuals are swayed to abstain or vote based on their interactions within the community. Furthermore, we find that the minority party's level of mobilization can skew the breakdown of election results, allowing for an unexpected winner to emerge. (Received July 21, 2023)

1192-91-26297
Emily Brooke Blevins, Morehead State University, Devyn Morgan Fleming*, Macalester College, Carl Hammarsten, Desales University, Rowan David Hess, Cornell University, Rahul Krishna Thomas, Stanford University. Exploiting Planar Preference Orders to Manipulate Elections.
Arrow's theorem famously claims that no "fair" voting system exists. We explore how a new candidate can be added to exploit various voting systems and manipulate the outcome of an election. We focus on the board election setting, where all voters are also candidates, and we use a planar representation of candidates to model their preferences. Specifically, each candidate is assigned a distinct point in the plane and ranks the others in order of proximity measured by Euclidean distance. With these assumptions, we can determine how a new candidate may manipulate a given voting system by considering their power over the outcome of a given election. In particular, we investigate how the new candidate may place themselves in the plane in order to win, swing, or otherwise change the election's results, depending on their motive. We show that in general, the new candidate can always place themselves to do at least as well as any chosen pre-existing candidate. Furthermore, we develop stronger results particular to each of the specific voting systems we explore.
(Received July 24, 2023)

1192-91-26298
Emily Brooke Blevins*, Morehead State University, Devyn Morgan Fleming*, Macalester College, Carl Hammarsten, Desales University, Rowan David Hess*, Cornell University, Rahul Krishna Thomas*, Stanford University. Manipulating Elections Based on Planar Preference Orders.
Arrow's theorem famously claims that no "fair" voting system exists. We explore how a new candidate can be added to exploit various voting systems and manipulate the outcome of an election. We focus on the board election setting, where all voters are also candidates, and we use a planar representation of candidates to model their preferences. Specifically, each candidate is assigned a distinct point in the plane and ranks the others in order of proximity measured by Euclidean distance. With these assumptions, we can determine how a new candidate may manipulate a given voting system by considering their power over the outcome of a given election. In particular, we investigate how the new candidate may place themselves in the plane in order to win, swing, or otherwise change the election's results, depending on their motive. We show that in general, the new candidate can always place themselves to do at least as well as any chosen pre-existing candidate. Furthermore, we develop stronger results particular to each of the specific voting systems we explore: plurality, anti-plurality, Borda count, Bucklin, minimax, and Copeland.
(Received July 24, 2023)

1192-91-27660
William E Gryc*, Muhlenberg College, Benedict Kohler, Muhlenberg College. Bidding Simulations with Different Equilibrium Strategies in an English Auction. Preliminary report.
An English auction is the typical auction format used by auction houses: think of a room full of bidders and an auctioneer soliciting bids from the crowd. The auction ends when no bids are forthcoming, in which case the last person bidding is the winner and pays their bid. In a famous 1982 paper, Milgrom and Weber mathematically modeled the English auction, found an equilibrium strategy for this auction, and showed that if bidders are risk neutral and their signals are affiliated, then the English auction model will generate more expected revenue than either a first-price or second-price auction. However, Milgrom and Weber's equilibrium strategy is far from unique; in a 2002 paper, Bikhchandani, Haile, and Riley derive a
continuum of alternate equilibrium strategies. When bidders all use the same equilibrium strategy, the outcome is fixed: the bidder with the highest signal wins the auction for the same high bid across all equilibria. However, when bidders are using different equilibria, outcomes are much less predictable. This unpredictability is to be expected as bidders using different strategies generally leads to unusual outcomes, but the ramifications for the English auction are especially dire. Since this auction model is played in multiple rounds and bidders update their information based on when other bidders drop out as filtered through their strategy, bad assumptions about other players strategies are magnified as the auction develops. We simulated the outcomes of thousands of auctions where bidders use various equilibria, bidder signals are uniformly distributed, and the value of the item is equal to the sum of all bidder signals (the so-called "wallet game" as it simulates bidders putting their wallets in a pile and then bidding on the cash that lies in all of the wallets). We will present the outcomes of those simulations at this talk. This work is the result of an undergraduate research independent study
(Received August 17, 2023)

## 1192-91-27845

Matthew I Jones*, Yale University. Nash Equilibrium in a Low-Information Vote Trading Game. Preliminary report. Groups are often asked to make decisions about a wide range of issues. If each issue is decided by a separate vote, voters are incentivized to give away their votes on issues they deem unimportant in exchange for additional votes on the most critical issues. This scenario leads to a vote trading game in which voters must decide which trades to offer to maximize their final utility. Using a continuous model of voter utility, we analyze the game by studying the Nash equilibria for players' trades and evaluate how the underlying utility distribution affects player behavior
(Received August 21, 2023)

Adam Graham-Squire*, High Point University, David McCune, William Jewell College. What can be learned from a large ranked-choice voting dataset?.
The Single Transferable Vote (STV) method is used for Scottish local government elections, with data available from 2007. The dataset contains over 1000 ranked-choice elections, by far the largest such freely available set of data on political rankedchoice elections. We will summarize our findings about what these elections can tell us about the frequency of voting anomalies, the ability of political parties to manipulate the STV voting method, and future research questions which could be answered by analyzing this data. (Received August 22, 2023)

## 1192-91-27973

Steven J Brams, New York University, Mehmet S. Ismail*, King's College London, D. Marc Kilgour, Wilfrid Laurier University. Fairer Shootouts in Soccer: The m-n Rule.
Winning the coin toss at the end of a tied soccer game gives a team the right to choose whether to kick either first or second on all five rounds of penalty kicks, when each team is allowed one kick per round. There is considerable evidence that the right to make this choice, which is usually to kick first, gives a team a significant advantage. To make the outcome of a tied game fairer, we suggest a rule that handicaps the team that kicks first (A), requiring it to succeed on one more penalty kick than the team that kicks second (B). We call this the $m-n$ rule and, more specifically, propose ( $m, n$ ) = (5, 4): For A to win, it must successfully kick 5 goals before the end of the round in which B kicks its 4 th; for B to win, it must succeed on 4 penalty kicks before A succeeds on 5 . If both teams reach $(5,4)$ on the same round-when they both kick successfully at $(4,3)$-then the game is decided by round-by-round "sudden death," whereby the winner is the first team to score in a subsequent round when the other team does not. We show that this rule is fair in tending to equalize the ability of each team to win a tied game in a penalty shootout. We also discuss a related rule that precludes the teams from reaching $(5,4)$ at the same time, obviating the need for sudden death and extra rounds.
(Received August 23, 2023)

## 1192-91-28363

Mason A Porter*, UCLA. Textual Analysis via Punctuation Sequences.
Whether enjoying the lucid prose of a favourite author or slogging through some other writer's cumbersome, heavy-set prattle (full of parentheses, em dashes, compound adjectives, and Oxford commas), readers will notice stylistic signatures not only in word choice and grammar but also in punctuation itself. Indeed, visual sequences of punctuation from different authors produce marvellously different (and visually striking) sequences. Punctuation is a largely overlooked stylistic feature in stylometry, the quantitative analysis of written text. In this talk, I examine punctuation sequences in a corpus of literary documents and ask the following questions: Are the properties of such sequences a distinctive feature of different authors? Is it possible to distinguish literary genres based on their punctuation sequences? Do the punctuation styles of authors evolve over time? Are we on to something interesting in trying to do stylometry without words, or are we full of sound and fury (signifying nothing)? In our investigation, we examine a large corpus of documents from Project Gutenberg (a digital library with many possible editorial influences). We extract punctuation sequences from each document in our corpus and record the number of words that separate punctuation marks. Using such information about punctuation-usage patterns, we attempt both author and genre recognition, and we also examine the evolution of punctuation usage over time. Our efforts at author recognition are particularly successful. Among the features that we consider, the one that seems to carry the most explanatory power is an empirical approximation of the joint probability of the successive occurrence of two punctuation marks. In our conclusions, we suggest several directions for future work, including the application of similar analyses for investigating translations and other types of categorical time series. This project is joint work with Marya Bazzi, Alexandra Darmon, and Sam Howison.
(Received August 30, 2023)

## 1192-91-28612

Yifeng Ding, Peking University, Wesley H Holliday*, University of California, Berkeley, Eric Pacuit, University of Maryland. An Axiomatic Characterization of Split Cycle.
The most notorious problem in voting theory since the 18 th century is that there can be majority cycles in an election: it can
happen, e.g., that a majority of voters prefer candidate $A$ to candidate $B$, a majority of voters prefer $B$ to candidate $C$, and a majority of voters prefer $C$ to $A$. Cycles with more than three candidates are also possible, as are overlapping cycles. A number of rules for resolving majority cycles in elections have been proposed in the literature. Recently, Holliday and Pacuit (Journal of Theoretical Politics 33 (2021) 475-524) axiomatically characterized one such cycle-resolving rule, dubbed Split Cycle: in each majority cycle, discard the majority preferences with the smallest majority margin. They showed that any rule satisfying five standard axioms, plus a weakening of Arrow's Independence of Irrelevant Alternatives (IIA) called Coherent IIA, is refined by Split Cycle. In recent work, we have gone further and shown that Split Cycle is the only rule satisfying the axioms of Holliday and Pacuit together with two additional axioms: Coherent Defeat and Positive Involvement in Defeat. Coherent Defeat states that any majority preference not occurring in a cycle is retained, while Positive Involvement in Defeat is closely related to a well-known axiom of Positive Involvement (as in J. Pérez, Social Choice and Welfare 18 (2001) 601-616). In this talk, I will explain this new axiomatic characterization, which provides an argument for resolving majority cycles according to the Split Cycle method.
(Received September 01, 2023)

## 1192-91-28759

Christoph Hauert*, The University of British Columbia. Spatial social dilemmas promote diversity.
Evolutionary branching and diversification in interactions with continuous behavioral traits is important for understanding the emergence of distinct, discrete, and coexisting strategies. In social dilemmas, this suggests an evolutionary pathway for the origin of distinct traits of cooperators and defectors. Continuous investments in social dilemmas can spontaneously diversify into stably coexisting high investing cooperators and and low investing defectors in unstructured, well-mixed populations. Here we extend the analysis to study emerging diversity in spatially structured populations through the invasion fitness of rare mutants, which yields a spatial adaptive dynamics framework. Based on this analytical framework and extensive individualbased simulations, we find that spatial structure has significant and varied impacts on evolutionary diversification in continuous social dilemmas. More specifically, spatial adaptive dynamics suggests that spontaneous diversification through traditional evolutionary branching is suppressed, but simulations show that spatial dimensions offer new modes of diversification that are driven by an interplay of finite-size mutations and population structures. Even though spatial adaptive dynamics is unable to capture these new modes, they can still be understood based on an invasion analysis. In particular, population structures alter invasion fitness and can open up new regions in trait space where mutants can invade, but those may not be accessible to small mutational steps. Instead, stochastically appearing larger mutations or sequences of smaller mutations in a particular direction in the trait space are required to bridge regions of unfavourable traits. The net effect is that spatial structure admits new modes of diversification that complement classical evolutionary branching. In particular, when selection is strong, diversification tends to occur more readily than in unstructured populations. (Received September 02, 2023)

## 1192-91-29152

Landon Gauthier*, Carthage College. Patterns in Political Polarization. Preliminary report.
The present political climate of the United States suggests that political partisanship has reached an all-time high. How can this be quantified? We investigate polarization in the U.S. Senate with methods previously used to study the U.S. House of Representatives. By measuring agreements both within and between members of political parties, we create a network structure defined by voting records. These networks display the range of senator behaviors, allowing us to track changes in partisanship on both an individual and a party level. We observe shifts in behavior over decades of voting history, highlighting trends and outlier events. Some of our findings may contradict what you hear on the news!
(Received September 05, 2023)

## 1192-91-29385

Matthieu Dufour, Université du Québec à Montréal, Silvia Heubach*, California State University Los Angeles. A family of Slow Exact $k$-Nim Games
Slow Exact $k$-Nim is a variant of the well-known game of Nim, in which two players alternately remove any number of tokens from one of $n$ stacks of tokens. In Slow Exact $k$-Nim, in each move, a player selects $k$ of the $n$ stacks and then removes a single token from each of the selected stacks. The last player to move wins. Nim and this variant are impartial combinatorial games, those without hidden information or randomness and where both players have the same moves available from a given position. In these types of games, each game position is either an $\mathcal{N}$ - or a $\mathcal{P}$-position. Assuming optimal play, an $\mathcal{N}$-position is one from which a player starting in that position has a winning strategy, while a player starting in a $\mathcal{P}$-position is bound to lose, no matter what moves s/he makes. Knowing the $\mathcal{P}$-positions therefore tells us all we need to know about the game and who can win. Previously, results for Slow Exact $k$-Nim were shown for some small values of $n$ and $k$ and for the two trivial families when play is on one stack or on all stacks, respectively. Here we prove results on the structure of the $\mathcal{P}$-positions for the infinite family of games where play is on all but one of the $n$ stacks, that is, when $k=n-1$. Not only is the solution for this family no longer trivial, it exhibits a rich structure that depends on the parity of $n$. We also extend this family of games to Slow Exact $\operatorname{Set}-\operatorname{Nim}(A)$, where the set $A$ gives the allowable selection of stacks for each move. We determine the structure of the $\mathcal{P}$ positions of the game Slow Exact Set-Nim ( $\{n-1, n\}$ ) where in each move, a player can decide to either play on all stacks or all but one stack. The set of $\mathcal{P}$-positions of this game is intimately related to those of Slow Exact $(n-1)$-Nim.
(Received September 06, 2023)

1192-91-29636
Radoslav Dimitric*, DBRI. Cobb and Douglas Dissected.
We examine circumstances surrounding the so-called Cobb-Douglas formula $P=b L^{\alpha} C^{1-\alpha}(*)$ that relates production $P$ with labor $L$ and capital $C$. It is well-known that the two variable function $f(L, C)=L^{\alpha} C^{1-\alpha}$ had been used at least as early as by Euler and that the relationship $\left(^{*}\right.$ ) was used in economics at least as early as with Walras and Wicksell. Even though it was noted early that the name of the formula was a misnomer the paper of Cobb and Douglas: A theory of production, published in 1928 was responsible for this misnomer to persist for almost hundred years. It is mostly due to the fact that the paper presumably gave a "proof" of the formula by supplying data that "proved" it (with $100 \%$ exactitude!). We bust that myth by carefully ("forensically") examining this paper and show that, contrary to popular belief, the authors did not prove the formula by fitting some existing data, but rather constructed, i.e. invented data in order to fit the formula, perfectly.

## 1192-91-29652

Sarah Kulas*, St. Norbert College. Present Bias in Group Work. Preliminary report
In this talk, we discuss how procrastination is affected in a group setting. We begin by reviewing why a single individual procrastinates using findings by Kleinberg and Oren. The general idea is that a person inflates the cost of having to do a task in the present because doing work right now feels much harder than putting off that same work until tomorrow. Yet, when we get to tomorrow, this idea still applies and we end up putting off the task until the last minute when we are forced to do it for a much larger cost. We investigated this procrastination problem by using a fan graph with a source vertex, sink vertex, and weighted edge costs. To find the optimal path for doing a task requires finding the cheapest path from the source to the sink. In this talk, our goal is to find the most expensive path that a group could take.
(Received September 07, 2023)

1192-91-29677
Erin Martin*, Brigham Young University, David McCune, William Jewell College. A Comparison of Sequential Ranked-Choice Voting and Single Transferable Vote.
Within the last 20 years, states such as Minnesota and California have adopted various forms of ranked-choice voting (RCV) to determine winners in elections. For multi-winner elections, there is a version of RCV known as single-transferable vote (STV) which has been used to elect members to city councils and school committees in Massachusetts, California, Michigan, and Minnesota. In 2019 two cities in Utah introduced a new multi-winner ranked-choice voting method called sequential RCV. We will explain and compare these methods using mathematical and empirical analysis.
(Received September 07, 2023)

## 1192-91-29776

Josep Freixas, Universitat Politècnica de Catalunya (Campus Manresa), Roger Hoerl, Union College, William S Zwicker*, Union College; Murat Sertel Center for Advanced Economic Studies, Istanbul Bilgi University. Axiomatic foundations of the scale-invariant Hirsch citation index,
A number of citation indices have been proposed for measuring and ranking the research publication records of scholars. The index proposed by Hirsch is particularly well known, and was designed to reward most highly those records that strike some balance between productivity (number of papers published), and impact (frequency with which those papers are cited). Hirsch's index has been criticized, however, for implicitly equating its chosen unit of productivity (a single publication) with its chosen unit of impact (a single citation). In response, two groups of scholars independently proposed a scale-invariant variant h' of Hirsch's original index h , arguing that the induced ranking behaves more fairly in the way it ranks scholars, especially within subdisciplines that more strongly violate Hirsch's scale assumptions. Our emphasis in this talk will be on a recent axiomatic characterization of $h^{\prime}$, with its striking parallels to John Nash's axiomatization of his solution to the two-person bargaining problem. We will focus in particular on connections between the axiom that has been called - though not by Nash himself - "Nash IIA" and the Max Bounded axiom, which selects the h' index from a wider class of scale invariant indices based on inscribed shapes other than squares or rectangles.
(Received September 07, 2023)

## 1192-91-29848

Ismar Volic*, Wellesley College. Simplicial complexes and political structures
Simplicial complexes and their topology are a natural tool for modeling interactions in a system and revealing its deeper underlying structures. We will discuss how simplicial complexes can be used to study political systems in which coalitions are represented by simplices. Some basic topological constructions can then easily be translated into political situations such as merging of parties and introduction of mediators, as well as supply a refined point of view on game-theoretic notions like the Banzhaf and Shapley-Shubik power indices. We will also discuss a generalization to hypergraphs which captures an even richer collection of political dynamics concepts.
(Received September 08, 2023)

1192-91-29923
Tian An Wong*, University of Michigan-Dearborn. The Mathematics of Policing: Theory and Applications. Preliminary report In 2020, following the murder of George Floyd, thousands of mathematicians signed a letter to the AMS agreeing to boycot collaborations with police departments. This raises the question: what exactly is the role of mathematics in, whether for or against, policing in the USA? In this talk, I will discuss recent and ongoing work studying the police and police violence from a mathematical point of view, using methods in complex networks, statistics, and reinforcement learning. I will also discuss potential pitfalls in focusing narrowly on mathematical models, community-driven research questions, and the limits on influencing policy.
(Received September 08, 2023)

Gillian Grindstaff, Oxford, Abigail Hickok*, UCLA, Benjamin Jarman, UCLA, Michael Johnson, UCLA, Jiajie Luo, UCLA, Mason A Porter, UCLA, Sarah Tymochko, UCLA. Applications of Topological Data Analysis to Spatial Systems: Case Studies in Polling-Place and Public-Park Accessibility.
Determining the accessibility of geographically-distributed "resources" (e.g., polling places, medical care centers, and public green spaces) is a complex task. In this talk, we will use TDA as a framework to measure and evaluate the equity of the geographic distribution of a given resource. Our aim is to identify geographic regions that are under-served ("holes in coverage"). A naive approach is to consider geographic distances from resource sites; one could calculate the percentage of people within some distance $D$ of the nearest resource site. However, such an approach poses three issues: (1) it requires choosing an arbitrary cutoff distance $D,(2)$ using only geographic distance fails to account for many other factors involved in
ease of access, such as population density and availability of public transportation, and (3) it cannot account for cases in which the resources have heterogeneous quality, utility, or capacity. Instead, we use persistent homology (PH) because it allows us to study holes in coverage at all scales without needing to choose an arbitrary cutoff distance. In our case study of access to polling places, we incorporate travel time and waiting times at the polls, which better capture the "cost" of accessing a polling site compared to using geographic distance. In our case study of access to public parks, we use multiparameter PH to incorporate the quality of each park.
(Received September 08, 2023)

## 1192-91-29934

D. Marc Kilgour*, Wilfrid Laurier University. The Borda-Weighted Bucklin Electoral Procedure. Preliminary report. The Borda-Weighted Bucklin (BWB) electoral procedure applies to ranked-ballot single-winner elections. It relies on the same weights as the Borda Count and indeed some of its properties are identical to the Borda system, though others are differentand in some cases better. For instance, in three-candidate spatial models, BWB achieves a Condorcet Efficiency of 100\%, while in Impartial Culture elections BWB and Borda apparently have the same Condorcet Efficiency. A detailed study of the BWB procedure explains these phenomena and provides some new insights into the Borda Count
(Received September 08, 2023)

## 1192-91-30009

Kaitlynn Addita Harrylal*, St. Edward's University. Assessing the Black-Scholes Model
Assets of different types are traded in financial markets and their prices fluctuate wildly. Anticipating these price fluctuations could make one a great amount of money very quickly, so the prediction of these fluctuations has been analyzed using the famous Black-Scholes model to determine option pricing. This does not work on every investment due to certain limitations but is one of the most used techniques for pricing options, and economists continue to amend the model to make it more realistic. We analyze the price movement of 480 stocks in the S\&P 500 during 2014 to assess the accuracy of the Black-Scholes model. We then apply this methodology to the year the pandemic began and compare the call options from the Black-Scholes model to market results.
(Received September 08, 2023)

1192-91-30304
Hari Sarang Sarang Nathan*, University of Rochester, Michael Orrison, Harvey Mudd College, Katharine Shultis, Gonzaga University, Jessica Sorrells, Converse University. ( $k, \mathcal{L}^{p}$ )-Approval Voting.
Approval voting is a common method of preference aggregation where: (1) each voter chooses a subset of $n$ candidates to "approve" of; (2) a candidate's score is the number of voters who approved of them; and (3) the winner(s) are those with the highest score(s). In approval voting, the degree to which a voter impacts a candidate's score depends only on if that voter approved of the candidate or not, i.e., it is independent of which, or how many, other candidates they approved of. Recently, there has been interest in satisfaction approval voting and quadratic voting, both of which include a trade-off between approving of more candidates and how much each vote impacts the scores of each candidate. Approval voting, satisfaction approval voting, and quadratic voting can all be viewed as voting where each voter submits a vector in $\{0,1\}^{n}$ which is then normalized using different norms $\left(\mathcal{L}^{\infty}, \mathcal{L}^{1}\right.$, and $\mathcal{L}^{2}$ respectively). This suggests a family of voting methods where different $\mathcal{L}^{p}$ norms are used. When using the $\mathcal{L}^{p}$ norm to select a committee of $k$ members, we call this $\left(k, \mathcal{L}^{p}\right)$-approval voting. Here, we look at examples of how changing $p$ can affect the outcome and properties of $\left(k, \mathcal{L}^{p}\right)$-approval voting. In addition, we look at properties used to justify voting methods and if they can, or cannot, be used to justify $\left(k, \mathcal{L}^{p}\right)$-approval voting.
(Received September 09, 2023)

## 1192-91-30309

Dhruv Chandra, Dartmouth College, Longmei Shu*, Dartmouth College. Dating, faithfulness, and monogamy. Online dating has gamified the dating process even further than before. Even after successful in person dates, or marriages, there is still a question about faithfulness of the partners involved. Aside from humans, other animals have demonstrated monogamous and non-monogamous family structures. For example, the side-blotched lizards of California, the males with orange throat have multiple wives and ones with blue throat are monogamous. The yellow-throat males look just like females and mate with the other two types' wives. Over a six year period, the population of the lizard cycles through dominance of each of the three types of males. We would like to analyze the games involved in all these processes and give some game theoretic explanation behind the morals regarding dating and faithfulness.
(Received September 09, 2023)

## 1192-91-30666

Stanley R. Huddy, Fairleigh Dickinson University, Michael Ivanitskiy, Colorado School of Mines, Michael A. Jones*, Mathematical Reviews | AMS. Equilibria for the Wallet Game and the Paradoxical Role of Zero.
Martin Gardner popularized the wallet game in which two players each place their wallets on a table. Whichever player has the smallest amount of money in his or her wallet, wins the money in the other player's wallet. Strategies for this game are either distributions over dollar amounts $\{0\} \cup \mathbb{N}$ or $\{0,1,2, \ldots, m\}$ with fixed mean $\mu$. When carrying 0 dollars, a player cannot lose any money. Zero plays a surprising role in Nash equilibria for this game. For $\mu \leq 1$, we show that there exists a unique Nash equilibrium that places weight on 0 (if $\mu<1$ ) and 1, whether the amount of money carried is bounded or not. For $\mu>1$, when distributions may be unbounded, we prove that there is no Nash equilibrium, improving on a result in the literature which had to introduce carrying $1 / 2$ to show the result. The proof hinges on constructing a strategy that places a high probability on 0 . The more interesting case is when distributions are bounded: for $\mu>1$, Nash equilibria place zero probability on 0 . To show that Nash equilibria in the bounded case exist, a separate probability result that all distributions are convex combinations of specified distributions is applied; this result may be of independent interest. Additional observations and conjectures about the structure of Nash equilibria will be discussed
(Received September 10, 2023)

## 1192-91-30689

Mark Curtis Wilson*, University of Massachusetts Amherst. Why does uniform swing work so well?. Preliminary report. There have been two basic models using election outcomes at time $t$ to predict outcomes at time $t+k$ : uniform swing and proportional swing, with the former having been given both a deterministic and a probabilistic form. Wilson and Grofman (2022) show that both uniform swing and proportional swing violate key theoretical axioms that we would want any model of inter-election shift to satisfy, and they suggest an alternative model. They also point out that these two models neglect important complications such as regression to the mean effects and differential electoral tides. Our focus here, however, is on the empirical puzzle of why, despite these theoretical failings, both uniform swing and proportional swing appear to work really well in fitting empirical data on two-party elections, and we also address the related puzzle of why the two models give such similar results despite the difference in their functional form. We show that various phenomena operate to cancel each other out to give these models a better fit, and we show that massaging the vote share data by truncating it at the extremes, a common practice in the U.S. literature, improves the fit of these models. But we also show that looking only to goodness of regression fit fails to test the key idea of uniform swing: namely that $m=1$ and $b=$ mean inter-election shift.
(Received September 10, 2023)

## 1192-91-30814

Will Fowlkes*, Ithaca College REU, Kate Sophia Hartke*, Ithaca College REU. Evolutionarily Stable Dispersal Timing: Effects of Dispersal Costs and Kin Competition.
Dispersal is the permanent relocation of individuals or propagules away from the natal habitat. Previous studies have investigated which biological factors most affect dispersal. However, few studies have addressed the timing of dispersal. We present a game-theoretical model of the time at which individuals should disperse from the natal area. This model includes the effects of kin competition (i.e., sibling rivalry) and dispersal costs such as time and risk of death. In our model, siblings in a brood decide sequentially whether to disperse from or remain in their natal area. This decision occurs each day of a finite breeding season, where the most 'dominant' sibling makes their decision first and every sibling has complete knowledge of all previous decisions. We find strict Nash Equilibria such that each sibling optimizes their own expected future reproductive success, demonstrating the evolutionarily stable dispersal timing for each sibling. Finally, we use sensitivity analysis to draw conclusions about how evolutionarily stable dispersal timing is affected by: semelparity/iteroparity, kin competition, parental care, dominance level, and inbreeding avoidance.
(Received September 11, 2023)

## 1192-91-30951

Emerson Arehart*, University of Pennsylvania, Anna Berryman, University of Oxford, Sara M Clifton, Kenyon College, Nicholas W. Landry, University of Vermont, Denis Tverskoi, University of Tennessee Knoxville, Alexandria Volkening, Purdue University. Effect of primary candidate performance statistics on general election turnout at the precinct level. Preliminary report.
In the United States, polling data are collected at the state level, but election results vary substantially within states (e.g., between rural and urban areas). Unlike polls, election results are often available at the precinct level, and these fine-scale data can provide insight into how primary elections, general elections, and voter turnout are related at a granular level. Using primary and general election data at the precinct level for past presidential races, we show the relationship between the degree of partisan consensus ('entropy') in the primaries and each party's under- or over-performance in the general election These patterns give insight into voter turnout decisions and provide a fine-grained method for forecasting general elections. (Received September 11, 2023)

## 1192-91-31015

Malavika Mukundan*, University of Michigan. On ranked choice voting. Preliminary report.
In this talk, we recall the basics of ranked choice voting and several variations based on adjusting ambient parameters. We give some results to suggest how changing these ambient parameters can affect the outcome of the election, and their impact on best strategic play for various voting blocs. We also introduce a new probabilistic model for elections with random ballots that are allowed to be incompletely ranked, and lay out a framework to analyze the utility of this model, in comparison with known models such as Plackett-Luce and Bradley-Terry.
(Received September 11, 2023)

## 1192-91-31063

Steven J Brams*, New York University. Extending Divide-and-Choose to the Envy-Free Allocation of Indivisible Items (if Possible): An Algorithm. Preliminary report.
An analogue to the cake-cutting procedure of "I Cut, You Choose" is presented, in which instead of a divisible item like cake or money, it is indivisible items that need to be divided. The algorithm for doing so assumes that two players can strictly rank the items from best to worst, and the items come up one at a time. Starting with the top item of each player and going down the ranks, if the items named by each player are different, each is allocated to the player who names it; all the other items go into a contested pile. The latter items are ordered the same by both players, and an algorithm for allocating them is presented such that if there is an "envy-free" allocation of these items-neither player would prefer the items of the other player-the algorithm finds it in a computationally simple way. Coupled with the envy-free allocation of the noncontested items, the overall allocation of the items is envy-free if there exists an envy-free allocation of the contested pile (Received September 11, 2023)
for cultural group selection to suggest that altruistic punishment can help to achieve cooperation via multilevel selection even when a group of altruistic punishers can be successfully invaded by defectors under individual selection alone. In this talk, we will discuss a way to formulate a partial differential equation model for multilevel selection that incorporates these assumption made by Boyd and coauthors on individual-level and group-level competition, which can be used to analytically characterize scenarios in which altruistic punishment and between-group competition can work in concert to promote long-time cooperative behavior. The particular form of between-group competition features pairwise competition between groups and a nonlinear group-level reproduction rate, extending existing work on PDE models for multilevel selection in which among-group competition does not depend on the strategic composition of the group-structured population.
(Received September 22, 2023)

## 1192-91-31269

Peter Michael Bigica*, Western Connecticut State University. Wavelet Based Financial Forecast Ensemble Featuring Hybrid Quantum-Classical LSTM Model.
One of the most sought-after goals in the financial world is a reliable method by which investors can predict a stock price movement consistently. Advancements in stock prediction via the use of machine learning have improved the accuracy of such predictions and yielded better ideas about value investments in the stock market. However, with the addition of an M-band wavelet transform as a preprocessing step, we can denoise our data set (prior stock prices) and refine it to make the forecast even more accurate. Following this preprocessing step, multiple neural networks are deployed to construct a robust, nonparametric hybrid forecasting model. To demonstrate the power of this method, we present a case study on stock price prediction employing a discrete 4-band wavelet transform integrated with a hybrid machine learning ensemble. Our results underscore the potential and importance of such ensemble methods in refining the accuracy and reliability of financial forecasts. Furthermore, our ensemble will feature a hybrid quantum-classical LSTM Model to demonstrate both the power of wavelets and quantum computing in stock forecasting.
(Received September 11, 2023)

1192-91-31411
Vicky Chuqiao Yang*, MIT Sloan School of Management, Massachusetts Institute of Technology. Can a group generate good decisions when some members don't think for themselves?.
It is well known that many voters in the US display political ignorance-they cannot answer basic questions about how the government functions or correctly identify the policies the candidates support. A key question concerning democratic elections is whether they can settle on the best available option when some members learn from others instead of evaluating the options on their own. This question is challenging to study, and previous research has reached mixed conclusions, because collective decision outcomes depend on the insufficiently understood complex system of cognitive strategies, task properties, and social influence processes. I will present a study that integrates these cognitive and social factors in one general yet partially analytically tractable mathematical framework using a dynamical system model. In particular, it investigates how the interplay of the proportion of social learners, the relative merit of options, and the type of conformity response affect collective decision outcomes in a binary choice. The model predicts that, when the proportion of social learners exceeds a critical threshold, a bistable state appears in which the majority can end up favoring either the higher- or lower-merit option, depending on fluctuations and initial conditions. Below this threshold, the high-merit option is chosen by the majority. The study helps reconcile disagreements about the effect of social learners on collective performance and proposes a mathematical framework that can be readily adapted to extensions investigating a wider variety of dynamics. Published paper:
https://www.pnas.org/doi/abs/10.1073/pnas.2106292118
(Received September 11, 2023)

1192-91-31727
Bret Benesh, College of Saint Benedict and St. John's University, Dana Ernst, Northern Arizona University, Marie Meyer*, Lewis University, Sarah Salmon, University of Colorado Boulder, Nandor Sieben, Northern Arizona University. Impartial Geodetic Building Games on Graphs.
A subset of vertices of a graph is geodetically convex if it contains every vertex on any shortest path between two vertices of the subset. The convex hull of a subset of vertices is the smallest geodetically convex set containing itself. We study two impartial games where two players take turns selecting previously-unselected vertices of a graph until the convex hull of the jointly-selected vertices becomes too large. In both versions, the last player to move is the winner. The achievement game ends when the convex hull contains every vertex. In the avoidance game, the convex hull is not allowed to contain every vertex. Our convex hull games are analogous to the geodetic closure games originally studied by Buckley and Harary. We apply structure theory to determine the nim-number of our games for several families of graphs including graphs with a unique minimal generating set, cycle graphs, hypercube graphs, grid graphs, complete multipartite graphs, and wheel graphs (Received September 12, 2023)

## 1192-91-32268

Philip Chodrow, Middlebury College, Joshua Daymude, Arizona State University, Priyanka Gautam, Kansas State University, Pushpi Paranamana, Saint Mary's College, Cara Sulyok, Lewis University, Alexander Wiedemann*, RandolphMacon College, Heather Zinn Brooks, Harvey Mudd College. Inferring Network Structure in Models of Opinion Dynamics. Understanding opinion dynamics within a networked society is crucial for understanding the collective decision-making processes that shape our world. In this work, we assume access to temporal opinion data for each user and knowledge of the underlying dynamics governing their opinions; that is, we know how users influence their neighbors. We do not, however, assume to know the structure of the network itself. We will discuss methods for inferring this network structure under certain assumptions. Several common opinion dynamics models will be considered, and we will discuss the accuracy of each inferred network by testing against several quantitative and qualitative measures.
(Received September 12, 2023)

Public Health Workers' Engagement with Violent Agents through Kinetic Equations.
In recent years, there has been a growing movement among criminologists and advocates to adopt public health strategies as a means to combat violence. This shift in discourse has resulted in the implementation of initiatives like Cure Violence, which are designed to help communities in their efforts to prevent violence. Within the past decade, there has emerged a novel approach applying the principles of kinetic equations to model interactions between law enforcement and criminal activities, often without a clear sociological foundation. This presentation aims to leverage the kinetic equation framework to create models that integrate both public health and classical methodologies. By doing so, we seek to gain a deeper understanding of the impact of these innovative programs on violence reduction. On the mathematical front, we will investigate both finite-time dynamics and long-term behaviors. Through this interdisciplinary exploration, we aim to bridge the gap between the worlds of public health, criminology, and mathematical modeling. By developing and analyzing these models, we hope to contribute valuable insights that can inform evidence-based decision-making, ultimately fostering safer and healthier communities. (Received September 12, 2023)

## 1192-91-32366

Izabel Pirimai Aguiar*, Stanford University, Ekaterina Landgren, University of Colorado, Boulder, Samantha Linn, University of Utah, Sam Zhang, University of Colorado Boulder. Modeling candidate momentum in U.S. primary elections using campaign contributions. Preliminary report
In the U.S., presidential campaigns in both primary and general elections require significant funding. The Federal Election Commission (FEC) tracks individual campaign contributions, including the date of the contribution, the donation amount, donation recipient, and the billing zip code of the contributor. The goal of this work is to use this rich data to better understand the relationship between candidate performance and the spatiotemporal dynamics of campaign contributions. We focus on analyzing the FEC data from campaign contributions during the 2016 Democratic and Republican U.S. presidential primaries. We discuss the key characteristics of these data and explore how time series analysis-like change-point detectioncan help identify important dates at which external events influence contributions. Accounting for differences across states, parties, and candidates, we leverage the structure of stochastic differential equations to propose a model of candidate momentum that captures key characteristics of noisy campaign donation trajectories.
(Received September 12, 2023)

## 1192-91-32438

Moon Duchin, Tufts University, Jack Gibson*, University of Chicago, David McCune, William Jewell College. Fractional vs Random Single Transferable Vote. Preliminary report.
The vote method of single transferable vote (STV) is widely used throughout the world in multiwinner elections. The most common implementation of STV involves transferring fractional votes when candidates achieve quota and their surplus votes are reallocated to other candidates. The city of Cambridge, Massachusetts uses an old version of STV (the "Cincinnati method") in which surplus votes are not transferred fractionally but are instead transferred at random. Using random selection to transfer votes means that the Cincinnati method is not really a vote method in the sense that the same ballot data can produce different winner sets. We explore how differently the Cincinnati method can behave from fractional STV, providing worst-case analyses and results from a large database of real-world elections.
(Received September 12, 2023)

## 1192-91-32669

Feng Fu, Dartmouth College, Alina Glaubitz*, Dartmouth College. The Other Side of the Coin: Recipient Norms and Their Impact on Indirect Reciprocity and Cooperation. Preliminary report.
The other side of the coin Human cooperation uniquely depends on indirect reciprocity. In this talk, we will extend upon previous work taking into account that within the prisoner's dilemma people might update not only the reputation of the donor but also the recipient. We develop a new mathematical framework to compare stochastic updating rules. We find that if two groups within our population use different probabilities of updating donor and recipient norms, the dynamics can become bistable. Assigning a 'bad' reputation to 'good' players who cooperate with 'bad' players can lead to the emergence of bistability. One cooperative equilibrium that takes updates donor and recipient norms and one equilibrium where only recipient norms are updated. This equilibrium can be invaded by defectors and cooperation collapses. In conclusion, our work reveals that the interplay between reputation dynamics and updating rules, particularly in the context of indirect reciprocity, can give rise to bistable dynamics, posing a critical challenge to cooperation within human societies. Understanding these nuances is paramount for devising strategies to sustain and fortify cooperative behavior in complex social systems. (Received September 12, 2023)

1192-91-32705
Duane A. Cooper*, Morehouse College. Countering Partisan Gerrymandering with Multimember Electoral Districts. Preliminary report.
In recent years, the mathematical community has given considerable attention and energy to matters of districting, especially in the United States. Much of this work has addressed the effects of gerrymandering on representation in legislatures, most of which are composed from single-member electoral districts. This work presents results showing how multimember electoral districts, in conjunction with supporting election methods - cumulative voting and others - can blunt the most extreme misrepresentation that can occur in gerrymandered single-member districts. The results showing decreased misrepresentation for two-parties are generalized over jurisdiction size and population fraction. Some classic examples are used to illustrate the results, and specific instances of current congressional districting in some US states are examined. We conclude by considering how these results analyzing fair representation to parties can be extended to examine fair representation for individual citizens.
(Received September 12, 2023)
from a talk called myths of math and poker given by Yan X Zhang. I adapted it into a math circle session, and from there added it to the class I now teach on game theory. At some point it spawned a research project with two undergrads in my class, which is now published as an expository article in Mathematics Magazine with the title "When to hold 'em." So, the research came more from students in the class than the math circle, and the outcome ended up classified as expository instead of research but the session was certainly a gateway into something interesting. The subject itself is well studied - lots of people want to know how to win a poker after all. But by focusing on simplified mathematical games as opposed to more "applied" poker strategy there is a vast store of things to explore, and plenty of insight to be gained into both poker and math. I believe there could be plenty to discover for future researchers, with the warning that, in such a well-studied field, new discoveries may turn out to be not so new. But for research projects with K12 and college students I tend to think that is ok. After all, each time an idea is rediscovered is a bit different, and there is always room to gain new insights from that.
(Received September 12, 2023)

1192-91-32881
Olivia Cannon*, University of Minnesota, Twin Cities, Daniel Brendan Cooney, University of Pennsylvania, Stephanie Dodson, Colby College, Rebecca Hardenbrook, Dartmouth College, Jeungeun Park, SUNY at New Paltz. Spatial Patterns in Population Aggregation. Preliminary report.
We study the roles of competition, motility, and altruism in population aggregation. We find that these factors, when balanced appropriately, lead to spatial patterning, both in an agent-based and mean-field formulation. We prove the existence of spatial patterns near onset in the mean-field formulation, and show agreement numerically between the mean-field and agent-based problems. We also study weak convergence of the agent-based model to a mean-field model in the large-population limit Ongoing questions include more complex spatial patterns in two dimensions and whether mutation can lead to natural selection towards altruistic traits.
(Received September 12, 2023)

1192-91-32892
Santiago O Guisasola*, Independent Researcher, Donald G. Saari, Retired, Un of California Irvine. Symmetries in Games: A Basis for Social Interaction. Preliminary report.
Game theory is a strong candidate for modeling complex social systems. But, as known, the complexity of game theoretic models increases exponentially by adding agents and strategies. For instance, adding just one agent to the simplest $2 \times 2$ game (i.e., creating a $2 \times 2 \times 2$ game) increases the dimensionality of analysis from 8 to 24 . To handle this complexity, simplifying assumptions often are employed that reduce models to just one or two dimensions. In reality, social interactions are far more complex than what most models presently capture. A message from experimental economics, for instance, is that agents need not behave as theory predicts. Even more: agents are diverse, may encounter different payoffs in the same strategic situations, and may interact at different scales-not just with their neighbors. A critical question is to understand when these complexities matter. Which results hold only by looking at the full picture? Which ones survive when simplifying assumptions are relaxed? Answering such questions-or at the very least having a framework to explore them-is crucial to broadening and deepening our understanding of game theory and its applications. A recent book by Jesse and Saari uses representation theory to translate game theoretic symmetries into bases for the space of games, which can be tailored to one's research needs. This talk expands on their work and shows how symmetries in a $2 \times 2$ game give rise to a basis that elucidates a myriad of game theoretic topics, including the Nash equilibrium, potential games, and tensions between individual and collective incentives (e.g., in prisoner's dilemma, battle of the sexes, and hawk dove games). It is shown how different solution concepts and measures of social welfare may emphasize different basis vectors, potentially causing an outcome that appears to be "good" to be "bad," or vice-versa. It is also shown how to extend this basis to more agents and more strategies and how the basis naturally integrates network topologies.
(Received September 12, 2023)

1192-91-33065
Daniel M Abrams, Northwestern University, Andrew Feig, Research Corporation for Science Advancement, Richard J Wiener, Research Corporation for Science Advancement, Emma R Zajdela*, Princeton University. Nonlinear Dynamics of Team Formation at Virtual and In-Person Conferences.
Forming new and innovative scientific collaborations is crucial to spurring the innovation necessary to address growing societal challenges. One of the main ways that collaborations are catalyzed is by gathering scientists together at conferences, but conferences amount to billions of dollars per year, not to mention environmental and opportunity costs. In this context, the COVID-19 pandemic sparked a debate about the value of in-person compared to virtual interaction. This talk will describe a nonlinear dynamical model for the origin of scientific collaborations at conferences, inspired by the physics of catalytic processes. Specifically, the model tracks the probability that conference participants form collaborations given their level of interaction throughout the conference. We test the model with a novel dataset tracking multi-year scientific conferences, including room-level participation data from four in-person and five virtual meetings, each with about 50 participants and extend the results to largescale conferences with $>10,000$ participants. The results show that formal interaction is a greater driver for team formation at virtual compared to in-person conferences, suggesting that organizers have more social control for engineering teams in a virtual environment. Our study underscores the critical role organizers must play for optimizing professional interactions, whether virtual or in-person.
(Received September 13, 2023)

1192-91-33121
Abhi Palikala, Ross Mathematics Program, Sammy Shankar, Ross Mathematics Program, Bruce Tang*, Ross Mathematics Program, Christina Yum, Ross Mathematics Program. Leveraging Entanglement Against Adversarial Jamming: A Quantum Game Theoretic Approach. Preliminary report.
Quantum communication networks offer exciting prospects for quantum information science, enabling large-scale distributed quantum computing and a secure global quantum internet. However, they must be designed to limit noise, prevent quantum decoherence, and defend against attacks from malicious agents. In an adversarial jamming attack, agents add frequency and phase noise with the aim of making the initial message difficult to decode, thus effectively jamming the link. We employ a modified two player sequential quantum Colonel Blotto game to model adversarial jamming across an $N$-repeater quantum
communications link. Player payoff landscapes have been previously characterized in the $N=1$ case (Borah). We utilize a multi-agent reinforcement learning framework to analyze optimal strategies in the $N=2$ case with nontrivial entanglement matrices. We survey the possibility of leveraging an entanglement-enabled Colonel Blotto to increase network resilience and present avenues for further investigation. This research was mentored by Professor Felix Leditzky of the University of Illinois Urbana-Champaign.
(Received September 13, 2023)

## 1192-91-33140

Daniel M Abrams, Northwestern University, Alicia Caticha, Northwestern University, Emily Kohlberg, Northwestern University, Jeremy White, Northwestern University, Emma R Zajdela*, Princeton University. Back in Fashion - Modeling the Cyclical Dynamics of Trends. Preliminary report.
Common wisdom holds that fashion is cyclical, with talk of trends coming "back," from bell-bottom jeans to miniskirts. Historically, a lack of quantitative data posed a barrier to explicit mathematical study of this system, however, newly digitized historical records now make such work possible. This talk will present analysis from a new database we constructed quantifying tens of thousands of images of clothing from 1869 to present day. It will describe approaches to modeling the cyclical dynamics of fashion observed in the dataset. Large-scale social phenomena such as fashion trends are of intrinsic interest themselves, but can also help elucidate the interplay of creativity, differentiation, conformity, and diffusion of ideas in broader human systems.
(Received September 13, 2023)

## 1192-91-33172

Olivia Jessica Chu*, Dartmouth College, Wai-Tong Fan, Indiana University, Atticus W McWhorter, Dartmouth College. Heterogeneous Preferences and Personality in Adaptive Network Models.
In human social systems, we observe that individuals' opinions influence and are influenced by their interactions.
Mathematically, it is common to represent such systems as networks, where nodes are individuals and edges between them denote a connection or association. Adaptive network models explore the dynamic relationship between node properties and network topology. In the context of opinion dynamics, these models often take the form of adaptive voter models, where there are two mechanisms (controlled with a single parameter, $\phi$ ) through which network changes can take place. Through homophily, an edge forms between two individuals who already agree. Through social learning, an individual adopts a neighbor's opinion. In these models, individuals are more frequently attached to those who share their opinion, which is seen through the formation of sub-communities of like-minded individuals. However, it is not always the case that individuals will only want to cluster into homogeneous groups. Instead, they might try to surround themselves with those who both agree and disagree with them to attain a balance of inclusion and distinctiveness in their social environments. In this work, we explore the effects that such heterogeneous preferences have on the dynamics of the adaptive voter model. We find that a broad range of dynamics arises in which the particular combination of $\phi$ and preference for distinctiveness determines the steady state behavior.
(Received September 13, 2023)

## 1192-91-33183

Olivia Jessica Chu*, Dartmouth College, Arturo F Serrano Borrero, Dartmouth College. Using Conviction-Moderated Adaptive Network Models to Understand Political Activism.
The formation of activist groups can spark social movements, coalitions, and revolutions. The creation of such groups can be influenced by social ties, network structures, ideology and culture, and the institutional environment. Yet, the relative importance of each of these factors, the mechanisms through which individuals develop or lose their commitment to a cause, and the channels through which likeminded individuals find each other and establish social connections are not thoroughly understood. In this work, we develop a theory that begins to explain two phenomena: 1) how a potential activist's conviction co-evolves with their immediate social network, and 2) how so-called "politically viable networks" tend to arise or disappear based on the distribution of potential activists and the sociopolitical environment. We illustrate this theory by modifying the adaptive voter model (AVM) with a conviction variable, which represents both the strength with which an individual holds on to their beliefs and the comfort of holding on to such beliefs in the context of their network, encapsulating the co-evolutionary dynamics of networks and sociopolitical attitudes. As is expected from empirical evidence in socio-politically stable countries, we find that activists are systematically discouraged by their exposure to disengaged individuals. However, some situations with increased interaction payoffs and strong homophily preferences favor the formation of politically viable networks. (Received September 13, 2023)

## 1192-91-33321

Aden Omar Ahmed*, Texas A\&M University-Kingsville. Octonions, Game Extension, and the Three-Player Game of Firms. Preliminary report.
We present an octonionic representation of the payoff function for the three-player, two-strategy Game of Firms. The octonionic representation provides a computational capability that we exploit to analyze and identify the potential Nash equilibria of the Game of Firms and its extensions.
(Received September 13, 2023)

## 1192-91-33456

Hai Van Le*, University of Washington. Game theory in the FIFA World Cup 2014-2022. Preliminary report.
This paper focuses on the concepts of mixed strategies in game theory within the setting of a football game using penalty kicks as the focal point. A penalty kick involves one kicker and one goalkeeper. The kicker must choose which direction (right, left, center) to shoot in order to score a goal, and the goalkeeper must choose which direction to dive (right, left, center) in order to stop the ball. Each makes a choice simultaneously without being aware of the other's decision. Thus, there is a unique mixedstrategy equilibrium with a well-defined strategy system. A goal is scored by the kicker or not (zero-sum structure because the goalkeeper makes a save or the kicker shoots high or wide. Given the availability of historical data, both the kickers and goalkeepers know about the past behaviors or preferred feet of their opponents, respectively. A number of studies have
assessed the empirical validity of Nash equilibrium for the two-person zero-sum game in both experimental and natural settings. This project re-examines one model developed by Chiappori et al. (2002) which specified a general game in which each player can choose one out of three possible strategies (right, left, center), and then evaluates their proposed assumptions via the payoff matrix (scoring probabilities) using data from recent penalty shootouts in the World Cup games of 2014, 2018, and 2022. We gathered the data by collecting information from live commentators on players' strategies and outcomes. With this data, we tested a new model which specifies a general game in which each player can choose one out of two possible strategies (upper or lower) and then evaluated our proposed assumptions using the payoff matrix. We found that scoring is more likely when kickers successfully place the ball in the upper corner of the net, a tactic that is also known as the "Postage Stamp." Our data partially support the assumptions made by Chiappori et al. (2002) and our new model. We conclude that the empirical results of our model could better inform kickers to be more effective, especially in the World Cup, the most popular football competition in the world, where the pressure is insurmountable.
(Received September 17, 2023)

1192-91-33467
Hai Van Le, Hollins University, Dulce Karina Lopez Cruz*, University of California, Berkeley. CEO Compensation and RiskTaking: S\&P 1500 Firms.
Executives in United States firms have experienced significant compensation gains since the late 1970s. From 1978 to 2020 , Chief Executive Officer (CEO) compensation alone increased by 1,322\%, while real wage growth for typical workers was a mere $18 \%$ during the same period (Mishel and Jori 2021). There exists no uniform earnings ceiling for executives since compensation is determined by the market, and debates over government intervention are ongoing. Compensation and risktaking are of interest, since compensation growth is significant and risk-taking is often fundamental in corporate strategies. We extrapolate from literature in the banking industry to publicly traded firms in order to comment on the extent to which compensation and risk-taking are related, and in particular, if compensation is associated with risky CEO behaviors. We use data from S\&P 1500 firms for the period of 1992-2022, sourced from Compustat and Execucomp databases. In this paper, we estimate the relationship between executive compensation and two measures of firm-level risk-taking behavior, Altman's Zsquare and stock volatility. Because compensation may be endogenous in this setting, we use a two-stage least squares approach, where we instrument for two measures of compensation using CEO tenure, firm size, and two sets of policy and asset pricing controls. Ultimately, we find a strong positive relationship between total compensation and risk-taking as represented by Altman's Z-score. The effect of stock options on risk-taking is unclear since contrasting results arise when using each measure of risk. Keywords: CEO compensation, firm risk, Altman's Z-score, stock volatility, financial crisis (Received September 18, 2023)

1192-91-33618
Paul Ge*, University of Illinois Urbana Champaign, Benjamin C Taylor, University of Illinois Urbana Champaign. Misère Game of Cycles. Preliminary report.
We explore some variations on the Game of Cycles, a zero-sum two-player combinatorial game introduced by Alvarado, et al, in 2020 and further explored by Lin in 2021. In the Game of Cycles, players take turns orienting edges of a simple connected planar graph, called a board, with the objective of either completing a cycle cell or making the last possible move. Previous work considers the game under normal play, whereas we primarily focus on Misére play. We exhibit a family of boards that are counterexamples to a parity conjecture originally made by Alvarado, et al, and subsequently refined by Lin. We prove that the mirror-reverse strategy, which is a winning strategy under normal play for boards with particular symmetries, is not in the Misére game. Finally, we exhibit winning strategies for certain boards that were previously studied under normal play. (Received September 21, 2023)

1192-91-33748
Amy Cochran, University of Wisconsin - Madison, Haley Kottler, University of Wisconsin - Madison, Peizhe Li*, University of Wisconsin - Madison, Jack Maloney, University of Wisconsin - Madison, Seungyeon Oh*, University of Wisconsin Madison, Jimmy Vineyard*, University of Wisconsin - Madison. Distinguishing Between Model-Free and Model-Based Learning with LoCA. Preliminary report.
A recent problem of interest in both reinforcement learning and psychology is distinguishing between model-free (MF) and model-based (MB) learning algorithms. However, there is no clear psychological definition of either class of algorithms. Some algorithms are purported to be one or the other, but there is not a generally accepted method for distinguishing between the two. Understanding the distinction between model-free and model-based approaches can help explain the process of human decision-making, as well as provide a better understanding of human learning in general. A group of researchers from New York University proposed a two-stage Markov decision process, which they claimed to be capable of distinguishing between model-free and model-based learning. It has been widely used, but has also been criticized, with some researchers suggesting it does not accurately distinguish between MB and MF learning. The purpose of our project is to propose a new task for distinguishing between model-free and model-based learning in humans. Due to the limitations of the two-stage Markov method, we propose adapting the local change and adaption framework (LoCA), a new framework used in characterizing reinforcement learning algorithms, for use in human tasks. This novel approach has the potential to advance our understanding of human cognition and contribute to the development of more accurate and realistic models of learning in psychology.
(Received September 25, 2023)

## 92 Biology and other natural sciences, behavioral sciences

## 1192-92-25346

Trachette Jackson*, University of Michigan. Mobilizing Mathematics for the Fight Against Cancer. Preliminary report. Mathematical oncologists apply mathematical and computational modeling approaches to every aspect of cancer biology, from tumor initiation to malignant spread and treatment response. Using data-driven computational models is a powerful and practical way to investigate the therapeutic potential of novel combinations of various strategies for clinical cancer treatment. This talk will showcase a suite of mathematical models designed to optimize the use of targeted drug treatment strategies in combination with immunotherapy. The goal is to gain a more robust understanding of how specific tumor mutations affect the
immune system and ultimately impact the efficacy of combination therapy. Combined with existing and newly generated experimental data, these mathematical models are poised to improve the ability to connect promising drugs for clinical trials and reduce the time and costs associated with transitioning novel therapeutic approaches from "equations to bench to bedside."
(Received May 05, 2023)

1192-92-25636
Oguntolu Abiodun Festus*, Federal University Technology, Minna. MODELLING THE TRANSMISSION DYNAMICS OF CHOLERA WITH OPTIMAL CONTROL AND COST-EFFECTIVENESS ANALYSIS. Preliminary report.
Cholera is an acute diarrhoeal infection caused by ingestion of food or water contaminated with the bacterium, Vibrio cholera and this accounts for a huge death burden in Africa. In this study, a mathematical model of cholera disease transmission dynamics with optimal control and cost-effectiveness shall be developed. Based on the epidemiological condition of individuals in the population, the total human population shall be divided into four compartments: susceptible, exposed, infected, and recovered humans. The bacterial population Vibrio Cholerae is also taken into account. It is assumed that the Cholera disease spread to humans by the consumption of contaminated food or water with the Vibrio Cholerae bacteria that can be found in the environment. Furthermore, it is assumed that the Vibrio Cholerae bacterium can be shedded in the environment via vomiting and feces of infected individuals. To achieve the best fit, we fit the proposed cholera model using accessible data from the Nigeria Centre for Disease Control (NCDC) from January to December 2022. This allows us to determine the accuracy of the model's representation of real-world data. The basic reproduction number R0 and equilibrium points will be computed. The global sensitivity and stability analysis can be investigated. To eradicate cholera disease from the human population, we extend the basic model into an optimal control problem and conduct cost-effective analysis of the control techniques: sanitation, educational campaigns, treatment, and vaccination. Biological interpretations of the findings obtained from the numerical simulations shall be presented.
(Received June 28, 2023)

## 1192-92-25649

Zhilan Feng, Purdue University, Katharine F Gurski*, Howard University, Olivia Prosper, University of Tennessee Knoxville, Miranda I Teboh-Ewungkem, Government. Application of a malaria model with non-exponential waiting times: the impact on treatment outcomes.
Most epidemiological models for malaria assume exponentially distributed residence times in disease stages to simplify the model formulation and analysis. However, these models do not allow for accurate description of the interaction between drug concentration and parasite load within humans and mosquitos. To improve this, we formulated a model by considering arbitrarily distributed sojourn for various disease stages. When the general distributions were replaced by gamma distributions fit to data, the system of integral equations can be reduced to a system of ODEs, which has some non-trivial characteristics which are only captured by non-exponential distributions for disease stages. We incorporate data for asexual parasite and gametocyte levels, drug concentrations, and recorded patient recovery times to the malaria model with gamma distributed infection and treatment stages. The goals are to study effects of timing of treatment, the impact of asymptomatic population, and the treatment drug with dosing protocol.
(Received June 30, 2023)

1192-92-26140
Ryka C. Chopra*, San Francisco Conservatory of Music. A Mathematical Approach for Optimizing Linguistically-Induced Analgesia.
In 2022, the Stanford-Lancet Commission on the North American Opioid Crisis estimated that without urgent intervention, 1.2 million people will die due to opioid abuse by the end of this decade. National Institute of Health statistics show that over the period 1999-2020, opioid prescriptions have increased 104-fold and opioid abuse related deaths have increased 4 -fold. While opioids are widely prescribed as our primary defense against pain, medical practitioners are advocating for alternatives to opioids, given the serious side-effects. One option that has gained traction in recent years is Music-Induced-Analgesia (MIA), or pain therapy using music. Theoretically, music is a somatosensory stimulant that acts on our sensory pathways to reduce pain, yet experimental evidence focusing on the melodic aspects of music have yielded little proof of its efficacy. In this project, I argue that the efficacy of MIA is largely dependent on the linguistic cues embedded in the lyrics of a song. To this end, I develop a mathematical model that shows how hopeful linguistic cues embedded in the lyrics of a song positively impacts a patient's affective-motivational state by manipulating the perceived discount rate of future pain alleviation. With a decline in the future discount rate, the expected net benefit of pain relief over a patient's lifetime increases, and this positive emotion triggers the nucleus accumbens to release additional dopamine, allowing better pain management. I test my theory by building a sentiment scoring computational linguistic algorithm utilizing natural language processing principles that assigns songs a sentiment score based on their lyrics. In a randomized control trial where 35 participants undergo a cold-pressor test under different musical stimuli, I find that hopeful futuristic lyrics, on average, increases pain tolerance by 18.3 percent and reduces pain intensity by 11.6 percent. In addition to pain relief, hopeful futuristic lyrics also improves the cardiovascular parameters. Further, I find that the optimal impact comes from "upbeat music" - a combination of hopeful, futuristic lyrics and happy melody of lower cortical complexity that also significantly lowers an individual's discount rate of the future, thus providing support for MIA's viability as an alternative to opioids.
(Received July 19, 2023)

Natalie Burton, Arizona State University, Steffen Eikenberry, Arizona State University, Claire Gan*, Arizona State University, Jules Perez*, Arizona State University, Marina Rodriguez, Arizona State University. Watering the West: Exploring Agricultural Water Use in the Colorado Basin States.
Water resources in the arid western US are under pressure, especially within the Colorado Basin states. Excluding thermoelectric withdrawals, these seven states account for nearly a third of all water withdrawals in the contiguous United States, with over three-fourths of this water going toward agriculture. Agricultural water use footprints and flows through the system were characterized using a variety of datasets from the United States Department of Agriculture (USDA), the United States Geological Survey (USGS), and the Bureau of Land Management (BLM), revealing that the majority of irrigation water
in the Colorado Basin goes toward vegetables (10.4\%), orchards (27\%), and especially animal feed crops (53.5\%) (i.e., hay, corn grain, corn silage, pasture, and soybeans). Outside California and Arizona, animal feed crops account for $72 \%$ of irrigated acreage and $44 \%$ of water usage within the basin. Furthermore, it was determined that most basin states are reliant upon feed imports, especially California. This investment in water yields 35.34 billion kilograms of animal products, amounting to $28 \%$ of milk, $15.5 \%$ of beef, $8.6 \%$ of egg, $3.1 \%$ of chicken, and $1.8 \%$ of pork production in the US. A $20 \%$ reduction in the two major feed crops, hay and corn silage, could reduce water consumption in the basin states by over 4 million acre-feet, which is sufficient to satisfy long-term Colorado River conservation needs, at the cost of under 9\% of milk and beef production. Moreover, re-orienting production to higher value horticultural crops could increase farm sales while lowering water use. (Received July 19, 2023)

1192-92-26343
Anne Joyce Shiu*, Texas A\&M University. Sign conditions for analyzing chemical reaction systems.
Reaction networks arising in applications - such as in systems biology, ecology, and chemical engineering - are often modeled with mass-action kinetics. This setup generates systems of polynomial ODEs involving many (typically unknown) parameters. Such systems may exhibit interesting dynamics, such as oscillations. Another important property is multistationarity, which refers to the capacity for two or more steady states (mathematically, this means two or more positive real solutions to a system of polynomials). This talk showcases how we can investigate these dynamical properties using sign conditions and sign-pattern matrices. In particular, results on multistationarity arise from necessary and sufficient conditions in terms of sign vectors for the injectivity of families of polynomial maps with arbitrary real exponents defined on the positive orthant. Additionally, analyses of Hopf bifurcations (leading to oscillations) are aided by sign-pattern matrices arising from Jacobian matrices. (Received July 25, 2023)

## 1192-92-26431

Daniel Alfonso-Travieso*, University of Central Florida. Examining the Effects of Routes of Exposure in a Physiologically Based Pharmacokinetic (PBPK) Model of Chloroform in Rodents. Preliminary report.
Physiological Based Pharmacokinetic (PBPK) models use ordinary differential equations to model the concentration of toxicants in an organism over time. This project aims to develop a multi-route PBPK model to predict the impact of different routes of exposure to chloroform- namely oral, dermal, intravenous, and inhalation- on concentration over time. Two simulations were conducted. A base model, where exposure was instantaneous, and a multi-route model using a more realistic time-release exposure. The base model showed that a dermal dose led to the highest concentration in most tissue compartments over time. The multi-route model showed intravenous and oral exposure were the most efficient, leading to higher concentrations in the organ and tissue compartments over time. Dermal and inhalation exposures led to a slower distribution of chloroform through the compartments. The multi-route model was further investigated by comparing simulated results to a published data set from an experiment on rats. Ultimately, the models accurately captured the pharmacokinetics of chloroform in both rats and mice, suggesting that PBPK modeling is an appropriate strategy for further research into the effects of chloroform. (Received July 28, 2023)

## 1192-92-26542

Christopher E Miles*, Department of Mathematics, UC Irvine. Decoding stochastic reaction-diffusion RNA dynamics from imaging data with point process inference. Preliminary report.
Quantitative models in biology largely partition into either classical mechanistic (e.g., differential equations) or statistical/machine-learned. The former tend to fail to faithfully incorporate noisy, heterogeneous data, whereas the latter struggle to extract underlying interpretable mechanisms. In this talk, I will share a humble case study in the pursuit of bridging these techniques in the context of a specific biological problem. Advances in microscopy have provided snapshot images of individual RNA molecules within a nucleus. Decoding the underlying spatiotemporal dynamics is important for understanding gene expression, but challenging due to the static, heterogeneous, and stochastic nature of the data. I will write down a stochastic reaction-diffusion model and show that observations of this process follow a spatial point (Cox) process constrained by a reaction-diffusion PDE. Inference on this data resembles a classical "inverse problem" but differs in the observations of individual particles rather than concentrations. We perform inference using variational Bayesian Monte Carlo with promising results. However, a number of seemingly open computational challenges remain in the development of scalable and extendable techniques for this inverse problem. This work is in collaboration with the Fangyuan Ding lab of Biomedical Engineering at UCI.
(Received July 31, 2023)

## 1192-92-26549

Christina Duron, Pepperdine University, Kamryn Kamps, Pepperdine University, John Mello*, Pepperdine University. Using Networks to Evaluate an Infrastructure's Condition During Seismic Activity: A Proposed Approach. Preliminary report. In 2013, the White House issued Presidential Policy Directive 21 (PPD-21) on Critical Infrastructure Security and Resilience to advance a nationally unified effort to ensure a secure, functioning, and resilient critical infrastructure. In 2017, US Senator Dianne Feinstein expressed her concern that inspections conducted on critical infrastructure may not adequately consider the safety consequences of increasingly extreme weather patterns linked to climate change or considerable seismic hazards. Engineers responded to these opportunities by updating surveillance and monitoring programs designed to capture actual infrastructure behavior during operations. While advances in modern instrumentation and computer technologies make it easier to acquire behavior during both normal and extreme operating conditions, interpreting the data for decision makers who ensure continued and safe infrastructure operations remains a challenge. Consequently, the need exists for the development of algorithms that can evaluate an infrastructure's existing condition based on measurements acquired on the infrastructure during normal and extreme operations. Key to achieving this, however, is developing indicators that relate a monitoring location's relative importance to conditions within the infrastructure. This poster describes the development of a network centrality-based evaluation procedure to assess which measured responses of an infrastructure are the most critical for monitoring applications. The procedure is described and applied to a case study involving measured seismic responses acquired during an actual earthquake on an embankment dam. Results and implications are discussed within the context of the mathematical procedure as well as against engineering judgment. If successful, the procedure has the potential to aid decision makers during periods of heightened operations and extreme conditions.
(Received July 31, 2023)

1192-92-26744
Helen Byrne, University of Oxford, Heyrim Cho, Department of Mathematics, University of California, Riverside, Allison L
Lewis*, Lafayette College, Kathleen M Storey, Lafayette College. Inferring tumor cell line interaction types using the LotkaVolterra model with various experimental designs.
The Lotka-Volterra model is widely used to model interactions between two species, but can also be used to describe microscale interactions such as those between two co-existing tumor cell lines: for instance, radiosensitive versus radioresistant tumor cells. In this study, we assess the practical identifiability of the Lotka-Volterra model in such a setting, comparing three different experimental designs with regards to their ability to inform both model parameters as well as the interaction type (e.g., whether the two cell lines interact in a competitive, mutualistic, or antagonistic manner). Additionally, we investigate how the predictive power of the model-that is, its ability to fit data from experiments in which the initial ratios of the two cell types differ from the data to which it was calibrated-is affected by the choice of experimental design. This study highlights the importance of conducting a rigorous experimental design analysis, in order to produce a robustly-calibrated model that can accurately represent the interaction mechanism between different cell populations when predicting how the tumor will evolve and respond to treatment.
(Received August 04, 2023)

1192-92-26815
Robert S Eisenberg, Illinois Institute of Technology, Huaxiong Huang, York University, Zilong Song*, Utah State
University, Shixin Xu, Duke Kunshan University. A Bubble Model for the Gating of Kv Channels. Preliminary report. Voltage-gated Kv channels play fundamental roles in many biological processes, such as the generation of the action potential. The gating mechanism of Kv channels is characterized experimentally by single-channel recordings and ensemble properties of the channel currents. In this talk, we propose a bubble model coupled with a Poisson-Nernst-Planck (PNP) system to capture the key characteristics, particularly the delay in the opening of channels. The coupled PNP system is solved numerically by a finite-difference method and the solution is compared with an analytical approximation. We hypothesize that the stochastic behaviour of the gating phenomenon is due to randomness of the bubble and channel sizes. The predicted ensemble average of the currents under various applied voltage across the channels is consistent with experimental observations, and the ColeMoore delay is captured by varying the holding potential.
(Received August 06, 2023)

1192-92-27105
Haridas K. Das*, Oklahoma State University, Lucas M. Stolerman, Oklahoma State University. Exploring the Impact of Network Geometry and Human Mobility on Epidemic Thresholds in Metapopulation Model.
The SIR-network mode is a metapopulation network model that describes the dynamics of an infectious disease in a city where each neighborhood is a network node, and the edges of a network represent the daily flux of people between the nodes. We utilize this metapopulation model to study the specific role of human mobility levels and network geometry and consider a range of geometric structures. For star-shaped networks where all nodes only connect to a center, we obtain the same epidemic threshold formula as previously found for fully connected networks, where all nodes have the same infection rate except one. Next, we analyze cycle-shaped networks that yield different epidemic thresholds than star-shaped ones. We then analyze more general classes of networks by combining the star, cycle, and other structures, obtaining classes of networks with the same epidemic threshold formulas. We present some conjectures on even more flexible networks and complete our analysis by presenting simulations to explore the epidemic dynamics for the different geometries.
(Received September 22, 2023)

## 1192-92-27637

William S. Hlavacek, Los Alamos National Laboratory, Yen Ting Lin, Los Alamos National Laboratory, Abhishek Mallela*, Los Alamos National Laboratory. Differential contagiousness of respiratory disease across the United States. The initial contagiousness of a communicable disease within a given population is quantified by the basic reproduction number, $\mathcal{R}_{0}$. This number depends on both pathogen and population properties. On the basis of compartmental models that reproduce Coronavirus Disease 2019 (COVID-19) surveillance data, we used Bayesian inference and the next-generation matrix approach to estimate region-specific values for 280 of 384 metropolitan statistical areas (MSAs) in the United States (US), which account for $95 \%$ of the US population living in urban areas and $82 \%$ of the total population. We focused on MSA populations after finding that these populations were more uniformly impacted by COVID-19 than state populations. Our maximum a posteriori (MAP) estimates for $\mathcal{R}_{0}$ range from 1.9 to 7.7 and quantify the relative susceptibilities of regional populations to spread of respiratory diseases.
(Received August 16, 2023)

1192-92-27833
Donggu Lee*, Konkuk University. Role of NK cells and ionizing radiation in regulation of lung cancer progression. Preliminary report.
Neutrophils play a crucial role in the innate immune response in many diseases. However, tumor-associated neutrophils (TANs) and STATs can either promote or inhibit tumor growth through cross-talk with the tumor microenvironment. In this study, we developed and analyzed mathematical models to investigate the dynamic transition between N1 and N2 TAN phenotypes and how the immune system impacts tumor growth in the microenvironment. We examined the dynamics of TANs and tumors in response to various biochemical stimuli, including TGF-beta, IFN-gamma, and NK cells. Our models yielded several optimal tumor prevention strategies, including ionizing radiation (radiotherapy) to kill tumors. Our model predictions aligned well with experimental data, and from our analysis, we derived new insights. We observed a positive correlation between the ratio of N2-to-N1 TANs and NK cells and tumor progression, suggesting the need for optimal combinations of anticancer treatments.
(Received August 21, 2023)

Zhilan Feng, National Science Foundation, John W Glasser*, The US Centers for Disease Control and Prevention. Mechanistic modeling of SARS-CoV-2 transmission in the United States.
During the COVID-19 pandemic, we endeavored to keep pace with understanding of biological phenomena that might affect SARS-CoV-2 transmission by modifying SEIR metapopulation models structured via age, location, or strain. With probabilities of infection on contact and initial conditions from a serial, cross-sectional survey of antibodies to nucleocapsid protein among commercial laboratory clients throughout the United States and all save one other parameter from the literature, our age- and location-structured model reproduced seroprevalence from this and another nationwide survey, of antibodies to spike as well as nucleocapsid protein among blood-donors, remarkably well. While those surveys were either discontinued before or refocused during 2022, our model reproduced available but inevitably less accurate subsequent observations less well. In this talk, we illustrate via analytical and simulation results that it is possible to a) model transmission from first principles (i.e. without fitting) and b) reliably answer counterfactual questions (about, e.g., the impact of control measures), but that c) strain replacement may be a multifactorial phenomenon whose mechanisms (i.e., effective combinations of factors) are not unique. (Received August 22, 2023)

## 1192-92-28039

Miriam Goldman*, UCSF, Katherine Pollard, Gladstone Institutes, UCSF, Chunyu Zhao, Gladstone Institutes, UCSF. Integrating population structure into bacterial GWAS using micro-GLMMs to increase the ability to find associations in the host-microbiome interactions. Preliminary report.
The human body is home to a complex community of microorganisms, known as the microbiome, which differs in composition between individuals and is correlated with various human diseases. When two people harbor the same microbial species, the cells within those populations are genetically diverse, underscoring the limitations of using species to gain insight into hostmicrobiome interactions. The loss and gain of specific microbiome genes is strongly associated with differences in host disease status. To identify disease-associated microbiome genes, we developed a statistical model that can be used to perform a microbiome-wide association study (MWAS) for any continuous or binary host trait. Building on the work done on generalized linear mixed-effects models from human genetics, we have developed microbiome generalized linear mixed-effects models (micro-GLMMs) to perform association tests that link microbiome gene copy number variation (CNVs) within each microbiome species to host traits while accounting for microbial relatedness using a genetic relatedness matrix estimated for a given species using single nucleotide variants within the core genome of that species. This micro-GLMM can be used in cases with unequal cases and controls, is scalable to thousands of hosts, and has a controlled type 1 error rate. (Received August 24, 2023)

## 1192-92-28070

Thomas Y Chen, Columbia University, Hyunwoo Park*, Carnegie Mellon University. Mathematical Overview of Hebbian Learning. Preliminary report.
Hebbian learning, a fundamental concept in both neuroscience and machine learning, plays a significant role in the process of synaptic plasticity. It refers to the modification of the strength of connections between neurons or artificial units based on the correlation of their activities. In this paper, we present a mathematical formulation of Hebbian learning that formalizes this process of synaptic plasticity and provides a quantitative framework for understanding its mechanisms. We derive a set of equations that describe how the weights of connections between neurons change as a function of their pre- and post-synaptic activities. We also investigate the properties of the Hebbian learning rule and how it relates to principal component analysis. This mathematical formulation of Hebbian learning provides a valuable tool for studying synaptic plasticity in both biological and artificial systems, and has the potential to facilitate the development of new algorithms for machine learning and artificial intelligence.
(Received August 25, 2023)

## 1192-92-28082

Jordan Collignon*, University of California, Merced, Wesley Naeimi, University of Massachusetts, Amherst, Tricia R Serio, University of Washington, Suzanne Sindi, University of California, Merced. [PSI]-CIC: A High-Throughput Deep-Learning Pipeline for Analysis and Annotation of Sectored Yeast Colonies. Preliminary report.
The $\left[P S I^{+}\right]$prion phenotype in yeast manifests as a white, pink, or red color pigment related to the specific prion variant. Experimental manipulations destabilize prion phenotypes, and allow colonies to exhibit [ $p s i^{-}$] (red) sectored phenotypes within otherwise completely white colonies. However, the mechanisms governing both size and quantity of sectors remain unknown. Images of experimental yeast colonies exhibiting sectored phenotypes offer an abundance of data to help uncover mechanisms of sectoring, yet the structure of sectored colonies is ignored in traditional biological pipelines. This is both because colony counting is labor intensive and procedures for characterizing sectored colonies have not been standardized. In this talk, we introduce a computational pipeline, $[P S I]$-CIC, designed to identify and characterize features of sectored yeast colonies. Due to a lack of annotated images, we use synthetic images of sectored yeast colonies to train our neural network and test it on a set of 11 expertly annotated images of $\left[P S I^{+}\right],\left[p s i^{-}\right]$, and sectored colonies. Our pipeline correctly predicts the colony state and frequency of sectors in approximately $89.5 \%$ of colonies detected in hand annotated experimental images. With this information, we plan to extend our approach to categorizing colonies grown under different experimental conditions, allowing for more meaningful and detailed comparisons between experiments performed on yeast. Our approach aims to streamline the analysis of sectored yeast colonies providing a rich set of quantitative metrics to compare with mathematical models of sector formation and provide insights into mechanisms driving the curing of prion phenotypes. (Received August 25, 2023)

Hassan Chehaitli, McMaster University, Tom Hurd, McMaster University, Weijie Pang*, Wentworth Institute of Technology. Impact of asymptomatic COVID-19 carriers on pandemic policy outcomes.
The speed, range and influence of the Covid-19 virus exceed any pandemic in history. To find reasons for the incredible fast spread, we divide the viral carriers in a typical SEIR pandemic model into two categories of ordinary viral carriers and
asymptomatic viral carriers. In addition, we include parameters to measure the effects of different public health policies on the spread in a pandemic model. By the simulation of this model, we analyze the effect of asymptomatic viral carriers to the spread and pandemic policy outcome and provide feasible suggestions of containment to regulators.
(Received August 26, 2023)

Keoni Castellano*, University of Nevada, Las Vegas, Rachidi B. Salako, University of Nevada, Las Vegas. On the multiplicity of endemic equilibria for a diffusive SIS epidemic model with mass-action transmission mechanism. In this talk, we show, under appropriate assumptions on the parameters, the existence of multiple endemic equilibria for a diffusive SIS epidemic model with mass-action transmission mechanism when the basic reproduction number, $\mathcal{R}_{0}$, is either less than one or greater than one. Previous studies have left open the question of extinction or persistence of disease when $\mathcal{R}_{0}<1$. Our results settle completely this open question
(Received August 27, 2023)

## 1192-92-28150

Robert Blackwell, Scientific Computing Core, Flatiron Institute, Adam Lamson, Center of Computational Biology, Flatiron Institute, Michael Shelley, Center of Computational Biology, Flatiron Institute, Zihan (Steven) Zhang*, Department of Applied Mathematics, University of Washington. Use Adaptive Fast Function Approximator in Motor-Filament Binding Kinetics. Preliminary report.
The cytoskeleton, consisting of biopolymer filaments, molecular motors, and passive crosslinking proteins, provides the internal structure of cells that facilitate movement, growth, and cell division. Understanding the microscopic motor-filament kinetics and dynamics is essential for comprehending macroscopic behaviors of reconstituted cytoskeletal assemblies, such as self-organized flow and active stress. In this study, we employ an adaptive fast Chebyshev approximation based on tree search alongside parallel computing to accurately recover the equilibrium distribution of crosslinking proteins, thus satisfying detailed balance in binding through kinetic Monte Carlo sampling, while maintaining cost-effectiveness. Additionally, we offer expandable features, including segregating the simulation process via pre-building and allowing the free-loading of different explicit formulations of the motor's potential energy. This research has the potential to better describe the evolution of cytoskeletal active matter.
(Received September 25, 2023)

## 1192-92-28185

Jiahui Chen*, University of Arkansas, Weihua Geng, Southern Methodist University, Johannes Tausch, Southern Methodist University. A Cartesian FMM-accelerated Galerkin boundary integral Poisson-Boltzmann solver.
The Poisson-Boltzmann model is an effective and popular approach for modeling solvated biomolecules in continuum solvent with dissolved electrolytes. In this paper, we report our recent work in developing a Galerkin boundary integral method for solving the linear Poisson-Boltzmann (PB) equation. The solver has combined advantages in accuracy, efficiency, and memory usage as it applies a well-posed boundary integral formulation to circumvent many numerical difficulties associated with the PB equation and uses an O (N) Cartesian Fast Multipole Method (FMM) to accelerate the GMRES iteration. In addition, special numerical treatments such as adaptive FMM order, block diagonal preconditioners, Galerkin discretization, and Duffy's transformation are combined to improve the performance of the solver, which is validated on benchmark Kirkwood's sphere and a series of testing proteins.
(Received August 27, 2023)

## 1192-92-28188

Parsa Seyfourian, University of British Columbia - Vancouver, Adhvaith Sridhar*, University of Minnesota Twin Cities. Modeling the Spread of the H5N1 Influenza for the Development of Public Health Response Policy.
The avian influenza, also known as bird flu, is a zoonotic infection that can spread to humans. The H5N1 strain of the Avian influenza is thought to be the cause of a recent epidemic in the United States. The infection of birds in poultry farms is imputed to migratory birds carrying the H5N1 influenza. Outbreaks of this flu variant could lead to significant human health concerns, economic loss, and a public health crisis. It is imperative to slow the transmission of the virus and mitigate the serious consequences of the outbreak through a coordinated and efficacious public health response. However, a successful response requires a well-informed, data-driven foundation in order to enable the efficient allocation of resources to regions that require them most and in a way that ensures the stop of the spread of cases. In order to enable the generation of accurate predictions that may be used to inform decisions about resource allocation and spread management, we have developed a SIR model (SIV-SIRV) and simulation to predict the number of infected individuals, vaccinations, recovered individuals, and other metrics in the case of an avian and human outbreak. The dynamics of the model are examined with the basic reproductive number. Numerical simulations and incorporation of stochastic elements are used to support the model's predictions, and sensitivity analyses are used to determine the most effective control and preventative measures. Our differential-equation model serves as a comprehensive tool to dictate the development, revision, and execution of a public policy-based response to the epidemic. Herein, we describe the applications of the model to the enactment of policy surrounding various facets of a public health response. We elucidate the predication of the effectiveness of these response components upon our model in conjunction with the revision of the model using new data collected from these responses. Our model-based public policy response recommendations address embedded inequities in healthcare access due to geographic, socioeconomic, cultural, and medical vulnerabilities and provide healthcare providers and public health departments with a tool that can quantitatively combat disparities in future public health scenarios as well. (Received August 29, 2023)

Christina Duron*, Pepperdine University. Incorporating Undergraduate Students in Research on Assessing Dam Safety Using Network Analysis. Preliminary report.
Modeling complex, relational data with networks is an excellent way to engage and excite students about the practicality and applicability of network analysis. Yet onboarding undergraduate students as research assistants can be challenging for both
faculty and students, especially when the students are non-mathematics majors. In this presentation, I will share my experience from the past summer of working with two undergraduate research assistants on using networks to evaluate a dam's condition during seismic activity. I will focus on lessons learned, especially related to student preparation, network construction, the "black-boxing" of code in R, and the use of R Markdown to summarize and present results.
(Received August 29, 2023)

## 1192-92-28301

Antoni Luque*, University of Miami. Predicting the geometrical landscape of viral capsids.
Viruses protect their infective genome in protein shells called capsids, which assemble from multiple copies of one main structural protein: The major capsid protein. Capsids and their proteins are an ideal antiviral target due to their numerous roles in the life cycle of viruses. However, the molecular investigation of viral capsids faces severe limitations because most viruses cannot be grown in laboratory conditions. This impediment is particularly problematic given the thread of new emerging viruses associated with climate change and the impact of human activity on ecosystems. In this contribution, I will share the progress of my research group to provide a solution to this problem, applying biophysical modeling, machine learning, and bioinformatics to data obtained from collaborations carrying environmental sampling and experiments. First, I will introduce the generalized geometrical theory of viral capsids, which provides a mathematical framework to navigate the possible landscape of architectures among viral capsids, including common symmetrical capsids based on icosahedral symmetry as well as asymmetric capsids like those observed among retroviruses. Second, I will show how we used the theory of viral capsids to predict the existence of capsids that had not been observed previously and how we validated their existence empirically. Finally, I will share our latest progress investigating the asymmetric capsids of HIV, which we have recently characterized thanks to the development of the triangular path framework, our latest addition to the generalized theory of viral capsids.
(Received August 29, 2023)

1192-92-28359
Sally Blower, University of California, Los Angeles, Justin Okano, University of California, Los Angeles, Joan Ponce*, UCLA. Geospatial modeling of access to antiretroviral therapy (ART). Preliminary report.
We use dynamic transmission models to assess the efficacy of interventions for infectious diseases. As epidemics evolve, it is beneficial to incorporate diverse models to gain a more comprehensive understanding of the disease dynamics and control measures. Differential equation models have successfully depicted long-term disease dynamics, but controlling lifelong endemic diseases like HIV requires consistent access to antiretroviral therapy (ART). In some areas in Sub-Saharan Africa healthcare is a limited resource. Measuring disparities in ART access through a model that focuses on spatial healthcare provision and precisely evaluates a community's access to this resource is crucial. In this talk, I introduce a spatial interaction model that utilizes geospatial data to estimate the spatial accessibility of ART in Malawi. This model allows us to identify inequities in the opportunity to access ART which could be due to differences in demand for the treatment of HIV, ART supplies, or the geographic accessibility of healthcare facilities that provide ART. (Received August 30, 2023)

## 1192-92-28473

Jonathan Shane Welker*, University of North Alabama. An Immuno-Epidemiological Model of Foot-and-Mouth Disease (FMD) in the African Buffalo Population with Carrier Transmission. Foot-and-mouth disease (FMD) is a highly contagious disease that spreads among cloven-hoofed animals. FMD is not deadly, but can cause delays in meat and dairy production. We present a novel immuno-epidemiological model of FMD in African buffalo host populations. Upon infection, hosts can become carriers or acutely infected. We divide the infectious population into these two stages to dynamically capture the immunological characteristics of both phases of the disease to better understand the carrier's role in disease transmission. We define the within-host viral-immune kinetics dependent epidemiological basic reproduction number $\mathcal{R}_{0}$ and show that it is a threshold condition for the local stability of the diseasefree equilibrium and existence of the endemic equilibrium. By using a sensitivity analysis approach developed for multi-scale models, we assess the impact of the acute infection and carrier phase immunological parameters on $\mathcal{R}_{0}$. Our numerical results show that the within-carrier infected host immune kinetics parameters and the susceptible individual recruitment rates play significant roles in disease persistence, which are consistent with experimental and field studies.
(Received September 22, 2023)

1192-92-28486
Qixuan Wang*, University of California, Riverside. A probabilistic Boolean model on cell fate regulation.
In tissues, proper regulations of cell fate decisions are important in maintaining the shape and functions of the tissue. In this talk, I will present our recent research in modeling cell fate regulation mechanisms, using mammalian hair follicles as a model system. Hair follicles are mini skin organs, and they are highly dynamic in the way that they can undergo cyclic growth during the lifespan of the organism. To maintain tissue homeostasis and functions of a hair follicle, the follicle transient amplifying epithelial cells need to make correct decisions among cell division, differentiation and apoptosis, instructed by various signals produced by the follicle itself as well as by the surrounding skin environment. We develop a probabilistic Boolean model based on both literature and published single-cell RNA sequencing data. Using both computational simulations and attractor analysis, we investigate how hair follicle epithelial cells respond to TGF- $\beta$, BMP and TNF, so to make the correct cell fate decision, and how signals cooperatively regulate hair follicle growth dynamics. In addition, model predictions of gene knockout and over-expressions could potentially guide future experiment designs. (Received August 31, 2023)

Chris M. Heggerud*, University of California, Davis. Coupling the socio-economic and ecological dynamics of cyanobacteria. In recent decades freshwater lakes have seen an increase in human presence. A common byproduct of this human presence is eutrophication, which readily results in harmful cyanobacteria (CB) blooms. In this work we propose a model that couples the socio-economic and ecological dynamics related to cyanobacteria systems. The socio-economic dynamics considers the choices
a human population makes regarding whether or not to mitigate their pollution levels. These choices are based on various costs related to social ostracism, social norms, environmental concern and financial burden. The coupled model exhibits bistable dynamics, with one stable state corresponding to high mitigation efforts and low CB abundance, and the other to low mitigation efforts and high CB abundance. Furthermore, we consider social interactions among a network of lakes and present dynamic outcomes pertaining to various associated costs and social situations. In each case we show the potential for regime shifts between levels of cooperation and CB abundance. Social ostracism and pressure are shown to be driving factors in causing such regime shifts.
(Received August 31, 2023)

1192-92-28532
Chris M. Heggerud*, University of California, Davis. Understanding transient crawl-bys in ecological timeseries.. Preliminary report.
Transient dynamics pose unique challenges when dealing with predictions and management of ecological systems yet little headway has been made on understanding when an ecological system might be in a transient state. In particular, given an ecological timeseries, it is difficult to detect what type of transient dynamic is occurring, or if one is occurring at all. Through a series of simplifications we analyze synthetic data known to exhibit crawl-by type transient dynamics. Using dynamical systems theory, we create metrics that predict the crawl-by and furthermore to understand how long the crawl-by will last. (Received August 31, 2023)

## 1192-92-28562

Rowan J Barker-Clarke*, Theory Division, Cleveland Clinic Lerner Institute, Cleveland, OH. Graph-based Representations of Cancer Evolution.
We introduce an innovative approach to comprehend mutational evolutionary processes in cancer, employing a graph-based representation. Genetic mutations, the driving force behind evolution, are visualized as edges linking genotypes within a hypercube structure. Our model characterizes evolution as diffusion on this graph, integrating the Fokker-Planck equation to encompass selection pressures and mutations. Through this fusion, we can both analyse topological features of evolutionary processes, and investigate the inherent timescales of mutational evolution on the hypercube. We use via mean first-passage time analysis, extending analogies from electrical network theory to abstract this process onto mutational graphs. This study enhances our understanding of timescales in genetic evolution and has potential applications in evolutionary control, genetics, and network theory.
(Received August 31, 2023)

## 1192-92-28601

Christian Parkinson*, University of Arizona. Compartmental Models for Epidemiology with Noncompliant Behavior In this work, we use a modified version of the SIR model for epidemics to study and quantify the effect of noncompliance to nonpharmaceutical intervention measures on the spread of an infectious disease. Our model distinguishes between symptomatic infected and asymptomatic infected populations, and treats noncompliant behavior as a social contagion. We theoretically explore different scenarios such as the occurrence of multiple waves of infections, and provide local asymptotic analysis. This work began as an REU project
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## 1192-92-28620

Iniya Anandan, University of Texas at Dallas, Benedict J Kolber, University of Texas at Dallas, Kayla Kraeuter, Duquesne University, Rachael L Miller Neilan*, Duquesne University, Carley Reith, Duquesne University. A 3-D Agent-Based Model of Pain-Related Neurons in the Amygdala.
Neuropathic pain is the predominant cause of pain for $20 \%$ of people and involves brain areas such as the central nucleus of the amygdala (CeA). Within the CeA, neurons expressing protein kinase c-delta (PKC $\delta$ ) are pro-nociceptive (i.e., increase pain) while neurons expressing somatostatin (SST) are anti-nociceptive (i.e., decrease pain). In this talk, I will describe our progress towards developing a 3-D agent-based model of PKC $\delta$ and SST neurons in the CeA and the use of this model to explore the pharmacological targeting of these two neural populations in modulating neuropathic pain. Our agent-based model includes a realistic 3-D spatial representation of the CeA and its subnuclei and a network of directed links that preserves the connectivity properties of PKC $\delta$ and SST neurons. The model is programmed in NetLogo3D and consists of 13,000 neurons (agents) with cell-type specific properties and behaviors estimated from laboratory data. During each model time step, neuron firing rates are updated based on an external stimulus, inhibitory signals are transmitted between neurons via the network, and a measure of pain-related output from the CeA is calculated as the difference in firing rates of pro-nociceptive PKC $\delta$ neurons and antinociceptive SST neurons. Model simulations were performed to explore differences in pain-related output for different spatial distributions of PKC $\delta$ and SST neurons. Furthermore, a sensitivity analyses was conducted to explore the impact of several key parameters on model output. Our results show that the localization of these neuron populations within CeA subnuclei is a key parameter in predicting pain-related output from the CeA and identifying spatial and cell-type pharmacological targets for pain.
(Received September 01, 2023)

David Christian Elzinga*, University of Wisconsin-La Crosse, Christopher Strickland, University of Tennessee, Knoxville. Baa-ttling Sore Mouth in Sheep with Mathematical Modeling. Preliminary report.
Sore mouth is a ubiquitous, highly contagious, enzootic disease in ruminants that has profound economic and cultural impacts throughout the world. Although sore mouth has been observed for centuries, advancements in our ability to prevent and mitigate outbreaks have been minimal, with some outbreaks achieving morbidity rates of nearly $100 \%$ in a matter of weeks. The animal science literature has proposed a number of recommendations to prevent sore mouth outbreaks such as the isolation of infectious sheep, vaccination, and pasture management; however, several open questions remain regarding the relative importance of these approaches, their possible interactions, and their implementation. As a result, the consequences of sore mouth outbreaks continue to degrade the prosperity of animal welfare, economic output, and cultural celebrations. We
developed two mathematical models of sore mouth in sheep to address these challenges, including (1) an ordinary differential equation model to simulate outbreaks in large, well-mixed flocks and (2) a cellular automata continuous time Markov chain model to simulate outbreaks in smaller flocks in spatially heterogeneous environments. We applied both models to measure the effectiveness of existing management strategies and to identify novel approaches that could either prevent or mitigate outbreaks. Our results suggest that (1) the routine surveillance of populations is essential to quickly isolate infectious sheep to prevent outbreaks, (2) shared infectious fomites can rapidly accelerate outbreaks and should be reduced or cleaned during a potential infection season, and (3) preventive vaccination with isolation can confer a degree of herd immunity that limits the impact of sore mouth outbreaks.
(Received September 01, 2023)

## 1192-92-28645

Margaret Cathcart*, University of South Carolina, Nihal Kumar*, Penn State University, Charles S Ohanian*, Muhlenberg College. Sex-Biased Predation in a Mating Model For Invasive Species Control.
Invasive species are nonnative species capable of causing economic and environmental harm and are very difficult and costly to eradicate. One of the strategies to control them is biocontrol, which involves the introduction of natural enemies (predators). There is evidence that some invasive species are being preyed based on sex-bias as a result of sexual dimorphism. In this work, we propose and investigate a predator-prey model where the prey population (invasive species) is structured by sex and the predators (unstructured) serve as agents that employ sex-biased predation for biocontrol. We provide conditions for the existence of equilibrium points and perform local stability analysis on them. We derive global stability conditions for the extinction state. We also show the possible occurrence of Hopf and saddle-node bifurcations. Multiple Hopf bifurcations are observed as the sex-biased predation rate is varied. This variation also shows the opposite consequences in the densities of the sex structured prey. Our results show that sex-biased predation can cause both stabilizing and destabilizing effects for certain parameter choices. Furthermore when intraspecific competition is minimized, it can aid in the extinction of the invasive species. We discuss the ecological implications of our results. This work was done under NSF grant number 1851948 at Ursinus College.
(Received September 01, 2023)

## 1192-92-28743

Jin Wang*, UTC. Multi-strain models for SARS-CoV-2 variants.
Although the World Health Organization has declared an end to the global COVID-19 pandemic, the emergence of SARS-CoV-2 new variants still poses a significant public health threat. We develop mathematical models to study the transmission and spread of COVID-19, with a focus on the interplay between different viral strains at the population level. We pay special attention to XBB, an Omicron subvariant of SARS-CoV-2 that was dominant in the first half of 2023. We validate the model outcomes with the surveillance data from the US.
(Received September 02, 2023)

## 1192-92-28745

Jin Wang*, UTC. Modeling cholera dynamics: within-host pathogen kinetics and population-level disease spread.
We propose a mathematical modeling framework for cholera, a severe waterborne infection caused by the bacterium Vibrio cholerae. At the in-host level, we model the pathogenesis of Vibrio cholerae and its interaction with the host immune response using ordinary differential equations. At the population level, we study the transmission dynamics of cholera, with a special emphasis on reaction-diffusion equations that are formulated to model the spatial spread of the disease. We discuss a few approaches to bridge the infection dynamics between these two different scales. We present both mathematical analysis and numerical simulation results.
(Received September 02, 2023)

## 1192-92-28748

Nikolaos Voulgarakis*, Washington State University. Self-Regulated Symmetry Breaking Model for Stem Cell Differentiation. In disorder-to-order phase transitions, a system changes from a state of high symmetry where all states have equal accessibility (disorder) to a state of low symmetry with a restricted number of available states (order). Often, this transition is triggered by adjusting a control parameter that represents the inherent noise of the system. The process of stem cell differentiation can be viewed as a series of such symmetry-breaking events. Pluripotent stem cells, known for their ability to mature into various specialized cell types, are regarded as highly symmetrical systems. In contrast, differentiated cells exhibit lower symmetry since they can only perform a limited range of functions. The main difference between traditional disorderorder phase transitions and the proposed hypothesis of cell differentiation is that cell populations must be able to self-regulate intrinsic noise and navigate through critical points where spontaneous symmetry breaking, i.e., differentiation, takes place. In this talk, we will discuss a mean-field model for stem cell populations that considers the interplay between cell-to-cell cooperation, variability, and finite-size effects. We will present a feedback mechanism capable of regulating intrinsic noise. This process drives stem cell populations through various bifurcation points that can be viewed as spontaneous symmetrybreaking events. Standard stability analysis demonstrates that the system can differentiate into multiple cell types, which can be mathematically described as stable nodes and limit cycles. The presence of a Hopf bifurcation in our model is discussed in the context of stem cell differentiation.
(Received October 27, 2023)

1192-92-28763
Wenjian Liu*, Queensborough Community College, CUNY. Bayesian Phylogenetic Inference of of DNA Evolution Models. Preliminary report.
The information reconstruction problem on an infinite tree, is to collect and analyze massive data samples at the nth level of the tree to identify whether there is non-vanishing information of the root, as goes to infinity. This problem has wide applications in various fields such as biology, information theory and statistical physics, and its close connections to cluster learning, data mining and deep learning have been well established in recent years. It is quite challenging with the techniques used including Markov chains, statistics, statistical physics, information theory, cryptography and noisy computation. Although
it has been studied in numerous contexts, the existing literatures with rigorous reconstruction thresholds established are very limited. Inspired by a classical deoxyribonucleic acid (DNA) evolution model, the F81 model, and also taking into consideration of the Chargaff's parity rule by allowing the existence of a guanine-cytosine content bias, we study the noise channel in terms of an $4 \times 4$-state asymmetric probability transition matrix with community effects, for four nucleobases of DNA. The corresponding information reconstruction problem in molecular phylogenetics is explored, by means of refined analyses of moment recursion, in-depth concentration estimates, and thorough investigations on an asymptotic 4-dimensional nonlinear second order dynamical system. We rigorously figure out the sharp reconstruction bound when the sum of the base frequencies of adenine and thymine falls in a specific interval, which is the first rigorous result on asymmetric noisy channels with community effects.
(Received September 02, 2023)

## 1192-92-28780

Claire E Plunkett, University of Utah, Grant R Poulson, University of Utah, Jody R Reimer*, University of Utah. Mean first passage time analysis of long transient dynamics in a stochastic ecological system.
We investigate long transient dynamics in ecological systems, where the system trajectory stays near a non-attracting state for a long time before suddenly undergoing a regime shift. We consider a simple model with a saddle-node bifurcation. For parameter values above the bifurcation threshold, the model exhibits bistability, while for values below it, the model has long transient dynamics near a so-called ghost attractor. In the deterministic case, these regimes have distinct dynamical properties. In the stochastic case, both regimes can produce similar time series, fluctuating around one state for several generations before suddenly switching to another. We quantify the expected timing of this switch using mean first passage time (MFPT) theory, extending it to the case of the ghost attractor. We show that the MFPT varies continuously with the bifurcation parameter, obscuring the difference between the bistable and long transient regimes.
(Received September 03, 2023)

## 1192-92-28785

Khanh Dinh*, Columbia University, Simon Tavaré, Columbia University, Ignacio Vázquez-García, Memorial Sloan Kettering Cancer Center. Modeling cancer evolution as driven by copy number aberrations and point mutations, informed by bulk and single-cell DNA sequencing data. Preliminary report.
Over the last decade, bulk DNA-sequencing has allowed us to appreciate the sheer amount and diversity of the genomic changes associated with cancer development. More recently, advances in single-cell DNA-sequencing has enabled profiling of copy number aberrations (CNAs) at high resolution in thousands of cells, and has uncovered the level of intra-tumor heterogeneity. These technologies promise quantitative measurements of tumor dynamics, and measurements of the rate of chromosomal aneuploidy, whole-genome duplications and replication errors in tumors. We have developed a state-of-the-art mathematical model and simulation algorithm for studying single-cell dynamics in a population of cells, incorporating clonal selection of somatic driver mutations and copy number aberrations (CNAs), as well as accumulation of neutral passenger mutations and CNAs. The simulator follows population dynamics as input by the user, generates the clonal evolution forward in time, where clones are defined by their copy number and driver mutation profiles. The phylogeny of a sample is then computed backward in time. The algorithm is designed to be efficient for large cell populations while maintaining statistical accuracy in the sampled cells. The mathematical model and simulation method provide an efficient framework for modeling genomic changes during tumor growth, and are easily adaptable to accommodate newly uncovered genomic alterration mechanisms. We present several applications of the simulator package. The first study uncovers the selection forces driving chromosomal CNAs. The selection rates estimated with our method can remarkably predict the prevalence of Whole-genome duplication in each cancer type. Moreover, the selection rates fitted for a pan-cancer dataset show a strong correlation with the chromosomal driver gene count and potency. Together, these observations prove the biological relevance of our algorithm. The second part of this presentation delves into the development of an inference method for single-cell DNA-seq data, which poses several challenges over bulk data. We analyze the statistics of a dataset that are most representative of the genomic landscape and history, hence are appropriate for inferring CNA rates and chromosomal selection rates. (Received September 03, 2023)

1192-92-28803
Abba Gumel*, University of Maryland. Mathematics modeling and analyis of Wolbachia-based biocontrol of dengue mosquitoes. Preliminary report.
The release of Wolbachia-infected mosquitoes into the population of wild mosquitoes is one of the promising biological control methods for combating the population abundance of mosquitoes that cause deadly diseases, such as dengue. In this lecture, a two-sex mathematical model for the population ecology of dengue mosquitoes and disease will be presented. The model will be used to assess the potential population-level impact of the periodic release of Wolbachia-infected mosquitoes (into the wild mosquito population in the community) on the population abundance of the dengue mosquitoes and disease.
(Received September 03, 2023)

1192-92-28805
Abba Gumel*, University of Maryland. Can malaria eradication be achieved despite widespread Anopheles resistance to available insecticides?.
Malaria, a deadly disease caused by protozoan Plasmodium parasites, is spread between humans via the bite of infected adult female Anopheles mosquitoes. Over 2.5 billion people live in geographies whose local epidemiology permits transmission of $P$. falciparum, responsible for most of the life-threatening forms of malaria. The wide-scale and heavy use of insecticide-based interventions, notably long-lasting insecticidal nets and indoor residual spraying), during the period 2000-2015, resulted in a significant reduction in malaria incidence and burden in endemic areas, prompting a renewed quest for malaria eradication (through major international initiatives, such as the Zero by 2040 initiative). Numerous factors, such as Anopheles resistance to all currently-available insecticides and anthropogenic climate change, potentially pose important challenges to the eradication efforts. In this talk, a genetic-epidemiology framework for assessing the impact of insecticide resistance on malaria transmission dynamics and control will be presented. Specifically, questions on whether eradication can be achieved using existing insecticide-based control resources will be addressed. If time permits, the potential impact of some of the gene drivebased biological interventions being proposed as plausible alternative pathways for achieving malaria eradication will be briefly discussed.

Abba Gumel*, University of Maryland. Mathematical assessment of the role of pre-exposure prophylaxis on HIV dynamics in an MSM population.
The use of pre-exposure prophylaxis (PrEP), where approved antivirals are administered to uninfected high-risk individuals, is universally regarded as a promising strategy to prevent susceptible high-risk individuals from acquiring HIV infection from their infected partners. A number of antiviral drugs (and their combinations) have been developed and are being used as prophylaxis against the HIV/AIDS pandemic globally. This talk presents a mathematical model, which takes the form of a deterministic system of nonlinear differential equations, for assessing the population-level impact of the use of PrEP in an MSM (men-who-have-sex-with-men) population. The asymptotic stability properties of the equilibria of the model, as well as characterization of the bifurcation types the model undergoes, will be discussed. Theoretical conditions for the effective control or elimination of the disease in the MSM population will be derived.
(Received September 03, 2023)

## 1192-92-28828

Heiko Enderling, Department of Radiation Oncology, MD Anderson Cancer Center, Thomas Hillen, University of Alberta,
Edmonton, Canada, Ashna Patel*, Northwestern University, Megan Rhodes, University of Alberta, Edmonton, Canada
Alexandra Shyntar, University of Alberta, Edmonton, Canada. Investigating the Tumor-Invasion Paradox through Agent Based Modeling.
The tumor invasion paradox is defined as tumors with higher probabilities of cell death having a direct correlation with increased invasion rates. This is likely a biological consequence of immortality within cancer stem cells causing an insusceptibility to treatment. To investigate this, agent-based modeling is utilized to examine the rates of invasion of simulated tumors under both continuous and intermittent treatment courses and evaluated through spatiotemporal density maps. (Received September 03, 2023)

## 1192-92-28835

Xiunan Wang*, University of Tennessee at Chattanooga. From HIV to SARS-CoV-2: Mathematical Modeling of Viral Dynamics.
In this presentation, we delve into the field of viral dynamics modeling within two distinct contexts: HIV and SARS-CoV-2. First, we investigate a groundbreaking HIV experiment that showcases the effectiveness of vectored immunoprophylaxis. Utilizing mathematical modeling, we unveil the concept of a backward bifurcation and explore its far-reaching implications. Additionally, we dissect the intricate interplay between antibodies and the virus, revealing a world of complex dynamics, including bistable behavior. Shifting our focus to SARS-CoV-2, we introduce a reaction-diffusion model that characterizes viral infection within a heterogeneous environment. Our discussion encompasses the profound impact of diffusion, spatial heterogeneity, and incidence types on SARS-CoV-2 infection dynamics. This presentation provides a valuable opportunity for both students and mathematicians to explore mathematical modeling of viral dynamics.
(Received September 04, 2023)

## 1192-92-28896

Brittany Bannish*, University of Central Oklahoma. Mechanisms for Supporting Undergraduate Research and Some Resulting Student Projects.
I will discuss four different mechanisms for supporting undergraduate research, along with specific projects funded by those mechanisms. 1) The University of Central Oklahoma's (UCO) year-long RCSA grant pays students for $5 \mathrm{hr} / \mathrm{wk}$ of research and provides a small supplies or travel budget. I will highlight an RCSA project on the spread of Toxoplasma gondii in cats. 2) UCO's College of Mathematics and Science CURE-STEM program gives faculty the funds to pay a student researcher up to 10 $\mathrm{hr} / \mathrm{wk}$. I will present a CURE-STEM project on the effect of alcohol on circadian rhythms. 3) The National Institutes of Health R15 award provides up to $\$ 300,000$ over 3 years. I will present a collaborative project (between mathematicians and experimentalists) on the polymerization and degradation of the fibrin fibers which make up blood clots. 4) Conference travel support gives students the opportunity to present their research nationally, but attending a conference can also inspire younger students to start research. I will discuss a measles model that was inspired by a student's attendance at the Nebraska Conference for Undergraduate Women in Mathematics. By discussing various funding mechanisms and projects, I hope to provide ideas and encouragement to faculty and students interested in undergraduate research.
(Received September 04, 2023)

## 1192-92-28901

Lauren M Childs, Department of Mathematics, Virginia Tech, United States of America, Christina Edholm, Scripps College, Denis Patterson, Durham University, Joan Ponce, Arizona State University, Olivia Prosper, University of Tennessee/Knoxville, Zhuolin Qu*, University of Texas at San Antonio, Lihong Zhao, Virginia Tech. Modeling Immunity to Malaria with an Age-Structured PDE Framework.
Malaria is one of the deadliest infectious diseases globally, causing hundreds of thousands of deaths each year. It disproportionately affects young children, with two-thirds of fatalities occurring in under-fives. Individuals acquire protection from disease through repeated exposure, and this immunity plays a crucial role in the dynamics of malaria spread. We develop and analyze an age-structured PDE model, which couples vector-host epidemiological dynamics with immunity dynamics. Our model tracks the acquisition and loss of anti-disease immunity during transmission and its corresponding nonlinear feedback onto the transmission parameters. We derive the basic reproduction number (R0) as the threshold condition for the stability of disease-free equilibrium and interpret R0 probabilistically as a weighted sum of cases generated by infected individuals at different infectious stages and ages. Numerical bifurcation analysis demonstrates the existence of an endemic equilibrium, and we observe a forward bifurcation in R0. Our model reproduces the heterogeneity in the age distributions of immunity profiles and infection status created by frequent exposure. Motivated by the recently approved RTS, S vaccine, we also study the impact of vaccination and its implementation in areas with different seasonal variations.
(Received September 04, 2023)

## 1192-92-28975

Ren-Yi Wang*, Rice University. A Countable-Type Branching Process Model for the Tug-of-War Cancer Cell Dynamics. We consider a time-continuous Markov branching process of proliferating cells with a countable collection of types. Amongtype transitions are inspired by the Tug-of-War process introduced in McFarland et al. (2014) as a mathematical model for competition of advantageous driver mutations and deleterious passenger mutations in cancer cells. We introduce a version of the model in which a driver mutation pushes the type of the cell $L$-units up, while a passenger mutation pulls it 1-unit down. The distribution of time to divisions depends on the type (fitness) of cell, which is an integer. The extinction probability given any initial cell type is strictly less than 1, which allows us to investigate the transition between types (type transition) in an infinitely long cell lineage of cells. The analysis leads to the result that under driver dominance, the type transition process escapes to infinity, while under passenger dominance, it leads to a limit distribution. Implications in cancer cell dynamics and population genetics are discussed.
(Received September 05, 2023)

## 1192-92-29047

Kwadwo Antwi-Fordjour*, Samford University, Kendall Bearden, Samford University, Sarah Westmoreland, Samford University. Dual Fear Phenomenon in an eco-epidemiological model with prey aggregation. Preliminary report. In this talk, we shall present an analysis of an eco-epidemiological model that integrates infectious diseases in prey, prey aggregation, and the dual fear effect induced by predators. We establish criteria for determining the existence of equilibrium points, which carry substantial biological significance. We establish the conditions for the occurrence of Hopf, saddle-node, and transcritical bifurcations by employing fear parameters as key bifurcation parameters. Furthermore, through numerical simulations, we demonstrate the occurrence of multiple zero-Hopf ( ZH ) and saddle-node transcritical (SNTC) bifurcations around the endemic steady states by varying specific key parameters across the two-parametric plane. We demonstrate that the introduction of predator-induced fear, which hinders the growth rate of susceptible prey, can lead to the finite time extinction of an initially stable susceptible prey population. Finally, we discuss management strategies aimed at regulating disease transmission, focusing on fear-based interventions and selective predation via predator attack rate on infectious prey. (Received September 05, 2023)

## 1192-92-29073

Nakul Chitnis, Department of Epidemiology \& Public Health, Swiss Tropical and Public Health Institute and the University of Basel, Jim Michael Cushing*, Department of Mathematics \& Interdisciplinary Program in Applied Mathematics, University of Arizona, Alex Farrell, Caris Life Sciences, Tempe, Arizona, Junpyo Park, Department of Applied Mathematics, College of Applied Science, Kyung Hee University. A Darwinian SI Model for the Evolution of Pathogen Resistance.
We consider a basic susceptible-infected (SI) model under the assumption that the infection causing pathogen is endemic in the absence of a certain medical treatment, but which will be eliminated if the fraction $\tau$ of infected individuals who receive successful treatment exceeds a certain threshold $\tau_{0}$. We derive a Darwinian (evolutionary game theoretic) version of the model in which it is assumed that the pathogen has a (continuous phenotypic) trait the promotes resistance to the treatment and that this trait is subject to evolution by natural selection. We analyze the existence and stability properties of equilibria with the goal of answering the following question: is the threshold level $\tau_{0}$ for the treatment level $\tau$ still adequate for elimination of the pathogen or will the pathogen manage to evolve sufficient resistance to remain endemic? The analysis reveals one of three outcomes are possible for this model: (1) the treatment threshold $\tau_{0}$ remains valid despite the pathogen's ability to evolve resistance; (2) the treatment will eliminate the evolving pathogen, but the threshold level is greater than $\tau_{0}$; or (3) there is no treatment threshold that will eliminate the pathogen, which will evolutionarily adapt enough resistance to become endemic for all treatment levels. We give conditions on the model parameters under which each of these possibilities occurs.
(Received September 05, 2023)

## 1192-92-29089

Daniel Fitzsimons, University of Idaho, Tuan Phan*, University of Idaho. Mathematical modeling of molecular cooperativity and force development in human cardiac muscles.
In this study, we present a comprehensive thin filament model consisting of four states of a regulatory unit to elucidate the kinetics of contraction and relaxation in cardiac muscles. The model integrates the ensemble effect of three different near neighboring interactions, namely RU-RU, XB-XB, and XB-RU. The model was fitted to our mechanical data obtained from porcine and murine ventricular myocardium. Our findings suggest that porcine myocardium exhibits greater sensitivity to calcium binding to Troponin C and cross-bridge binding to actin, as well as greater reliance on cross-bridge cooperativity during force redevelopment compared to murine myocardium. (Received September 05, 2023)

## 1192-92-29117

Mason R Kelsey*, Western Connecticut State University, Samuel R Miller, Western Connecticut State University. M-Band Wavelet Based Time Dependent Machine Learning Model for Analyzing Spotted Lanternfly. Preliminary report The invasive Spotted Lanternfly (Lycorma delicatula) poses a significant threat to agriculture and ecosystems in North America. Its rapid spread has raised concerns among farmers, researchers, and policymakers. In this research, we propose a comprehensive time series modeling analysis using the state of art tool, Discrete M-Band Wavelet transform (DMWT) based machine learning methods to understand and predict the expansion of the Spotted Lanternfly in the New England region. Leveraging a dataset spanning several years of citizen reported sightings, we apply DMWT for preprocessing our data to decompose our data into multiscale components and then employ various time series modeling techniques including seasonal autoregressive integrated moving average (SARIMA) and machine learning-based approaches to capture the dynamic nature of the Spotted Lanternfly's aggressive spread across the area. This research will be the first in the area using machine learning methods and will shine a light for further study.
(Received September 22, 2023)

Jeffrey West*, Moffitt Cancer Center. Designing cancer treatment schedules using the principles of convexity and concavity. Dose response curves are ubiquitously used in medicine to measure response to varied drug dosage. Classical dose response models (e.g. Hill function) are nonlinear by nature; however these models are typically used to measure differential response in first-order effects (mean value of drug dose delivered), while second-order effects (variance of drug dose) are often neglected. Here, we explore the practicality of guiding treatment scheduling based on second-order effects. For example, nonlinear convex or concave curvature of a exposure-response function provides a straightforward comparison of continuous treatment and high-dose / low-dose alternatives. Concave exposure-response functions predict that the daily administration of a dose of $x$ may be less efficacious than a regimen that switches equally between 120Thus, functional nonlinearity maps the variance in input distribution to the skew (left or right tailed) of the output distribution. We advocate the point of view that second-order effects must be considered when designing treatment schedules, such as a comparison of continuous administration against high / low intermittent therapies. In this talk, we consider a variety of classical theoretical cancer models including 1) growth-response curves, 2) drug pharmacokinetic models, 3) gradual, multi-factorial resistance models, 4) game-theoretic cell competition/cooperation models, and 5) radiation therapy hyper- and hypofractionation models. Finally, we present analysis of experimental and clinical data to validate the effectiveness of this approach in lung cancer (ALK-inhibitors) and prostate cancer (CYP17A1-inhibitor) treatment. (Received September 05, 2023)

## 1192-92-29124

Lauren M Childs, Department of Mathematics, Virginia Tech, United States of America, Kyle Dahlin*, Virginia Tech,
Michael Andrew Robert, Virginia Tech. Once bitten, twice shy: modeling trade-offs between mosquito biting persistence and vertebrate host defensive behaviors. Preliminary report.
Despite evidence of its importance to mosquito feeding success, models of mosquito biting have largely ignored the effects of host defensive behaviors on blood-feeding and transmission. For example, it is known that host defensive behaviors can increase transmission opportunities by inducing multiple blood feeding. On the other hand, transmission may be reduced as host defensiveness can lead to increased mortality in mosquitoes. These trade-offs may have led different mosquito species to adapt differing levels of biting persistence: while some mosquitoes fly away in response to host defensive behavior, more persistent species remain attached to their host until their blood meal is finished. We investigate the implications of these trade-offs for mosquito-borne pathogen transmission using systems of ordinary differential equations. We consider various models of i) mosquito responses to defensive behavior and ii) the effects of biting persistence on mosquito fitness. By evaluating their basic reproduction numbers, we measure the transmission potential of transmission systems across levels of biting persistence and host defensiveness. We find that biting persistence only has a strong impact on transmission when more persistent mosquitoes take smaller blood meals. On the other hand, additional mortality due to multiple blood feeding has little effect on transmission. These findings suggest that biting persistence is a critical determinant of the outbreak potential of a mosquito population.
(Received September 05, 2023)

## 1192-92-29132

Azmy S Ackleh, University of Louisiana at Lafayette, Md Istiaq Hossain*, Stephen F. Austin State University, Amy Veprauskas, University of Louisiana at Lafayette. Impacts of prey stage structure and overcompensatory prey density dependence on predator-prey dynamics. Preliminary report.
We develop and investigate variations of discrete-time predator-prey models to examine the impacts of prey developmental stage structure (juvenile and adult) and different types of prey density-dependence (i.e., undercompensatory or overcompensatory) on the overall dynamics of predator and prey populations. We establish the local and global stability of the model equilibria, system persistence, and the existence of 2 -cycles that arise due to prey structure in these models. Our analysis suggests that introducing stage structure in the prey when the prey projection matrix is primitive or close to primitive may result in unstable cycles in the absence of the predator that are stabilized with high predator density. Moreover, when we consider overcompensatory prey density dependence the predator may have a stabilizing effect on the system dynamics with increasing predator density reversing the period doubling bifurcations generally occurring with Ricker-type nonlinearities. We also observe a stabilizing effect of the predator when structure is introduced into the prey population with overcompensatory prey density dependence.
(Received September 05, 2023)

1192-92-29149
Richard Allen, Pfizer Inc., Kathryn Link*, Pfizer Inc, C.J. Musante, Pfizer Inc.. Quantitative systems pharmacology modeling of respiratory syncytial virus and novel oral anti-infectives..
Objectives: Respiratory syncytial virus (RSV) is a common cause of lower respiratory tract infections (LRTIs) across all age groups but most severely impacts children and vulnerable adult populations, including the elderly and immunocompromised. In young children with LRTIs, RSV is the most commonly identified pathogen, causing significant morbidity and mortality and therefore there remains a need for effective and affordable treatments for RSV. A quantitative systems pharmacology (QSP) model of the pathogenesis and treatment of RSV infection could streamline and accelerate the development of innovative medicines by informing time to treatment initiation and dosing duration. Methods: We developed a QSP model to support the clinical development of novel oral anti-virals for the treatment of RSV. The mathematical model expands on a published model of RSV viral dynamics5 by incorporating the effects of both innate and adaptive immune responses along with the drug effect of an F-protein inhibitor. We trained a virtual population against published reports from human challenge studies to develop our understanding of RSV pathophysiology and predict treatment response. The model was extended to project the rate of hospitalization within a population by using immune-related surrogate biomarkers of disease severity. Lastly, the model was calibrated to published viral challenge data (for F-protein and N-protein inhibitors) and pharmacokinetic (PK) data. Results: The virtual population quantitatively matches viral load from both the placebo and treated responses in viral challenge studies. Analysis of the terminal viral shedding phase suggests that placebo groups in RCTs shed virus slower than observed in viral challenge studies. By selecting a subset of the virtual population, we can match this slower rate. Conclusions: Our QSP model of RSV captures key viral load and severity endpoints from human challenge studies of therapeutic interventions targeting the viral dynamics of RSV. By translating the virtual population to match the terminal viral shedding, the model can support the development of novel oral anti-virals for the treatment of RSV by informing.
(Received September 05, 2023)

## 1192-92-29163

Joel Ireta, Universidad Autonoma Metropolitana, David Orbe*, Universidad Autonoma Metropolitana. Protein structure: An approach through dual quaternions. Preliminary report.
A protein is a set of a main chain and particular side chains. The main chain is a set of amino acids linked by peptide bonds in a specific sequence. Each amino acid consists of a carbon atom (called alpha carbon, $\mathrm{C}_{\alpha}$ ) attached to a carboxyl group ( COOH ), an amino group $\left(\mathrm{NH}_{3}^{+}\right)$, a hydrogen atom, and a particular side chain. It is worth pointing out that the main chain could show characterized conformations through the method based on dihedral angles. However, the method assumes that each peptide unit is not distorted. One may ask whether this already characterized conformations still hold accurate if we change the method and emphasized the fact that each peptide unit is distorted. Motivated by the above, in this work we implicate two elements: conformations of the main chain of the protein and the dual quaternions. Geometrically speaking, the basic idea is to apply the dual quaternion in the representation of two displacements: rotation and translation. Our methodology and algorithm provide a natural and intrinsic characterization of the entire main chain of proteins. Our examples show that it is possible to use the protein main chain and dual quaternions to provide geometric characterization of the main chain of proteins. (Received September 05, 2023)

## 1192-92-29179

Lale Asik*, University of the Incarnate Word, Rebecca A. Everett, Haverford College, Angela Peace, Texas Tech University, Eric Seabloom, University of Minnesota. Dead or alive: carbon as a currency to integrate disease and ecosystem ecology theory.
Death is a common outcome of infection, but most disease models do not track hosts after death. Instead, these hosts disappear into a void. !is assumption lacks critical realism, because dead hosts can alter host-pathogen dynamics. Here, we develop a theoretical framework of carbon-based models combining disease and ecosystem perspectives to investigate the consequences of feedbacks between living and dead hosts on disease dynamics and carbon cycling. Because autotrophs (i.e plants and phytoplankton) are critical regulators of carbon cycling, we developed general model structures and parameter combinations to broadly reflect disease of autotrophic hosts across ecosystems. Analytical model solutions highlight the importance of disease-ecosystem coupling. For example, decomposition rates of dead hosts mediate pathogen spread, and carbon flux between live and dead biomass pools are sensitive to pathogen effects on host growth and death rates. Variation in dynamics arising from biologically realistic parameter combinations largely fell along a single gradient from slow to fast carbon turnover rates, and models predicted higher disease impacts in fast turnover systems (e.g. lakes and oceans) than slow turnover systems (e.g. boreal forests). Our results demonstrate that a unified framework, including the effects of pathogens on carbon cycling, provides novel hypotheses and insights at the nexus of disease and ecosystem ecology.
(Received September 05, 2023)

## 1192-92-29184

Heather L Cihak*, University of Colorado Boulder, Zachary P Kilpatrick, University of Colorado Boulder. Synaptic mechanisms for resisting distractors in neural fields. Preliminary report.
Persistent neural activity has been observed in the non-human primate cortex when making delayed estimates. Organized activity patterns according to cell feature preference reveals "bumps" that represent analog variables during the delay. Continuum neural field models support bump attractors whose stochastic dynamics can be linked to response statistics (estimate bias and error). Models often ignore the distinct dynamics of bumps in both excitatory/inhibitory population activity but recent neural and behavioral recordings suggest both play a role in delayed estimate codes and responses. In past work, we developed new methods in asymptotic and multiscale analyses for stochastic and spatiotemporal systems to understand how network architecture determines bump dynamics in networks with distinct E/I populations and short term plasticity. The inhibitory bump dynamics as well as facilitation and diffusion impact the stability and wandering motion of the excitatory bump. Our current work moves beyond studying ensemble statistics like variance to examine potential mechanisms underlying the robustness of working memory to distractors (irrelevant information) presented during the maintenance period. The relative timescales of the E/I populations, synaptic vs activity dynamics, as well as short term plasticity are all expected to play an important role which we aim to reveal via an advanced set of asymptotic methods to accurately identify the low-dimensional dynamics of stochastically evolving patterns in integrodifferential equations.
(Received September 05, 2023)

1192-92-29187
Brandon Jerome Legried*, Georgia Institute of Technology. Probabilistic methods for evolution of nucleic acid sequences. Genome sequencing methods have been important to identify mutations within DNA and their selective impact. Notable tasks involve the construction or use of a sequence alignment depicting homology. Pairwise and multiple sequence alignment have close relations to other genetic projects, including phylogenetic reconstruction and ancestral state reconstruction. In this presentation, I go over recent results in this area for models with and without insertion and deletion (indel) mutations. (Received September 05, 2023)

## 1192-92-29195

Ibrahim Halil Aslan*, Stanford University, Giulio A De Leo, Stanford University, Kaitlyn Rose Mitchell, Stanford University, Julie D Pourtois, Stanford University. Thermal sensitive mathematical model of schistosomiasis. Preliminary report
Schistosomiasis, a snail-borne disease, is a significant global health challenge with more than 200 million active infections and about 800 million people at risk in more than 70 countries, the vast majority in sub-saharan Africa. Temperature plays a pivotal role in driving the transmission cycle of schistosomiasis due to its influence on the life history traits (LHT) of the intermediate host snails and the free living stages of Schistosoma spp. parasites. To comprehensively assess the thermal response of schistosomiasis transmission risk, we developed a mathematical model with a system of ordinary differential equations, whose parameters are functions of temperature. Specifically, we first derived the thermal response of each model parameter following an extensive literature review to identify a comprehensive set of experimental studies reporting the temperature effect on snail's and parasite's LHTs. We then derived the basic reproduction number and long-term mean parasite burden as
a function of temperature. We found that the thermal optima for transmission of S. mansoni and S. haematobium falling within the range of 23.1-27.3 and 23.6-27.9 degree celsius, respectively is remarkably higher than previously estimated. We also found that, if we accounted for the increasing water contact rate of people at the transmission sites as temperature increases thermal optimum for both Schistosoma parasites shifts even more toward higher temperatures. Our model's predictions are supported by a large set of schistosomiasis prevalence data in Africa. The utilization of these refined nonlinear thermal response models is poised to enhance our understanding of the impacts of current and future temperature trends on schistosomiasis transmission risk in the context of Climate Change
(Received September 08, 2023)

## 1192-92-29208

Tahmineh Azizi*, University of Wisconsin-Madison. Time varying analysis of dynamic resting state functional brain network to unfold memory function. Preliminary report.
Recent advances in brain network analysis are largely based on graph theory methods to assess brain network organization, function, and malfunction. Although functional magnetic resonance imaging (fMRI) has been frequently used to study brain activity, however, the nonlinear dynamics in resting-state (fMRI) data have not been extensively characterized. In this work, we aim to model the dynamics of resting-state (fMRI) and characterize the dynamical patterns in resting-state (fMRI) time series data in left and right hippocampus and inferior frontal gyrus. We use Sliding Window Embedding (SWE) method to reconstruct the phase space of resting-state (fMRI) data from left and right hippocampus and orbital part of inferior frontal gyrus. The complexity of resting-state MRI data is examined using fractal analysis. The main purpose of the current study is to explore the topological organization of hippocampus and frontal gyrus and consequently, memory. By constructing restingstate functional network from resting-state (fMRI) time series data, we are able to draw a big picture of how brain functions and step forward to classify brain activity between normal control people and patients with different brain disorders. (Received September 05, 2023)

## 1192-92-29217

Robin Bachelder, University of California, San Diego, Trey Ideker, University of California, San Diego, JungHo Kong, University of California, San Diego, Sungjoon Park, University of California, San Diego, Akshat Singhal*, University of California, San Diego, Xiaoyu Zhao, University of California, San Diego. Interpretable Deep Learning Identifies a Constellation of Molecular Assemblies to Predict Response to Replication Stress.
Rapid proliferation is a hallmark of tumor cells, associated with sensitivity to therapeutic agents that cause DNA replication stress (RS). RS-inducing therapies target several distinct pathways, many of which are incompletely understood. In addition, tumors acquire genetic alterations that disrupt these pathways causing resistance to RS agents. Conventional statistical analyses to identify such alterations have met with limited success because most of these alterations are rarely represented in cancer patients. To address these challenges, we present \underline \{an ensemble of deep-learning models that assess genetic alterations in tumor cells to predict sensitivity or resistance to six common replication stress agents.\} The models implement two emerging machine learning (ML) methodologies. The first, known as interpretable ML, incorporates a hierarchical map of protein assemblies in cancer to guide the internal architecture of the neural network. The map integrates alterations in hundreds of genes, both common and rare, and distributed across a constellation of oncogenic pathways, allowing for both genetic and mechanistic interpretation. The second is a form of transfer learning, called multi-task learning, which allows us to capture both drug-specific and general features of RS agents. Model accuracy is assessed using five-fold nested crossvalidation, yielding odds ratios (OR) of 3.0 to 4.9 and significantly better performance than black-box neural networks (OR $=$ 1.8 to $3.3 ; \mathrm{p}=1.1 \times 10^{-6}$ ). Initial studies in tumor cells identify \underline $\{28$ molecular assemblies, covering roles in transcription, repair, cell-cycle checkpoint, and growth signaling, of which 12 are confirmed by loss of function screenings to regulate drug sensitivity as well as H2AX/Edu.\} The models translate to cisplatin-treated cervical cancer patients by accurately predicting patients with worse progression-free survival to be resistant (Hazard Ratio (HR) $=2.5, \mathrm{p}=0.008$ ) and identifying the role of an RTK-JAK-STAT assembly in cisplatin resistance ( $\mathrm{HR}=2.8, \mathrm{p}=0.004$ ). Overall, this study defines a compendium of predictive markers that facilitate a precise and dependable assessment of a patient's anticipated drug response, thus driving the progress of precision medicine
(Received September 05, 2023)

## 1192-92-29226

Olusegun M. Otunuga*, Augusta University. Tumor growth and population modeling in a toxicant-stressed random environment.
By analyzing the effects of stress on a population (or volume of a tumor growth) in a random environment, we develop stochastic models describing the dynamics of the population (or tumor growth) based on random adjustments to the population's intrinsic growth rate, carrying capacity, and harvesting efforts (or tumor treatments). The distribution of the stressed population size with or without harvesting (or treatments) is derived and used to calculate the maximum expected amount of harvest that can be taken from the population without depleting resources on the long run (or the minimum amount of chemotherapy needed to cause shrinkage or eradication of a tumor). The work done is applied to analyze tumor growth using published data comprising of the volume of breast tumor obtained by orthotopically implanting LM2-4 ${ }^{L U C+}$ cells into the right inguinal mammary fat pads of 6-to 8-week-old female Severe Combined Immuno-Deficient mice.
(Received September 06, 2023)

1192-92-29227
Jan Strydom, Kennesaw State University, Jesse Todd, Kennesaw State University, Pengcheng Xiao*, Kennesaw State University, Zeyu Zhang, The University of Texas at Arlington. Epidemiological Modeling of MisInformation Diffusion on social network. Preliminary report.
Characterizing the misinformation diffusion on social networks enables us to understand the properties of underlying media and model communication patterns. In this research work, we use SEIZ epidemiological model to study the spread of misinformation(rumors). The misinformation diffusion process were verified based on the dataset which includes top events across the world with different types.
(Received September 06, 2023)

## 1192-92-29250

Ethan Levien*, Dartmouth College. From individual to population-level dynamics: the role of extremal statistics. Preliminary report.
Many questions in biology and physics concern how variation at the level of individuals components (e.g. cells, molecules, etc.) shapes the macroscopic (e.g. population level) behavior of a system. In this talk I will discuss examples from population dynamics where extremal statistics dictate the macroscopic behavior. In these systems, there are two critical levels of individual-to-individual variation between which population has a law of large numbers, but not a central limit theorem. These will serve to illustrate a more general connection between large deviations and heavy-tailed distributions, as well as a connection to models from disordered systems.
(Received September 06, 2023)

## 1192-92-29253

Alexandru Hening*, Texas A\&M, Weiwei Qi, University of Alberta, Zhongwei Shen, University of Alberta, Yingfei Yi, University of Alberta. Population dynamics under random switching.
An important question in ecology is the relationship between the coexistence of species and environmental fluctuations. A natural way to model environmental fluctuations is to use piecewise deterministic Markov processes (PDMP). In a PDMP, the environment switches between a fixed finite number of states to each of which we associate an ordinary differential equation (ODE). In each state the dynamics is given by the flow of its associated ODE. After a random time, the environment switches to a different state, and the dynamics is governed by the ODE associated with the new state. I will look at two and three species examples of PDMP and explain how randomness can lead to some very interesting and counterintuitive behavior. (Received September 06, 2023)

1192-92-29260
Alexander Joseph Diefes*, Duke University, Miranda Ijang Teboh-Ewungkem, Lehigh University. Modeling the Effects of Temperature on Within-Mosquito Malaria Parasite Transitions and Sporozoite Load.
Human malaria is caused by Plasmodium parasites and is transmitted via female Anopheles mosquitoes from one human to another. The asexual stage of the parasite's life cycle takes place in an infected human, and the sexual stage is carried out in an infected mosquito. Several factors can inhibit or accelerate and enhance the successful completion of the parasite's life cycle, which is required for malaria to be transmitted. Temperature is one of the most influential factors and is known to affect within-mosquito parasite forms and dynamics. Here, we extend the first mathematical model of within-mosquito parasite dynamics proposed by Teboh-Ewungkem et al. in 2010 by incorporating the effects of temperature. We integrate experimental data associated with certain transition rates of the sporogonic phase, which occurs within infected female mosquitoes. This phase encapsulates processes from male and female gamete fertilization to sporozoite formation in the salivary glands. Using constructed cubic splines, we incorporate regional average monthly temperature from selected African regions by mapping the experimental temperature-dependent sporogonic traits to time-varying model parameters; we embed these converted temperature-to-time varying parameters into our mathematical model, yielding a non-autonomous system of differential equations. The system is then used to study the impact of temperature on the sporogonic malaria cycle. We also propose appropriate approximations for these constructed spline functions and investigate how the overall timeline of the sporogonic process is influenced by temperature. We conclude by discussing broader implications of rising global temperatures on malaria transmission dynamics. (Received September 06, 2023)

## 1192-92-29263

Kiel Daniel Corkran*, University of Missouri- Kansas City's Midwest Virtual Laboratory. The Mathematics Behind Infectious Disease Spread in Nursing Homes.
Estimating the prevalence and hazard ratio for mortality and morbidity of infection could help administrators better understand the force of infection. Research shows that most COVID-19 mortalities in the United States came from elderly citizens who resided in the nursing home. It is also known that many caregivers work in more than one nursing home at a time. The goal of this study was to highlight the importance of maintaining a threshold level of shared nursing home staff. To achieve this goal, we built an agent-based model to account for transmission dynamics within and between nursing homes. The prevalence and hazard ratios were estimated from the model simulations for different percentages of shared staff. Our analysis showed that increasing the percentage of shared staff resulted in a gradual increase in prevalence among residents, but after setting the percentage value to 20
(Received September 22, 2023)

## 1192-92-29274

Jianjun Paul Tian*, New Mexico State University. Infiltrating dynamics of immune cells into solid tumors. Preliminary report. Tumor-infiltrated immune cells compose a significant portion of many solid tumors. Their impact on tumor growth has been controversial. However, the regulation of infiltration of immune cells into tumors is important. In this talk, I will report our experimental results with mathematical modeling using PDGF-driven gliomas in mice. Our mathematical model is a parabolic PDE free boundary problem. Our numerical results show muIDH1 mice recruit lesser immune cells while having the same amount of tumor cells as wtIDH1 mice do when they die. Chemoattractant gradient field produced by tumor cells facilitates immune cells migration to the tumor cite. Infiltrating dynamics is largely determined by the chemoattractant production rate and chemotactic coefficient. These two parameters may distinguish wtIDH1 and muIDH1 tumors. We reduce the PDE model to a system of Ito stochastic differential equations to study those two parameters, and detailed analysis confirms their decisive role in immune infiltration dynamics even in noisy environemnts. This a joint work with Eric Holland, Tuan Phan, et al. (Received September 06, 2023)

In the last three years, the SARS-CoV-2 epidemic posed devastating effects on public health, and there is growing concern about the proliferation of misinformation and the role awareness plays in altering the dynamics of an epidemic. We formulate an SEIR-type model of SARS-CoV-2 with three susceptible compartments, consisting of individuals who are unaware, wellinformed, and misinformed of SARS-CoV-2 spread and control. We derive the disease-free equilibrium and the mediadependent reproduction number of our model. The global stability of the disease-free equilibrium and the uniform persistence of SARS-CoV-2 are established. Unknown parameter values of our model are estimated, using data on the cumulative number of cases of symptomatic infections in the state of Georgia, and sensitivity analysis of parameters in the media-dependent reproduction number is studied. Numerical simulations suggest a decrease in prevalence when susceptible humans transition from misinformed to well-informed susceptible humans, and when well-informed susceptible humans send media-related messages on the spread and control of SARS-CoV-2. Furthermore, there is an increase in the media-dependent reproduction number with an increase in misinformation. Finally, an optimal control of awareness and misinformation problem is formulated and analyzed.
(Received September 06, 2023)

## 1192-92-29348

Daniel M Anderson, George Mason University, Rayanne A Luke*, George Mason University. A Mathematical Model of Drug Delivery via a Contact Lens During Wear. Preliminary report.
Contact lenses have been studied as a potential method of ophthalmic drug delivery since at least the 1970s, but no therapeutic option has been marketed to date. Barriers to commercial implementation include lack of in vivo and in vitro studies, and the absence of a complete understanding of the underlying fluid dynamics. To join theoretical and experimental information, we design a compartmental model of contact lens drug delivery to align with an experimental in vitro model eye system. The model couples a partial differential equation for linear diffusion of drug inside the contact lens with ordinary differential equations governing the dynamics of the pre- and post-lens tear films and the eyelid region. The model simulates tear film dynamics during blinking over multiple hours. We compute the cumulative amount of drug released from the contact lens and compare our results to experimental model eye data. By isolating certain mechanisms or combinations of mechanisms in our model during this comparison, we attempt to better understand the mechanics of contact lens drug delivery.
(Received September 06, 2023)

## 1192-92-29360

Victoria Chebotaeva*, University of South Carolina, Paula A Vasquez, University of South Carolina. Erlang-Distributed SEIR Epidemic Models with Cross-Diffusion. Preliminary report.
We examine the effects of cross-diffusion dynamics in epidemiological models. Using reaction-diffusion dynamics to model the spread of infectious diseases, we focus on situations in which the movement of individuals is affected by the concentration of individuals of other categories. In particular, we present a model where susceptible individuals move away from large concentrations of infected and infectious individuals. Our results show that accounting for this cross-diffusion dynamics leads to a noticeable effect on epidemic dynamics. It is noteworthy that this leads to a delay in the onset of epidemics and an increase in the total number of people infected. This new representation improves the spatiotemporal accuracy of the SEIR Erlang model, allowing us to explore how spatial mobility driven by social behavior influences the disease trajectory. One of the key findings of our study is the effectiveness of adapted control measures. By implementing strategies such as targeted testing, contact tracing, and isolation of infected people, we demonstrate that we can effectively contain the spread of infectious diseases. Moreover, these measures allow achieving such a result, while minimizing the negative impact on society and the economy.
(Received September 06, 2023)

## 1192-92-29372

Daniel Fregoso, Department of Dermatology, School of Medicine, UC Davis, Sacramento, CA 95816, United States of America, Marcella Gomez, Department of Applied Mathematics, University of California, Santa Cruz, Santa Cruz, CA, 95060, United States of America, Rivkah Isseroff, Department of Dermatology, School of Medicine, UC Davis, Sacramento, CA 95816, United States of America, Manasa Kesapragada*, Department of Applied Mathematics, University of California, Santa Cruz, Santa Cruz, CA, 95060, United States of America, Cynthia Recendez, Department of Ophthalmology \& Vision Science, School of Medicine, University of California, Davis, Sacramento, CA, 95817, United States of America, Yao-Hui Sun, Department of Ophthalmology \& Vision Science, School of Medicine, University of California, Davis, Sacramento, CA, 95817, United States of America, Hsin-ya Yang, Department of Dermatology, School of Medicine, UC Davis, Sacramento, CA 95816, United States of America, Min Zhao, Department of Dermatology, School of Medicine, UC Davis, Sacramento, CA 95816, United States of America. Data-driven classification of cell subtypes based on time-lapse microscopy of single cells. Single-cell time-lapse microscopy allows researchers to track the dynamic response of cellular processes in real time. Observing these dynamics to external stimuli can help researchers understand the regulation mechanisms underlying complex biological processes. When studying the evolution of cell subtypes, it can become difficult to identify the cell types. Cell size and shape have been used to characterize these cell subtypes, but quality images that can provide cell morphology are difficult to come by. We propose that motility properties can be mapped to cell morphology and, hence, cell subtype. We applied this to macrophages, critical players in our body's defense and wound healing. We found that different types of macrophages, move uniquely. We developed methods to track single cells and linked cell movement to its shape using machine learning. Through this study, we demonstrate that mapping migratory patterns and motility properties to cell morphology can inform the classification of cell subtypes.
(Received September 06, 2023)

Miriam Di Salvatore, Toronto Metropolitan University, Suzan Farhang-Sardroodi, University of Manitoba, Joey Kapusin, Toronto Metropolitan University, Michael La Croix, York University, Kathleen P Wilkie*, Toronto Metropolitan University. Cancer Cachexia: Insights from Modelling and Analysis of the Wasting Condition. Preliminary report.
Cancer cachexia is the loss of muscle and adipose tissues that directly correlates with patient mortality rates, and that causes fatigue, weakness, pain, and poor quality of life. Chemotherapy is a standard cancer treatment with notorious side effects, including the unfortunate loss of muscle mass. Cancer presence itself may induce cachexia, leading to a double-barrelled
attack on healthy lean mass, and thus patient quality of life. In this talk I will present recent work on modelling and analysis of cancer cachexia. We have developed a framework to describe the response of muscle tissue to cancer-derived signals and to chemotherapy. We model the role of stem cells in tissue maintenance and use the model to examine mechanisms of cancer and chemotherapy induced muscle loss, including disruption of the differentiation pathway. We simulate various treatment regimes to explore the trade off between targeting the tumour and preserving lean mass. Additionally I will present a network approach to examine gene expression data for cachectic muscle. Using mutual information we inferred a network structure and then applied community detection and clustering techniques to identify regions of the network that may target particular biological processes in the diseased tissue. Our new approach suggests biological activities not identified with the standard statistical approach, and may shed new light on tissue level alterations in cachectic muscle.
(Received September 06, 2023)

## 1192-92-29378

Lingyun Xiong*, University of California Los Angeles. Sexual dimorphism in renal metabolism, hemodynamics and diseases. Preliminary report.
Mammalian organs exhibit distinct physiology, disease susceptibility and injury responses between the sexes. Especially, agerelated decline of renal function is faster in males than in age-matched females, manifesting in increased susceptibility to both chronic and acute kidney diseases among males. In the mouse kidney, sexually dimorphic gene activity maps predominantly to proximal tubule (PT) segments, where most reabsorption of water and salt happens, with a high energetic demand. Recent work suggests that male PTs undergo excessive oxidative stress to meet the high energetic demand, while female PTs exhibit an anti-oxidation state. However, it remains elusive how the observed molecular differences relate to sex disparities in renal physiology and pathophysiology. Widely known as an indicator of renal function, glomerular filtration rate (GFR) is tightly regulated by tubuloglomerular feedback (TGF) within the nephron to optimize ultrafiltration of plasma and reabsorption of essential molecules. Importantly, GFR shows a sustained oscillatory pattern over time with a period of 30-45 seconds in rodents, and loss of GFR oscillations is associated with cessation of reabsorption activities and ischemia-reperfusion injuries in the kidney, as well as systemic hypertension. To study the relationship between intracellular events and organ physiology, we developed a mathematical model of TGF linking metabolic regulation within PT cells to fluid handling and salt reabsorption in the nephron, using intra-vital imaging data from both sexes. Bifurcation analysis revealed that due to heightened oxidative profile to supply energy for reabsorption, male PTs operate near the critical boundary between sustaining and losing oscillatory behavior in the system, hence they are prone to damage upon metabolic stress. In contrast, female PTs situate further away from the boundary in the parameter space and are more resilient to failure. Combining single-cell genomics, physiological measurement and mathematical modeling of renal hemodynamics, this study provides a mechanistic explanation for sexual dimorphism in kidney diseases.
(Received September 06, 2023)

## 1192-92-29396

Julie Blackwood*, Williams College. Using Mathematical Models to Evaluate Strategies for Managing White-Nose Syndrome in Little Brown Bats.
The development of public health policy is inextricably linked with governance structure. In our increasingly globalized world, human migration and infectious diseases often span multiple administrative jurisdictions that might have different systems of government and divergent management objectives. In this talk, I will introduce mathematical models of infectious diseases that account for both spatial interactions and multiple strategies for disease mitigation. We show that successful management may depend on both the actions of multiple managers and their desired objectives.
(Received September 06, 2023)

## 1192-92-29405

Julie Blackwood*, Williams College. Uncovering the Drivers of Spatial Synchrony of Periodical Cicades in the U.S.. White-nose syndrome (WNS) is a highly lethal pathogen impacting hibernating bats and is caused by the psychrophilic fungus Pseudogymnoascus destructans (Pd). WNS was first documented in 2006 near Albany, NY and it has now been identified in 40 U.S. states as well as 8 Canadian provinces. Managing WNS requires coordination of multiple agencies as well as the use of several forms of mitigation strategies. In this talk, I will introduce a mathematical model that captures the transmission dynamics of WNS in little brown bats. The model will account for multiple methods for disease control, and I will discuss how effective control depends on complexities associated with the pathogen (such as the transmission route). (Received September 06, 2023)

## 1192-92-29431

Richard Bertram, Florida State University, Department of Mathematics, Nicole Bruce*, Florida State University, Department of Mathematics, Michael Roper, Florida State University, Department of Chemistry and Biochemistry, I An Wei, Florida State University, Department of Chemistry and Biochemistry. Investigating the Bistability in Coordinated Oscillations of Pancreatic Islets. Preliminary report.
Insulin is secreted in pulses by beta-cells located within cell clusters in the pancreas called islets of Langerhans. The pulsatility is evident in blood insulin measurements, indicating that the islet population is synchronized. While the exact mechanism for this synchronization is unknown, one possibility is that synchronization is achieved through a negative feedback loop between the pancreas and liver hepatocytes. Specifically, the action of hepatocytes to adjust blood glucose in response to insulin secretion from beta-cells acts as a global coordinating signal to the islets. Previously, we have tested this mechanism using mathematical modeling and microfluidic experiments, focusing on incorporating a time delay between when the liver cells uptake glucose and when this change in glucose concentration is sensed by the islets. With a nonzero time delay, the closed loop system is bistable, able to produce both fast and slow coordinated oscillations. In this presentation, we investigate the origin of this bistability. Do the same oscillation patterns persist for larger islet populations and larger time delays? Can we capture and explain the dynamics of the closed loop system using a reduced mathematical model? (Received September 06, 2023)

Owen L Lewis*, University of New Mexico. Principles of slowed hydrogen diffusion through a mucus layer.
The control of transport through mucus layers is a ubiquitous phenomenon in physiological systems. Mucus is often tasked with the mediation of passive, diffusive transport of small ionic species. However, questions remain regarding how mucin gel characteristics (charge density of the polymeric network, binding affinity of ions with mucus) govern the rate at which ions diffuse through mucus layers. Experimental studies measuring hydrogen diffusion through gastric mucus have provided conflicting results, and it is not clear if the rate of ionic diffusion through mucus layers is appreciably different than in aqueous environments. Here, we present a mathematical analysis of electrodiffussion of two ionic species (hydrogen and chloride) through a mucus layer. In addition to accounting for the chemical binding of hydrogen to the mucus network, we calculate the Donnan potential that occurs at the edge of the mucus layer. The model predicts the steady-state fluxes of ionic species and the induced potential across the layer. We characterize the dependence of these quantities on the chemical properties of the mucus gel and the composition of the bath solution. Finally, we show that the model predictions are consistent with a large portion of the experimental literature, and possibly shed light on the cause of discrepancies in reported values. Our analysis predicts that mucus layers generically slow the diffusive transport of hydrogen, but that chemical binding with the network attenuates this effect.
(Received September 06, 2023)

1192-92-29463
Jennifer Mawuyo Aduamah, University of Delaware, Angelica Davenport, Florida State University, Katharine F Gurski, Howard University, Yeona Kang, Howard University, Kathryn Link, Pfizer Inc, Yanping Ma*, Loyola Marymount University, Los Angeles CA USA. A mathematical model to study the extended-release Pre-Exposure Prophylaxis and drug-resistant HIV. The pharmacologic tail of long-acting cabotegravir (CAB-LA), an injectable Pre-Exposure Prophylaxis (PrEP), allows for months-long intervals between injections, but it may facilitate the emergence of drug-resistant human immunodeficiency virus (HIV) strains during the acute infection stage. In this study, we present a within-host, mechanistic ordinary differential equation model of the HIV latency and infection cycle in CD4 ${ }^{+}$T-cells to investigate the impact of CAB-LA on drug-resistant mutations in both humans and macaques. We develop a pharmacokinetic/ pharmacodynamic model forCAB-LA to correlate the inhibitory drug response with the drug concentration in plasma. After validating our model against experimental results, we conduct in-silico trials. First, we separately administer CAB-LA to the in silico macaque and human patients prior to and post-simian-human immunodeficiency virus (SHIV)/HIV exposure, to observe SHIV and HIV infectivity dynamics, respectively. Although the model does not incorporate a mechanism for CAB-LA-induced HIV mutations, we analyze the outcomes when mutations occur naturally. Our findings suggest that CAB-LA may enhance the growth of drugresistant strains over the wildtype strains during the acute stage. The in-silico trials demonstrate that the effectiveness of CAB-LA against mutations and the fitness of the drug-resistant strain to infect T-cells determine the course of the mutated strain. First, we separately administer CAB-LA to the in-silico macaque and human patients prior to and post-simian-human immunodeficiency virus (SHIV)/HIV exposure to observe SHIV and HIV infectivity dynamics, respectively. Although the model does not incorporate a mechanism for CAB-LA-induced HIV mutations, we analyze the outcomes when mutations occur naturally. Our findings suggest that CABLA may enhance the growth of drug-resistant strains over the wild-type strains during the acute stage. The in-silico trials demonstrate that the effectiveness of CAB-LA against mutations and the fitness of the drug-resistant strain to infect T-cells determine the course of the mutated strain.
(Received September 06, 2023)

1192-92-29480
Jim Michael Cushing*, Department of Mathematics \& Interdisciplinary Program in Applied Mathematics, University of Arizona. On a Darwinian Version of Leslie's Age-Structured Logistic Model and an Application to Evolutionarily Stable Life History Strategies. Preliminary report.
In his seminal 1948 paper on matrix models, P. H. Leslie introduced a logistic version of his famous discrete time matrix model for the dynamics of an age-structured population. I summarize the global asymptotic dynamics of this model and then derive a Darwinian (evolutionary game theoretic) version of the model that accounts for evolutionary adaptation by natural selection of an individual's inherent (density-free) fertility and survival rates and its (density dependent) intraspecific competition ability. A specific example, based on an adaptive trade-off between adult fertility and post-reproductive survival, illustrates the modeling methodology and analysis and also provides biological punch lines concerning semelparity or iteroparity as evolutionary stability life history strategies (ESS).
(Received September 06, 2023)

1192-92-29488
Edward Castillo, University of Texas at Austin, Richard Castillo, Emory University, Jorge Cisneros*, University of Texas at Austin, Aaron Luong, University of Texas at Austin, Amanda Nowacki, University of Texas at Austin, Yevgeniy Vinogradskiy, Thomas Jefferson University. Deep generative model optimization for 3DCT artifact removal. We present a novel approach to enhance the clinical integration of 4DCT-ventilation, a novel lung function imaging modality aimed at improving the outcomes of lung cancer patients undergoing radiation therapy. Despite its potential benefits, clinical application of 4DCT-ventilation is hindered by motion-induced or phase-binning artifacts, adversely affecting image fidelity. To address this, we introduce a coupled deep learning (DL) and optimization framework that removes motion-based artifacts from 3DCT scans, optimizing their utility for generating 4DCT-ventilation maps. Our pipeline leverages the power of generative adversarial networks (GANs) and various optimization techniques to inpaint artifact-affected regions with anatomically accurate information, ultimately enhancing the accuracy of 4DCT-ventilation imaging. Our proposed pipeline consists of (1) preprocessing the artifact-ridden 3DCT scan, (2) blocking out the majority of the z-slices where artifacts reside while allowing a small percentage of random voxels to pass through to aid the optimizer in preserving the underlying structure, (3) solving an unconstrained minimization problem to determine the best-fit artifact-free 3DCT scan from a DL pre-trained generative model, and (4) inpainting the block of voxels from the artifact-free 3DCT into the affected region of the input image via a feathering mask. We sample and evaluate a diverse set of objective functions and stochastic solvers to determine the best combination that corrects motion-based artifacts and retains physiological accuracy. We validate the corrected 4DCT scans by assessing the 4DCT-ventilation maps against ground-truth data, affirming the improved accuracy of our corrected images. Our pipeline not only prevents the need for costly rescans but also enhances functional avoidance radiotherapy planning based on 4DCTventilation, benefiting lung cancer patients. Future directions involve refining the technique to address additional artifacts and objects present in a 3DCT scan and integrating DL models for artifact detection. Our findings present a significant advancement in imaging sciences, with potential applications in other imaging modalities.

Steven M. Baer*, Arizona State University. Dendritic Spine Modeling: Application to Structural Plasticity and Retina
Circuitry.
Dendritic spines are small protrusions that stud the dendritic shaft of different neurons in the brain, including the retina. They are considered to be an important locus for plastic changes underlying memory and learning processes. Spines exist in great numbers: e.g., over 200,000 spines per neuron on Purkinje cells. In this talk, continuum spine modeling is used to study the spatiotemporal dynamics of spine restructuring and spine population dynamics. Next, we show how the continuum approach can be generalized to formulate a multiscale continuum model of the neural subcircuits in the vertebrate outer retina. We conclude by introducing a new continuum model for the distribution of branches, rather than spines, over the primary neurite of the Drosophila MN5 motoneuron.
(Received September 07, 2023)

## 1192-92-29587

Megan Olivia Powell*, University of North Carolina Asheville. Impulse Vaccination Model for the Control of Devil Facial Tumor Disease.
Devil facial tumor disease (DFTD) is a cancer that affects Tasmanian devils and that has caused the devil population to grossly decline since 1996. We present an SEIVR model to explore if recent advances in DFTD vaccines can help the wild population recover. Considering both and bi-annual impulse of vaccinating wild devils through food drops and introducing vaccinated captive-bred devils into the population, we explore the vaccine efficacy, percent of healthy devils receiving the vaccine, and years of campaign necessary for the devil population to have a long-term recovery. Based on our initial parameter estimations, we find a stable population can be reached after 8 years of bi-annual bait drop vaccine campaigns and introduction of 2 captive-bred vaccinated devils into the wild population.
(Received September 07, 2023)

## 1192-92-29588

Grace Branshaw*, Siena College, Scott W Greenhalgh, Siena College, Megan Perry*, Siena College, Kurad Tosun, Siena College. A Mathematical Model of Marburg Virus Using Differential Equations. Preliminary report.
Marburg Virus Disease (MVD) is a very rare yet deadly disease, often caused by a spillover event from the Rousettus aegyptiacus, commonly known as the Egyptian rousette or Egyptian fruit bat. This species of megabat is known as the natural reservoir for Marburg Virus Disease. These bats live throughout Africa, India, the Mediterranean and the Middle East. All known outbreaks have taken place or come from Africa. Spillover events do not occur very often and many times records are not clear. In several outbreaks, medical practices, record keeping and cultural practices have made the disease more difficult to study. Our goal was to mathematically model how Marburg can spread in a population. We also touch on the effects of using proper intervention such as isolation, safe burials, and PPE versus resisted or no intervention. Our results show that while Marburg has a very high fatality rate, it does not spread very easily. They also show, using proper interventions can lead to a shorter outbreak, saving many lives, if not, many will be affected and the outbreak can last several years. (Received September 07, 2023)

## 1192-92-29590

Jack Farrell*, Siena College, Scott W Greenhalgh, Siena College. Stage-structured population models: Using an ODE approach based on reproductive potential. Preliminary report
The mathematical modeling of population dynamics has provided researchers and ecologists with critical insights into the progression, control, and preservation of many species. Classically, these mathematical models are composed of systems of differential equations that abstractly categorize a species into distinct stages. However, these models have a significant shortcoming in that their stage-structured formulations are typically unable to account for the variability in reproductive potential across a population. As the misspecification of such a biological characteristic is known to affect predictions, there is a need to develop new modeling approaches that capture this information. So, we illustrate a new class of stage-structured models that capture the variability in the reproductive potential of a species, while maintaining the convenient ODE model formulations that are the preferred choice of ecological modelers and population dynamics researchers. (Received September 07, 2023)

1192-92-29593
Scott W Greenhalgh, Siena College, Owen Spolyar*, Siena College. On the persistence of recurrent epidemics: Exploring the effects of the duration of infection. Preliminary report.
Recurrent disease outbreaks are commonplace in everyday society. For instance, every winter in the US novel variants of influenza arise, which infects millions of people, causing thousands of deaths, and creating financial hardship on the US healthcare system. In part, these hardships occur because novel variants of influenza feature genetic mutations that enable them to potentially evade individual immunity, and thereby spread throughout a population. While these new variants are detrimental to the health and well-being of the population, they may carry an unseen benefit. Namely, their spread may inhibit other, potentially more virulent, variants from propagating. However, insight on the relationship of the particular traits of variants that cause recurrent epidemics and the non-emergence of other highly virulent variants is limited. So, we develop a generalized Susceptible-Infectious-Recovered (gSIR) model, which can describe recurrent epidemics through its time-varying recovery rates, to inform on conditions that promote or prevent seasonal outbreaks. For our model, we determine the stability properties of all equilibria using the standard Jacobian approach, identify a potential periodic solution, and evaluate its stability properties using Floquet theory. We found the basic reproductive number by using the next generation method. Our findings show that the recurrent outbreak, namely the periodic solution, is locally stable provided that the basic reproductive number is greater than 1. By understanding the conditions that permit the formulation of recurrent outbreaks, health professionals can improve their capacity to make informed decisions on how to better combat future novel variants of disease. (Received September 07, 2023)

## 1192-92-29606

Scott W Greenhalgh*, Siena College. A generalized ODE susceptible-infectious-susceptible compartmental model with potentially periodic behavior. Preliminary report.
Differential equation compartmental models are crucial tools for forecasting and analyzing disease trajectories. Among these models, those dealing with only susceptible and infectious individuals are particularly useful as they offer closed-form expressions for solutions, namely the logistic equation. However, the logistic equation has limited ability to describe disease trajectories since its solutions must converge monotonically to either the disease-free or endemic equilibrium, depending on the parameters. Unfortunately, many diseases exhibit periodic cycles, and thus, do not converge to equilibria. To address this limitation, we developed a generalized susceptible-infectious-susceptible compartmental model capable of accurately incorporating the duration of infection distribution and describing both periodic and non-periodic disease trajectories. We characterized how our model's parameters influence its behavior and applied the model to predict gonorrhea incidence in the US, using Akaike Information Criteria to inform on its merit relative to the traditional SIS model. The significance of our work lies in providing a novel susceptible-infected-susceptible model whose solutions can have closed-form expressions that may be periodic or non-periodic depending on the parameterization. Our work thus provides disease modelers with a straightforward way to investigate the potential periodic behavior of many diseases and thereby may aid ongoing efforts to prevent recurrent outbreaks.
(Received September 07, 2023)

## 1192-92-29655

Anna Bauer*, St. Norbert College. AI Powered Molecular Analysis. Preliminary report.
In order to predict a previously unknown protein structure we leveraged an AI powered methodology called AlphaFold. This method allows us to predict this structure based on amino acid side chain interactions locally and globally.
(Received September 07, 2023)

## 1192-92-29696

John Palacios*, Virginia Commonwealth University, Rebecca A Segal, Virginia Commonwealth University. Exploration of Differential Equation Models for Phage-Bacteria Population Dynamics.
Bacteriophages are viruses that infect and replicate within bacteria. Lytic phages will eventually cause the bacteria cell to burst, killing the bacteria. These types of phages can be used to treat patients with antibiotic resistant bacteria infections. As a step in developing successful treatment protocols, we aim to understand the population dynamics of phages and bacteria using an in-vitro model. We model the dynamics by using the Campbell model, which consists of two delay differential equations (DDE), as a base model. We extended the model by including the emergence of phage resistance. We then made a comparison of the DDE model to a parallel ordinary differential equation (ODE) model. We apply several parameter estimation methods to the ODE model such as local sensitivity analysis, sensitivity based identifiability, and using a local optimization method to fit experimental data. Finally, we compare best- fit parameters found from the ODE model to the DDE model and found that there are no significant differences between the two models, allowing us to move forward with the ODE formulation. The interaction dynamics are then extended to a preliminary biofilm flow model to benchmark experimental data.
(Received September 07, 2023)

1192-92-29708
Laura Storch*, Bates College. Topological data analysis and early warning signs of spatial population extinction.
Many populations and ecosystems are experiencing decline due to habitat degradation, global climate change, and increasing human pressures such as overfishing. Populations undergoing decline can experience a critical transition - an abrupt, irreversible shift in the dynamics of the population. It is particularly challenging to detect critical transitions in spatially explicit populations, e.g., a grassland. Here, we use computational algebraic topology to quantify features in a population distribution pattern and observe how those features change over time during a critical transition such as an extinction event. The population distribution patterns are generated via a spatially explicit discrete time population model. In this way, we identify topological signatures of impending population collapse. (Received September 07, 2023)

1192-92-29735
Farjana Mukta*, University of Kentucky, Duc Duy Nguyen, University of Kentucky, Md Masud Masud Rana, University of Kentucky. Advanced Mathematical Graph-Based Models in Drug Design. Preliminary report
Drug discovery is a highly complicated and time-consuming process. One of the main challenges in drug development is predicting whether a drug-like molecule will interact with a specific target protein. This prediction is crucial in expediting the validation and discovery of targets, and it enables biochemists and pharmacists to accelerate the drug development process. In recent studies of biomolecular sciences, the application of algebraic graph-based models to accurately represent molecular complexes and predict drug-target binding affinity has generated significant interest among researchers. Here, we present algebraic graph-based molecular representations to form data-driven scoring functions (SF) named AGL-EAT-Score featuring extended atom types to capture wide-range interactions between the target and drug candidate. Our model applies multiscale weighted colored subgraphs for the protein-ligand complex where the graph coloring is based on SYBYL atom-type and ECIF atom-type interactions. Furthermore, combined with the gradient-boosting decision tree (GBDT) machine-learning algorithm, our newly developed SF has outperformed numerous state-of-the-art models in PDBbind benchmarks for binding affinity scoring power, and the D3R dataset, a worldwide grand challenge in drug design. Finally, we have constructed non-redundant datasets to ensure that our model is robust against any hidden biases present in the benchmarks. (Received September 07, 2023 )

Spatial fishery models are discussed with an emphasis on optimal management strategies. The models incorporate nonlinear, parabolic partial differential equations with density-dependence in the growth rate of the fish stock on a spatial domain. The role of feature heterogeneities such as habitat properties, spatial boundaries, and destructive fishing practices will be discussed. We also introduce multiple harvesters in the context of traditional federalism questions. The objective is to find the dynamic distribution of harvest effort that maximizes the discounted net present value of stock while minimizing costs. Optimal management strategies are found numerically. (Received September 12, 2023)

Garri Davydyan*, Appletree Medical Group. Homeostasis, non-equilibrium dynamics and cyclic transformations of base functional mechanisms of biologic systems. Preliminary report.
Stability of biologic functions is closely related to homeostatic mechanisms. Physiologically the programmed destruction (apoptosis) and synthesis of biological cells are two opposite processes organized in a cycle. Because of the physiologic principle to keep the wholeness of complex biologic systems, these must be a basis for the space of regulatory elements providing this principle. Negative feedback, positive feedback and reciprocal links (PNR) are the suggested base regulatory patterns of inner functional structure of a biologic system. The property of a biologic system to return disturbed functional states to the equilibrium (physiologic constants) can be modeled by a mathematical pendulum representing a physical system. This model may also demonstrate mechanisms of physiologic interactions of base elements. It shows that the low amplitude swings of the pendulum are described by phase images corresponding to biologic negative feedback. With the added environmental factor related to dissipative forces and making the system unstable the swings will increase their amplitudes until the system reaches its threshold in potential energy. Further increase of the energy of the system will transform the swings into rotations. It corresponds to the bifurcation of the phase portrait of the system and appearing "saddle" after "node" which is a phase image of positive feedback. Physiologic sense of this change is moving the system to the higher energy level required for the new system's actions. In order to keep morphologic wholeness the system will split into two subsystems. It corresponds to similarity transformation (rotation) of the matrix related to the positive feedback to the ones of reciprocal links. After that each daughter system related to one-dimensional invariant subspace can begin its own cycle of functional changes. Moreover, two newly appearing functional cycles in certain conditions can be united by negative feedback loops. Cyclic changes prevent the system from chaotic behavior. Therefore, three base regulatory elements span the regulatory space of biologic systems. This is the minimal basis for maintaining morphology and functions of biologic objects, i.e. homeostasis. Cyclic changes confirm the necessity of PNR related signaling substances to be presented in a genome.
(Received September 07, 2023)

1192-92-29778
Vincent L Cannataro, Emmanuel College, Boston, MA, Caralynn Elizabeth Hampson*, Emmanuel College. Quantifying selection intensity and epistatic interactions among gene variants within angiosarcoma..
Evolution underlies cancer development. Mutations occur and accumulate in somatic tissues, and some mutations result in increases in cell division or survival and fix in cell populations more often than expected by chance. Computational techniques that account for differences in mutation rate across the genome and between tissues have helped pinpoint which genes are contributing to tumor growth within a wide array of cancer types. However, rare cancers, such as angiosarcoma with approximately 300 new cases a year in the US, are often understudied due to lack of funding, availability of data, and applicable tools that are capable of incorporating data from different sources. In this study, we incorporate whole-genome, whole-exome, and targeted sequencing data from different sources-accounting for study-specific differences in genome coverage-into an evolutionary mathematical model of angiosarcoma tumorigenesis. We calculate expected rates of mutation at every position in the genome by combining predictions of synonymous substitution rates across the genome with rates of tumor-specific trinucleotide mutation. We utilize a maximum likelihood framework to estimate the intensity of selection for the variants driving increases in cellular division and survival within angiosarcoma, and complement existing studies that have previously determined KDR and TP53 mutations to be a driver, and we also implicate novel putative drivers that have high calculated selection intensity, including BRAF V600E, IGDCC4 S1020A, and SETD2 Q1152E. Additionally, we evaluated epistatic interactions among genes and found that selection for TP53 is increased after mutation of other genes such as MUC16, FAT1, and NOTCH2. This study demonstrates the feasibility of applying mathematical models and computational approaches informed by evolutionary theory to a rare disease, and identifies gene variants that likely have a large impact on angiosarcoma tumorigenesis.
(Received September 25, 2023)

1192-92-29899
Yeon Hyang Kim, Central Michigan University, Hiruni Pallage*, Central Michigan University. $k$ - NN regression in $Q$-ball Imaging. Preliminary report.
In this talk, we study q-ball imaging (QBI) that reconstructs an orientation distribution function (ODF). ODF is a spherical function, capturing the angular information regarding the probability of diffusion displacement for water molecules and having peaks that align with the underlying fiber distribution. As per Tuch (2004), this ODF can be directly derived from the raw signals on the sphere in q -space using the Funk-Radon transform (FRT) $\mathcal{G}$. Mathematically, this can be expressed as $\mathcal{G}[S](\mathbf{v})=\int_{|\mathbf{w}|=1} \delta\left(\mathbf{v}^{T} \mathbf{w}\right) S(\mathbf{w}) d \mathbf{w}$. Analytical solutions for the FRT employ Spherical Harmonic (SH) interpolation of the signal. We also adopt Tikhonov regularization with Laplace Beltrami Operator. However, the requirement of signal in numerous diffusion gradient (DG) directions leads to lengthy acquisition times in QBI. To address this challenge, we propose to measure the signal only at a relatively small number of DG directions and predict the signal at other DG directions using the knearest neighbors (k-NN) regression.
(Received October 11, 2023)

1192-92-29900
Ahmad Amir*, Siena College, Scott W Greenhalgh, Siena College, Kurad Tosun, Siena College. Does mask-wearing affect slowing COVID-19 spread in a population with a high vaccination rate?. Preliminary report.
This study explores the impact of mask-wearing on slowing the spread of COVID-19 in a population with a high vaccination rate. The research utilizes a compartmental model divided into susceptible, exposed, infective, and recovered groups. Disease
transmission dynamics are described using a system of nonlinear differential equations. The vaccinated population is further distinguished between those wearing masks and those not wearing masks. Parameters include infection rates, death rates, recovery rates, and probabilities of mask-wearing behaviors. Through stability analysis and calculation of the basic reproduction number (R0), the study assesses the potential of mask-wearing to mitigate the spread of the virus. The stability of equilibrium points is determined by the eigenvalues of the Jacobian matrix. The eigenvalues provide insight into local asymptotic stability and potential spread or control of the disease. Simulation results are presented for different population sizes and various probabilities of mask-wearing behaviors. The study indicates that when a substantial proportion of exposed individuals opt to wear masks after getting sick, the impact of mask-wearing by vaccinated individuals on the duration of the infection is negligible. However, the number of deaths is significantly influenced by the mask-wearing behavior of both vaccinated and exposed individuals. The findings suggest that high mask-wearing rates among both groups can lead to reduced mortality rates while maintaining similar infection durations. Overall, this research sheds light on the intricate interplay between vaccination, mask-wearing behaviors, and disease spread, offering insights into effective public health strategies to combat COVID-19 in populations with high vaccination coverage.
(Received September 08, 2023)

## 1192-92-29919

Joseph D. Butner*, Mathematics in Medicine Program, Houston Methodist Research Institute, Houston, TX 77030, USA, Zhihui Wang, Mathematics in Medicine Program, Houston Methodist Research Institute, Houston, TX 77030, USA. Predicting cancer patient survival by hybridizing mechanistic mathematical modeling and deep learning methods. Preliminary report. Mechanistic mathematical modeling enables calculation of quantities that are often indicative of disease prognosis or treatment outcome, but is inherently limited to reliance on only quantifiable factors or processes that have a known mechanistic link to the outcome of interest. Commonly, this results in inclusion of variables that are unmeasurable, rendering a model impractical beyond hypothesis-generation, while available information that does not directly relate to the mechanisms described in the model is excluded. It is likely that some clinical measures that are not easily included in mechanistic descriptions also contain valuable information that is indicative of the outcome being predicted (e.g., patient age in survival prediction). To overcome this limitation, we have hybridized mechanistic modeling with existing deep machine learning methods to predict cancer patient survival under immune checkpoint inhibitor therapy, enabling inclusion of all information in one model. This also facilitates probability-based survival prediction (e.g., proportional hazards, partial likelihood, probability mass function) that accounts for stochasticity inherent to future events. This hybrid deep learning model was found to predict patient survival with higher accuracy than when trained on only mathematical model-derived or clinical measures by both event-time concordance and Briar score-based criteria. Extensive feature importance analysis revealed that both mathematical and clinical data make notable contributions to how the deep learning model makes predictions and to the accuracy of the predictions, offering further evidence that their combination is advantageous. We expect that this hybrid approach may be used with other mechanistic models and clinical measures to improve the accuracy of event prediction based on more complete data than may be practical using either method alone.
(Received September 08, 2023)

## 1192-92-29928

Cash Bortner*, California State University, Stanislaus, Nicolette Meshkat, Santa Clara University. Graph-based sufficient conditions for indistinguishability of linear compartmental models. Preliminary report.
An important problem in biological modeling is choosing the right model. Usually, given an experiment or experimental data, one is supposed to find the best mathematical representation to describe that real-world phenomenon. However, there may not be a unique model representing that real-world phenomenon. Two distinct models could yield the same exact dynamics. In this case, these models are called indistinguishable. In this talk, we consider the indistinguishability problem for linear compartmental models, which are used in many areas, such as pharmacokinetics, physiology, cell biology, toxicology, and ecology. We exhibit sufficient conditions for indistinguishability for models with a certain graph structure: paths from input to output with "detours". The benefit of our results is that indistinguishability can be proven using only the graph structure of the models, without the use of any symbolic computation. This can be very helpful for medium-to-large sized linear compartmental models. To our knowledge, these are the first sufficient conditions for indistinguishability of linear compartmental models based on graph structure alone, as previously only necessary conditions for indistinguishability of linear compartmental models existed based on graph structure alone.
(Received September 08, 2023)

## 1192-92-29931

Stanca Ciupe, Virginia Tech, Nisha Duggal, Virginia Tech, Nora Grace Heitzman-Breen*, Virginia Tech, Yuganthi
Liyanage, Florida Atlantic University, Necibe Tuncer, Florida Atlantic University. Multiscale models of Usutu virus infection and transmission.
Usutu virus is a mosquito-borne flavivirus maintained in wild bird populations, with occasional spillover in humans. Understanding USUV epidemiology requires characterization of host-virus ecology at many scales, including individual avian host infections, bird-to-vector transmissions, and USUV incidence in bird and vector populations. To provide insight into the intrinsic complexity of host-virus processes in individual birds, we develop a within-host mathematical model of Usutu virus infection from chicken host data, taking into consideration variations in virus strain and host immune competency. We then investigate bird-to-vector transmission and incidence predictions based on laboratory-type inoculation and transmission experiments combined with dynamical mathematical modeling. Finally, we develop a multiscale vector-borne epidemiological model of Usutu virus infection in birds and mosquitoes, using individual within-host viral load data from sparrows and host-tovector probability of transmission data to predict USUV incidence in bird and mosquito populations exposed to two different Usutu virus strains. Within-host we observe differences in virulence amongst virus strains, and find immune priming coupled with competent response leads to control of viral spread. Through our multiscale approach we find within-host peak viremia does not always correlate with infection incidence levels in host and vector populations, and that uncertainty in predictions at one scale may change predicted results at another scale. We demonstrate that optimal experimental design and increased frequency of data collection vastly improve the characterization of virus dynamics and transmission probabilities. These results may be useful for predicting spillover events.
(Received September 08, 2023)

Jorge Arroyo-Esquivel*, Carnegie Institution for Science, Christopher A Klausmeier, Michigan State University, Elena
Litchman, Carnegie Institution for Science. Inferring ecological niches from time series using neural ordinary differential equations. Preliminary report.
Functional diversity is one of the factors that determine the stability of a real ecosystem. Determining functional diversity of wild populations is a challenging task as it may require extensive experimental work to determine the individual functional traits for many species. Trait-based ecological models incorporate functional traits into the dynamics of a community, and could be parameterized to identify these traits from a time series of monitoring data. However, traditional methods to identify parameters of dynamical systems become computationally expensive even for small communities due to the curse of dimensionality. Neural Ordinary Differential Equations (NODEs) have showed up on the computational literature as a method to estimate the rates of change of a time series using neural networks. Getting the rates of change this way turns the parameter estimation problem into an easier to solve convex optimization problem. In this talk I will present the preliminary results of our work using NODEs to identify traits using trait-based ecological models. We evaluate these capabilities using synthetic data by estimating the trait parameters of the ground-truth model. We also evaluate the performance of these methods in the case where only the trait-dependent terms of the model are known, and when only an approximation of these terms exists. We hope this tool will be useful to determine functional diversity from available ecosystem monitoring projects. (Received September 08, 2023)

## 1192-92-29992

Stephen D. Pankavich*, Colorado School of Mines. Three-stage modeling of HIV infection and implications for antiretroviral therapy. Preliminary report.
We consider a deterministic model of HIV infection that involves macrophages as a long-term active reservoir to describe the three stages of the disease process: the acute phase, chronic infection, and the transition to AIDS. A dynamical analysis is performed to illustrate how infected macrophages can explain the progression to AIDS and provoke viral explosion, while previous models do not. Moreover, we explore the implications of the model for the administration of antiretroviral therapy (ART) and provide quantitative estimates that emphasize the time sensitive nature of treatment initiation and the level of drug efficacy. Finally, we study the effects of treatment interruption on the disease dynamics predicted by the model and elucidate the influence of both interruption time and duration.
(Received September 08, 2023)

## 1192-92-30029

Kimberlyn Eversman*, University of Tennessee Knoxville. Modeling the Opioid Epidemic in the US. Preliminary report. Opioid addiction is a national epidemic that has been ongoing for decades. A rise in prescribing opioids in the 1990s led to rises in non-medical heroin and, more recently, fentanyl use. The increased availability of prescription and illicit opioids has led to fatal and nonfatal overdoses throughout the country. We present a two-part compartmental model for prescription opioid and fentanyl/heroin use disorder and overdoses which aims to better understand how two specific factors affect opioid use in a vulnerable subpopulation: (1) the prevalence of opioid use within the larger community and (2) the availability of resources that specifically target the subcommunity. Mathematical analysis of the model and comparison with population-level data reveal how the subcommunity compares to and is influenced by their community at-large.
(Received September 08, 2023)

1192-92-30031
Duc Duy Nguyen, University of Kentucky, Md Masud Rana*, University of Kentucky. Differential geometry and graph theory-based machine-learning model for biomolecule: application to structure-based drug design.
The fundamental step in the drug design and discovery process is understanding and accurately predicting the binding affinity between a drug candidate (ligand) and its receptor protein. Machine learning-based methods have become increasingly popular in this regard due to their efficiency and accuracy, as well as the growing availability of data on the structure and binding affinity of protein-ligand complexes. In molecular and biomolecular studies, differential geometry and graph theory are widely used to analyze vast, diverse, and complex datasets. Using molecular surface representation, crucial chemical and biological data can be encoded in differentiable manifolds that can reduce dimensionality. Graph theory is extensively used in biomolecular research because molecules or molecular complexes can be naturally modeled as graphs where graph vertices typically represent atoms and graph edges represent possible interactions. Here, I will present several models based on differential geometry and graph theory that can be combined with advanced machine learning techniques to predict proteinligand binding affinity with high accuracy. Our proposed models have demonstrated superior performance compared to numerous state-of-the-art models on established benchmark datasets.
(Received September 08, 2023)

## 1192-92-30038

Erika Tatiana Camacho, The University of Texas at San Antonio, Atanaska Dobreva*, Augusta University, María Miranda Sanz, CEU Cardinal Herrera University. Examining oxidative stress factors in control and diseased retinas with mathematical model for the glutathione redox system.
The retina houses some of the most metabolically active cells in the body, and it is constantly exposed to light and high oxygen levels in the choroidal circulation. This leads to the generation of harmful reactive oxygen species (ROS), which damage proteins and lipids and disrupt the normal processes of retinal cells. The glutathione (GSH) redox system is crucial in providing defense for retinal cells against ROS. This talk will present the development of the first mathematical model for dynamics and interactions of ROS and GSH system in the retina. We calibrate and validate the model using experimental data from control mice and from the rd1 mouse model for the disease retinitis pigmentosa. Then, we conduct global sensitivity analysis to determine which processes have the greatest impact on oxidative stress in normal compared to diseased retinas. (Received September 22, 2023)

## 1192-92-30054

and Cell Adhesion.
Cell aggregation and cell sorting mediated by adhesion is a fundamental process in development and in organisms that switch between unicellular and multicellular modes of life, such as myxobacteria. In many such cases, adhesion and other biophysical cellular properties interact with intracellular oscillatory processes. We consider mathematical models of cell-cell and cellmedium adhesion and how to modify them to account for the effects of intracellular oscillations. This includes continuum (PDE) and discrete approaches (Lattice-Gas Cellular Automata and the Cellular Potts Model). We discuss two specific models: the effect of Hes1 oscillations on precartilage condensations in embryonic chick limb cell cultures, as well as a model of aggregation and synchronization motivated by the behavior of unicellular organisms where cell-cell adhesion depends on relative clock phases.
(Received September 08, 2023)

## 1192-92-30070

Saber Elaydi*, Trinity University. Ititle \{Global Dynamics of Discrete Mathematical Models of Tuberculosis\}.
labstract In this talk, we develop discrete models of Tuberculosis (TB). This includes SEI endogenous and exogenous models without treatment. These models are then extended to a SEIT model with treatment. We introduce two types of net reproduction numbers: the traditional $\mathcal{R}_{0}$ based on the disease-free equilibrium, and a new net reproduction number $\mathcal{R}_{0}\left(\mathcal{E}^{*}\right)$ based on the endemic equilibrium. We show that the disease-free equilibrium is globally asymptotically stable if $\mathcal{R}_{0} \leq 1$ and unstable if $\mathcal{R}_{0}>1$. Moreover, the endemic equilibrium is locally asymptotically stable if $\mathcal{R}_{0}\left(\mathcal{E}^{*}\right)<1<\mathcal{R}_{0}$. \end } \{ abstract \} par
(Received September 09, 2023)

1192-92-30095
Jessica M Conway, Penn State, Ruian Ke, Los Alamos National Laboratory, Jasmine Kreig, Los Alamos National Laboratory, Nicole Pagane, Massachusetts Institute of Technology, Alan S Perelson, Los Alamos National Laboratory, Tin Phan*, Los Alamos National Laboratory, Ruy M Ribeiro, Los Alamos National Laboratory, Narmada Sambaturu, Los Alamos National Laboratory. Feasibility of using dynamic models with an effector cell response to predict early viral rebound dynamics following HIV-1 antiretroviral therapy interruption.
Most individuals living with HIV-1 experience rapid viral rebound once antiretroviral therapy is interrupted; however, a small fraction remain in viral remission for an extended duration. Understanding the factors that determine whether viral rebound is likely after treatment interruption can enable the development of optimal treatment regimens and therapeutic interventions to potentially achieve a functional cure for HIV-1. We build upon the theoretical framework proposed by Conway and Perelson to construct dynamic models of virus-immune interactions to study factors that influence viral rebound dynamics. We evaluate these models using viral load data from 24 individuals following antiretroviral therapy interruption. The best performing model accurately captures the heterogeneity of viral dynamics and highlights the importance of the effector cell expansion rate. In particular, our results show that post-treatment controllers and non-controllers can be distinguished based on the effector cell expansion rate in our models. Furthermore, these results demonstrate the potential of using dynamic models incorporating an effector cell response to understand early viral rebound dynamics post antiretroviral therapy interruption. (Received September 08, 2023)

## 1192-92-30099

Zarif Ahsan*, Stanford University, Xiran Liu, Stanford University, Noah A. Rosenberg, Stanford University. Mathematical properties of allele-sharing dissimilarity statistics in population genetics.
Allele-sharing statistics for a genetic locus measure the dissimilarity between two populations as a mean dissimilarity between random pairs of individuals, one from each population. Motivated by the study of allele-sharing dissimilarities in population genetics, we consider the dissimilarity between a pair of draws from discrete probability vectors on finite sets of objects. Consider two probability distributions on a set of I objects, and consider two random unordered draws of size K, one from each probability distribution. We define an equivalence relation for pairs of draws and enumerate all possible equivalence classes under this relation (for $\mathrm{I} \geq 2 \mathrm{~K}$ ). The enumeration relies on a series of actions by permutation groups. Considering probabilities of all possible pairs of draws in terms of the specified probability distributions, we evaluate the expectation of the dissimilarity between the two random draws - representing the expected allele-sharing dissimilarity between two populations with specified allele-frequency vectors. The combinatorial, probabilistic, and algebraic properties of the evaluation of allele-sharing dissimilarities are reflected in numerical computations with actual allele frequencies in human populations, helping to clarify the mathematical basis for previously unexplained empirical phenomena. (Received September 08, 2023)

1192-92-30130
Lily Agranat-Tamir*, Stanford University, Noah A. Rosenberg, Stanford University. Enumeration of rooted binary unlabeled galled trees.
Rooted binary galled trees generalize rooted binary trees to allow a restricted class of cycles, known as galls. These kinds of trees are relevant in phylogenetics, where the cycles can represent several biological or demographic events. We build upon the Wedderburn-Etherington enumeration of rooted binary unlabeled trees with $n$ leaves to enumerate rooted binary unlabeled galled trees with $n$ leaves, also enumerating rooted binary unlabeled galled trees with $n$ leaves and $g$ galls, $0 \leq g \leq\left\lfloor\frac{n-1}{2}\right\rfloor$. The enumerations rely on a recursive decomposition that considers subtrees descended from the nodes of a gall. We present numerical and asymptotic analyses of the resulting recursions, finding that the number of rooted binary unlabeled galled trees with $n$ leaves is far greater than the corresponding number of rooted binary unlabeled trees without galls. Numerical analysis also shows that for a fixed number of leaves $n$, the number of galls $g$ that produces the largest number of rooted binary unlabeled galled trees lies intermediate between the minimum of $g=0$ and the maximum of $g=\left\lfloor\frac{n-1}{2}\right\rfloor$.
(Received October 11, 2023)

Azmy S Ackleh*, University of Louisiana at Lafayette, Neerob Basak, University of Louisiana at Lafayette, Narendra Pant, University of Louisiana at Lafayette, Amy Veprauskas, University of Louisiana at Lafayette. The impact of predator evolution and periodic reproduction on the dynamics of a discrete-time predator-prey model. Preliminary report.
We extend the predator prey model developed in Ackleh et. al (2019) to account for evolution in the predator to resist toxicant effects as well as periodic reproduction due to seasonality. We first model the predator evolution to resist the effect of toxicants, assuming a trade-off between toxicant resistance and the ability of the predator to capture prey and survive. We study the dynamics of the resulting predator-prey model and compare the results with those in Ackleh et al. (2019), which assumed instead that the prey evolves to resist toxicants. Then, we model the reproduction as a periodic function of period-two and show the predator-prey model attains an interior 2-cycle. We study the local asymptotic stability of the periodic solution and establish persistence of the system and compare our results with those in Ackleh et al. (2019), which assumed that reproduction in continuous.
(Received September 09, 2023)

## 1192-92-30187

Folashade B. Agusto, University of Kansas, Kwadwo Antwi-Fordjour*, Samford University, Isabella Kemajou-Brown, Morgan State University. Modeling the effect of lethal and non-lethal predation on the dynamics of tick-borne disease. Preliminary report.
In this presentation, we will explore how predation, both lethal and non-lethal, affect the dynamics of a tick-borne disease using systems of differential equations. While these predation effects are commonly observed in ecological settings, they haven't been widely studied in the context of tick-borne diseases. We investigated the conditions under which the disease-free state remains stable using the reproduction number calculated via the next generation operator method. Remarkably, our results align with classical findings observed in real-world ecological experiments. Furthermore, we discovered that the combined impact of both lethal and non-lethal predation sets off a cascading chain reaction. This reaction ultimately leads to a reduction in the number of ticks, which in turn helps control the spread of tick-borne diseases. (Received September 09, 2023)

## 1192-92-30190

Louis J Gross*, University of Tennessee, Knoxville. Human Behavior Modeling, the Exposome and Disease Progression. Preliminary report.
The integration of human behavior modeling with classic structured population models for infectious diseases has been ongoing for several decades. Despite the tremendous impact on human health of noncommunicable diseases which make up by far the majority of causes of mortality in much of the world, there has been relatively little connection between human behavior modeling and models in areas such as cancer, heart disease, stroke and other physiologically-connected diseases. Human behavior directly impacts exposure at the individual and population level to known factors which impact noncommunicable disease progression, including diet, activity-level and environmental toxicants. The term used to describe the exposure throughout an individual's lifetime to factors which affect health is the exposome. Much of environmental risk assessment deals with models for the fate and effects of toxicants in the environment, though the magnitude of exposures included in these models doesn't account for social factors which impact behaviors. My objective is to discuss how the various theories derived from social psychology which have been incorporated in human behavior models and linked to climate and infectious disease modeling can provide insights about impacts on the exposome at the individual level, how these may be aggregated at population scale and the implications for disease progression.
(Received September 09, 2023)

## 1192-92-30196

Jordan Pellett*, University of Tennessee, Knoxville, Olivia Prosper, University of Tennessee/Knoxville. A within-host model for the pharmacokinetics and pharmacodynamics of malaria. Preliminary report.
Malaria is a significant public health concern, especially as the increasing presence of drug resistance creates challenges for control efforts. Over the last several decades, mathematical modeling has been used to gain insight into the within-host dynamics of malaria parasites. Within the human host, the parasite undergoes several life stages, including an asexual replication stage associated with the onset of symptoms, and a sexual stage of parasites (the gametocytes) which can be transmitted to mosquitoes. Of particular importance is the incorporation of pharmacokinetics and pharmacodynamics to better understand how different actions of treatment on malaria parasites impact gametocyte levels and the potential to select for drug-resistant parasites. In this talk we discuss a within-host model that incorporates anti-malarial treatment and explore the impact of different dosing regimes and drug actions on parasite load and probability of forward transmission. (Received September 09, 2023)

## 1192-92-30229

Polly Y. Yu*, NSF-Simons Center for Mathematical and Statistical Analysis of Biology. Homeostasis, biphasic response curve, perfect adaptation: arbitrary system vs. chemical systems. Preliminary report.
There has been increased interest in modelling (approximately) constant steady state value when perturbing a parameter (e.g., rate constant), especially finding graph-theoretic conditions that are necessary for such behaviour. In this talk, we will review an algebraic condition that models this phenomenon, and how important is the assumption of "reasonable kinetics" (e.g., massaction, Michaelis-Menten, power-law kinetics). We will further discuss how the algebraic condition manifest itself in a bipartite signed graph, the species-reaction graph, for chemical systems with "reasonable kinetics".
(Received September 09, 2023)

1192-92-30400
Ami Bhatt, Stanford University, Angela Hickey, Stanford University, Ivan Liachko, Phase Genomics, Danica Schmidtke*, Stanford University, Gavin Sherlock, Stanford University. Culturing an Abundant and Prevalent Gut Bacteriophage and Identifying Invertible Regions in its Genome.
Bacteriophages in the human gut microbiome outnumber bacteria by at least $2: 1$, yet, we know very little about them (Pinto and Chakraborty, 2023). For example, the prototypic crAssphage (p-crAss) was computationally discovered in 2014, is one of
the most prevalent and abundant gut phages (Dutilh, 2014), but remains uncultured and its bacterial host and lifestyle are unknown. We propagated p-crAss by culturing human stool samples, and using quantitative PCR (qPCR) to track its copy number over time. We observed that p-crAss can replicate to high titers in stool-based culture. We used meta-Hi-C sequencing to predict the bacterial host (Bacteroides vulgatus), and applied the supernatant from stool culture to B. vuglatus. Using qPCR to measure phage copies over time, we observed replication of the phage after 30 hours of culturing. We were unable to recover plaques of p -crAss on $B$. vulgatus, and the closest cultured relative of p -crAss (crAss002) also fails to plaque. To understand p-crAss's non-plaquing lifestyle, we first took a genomics-based approach. The genome is separated into two distinct regions with opposing gene orientation, where the forward strand is largely composed of genome replication related genes, and the reverse strand is largely structural and cell lysis related genes. We wondered whether a regulatory element might exist in between the forward and reverse stranded genes which impacts the lifestyle of the phage. Given that its bacterial host uses invertible regions of DNA (invertons) for gene regulation, we computationally searched for an identified an inverton in this region of the phage genome. We computationally predicted a promoter in the invertible region and found that the ratio of the two inverton orientations varies over time in the culture. We hypothesize that when flipped this inverton either promotes genome replication or the cell lysis.
(Received September 09, 2023)

1192-92-30443
Aaron Fogelson, University of Utah, Keshav B Patel*, University of Utah. A Spatially Averaged Model for Platelet Cohesion by Von Willebrand Factor and Fibrinogen. Preliminary report.
In this talk, we will discuss steps toward building a spatially-averaged model of platelet aggregation to inform PDE models and efficiently determine essential pathways involved in aggregate formation. High shear rate conditions in arterial blood flow result from several pathologies, such as cardiovascular disorders, cholesterol buildup, and stenosis. In these environments, mechanically sensitive protein Von Willebrand Factor (vWF) mediates the initial adhesion of platelets to the wall or pathological structure. CFD simulations provide a highly detailed description of the process, but computational complexity limits our ability to examine various physiological conditions that could impact aggregation, like shear rate, vessel geometry, and injury size. This purely dynamical systems framework incorporates an averaged flow through porous media, imparting a drag force that is balanced by crosslink bonds. Crosslinks are formed by vWF and fibrinogen and break in a force-dependent manner. We will show how development of the aggregate due to vWF crosslinking affects which pathway is primarily responsible for platelet activation over time. We will also investigate how the properties of vWF affect the growth rate and stability of the aggregate, particularly at high shear rate. These results have implications for aggregation in stenotic regions as well as in disease states such as Von Willebrand's Disease.
(Received September 22, 2023)

1192-92-30469
Esteban A. Hernandez Vargas*, University of Idaho. Topological data analysis informing parameter estimation of differential equations.
Mathematical modeling based on differential equations is an important tool in biomathematics for understanding the dynamics of infectious diseases and immune responses in the host. However, the variability in host diseases and immune responses plays a critical role in severity. Informing mathematical models with different levels of heterogeneity can improve the quantitative understanding. Topological data analysis (TDA) can uncover information from high-dimensional databases, which are complex to study with traditional methods. Here, we apply the mapper algorithm to antibody kinetic data of severe and non-severe COVID-19 patients. TDA shows differences in the shape of antibody responses that further classify COVID-19 patients into nonsevere, severe, and intermediate-severity cases. Based on this classification, parameter fitting is applied to represent the dynamics between the different severity groups. Our results show that TDA can suggest a more educated parameter estimation in differential equations to quantify immune mechanisms among severity groups.
(Received September 10, 2023)

## 1192-92-30537

Paola Vera-Licona*, Center for Cell Analysis and Modeling, UConn Health. Environmental and signaling factors regulating transcriptional dynamics of hypoxia-inducible factor-1 (HIF-1) within a tumor microenvironment. Preliminary report. Hypoxia, a state of low oxygen availability, is a hallmark feature of solid tumors that has been shown to influence tumor progression, metastasis, and treatment resistance significantly. Cellular adaptation to hypoxia is primarily mediated by a family of transcriptional regulators, the hypoxia-inducible factors (HIFs). Canonical response to hypoxia involves stabilization of HIF-1 $\alpha$, which acts as a key transcriptional factor, regulating the expression of more than 1000 gene products indirectly and influencing critical steps in cancer progression. However, recent evidence suggests that cancer cells exhibit substantial heterogeneity in their response to hypoxia, with some cells adapting and proliferating while others undergo HIF-1-mediated cell cycle arrest. Furthermore, the metabolic milieu in the tumor microenvironment can influence the heterogeneous presentation of HIF-1 activity. Because HIF-1a plays a central role in regulating transcription in hypoxic tumors, changes in its dynamics and heterogeneity within the cellular population may profoundly affect tumor evolution, growth, and metastasis. In this talk, we will discuss our work building a multiscale model to better understand the environmental and signaling drivers of HIF-1 $\alpha$ abundance and dynamics in a breast cancer model with the ultimate goal of identifying targets to reduce heterogeneity in HIF-1 transcriptional response and its effect on cancer growth and progression. (Received September 10, 2023)

## 1192-92-30609

Hum Nath Bhandari, Rogers William University, Christopher Burtner, Roger Williams University, Colton James
Pelletier*, Roger Williams University. Searching For Longevity Inducing Pharmaceuticals: A Case Study of Deep Learning Application in Computer Vision. Preliminary report.
Deep learning has proven a valuable tool to solve a wide variety of real-world problems. An important application includes solving computer vision problems, such as image detection and video classification. This study aims to implement deep learning techniques in the biology of aging problems, particularly in predicting the lifespans of Rotifers. The biology of aging is a specific field looking to understand how organisms develop in hopes of extending life spans. Rotifers are a relatively new model organism in this field; however, they have quickly proven their relevance by exhibiting various favorable traits, including a short natural life span, parthenogenic reproduction, ease of culture, and a recently sequenced genome. With
preliminary data showing that B. plicatilis life span can be extended with low concentrations of treatments such as rapamycin, we intend to search for additional pharmaceuticals with this effect. We assembled a robotic rotifer life span machine called 'Rotibot' to run through an FDA- approved drug library. It tracks the activities of rotifers as a lifespan video by capturing their images twice at 5 seconds apart at every 10 -minute intervals. The input data set is prepared after various data exploration and visualization steps, such as frame extrac- tion, image subtraction, and cropping. Several strategies are implemented to create input-output data sets to train deep learning models, particularly CNN-LSTM. The goal is to predict the rotifers' lifespans by capturing the spatial-temporal relationships of the given images. Furthermore, several hyper-parameter tuning and model selection strategies are implemented to identify the best-performing model configuration.
(Received September 10, 2023)

## 1192-92-30611

Hum Nath Bhandari, Rogers William University, Christopher Burtner, Roger Williams University, Colton James
Pelletier*, Roger Williams University. Rotifer Life Span Analysis Using Machine Learning Techniques. Preliminary report.
Rotifers are becoming a valuable model organism for the biology of aging with many ideal traits such as a short natural life span, parthenogenic reproduction, ease of culture, and a recently sequenced genome. Preliminary data suggests B. pli- catilis life span can be extended by pharmacological treatments known to extend the life span in other models, including C. elegans and fruit flies. With this knowledge, we aim to use machine learning techniques to analyze rotifer life spans being subjected to an FDA-approved drug library. Initially, life span data is gathered via a robotic sys- tem, taking images of experiments twice every ten minutes. Images are extracted and paired with their counterpart images taken five seconds apart. We then perform ap- propriate data analytics on those images, including image resizing, image subtraction, normalization, and cropping, so that any live rotifers (moving) appear on the output image and eliminate any detections outside the area of interest. Resulting images are then run through a trained YOLO machine learning algorithm for object detection, predicting rotifer counts. Further attempts are made to create input-output data sets to train machine learning models, to predict the rotifers' lifespans by capturing the spatial-temporal relationships of the given images.
(Received September 10, 2023)

1192-92-30644
Michael Andrew Robert*, Virginia Tech. Investigating impacts on malaria transmission of altered bioamine levels in Anopheles mosquitoes.
Malaria is caused by Plasmodium species that are transmitted to humans primarily by Anopheles mosquitoes. Currently, over half of the world's population is at risk of malaria, and after decades of progress towards eradication led to declines in cases reported globally, cases have increased since 2020, with 247 million cases and 619,000 deaths reported in 2021. Malaria infection is known to influence levels of biogenic amines in human blood. Specifically, individuals with severe malaria may exhibit increased concentrations of histamine and/or decreased concentrations of serotonin. The altered amine levels may also impact the biology and behavior of Anopheles mosquitoes that ingest them in bloodmeals, but it remains to be seen what role these changes may have on mosquito population dynamics and malaria transmission. We developed a stage-structured discrete time mathematical model of mosquito population dynamics coupled with population-level malaria transmission dynamics to investigate how these altered amine levels may play a role in the malaria transmission cycle. We incorporated demographic, behavioral, and parasite reproduction data into the model and explored scenarios that consider different possible concentrations of histamine and serotonin in bloodmeals and the effects thereof by studying impacts on mosquito population size and malaria incidence and prevalence. We explore different possible extensions of the model and discuss our findings in the context of malaria control as well as future experimental work.
(Received September 11, 2023)

## 1192-92-30681

Marek Kimmel*, Rice University. Branching processes with denumerable collections of types, inspired by models of cell evolution in cancer. Preliminary report.
Mutation processes in proliferating cancer cells lead to populations with highly diversified genomes. A natural simple tool to describe the resulting stochastic phenomena are branching processes (bp) with countable collections of types. I will review several such models I worked on over a number of years. These include processes such as (i) gene amplification (increase in copy count) in response to chemotherapy (Kimmel and Stivers, 1994), (ii) uneven replication and segregation of chromosomes (Kimmel, 1997), and (iii) bp's with randomly changing lifetime distributions (Ernst et al. 2019). Although these are biologically and mathematically different models, they share some "exotic" properties, which result from nontrivial interaction between branching and type transitions. Among other, they result in limit growth laws that can be exponential modified by negative fractional power, and unexpected relations between criticality and finite time moment explosions consistent with increasingly heavy-tail distributions of cell counts. Other interesting properties are present. This review illustrates diversity of behaviors of such models.
(Received September 10, 2023)

## 1192-92-30698

Lauren Sugden*, Duquesne University. On CNNs for Population Genetics: How much complexity is really necessary?. Convolutional neural networks (CNNs) have taken the research world by storm as a powerful approach across a broad swath of scientific disciplines. Here we investigate their use in population genetics for the particular problem of identifying genomic regions undergoing positive selection, a scenario which is well-studied, and therefore has a deep well of theory. Since multiple recent publications have shown CNNs to be effective for this problem, we investigate here what, if anything, we can learn from the trained model itself, how its learning differs or aligns with known theoretical patterns, and how human-dependent preprocessing sways the model to find certain patterns. Our findings indicate that human processing upstream of the model itself plays a crucial role, and that the typical complexity of CNNs is likely unnecessary for this particular problem. (Received September 10, 2023)

UT Austin. Rate-Independent Computation in Continuous Chemical Reaction Networks.
Coupled chemical interactions in a well-mixed solution are commonly formalized as chemical reaction networks (CRNs). CRNs are widely used to describe information processing occurring in natural cellular regulatory networks, and with upcoming advances in DNA nanotechnology, CRNs are a promising language for the design of artificial molecular control circuitry However, despite the widespread use of CRNs in the natural sciences, the range of computational behaviors exhibited by CRNs is not well understood. Here we study the following problem: what functions $f: \mathbb{R}^{k} \rightarrow \mathbb{R}$ can be computed by a CRN, in which the CRN eventually produces the correct amount of the "output" molecule, no matter the rate at which reactions proceed? This captures a previously unexplored, but very natural class of computations: for example, the reaction $X_{1}+X_{2} \rightarrow Y$ can be thought to compute the function $y=\min \left(x_{1}, x_{2}\right)$. Such a CRN is robust in the sense that it is correct no matter the kinetic model of chemistry, so long as it respects the stoichiometric constraints. We develop a reachability relation based on "what could happen" if reaction rates can vary arbitrarily over time. We define *stable computation* analogously to probability 1 computation in distributed computing, and connect it with a seemingly stronger notion of rateindependent computation based on convergence under a wide class of generalized rate laws. We also consider the "dual-rail representation" that can represent negative values as the difference of two concentrations and allows the composition of CRN modules. We prove that a function is rate-independently computable if and only if it is piecewise linear (with rational coefficients) and continuous (dual-rail representation), or non-negative with discontinuities occurring only when some inputs switch from zero to positive (direct representation). The many contexts where continuous piecewise linear functions are powerful targets for implementation, combined with the systematic construction we develop for computing these functions, demonstrate the potential of rate-independent chemical computation.
(Received September 11, 2023)

1192-92-30774
Badal Joshi, California State University San Marcos, Tung D. Nguyen*, Texas A\&M University. Identifying the locus of bifunctional enzyme action in enzymatic reaction networks of arbitrary size and complexity. Preliminary report.
Shinar and Feinberg in 2010 introduced the concept of absolute concentration robustness (ACR) in biochemical reaction systems to mean the concentration of a certain species at positive steady state does not depend on the initial conditions. Algebraically, this means all positive steady states of the reaction system lie on a hyperplane of the form $x_{i}=c$ for some positive constant $c$. Shinar and Feinberg gave sufficient conditions for the existence of a robust species; that is the underlying reaction network must have a deficiency of one and there are two non-terminal complexes differing in the robust species. While the former condition (deficiency one) does not have a clear biological meaning, the latter condition is related to the "bifunctionality" of an enzyme. When isolated, the bifunctional enzyme promotes production of the robust species while when bound, the same enzyme facilitates degradation of the robust species. We develop a graphical framework to define bifunctionality in a large class of enzymatic reaction networks of arbitrary size and complexity, and prove that the presence of bifunctionality causes concentration robustness in certain target species. (Received September 11, 2023)

## 1192-92-30786

Brendan Shrader*, University of Central Florida, Zhisheng Shuai, University of Central Florida. Global dynamics of an SIR model with post-infection mortality and partial immunity preliminary report. Preliminary report.
The recent COVID-19 pandemic highlights the importance of understanding how post-infection mortality affects disease dynamics. This is because a significant proportion of COVID-19 survivors continue to experience persistent symptoms, known as "Long-Covid," enduring for years after initial infection. In this paper, we discuss an epidemiological model that incorporates post-infection mortality and partial immunity. Additionally, our model assumes the disease is transmitted by the standard incidence rate. We prove the existence and uniqueness of the endemic equilibrium and provide preliminary results on the global stability of the endemic equilibrium and the disease-free equilibrium. We compare our model to a similar model that uses the mass-action incidence rate, with particular attention to the existence of limit cycles. We find that in cases where limit cycles occur in the mass-action, they do not occur in our standard incidence model. Our findings highlight how incidence rates qualitatively change the dynamics of disease models and how partial immunity impacts disease severity. (Received September 11, 2023)

## 1192-92-30840

Dimos Gkountaroulis*, Baylor College of Medicine, Erez Lieberman Aiden, Baylor College of Medicine. The shapes of DNA. Preliminary report.
Stretched out from end-to-end, the DNA in a single human cell is two meters long. And yet this genome folds up to fit inside a nucleus that is only a few micrometers wide - all while remaining fully functional, serving as a repository of 6 billion chemical letters of genetic information that collectively determine a cell or organism's traits. How is this accomplished? In our lab we study the genome, at work, in 3D. One key tool we use is Hi-C (Lieberman-Aiden et al., Science, 2009; Aiden, Science, 2011) an experiment that measures how frequently all of the sequences in a genome collide into one another in 3D. Such experiments allow us to deduce what the 3D structure must be. A second key modality we employ is microscopy, whereby we can create images of the genome in true 3D. Together, these methods allow us to explore the geometry, topology, physics, and biochemistry of genomes - across species, tissues, and time; in health and in disease. In this talk, I will give a broad overview of the many shapes of DNA, with a focus on efforts to interrogate the geometry and topology of genomes by direct microscopic visualization.
(Received September 11, 2023)

## 1192-92-30861

Nicholas Linthacum*, Gonzaga University. Machine Learning Priors for Electrical Impedance Tomography.
The internal conductivity of a medium can be determined by applying a current to the boundary and measuring the resulting voltage. The inversion process to reconstruct the conductivity is the basis of Electrical Impedance Tomography (EIT). A common application of EIT is medical imaging because it is non-invasive, low-cost, portable, and allows for real-time imaging. Previous advances have allowed for a priori spatial data in the form of organ boundaries to be included in the reconstruction, yielding more stable and sharper reconstructions. The prior information has historically been manually extracted from previous scans, but this is a labor-intensive process, requires a previous scan, and introduces human bias. This talk
demonstrates the use of machine learning to generate a priori data, solving the aforementioned issues with manual extraction. (Received September 11, 2023)

1192-92-30914
Sonia Durr, Department of Biomedical Engineering, San Jose State University, Antti Elonen, Department of Computer Science, Aalto University, Seth Gonzalez, Department of Biomedical Engineering, San Jose State University, Hao Legaspi, Department of Biomedical Engineering, San Jose State University, Abdulmelik Mohammed*, San Jose State University, Han Nguyen, Department of Biomedical Engineering, San Jose State University, Johnny Nguyen, Department of Computer Science, San Jose State University, Dung Pham, Department of Biomedical Engineering, San Jose State University, Matthew Vu, Department of Biomedical Engineering, San Jose State University. Unknotted 3D Tracing of DNA Strands for the Design of Toroidal Nanopolyhedra. Preliminary report.
The unknotted tracing of circular DNA scaffold strands along the edges of linearly embedded graphs is the most complex step in the automated design of wireframe DNA origami. Previously proposed A-trail scaffold routings can be knotted for nonspherical polyhedral graphs, severely limiting the class of DNA origami wireframes that can be assembled based on the efficient Eulerian routings. We propose a method to double at most one-third of the edges so that an unknotted scaffold routing can be found in an augmented polyhedral graph. To this end, we propose and implement an algorithm that takes a polyhedron in a PLY format and outputs the set of double edges that make the faces of the augmented polyhedron checkerboard colorable. In addition, we propose and implement a branch-and-bound algorithm to search for a covering-tree of the checkerboard colorable augmented polyhedral graph, from which an unknotted A-trail scaffold routing can be obtained by going around the covering tree.
(Received September 11, 2023)

## 1192-92-30915

Khanh Dao Duc*, University of British Columbia. The mathematics of ribosome heterogeneity.
The ribosome is a molecular machine made of RNA and protein complexes, that mediates the translation of mRNA into protein. While ribosomes were traditionally thought to be homogeneous, recent work has discovered that their composition is highly heterogeneous, with important consequences on the regulation of translation. By using a variety of mathematical and computational approaches, I have recently quantified the extent of such heterogeneity and its impact across multiple scales, with a specific focus on the ribosome exit tunnel, a narrow cavity of the ribosome that contains the nascent protein. At the atomic level, we developed an online database that provides organized access to all the ribosome structures available from cryogenic electron microscopy (cryo-EM), with specialized visualization tools. By extracting and comparing the geometry of the ribosome exit tunnel across species, we found some key differences between eukaryotes and prokaryotes, and further investigated the functional implications of the differences between tunnels by using molecular dynamics (MD) simulations of the exit time of polypeptide chains. To facilitate the comparison of different EM maps, we also developed a new general methods for 3D alignment, that relies on optimal transport, as an alternative non linear metric for comparing 3D shapes. At the gene level, I'll show how using a stochastic model that generalizes the Totally Asymmetric Simple Exclusion Process (TASEP) allowed us to infer gene specific translation rates from sequencing data, with some applications for differential expression analysis and unraveling the regulation of translation by the FMR X gene associated with autism.
(Received September 11, 2023)

## 1192-92-30916

Javier Arsuaga*, University of California, Davis, Gina Gonzalez, UC Davis, Mariel Vazquez, University of California, Davis. Using machine learning to detect coronaviruses potentially infectious to humans.
We address the challenge of identifying non-human animal coronaviruses that may infect humans by creating an artificial neural network model. This model learns from spike protein sequences of alpha and beta coronaviruses and their binding annotation to their host receptor. We identify three viruses, previously unknown to bind human receptors: Bat coronavirus $\mathrm{BtCoV} / 133 / 2005$ (a MERS related virus), Rhinolophus affinis coronavirus isolate LYRa3 (a SARS related virus) and Pipistrellus abramus bat coronavirus HKU5-related. We further analyze the binding properties of these viruses using molecular dynamics. To test whether this model can be used for surveillance of novel coronaviruses, we re-trained the model on a set that excludes SARS-CoV-2 and all viral sequences released after the SARS-CoV-2 was published. The results predict the binding of SARS-CoV-2 with a human receptor, indicating that machine learning methods are an excellent tool for the prediction of host expansion events.
(Received September 11, 2023)

## 1192-92-30974

Timothy D Comar*, Benedictine University. Introductory Student Research Projects In Mathematical Biology.
This presentation will discuss some ongoing student research projects in mathematical biology initially designed for students with only introductory experience in mathematics and no prior experience in mathematical biology. The projects are based on the development and analysis of integrated pest management models or vaccination models. The modeling frameworks included agent-based models and impulsive differential equations. We will discuss how the students begin to explore problems and then progressively learn to ask and investigate further questions. Project results will also be presented. (Received September 11, 2023)

1192-92-31013
Jim Elser, University of Montana, Puni Jeyasingh, Oklahoma State University, Yang Kuang, Arizona State University, Tin Phan*, Los Alamos National Laboratory, Clay Prater, University of Arkansas. Resource imbalances, organismal growth, and population dynamics.
The growth of an organism is intricately governed by the dynamic imbalances of multiple resources, which also has profound implications at the population level. Through the lens of Liebig's law of minimum, single resource limitation has historically been used in place of multiple resource limitation to gain insights into the complex dynamical processes. However, an increasing number of experimental studies have demonstrated that organisms, such as E. coli, can in fact dynamically adapt their resource allocation strategies in response to different resource limitations. This suggests the interconnected nature of
different resources and an organism's resource allocation strategy. Thus, a more encompassing framework to study the impact of multiple resource limitations and its influence on population dynamics is necessary. In this presentation, I will present two simple modeling frameworks that link resource imbalances, organismal growth, and population dynamics. These modeling frameworks not only can recapitulate experimental observations, but also provide a fresh perspective on the complex multiple resource limitations and their implications for the population dynamics.
(Received September 22, 2023)

## 1192-92-31017

Hem Raj Joshi*, Xavier University, Cincinnati. Covid-19 inter-provincial disparities and clinical trails. Preliminary report. The COVID-19 pandemic has had a huge impact on the lives of all, the pandemic started towards the end of 2019 and there are currently more that 770 million confirmed cases worldwide and almost seven million deaths. This disease is still a global threat and different strains emerge in different part of the world time and again. We also studied the inter-provincial disparities of important indicators such as movement data, epidemic trend, epidemic growth rate, and reproduction numbers. Furthermore, we extended our analysis to identify prevention and control policies that are effective in altering these indicators. Vaccination is perhaps the most consequential intervention for any pandemic and clinical trials are an important requirement before this intervention gets to the people. In this work, we also model a clinical trial for COVID-19 vaccines and present selected simulation results. (Received September 11, 2023)

1192-92-31024
Ross Cressman, Wilfred Laurier University, Theodore E. Galanthay*, Ithaca College, Vlastimil Krivan, Czech Academy of Sciences, Tomás A. Revilla, Czech Academy of Sciences. Evolution of aggression in consumer-resource models. Are animals in resource-poor environments more or less aggressive than animals in resource-rich environments? Dynamic game-theoretical models provide conflicting answers to this ecological question. We propose a differential equation model that resolves this conflict. In this talk, we explain how the assumptions of a model influence the answer, and we highlight the dynamical behavior of our model.
(Received September 11, 2023)

1192-92-31025
Samantha Brozak, Arizona State University, Ruian Ke, Los Alamos National Laboratory, Yang Kuang, Arizona State University, Bruce Pell, Lawrence Technological University, Alan S Perelson, Los Alamos National Laboratory, Tin Phan*, Los Alamos National Laboratory, Ruy M Ribeiro, Los Alamos National Laboratory, Fuqing Wu, University of Texas Health Science Center at Houston. Integrating wastewater surveillance data in multi-scale epidemiological models.
Wastewater-based surveillance (WBS) data has been proven to be a valuable tool to track the COVID-19 pandemic and many other diseases. WBS data bypasses the issues of time lag and underreporting in case report data, thus enhancing the utility and applicability of dynamic models. However, epidemiological inference from WBS data remains understudied with the vast majority of literature revolving around establishing the correlation between WBS data and case report data. In this presentation, I will first present a simple extension of the classical SEIR-type model using within-host viral dynamics and demonstrate the capability of the proposed model to integrate WBS data to make epidemiological inferences. Then, using this framework, I will highlight existing challenges associated with the usage of WBS data and how mathematical models can help addressing these limitations.
(Received September 11, 2023)

## 1192-92-31028

Erica Marie Rutter*, University of California, Merced. Hybridizing Machine Learning and Dynamical Systems to Model Biological Systems.
Traditional mathematical modeling approaches to biology are interpretable and generalizable, but it may take years to iteratively develop accurate mathematical models. Machine learning approaches are quick and easy to train, but often suffer in predicting new data, are rarely interpretable, and require large amounts of data. I will explore a hybrid framework that combines elements from dynamical systems and machine learning that is interpretable and requires little data. In particular, I will present a robust method for learning the underlying dynamical systems from sparse noisy spatiotemporal data, drawing from examples in cell migration, physiology, and cancer.
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## 1192-92-31051

Aaron Fogelson, University of Utah, Anna Nelson*, Duke University. Towards a mathematical model of platelet aggregation and fibrin polymerization under flow. Preliminary report.
The formation of a blood clot involves complex biophysical and biochemical processes that occur under flow. In the event of an injury, platelets become active and weakly aggregate to form a plug at the injury site. Activated platelet allow for surfacebound complexes to convert zymogen prothrombin into enzyme thrombin. Thrombin further activates platelets and converts fibrinogen molecules in the blood to fibrin monomers, which polymerize and form a space-filling gel that supports the growing clot. Fibrin(ogen) interacts with activated platelets, and can facilitate platelet adhesion and cohesion through bonds mediated by fibrin(ogen). In this work, we propose a mean field mathematical model of fibrin branch formation which tracks fibrinogen fibrin, and platelet species in either a bound or unbound state. Fibrin oligomers form in both the bulk and on the surface of platelets, and flow-mediated transport will affect fluid-phase species. A kinetic fibrin polymerization model is used to model fibrin gel formation; this model is studied up until gelation, which is defined as the emergence of an oligomer of infinite size. In this presentation, we show how the gel time depends on model parameters and how platelet-fibrin(ogen) interactions affect the gel structure. Finally, we discuss on going work of incorporating biological parameters into our modeling framework and matching our existing mathematical model to experimental data.
(Received September 11, 2023)

Andrew Nevai, University of Central Florida, Connor Shrader*, University of Utah. Predation and Harvesting in Spatial Population Models.
Predation and harvesting play critical roles in maintaining biodiversity in ecological communities. Too much harvesting may drive a species to extinction, while too little harvesting may allow a population to drive out competing species. The spatial features of a habitat can also significantly affect population dynamics within these communities. Here, we formulate and analyze three ordinary differential equation models for the population density of a single species. Each model differs in its assumptions about how the species is harvested. We then extend each of these models to analogous partial differential equation models that more explicitly describe the spatial habitat and the movement of individuals using reaction-diffusion equations. We study the existence and stability of non-zero equilibria of these models in terms of each model's parameters. Biological interpretations for these results are discussed.
(Received September 11, 2023)

## 1192-92-31099

Alan Hastings*, Santa Fe Institute. Spatial dynamics in relation to movement of suitable habitat. Preliminary report. Global change is moving the location of suitable habitat for many organisms. Simple spatial models, based on reactiondiffusion or integro-difference equations, can provide insights into the effect on dynamics and persistence of the ecological population. The basic questions are whether the combination of dispersal by the organism and the local growth rates permit persistence or lead to extinction. I will begin with a review and overview of existing results obtained using these approaches. I will then present an even simpler formulation that is discrete in both time and space. I will first show that this simple model can easily duplicate the ecological insights of the murre complex models. I will then show that the simpler formulation is ideal for considering different management approaches, including the idea of actively moving organisms in response to threats to persistence.
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## 1192-92-31106

Junping Shi*, College of William \& Mary, Yiwen Tao, Zhengzhou University. A reaction-diffusion-advection model for glucose metabolism. Preliminary report.
A reaction-diffusion-advection model for glucose metabolism is proposed to describe the spatiotemporal behaviors of glucose in the pancreatic islet. We investigate the global existence and boundedness of the model's solution, as well as establishes the uniqueness of its positive steady state. We also propose formulas of spatiotemporal sensitivity index and correlation index to identify high-impact physiological factors and illustrate parameter inter-dependency. Additionally, we simulate different stages such as hyperinsulinemia, hypoglycemia, euglycemia, and diabetes to analyze the system's dynamics under varying physiological conditions. These findings provide valuable guidance in the therapeutic process, aiding in the development of effective interventions.
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1192-92-31111
Rima Arnaout*, University of California, San Francisco. Combining machine learning with clinical priors to solve problems in medical imaging. Preliminary report.
Deep learning (DL) has potential for improving diagnosis and management decisions in medical imaging. To date, however, DL can be inefficient and can fail at challenging, high-stakes tasks. We hypothesize that using mathematical modeling to curate and/or create medical imaging data can improve DL's efficiency and performance. One example of this approach concerns instance selection to prioritize using those images that are most likely to improve a model's performance. It is natural to prefer straightforward, robust, quantitative metrics as the basis for prioritization for instance selection. However, in current practice, such metrics are not tailored to, and almost never used for, image datasets or DL. To address this problem, we introduce ENRICH-Eliminate Noise and Redundancy for Imaging Challenges-a customizable method that prioritizes images based on how much diversity each image adds to the training set. We show that medical datasets are special in that in general each image adds less diversity than in nonmedical datasets. We demonstrate that ENRICH achieves nearly maximal performance on classification and segmentation tasks on several medical image datasets using only a fraction of the available images and without up-front data labeling. ENRICH outperforms random image selection, the negative control. Finally, we show that ENRICH can also be used to identify errors and outliers in imaging datasets. ENRICH is a simple, computationally efficient method for prioritizing images for expert labeling and use in DL. (Received September 11, 2023)

## 1192-92-31119

Romuald Lipcius, Virginia Institute of Marine Science, Gwen Sargent, William \& Mary, Leah Shaw, William \& Mary, Junping Shi*, College of William \& Mary. Seasonal disease model of blue crab population in the Chesapeake Bay. Preliminary report.
The emergence of pathogens in marine systems affects us all by damaging fisheries and their supporting ecological communities. Climate change magnifies the disease spreading through direct and indirect effects on both the host and pathogen dynamics. A stage-structured epidemic model is constructed to study the impacts of density-dependent predation, cannibalism, fishing, and Hematodinium infection on the blue crab population in the Chesapeake Bay. It is shown that extinction, disease-free and disease-outbreak dynamics can occur under different parameter conditions, and the diseaseoutbreak could happen in different frequencies.
(Received September 11, 2023)

1192-92-31128
Alan Hastings*, Santa Fe Institute. Transient dynamcis and the management of ecological systems. Preliminary report. Ecological dynamics typically play out on relatively short time scales, so asymptotic behavior of models is often not relevant and instead transient dynamics are relevant. This is particularly true for management actions, and with global change. I will present an overview of general principles related to the idea that managing an ecological system may involve maintaining a
transient, or, alternatively, shortening a transient. In either case recognition of the transient is the first important step. I will discuss how this applies to specific examples, including coral-algal grazer dynamics.
(Received September 11, 2023)

## 1192-92-31130

Simone Bianco, Altos Labs, Lulin Jiang, Altos Labs, Chaitanya Mokashi, Altos Labs, Robyn Shuttleworth*, Altos Labs. Mathematical Modeling of the Blood-Brain Barrier. Preliminary report.
The formation of the blood-brain barrier ( BBB ) is a key process during both early development and recovery after injury. To analyze the functions of the BBB, experiments are run on manufactured brain tissue. This process involves mixing the three key cell types; endothelial cells, astrocytes, and pericytes, under specific experimental conditions to enable in vitro vascularization. We use a mathematical model to simulate this system and explore the dynamics of an entire cell population throughout space and time. Using an agent-based model, we define a set of rules by which each cell behaves, as well as describing cell-cell communication through interaction dynamics. We simulate many different experimental conditions and identify key cellular processes, with the aim of optimizing both the state and structure of the underlying media and cell distributions necessary to form a stable vasculature.
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## 1192-92-31140

Vincent Jodoin, University of Tennessee, Knoxville, Suzanne Marie Lenhart, University of Tennessee, Knoxville, Leigh Bennett Pearcy*, University of Pittsburgh, Owen Queen, Harvard Medical School, Christopher Strickland, University of Tennessee, Knoxville. The Role of Social Networks on the Prescription Opioid Epidemic. Preliminary report. Substance use disorder (SUD) is a complex condition that leads to problematic drug use despite the adverse consequences to one's health and social functioning. Unlike a pathogenic disease, which spreads solely through social contact, the mechanics of SUD development are fundamentally different. For example, prescription drugs and social health play a major role in the development of opioid use disorder (OUD). Our work takes an individual-level approach by using a cellular automata on a social network to model OUD. We explore the relative influence of a network-based social contagion on the spread of prescription- and illicitly-sourced opioids by creating a minimization algorithm to fit the stochastic cellular automata models to a data-driven, deterministic differential equations model by Phillips et al. using two approaches: (1) A direct fit using a meanfield approximation, and (2) a maximum likelihood estimation given specified distributions. Most notably, we find that the transition probability between OUD classes increases dramatically, implying that individuals with prescription-based OUD and connections to illicit fentanyl users very quickly transition to using fentanyl themselves.
(Received September 22, 2023)

## 1192-92-31149

Daniel Cicala*, Southern Connecticut State University, Yi Jiang, Georgia State University, Kristin Marie-Dettmers Kurianski, California State University Fullerton, Glenn Ledder, University of Nebraska-Lincoln, Jane Hyo Jin Lee, Stonehill College, Yanping Ma, Loyola Marymount University, Los Angeles CA USA. Flexible Vaccine Ideology for Infectious Disease Models. Preliminary report
Differing ideologies about receiving vaccinations emerged from the vaccine roll out during the COVID-19 outbreak. To study disease dynamics with differing vaccine ideologies, Ledder introduced a modified SIR-style model incorporating the ideological effects that partitions the susceptible population into two states-a pro-vaccine state who will become vaccinated and an unwilling state who will not get vaccinated-and sets the recovery state to include those who have recovered from the disease and those who have received a vaccination. In Ledder's model, the vaccine ideology is fixed, in that one cannot move between pro-vaccine and unwilling states. In this talk, we present a modification of Ledder's model that incorporates the ability to move between the pro-vaccine and unwilling states with a rate depending on the disease prevalence. Additionally, we discuss the existence and stability of the endemic disease equilibrium, including conditions that lead to a limit cycle rather than a stable end state. Finally, we present numerical simulations of this new model. (Received September 22, 2023)

1192-92-31193
Daniel Brendan Cooney*, University of Pennsylvania, Simon A Levin, Department of Ecology and Evolutionary Biology, Princeton University, Dylan H. Morris, University of California, Los Angeles, Fernando W. Rossine, Harvard University. A PDE Model for Protocell Evolution and the Origin of Chromosomes via Multilevel Selection
The origin of chromosomes was a major transition in the evolution of complex cellular lie. In this talk, we model the origin of chromosomes by considering a simple protocell composed of two types of genes: a "fast gene" with an advantage for gene-level self-replication and a "slow gene" that replicates more slowly at the gene level, but which confers an advantage for protocelllevel reproduction. Protocell-level replication capacity depends on cellular composition of fast and slow genes. Using a PDE to describe how the composition of genes within protocells evolves over time under within-cell and between-cell competition, we find that the gene-level advantage of fast replicators casts a long shadow on the multilevel dynamics of protocell evolution: no level of between-protocell competition can produce coexistence of the fast and slow replicators when the two genes are equally needed for protocell-level reproduction. By introducing a "dimer replicator", a linked pair of the slow and fast genes, we show that coexistence between the two genes can be promoted in pairwise multilevel competition between fast, slow, and dimer replicators. Our results suggest that dimerization, or the formation of a simple chromosome-like dimer replicator, can help to overcome the shadow of lower-level selection and work in concert with deterministic multilevel selection to allow for the coexistence of two genes that are complementary at the protocell level but compete at the level of individual gene-level replication.
(Received September 11, 2023)

1192-92-31243
Maria Gabriela Navas Zuloaga*, University of California, San Diego. The role of synaptic connectivity on sleep dynamics: spatio-temporal properties of cortical slow oscillations in a large-scale thalamocortical network model of the human brain. Preliminary report.

Slow-wave ( $<1 \mathrm{~Hz}$ ) sleep, characterized by cortical slow oscillations (SO) of alternating active (up) and silent states (down), is critical for memory consolidation. However, understanding the emergence of global SO from micro-scale neuron dynamics and network connectivity remains unclear. We establish a multi-scale, biophysically realistic "whole-brain" thalamocortical network model of slow wave sleep, and investigate the role of connectivity on SO generation and propagation. Within a broad range of biologically reasonable values of conduction delays, long-range propagation and synchrony of SO is essentially unchanged. In contrast, as connection range is reduced, specific regions exhibit local SO that fails to spread through the cortex. This results in reduced global SO amplitude as long-range synchronicity between locally oscillating neural populations is impaired. Conversely, synchronized up and down states are maintained in sparser or more weakly connected networks. However, they exhibit reduced overall activity leading to lower frequency and amplitude of SO. Lastly, reduction of long-range connection strength resulted in SO with moderate propagation from multiple sources, which most closely resembles experimental results. Our work elucidates how spatio-temporal properties of SO emerge from global cortical connectivity and provides a framework to further explore slow-wave sleep disruptions and diseases.
(Received September 11, 2023)

## 1192-92-31256

Diana T White*, Clarkson University. Mathematical modeling of the growth, spread, and control of invasive watermilfoil. Eurasian watermilfoil (EWM) and Variable-Leaf Watermilfoil (VLM) are two highly invasive varieties of the aquatic plant watermilfoil, which is found throughout the US and in Southern Canada. EWM is one of the most invasive plants in the US, and is believed to have been introduced into North America in the 1940s (originating from Europe, Asia, and North Africa). VLM is native to the Southern US, but is invasive (and spreading rapidly) in northern regions. Like other invasive plants, invasive watermilfoil (IWM) has the ability to grow and spread quickly, forming dense monocultures, due to its ability to out-compete many native aquatic plants. Expanding on previous theoretical models, we use an ordinary differential equation approach to model the growth of IWM in a dense stand, and a partial differential model to account for seasonal diffusion of the plants roots. Our project goals are to (1) help understand how IWM is growing and spreading in a given lake/domain in a natural setting, and (2) to understand how to reduce IWM invasion in a lake by simulating handpulling and mat laying. Handpulling is a "instantaneous" control, whereas mats can be placed for long time periods over a growing season, requiring the introduction of new boundary conditions along patches in the lake's interior. Here we will show the results for IWM growth and spread under different handpulling and may laying control simulations to provide insight into the efficacy of this approach in real lakes. (Received September 11, 2023)

1192-92-31275
Timothy W. Secomb*, University of Arizona. Mathematical models for the development of microvascular networks: angiogenesis, remodeling and pruning.
Physiological transport of materials in the body is accomplished mainly by convective transport in blood together with diffusion into surrounding tissue. This requires hierarchical network structures to allow efficient flow distribution, together with many small terminal vessels to allow short diffusion distances. Such structures emerge by local responses to available biological signals, through the processes of angiogenesis, remodeling and pruning. Mathematical models of these processes have been developed, including molecular diffusion of oxygen and growth factors, growth of new vessels by stochastic sprouting angiogenesis, and remodeling and pruning of vessels in response to local levels of growth factors. Simulations using these models demonstrate development of functional network structures, and show how defects in vascular function can lead to impaired tissue oxygenation.
(Received September 11, 2023)

## 1192-92-31284

Simon Roux*, DOE Joint Genome Institute, Lawrence Berkeley National Laboratory. Viral ecogenomics: how and why our collective exploration of viral diversity and viral ecology has been transformed by high-throughput sequencing. Virology historically focused on viruses as pathogenic agents responsible for some of the major infectious diseases in human, cattle, and plant populations. These viruses were then studied in the laboratory, most often based on cell cultures or experimental models, and provided the foundational basis for our current understanding of viral biology. However, they represent only a tiny fraction of the global diversity of viruses on Earth. Indeed, all forms of life on the planet, including microbes, are infected by viruses which can strongly constrain their ecology and evolution. The world's oceans, for example, harbor an estimated $10 \backslash$ textsuperscript $\{30\}$ virus particles, with an estimated 1 (microbial) cell out of 3 infected at any given time. Understanding the key drivers of viral diversity and ecology across hosts and ecosystem is thus critical to fully comprehend the multiple impacts that viruses exert on biological processes. Until recently, technical challenges limited our ability to even catalog the global virosphere, leading to the denomination of these seen-but-uncharacterized viruses as "dark matter of the biological universe". In the past three years alone however, metagenomic approaches, i.e. the direct sequencing of genetic material from an environmental sample, increased viral genome databases by $>1,000$ times. While still incomplete and unevenly representing the whole viral diversity, this comprehensive catalog of uncultivated virus genomes represents an invaluable resource to evaluate ecological and evolutionary patterns in the viral world. Our current work presented here involves the development of new approaches to maximize the recovery of viral genomes from metagenomes and make these available to the broader community of researchers, as well as examples of how these data can be leveraged to further understand evolutionary constraints on viral particle structure as well as ecological drivers of virus-host dynamics. Eventually, we envision that a full viral ecogenomics toolkit will empower researchers to characterize viral communities and virus-host interactions in nature with an unprecedented level of details and resolution, enabling us to revisit long-standing biological questions and possibly enabling robust and predictive modeling of viral diversity and evolution. (Received September 11, 2023)

Ethan T Holleman*, UC Davis. Single-strand nicks and single-strand binding protein dramatically alter $R$-loop formation landscape. Preliminary report.
R-loops are three stranded nucleic acid structures that are formed co-transcriptionaly by RNA polymerase when the nascent RNA strand hybridizes to the DNA template strand, displacing the template strand. Utilizing an in vitro transcription system and single molecule R-loop footprinting techniques we have demonstrated that the frequency and pattern of R-loop formation is dramatically altered by the presence of single strand breaks and single strand binding protein (SSB) during transcription,
the effects of which provide new insights into the dynamics of R-loop formation, stability and energetics.
(Received September 11, 2023)

1192-92-31343
Jonathan Erwin Forde*, Hobart and William Smith Colleges. Resource constraints and control in epidemiological models. Preliminary report.
During emergent outbreaks of viral infections, public health policy decisions are made on the basis of incomplete information in a changing landscape of scientific knowledge and budgetary and infrastructure constraints. We developed a multi-scale immunoepidemiological model of the spread of COVID-19 to explore efficacy of testing and vaccination for preventing outbreaks. Our model indicates that in budget constrained situations, less sensitive but cheaper tests can be more effective at outbreak preventions. Our findings further indicate that population vaccination prevalence has a strong influence on the efficacy of testing regimes. In emerging pandemic scenarios, limited public health resources must also be divided between different control methods. Accounting for the trade-offs necessitated by the resource limitation is essential when formulating an optimal policy response. Using our immunoepidemiological model, we pose optimal control problems to explore the implications of several such trade-offs, focusing on testing vs. vaccination and long-term vs. short-term public health objectives. We also explore the how these optimal controls are influenced by the efficacy of the interventions and the frequency with which policy changes can be made.
(Received September 11, 2023)

1192-92-31353
Feng Wang*, University of Arkansas, Fayetteville. Properties of supercooled water studied with microsecond simulations using a first principles based model potential.
The temperature of maximum density of liquid water can be explained by the existence of a liquid-liquid phase transition in the supercooled regime. Atomic models being used to model this regime are typically hampered by the inability to properly model ice and liquid equilibrium, thus unable to properly simulate supercooled condition. With the Adaptive Force Matching (AFM) method, a coupled cluster quality potential energy surface is mapped into simple molecular mechanics energy expressions. The model gives a good description of both ice and water and is able to access the supercooled regime with high confidence. Microsecond time scale simulations with this model showed clear evidences for liquid-liquid phase transitions. Also, the viscosity of super cooled water shows a strong to a fragile dynamic crossover. The data suggests a new equation for the surface tension of supercool cooled water, which indicates new physics emerging as water is supercooled far below the normal freezing temperature.
(Received September 11, 2023)

## 1192-92-31364

Francesca Bernardi*, Worcester Polytechnic Institute, Shankar Chellam, Texas A\&M University, Nick Cogan, Florida State University. Monte Carlo simulations of $2 D$ flat-sheet membrane filters for water purification. Preliminary report. Municipal facilities performing potable water purification in the United States utilize micro- or ultra-filtration. In these systems, wastewater is pushed through membranes with micro or nanometer-scale pores to filter it; foulants and pollutants are captured by the membrane, allowing clean water to exit. Periodically, filters are cleaned to maintain constant flux operations through a process called "backwashing." The flow is reversed, and clean water is sent through the filters to dislodge impurities from the occluded pores. Understanding the fluid dynamics of filtration and foulant accumulation would allow for better control of the fouling process and improved filtration efficiency. In this talk we introduce a 2D flat-sheet model for the filtration membranes and perform Monte Carlo simulations to qualitatively capture fouling and backwashing mechanisms. Foulant dynamics are described by a two-state system: particles are either being transported by the background plug flow or are captured by the membrane and fixed in situ. Initial simulations show encouraging results describing the dynamics of foulant accumulation during forward filtration and dislodgment during backwashing operations. Next steps and potential benchmarking with real-world data will be discussed.
(Received September 11, 2023)

## 1192-92-31440

Omar Saucedo*, Virginia Tech. Exploring the emerging risk of tick-borne diseases in Virginia. Preliminary report. Ticks are widespread throughout the Commonwealth of Virginia, and they produce a significant burden to the public health community with regards to prevention and detection. Although Lyme disease is the most prevalent tick-borne disease (TBD), other TBDs are starting to appear such as Bourbon Virus (BRBV), Heartland Virus (HRTV), and Powassan Virus (POWV). These emerging TBDs are a cause for concern has an infection can lead to several health issues, particularly in wild life. While surveillance programs generate insightful information on the prevalence of TBDs, we will develop a compartmental tick-borne disease model to determine the prevalence of (BRBV), (HRTV), and (POWV) in Virginia.
(Received September 11, 2023)

## 1192-92-31469

Maruf A Lawal*, University of Tennessee. Modelling COVID-19 Dynamics Incorporating Vaccine Hesitancy. Preliminary report.
COVID-19 continues to pose a significant challenge to human lives globally with about 800 million confirmed cases and about 7 million deaths reported to the World Health Organization (WHO) globally as at today. Many efforts have been made to develop vaccines to control the spread of this virus and to this effect, about 14 billion doses of vaccines have been administered already worldwide. However, with the arrival of the COVID-19 vaccine, there is hesitancy and mixed reactions towards getting the vaccine. In this work, we propose and analyze a model of COVID-19 incorporating vaccine hesitancy using a system of ordinary differential equations. We will present an Ro analysis of the model and show the effects of key time-varying parameters using numerical simulations. We also compare our numerical results with data to better understand the factors contributing to the differences in hesitancy behaviors for selected countries of interest, including the United States, France, and South Africa. (Received September 11, 2023)

Alina Dubovskaya, University of Limerick, Kristin Marie-Dettmers Kurianski, California State University Fullerton, Anna Nelson, Duke University, Mason A Porter, UCLA, Filippo Riscica, University of Hamburg, Yang Yang, Ohio State University, Lihong Zhao*, Virginia Tech. Coupling Disease Dynamics with Behavior and Opinion Dynamics. Preliminary report
Social interactions are key factors in the transmission of infectious diseases. The behavior and opinion of an individual are rarely binary. They could affect the behavior and/or opinions of others and thus influence the disease dynamics. Additionally, disease dynamics can have a major effect on people's behavior and opinions. The interplay of disease and behavior and opinions dynamics can lead to epidemic outcomes that are different from the ones predicted based solely on disease dynamics. In this talk, we present how we use a multilayer network model that couples a compartmental model of disease dynamics and a kinetic model for behavioral/opinion dynamics to explore the feedback between infection dynamics and the spread of behavior and opinions through social interactions.
(Received September 11, 2023)

## 1192-92-31573

Jemal S Mohammed-Awel*, Department of Mathematics, Morgan State University. Can insecticide resistance increase malaria transmission? A genetics-epidemiology mathematical modeling approach
In this study, we address one of the key questions in malaria ecology, namely whether insecticide resistance increases malaria transmission. We developed a genetics-epidemiology modeling framework that incorporates a detailed genotype structure of the gene that confers insecticide resistance in mosquitoes, malaria epidemiology in mosquitoes and humans (stratified based on whether or not they are protected by Long-lasting insecticide-treated nets (LLINs) indoors), genotype-specific mosquito repellance property of LLINs and mosquito biting behavior (indoor and outdoor bites). Conditions for the existence and local asymptotic stability of the various disease-free equilibria (by genotype) of the resulting genetic-epidemiology model are derived. This study identifies four parameters of the model that play a crucial role in quantifying the impact of insecticide resistance on malaria transmission. We showed that, depending on the values of these four identified parameters, insecticide resistance can increase, decrease, or have no effect on malaria transmission. Our simulations show that malaria eradication can indeed be achieved using the currently available chemical insecticides, even in the wake of the prevailing widespread insecticide resistance in malaria-endemic areas, if the insecticide-based interventions implemented can result in the attainment of the optimal values of the four identified parameters in malaria-endemic areas.
(Received September 11, 2023)

1192-92-31582
Marco V Martinez, Associate Professor, Department of Math \& Actuarial Science, North Central College, Maisha Marzan*, Undergraduate Student, Department of Applied Mathematics, North Central College, Gregory Ruthig, Professor, Department of Biology, North Central College. Multihost Pathogen Creates Ecological Links Between Hosts and Influence Host Population Dynamics.
Pathogens infecting multiple species create ecological links between species, even if these species don't directly interact. These different hosts with varying susceptibility to the pathogen can have indirect negative effects on another host by amplifying the number of infective propagules. This interaction is like "apparent competition" and one host species can end up decreasing the abundance of another host species via this shared enemy. Furthermore, if a multihost pathogen can infect any of its host saprophytically it can persist in the system even after host death and further increase infection pressure on other susceptible hosts in the system which can possibly drive a host species to extinction. We mathematically model such an ecological community with one common pathogen and multiple hosts, including hosts only being infected after death (saprophytic) and hosts tolerant to the infection. Our objective is to determine if the presence of hosts with varying susceptibility including saprophytic host can increase infection burden on specific susceptible hosts. We have completed a conclusive deterministic mathematical model for the population of susceptible, infected, and dead hosts, along with the pathogen, using a $4 \times 4$ linear system of ordinary differential equations. Using this model, we can conclude that increasing number of host species have indirect negative effects on another host population size by amplifying the number of infective propagules. We also calculate the community Basic Reproductive Number, $R_{0}$, utilizing the Next Generation Matrix to conclude which host species can have the largest effect in propagating the infection and which can aid in policy development and conservation.
(Received October 11, 2023)

## 1192-92-31625

Junling Ma*, University of Victoria, Mingran Zhang, University of Victoria. Regulation of CD4+ T Cell Proliferation During an Immune Response Leads to Original Antigenic Sin.
Original antigenic sin (also called antigenic imprinting) is a phenomenon that an earlier infection leads to an interference of immune responses to a secondary infection by a mutant viral strain, such that the immune response gears towards the strain of the first viral infection instead of the challenging mutant strain. In this talk, we propose a new mechanism that leads to this phenomenon. The autocrine reaction of chemokine IL-2 and CD4 + T cells leads to positive feed back loop of CD4+ T cell proliferation. This proliferation is regulated by the induced regulatory T cells (Tregs). The signals controlling the proliferation and regulation processes are not antigen specific, so a faster proliferation of memory CD4 +T cells may leads to an earlier regulation, which may interfere with the proliferation of the activated naive CD4+ T cells, leading to original antigenic sin. We develop a mathematical model to that incorporate these biological components, and calibrate the model to real data. Our results demonstrate the original antigenic sin phenomenon.
(Received September 11, 2023)

## 1192-92-31661

Daniela A. Florez Pineda*, Tulane University, Caroline Franco, University of Aberdeen, Carrie A. Manore, Los Alamos National Laboratory, Kaitlyn Martinez, Los Alamos National Laboratory, Ethan Romero-Severson, Los Alamos National Laboratory. Assessing the Risk of Dengue Outbreaks Across Continental Biomes in Brazil. Preliminary report.

Dengue is a mosquito-borne disease that causes an estimate of 390 million new infections every year and has been historically limited to tropical regions. Current evidence suggests that already occurring climate change is altering the distribution of mosquito-borne diseases. These changes will strain the global public health system by introducing old diseases into naive populations. Consequently, it is relevant to characterize the behavior of dengue transmission in the context of climate change. The central challenge of understanding the relation between dengue incidence and temperature variation relies on the nonlinear trend it exhibits, along with the complex interactions among environmental variables and the biology in the spread of this disease. To better capture the dynamics of temperature-dependent dengue risk, we propose a SEIR-type model to analyze its behavior across tropical eco-zones. This model integrates statistical methodologies to infer the time dependence on the mosquito biting rate, carrying capacity, and growth rates. In addition, temperature dependent mosquito-life traits are included as distributional forms into our mechanistic approach. Because of their well-differentiated ecological diversity, we have selected Brazil's continental biomes to validate our model output and illustrate better the effects of spatial and temporal temperature heterogeneity on dengue dynamics.
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1192-92-31666
Erin N Bodine*, Rhodes College. Modeling Growth \& Reproduction in Bromeliads: An Ongoing Interdisciplinary Collaboration with Multiple Undergraduate Research Projects.
The plant family Bromeliaceae contains over 3000 species of rosette-structured flowering plants (commonly known as bromeliads) and includes the pineapple plant and Spanish moss. The long lifespan of many Bromeliaceae species (up to 100 years in some species) can make it difficult to in situ study the growth and reproduction of individual rosettes over their lifetime. However, this provides fertile ground for developing mathematical and computational models that can simulate and predict growth, reproduction, and population dynamics over many decades. These models have the additional benefit of allowing for simulations that consider the impact of changing environmental conditions, such as climate change or the introduction of invasive species. In this talk, we will tour a selection of mathematical models of bromeliad growth \& reproduction that have been developed and analyzed in collaboration with research botanists and undergraduate students from a variety of institutions. From simple single equation continuous functions and discrete difference equations to more intricate models of systems of differential equations and agents-based models, each model provides a different lens from which to view and understand bromeliad growth and reproduction. Additionally, this talk will reflect on the elements of this collaboration that have enabled it to persist and evolve over time, and to provide research opportunities for over 20 undergraduate students.
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1192-92-31733
Adan Baca*, University of Arizona, Diego Raul Gonzalez, University of La Verne, Alonso Ogueda Oliva, George Mason University, Padmanabhan Seshaiyer, George Mason University. Mathematical Modeling, Analysis and Simulation of Patient Journey in Drug Addiction.
The COVID-19 pandemic significantly challenged our society, consequently resulting in increased levels of substance abuse Addiction is often stigmatized as a moral failing and as a burden on society that presents dangers and issues to everyone. However, conceptualizing addiction as a disease that spreads can help to create a better understanding of the complexity of the disease, and observe it from a scientific point of view. Our research aims to develop a mathematical model towards viewing and treating addiction as an infectious disease, using a modified Susceptible-Infected-Recovered (SIR) model. Our Susceptible-Exposed-Infected-Hospialized-Recovered (SEIHR) model takes into account unique factors that make addiction a unique disease, demonstrating how it perpetuates in communities and populations. Our mathematical model studies the impact of different types of treatment interventions on reducing addiction rates, and demonstrates the efficacy of recovery and treatment programs in lowering these rates. The predictive capability of our SEIHR model holds promise for informing decisions in public health planning and resource allocation.
(Received September 12, 2023)

1192-92-31757
Caleb Erickson*, National Science Foundation. An Androgen-Dependent Mathematical Model of the Menstrual Cycle. Preliminary report.
The condition PCOS (Polycystic Ovarian Syndrome) is an affliction that affects approximately $10 \%$ of those who menstruate The defining characteristics of PCOS are hyperandrogenism, irregular menstruation, and polycystic morphology in the ovaries caused by hormonal imbalances during menstruation. Since there is a focus on hyperandrogenism as a symptom, the research modeled the hormonal imbalances with a focus upon testosterone, the prevalent androgen during menstruation, and the introduction of anti-androgens. The modeling of PCOS hormones in an attempt to find an effective treatment is a relatively new endeavor, thus modeling these hormonal imbalances to find ways to lessen symptoms is imperative. Previously, research towards PCOS treatment has mainly focused upon insulin treatments, which have been shown to improve symptoms, and attempting to model the effects of an anti-androgen treatment has been less studied. In order to model these hormonal levels, we modified the differential equations, that Erica Graham put forth in her paper, to focus on pre-menstrual testosterone dependent follicle growth, in an attempt to better model testosterone levels during the menstrual cycle. A bifurcation analysis was unable to be performed on our model due to system and time limitations, which resulted in the model being unstable. Once a proper bifurcation analysis has been done, and if it produces a stable result one can model hyperinsulinemia or one can model PCOS.
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1192-92-31790
Azadeh Aghaeeyan, Brock University, Mark A Lewis*, University of Victoria, Pouria Ramazi, Brock University. Revealing the unseen: Likely half of the Americans relied on others' experience when deciding on taking the COVID-19 vaccine. Preliminary report.
Efficient coverage for newly developed vaccines requires knowing which groups of individuals will accept the vaccine immediately and which will take longer to accept or never accept. Of those who may eventually accept the vaccine, there are two main types: success-based learners, basing their decisions on others' satisfaction, and myopic rationalists, attending to
their own immediate perceived benefit. We used COVID-19 vaccination data to fit a mechanistic model capturing the distinct effects of the two types on the vaccination progress. We estimated that 47 per cent of Americans behaved as myopic rationalists with a high variation across the jurisdictions, from 31 per cent in Mississippi to 76 per cent in Vermont. The proportion was correlated with the vaccination coverage, proportion of votes in favor of Democrats in 2020 presidential election, and education score.
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## 1192-92-31811

David Christian Elzinga, University of Wisconsin-La Crosse, W. Christopher Strickland*, University of Tennessee, Knoxville. Generalized Stressors on Hive and Forager Bee Colonies.
Hive-forming bees play an integral role in promoting agricultural sustainability and ecosystem preservation. The recent worldwide decline of several species of bees, and in particular, the honeybee in the United States, highlights the value in understanding possible causes. Over the past decade, numerous mathematical models and empirical experiments have worked to understand the causes of colony stress, with a particular focus on colony collapse disorder. We integrate and enhance major mathematical models of the past decade to create a single, analytically tractable model using a traditional disease modeling framework that incorporates both lethal and sublethal stressors. On top of this synthesis, a major innovation of our model is the generalization of stressor attributes including their transmissibility, impairment level, lethality, duration, and temporaloccurrence. Our model is validated against numerous emergent, biological characteristics and demonstrates that precocious foraging and labor destabilization can produce colony collapse disorder. The thresholds for these phenomena to occur depend on the characteristics and timing of the stressor, thus motivating further empirical and theoretical studies into stressor characteristics.
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## 1192-92-31831

Logan Smith Dudney*, Eckerd College. Optimal Treatment of Stony Coral Tissue Loss Disease in Coral Reefs in the Florida Keys.
Stony coral tissue loss disease (SCTLD) is a new disease first discovered off the coast of Miami in 2013 and has rapidly spread south to the Florida Keys and subsequently to the Caribbean Sea. This disease leads to an extremely high mortality rate among some stony coral species, and its primary cause is still unknown. Although there currently isn't a cure for SCTLD, Mote Marine Laboratory and Aquarium has developed a topical treatment that limits the progression and transmission of the disease. In this study, we constructed a node-based transmission model to optimize the amount of treatment used to mitigate the number of cases of SCTLD. Our model is built on data supplied from previous Mote research. A node-based transmission model is like an agent-based model, which keeps track of an individual specimen's state, allowing for heterogeneity and tracking an individual's disease progression. The model connects all coral that are within one meter of each other, and every node had its disease state as "susceptible" or healthy, "infected", or "treated". Neighboring susceptible coral to an infected coral had a chance of being infected each time step. Infected coral was treated by choosing the infected coral with the most susceptible neighbors, and then treating the coral with the highest number of connections. The research and data produced by our study will help inform and potentially optimize treatment of coral specimens in the Florida Keys and help mitigate the spread of SCTLD until a cure prevention method can be developed.
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1192-92-31842
J. T. Cronin, Louisiana State University, Jerome Goddard II*, Auburn University Montgomery, Ratnasingham Shivaji, University of North Carolina Greensboro. Exploring density-dependent dispersal and habitat fragmentation via reactiondiffusion equations.
Dispersal is broadly defined as movement from one habitat patch to another and typically is considered to encompass three stages: 1) emigration, 2) inter-patch movement, \& 3) immigration. Dispersal can have both beneficial and detrimental effects on the persistence of spatially structured systems. Recent empirical results indicate that certain organisms' emigration from a patch is dependent on density of their own species or even an interacting species-known as density dependent emigration. To date, little is known about the patch-level consequences of such dispersal strategies. In this talk, we will give a brief overview of density dependent emigration and its modeling history, discuss a framework built upon reaction diffusion equations designed to model patch-level effects of density dependent emigration, and share some recent advances.
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1192-92-31846
J. T. Cronin, Louisiana State University, Jerome Goddard II*, Auburn University Montgomery, Ratnasingham Shivaji, University of North Carolina Greensboro. Ecological release and patch geometry can cause nonlinear density-area relationships.
A primary driver of species extinctions and declining biodiversity is loss and fragmentation of habitats owing to human activities. Many studies spanning a wide diversity of taxa have described the relationship between population density and habitat patch area, i.e., the density-area relationship (DAR), as positive, neutral, negative or some combination of the three. However, the mechanisms responsible for these relationships remain elusive. In this talk, we will discuss implementation of a reaction-diffusion framework with absorbing boundary conditions to model a habitat specialist dwelling in islands of habitat surrounded by a hostile matrix. We consider patches with both a convex and non-convex geometry. Our results show overall DAR structure can be either 1) positive, 2) positive for small areas and neutral for large, or 3) hump-shaped, i.e., positive for area below a threshold and negative for area above. We will also discuss comparison of our theoretical results with two empirical studies. Close qualitative agreement between theoretical and observed DAR indicates that our model gives a reasonable explanation of the mechanisms underpinning DAR found in those studies.
(Received September 12, 2023)

University Montgomery, Safa Motallebi*, Auburn University Montgomery, Ratnasingham Shivaji, University of North Carolina Greensboro. Exploring effects of patch size and matrix quality on the evolution of dispersal.
Degradation of habitats and their fragmentation resulting from human activities often lead to a landscape characterized by spatial heterogeneity, where isolated patches frequently find themselves encircled by inhospitable surroundings. The composition and quality of the surrounding matrix play crucial roles as landscape features with substantial impacts on a range of ecological phenomena, including animal movement behavior. This talk explores application of reaction-diffusion models to investigate the consequences of patch size and matrix hostility on the evolution of dispersal when landscape features such as patch size and matrix quality are considered.
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1192-92-31877
J. T. Cronin, Louisiana State University, Jacob Garrett*, Auburn University Montgomery, Jerome Goddard II, Auburn University Montgomery, Ratnasingham Shivaji, University of North Carolina Greensboro. Modeling effects of harvestingmediated emigration on population persistence.
Trait-mediated behavioral responses (an indirect effect) to other species can affect population dynamics significantly. One example of such a response is modification of emigration probability, which has the potential to change single species persistence, as well as interactions and community structure. Habitat loss and fragmentation due to anthropogenic activities creates landscape-level spatial heterogeneity where remnant patches are often surrounded by a hostile matrix. Matrix composition or hostility is an important component of a landscape and can have profound effects on species movement and boundary behavior, persistence of a single species, and coexistence of interacting species. In this talk, we will introduce a modeling framework to explore effects of harvesting-mediated emigration on population persistence and share some recent results.
(Received September 12, 2023)

## 1192-92-31912

Najat Ziyadi*, Morgan State University. Local and global sensitivity analysis in a mathematical model of human papillomavirus (HPV) and cervical cancer. Preliminary report.
This talk addresses the problem of local and global sensitivity analysis in a mathematical model of human papillomavirus (HPV) and cervical cancer with application. The mathematical model of human papillomavirus (HPV) and cervical cancer is presented as a system of ordinary differential equations. The basic reproduction number is computed using the next generation method. Local and global sensitivity analysis will be used to illustrate the impact of model parameters on the model. (Received September 12, 2023)

1192-92-31923
Samantha Erwin*, Pacific Northwest National Laboratory. Distilling Mechanistic Models From Multi-Omics Data. High-dimensional multi-omics data sets are increasingly accessible and now routinely being generated as part of medical and biological experiments. However, the ability to infer mechanisms of these data remains low due to the abundance of confounding data. The gap between data generation and interpretation highlights the need for strategies to harmonize and distill complex multi-omics data sets into concise, mechanistic descriptions. To this end, a four-step analysis approach for multiomics data is herein demonstrated, comprising: filling missing data and harmonizing data sources, inducing sparsity, developing mechanistic models, and interpretation. This strategy is employed to generate a parsimonious mechanistic model from high-dimensional transcriptomics and metabolomics data collected from a murine model of Clostridioides difficile infection. This approach highlighted the role of the Stickland reactor in the production of toxins during infection, in agreement with recent literature. The methodology present here is demonstrated to be feasible for interpreting multi-omics data sets and it, to the authors knowledge, one of the first reports of a successful implementation of such a strategy. (Received September 12, 2023)

## 1192-92-31952

Tyler Skorczewski*, University of Wisconsin Stout, Keith Wojciechowski, University of Wisconsin Stout. Crop per drop: using ODE models to find relationships between irrigation practices and kidney bean yield.
Chippewa Valley Bean, located in Wisconsin, USA, is the world's largest processor of dark red kidney beans and works with farmers over several states. Current trends in farming are pressuring producers to generate higher yields with fewer resources, particularly water resources. This project describes our work creating ODE models that describe the relationship between irrigation inputs, soil parameters, and kidney bean yields that CVB can use to advise farmers for productive yet sustainable practices. (Received September 12, 2023)

## 1192-92-31964

Dana Droz*, North Carolina State University. Optimal control of BK virus infection in kidney transplant recipients. Preliminary report.
Kidney transplant recipients are put on an immunosuppressant drug regimen to suppress their immune system which prevents allograft rejection. These drugs make the patient susceptible to infections. The opportunistic BK virus (BKV) infection has no effective antiviral treatment. The standard clinical practice to treat BKV is to reduce immunosuppression which in turn increases the risk of rejection. We use a mathematical model to predict the amount of BKV and develop an optical control method to provide individualized treatment to patients. We present the implementation of a receding horizon control approach on patient data from both the Duke Transplant center and CTOT.
(Received September 12, 2023)

University. The many proofs of the reduction phenomenon. Preliminary report.
In this talk we consider several proofs, old and new, of the reduction phenomenon: Many mathematical models of biological systems that incorporate dispersal and reproduction exhibit the phenomenon that increased dispersal reduces overall growth. Karlin first proved this for basic, parameterized linear discrete-time model in 1975, and many other proofs have been discovered since then. We review some of these proofs, indicate how they are connected and also provide streamlined, novel proofs of this celebrated biological principle.
(Received September 12, 2023)

## 1192-92-31981

Yishay Pinto*, Department of Medicine, Stanford. Identification of diverse human-bacteriophage interactions in human health.
The human gut microbiome is a diverse ecosystem that encompasses multiple kingdoms of life and plays a vital role in human health. Unfortunately, due to technical limitations, most studies have focused on gut prokaryotes, overlooking gut viruses. The most common method to profile viruses is to assemble shotgun metagenomic reads and identify viral genomes de novo. While valuable, this resource-intensive and reference-independent method has limited sensitivity. Leveraging recently published catalogs of gut viral genomes, we developed a workflow that profiles human gut metagenomes in a viral-aware manner, directly from short reads. Based on simulations, the workflow is fast and accurate with respect to both prokaryotes and viruses, minimizing false positive identifications using a genome coverage-based strategy. When applied to metagenomes from healthy adults, the workflow identified 200 viral species per sample, 5 x more than the standard assembly-based methods. Notably, we observed a 2:1 ratio between gut viruses and bacteria, with higher inter-individual variability of the gut virome. Finally, we used our method to tandemly profile gut viruses and bacteria in hundreds of samples from humans with varied age, immune status, and immunotherapy responses to identify thousands of cross-domain interactions with likely relevance to human health.
(Received September 12, 2023)

## 1192-92-32028

Smita Iyer, UC Davis, Audrey Oliver*, San Diego State University, Naveen K. Vaidya, San Diego State University. Modeling the Spatiotemporal Distribution of HIV Infection in the Brain.
In standard clinical practice, only plasma viral load and CD4 counts are measured to keep track of disease status and progression in Human Immunodeficiency Virus (HIV)-infected individuals. However, viruses reside in the brain, causing neurocognitive disorders and an obstacle to a cure, despite virus control in plasma with antiretroviral therapy. Therefore, tracking the virus distribution across different brain compartments is essential for disease management in HIV-infected individuals. In this study, we first performed a correlation network analysis of RNA in the brain with plasma and CSF (Cerebrospinal fluid) to identify whether plasma or CSF viral loads can infer the viral burden in the brain. Secondly, we performed a correlation network analysis of viral RNA among different brain regions to identify the brain's essential regions related to viral burden within the brain. Thirdly, we built a mathematical model that explains the spatiotemporal distribution of HIV in the brain using the essential brain regions obtained from our correlation analysis. Our model was validated using data collected from the brain of the simian immunodeficiency virus (SIV)-infected macaques. We analyzed the model and performed parameter sensitivity to get insights into the distribution and replication of HIV throughout the different brain regions, as well as evaluate the reproduction number to determine the persistence of the virus. (Received September 12, 2023)

1192-92-32072
Alexandria Volkening*, Purdue University. Topological data analysis of biological pattern formation.
Mathematical models of collective movement and pattern formation come in many different forms, including macroscopic PDE approaches and on- or off-lattice microscopic models. Especially in the case of spatial dynamics and pattern formation, it can be difficult to compare models and data, or even the output of different types of models. The presence of variability in stochastic simulations and in vivo data adds further difficulty, since we may need to view many patterns in order to quantify how different models are related. To help address this challenge, I will discuss how to adapt methods from topological data analysis to robustly characterize different models of the same biological system. To illustrate concrete challenges, I will focus on the specific example of zebrafish-skin pattern formation, and I will show how to quantitatively describe patterns that arise from on- and off-lattice cell-based models in this setting. Applying persistent homology to quantify patterns involves choices, and I will highlight the role of hyper-parameters in our process. (Received September 12, 2023)

## 1192-92-32075

Weitao Chen*, University of California, Riverside. Multiscale Modeling to Understand Robustness Mechanisms of Stem Cell Maintenance.
The regulation and interpretation of transcription factor levels is critical in spatiotemporal regulation of gene expression in development biology. However, concentration-dependent transcriptional regulation, and the spatial regulation of transcription factor levels are poorly studied in plants. WUSCHEL, a stem cell-promoting homeodomain transcription factor was found to activate and repress transcription at lower and higher levels respectively. The differential accumulation of WUSCHEL in adjacent cells is critical for spatial regulation on the level of CLAVATA3, a negative regulator of WUSCHEL transcription, to establish the overall gradient. However, the roles of extrinsic spatial cues in maintaining differential accumulation of WUSCHEL are not well understood. We have developed a 3D cell-based computational model which integrates sub-cellular partition with cellular concentration across the spatial domain to analyze the regulation of WUS. By using this model, we investigate the machinery of the maintenance of WUS gradient within the tissue. We also developed a stochastic model to study the binding and unbinding of WUS to cis-elements regulating CLV3 expression to understand the concentration dependent manner mechanistically. The robustness mechanism and the concentration-dependent machinery discovered by the modeling analysis can be general principles for stem cell homeostasis in different biological systems.
(Received September 12, 2023)

Vitaliy A Kurlin*, University of Liverpool (UK). The Crystal Isometry Principle.
All solid crystalline materials can be modelled by periodic sets of points at atomic centres. Since crystal structures are determined in a rigid form, the strongest equivalence between crystals in practice is rigid motion (a composition of translations and rotations). A slightly weaker isometry allows mirror reflections. Conventional crystal representations use a motif of atoms in a periodically repeated unit cell, which can be fixed in infinitely many ways and is discontinuous under atomic perturbations. This data ambiguity motivates invariants that are continuous under noise and allow an explicit reconstruction. Our Pointwise Distance Distribution (PDD) is based on distances to atomic neighbors and can be computed in a near-linear time in the input size. We have completed 200+ billion PDD comparisons of all 660+ thousand periodic crystals in the world's largest collection of real materials (Cambridge Structural Database) over two days on a modest desktop. Unexpectedly, we found five pairs of duplicates that were geometrically identical (with all equal parameters) but differed by one atomic type, which seems physically impossible without perturbing geometry. Several journals are investigating the integrity of the relevant papers. The more important conclusion is the Crystal Isometry Principle meaning that all real periodic crystals have unique geographic-style positions in a common continuous space. All relevant papers are at http://kurlin.org/research-papers.php\#Geometric-Data-Science
(Received September 12, 2023)

1192-92-32093
Renee Brady, Department of Integrated Mathematical Oncology, H. Lee Moffitt Cancer Center and Research Institute, Alexandria Victoria Johnson*, H. Lee Moffitt Cancer Center \& Research Institute. An In-Silico Study Investigating RaciallyDriven Response Differences in Metastatic Prostate Cancer. Preliminary report.
African American men have the highest incidence and mortality rates of prostate cancer (PCa) compared to any other racial group. The increased incidence as well as mortality are likely due to socioeconomic factors, environmental exposure, access to care, and biological variations. Deciphering the specific drivers of increased incidence and mortality is difficult due to a scarcity in available data from African American patients. In silico modeling can be used to generate pseudo patient data that can be used to compare response dynamics between groups. Here, we use propensity score matching to conduct an in silico study of hormone treatment in African American (AA) and European American (EA) PCa patients. The data included longitudinal prostate-specific antigen (PSA) data from 57 metastatic PCa patients ( $\mathrm{N}=47 \mathrm{EA}, \mathrm{N}=10 \mathrm{AA}$ ) receiving continuous androgen deprivation therapy (ADT). To compensate for the lack of data for AA patients, propensity score matching was used to select 15 EA patients that were most alike to the 10 AA patients. A simple PCa stem cell model was calibrated to the PSA data from these 25 patients. This was done to investigate any possible difference in PSA dynamics between AA and EA patients. Statistical analysis found that AA patients had a significantly higher stem cell self-renewal rate than the EA patients. We hypothesize that this is primarily responsible for earlier progression in AA patients. An in silico study was performed by sampling from the parameter sets from each racial group to create 100 in silico patients ( $\mathrm{N}=50 \mathrm{EA}, \mathrm{N}=50 \mathrm{AA}$ ). The model parameters were then used to investigate whether adaptive therapy, whereby treatment is cycled on and off using patientspecific treatment triggers, could extend time to progression when compared to continuous therapy. Simulations showed that adaptive therapy provided a greater benefit for AA patients, compared to EA patients. Our results show that stem cell selfrenewal is potentially driving racial differences in PSA dynamics between EA and AA patients. Adaptive therapy might be a step in addressing the higher stem-cell self-renewal rate in AA patients and narrow the health disparity gap between AA and EA PCa patients. This valuable information can be used in the future to design race-specific treatment modalities to maximally delay time to progression.
(Received September 12, 2023)

1192-92-32097
Marielle D. Friedman*, University of Central Florida: Institute for Simulation and Training, Armando Roldan, Moffitt Cancer Center: Department of Integrated Mathematical Oncology. Circadian Rhythm Dynamics for Personalized Dosing Times. Circadian rhythm (CR) is the natural oscillation of processes that occurs in humans on a 24 -hour cycle. We depend on our CR to maintain physiological homeostasis, and disruptions to this cycle are linked to several cancer hallmarks. The research surrounding the alignment of CR disruptions to cancer progression has shown biological instances in which CR dosing is effective in some areas of medicine. The applications of understanding the relationship between these mechanisms can lead to increased efficacy of chemotherapy and other cancer treatments. By understanding the effects of timing on treatment, dosing can be individually modified to reduce relative toxicity and mitigate the harsh side effects. Here we aim to explore the dynamics of CR in personalized cancer treatments through simulated cancer trajectories.
(Received September 12, 2023)

## 1192-92-32116

Susan Rogowski*, Florida State University. Parameter Estimation for Bacteria Models.
It is a common problem within biomathematics that one is given a noisy or sparse data set and must estimate realistic parameters for some mathematical model. In this presentation we will present two parameter estimation techniques on bacteria models and make some comparisons. The two parameter estimation techniques we are interested in is data assimilation, or more specifically nudging, and Markov Chain Monte Carlo (MCMC). We will start with a simple logistic growth model and estimate the growth rate from both sparse and noisy synthetic data sets. Then, we will move to a model of three interconnected chemostats, often referred to as a gradostat. In this model, we will consider the scenario where only one chemostat tank can be observed and work on recovering the dynamics of our other two tanks. We will conclude with some comparisons and insights on best practices for parameter estimation applied to bacteria models of differing complexities. (Received September 12, 2023)

1192-92-32119
Christine Heitsch*, Georgia Institute of Technology, Svetlana Poznanovic, Clemson University. Can geometric combinatorics improve RNA folding predictions?.
Branching is a critical characteristic of RNA folding, yet challenging to predict with thermodynamic optimization approaches. Using combinatorial methods (convex polytopes and their normal fans), we can improve RNA base pairing prediction accuracy on well-defined families while also illuminating why the general problem is so difficult.

## 1192-92-32131

Christine Heitsch*, Georgia Institute of Technology. Spaces of RNA branching configurations.
Understanding the folding of RNA sequences into three-dimensional structures is a fundamental scientific challenge. For example, the branching of an RNA secondary structure is an important molecular characteristic yet difficult to predict correctly, especially for sequences on the scale of viral genomes. However, combinatorial models, methods, and analyses can yield insight into RNA structure formation, and suggest new directions in viral capsid assembly.
(Received September 12, 2023)

1192-92-32158
Benjamin Levy*, Fitchburg State University. Analyzing Our Ability to Monitor Fishery Population Trends Under the Pressures of Climate Change.. Preliminary report.
Many stock assessments in the United States use abundance estimates derived from stratified random bottom trawl data. To accurately represent true abundance, catches of a species must contain a low enough noise level to allow for a discernible pattern and the proportion of the population that is sampled should be consistent over time. These assumptions could be violated given enough noise in the sampling process and/or climate change causing a population to increase its abundance in strata that are not sampled. Using the R package MixFishSim, we have developed data-driven spatial models for Yellowtail Flounder, Cod, and Haddock in the western Atlantic Ocean to allow examination of these assumptions through simulation. Movement rates combine species-specific static habitat preferences with temperature tolerances. Habitat preferences were derived from niche models relating bottom trawl catches to environmental covariates. A repeating yearly temperature pattern produces repeating spatial biomass distributions in a given week, while a temperature gradient that increases on average over time results in spatial preferences that evolve throughout a given simulation. We created simulated spatial time series datasets for each species for several temperature scenarios and population trends. Using stratified random sampling on model output we were able to compare abundance estimates derived from the design-based stratified mean to those using a spatio-temporal model-based approach that allows inclusion of environmental covariates (VAST). Our focus was on the ability of contemporary indexing methods to track population trends under shifting spatial preferences.
(Received September 12, 2023)

1192-92-32184
Shakhawat Bhuiyan, Jarvis Christian University, Widodo Samyono*, Jarvis Christian University. Jarvis Summer Undergraduate Research Experience in Computational and Mathematical Biology.
This work is supported by NSF three years funding (July 1, 2019 - June 30, 2022) with Award\# 1912196 under Targeted Infusion Projects (TIP) as a subprogram of Historically Black College and University Undergraduate Program (HBCU-UP) entitled "Targeted Infusion Project: Innovative Jarvis Undergraduate Mathematics Program (I-JUMP): Embedding Computational and Mathematical Biology into Life STEM." The main goal of the project is to expose underrepresented undergraduate students in career and graduate studies in Computational and Mathematical Biology. This talk discusses the challenges, opportunities, and strategies to recruit, expose, and retain underrepresented undergraduate students in career and graduate studies in Computational and Mathematical Biology during and after the commencement of the project. (Received September 12, 2023)

## 1192-92-32189

Veronica Ciocanel, Duke University, Scott McKinley, Tulane University, Anna Nelson*, Duke University. Modeling mechanisms of microtubule growth dynamics and polarity in neurons. Preliminary report.
In neurons, the polarity and stability of the microtubule cytoskeleton is required for long-range, sustained intracellular transport of cargo such as proteins and mRNA. In fruit fly neurons, the healthy microtubule cytoskeleton has a specific polarity, where depending on the region of the neuron, the microtubules are either all minus-end out or all plus-end out. However, these microtubules are dynamic and rearrange their orientation in the event of an injury. It is unknown how these mechanisms can maintain both dynamic rearrangement and sustained function. To better understand these mechanisms, we introduce a spatially-explicit mathematical model of microtubule growth using parameters informed by experimental data. We explore several mechanisms that control microtubule length using both a stochastic model and a continuous model, and validate such mechanisms with fluorescence experiments and other experimental data. The results from this study can then be used to better understand how microtubule polarity is maintained or rearranged in fruit fly dendrites and axons. (Received September 12, 2023)

1192-92-32203
Reidun Twarock*, University of York. The regulatory roles of phage genomes in assembly and genome release. Preliminary report.
In many viral families, including bacteriophages, viral genomes encode assembly instructions that direct (nucleo)capsid assembly along the most efficient pathways. In this talk, I will demonstrate how a combination of mathematical insights into virus architecture, stochastic simulations, and experiment enabled the discovery of these genome-encoded virus assembly instructions. The mechanism relies on multiple sequence/structure motifs, called packaging signals, that are dispersed across the viral genome and act collectively in regulating its efficient encapsidation. Based on recent work for bacteriophage MS2, I will demonstrate that the regulatory roles of these signals extend beyond the assembly step, regulating the timely release of the viral genome at the required time during the viral life cycle. I will also show that variants of this mechanism are widespread in nature. For example, this mechanism evolved spontaneously during directed evolution of a bacterial enzyme that was engineered to package its own mRNA, explaining the vastly increased mRNA packaging efficiency during later rounds of directed evolution. These results shed new light on the early stages of viral evolution and inform the design of protein nanocontainers for a host of applications.
(Received September 12, 2023)

Chiara Mattamira*, University of Tennessee Knoxville, Olivia Prosper, University of Tennessee Knoxville. Practical Identifiability of Epidemiological Models. Preliminary report.
When estimating model parameters it is crucial to investigate whether, given a model and accompanying data, parameters can be uniquely recovered; that is, whether model parameters are identifiable. The emphasis of this talk will be on practical identifiability, its relationship with available data, and the methods used to assess it. I will first demonstrate the consequences of making conclusions from models that are practically unidentifiable. I will then present a commonly used Monte Carlo algorithm and I will discuss the underlying assumptions and limitations of this type of identifiability assessment tool. Time permitting, I will touch on likelihood-free methods such as approximate Bayesian computation and how they compare with the more commonly used likelihood-dependent approaches.
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## 1192-92-32274

Doug Boyer*, Duke University, Ingrid Daubechies, Duke University, Shira Faigenbaum-Golovin, Duke University, Tingran Gao, Radix Trading LLC, Robert J. Ravier, Sarcos Technology and Robotics Corporation, Shan Shan, University of Southern Denmark. Automated registrations based on sample variation are needed for comparative morphology.
In natural sciences, genetic, developmental, evolutionary and ecological phenomena are reflected in organismal anatomy. The field of comparative morphology investigates the covariation between these phenomena and anatomical form in different individuals or species. Since all life is related through a branching evolutionary tree of ancestors and descendants, an ultimate goal of comparative morphology, within major phyla, is to reconstruct the evolutionary transformations that have occurred since all living and extinct species diverged from their common ancestor. Understanding the time and place, as well as the genetic, developmental, and ecological drivers of those transformations is also a goal. Exploration and documentation of morphological variation on this very large scale is hindered by subjective manipulations that underly almost all quantitative methods. These subjective manipulations impose an initial correspondence between different instances of the same bone, organ or organism to be compared. While adequate for small scale questions, these approaches fatally limit analyses hoping to discover major transformations in body form. Thus, qualitative descriptions of major transformations supplant quantitative demonstrations of them. Nevertheless, a hypothesis of correspondence (or registration) is essential for comparing biological shapes because different patterns of correspondence lead to drastically different conclusions about the details and major features of the evolutionary process. To address these gaps, our interdisciplinary team has been attempting to develop algorithmic pipelines that automatically generate high quality registrations on geometrically diverse samples of the same bone, organ or organism by drawing on properties of variation in the sample as a whole. In this talk, we discuss some of the principles, results, challenges and future goals of this work. (Received September 12, 2023)

## 1192-92-32278

Michelle Girvan, University of Maryland, College Park, Abba Gumel, University of Maryland, Alice Oveson*, University of Maryland, College Park. Towards a novel behavior-epidemiology modeling framework for pandemics of respiratory pathogens. The recent COVID-19 pandemic, caused by SARS-CoV-2, has highlighted the importance of explicitly incorporating human behavior into mathematical models for the transmission dynamics and control of emerging, resurging and re-emerging respiratory pathogens. Models that did not explicitly account for the impacts of human behavior changes during the epidemic often had limited predictive power due to distinctly human phenomena such as masking fatigue. In this lecture, I will present a new mathematical framework for incorporating human behavior changes and social influences (as a driver for positive or negative human behavior changes with respect to adherence to control and mitigation measures or overall attitude towards the pandemic) on the transmission dynamics of a respiratory pathogen of major public health significance. The models to be developed will be parameterized using available epidemiological data, as well as contact and influence data from a certain demographic in the United States. I will discuss the merits and demerits of various modeling types within this framework, and how they can be combined to potentially enhance the predictive capacity of the modeling framework.
(Received September 12, 2023)

1192-92-32297
Robert Stephen Cantrell*, University of Miami. Spatial Ecology via Reaction-Diffusion Equations: A 2003 to 2023 Space and Time Odyssey.
2023 marked the 20th anniversary of the publication of the book "Spatial Ecology via Reaction-Diffusion Equations". In conjunction with the theme of this session, in this talk, I will survey what I think are some interesting current topics in papers now citing the book.
(Received September 12, 2023)

## 1192-92-32301

Tom Chou, UCLA, Sara Habibipour, UCLA, Xiangting Li*, UCLA, Otto O. Yang, UCLA. The role of APOBEC3-induced mutations in the differential evolution of monkeypox virus.
Recent studies show that newly sampled monkeypox virus (MPXV) genomes exhibit mutations consistent with Apolipoprotein B mRNA Editing Catalytic Polypeptide-like3 (APOBEC3)-mediated editing, compared to MPXV genomes collected earlier. It is unclear whether these single nucleotide polymorphisms (SNPs) result from APOBEC3-induced editing or are a consequence of genetic drift within one or more MPXV animal reservoirs. We develop a simple method based on a generalization of the General-Time-Reversible (GTR) model to show that the observed SNPs are likely the result of APOBEC3-induced editing. The statistical features allow us to extract lineage information and estimate evolutionary events.
(Received September 12, 2023)

1192-92-32310
Amanda Laubmeier*, Texas Tech University. Modelling ecosystem services by arthropods in agricultural landscapes. Preliminary report.
In agricultural ecosystems, insect communities provide important services such as biological pest control and crop pollination.

The value of these services is shaped by the abundance and diversity of resident insect communities, which form complex networks of complementary and adversarial interactions. Importantly, the prevalence of different insects is shaped by migration from surrounding landscapes, and the ideal management of these landscapes is a persistent question in agricultural applications. In this work, we consider the interface of conservation reserve land with a cotton crop. We begin with an ordinary differential equation model across these two landscape patches. We are primarily interested in pollinator abundance and interactions with crop, but we expect feedback with crop development as determined by pest colonization and pressure from natural predator communities. We also expect that analogous interactions with weedy, natural plants in the reserve will affect migration into the field. Collaborators have obtained pilot data for the insect community in our study system, which provides information about potential migration rates and reasonable insect abundance. For other model parameters, we investigate the effect of different values on the expected ecosystem services within the agricultural field. We use these preliminary findings to explore how management decisions about surrounding landscape may affect crop productivity. (Received September 12, 2023)

## 1192-92-32328

Jared Barber, Indiana University-Purdue University Indianapolis, Nigar Karimli*, Indiana University-Purdue University Indianapolis, Luoding Zhu, Indiana University- Purdue University. Mathematical Modeling of an Osteocyte Interacting with Its Surrounding Flow in a Lacuno-Canalicular Network. Preliminary report.
Osteocytes are bone cells that play a key role in bone remodeling and reside in lacunae, interconnected cavities surrounded by hard bone matrix. These cells receive mechanical signals, transform them into biochemical signals, and transmit those signals to regulate bone tissue formation and resorption. The cells reach out processes, or arms, to connect with each other through narrow channels known as canaliculi. The lacuna and canaliculi containing these cells form a complex lacuno-canalicular network. Because investigations within this rigid bone environment can be difficult, computational models have become an appealing alternative. In this study we focus on a single osteocyte to advance our understanding of osteocyte behavior and its interaction with the surrounding flow. We present a 3D mathematical model that considers an osteocyte as an interconnected viscoelastic element network surrounded by viscous fluid that, in turn, is surrounded by hard bone material. Beyond viscoelastic forces, our model includes additional cell properties: bending rigidity, membrane area conservation (both local and global), and total volume conservation. To simulate flow, we use the lattice-Boltzmann method (D3Q19 model) coupled with the immersed boundary method to investigate the flow dynamics in the lacuno-canalicular network. This coupling allows investigation of the fluid-structure interaction between the osteocyte and the surrounding flow. In addition to our modeling approach, we present results related to stress computations on the osteocyte's membrane. We utilize radial basis functionbased interpolation to estimate stress distributions on the cell membrane in order to gain insights into the mechanosensitive behavior of osteocytes. The main goal of this work (supported by NSF-DMS 1951531) is to develop feasible tools that can be used to better understand how the geometry of the lacuno-canalicular network can affect osteocyte-fluid interactions and corresponding cellular responses. The main goal of this work is to develop feasible tools that can be used to better understand stress and strain distribution, concentration, and amplification on the osteocytes in the lacuno-canalicular network. (Received September 12, 2023)

## 1192-92-32339

Vittorio Addona, Department of MSCS, Macalester College, Darko Bosnakovski, Department of Pediatrics, University of Minnesota, Lauren Mills, Department of Pediatrics, University of Minnesota, Leslie Myint, Department of MSCS, Macalester College, Wenxuan Zhu*, Department of MSCS, Macalester College. Exploring Surfaceome of CIC-DUX4 Sarcoma. Preliminary report.
CIC-DUX4 sarcoma (CDS) is a rare and aggressive subtype of undifferentiated round-cell sarcomas primarily affecting adolescents and young adults, with a median survival of less than 2 years. CDS is characterized by the CIC-DUX4 oncogene, resulting from a genetic translocation between CIC and DUX4 genes. Our understanding of CDS, particularly regarding its surfaceome (surface membrane proteins), remains limited, yet knowledge of the cancer's surfaceome is essential for comprehending how tumors interact with their surroundings and lays the foundation for potential targeted treatments. In our prior research, we established that the malignancy of CDS relies on P300/CBP. We identified iP300w as a potent inhibitor of P300/CBP, effectively suppressing CIC-DUX4's oncogenic activity. In this study, we delved into the CDS surfaceome by analyzing RNA-Seq data from CDS cell lines treated with iP300w. Through a range of informative visual aids, we pinpointed a distinct subset of 19 genes encoding surface proteins that are intricately linked to CIC-DUX4/P300/CBP activity. These proteins signify an intersection of critical biological features and serve as potential targets of CIC-DUX4 sarcoma. (Received September 12, 2023)

## 1192-92-32404

Christina Edholm*, Scripps College. Control Theory Implemented for Management of Biological Systems
Often when building a model and exploring a biological question, we consider how to implement management options and what is the best way to optimize. There are many different methods for incorporating management into a model, we will focus on aspects of control theory. In this talk we will discuss different biological questions which are analyzed using different control schemes and then deduce the management suggestions.
(Received September 12, 2023)

1192-92-32407
Nick Battista, The College of New Jersey, Yanyan He*, University of North Texas, Lindsay Waldrop, Chapman University. Mixed Uncertainty Analysis on Pumping by Peristaltic Hearts using Dempster-Shafer Theory. Preliminary report In this talk, we introduce the numerical strategy for mixed uncertainty propagation based on probability and Dempster-Shafer theories, and apply it to the computational model of peristalsis heart pumping system. Specifically, the stochastic uncertainty in the system is represented with random variables while epistemic uncertainty is represented using non-probabilistic uncertain variables with belief functions. The mixed uncertainty is propagated through the system, resulting in the uncertainty in the chosen quantities of interests (QoI, such as flow volume, cost of transport and work). With the introduced numerical method, the uncertainty in the statistics of QoIs will be represented using belief functions, and compared between sinewave and Gaussian-peak peristalsis models. To reduce the computational cost, physics constrained generalized polynomial chaos method is adopted to construct cheaper surrogates as approximations for the full simulation.

Bren Case*, University of Georgia. Identifiability and geometry of epidemiological models under preferential sampling and noisy diagnostic tests. Preliminary report.
The onset of the COVID-19 pandemic led to an explosion of research fitting epidemiological models to data, and concurrently a growing recognition of the nuanced limitations behind different sources of data. Daily reported cases are among the most important data sources for calibrating infectious disease models, and have been successful in informing critical public health response especially when combined with other data streams such as hospitalizations. However, there remains a lack of formal analyses into how different sources of bias associated with diagnostic testing impact our ability to derive reliable estimates. Here we quantify the effect of several forms of bias common in reported case counts, including preferential sampling of symptomatic individuals, delays in reporting, and sensitivity of different assays depending on viral load. We propose a generalization of the classic Susceptible-Infectious-Recovered model using partial differential equations, which allows us to study the effect of these complex observational processes through a minimal set of mechanisms. To visualize and provide insights into how these biases impact the space of viable estimates, our methods incorporate tools from identifiability analysis and embedding theory that are widely applicable for understanding how models from mathematical biology interact with noisy data.
(Received September 12, 2023)

## 1192-92-32442

Elena S Dimitrova*, Cal Poly, San Luis Obispo. An algebraic approach to reverse engineering and data selection for network identification
Due to time and cost concerns, it is optimal to gain insight into the connectivity of biological networks using as few experiments as possible. Data selection for unique network connectivity identification has been an open problem since the introduction of algebraic methods for reverse engineering for almost two decades. In this talk we determine what data sets uniquely identify the unsigned (activation and inhibition not distinguished) wiring diagram corresponding to a system that is discrete in time and space. Furthermore, we answer the question of uniqueness for signed (activation and inhibition distinguished) wiring diagrams for Boolean networks. Computationally, unsigned and signed wiring diagrams have been studied separately, and in this talk we show that there exists a polynomial ideal capable of encoding both unsigned and signed information. This provides a unified approach to studying reverse engineering that also gives significant computational benefits.
(Received September 12, 2023)

## 1192-92-32447

Elyssa Sliheet*, Southern Methodist University. Predicting Biophysical Properties of Proteins with Electrostatics and Machine Learning. Preliminary report.
We report our recent work for the prediction of protein binding affinity for the purpose of drug discovery/development. We utilize convolutional neural networks (CNNs) which process uniform features from both topological data analysis and electrostatics from charged protein-ligand complexes. Specifically, we study the Poisson-Boltzmann model including various parallelization schemes using the Message Passing Interface (MPI). Our methods overcome the difficulties of involving electrostatics, which are expensive to compute due to its long range and pairwise natures. The simulation results show accurate prediction of binding affinity and meanwhile demonstrate the significance of involving electrostatics in the machine learning framework.
(Received September 12, 2023)

1192-92-32450
Chadi M Saad-Roy*, University of California, Berkeley. Immune uncertainties, individual behavior, and the dynamics of COVID-19.
In this talk, I will highlight the potential effects of remaining immune uncertainties on the future immuno-epidemiological dynamics of SARS-CoV-2. I will also examine the dynamics of individual decision-making with respect to adherence to an intervention.
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1192-92-32452
Frederick R Adler, University of Utah, Anuraag Bukkuri*, Moffitt Cancer Center. Of Criminals and Cancer: The Importance of Social Bonds and Innate Morality on Cellular Societies. Preliminary report.
The current dogma in cancer biology contends that cancer is an identity problem: mutations in a cell's DNA cause it to "go rogue" and proliferate out of control. However, this largely ignores the role of cell-cell interaction and fails to explain phenomena such as cancer reversion, the existence of epigenetic cancers, and foreign-body carcinogenesis. In this conceptual paper, we draw on criminology to propose that cancer may alternatively be conceptualized as a relational problem: Although a cell's genetics is essential, the influence of its interaction with other cells is equally important in determining its phenotype. We create an agent-based network model of interactions among normal and cancer cells to demonstrate this idea. We find that both high mutation rates and low levels of connectivity among cells can promote oncogenesis. Viewing cancer as a breakdown in communication networks among cells in a tissue complements the gene-centric paradigm nicely, and provides a novel perspective for understanding and treating cancer.
(Received September 12, 2023)

## 1192-92-32478

Ari Seth Freedman*, Department of Ecology and Evolutionary Biology, Princeton University. Superspreading and the degree distribution of infected nodes in a network epidemic.

Superspreading is a pervasive and commonly modeled aspect of epidemics with substantial consequences for infectious disease dynamics. Theoretical models accounting for superspreading often rely on a transmission network on which the epidemic spreads, measuring superspreading by fixed characteristics of the network's degree distribution. In reality, however, the impact of superspreading on disease dynamics is always changing, even when the transmission network itself is fixed. I investigate the dynamic impact of superspreading through the "infected degree distribution", which captures the degree distribution of only infected nodes at a moment in time, and characterize the infected degree distribution's moments and longterm behavior. We show that the impact of superspreading, as measured by the infected degree distribution's moments, peaks exactly once during an SIR epidemic, and that the timing of this peak is always early relative to the peak of disease incidence. We discuss the implications these findings have for network-based epidemic control measures.
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## 1192-92-32505

Mike Boots, Department of Integrative Biology, University of California Berkeley, CA, Anna-Liisa Laine, Department of Evolutionary Biology and Environmental Studies, University of Zurich, Switzerland, Graham Richard Northrup*, Center for Computational Biology, College of Engineering, University of California, Berkeley, CA, Steven R Parratt, Department of Ecology, Evolution and Behaviour, University of Liverpool, Liverpool, UK, Carly Rozins, Department of Science and Technology Studies, Division of Natural Science, York University, Toronto, Ontario, Canada, Andy White, Maxwell Institute for Mathematical Sciences, Heriot-Watt University, Edinburgh, UK. The Evolutionary Dynamics of Hyperparasites. Evolutionary theory has typically focused on pairwise interactions, such as those between hosts and parasites, with relatively little work having been carried out on more complex interactions including hyperparasites: parasites of parasites. Hyperparasites are common in nature, with the chestnut blight fungus virus CHV-1 a well-known natural example, but also notably include the phages of important human bacterial diseases. Theory on hyperparasitism has mostly focused on their impact on the evolution of virulence of their parasite host and relatively little is known about evolutionary trajectories of hyperparasites themselves. Our general modeling framework highlights the central role that the ability of a hyperparasite to be transmitted with its parasite plays in their evolution. Hyperparasites which transmit with their parasite hosts (hitchhike) will be selected for lower virulence, trending towards hypermutualism or hypercommensalism and thus not reducing parasite virulence. We examine the impact on the evolution of hyperparasite systems of a wide range of host and parasite traits showing, for example, that high parasite virulence selects for higher hyperparasite virulence resulting in reductions in parasite virulence when hyperparasitized. Our results have implications for hyperparasite research, both as biocontrol agents and for understanding of how hyperparasites shape community ecology and evolution.
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1192-92-32528
Calina Copos, Northeastern University, Katherine Levandosky*, Northeastern University. Modeling Collective Symmetry Breaking in Cells. Preliminary report.
Collective cell migration is necessary in several important physiological processes including embryonic development and immune system response. Prior to migration, a cell must break its symmetry and establish a front-to-rear directional axis. There are several theories proposed for symmetry breaking, including Turing pattern formation [1]. Here, we use an established deterministic-stochastic model that couples the noisy kinetics of a biochemical network to the dynamics of mechanical structures in cells [2]. In the model, through mechanochemical reinforcement, chemicals and mechanical structures simultaneously segregate leading to symmetry breaking in an individual cell. But how do groups of cells synchronize symmetry breaking so that their directional axes point in the same direction? We extend the existing theoretical framework to investigate the specific chemical and mechanical interactions in the cell-cell region that ensures both cells' directional axes align parallel to each other. We test over 100 different interaction rules by modifying the reaction rates in the biochemical networks or the growth rates of the mechanical structures. We find that interaction rules which regulate the signaling kinetic rates differentially and dependently on the local mechanical forces are the most successful in achieving high probability of coaligned directional axes in the paired cells. Additionally, we ask which interaction rules improve co-alignment in the presence of an external signal, which biases a cell's directional axis. While most interactions show similar results with an external signal, other possible pathways emerge, some of which have been biologically speculated. [1] Goryachev AB, Leda M (2017) Many roads to symmetry breaking: molecular mechanism and theoretical models of yeast cell polarity. Molecular Biology of the Cell 28, 370-380. [2] Copos C, Mogilner A (2020). A hybrid stochastic-deterministic mechanochemical model of cell polarization. Molecular Biology of the Cell 31, 1637-1649. (Received September 12, 2023)

1192-92-32547
Sebastian J. Schreiber*, University of California, Davis. Joint impacts of spatial and temporal variation in demography and dispersal on population growth.
All populations experience spatial and temporal variation in environmental conditions. This variation drives population level variation in survival, reproduction, and movement. To better understand the interactive effects of these different sources of variation on population growth, I consider a class of stochastic difference equations accounting for discrete spatial structure, negative density-dependence, and temporally autocorrelated fluctuations. For these models, the origin is a fixed point corresponding to extinction and the long-term fate of the population is determined by a Lyapunov exponent at this fixed point When the Lyapunov exponent is negative, the population tends exponentially quickly to the origin. When the Lyapunov exponent is positive, the system is stochastically persistent - the fraction of time spent arbitrarily near the origin is arbitrarily small. To understand the sign of the Lyapunov exponent, I use a diffusion style approximation with respect to spatial and temporal variation in the parameters associated with demography and dispersal. This approximation highlights how different sources of variation differentially impact the Lyapunov exponent. For example, pure temporal variation always has a negative impact, while pure spatial variation can have a negative or positive impact depending on the form of dispersal. Alternatively, positive co-variation between dispersal and reproduction always increases the Lyapunov exponent. I will illustrate these conclusions with empirically based models of Paramecia dispersing between flasks and Northern Pike dispersing between basins of Lake Windermere.
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Libin Rong*, University of Florida. HIV infection dynamics and viral rebound: Modeling results from humanized mice. Despite years of combined antiretroviral therapy (cART), HIV persists in infected individuals. The virus also rebounds after the cessation of cART. The sources contributing to viral persistence and rebound are not fully understood. When viral rebound occurs, what affects the time to rebound and how to delay the rebound remain unclear. We started with the data fitting of an HIV infection model to the viral load data in treated and untreated humanized myeloid-only mice (MoM) in which macrophages serve as the target of HIV infection. By fixing the parameter values for macrophages from the MoM fitting, we fit a mathematical model including the infection of two target cell populations to the viral load data from humanized bone marrow/liver/thymus (BLT) mice, in which both CD4+ T cells and macrophages are the target of HIV infection. Data fitting suggests that the viral load decay in BLT mice under treatment has three phases. The loss of infected CD4+ T cells and macrophages is a major contributor to the first two phases of viral decay, and the last phase may be due to the latent infection of CD4+ T cells. Numerical simulations using parameter estimates from the data fitting show that the pre-ART viral load and the latent reservoir size at treatment cessation can affect viral growth rate and predict the time to viral rebound. Model simulations further reveal that early and prolonged cART can delay the viral rebound after cessation of treatment, which may have implications in the search for functional control of HIV infection.
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1192-92-32571
G. F. Webb, Vanderbilt University, Xinyue Zhao*, University of Tennessee, Knoxville. Population Models of Epidemics with Infection Age and Vaccination Age Structure.
A population dynamics epidemic model is developed that incorporates age of infection and age of vaccination. The model analyzes pre-symptomatic and symptomatic periods of an infected individual in terms of infection age. The model analyzes the efficacy of vaccination in terms of vaccination age. The model is applied to the 2003 SARS epidemic in Taiwan and the current COVID-19 epidemic in New York State.
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1192-92-32579
Maria G Emelianenko, George Mason University, Tracey G Oellerich*, George Mason University. Inferring Conservation Laws from Data. Preliminary report.
Conservation laws are an inherent feature in many systems modeling real world phenomenon, in particular, those modeling biological and chemical systems. If the form of the underlying dynamical system is known, one can use methods in linear algebra and algebraic geometry to identify the conservation laws. Our work focuses on using data-driven methods to identify the conservation law(s) and the limitations therein. Building upon previous work, we expand the idea of using a Singular Value Decomposition (SVD) to identify conservation law(s) within a system using only data. We will discuss features we look for in the recovery process as well as how it is affected by the number of available points and the noise level of the system. Finally, we will consider how identified conservation laws can be used within other data-driven dynamics recovery algorithms, such as Sparse Identification of Non-linear Dynamics (SINDy). (Received September 22, 2023)

## 1192-92-32588

Julio Cesar Enciso-Alva*, The University of Texas at Arlington, Jianzhong Su, University of Texas at Arlington. New Methods in EEG Source Localization based on EEG and Post-Mortem Pathology Data.
A central task for neuroscience is to determine the location of electrical activity inside the brain. Such electrical signals can be recorded at a high resolution in time (sub-millisecond) but low resolution in space, thus making it difficult to locate its source unambiguously. Further assumptions must be incorporated into the electrical source models to reliably locate this electrical activity inside the brain. One such assumption is to consider the current density distribution and assume that, among all possible configurations, the ones with minimal energy are more likely to be correct. The specifics of implementing this assumption have led to a multitude of methods. However, these minimal-norm methods are limited to the quality of electrical recordings and their low resolution in space. On the paradigm of multi-modal data fusion, the electrical source localization methods are enhanced by considering data from additional imaging modalities. In this work, we propose a simple model using binarized pathology data to enhance electrical source imaging from electroencephalography (EEG) recordings. This study is motivated by post-mortem data on hypoxia due to ischemic stroke, but it may use data derived from fMRI, NIRS, and CT, among others.
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1192-92-32593
Anne E. Yust*, University of Pittsburgh. Empowering Non-Mathematics Students to Reason Mathematically Through Modeling in NetLogo. Preliminary report.
This presentation highlights the application of mathematical modeling in NetLogo to cultivate mathematical and quantitative reasoning in non-mathematics undergraduate students. We discuss exploratory projects spanning diverse biological domains, including modeling COVID-19 transmission with a pre-med student, investigating bacterial growth rates with science undergraduates, and engaging liberal arts students in various dynamical modeling contexts within the NetLogo environment. Throughout, we illustrate how mathematical modeling in NetLogo equips these students with valuable analytical skills for their future careers.
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Eva Marie Strawbridge*, James Madison University. Modelling Bacterial Carpets.
There are many biologically relevant situations which involve long slender bodies (e.g. worms, flagella, bacterial bodies, etc.) where it is important to understand the dynamic interactions of the body and the low Reynolds number fluid in which it moves. In this presentation, I will be discussing applications of the method of regularized stokeslets to periodically moving bodies in fluids. These models have applications to the study of locomotion as well as fluid mixing.

Sabrina H Streipert, University of Pittsburgh, Gail SK Wolkowicz*, McMaster University. Modeling Discrete or Distributed Delay in Population Growth Difference Equation Models.
We introduce a class of higher order single species difference equation models that include a distributed delay in the reproductive process and a survival function accounting for survival pressure during the delay period. The resulting delay recurrences are aimed at modeling the mature population for species in which individuals reach maturity after at least $\tau$ and at most $\tau+\tau_{M}$ breeding cycles. This differs from existing discrete population models with distributed delay that consider delay in the fitness (per-capita) growth rate. For the general set-up, but under realistic model assumptions, we prove the existence of a critical delay threshold $\widetilde{\tau}_{c}$. If each individual takes at least $\widetilde{\tau}_{c}$ to reach maturity (i.e., $\tau>\widetilde{\tau}_{c}$ ), then the population goes extinct; a reasonable biological consequence. In the case of a constant reproductive rate, we provide an equation to determine $\widetilde{\tau}_{c}$. We show that the positive equilibrium is decreasing in both $\tau$ and $\tau_{M}$. We investigate the effect of different kernels and prove relationships between their respective critical delay thresholds. We then apply the derivation technique to model a species for which the adults are assumed to satisfy a Beverton-Holt type reproductive process and the juveniles a Beverton-Holt type function accounting for survival pressure during the delay period. We discuss the global dynamics and its dependence on the delay relevant model parameters $\tau, \tau_{M}$, and the delay kernel. When $\tau_{M}=0$ (discrete delay), we also apply the derivation technique to model a species for which the adults are assumed to satisfy a Ricker type reproduction process and the juveniles a Beverton-Holt type function accounting for survival pressure during the delay period. We also show how these models are related to age-structured population growth models.
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Maya Bocanegra, California State University Northridge, Lauren M Childs, Department of Mathematics, Virginia Tech, United States of America, David W Dick, York University, Fatma Djellouli, University of California Los Angeles, Zhilan Feng, National Science Foundation, Jane M Heffernan, York University, Jing Li*, California State University Northridge, Gergely Röst, University of Szeged. Mathematical Modeling of Waning and Boosting of COVID-19 Immunity through Infection and Vaccination to Predict Seroprevalence in Hungary. Preliminary report.
The emergence of Covid-19 has had enormous impacts on global health. In the past two years, we have successfully extended our previous work, which addressed problems related to estimating the impact of COVID-19 vaccination strategies in the study of the outbreak in Canada and France, by formulating an age-structured model based on an SEIV compartmental modeling framework to study the waning and boosting of COVID-19 through infection and vaccination in Hungary. Our model directly takes into account non-pharmaceutical mitigation strategies that impact activity and contact rates. We identify one important parameter which measures factors including compliance to non-pharmaceutical interventions (masks, social distancing), weather conditions (seasonality), shifts in testing and contact rates, and effects of different strains. We identify this parameter value by fitting our model solution to hospitalization data from Hungary using a genetic algorithm. We also use vaccination data to inform the vaccination rate. The model results enable us to calculate the attack rate and immunity seroprevalence of SARS-CoV-2. Our predictions can provide useful information for future disease surges and can be valuable in guiding public health policies.
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## 1192-92-32613

Lora Bailey*, Grand Valley State University, Alessandra Pantano, University of California, Irvine, Deborah Tonne,
University of California, Irvine. Intertwined Summer Research Experiences for High School and Undergraduate Students. Preliminary report.
Since 2018, the University of California, Irvine has jointly hosted two intertwined summer research experiences for undergraduate and high school students in the area of mathematical and computational biology, called UCI Math BioU and UCI MathExpLR, respectively. Undergraduate and high school students work together in small groups with a mentor to explore a research problem. The six-week program includes short classes in writing LaTeX and MATLAB code, math modeling topics, and "soft skills" such as how to give a research presentation. At the end of the six-week program, students have done substantial research into their topic and present their findings to their peers. The program has survived multiple modalities (in-person, online, and hybrid) and has adapted to thrive despite COVID restrictions. In this talk, we describe how the program has evolved over the course of six years and how outcomes vary by modality of instruction and by typology of students (high school or undergraduate students). Another important question we explore is what advantages and disadvantages are created by mixing undergraduate and high school students in a summer research experience in mathematical and computational biology.
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1192-92-32630
Prajakta Purushottam Bedekar*, Johns Hopkins University, Raquel Binder, UMass Chan Medical School, Anthony Kearsley, National Institute of Standards and Technology, Paul Patrone, National Institute of Standards and Technology. Modeling and Optimization for Standardization in Diagnostics.
Antibody diagnostic samples measured using different equipment need not be comparable ipso-facto, but can be made commensurate by using standard dilution series data. We demonstrate that modeling and optimization are crucial tools for such an interpretation and standardization of diagnostic measurements. We model the thermodynamics of the measurement process as well as the the uncertainties accumulated in dilution, and find a weighted best-fit curve to minimize variability of antibody measurements. Further, we use dilution-dependent averaging techniques and constrained optimization over relevant function spaces, enabling us to compare results across experiments. The results will be demonstrated by using SARS-CoV-2 standards.
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Paul Hurtado*, University of Nevada, Reno. Finding reproduction numbers for epidemic models and predator-prey models of arbitrary finite dimension using the generalized linear chain trick.
Reproduction numbers, like the basic reproduction number R0, play an important role in the analysisand application of dynamic models, including contagion models and ecological population models. One difficulty in deriving these quantities is that they must be computed on a model-by-model basis, since it is typically impractical to obtain general reproduction number expressions applicable to a family of related models, especially if these are of different dimensions (i.e., differing numbers of state variables). For example, this is typically the case for SIR-type infectious disease models derived using the linear chain trick (LCT). Here we show how to find general reproduction number expressions for such model families (which vary in their number of state variables) using the next generation operator approach in conjunction with the generalized linear chain trick (GLCT). We further show how the GLCT enables modelers to draw insights from these results by leveraging theory and intuition from continuous time Markov chains (CTMCs) and their absorption time distributions (i.e., phase-type probability distributions). These results highlight the utility of the GLCT for the derivation and analysis of mean field ODE models, especially when used in conjunction with theory from CTMCs and their associated phase-type distributions.
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## 1192-92-32689

Daniel Nicholas Li, Western Connecticut State University, Luke Zhiyu Peng*, Western Connecticut State University. Wavelet Transforms of scRNA-seq Data Enhance Early-Stage Lung Adenocarcinoma Separation in Lung Tumor Microenvironment Ecotyping Analysis.
Tumors can be viewed as ecosystems consisting of diverse cells within a host environment. This perspective allows us to study cancer through an ecological lens, offering novel approaches for tumor research. In this study, we delve into the ecotyping of the tumor microenvironment. We leverage the proven benefits of wavelet transforms in pattern recognition and apply them to a publicly available single-cell RNAseq dataset of 58 lung adenocarcinoma and lung squamous carcinoma at various stages. We analyzed 130,000 single-cell transcriptomes encoded in more than 20,000 human genes. Based on histological phenotypes, we identified cell type composition clusters in both the untransformed and the wavelet-transformed data. Clustering patterns of the tumors were visualized through dendrograms and ordination plots. Importantly, wavelets-based ordination plots provide a clear spatial separation between early adenocarcinoma tumors and advanced adenocarcinoma and squamous tumors. In contrast, traditional untransformed data-derived ordination plots fail to clearly separate early adenocarcinoma from advanced cancer groups. This difference in separation between wavelet-transformed and non-transformed methods was confirmed by both independent wavelet transform models (Approximation and Detail). Based on cell composition data, the separation occurred as a result of varied cell type proportions; in particular, both wavelet-based methods identified a higher percentage of dendritic cells in early adenocarcinoma. Moreover, the detail wavelet-transformed model supported the existence of an ecological subtype of advanced adenocarcinoma tumors identified in the untransformed ordination plot. Our wavelet-based composition data also revealed novel cell type signatures between tumor histologies, with wavelet-based signatures demonstrating higher significance. Taken together, our approach of utilizing wavelet transforms of sc-RNAseq data enhances early-stage lung adenocarcinoma prediction, which adds value to future early diagnosis and precision treatment. (Received September 12, 2023)

## 1192-92-32693

Gillian Grindstaff*, Oxford. Metric perturbation in phylogenetic tree spaces. Preliminary report.
Given $n$ related species, the moduli space of evolutionary branching histories is known as the phylogenetic tree space. Phylogenetic tree space can be endowed with a wide range of metrics, both intrinsic and extrinsic via embeddings, each of which has distinct geometric and topological properties. The Billera-Holmes-Vogtmann [BHV] metric is built piece-wise from the Euclidean distances between edge length vectors on trees with the same topology. BHV space is popular for point cloud analysis, as it is a CAT( 0 ) cube complex, and therefore has unique geodesics that allow for the computation of Frechet means and piecewise-Euclidean statistics. On the other hand, probability metrics arise from the evolutionary model itself, such as the Jukes-Cantor model and related spaces such as the edge-product space of Moulton and Steel. The probability simplex has a metric which aligns closely with the amount of information available in the data, and the amount of expected measurement noise. We link these two settings by analyzing local distortion coefficients in the Jukes-Cantor model as a map from BHV space to distributions on the probability simplex, and we show that practical identifiability depends on tree topology and size. Together, these provide a geometric foundation for representing measurement noise in BHV space for analysis of error propogation through the phylogenetic data pipeline.
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## 1192-92-32698

Carina Curto, Penn State, Enrique Hansen, Ecole Normale Superieure, Nicole Sanderson*, The Pennsylvania State University, German Sumbre, Ecole Normale Supérieure. Structure in neural correlations during spontaneous activity: an experimental and topological approach. Preliminary report.
Calcium imaging recordings of 1000s of neurons in zebrafish larvae optic tectum in the absence of stimulation reveal spontaneous activity of neuronal assemblies that are both functionally coordinated and localized. To understand the functional structure of these assemblies, we study the pairwise correlations of the calcium signals of assembly neurons using techniques from topological data analysis (TDA). TDA is an emerging subfield of mathematics that can bring new insights to neuroscientific data. This is particularly true when analyzing pairwise correlations, as many common techniques to do so, like spectral analyses, are sensitive to nonlinear monotonic transformations introduced in measurement. In contrast, a TDA construction called the order complex is invariant under monotonic transformations and can capture higher order structure in a set of pairwise correlations. We find that topological signatures derived from the order complex can identify distinct correlation structures during spontaneous activity. Our analyses further suggest a variety of possible assembly dynamics around the onset of spontaneous activation.
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1192-92-32710
Abby Pekoske Fulton*, Wentworth Institute of Technology. A dynamical approach to studying of the energy landscape of

DNA trefoils with excess linking number.
Motivated by knot energy landscapes, the goal of this work is to describe the dynamics of a DNA trefoil knot with excess linking number in an ionized fluid. We consider two approaches: a deterministic approach and a stochastic approach. To study the deterministic dynamics, we use the Generalized Immersed Boundary (GIB) Method - a deterministic numerical method that accounts for the fluid structure interaction of an immersed DNA molecule in an ionized fluid. Using the GIB method, we found stable equilibrium configurations of DNA trefoil knots with excess linking number, analyze their symmetries, and approximate saddle configurations. We also analyze the elastic energy of the DNA trefoil throughout the deterministic process. To study the stochastic dynamics, we use the Stochastic Generalized Immersed Boundary (SGIB) Method - an extension of the GIB method which takes into account the random thermal fluctuations within the fluid - to numerically explore the knot energy landscape. For each fixed linking number, we consider the set of equilibrium configurations as the state space for a continuous time, discrete space, Markov chain, and find boundaries in the energy landscape using the Procrustes distance. Finally, given a fixed linking number, we obtain the steady state distribution for the Markov process and compare this to energy estimates obtained from the deterministic process.
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1192-92-32714
Daniel Nicholas Li*, Western Connecticut State University, Luke Zhiyu Peng*, Western Connecticut State University. Utilizing Wavelet Transformations to Improve Early-Stage Lung Adenocarcinoma Differentiation during Lung Tumor Microenvironment Ecotyping Analysis.
Tumors can be viewed as ecosystems consisting of diverse cells within a host environment. This perspective allows us to study cancer through an ecological lens, offering novel approaches for tumor research. In this study, we delve into the ecotyping of the tumor microenvironment. We leverage the proven benefits of wavelet transforms in pattern recognition and apply them to a publicly available single-cell RNAseq dataset of 58 lung adenocarcinoma and lung squamous carcinoma at various stages. We analyzed 130,000 single-cell transcriptomes encoded in more than 20,000 human genes. Based on histological phenotypes, we identified cell type composition clusters in both the untransformed and the wavelet-transformed data. Clustering patterns of the tumors were visualized through dendrograms and ordination plots. Importantly, wavelets-based ordination plots provide a clear spatial separation between early adenocarcinoma tumors and advanced adenocarcinoma and squamous tumors. In contrast, traditional untransformed data-derived ordination plots fail to clearly separate early adenocarcinoma from advanced cancer groups. This difference in separation between wavelet-transformed and non-transformed methods was confirmed by both independent wavelet transform models (Approximation and Detail). Based on cell composition data, the separation occurred as a result of varied cell type proportions; in particular, both wavelet-based methods identified a higher percentage of dendritic cells in early adenocarcinoma. Moreover, the detail wavelet-transformed model supported the existence of an ecological subtype of advanced adenocarcinoma tumors identified in the untransformed ordination plot. Our wavelet-based composition data also revealed novel cell type signatures between tumor histologies, with wavelet-based signatures demonstrating higher significance. Taken together, our approach of utilizing wavelet transforms of sc-RNAseq data enhances early-stage lung adenocarcinoma prediction, which adds value to future early diagnosis and precision treatment. *These two authors contributed equally to this work. Names are listed in alphabetical order.
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1192-92-32764
Sarah Brownstein*, William \& Mary, Romuald Lipcius, Virginia Institute of Marine Science, Leah Shaw, William \& Mary, Junping Shi, College of William \& Mary. Metapopulation Model for Oyster Restoration with Larval Transport. Preliminary report.
Oyster populations have suffered a dramatic decrease in recent years, so restoration efforts are needed. We build on a previous differential equation model for dynamics of juvenile oysters, adult oysters, dead shells, and sediment at a single location. To help select locations for the construction of restoration reefs, we need a metapopulation model in which the connectivity is based on transport of oyster larvae between locations. We explore how a population behaves with internally produced larvae compared with larvae from other locations. External larval input primarily controls if a population is able to succeed, while internal larval input controls population size once a reef is established. We also consider a metapopulation with connectivity motivated by tidal transport of larvae.
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## 1192-92-32768

Chirantha T Bandara*, University of Florida, Maia Nenkova Martcheva, University of Florida, Calistus N Ngonghala University of Florida. Mathematical Model on HIV and Nutrition.
HIV continues to be a major global health issue, having claimed millions of lives in the last few decades. While several empirical studies support the fact that proper nutrition is useful in the fight against HIV, very few studies have focused on developing and using mathematical modeling approaches to assess the association between HIV, human immune response to the disease, and nutrition. In this talk, we present a novel within-host model for HIV that captures the dynamic interactions between HIV, the immune system and nutrition. We will explore the relationship between serum protein levels, and viral loads, viral production rates and the enhancement rate of protein by the virus in HIV infected individuals. We will conclude our presentation with a discussion on correlation between dietary protein intake and serum protein levels in HIV-infected individuals.
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1192-92-32793
Erin Angelini*, University of Washington. A model for the intrinsic limit of cancer therapy: Duality of treatment-induced cell death and treatment-induced stemness.
Intratumor cellular heterogeneity and non-genetic cell plasticity in tumors pose a recently recognized challenge to cancer treatment. Because of the dispersion of initial cell states within a clonal tumor cell population, a perturbation imparted by a cytocidal drug only kills a fraction of cells. Due to the dynamic instability of cellular states, the cells not killed are pushed by the treatment into a variety of functional states, including a "stem-like state" that confers resistance to treatment and regenerative capacity. This immanent stress-induced stemness competes against cell death in response to the same perturbation and may explain the near-inevitable recurrence after any treatment. This double-edged-sword mechanism of
treatment complements the selection of preexisting resistant cells in explaining post-treatment progression. Unlike selection, the induction of a resistant state has not been systematically analyzed as an immanent cause of relapse. Here, we present a generic elementary model and analytical examination of this intrinsic limitation to therapy. We show how the relative proclivity towards cell death versus transition into a stem-like state, as a function of drug dose, establishes either a window of opportunity for containing tumors or the inevitability of progression following therapy. The model considers measurable cell behaviors independent of specific molecular pathways and provides a new theoretical framework for optimizing therapy dosing and scheduling as cancer treatment paradigms move from "maximal tolerated dose," which may promote therapy inducedstemness, to repeated "minimally effective doses" (as in adaptive therapies), which contain the tumor and avoid therapyinduced progression.
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## 1192-92-32837

Kelly Buch*, Austin Peay State University, Nina H Fefferman, University of Tennessee, Knoxville, Justin Wright, University of Tennessee, Knoxville. A Mathematical Representation of the Reactive Scope Model.
Physiologists have long sought to understand and predict an animal's response to stressful stimuli. Since the introduction of the concept of homeostasis, a variety of model frameworks have been proposed to describe what is necessary for an animal to remain within this stable physiological state and the ramifications of leaving it. The reactive scope model provides a novel conceptual framework for the stress response that assumes an animal's ability to tolerate a stressful stimulus may degrade over time in response to the stimulus. In this talk, we provide a mathematical formulation for the reactive scope model using a system of ordinary differential equations and show that this model is capable of recreating existing experimental data. We also provide an experimental method that may be used to verify the model as well as several potential additions to the model. (Received September 12, 2023)

## 1192-92-32838

Naveen K. Vaidya*, San Diego State University. Modeling drug release from preventive nanomedicine to obstruct HIV movement in vaginal mucus.
The majority of all female Human Immunodeficiency Virus (HIV) infections are the result of heterosexual intercourse with infected male partners. To establish infection, viruses in semen from HIV-infected partners must cross the vaginal mucus, which offers an opportunity to destroy HIV particles by nanomedicine, before they meet underlying CD4+ target cells. In this talk, I will present experimentally validated mathematical models to describe drug release kinetics from preventive nanomedicine and how these preventive therapies obstruct HIV movement in vaginal mucus. Using the model-informed spatiotemporal distribution of viruses and nanoparticles in vaginal mucus, we estimate the time required for the viruses and nanoparticles to penetrate the mucus layer and explore how acidity affects these phenomena. Our results show that the proper implementation of nanoparticle-based therapy can significantly control virus entry through vaginal mucus, thereby avoiding the establishment of HIV infection. Such preventive approaches can help curb the global HIV epidemic.
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1192-92-32846
Robert S Eisenberg, Illinois Institute of Technology, David Minh, Illinois Institute of Technology, Stanley Nicholson*, Brown University. Linear Community Identification in Molecular Dynamics.
Most proteins reduce the complexity of atomic motion to stable and coherent structures. Molecular dynamics (MD) has provided swaths of trajectory data of proteins. We analyze these trajectories using classical stochastic signal analysis, well established and utilized by engineers. Linear systems analysis operates to uncover linearities given an input and output signal. The coherence function says an input and output are linearly related if and only if the coherence equals one. Analyzing protein motion in the frequency domain allows us to extract a frequency function relating the modes of motion as determined by atomic power spectra. We search for large groups of linearly related atoms within two protein systems: crambin and the Mu Opioid Receptor (MOR). We extend our results to all pairwise interaction and determine coherent communities of atoms within the MOR. We present various community detection algorithms and demonstrate their validity using common metrics in MD. Various other graph metrics are presented to elucidate the effective protein structure. Identifying rigid and tightly correlated regions of the protein offers great potential in coarse graining protein structure and understanding protein motion. (Received September 12, 2023)

## 1192-92-32864

Megan Gorringe Dixon*, Brigham Young University. Dimensional Dependence of Binding Kinetics. Preliminary report. For protein binding, most experimentally measured dissociation constants are measured in solution and reported in units of volume concentration. However, many protein interactions take place on membranes suggesting the need for two-dimensional dissociation constants to appropriately model these interactions. I will describe a novel, stochastic approach to understanding the dimensional-dependence of protein-protein interactions in one, two, and three dimensions. I will also present a conversion for transforming traditional three-dimensional dissociation constants to two-dimensional dissociation constants and, using the example of ESCRT proteins, demonstrate that binding on membranes can lead to more stable interactions than binding in solution.
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1192-92-32883
Renee Brady, Department of Integrated Mathematical Oncology, H. Lee Moffitt Cancer Center and Research Institute,
Armando Roldan*, Moffitt Cancer Center: Department of Integrated Mathematical Oncology. Generative Adversarial Networks for Synthetic Patient Trajectories. Preliminary report.
Data disparity remains a persistent challenge for the broader translational science community. At present, models working with observational data frequently encounter difficulties stemming from inconsistent measurement frequencies and insufficiently diverse patient populations. We approach this issue by utilizing a modified generative adversarial network to generate synthetic temporal dynamics in nonlinear systems. We take inspiration from previous work and incorporate an autoencoder network into the traditional GAN framework to better explore the underlying latent-space and capture important
temporal features. We demonstrate this technique in both known system dynamics, as well as provide motivation for translational application in oncologic science.
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## 1192-92-32899

Andreas Buttenschoen*, University of Massachusetts Amherst. Cell Entrainment in a Mechano-Chemical Model of Collective Cell Migration.
Small GTPases, such as Rac and Rho, are well known central regulators of cell morphology and motility, whose dynamics also play a role in coordinating collective cell migration. Experiments have shown GTPase dynamics to be affected by both chemical and mechanical cues, but also to be spatially and temporally heterogeneous. This heterogeneity is found both within a single cell, and between cells in a tissue. For example, sometimes the leader and follower cells display an inverted GTPase configuration. While progress on understanding GTPase dynamics in single cells has been made, a major remaining challenge is to understand the role of GTPase heterogeneity in collective cell migration. Motivated by recent one-dimensional experiments (e.g. micro-channels) we introduce a one-dimensional modelling framework allowing us to integrate cell biomechanics, changes in cell size, and detailed intra-cellular signalling circuits (reaction-diffusion equations). Using this framework, we build cell migration models of both loose (mesenchymal) and cohering (epithelial) tissues. We use numerical simulations, and analysis tools, such as bifurcation and local perturbation analysis, to provide insights into the regulatory mechanisms coordinating collective cell migration. We show how feedback from mechanical tension to GTPase activation lead to a variety of dynamics, resembling both normal and pathological behavior.
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1192-92-32900
Erika Tatiana Camacho*, Arizona State University, Matthias Kawski, Arizona State University, Thierry Léveillard, Institut de la Vision de Paris - Sorbonne University/ INSERM, Kathryn Wifvat, Nuubi, Stephen Wirkus, University of Texas at San Antonio. Optimal Control with RdCVFL for Degenerating Photoreceptors. Preliminary report.
Both the rod and cone photoreceptors, along with the retinal pigment epithelium (RPE) have been experimentally and mathematically shown to work interdependently to maintain vision. Further, the theoredoxin-like Rod-derived Cone Viability Factor (RdCVF) and its long form (RdCVFL) have proven to increase photoreceptor survival in experimental results. Aerobic glycolysis is the primary source of energy production for photoreceptors and RdCVF accelerates the intake of glucose into the cones. RdCVFL helps mitigate the negative effects of reactive oxidative species (ROS) and has shown promise in slowing the death of cones in mouse studies. However, this potential treatment and its effects have never been studied in mathematical models. In this work, we examine an optimal control with the treatment of RdCVFL. We mathematically illustrate the potential this treatment might have for treating degenerative retinal diseases such as retinitis pigmentosa (RP), as well as compare this to the results of an updated control model with RdCVF.
(Received September 12, 2023)

## 1192-92-32906

Vivek K Mutalik*, Lawrence Berkeley National Laboratory. Systematic phage-host interaction datasets for building predictive models of phage susceptbility. Preliminary report.
Microbial communities drive and are driven by significant environmental processes, affect agricultural output, and impact human and animal health. Complex interactions among themselves, their hosts and environments are important for these effects. The virome - the collection of viruses that parasitize these microbial communities- are a critical feature of microbial community dynamics, activity and adaptation. Bacterial viruses (bacteriophage or phages), represent the most abundant biological entities on earth attack exceptionally specific bacterial hosts. However, the mechanisms underlying this specificity are deeply under-characterized and studies have largely focused on handful of individual bacterium-phage systems. The lack of insights into phage specificity and the breadth of bacterial responses to different phages has limited our ability to build models that can predict which phages have the potential to infect specific bacterial strains. Towards the grand goal of understanding phage suscepibility, we are working on building datasets that capture a natural genetic variation in bacterial strains and associated phages using scalable susceptibility assays and high throughput genetics to create a predictive model that connects bacterial genotype to phage susceptibility phenotype. I will present some of our recent efforts towards this goal. (Received September 12, 2023)

## 1192-92-32931

Julie Blackwood*, Williams College. The Role of Spatial Interactions in Managing Ecological Systems: Insights From Mathematical Models.
Policy decisions for managing ecosystems are often decided by political, rather than ecological, boundaries. For example, invasive insects might disperse across state borders and fish can freely move in and out of marine protected areas. Humans migrate across administrative jurisdictions thereby potentially impacting the outcome of policies designed to mitigate the spread of an infectious disease. In this talk, I will introduce several mathematical models of ecological systems that demonstrate that spatial coordination of management efforts may be critical. My primary focus will be on applications to infectious diseases of both humans (e.g. Ebola) and wildlife (e.g. rabies in vampire bats).
(Received September 13, 2023)

1192-92-32973
Ranjini Bhattacharya*, Moffitt Cancer Center, Joel S Brown, Moffitt Cancer Center, Anuraag Bukkuri, Moffitt Cancer Center. Angiogenesis: A Tragedy of Commons.
Cancer progression is the result of evolution in the tumor microenvironment. To meet their increasing growth requirements, cancer cells elevate the expression of pro-angiogenic factors like vascular endothelial growth factor (VEGF). VEGF expression induces angiogenesis- the formation of blood vessels- towards the tumor. Angiogenesis, thus, is an attempt to optimize resource uptake in the tumor habitat. Traditionally, angiogenesis has been viewed as a cooperative phenomenon in which VEGF- producing cells benefit the entire cancer population by redirecting resources to the tumor, albeit at an additional cost to themselves. This fosters the evolution of free-loader cells that exploit the redirected resources without contributing any

VEGF. Using a game theoretic framework, we model VEGF production as an evolutionary strategy and show that cancer cells over-produce VEGF to outcompete neighboring cells, ultimately resulting in a tragedy of the commons. A cell's investment in VEGF hinges on the 'degree of resource sharing' with its neighbors. If resources are evenly distributed, an individual cell's incentive to produce VEGF is low. However, if cells acquire resources in proportion to their VEGF production, they are incentivized to produce more. We investigate a spectrum of strategies ranging from cooperation to competition, contingent on the level of resource sharing in the cell's neighborhood. Our simulations predict that cancer cells can produce 100 times more VEGF than typically seen in normal cells, and what would be their collective team optimum. This means that VEGF production by a cancer cell aims to co-opt nutrients from neighboring cells resulting in an evolutionary arms race. An increase in the number of neighbors results in lower VEGF production per-cell, while exacerbating the tragedy of the commons collectively Additionally, we incorporate anti-angiogenic therapy and different modes of therapy-resistance into our model and discover that the cancer population's resistance strategy is also dependent on the degree of resource sharing in the neighborhood. In summary, the study explores how varying degrees of resource sharing influence the evolution of VEGF production in cancer and offers fresh insights into the different modalities through which resistance to anti-angiogenic therapy evolves. (Received September 22, 2023)

## 1192-92-32981

Joseph Canavatchel*, Manhattan College. The effect of TTX on the Recovery of Amblyopia. Preliminary report Amblyopia is a condition where one of the eyes is not properly stimulated, causing the nerves associated with the eye to be weaker than those of the fellow eye. One treatment is to cover the stronger eye to allow the weaker one to grow stronger, however this treatment does not work after what is known as the critical period. This last point was motivation for experiments on mice in which it was found that by injecting tetrodotoxin (TTX) into the stronger eye, recovery after the critical period is enhanced. Our goal is to make a mathematical model of these results using a setup where one binocular visual cortex neuron can receive controlled inputs from both eyes. We divided up the runtime of the model into 4 stages: a normal rearing stage during which the neuron receives input from both eyes; a monocular deprivation stage during which one of the eyes is weakly stimulated; a TTX stage during which the pooly stimulated eye is allowed to recover while the stronger eye is inactivated; and normal competition stage during which the TTX has worn out. Our simulations suggest that the effect of the TTX is to reduce the firing threshold of the neuron during recovery thereby allowing lower input level from the weak eye to still activate the neuron and help it detect stimuli.
(Received September 13, 2023)

## 1192-92-32982

Rowan J Barker-Clarke*, Theory Division, Cleveland Clinic Lerner Institute, Cleveland, OH. Topological analysis of multichannel images in high-grade serous ovarian cancer..
Cancer evolution encompasses multiple stochastic processes across various scales. In addition to genomic and transcriptomic changes, hallmarks of cancer within solid tumors involve the malignant co-evolution of the tumor microenvironment. Several features of these ecosystems, including cellular neighborhoods, have previously demonstrated clinical relevance by being linked to prognosis. A particularly effective and recent methodology for extracting spatial patterns from clinical images is topological data analysis(TDA). TDA is a discipline focused on investigating the structural properties and connections within data, drawing upon principles from geometric and algebraic topology. Recent advancements in topological data analysis have enabled the extraction of topological characteristics from images. These characteristics have shown correlations with clinical outcomes in studies involving H\&E images in prostate cancer and CT images in lung cancer analysis. To explore the intricate high-dimensional relationships among interacting cells within high-grade serous ovarian cancer, we employed imaging mass cytometry to assess the expression of 32 proteins with subcellular spatial precision in tumors from the BRITROC cohort. Using cell segmentation for single-cell analysis, we identified cell phenotypes encompassing epithelial, stromal, and immune categories. We conducted a systematic examination of both single-cell phenotypic characteristics and spatial relationships using persistent homology. For the first time, we also applied cubical persistence methods directly to images obtained through immuno-metal conjugated (IMC) imaging. By harnessing the capabilities of Topological Data Analysis (TDA) and persistent homology within high-dimensional data, we were able to investigate the distribution and interrelation of topological features in tumors, integrating spatial information across a large number of lineage and cell-activation markers. (Received September 13, 2023)

Deborah Tonne*, University of California, Irvine. Mathematical Modeling of Word-Meaning Association. Preliminary report. Primates communicate via proto-typical innate calls and composite "words" that contain combinations of such calls. In this work, we study the emergence of these new words and how the population's language reacts to such innovations. We have created a stochastic agent-based model to describe the process of learning in a population and a system of ODEs that captures important properties of this system. Using both numerical simulations and analyses of the ODEs we investigated situations where a new word is introduced to a population. Our results for a two-word two-event system, where an event represents a unique word-meaning association, indicate the ability of individuals to regularize linguistic input is at the core of the population's ability to form and maintain a shared language. We find that higher degrees of regularization ability, as well as a more balanced relative frequencies of the two events result in a language where event-word pairs are deterministic and nonambiguous.
(Received September 13, 2023 )

1192-92-33022
Lawrence C Udeigwe*, Manhattan College \& MIT. Modeling Recovery from Amblyopia. Preliminary report.
Amblyopia, also known as lazy eye, is a vision condition where the brain does not properly recognize vision from one eye. The affected eyes is poorly stimulated becuase the nerves associated with it are weaker than those of the fellow eye. One treatment is to cover the stronger eye to allow the weaker one to grow stronger, however this treatment does not work after what is known as the critical period. This last point was motivation for experiments on mice in which it was found that by injecting tetrodotoxin (TTX) into the stronger eye, recovery after the critical period is enhanced. Our goal is to build a mathematical model of these results using a setup where a binocular visual cortex neuron can receive controlled inputs from both eyes. We divided up the runtime of the model into 4 stages: a normal rearing stage during which the neuron receives input from both eyes; a monocular deprivation stage during which one of the eyes is weakly stimulated; a TTX stage during which the pooly
stimulated eye is allowed to recover while the stronger eye is inactivated; and normal competition stage during which the TTX has worn out. Our simulations suggest that (1) the effect of the TTX is to reduce the firing threshold of the neuron during recovery thereby allowing lower input level from the weak eye to still activate the neuron and help it detect stimuli (2) the effect of the TTX is to increase the firing threshold of an associated inhibitory interneurons thereby making it harder for the interneuron to inhibit the activity of the binocular neuorn, so that input from the weak eye is high enough to help activate the neuron to detect stimuli.
(Received September 13, 2023)

## 1192-92-33030

Zhisheng Shuai*, University of Central Florida. Population Dynamics in Stream Networks.
We revisit population dynamics of ecological and epidemiological models over a stream network. We highlight the impact of the network topology and heterogeneity on the threshold values of the population persistence (the network growth rate or basic reproduction number). Our recent studies on metapopulation models over steam networks demonstrate: 1.
Concentration of resources in one of the most downstream patches tends to increase the network growth rate; 2.
Concentration of resources in the most upstream patches tends to increase the population biomass; 3. A larger drift-diffusion ratio could promote population persistence.
(Received September 13, 2023)

## 1192-92-33037

Surl-Hee Ahn, University of California, Davis, Javier Arsuaga, University of California, Davis, Tamara Christiani*, University of California, Davis, Mariel Vazquez, University of California, Davis. Molecular modeling of the bacteriophage portal protein complex. Preliminary report.
Bacteriophages continue to be an attractive model for understanding powerful molecular motors. These viruses depend on this protein composite to work in synchrony to translocate its DNA into preformed procapsids. During the production pathway, accurate assembly of the bacteriophage motor is vital to generating viruses that will be able to transmit its genetic material into bacteria. A critical element of this molecular motor, which is present during DNA packaging and ejection, is the portal protein complex. This toroidal protein complex sits at a unique vertex of the icosahedral capsid and connects to the tail proteins after packaging is completed. Despite the extensive knowledge on the structure of the portal protein complex in DNA bacteriophages, the details of its mechanical role remains understudied. To address this gap in knowledge, we performed cryoelectron microscopy (cryo-EM) 3D single particle analysis and whole atom molecular dynamics (MD) simulations that identify interactions between DNA and bacteriophage P2/P4 portal protein (gpQ). Our results reveal the mechanistic workings and energies of the gpQ portal complex and begin to illuminate the role of the portal complex in packaging, maintenance and delivery of bacteriophage P4 DNA.
(Received September 13, 2023)

## 1192-92-33045

Fazal Abbas, Assistant Professor, St Mary's College of California, Naeem Mustafa Khan, Universal College Islamabad, Petko Kitanov, Wells College, Breanna Shi*, Georgia Tech. A Mathematical Study of Modeling the Bioaccumulation of Methyl-Mercury in Aquatic Systems.
Three models for the bioaccumulation of methyl-mercury in an aquatic ecosystem are described. each model combines predator-prey equations for interactions across a system of trophic levels with toxicokinetic equations for toxin elimination at each level. The models consider food preference and competition within trophic levels. A stability test and sensitivity analysis are conducted for each model. Using known elimination constants for methyl mercury in various fish species and known sources of input from literature, the model predicts toxin levels at the three trophic levels that are compared to samples in the Tampa Bay. The data shows that the competition model is the most accurate of the models produced, and further that actual Methyl-Mercury concentrations are higher than predicted in all models.
(Received October 11, 2023)

## 1192-92-33051

Sebastian J. Schreiber*, University of California, Davis. Community assembly via invasion graphs: Mathematical rigor meets empirical realism. Preliminary report.
Community assembly is the study of ecological processes that determine the species composition of ecological communities One conceptualization of community assembly is rare invasions of species from a regional pool into a focal area. Successful invasions will shift the local community from one configuration of species to another configuration. In this talk, I will introduce a mathematically rigorous framework for representing the process of community assembly. This framework builds on work with Josef Hofbauer on invasion graphs for Lotka-Volterra models. The vertices of these invasion graphs corresponds to equilibria of the Lotka-Volterra models and directed edges correspond, roughly, to potential connecting orbits between pairs of equilibria. Under appropriate assumptions, these invasion graphs determine which equilibria correspond to permanent communities. I will show how an appropriate pruning of these invasion graphs yields community assembly graphs (CAGs). that represent the community assembly process. To illustrate their utility, I compute CAGs for 35 empirically parameterized LotkaVolterra models of 3 to 13 interacting species. Analyzing these empirically derived CAGs, I address several fundamental questions in community ecology: How often does community assembly lead to a single end state, multiple end states, or endless cycling? What is the relationship between regional species richness and local community richness? How many paths are there to end states and what is the distribution of their lengths? Future directions and open questions will be discussed. (Received September 13, 2023)

## 1192-92-33067

Ami Radunskaya*, Pomona College. Cross-talk: mathematical modeling of the Gut-Brain axis. Preliminary report.
The connection between our brain and our guts has been known for a long time, and the role that the the gut microbiome plays in maintaining our health has been the subject of extensive research. New experimental platforms such as engineered 3-D tissues and organs-on-a-chip have generated useful data for better understanding the kinetics of the processes involved. In this talk we will present preliminary work on modeling the pathways that make up the gut-brain axis. These dynamic models can
provide another tool towards understanding the dynamics of the interactions between microbiome, gut and brain, and help in the exploration of new microbiome-based therapies.
(Received September 13, 2023)

## 1192-92-33114

Indunil M. Hewage*, Washington State University, Dylan Hull-Nye, Washington State University, Elissa Schwartz,
Washington State University. The dynamics of COVID-19 infection with vaccination and waning immunity.
Three years into the COVID-19 pandemic, new cases, hospitalizations, and mortality have reduced substantially due to the availability of effective vaccines. However, the waning of immunity has been a topic of particular interest in relation to disease control. Therefore, the objective of this study is to develop a model to investigate the impact of the waning of vaccine-induced immunity $\left(\omega_{1}\right)$ and the immunity acquired upon recovery from a previous infection $\left(\omega_{2}\right)$. We mathematically analyze the model and show that the bifurcation at $R_{0}=1$ is backward. The parameter $\omega_{1}$, but not $\omega_{2}$, appears in $R_{0}$. However, $\omega_{2}$ seems to largely control the endemic level of disease. The results of local sensitivity analyses and numerical simulations indicate that the waning of either immunity increases disease burden. We also conduct a global sensitivity analysis using Latin hypercube sampling and partial rank correlation coefficient analysis to assess the impact of parameters with uncertain values. We show which parameters are most strongly correlated with endemicity as well as how variation in $\omega_{1}$ and $\omega_{2}$ affect the epidemic. The implications of our results highlight the importance of developing vaccines that provide long-term immunity.
(Received September 13, 2023)

1192-92-33155
Mckayla Davis, Brigham Young University, Robert Davis, Brigham Young University, Erika Ibarra, Brigham Young University, Tyler J. Jarvis, Brigham Young University, Bryce Lunceford, Brigham Young University, Gwen Martin, Brigham Young University, William Terry, Brigham Young University, Lydia Tolman*, Brigham Young University, Andrew Williams, Brigham Young University. Mathematics of Noninvasive Glucose Sensing.
We describe several new mathematical techniques for analyzing signals and extracting information about glucose and other analytes from noninvasive sensors, including a miniaturized spectrometer and bioimpedance device. These techniques arise from combining probabilistic models and an understanding of physical laws related to the propagation of infrared light and electrical signals in the body. We also share some results from clinical trials using these techniques. (Received September 13, 2023)

## 1192-92-33157

Jude Kong*, Africa-Canada Artificial Intelligence and Data Innovation Consortium (ACADIC), York University. The
Phytoplankton competition for nutrients and light in a stratified lake: a mathematical model connecting epilimnion and hypolimnion.
In this talk, I will present several mathematical models describing the vertical distribution of phytoplankton in the water column. In particular, I will introduce a new mathematical model connecting epilimnion and hypolimnion to describe the growth of phytoplankton limited by nutrients and light in a stratified lake. Stratification separates the lake with a horizontal plane called thermocline into two zones: epilimnion and hypolimnion. The epilimnion is the upper zone which is warm (lighter) and well-mixed, and the hypolimnion is the bottom colder zone which is usually dark and relatively undisturbed. The growth of phytoplankton in the water column depends on two essential resources: nutrients and light. The critical thresholds for the settling speed of phytoplankton cells in the thermocline and the loss rate of phytoplankton are established, which determine the survival or extirpation of phytoplankton in epilimnion and hypolimnion. This is joint work with Jimin Zhang (Heilongjiang University), Junping Shi (William \& Mary) and Hao Wang (University of Alberta).
(Received September 13, 2023)

1192-92-33176
Brittany Stephenson*, Lewis University. Mathematical Epidemiology Research with Undergraduates: Stories of Success and Lessons from Failures.
My experience mentoring undergraduates in mathematical epidemiology research includes participation in summer research programs, collaboration with colleagues for internal grants to support student researchers, and student enrollment in research course credits. Project topics have involved either (1) modeling with ordinary differential equations and subsequent follow-up analysis through parameter estimation, sensitivity analysis, and/or optimal control theory or (2) simulations using agent-based models in NetLogo with BehaviorSpace analysis. Together with students I have studied the transmission of COVID-19, Clostridioides difficile infection, and measles. In this talk, I will discuss the ups and downs I have experienced in the mentoring process. In particular, I will provide details about the projects themselves, students' backgrounds and preparation, and awards received for student work.
(Received September 13, 2023)

1192-92-33194
Parsa Seyfourian*, University of British Columbia - Vancouver, Adhvaith Sridhar, University of Minnesota Twin Cities, Jeffrey Wang, Carleton College. The Versatility of Mathematical Modeling and Math Modeling Education: An Application in Predicting Flu Spread.
Mathematics, mathematical modeling, and simulations can be a versatile asset in today's educational framework. Their quantitative, data-driven nature makes mathematical modeling relevant to many fields and contexts and has redefined our approach to complex problems. Herein, we demonstrate this through our exploration of the role of math modeling in healthcare, chiefly epidemiology. We formulate a comprehensive model geared towards predicting the spread of emerging flu strains. The intrinsic nature of the flu, characterized by its cyclical rotation of strains, demands a model that is not only accurate but perpetually relevant. Our design not only meets this criterion but, due to its modular architecture, can also be repurposed for diseases with similar cyclic attributes. Central to our model is the SIR (Susceptible, Infected, Recovered) framework. This analytical tool processes flu case data, offering a real-time estimate of those potentially exposed to the virus. One key feature of our system is its ability to instantaneously adjust its numerical output with new data entry. Our initial results have shown that this computational prowess can be harnessed to generate a visual map, illustrating the potential
spread of the disease, thereby acting as a supportive asset for healthcare professionals and policymakers. Moreover, the alliance of mathematical modeling with AI/ML technologies amplifies its versatility, extending its applications far beyond healthcare. As a step towards ensuring the next generation is equipped with these multifaceted tools, we propose a learning model for use in undergraduate and high school contexts that utilizes case studies and project-based learning methodologies to impart the skills of mathematical modeling and simulations and underscore the value of mathematical modeling regardless of a student's career trajectory.
(Received September 13, 2023)

## 1192-92-33203

Heiko Enderling*, MD Anderson Cancer Center. Personalization of patient specific cancer radiotherapy dose and dose fractionation using volumetric tumor dynamics.
One shortcoming of current clinical practice is that radiation protocols do not consider patient-specific dynamic behaviors that may influence outcome. Mathematical modeling and computer simulations of tumor volume dynamics can reveal an emergent description of tumor radiosensitivity. We present a quantitative framework to estimate a personalized radiation dose for individual patients, based on pre- and early on-treatment tumor volume dynamics. We the discuss an in silico trial of this dose personalization using retrospective data from a combined cohort of $n=39$ head and neck cancer patients from the Moffitt and MD Anderson Cancer Centers that received 66-70 Gy RT in 2-2.12 Gy weekday fractions. The mathematical model of tumor growth, radiation response, and patient-specific tumor carrying capacity can fit the clinical data with high accuracy. Selected ranges of patient-specific carrying capacity predicts superior responses to hyperfractionation protocols. Early treatment response dynamics predict total dose to provide tumor control and disease-free survival. We demonstrate the feasibility of using tumor volume dynamics to inform dose personalization and stratification for dose escalation and de-escalation. The dose personalization methodology could potentially be applied in any treatment setting where fractionated RT is used with a potential for dynamics informed dose adaptation. The advantage of applying this methodology to fractionated delivery of RT is that this context allows for enough time to observe, calibrate the model, make forecasts, and then adjust the treatment course. While the focus of the presented work is on head and neck cancer, it is conceivable that the dynamics adapted radiation dose (DARD) framework is translatable to other cancer types.
(Received September 13, 2023)

## 1192-92-33205

Liana F Lareau*, University of California, Berkeley. Identifying cell state-associated regulation of mRNA processing via autocorrelation. Preliminary report.
Alternative splicing shapes the output of the genome and contributes to each cell's unique identity, but single-cell RNA sequencing has struggled to capture its impact. We have shown that low recovery of mRNAs from single cells can lead to misleading conclusions about alternative splicing and its regulation. To address this, we have developed a method, Psix, to confidently identify splicing that changes across a landscape of single cells, using a probabilistic model that is robust against the data limitations of scRNA-seq. Its autocorrelation-inspired approach finds patterns of alternative splicing that correspond to patterns of cell identity, such as cell type or developmental stage, without the need for explicit cell clustering, labeling, or trajectory inference. Psix reveals cell type-dependent splicing patterns and the wiring of the splicing regulatory networks that control them, enabling scRNA-seq analysis to go beyond transcription to understand the roles of post-transcriptional regulation in determining cell identity.
(Received September 13, 2023)

## 1192-92-33214

Deena R. Schmidt*, University of Nevada, Reno. Modeling of biological networks with undergraduate research collaborators. Many biological systems in nature can be represented as a dynamic model on a network. Examples include gene regulatory systems, neuronal networks, social networks, epidemics spreading within a population described by a contact network, and many others. A fundamental question when studying a biological process represented as a dynamic model on a network is to what extent the network structure is contributing to the observed dynamics. I will give an overview of my work with undergraduate students that addresses this question, and then I will focus on two recent projects. The first project investigates the spread of norovirus (stomach flu) within an age-structured population using a stochastic adaptive network model. The second project looks at mammalian sleep-wake regulation at different stages of development using an integrate-and-fire neuronal network model. Lastly, I will also talk about funding received by my undergraduates to support their research. (Received September 13, 2023)

## 1192-92-33234

Natasha Jonoska*, University of South Florida, Van Pham, University of South Florida, Masahico Saito, University of South Florida. Mathematical model for DNA building blocks used in crystallographic nanaostructures.
Bottom-up self-assembly of DNA nanostructures have been proposed for variety of biotech uses and the rationally-designed 3D DNA motif, the tensegrity triangle, is the first molecule used to provide DNA crystallographic assemblies. The sequence design possibilities of these building blocks give ever-increasing geometric complexities to form vast arrays of three-dimensional structures. We show a mathematical model that explains which tensegrity triangle structures can form and in which chiral topology they can form, left- or right-handed. The mathematical model is also experimentally verified.
(Received September 13, 2023)

1192-92-33250
Paul Hurtado*, University of Nevada, Reno. The Joys of Undergraduate Research: From New Model Derivation Tools to The Mathematics of Recreational Bird Watching.
One of my favorite aspects of undergraduate research is working on projects that are of interest to students. In this talk, I'll give a brief overview of two very different projects: The first focuses on a new approach to the derivation and interpretation of state transition models (ODEs) from stochastic model first principles via the generalized linear chain trick (GLCT). The GLCT allows one to quickly derive a mean field ODE model from first principles by framing the underlying stochastic model assumptions in a standard continuous time Markov chain (CTMC) framework. The resulting matrix-vector parameterization of
that CTMC can then be used to immediately write down the corresponding mean field ODEs via the GLCT. In the second part of the talk, I'll talk about an ongoing project looking to quantify the bird watchers' notion of rarity, and using that concept to try and remove bias in citizen science datasets (using eBird data). Here we use spatiotemporal clustering tools to tackle overreporting bias of rarities by multiple observers. I'll present some of the key results from both projects, as well as some background on how the research was carried out with multiple undergraduate student collaborators.
(Received September 13, 2023)

## 1192-92-33263

Sumitabha Brahmachari*, Rice University. Modeling the mechanics of transcription and its regulation via DNA supercoiling. DNA mechanics plays an integral role at the length scales of protein-DNA interactions. How the mechanical properties of DNA affect RNA Polymerase (RNAP) translocation and thus regulate transcription is not well understood. I will discuss a theoretical framework that integrates DNA mechanics with RNAP motion using dynamical equations that enforce torque balance. Within this framework, experimental observations, like cooperation between co-transcribing RNAPs and orientation-dependent interaction between genes, appear as emergent phenomena controlled by DNA twisting or supercoiling. This model makes testable predictions, revealing how the mechanical interplay between RNAPs and the transcribed DNA can govern transcriptional dynamics. I will further talk about expanding the framework to incorporate the effect of nucleosomes, and the corresponding function of nucleosomes in regulating eukaryotic transcription.
(Received September 13, 2023)

## 1192-92-33310

Javier Arsuaga, University of California, Davis, Tamara Christiani, University of California, Davis, Michael Keith*, University of California, Davis, Mariel Vazquez, University of California, Davis. Understanding Reduced Knotting Probability in Bacteriophage P4 Using Cryo-EM. Preliminary report.
The P4 bacteriophage is a satellite virus of bacteriophage P2 that infects Escherichia coli. Its genome is a linear dsDNA molecule with cohesive ends that forms knotted structures upon circularisation. DNA extracted from mature bacteriophage P4 and its tailless mutants show a high proportion of knots. Interestingly, the number of knotted molecules differs when DNA is extracted from the capsids of mature phages and tailless mutants. More specifically, the knotting probability of DNA extracted from tailless mutants is 0.96 , while it is only 0.6 when extracted from mature phages. Although the exact mechanism that accounts for this discrepancy in knotting probabilities is undetermined, it is believed the DNA in mature phages is located in such a way that the probability of knot formation is minimised. Previous studies suggest that one of the DNA's cohesive ends in lambda, P2, and P4 phages is located in the tail region, providing a potential DNA-protein interaction site that mitigates circularisation. To validate this hypothesis, we produce cryo-electron microscopy reconstructions of the bacteriophage tail, identify the location of the cohesive end, and elucidate the interactions between the tail and the DNA cohesive end. Our results therefore suggest that DNA knotting in mature P4 bacteriophages is partially prevented by keeping one of the cohesive ends outside the capsid.
(Received September 13, 2023)

1192-92-33325
Stephanie Chang, Massachusetts Institute of Technology, Tong Liu, Tsinghua University, Adeethyia Shankar*, Danbury Math Academy, Xiaodi Wang, Western Connecticut State University, Yongzhong Zhao, Frontage Labs. A Wavelet-Based Multi-omics Approach Reveals Host-Cell-Type-Specific Networks in Inflammatory Bowel Disease.
The interplay between microbiome metabolites and human cells is crucial and mechanistically linked to human health and disease. Inflammatory bowel disease (IBD) is linked to microbiome metabolite and host gene expression. However, details of the microbiome and host interplays remain elusive. We carry out microbiome metabolome-wide and host transcriptome-wide association studies of IBD with microbiome metabolite targeted cell types discovery via leveraging the publicly available IBD data sets from Human Microbiome Project 2 (HMP2). By performing deconvolution on the transcriptomic data and applying discrete wavelet transform (DWT), we obtained cell type-metabolite correlations, which we visualize in the form of heatmaps and networks. We also carried out both targeted and untargeted approaches by mean of correlating the microbiome data matrix to host transcriptomic data. Given the limited sample size, in addition to visualizing a global picture of the interplay landscape between microbiome metabolites and host genes alongside distinct clusters of IBD and healthy controls with UMAP and t-SNE, we found a set of microbiome metabolites most likely linked to IBD and the transcriptomic signature of IBD. For the targeted approach, we also refer to the single-cell gene signature dataset, i.e., the MSigDB C8, uncovering a bile acid, namely, lithocholate, targeted cell types, including intestine lymphoid cells and enterocytes. Moreover, we utilized Mendelian Randomization causality tests with ursodeoxycholic acid (UDCA), RUNX1 gene, and IBD, resulting in a putative causality network of RUNX1, UDCA, and IBD. Taken together, our approaches shed light on the mechanistic interplay of microbiome metabolites and host cells in human health and disease.
(Received September 13, 2023)

## 1192-92-33349

## Bob Palais*, University of Utah, Annie Staker, Co-Dx. Math For Multiple Targets in One PCR-Melting Assay. Preliminary

 report.We will discuss mathematical aspects used to design and analyze PCR-Melting assays that detect and distinguish several target sequences simultaneously. These assays leverage thermal and spectral differences of oligonucleotides and fluorophore labels and dyes to extract more diagnostic information from amplification or melting curves, or combinations of both. (Received September 13, 2023)

1192-92-33354
Pei Liu*, Florida Institute of Technology. Helical organization of DNA-like liquid crystal filaments in cylindrical viral capsids. We study equilibrium configurations of ds-DNA in a cylindrical viral capsid. The state of the encapsidated DNA consists of a disordered inner core enclosed by an ordered outer region, next to the capsid wall. The DNA configuration is described by a unit helical vector field, tangent to an associated center curve, passing through properly selected locations. We postulate an expression for the energy of the encapsulated DNA based on that of columnar chromonic liquid crystals. A thorough analysis of
the Euler-Lagrange equations yields multiple solutions. We demonstrate that there is a trivial, non-helical solution, together with two solutions with nonzero helicity of opposite sign. Using bifurcation analysis, we derive the conditions for local stability and determine when the preferred coiling state is helical. The bifurcation parameters are the ratio of the twist versus the bend moduli of DNA and the ratio between the sizes of the ordered and the disordered regions (Received September 13, 2023)

1192-92-33363
Sarah Elizabeth Ritchey Patterson*, Virginia Military Institute, Ashleigh Smythe, Virginia Military Institute. Particle Model for Deterring the Spread of Meloidogyne spp.. Preliminary report.
Plant parasitic nematodes cause an estimated $\$ \mathrm{U} 100$ billion in annual crop losses worldwide, with root knot nematodes in the genus Meloidogyne causing the greatest damage. We created a particle model to investigate how the infective juvenile stage (J2) of Meloidogyne spp. moves through different types of soil. This project investigates the effects sand, silt, clay, and soil composition on the motility and spread of the model nematodes in three-dimensional domains. The results have implications on green loss prevention in agriculture.
(Received September 25, 2023)

1192-92-33366
Joe Bondy-Denomy*, UC San Francisco. What to do when CRISPR fails? Innovation in bacteriophage defense. Preliminary report.
Bacteria use "front-line" defenses like CRISPR-Cas and restriction-modification (RM) systems to protect themselves from bacteriophage infection. However, phages have evolved mechanisms to thwart these systems including anti-CRISPR proteins, anti-RM proteins, DNA modifications, and nucleus-like structures, necessitating back-up plans for the host. Here, we have identified protective back-up mechanisms including a novel and widespread immune system that targets jumbo bacteriophages prior to the assembly of their CRISPR-resistant phage nucleus. Bacteria additionally possess a cGAS-STING-like immune system, called cyclic-oligonucleotide based anti-phage system (CBASS), which normally blocks phage maturation after frontline defenses fail. We have identified phage evasion mechanisms for this system as well, including a potent inhibitor protein that sequesters nearly all known cyclic nucleotides. In summary, numerous cell processes stand in the way of phage success beyond the front-line CRISPR-Cas and RM systems. Bacteria therefore use a multi-layered and diverse immune repertoire to protect themselves from phage killing.
(Received September 14, 2023)

## 1192-92-33372

Catherine Li*, MIT. Spatiotemporal risk prediction for infectious disease spread and mortality.
With the outbreak of the COVID-19 pandemic, various studies have focused on predicting the trajectory and risk factors of the virus and its variants. Building on previous work that addressed this problem using genetic and epidemiological data, we introduce a method, GeoScore, that also incorporates geographic, socioeconomic, and demographic data to estimate infection and mortality risk by region and time. We employ gradient descent to find the optimal weights of the factors' significance in determining risk. Such spatiotemporal risk prediction is important for informed public health decision-making so that individuals are aware of the risks of travel during an epidemic or pandemic, and, perhaps more importantly, so that policymakers know how to triage limited resources during a crisis.
(Received September 14, 2023)

## 1192-92-33393

Maxwell Joseph Fox*, University of Alabama in Huntsville. Sterile Insect Technique: Stability Analysis and Dynamics of a Mosquito Population.
A well proposed ODE system that models the interactive dynamics of sterile and wild mosquito populations with a saturated release rate of sterile mosquitoes can have up to four positive equilibria, but only two positive equilibria have been confirmed to exist in the literature. In this note we establish that for a range of parameters this system does have four positive equilibria. We also obtain the stability of these equilibria.
(Received September 14, 2023)

## 1192-92-33406

Marco V Martinez, Associate Professor, Department of Math \& Actuarial Science, North Central College, Maisha Marzan*, Undergraduate Student, Department of Applied Mathematics, North Central College, Gregory Ruthig, Professor, Department of Biology, North Central College. Multihost Pathogen Creates Ecological Links Between its Hosts and Influences Host Population Dynamics.
Pathogens infecting multiple species create ecological links between species, even if these species do not directly interact. These different hosts with varying susceptibility to the pathogen can have indirect negative effects on another host by amplifying the number of infective propagules. This interaction is like 'apparent competition' and one host species can end up decreasing the abundance of another host species via this shared enemy. Furthermore, if a multihost pathogen can infect any of its host saprophytically it can persist in the system even after host death and further increase infection pressure on other susceptible hosts in the system which can possibly drive a host species to extinction. We mathematically model such an ecological community with one common pathogen and multiple hosts, including hosts only being infected after death (saprophytic) and hosts tolerant to the infection. Our objective is to determine if the presence of hosts with varying susceptibility including saprophytic hosts can increase infection burden on specific susceptible hosts. We have completed a conclusive deterministic mathematical model for the population of susceptible, infected, and dead hosts, along with the pathogen, using a $4 \times 4$ linear system of ordinary differential equations. Using this model, we can conclude that increasing number of host species has indirect negative effects on another host population size by amplifying the number of infective propagules. We also calculate the community Basic Reproductive Number, $R_{0}$, utilizing the Next Generation Matrix to conclude which host species can have the largest effect in propagating the infection and which can aid in policy development and conservation
(Received September 14, 2023)

Jonathan Acuna-Robles*, Augusta University. Modeling of Vaccine Breakthrough and Rebound Infection: Analysis for the U.S. Department of Health and Human Services Regions. Preliminary report.

In this work, we analyze vaccine breakthrough infections and rebound infections of the COVID-19 virus, specifically Omicron and its lineages in each of the ten HHS regions in the United States. To accomplish this task, we create Susceptible-Vaccinated-Exposed-Infected-Recovered (SVEIR) epidemic models to describe the transfer of infections in a population. Our model will account for those who are vaccinated against any of the strains and assumes that an individual who recovers from, or vaccinated against, a particular strain can still be infected by emerging strains. Equilibrium points and reproduction numbers for each variant are calculated to examine the possibility of each variant becoming endemic. The result derived in this work is used to analyze the infectivity of the COVID-19 virus in the ten HHS regions in the United States. (Received September 18, 2023)

## 1192-92-33426

Courtney Davis, Pepperdine University, Relena Pattison*, Pepperdine University. Mathematically modeling how trapping regimes that target specific crayfish life stages impact removal efficacy. Preliminary report.
The red swamp crayfish, Procambarus clarkii, is an invasive species introduced into several streams within the Santa Monica Mountains (SMM) in California. Crayfish predation decimates native aquatic species. The Mountains Restoration Trust (MRT) has worked to remove crayfish through regular trapping in Malibu Creek. To aid conservation efforts, former student Dev Patel developed a mathematical model of crayfish removal efficacy. His discrete model of the crayfish life cycle newly accounted for cannibalism but was not yet parameterized to stream data. We expand Patel's model to better predict the efficacy of crayfish removal efforts in the SMM. We separate crayfish based upon life stage and total length: eggs, two monthly juvenile stages, small non-reproductive adults, medium adults, and large adults. We construct and parameterize this preliminary predictive model of crayfish population levels with and without trapping. We use literature and crayfish removal data provided by MRT to fit the model to the Middle Las Virgenes portion of Malibu Creek. We numerically simulate how the crayfish population changes over time and find that the model dynamics are highly sensitive to juvenile monthly survivorship. We determine the best crayfish life stages to trap to most efficiently decrease crayfish population size; specifically, model predictions suggest that a smaller mesh size that traps juveniles in addition to adults is necessary to reduce crayfish populations to local extinction.
(Received September 15, 2023)

## 1192-92-33452

Jasmine Bunag Carpena*, University of Hawaii at Manoa. Network Sampling.
In this research project, we study graphical models, with a particular emphasis on preserving the maximum likelihood threshold. The background of our investigation lies in the crucial role of graphical models, which are used for representing complex relationships among variables. For example, in biology they are routinely used to infer gene regulatory networks from gene expression data and to infer co-occurrence networks from microbiome data. A pivotal aspect of these models, which we intend to explore, is that the existence of a maximum likelihood estimate (MLE) is contingent upon having a sufficient number of data points relative to the number of variables in the model. Our primary hypothesis and objective revolve around the notion that by harnessing combinatorics and algebraic statistics, we can optimize model selection routines to accommodate relatively small datasets. These datasets, characterized by fewer observations than random variables, are frequently encountered in practical applications. We aspire to uncover strategies and techniques that enable us to adapt model selection algorithms to the constraints of small-sample scenarios in a way that maintains the validity of maximum likelihood estimation. (Received September 16, 2023)

1192-92-33472
Vincent L Cannataro, Emmanuel College, Boston, MA, J Nic Fisk, Yale University, New Haven, CT, Kira Alexa Glasmacher*, Emmanuel College, Boston, MA, Mia Jackson, Yale University, New Haven, CT, Jeffrey D Mandell, Yale University, New Haven, CT, Jeffrey P Townsend, Yale University, New Haven, CT. Mutation of NOTCH1 is selected within normal esophageal tissues, yet leads to selective epistasis suppressive of further evolution into cancer.
Normal tissues acquire genetic variants that are also commonly found in cancers arising from the same tissues. Our understanding of how mutations that are commonly found in normal tissue can contribute to tumorigenesis is limited: mutations may or may not confer phenotypes compatible with oncogenesis. However, we can use mathematical modeling approaches based on evolutionary principles to quantify the strength of selection for variants present in normal and cancerous tissues in a stepwise manner. We developed a maximum likelihood framework to calculate distinct selection intensities for an arbitrary number of steps in an evolutionary process and applied these methods to study the evolutionary trajectory from conception through adult normal esophageal tissue to esophageal squamous-cell carcinoma (ESCC) by analyzing genetic sequencing data to reveal the stepwise contributions of somatic mutations to increased cellular division and survival. We also calculated pairwise selective epistasis between mutated genes that may lead to stepwise substitution patterns. We found that NOTCH1 substitutions are at high prevalence in normal tissue, and scaled selection coefficients indicate that NOTCH1 mutations are highly selected in normal esophageal tissue. In contrast, there is little to no positive selection for NOTCH1 mutations in ESCC, suggesting that NOTCH1 substitutions do not drive tumorigenesis. Furthermore, we calculate that mutations in NOTCH1 exhibit antagonistic epistasis with well-known cancer drivers, including TP53, reducing selection on progressive mutations in tumorigenesis. This antagonistic epistasis likely underlies the low likelihood of tumor progression in the presence of NOTCH1 mutations in the esophagus. (Received September 18, 2023)

Alexander Wiedemann*, Randolph-Macon College. Computational Complexity of Genome Rearrangement. Large-scale mutations which occur when the order of genes within a genome is altered fall under the heading of genome rearrangement. When creating mathematical models to study such mutations, biological relevance must be balanced with computational approachability. Even for relatively simple models, the sheer number of potential evolution histories between two genomes makes it infeasible to analyze each one for hypothesis testing. In this talk, we will summarize the state-of-the-art
in computational complexity and uniform sampling questions for several genome rearrangement models, focusing on the Single Cut-and-Join model.
(Received September 19, 2023)

## 1192-92-33578

Valen Michael Feldmann*, Hope College, K. Greg Murray, Hope College, Brian Yurk, Hope College. Factors that influence Pioneer Plant Survival in Monteverde Cloud Forest Reserve.
We used logistic regression to explore the influence of the light environment on the survival rates of six different pioneer plant species in the cloud forest of Monteverde, Costa Rica. We used data from annual censuses of canopy gaps and plants that began in 1983 and 2000, respectively. We explored predictors of survival related to the characteristics of the plants (e.g., height, diameter, age) or to the canopy gaps they were growing in (e.g., gap area). We found that for most species, taller plants had a higher probability of survival. However, in Phytolacca rivinoides shorter individuals had a higher survival probability. For larger plants, a smaller gap led to a higher survival probability, and the opposite was true for smaller plants. (Received September 23, 2023)

## 1192-92-33590

Cameron Davis*, Fitchburg State University, Benjamin Levy, Fitchburg State University. Incorporating Sandfly Population Dynamics into a Compartmental Disease Model for Visceral Leishmaniasis. Preliminary report.
Visceral Leishmaniasis (VL) is a neglected tropical disease found primarily in the Indian subcontinent region. The disease is deadly if left untreated, and spread via sandflies. A typical way to explore disease spread is to formulate a compartmental system of ordinary differential equations where the rate of new cases is governed by the interaction of susceptible and infected individuals. While several compartmental models for the disease have been formulated and analyzed, they often fail to reproduce the annual periodic behavior seen in VL case data for the region. Since sandflies are the primary vector for the disease, we suspect this oscillating behavior is likely driven by annual sandfly population oscillations. This paper adapts an existing model for the region by adding a sandfly population sub-model to more accurately describe VL case data. This process included fitting a periodic population sub-model to sandfly data, before building it into the larger compartmental model, and then fitting the full system to VL case data from the region. Parameter fitting was carried out in the programming software MATLAB by solving a constrained least squares optimization problem. After parameterizing the model, we simulate the impact of various sandfly interventions on disease dynamics.
(Received September 21, 2023)

1192-92-33623
Alexandra Galitsyna, Mentor, Leonid Mirny, Mentor, Henrik Dahl Pinholt, Mentor, Elizaveta Rybnikova*, Student. Exploration of the Hi-C patterns through computer simulations. Preliminary report.
DNA in the nucleus of eukaryotic cells is a compact structure that forms a variety of local structures - domains, fountains, and stripes - responsible for the regulation of genes and other crucial biological processes. In mammals, these structures are known to be formed by the mechanism of loop extrusion. In this process, a protein binds to DNA, connecting two regions of DNA together in a loop. This loop is processively extruded by the extruder protein until it falls off randomly. In this work, we build a quantitative model to characterize the features of 3D DNA that emerge through this probabilistic mechanism. We compare our output against the experimental contact map of the mammalian genome (the readout of the Hi-C sequencing technique). Our polymer simulation model predicts the shape of fountains, domains, and stripes, which we further validate with a convolutional neural network. The resulting database systematizes the variety of local 3D genome structures, providing an understanding of the parameters of loop extrusion and helping to reveal its biological significance.
(Received September 21, 2023)

## 1192-92-33690

Austin Teddy Barton*, Georgia Institute of Technology, Jonathan Greer*, Howard University, Jordan Klein*, Appalachian State University. Incorporating Adaptive Human Behavior into Epidemiological Models using Equation Learning. Mathematical models have been shown to be valuable tools for forecasting and evaluating public health interventions during epidemics such as COVID-19. Covasim is an open-source agent-based model (ABM) that was recently developed to simulate the transmission of COVID-19. Covasim has been validated with real-world data and utilized for simulating the potential effect of public health interventions. Covasim's base model does not implement adaptive behaviors; however, we can utilize its resources to generate data for scenarios where human behavior can adapt based on the current state of the model, subsequently affecting the epidemic. Human behaviors, such as compliance to masking guidelines, have been shown to depend on the state of the epidemic and can have strong effects on the disease spread. We extended the Covasim model to incorporate adaptive masking behavior to investigate its effect on Covasim's predicted forecast. Using an existing compartmental model, we processed the data generated from this extended ABM through Biologically-Informed Neural Networks (BINNs) and sparse regression techniques to estimate parameters and obtain an ordinary differential equation (ODE) approximation of this model. The extended ABM and equation learning computational pipeline we developed is open-source to provide a quantitative framework for incorporating adaptive behaviors into forecasting future epidemics and other similar computational models. (Received September 23, 2023)

1192-92-33706
Michelle Bartolo, North Carolina State University, Darsh Gandhi*, North Carolina State University, Alexandria Johnson*, North Carolina State University, Mette S Olufsen, North Carolina State University, Emma Nicole Slack*, North Carolina State University, Isaiah Stevens*, North Carolina State University, Zachary Turner*, North Carolina State University. Optimization of Lesion Removal and Uncertainty of Image Segmentation on Pulmonary Hemodynamics. Preliminary report. Chronic thromboembolic pulmonary hypertension (CTEPH) is a fatal but curable disease, causing high blood pressure and unresolved lesions in the pulmonary arteries. One way to reduce lesions is via balloon pulmonary angioplasty (BPA), inserting micro-balloons into the arteries to reduce lesion size. Patients typically undergo several treatments each inserting three balloons. However, there is currently no objective way to identify which lesions to treat to minimize blood pressure and maximize perfusion of the lung. To inform this treatment, we develop a patient-specific fluid mechanics model predicting
hemodynamics in geometries extracted from computed tomography (CT) images. Using 3D Slicer, we generate a threedimensional (3D) volumetric surface from the CT image, and using the Vascular Modeling Toolkit, we extract centerlines and junctions creating a tree. Tree labels including vessel radii and length and their uncertainty are obtained using a statistical change point algorithm. This algorithm identifies the segment along each vessel that best represents the vessel radius. We generate 1000 networks by sampling from the distribution of radii from each vessel. For each network configuration, we predict blood pressure and flow in each vessel by solving the one-dimensional Navier-Stokes equations. The microvasculature is represented by an asymmetric bifurcating structured tree in which daughter vessels are scaled relative to their parent. These microvascular trees are attached at the terminal branches of the imaged informed networks, and we solve a linearized wave equation to predict hemodynamics. Perfusion is determined by projecting average flow and pressure at the end of each vessel onto the 3D lung volume. Results show segmentation quality, network size, and changes in radius and length significantly impact hemodynamics. Future work includes optimizing BPA strategies for CTEPH patients, simulating lesion reduction and its hemodynamic impact.
(Received September 23, 2023)

1192-92-33726
Shelby Noelle Horth*, Wake Forest University. Modeling Multiple Capillary Layers in the Human Retina. Preliminary report Impaired vascular factors have been associated with many ocular diseases, including glaucoma. For example, decreased capillary density has been observed in both early and advanced stages of the disease. Mathematical modeling can be used to assess the impact of vascular factors on retinal blood flow and oxygenation. The retina has a complex three-layer capillary network; our study extends a previous model to incorporate these three layers in five possible orientations. This model includes flow regulation and oxygen transport mechanisms and is used to predict how venous oxygen saturation and retinal blood flow are affected by changes in capillary density. Simulating a $10 \%$ capillary density reduction (as observed clinically), our model predicted a $5.6 \%$ decrease in venous saturation. Combining this simulation with elevated intraocular pressure led to a $9.8 \%$ decrease in venous saturation. Ultimately, our model demonstrates the importance of considering distinct capillary layers for realistic retinal oxygenation predictions. These modeled capillary layers will be integrated into an existing model accounting for arteriolar vascular network heterogeneity to predict tissue regions at high risk of the low oxygenation characteristic of glaucoma. (Received September 24, 2023)

## 1192-92-33739

Tat Chung Dawson Chan*, University of California, Berkeley, Michael Lindstrom, The University of Texas Rio Grande Valley, Elliot M Miller, The University of Alabama, Carlos Montes-Matamoros, The University of Texas Rio Grande Valley, Laurent Pujo-Menjouet, Université Claude Bernard Lyon 1, Omar Sharif, The University of Texas Rio Grande Valley. Oscillations in neuronal activity: a neuron-centered spatiotemporal model of the Unfolded Protein Response. labstract Many neurodegenerative diseases (NDs) are characterized by the slow spatial spread of toxic protein species in the brain. The toxic proteins can induce neuronal stress, triggering the Unfolded Protein Response (UPR), which slows or stops protein translation and thus indirectly reduces the toxic load. However, the UPR may also trigger processes leading to apoptotic cell death and the UPR is implicated in the progression of several NDs. In this paper, we develop a novel mathematical model to describe the spatiotemporal dynamics of the UPR mechanism for prion diseases. Our model is centered around a single neuron, with representative proteins P (healthy) and S (toxic). The model takes the form of a coupled system of nonlinear reaction-diffusion equations with a delayed, nonlinear flux for $P$. Through the delay, we find parameter regimes that exhibit oscillations in the $P$ and S protein levels. We find that oscillations are more pronounced when the $S$ clearance rate and $S$ diffusivity are small in comparison to the $P$ clearance rate and $P$ diffusivity, respectively. The oscillations become more ubiquitous as delays in initiating the UPR increase. In the model system, decreasing the production of $P$, increasing the clearance of S, and reducing the recruitment rate appear to be the most powerful mechanisms to reduce the mean UPR intensity and eventually moderate the disease progression. \endabstract
(Received September 25, 2023)

1192-92-33743
Bard Ermentrout, University of Pittsburgh, Neil Christopher Maclachlan*, University of Pittsburgh. Analysis and Simulation of Spatiotemporal Spiral Waves in Neuronal Networks. Preliminary report. Our work is motivated by the discovery of spiral waves in fMRI data from the human cortex, which vary in distribution and chirality based on cognitive task. We propose a biophysically motivated firing rate model of excitatory and inhibitory neurons on a 2D lattice with isotropic distance dependent connectivity. This approach is novel since phase models, previously used to study spirals, do not include inhibition. Mean field theory yields a planar synaptic activity model, allowing bifurcation analysis. Simulations show spirals forming from random initial conditions. Local excitatory connectivity yields smaller, denser spirals, while long-range excitatory connections produce a few large spirals. Long range inhibitory connectivity causes spirals to be sparse as the surrounding medium goes to synchrony, aligning with the hypothesis that such connections disrupt spiral formation. Spirals persist through the Hopf bifurcation, supporting analysis in the oscillatory regime. We derive a principled phase model using weak coupling theory, and approximate it via its Fourier transform. We discover the quadratic sine coefficient in a simple phase model controls spiral activity and this coefficient can be tuned in the principled phase model through the applied current to inhibitory neurons.
(Received September 25, 2023)

1192-92-33762
Sasha Hyacinthe*, Emmanuel College, Theodore Kelly*, Emmanuel College, Jacob Wayne Proulx*, Emmanuel College. Modeling Evolutionary Dynamics of Collective Action on Graphs. Preliminary report. Organisms across all forms of life partake in an array of collective behaviors, either to offer assistance or to inflict harm upon others. We investigate a mathematical model describing the evolution of collective behaviors, where individuals are located at nodes connected by edges weighted by distance. A specific set of "actor" nodes may choose to pay a cost to contribute to an action. If all nodes in this "actor" set contribute, then the group collectively helps or harms a specific "target" node. Selection between "Contributor" and "Non-contributor" types proceeds according to a spatial death-Birth process. We derive conditions under which contribution is favored based on the topology of the graph and choice of "actor" set or "target" node. We apply
these results to a variety of theoretical graphs and empirical networks, such as the spatial structure of sociable weaver bird nests and the growth of cancer cells. Overall, we find that smaller isolated sub-communities are most favored to collectively help their neighbors and that spatial structure promotes cooperation in public goods scenarios.
(Received September 25, 2023)

Poly Hannah da Silva, Columbia University, Arash Jamshidpey, Columbia University, Nafisa Anzum Raisa*, Fordham University. Mathematical Models of Genome Rearrangements and their Effects on Evolution. Preliminary report.
A molecule of DNA can be evolved by point and large-scale mutations. Large-scale mutations are often called genome rearrangements or structural variations. As their name suggests, these genomic operators are responsible for reorganizing the structure of a genome. The most popular genome rearrangements include reversal, translocation, transposition, and insertion/deletion. Any specific set R of genome rearrangements induces an "edit" genomic distance between a pair of genomes A and B which is defined to be the minimum number of genome rearrangements of type R needed to transform A into B. Edit distances are extensively used in gene-order phylogeny methods. We review the mathematical properties of the main genomic operators and distances. In particular, we study the effects of genome rearrangements on the evolution of genomes. (Received September 26, 2023)

## 1192-92-33825

Trevor Daniel Overton*, Colorado State University. Implementation of a Schur Complement post processing technique for Electrical Impedance Tomography imaging of 2D neonatal data.
Electrical Impedance Tomography (EIT) is a nonionizing, noninvasive pulmonary imaging technique for bedside monitoring. Functional images of conductivity are formed by measuring voltages that arise on electrodes from applied current patterns. Due to the severe ill posedness of the inverse problem, spatial resolution is the primary challenges in the field. One approach to improve resolution is to apply a statistical post processing method that uses the Schur complement to include information from an anatomical atlas of simulated EIT data. In this work, an anatomical atlas was constructed from CT scan data of infants. A computationally efficient Schur Complement method was implemented and applied to a verification data set to measure accuracy. Results showed significant improvement in resolution in various evaluation metrics.
(Received September 26, 2023)

## 1192-92-33849

Ryan Syed*, Lewis University. Tile-based Modeling of DNA Self-Assembly for Caveman Graphs.
Developments in nanotechnology and the ability for double-stranded DNA molecules to form such nanostructures motivates the application of formal graph theory to model self-assembling DNA complexes. Tiles are used to represent branched-junction DNA molecules with free cohesive ends to make up discrete graphs, which can then be analyzed through the lens of graph theory and linear algebra. We present the results of applying these techniques for constructing caveman graphs. (Received September 26, 2023)

## 93 Systems theory; control

## 1192-93-25470

Elif Demirci, Ankara University, Fatma Karakoc*, Ankara University, Ayşen Kütahyalığlu, Ankara University. Stability of Hopfield Neural Networks with Caputo-Hadamard Fractional Derivative. Preliminary report.
Recently, artificial neural networks have been created by modeling biological neural networks and have been used in the area of image processing, pattern recognition, associative memory, optimization problems, etc. Within the examinations of artificial neural network systems, one of the aims is to state the conditions for the existence and stability of the unique equilibrium. In this talk, a Hopfield-type neural network system with Caputo-Hadamard fractional derivative is discussed. First, the existence and uniqueness of the equilibrium is proved. Then, sufficient conditions for the Mittag-Leffler stability of the equilibrium is obtained using the Lyapunov direct method.
(Received June 14, 2023)

1192-93-28155
Hongjiang Qian, University of Connecticut, Gang George Yin*, University of Connecticut, Qing Zhang, University of Georgia. Computational Nonlinear Filtering: A Machine Learning Approach.
Nonlinear filtering is a fundamental problem in signal processing, information theory, communication, control and optimization, and systems theory. In the 1960s, celebrated results on nonlinear filtering were obtained. Nevertheless, the computational issues for nonlinear filtering remained to be a long-standing and challenging problem. In this talk, in lieu of solving the stochastic partial differential equations to obtain the conditional distribution or conditional measure, we construct finite-dimensional approximations using machine learning and neural networks for approximating the optimal weights. Two recursions are used in the algorithm. One of them is the approximation of the optimal weight and the other is for approximating the optimal learning rate.
(Received August 27, 2023)

1192-93-29122
Frank Yingjie Huo*, George Washington University. The new mathematics of online (mis)behavior.
Social media platforms and policymakers worldwide are struggling to control online harms caused by extreme "anti-X" communities, where "X" can stand for science and medicine through to particular races and genders. Can mathematics help? And if so, what does it look like? In this talk, I show how our recent work [1-3] answers the first question with a resounding "yes", and that the answer to the second question leads to new mathematics in the area of shocks and generalized fluid dynamics. I present a first-principles theory that describes analytically the dynamics of a heterogeneous population
undergoing fission-fusion dynamics online. Integral transformations of this mesoscale model yields novel generalizations of non-linear fluid equations at the system level. Their solutions can take the form of a shockwave and agree with empirical observations of "anti-X" activities in cyberspace [3]. Moreover, as additional mechanisms are incorporated (e.g. higher-order interactions represented by simplices and hypernetworks) a new playground emerges for exploring generalized Burgers equations - and these are already revealing some nice mathematical surprises. [1] Pedro D. Manrique, Frank Yingjie Huo, Sara El Oud, Minzhang Zheng, Lucia Illari, and Neil F. Johnson. Shockwavelike Behavior across Social Media. Physical Review Letters, 130, 237401 (2023). See https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.130.237401 [2] Elisabeth Wilhelm. A New Science for Describing Unhealthy Online Environments. Physics, June 5, 2023. See
https://physics.aps.org/articles/v16/89 [3] Dynamic Online Networks Lab. https://donlab.columbian.gwu.edu (Received September 06, 2023)

## 1192-93-31335

Weiwei Zhang*, King's College, Wilkes-Barre, PA. Optimal Design for Boolean Control Networks with Time Delays.
The Smith predictor approach is applied to the control design of Boolean control networks with delays. The objective is to steer the initial state to the desired state for Boolean control networks with the time optimal controls. The Smith predictor and the modified Smith predictor designed on the nominal controller are introduced. A numerical example is presented to illustrate the efficiency of the obtained results.
(Received September 11, 2023)

1192-93-31406
Bozenna Pasik-Duncan*, University of Kansas. Stochastic Adaptive Control \& Stochastic Differential Equations. Abstract: Stochastic adaptive control denotes the control of a partially known stochastic system which can be described by a stochastic differential or by a stochastic partial differential equation. While stochastic models are developed from physical systems, typically some parameters of the model are unknown but the systems have to be controlled. Thus there is a problem of stochastic adaptive control. A solution to it consists of parameter estimation and control. The noise processes for the models include some that have been empirically identified in cognition, that is, fractional Brownian motions. Stochastic control and stochastic adaptive control problems are described for systems with a noise modeled by fractional Brownian motions and other stochastic processes. Applications of stochastic adaptive control include telecommunication, biomedicine, and finance. (Received September 11, 2023)

1192-93-31498
Vijay Kumar, University of Pennsylvania, Matthew D Kvalheim, University of Maryland, Baltimore County, Jake Welde*, University of Pennsylvania. Differential Flatness and Geometric Hierarchy in Underactuated Mechanical Systems with Symmetry.
In this talk, we consider the problem of identifying a differentially flat output of an underactuated mechanical system with Noetherian symmetry. We describe sufficient conditions for the construction of a global or almost global flat output that preserves this invariance. With respect to the principal bundle describing the symmetry, our flat outputs consist of the group variables of a trivialization satisfying certain geometric criteria. In this trivialization, the dynamically feasible trajectories of the system as a whole can be parametrized by sufficiently smooth curves in the symmetry group. Our results stand in contrast to purely local approaches that may also fail to preserve the system's symmetry, and our perspective reveals a geometric hierarchy that we seek to exploit in the design of cascaded controllers.
(Received September 11, 2023)

## 1192-93-31756

Nathan Murarik*, Pennsylvania State University, Gino Rotellini*, Pennsylvania Western University, Tatiana Sosnovsky*, California State Polytechnic University, Pomona, Nick Wintz, Lindenwood University. The Conformable Kalman Filter. Preliminary report.
In this project, we formulate the Kalman-Bucy filter for linear, continuous conformable control systems corrupted by white noise. Here, we use the system first introduced by Khalil, et al. We obtain a state transition matrix via a Peano-Baker expansion that allows us to calculate our error propagation through a Riccati equation. In addition, we show the duality between the conformable Kalman filter and its associated conformable linear quadratic regulator (CLQR) problem is preserved. Finally, we provide numerical simulations for relevant applications. (Received September 12, 2023)

## 1192-93-31925

Cesar Contreras, Texas A\&M International Univ, Tomoki Ohsawa*, University of Texas at Dallas. Stabilization of Mechanical Systems on Semidirect Product Lie Groups with Broken Symmetry via Controlled Lagrangians.
We extend the method of controlled Lagrangians with kinetic shaping to those mechanical systems on semidirect product Lie groups with broken symmetry, more specifically to the Euler-Poincaré equations with advected parameters. We find a matching condition for the controlled Lagrangian for such systems whose configuration manifold is a general semidirect product Lie group $G \ltimes V$. Our motivating examples are a bottom-heavy underwater vehicle and a top spinning on a movable base. Their configuration space is the special Euclidean group $\operatorname{SE}(3)=S O(3) \ltimes \mathbb{R}^{3}$, where the $\mathrm{SE}(3)$-symmetry is broken by the gravity. The controls resulting from the matching condition stabilize unstable equilibria of these examples. Furthermore, the matching helps us find additional dissipative controls that asymptotically stabilize those unstable equilibria. (Received September 12, 2023)

## 94 Information and communication, circuits

Alexandra Koletsos*, Columbia Univeristy, Kerry Seekamp*, Smith College, Adrian Thananopavarn*, Princeton Univeristy. Quantum Error Correcting Codes from Multidimensional Circulant Graphs. Preliminary report.
Circulant graphs have been extensively studied for their ability to generate quantum error-correcting codes (QECCs). In this work, we extend this property to multi-dimensional circulant graphs (MDCs), a generalization of circulant graphs on multiple coordinates. We explore the graph-theoretic properties of these new graphs, establishing isomorphism characteristics within a broader class of vertex-transitive graphs. Using these results, we reduce the search space for new QECCs, and show with an exhaustive computer search that MDCs can produce comparable codes to circulant graphs. We discovered two new 0dimensional QECCs of lengths 77 and 90 with respective minimum distances 19 and 22, improving upon previous best known values by one. Utilizing a similar approach, we were able to construct 5 new ternary QECCs for the first time with parameters $[[51,0,16]]_{3},[[52,0,16]]_{3},[[53,0,16]]_{3},[[55,0,17]]_{3}$ and $[[57,0,17]]_{3}$.
(Received August 12, 2023)

## 1192-94-28401

Kenza Guenda, USTHB, Algeria, Padmapani Seneviratne*, Texas A\&M University-Commerce. LCD Codes over Finite Fields. Preliminary report.
We introduce several new construction methods of linear complimentary dual (LCD) codes over finite fields. First, we show that punctured code of an LCD code with a transitive permutation group is LCD. Further, we prove several important families including anti-primitive BCH codes, punctured McDonald codes, and extended Gabidulin codes are LCD. (Received August 30, 2023)

Mackenzie Bookamer*, Tulane University, Susana Jaramillo, Whittier College, Lani Southern, Willamette University. Schubert-Polar On the Grassmannian. Preliminary report.
The Grassmannian $\mathcal{G}_{l, m}$, is the collection of all $l$ dimensional subspaces of a vector space of length $m$. It is one of the most widely studied objects in Algebraic Geometry and has interesting algebraic, geometric, and combinatorial properties. We use a class of linear codes, known as Grassmann codes, to study the Grassmannian. We applied Schubert and polar conditions, which have been studied independently but not concurrently, to create a subvariety of the Grassmann code. We determined the minimum distance and found minimum weight codewords for several variations of the Schubert and polar conditions. (Received August 30, 2023)

## 1192-94-28498

Ken R. Duffy*, Northeastern University, Moritz Grundei, TU Munich, Muriel Medard, MIT. Universal soft detection decoding in channels with memory - ORBGRAND-AI.
To meet the Ultra Reliable Low Latency Communication (URLLC) needs of modern applications, there have been significant advances in the development of short error correction codes and corresponding soft detection decoders. A substantial hindrance to delivering low-latency is, however, the reliance on interleaving, the intentional delay in transmission of neighbouring data in order to break up omnipresent channel correlations, solely to ensure that decoder input matches decoder assumptions. Using ideas from statistics, combinatorics, and mathematical physics, here we describe ORBGRAND-AI (Ordered Reliability Bits Guessing Random Additive Noise Decoding-Approximate Independence), a soft-detection decoder that can efficiently decode any moderate redundancy code and overcomes the limitation of existing decoding paradigms by leveraging channel correlation and circumventing the need for interleaving. By making use of correlation, not only is latency reduced, but error correction performance can be significantly enhanced, while decoding complexity is also reduced.
(Received August 31, 2023)

## 1192-94-28598

Carolyn Mayer*, Sandia National Laboratories. Communication with Misspecified Input Distributions. Preliminary report. The capacity of a communication channel is the supremum of the rate at which information can be reliably transmitted over the channel. This supremum is over possible input probability distributions, and applications may give restrictions on the set of possible distributions. In this talk, we will explore some examples of the impact of using misspecified input probability distributions. This work was supported by the Laboratory Directed Research and Development program at Sandia National Laboratories, a multimission laboratory managed and operated by National Technology \& Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.
(Received August 31, 2023)

1192-94-28760
Sulani Thakshila Kottal Baddhe Vidhanalage*, Florida Atlantic University. Fiat-Shamir Signatures based on ModuleNTRU.
NTRU(Number Theoretic Research Unit) problem is a computational problem which recognize as a hard problem to solve in Lattice-Based Cryptography. The NTRU problem is defined as for a given $h$ polynomial, finding two small polynomials $f, g$ such that $h=f / g\left(\bmod x^{n}+1\right)$ in the quotient polynomial ring $\mathbb{Z}_{q}[x] /\left\langle x^{n}+1\right\rangle$ where $n$ is a power of 2 and $q \equiv 1(\bmod 2 n)$. Module-NTRU(MNTRU) problem is a generalization of NTRU problem and MNTRU lattices brings out an extra flexibility in the choice of parameters for the underlying ring dimension. Then, we generalized MNTRU problem in to the inhomogeneous MNTRU(iMNTRU) problem. Our work is to construct two Digital Signature Schemes(DSS) based on iMNTRU problem by using the Fiat-Shamir(FS) transformation which was applied to an Identification Scheme to construct the DSS. The first signature scheme is the Lossy signature scheme which is provable secure and have the smallest signature size 4400 bytes when compared with the Quantum Random Oracle Model(QROM) Dilithium DSS which was constructed by Ducas, Kiltz, Lepoint Lyubashevsky, Schwabe, Seiler, Stehl\'\{e\}(IACR 2018). The Lossy DSS has proved for tight security under QROM with 128 -bit security. The second signature scheme is the (Bimodal Lattice Signature Scheme)BLISS-like DSS which is based on the rejection sampling technique and provable secure under Random Oracle Model(ROM). We assume the iMNTRU problem which was used for proving the hardness and the security of two DSS are as same strength as the search and decisional standard

MNTRU problems. The construction of our Lossy DSS followed by the work on tightly secure Lossy Identification Schemes by Abdalla, Fouque, Lyubashvesky and Tibouchi(EUROCRYPT 2012) and tight security under QROM of the Lossy DSS is proved by Kiltz, Lyubashevsky and Schaffner (EUROCRYPT 2018). The BLISS-like signature scheme is followed by the work on BLISS DSS which was constructed by Ducas, Durmus, Lepoint and Lyubashvesky (CRYPTO 2013). \par
(Received September 02, 2023)

1192-94-29114
Daniel P. Bossaller*, University of Alabama in Huntsville, Hiram H. Lopez, Virginia Tech. Repair Schemes for Linear Codes over Galois Rings. Preliminary report.
Reed-Solomon codes over finite fields are ubiquitous in a wide variety of applications, from communication to distributed data storage. In 2017, Guruswami and Wootters developed a linear repair scheme for Reed Solomon codes which significantly improved on the naïve repair scheme. Since then, other researchers have further refined the theory of linear and nonlinear repair schemes for distributed data storage. In 2013, Quintin et al. defined Reed-Solomon codes over commutative and noncommutative rings. In this talk, I will explore the repair problem for such Reed Solomon codes, and, time permitting, I will discuss possible applications to problems of data storage. This talk reports on joint work with Hiram H. López (Received September 05, 2023)

## 1192-94-29146

Hsin-Po Wang*, University of California, Berkeley. Channel Manipulation as a Coding Technique
Mathematical breakthroughs are often made by making the abstract concrete. For example, additive continuous map from $\mathbf{R}^{m}$ to $\mathbf{R}^{n}$ sounds complicated, but $m \times n$ matrix sounds a lot better. In coding theory, there is a similar breakthrough:
Traditionally, channel coding is about how to manipulate bits and symbols. However, we can see it as a way to manipulate channels as if channels were concrete objects. By treating channels as concrete objects, we can study the convergence of a sequence of channels, partial orders on channels, and even distributions on the space of all channels. This viewpoint is proved powerful for design and analysis of polar codes and low-density parity-check (LDPC) codes which achieve Shannon capacity and are the two pillars of the 5G standard. In this talk, we give an overview of recent advanced results regarding polar and LDPC codes.
(Received September 05, 2023)

1192-94-29523
Natalia Luna*, Saint Mary's College. Belief Evolution Over Time in Social Networks. Preliminary report.
Evolution of beliefs of a society is a result of interactions between people in the society over generations. We analyze the long term dynamics of belief evolution by combining people's prior beliefs, social dynamic network structures and the confusion that occurs between beliefs. The main contribution of this work is threefold. First, we explore the belief evolution using existing network models such as scale free networks and small world networks to create social network structures and belief confusion structures. Second, we model the belief evolution with homophily based models using different statistical distances. We compare the individual and societal belief distributions and trends obtained from different models. Third, we explore the evolution of religious affiliations in different countries; both large and small in size, located in different continents. We use a homophily based model to fit religious affiliation data to model the dynamics of religious beliefs of Australia, Canada and Ireland over time. Using existing network models, we observe that the society evolves to a homogeneous belief system across all of the different models. However, the formation of heterogeneous belief systems such as social groups that share the same beliefs and isolated individuals can be observed using homophily based models. Moreover we can see how those formations change based on different distance measures. Finally, the successful implementation of the real world data justifies the theoretical formulation of the model and this allows us to interpret the social dynamics over time and the future implications. (Received September 07, 2023)

## 1192-94-29560

Cecília Salgado, University of Groningen, Anthony Varilly-Alvarado*, Rice University, Felipe Voloch, University of Canterbury. Rational surfaces and locally recoverable codes.
Motivated by large-scale storage problems around data loss, a budding branch of coding theory has surfaced in the last decade or so, centered around locally recoverable codes. These codes have the property that individual symbols in a codeword are functions of other symbols in the same word. If a symbol is lost (as opposed to corrupted), it can be recomputed, and hence a code word can be repaired. Algebraic geometry has a role to play in the design of codes with locality properties. In this talk I will explain how to use algebraic surfaces birational to the projective plane to both reinterpret constructions of optimal codes already found in the literature, and to find new locally recoverable codes, many of which are optimal (in a suitable sense). This is joint work with Cecília Salgado and Felipe Voloch.
(Received September 07, 2023)

## 1192-94-30051

Duncan Koepke*, University of Wisconsin-Eau Claire. Codebook Creation for Partial Correction.
In coding theory, a multiple access channel is one where the messages of two or more senders are combined and then sent for a receiver to decode. After the senders select their messages, there is a window for a malicious adversary to act. We investigate ways to model a particular channel structure and design codes.
(Received September 08, 2023)

1192-94-30052
Allison Beemer*, University of Wisconsin-Eau Claire, Duncan Koepke, University of Wisconsin-Eau Claire, Michaela Schnell, University of Wisconsin-Eau Claire, Madelyn St Pierre, University of Wisconsin - Eau Claire. Authenticated Partial Correction over Adversarial MACs.
Adversarial multiple-access channels (MACs) model scenarios in which multiple users transmit to a single receiver in the
presence of a malicious adversary. Previous work demonstrated that in the two-user case, given a set of channel conditions, it may be possible to correct one of the users' messages even if the other is too corrupted to decode. In other words, positive rate pairs are (at least sometimes) achievable for authenticated partial correction in the two-user case. In this talk, we extend these results to a larger number of users and discuss coding strategies for authenticated partial correction in this setting. (Received September 08, 2023)

1192-94-30053
Madelyn St Pierre*, University of Wisconsin - Eau Claire. Extending Block Length Over an Adversarial Multiple Access Channel.
Every day, people attempt to send messages over the internet, but those messages are only useful if they can be properly read by the receiver. Thus, it is important to be able to detect or correct errors in the message caused by a hostile adversary. In an adversarial multiple access channel, two or more users are sending information to a single receiver, and an adversary can attack the messages. In this setting, our goal is to authenticate messages and correct as many as possible. We look at extending the block length of codewords using short block length codebooks that are capable of successfully authenticating and partially correcting. Our construction determines how we can partially correct the longer messages and results in bounds on code rate.
(Received September 08, 2023)

## 1192-94-30057

Jordan Hebert*, University of Wisconsin Eau-Claire. Universal Hashing For Message Authentication.
In coding theory, Hashing and Hash functions are used to map messages to a short fingerprint (a hash) that allows users to authenticate the identity of the code's sender. Hashing can be an optimal choice in securing messages against an adversary as long as the probability of two messages returning the same hash (a collision) is low. In my research I investigate Universal Hashing, a method of selecting random Hash functions from a Hash family with a guaranteed bound on collision rate. By applying Universal Hashes to previous message authentication strategies that have utilized different Hashes, I will investigate whether Universal Hashing provides fewer collisions and optimizes efficiency by decreasing the complexity in decoding. (Received September 08, 2023)

## 1192-94-30499

Sofia Celi*, Brave. Sigma and proof of knowledge protocols: state, limitations and future. Preliminary report.
$\Sigma$ protocols and proofs of knowledge (PoK) are a class of protocols that are increasingly more interesting in real-world applications, especially in regards to creating cryptographic schemes for privacy-preserving systems. However, while simple and straightforward, complex composition of these protocols can be done in a non-accurate manner, especially in regards to proving their properties (soundness, zero-knowledge and completeness). In this talk, we will look at two schemes in the literature and show how their soundness proof contains invalid assumptions, which diminishes the security of the schemes. We will see what a correct proof should look like. This talk will give a brief overview of $\Sigma$ protocols and PoKs. It will explore the challenges and limitations of them, especially in regards to achieving their properties. It will show how proving their properties has to be done in a detailed manner, as invalid assumptions can be introduced (as we will show in the explored schemes). We will conclude by looking at why they are important, which kind of privacy applications they have, and where they stand.
(Received September 10, 2023)

## 1192-94-30517

Jacob Hume*, University of Michigan, Danny McDonald*, University of Pennsylvania, Allan Thomas Newman*, Indiana State University. Edge-Informed Estimations for Blurring Parameters in Convolutional Models. Preliminary report. Let $f \in L^{2}\left[-\frac{1}{2}, \frac{1}{2}\right)$ be a piecewise smooth function with finitely many well-separated jump discontinuities and let $g$ be a Gaussian function with an unknown shape parameter, $\alpha$. The convolution $f * g$ models the blurring process that is intrinsic to numerous imaging modalities in biomedicine and astronomy. Given the Fourier coefficients, $(\widehat{f * g})(n), n \in[-N, N] \cap \mathbb{Z}$, we have developed and analyzed a method for recovering $\alpha$, and consequently obtained an approximation, $\tilde{g}$ of $g$. Our construction involves convolutional edge detection kernels and Gaussian curve fitting. For $f$ belonging to a certain family of functions, we show bounds of the form $\|\tilde{g}-g\|_{2} \leq C e^{-c N}$, where $C, c>0$ are constants independent of $N$. Furthermore, we provide preliminary results that extend our work to the two-dimensional setting. These findings - confirmed by numerical simulations demonstrate the feasibility of recovering accurate estimates of the blurring parameter, which is an important first step in solving the challenging inverse problem of blind deconvolution.
(Received September 10, 2023)

## 1192-94-30670

Nathan Kaplan*, University of California, Irvine, Jon-Lark Kim, Sogang University. Hulls of projective Reed-Muller codes. Projective Reed-Muller codes and constructed from the family of projective hypersurfaces of a fixed degree over a finite field. We consider the relationship between projective Reed-Muller codes and their duals. We determine when these codes are selfdual and when they are self-orthogonal. For a wide range of parameters we determine the dimension of the hull of the projective Reed-Muller code.
(Received September 10, 2023)

1192-94-30810
Anoosheh Heidarzadeh*, Santa Clara University. Private Linear Computation with Side Information.
In this talk, we present an overview of the problem of Private Linear Computation with Side Information (PLC-SI). We will discuss lower and upper bounds on the capacity of the PLC-SI problem under different privacy guarantees and explore the optimality and tightness of these bounds.

Steve Szabo*, Eastern Kentucky University. Duality Preserving Bases and Connections to Quantum Codes. Preliminary report. Given a finite ring $A$ which is a free left module over a subring $R$ of $A$, two types of $R$-bases, pseudo-self-dual bases (similar to trace orthogonal bases) and symmetric bases, are defined which in turn are used to define duality preserving maps from codes over $A$ to codes over $R$. Along with general examples, connections to quantum codes will be discussed.
(Received September 11, 2023)

## 1192-94-31062

Christine Ann Kelley, University of Nebraska - Lincoln, Meraiah Martinez*, Benedictine College, Tefjol Pllaha, University of Nebraska - Lincoln. Minimum Distance and Other Properties of Quasi-n-adic Parity Check Codes.
A quasi-dyadic (QD) parity check code is a code whose parity check matrix representation is block structured with dyadic matrices as blocks. These can be generalized to quasi- $n$-adic (QN) matrices for other values of $n$. Depending on the number of nonzero positions in the leading row of each block, these codes may be low density or moderate density. Moreover, the blocks are reproducible, giving QN codes advantages similar to quasi-cyclic (QC) codes. In this presentation, we examine results on some properties of these codes.
(Received September 11, 2023)

## 1192-94-31201

Olivia Del Guercio*, Rice University. A Family of Error-Correcting Codes. Preliminary report.
In this talk we explain the fundamentals of error correcting codes, building up to a new area of research: Repeat Accumulate Accumulate (RAA) codes. Sending binary strings, or messages, across space and time is a fraught endeavor. Our messages are unfortunately often corrupted before they are received. We can ensure that the receiver can understand our message (despite possible errors or bit flips) by adding redundancy. The set of all of these new redundant messages is called a code. Adding redundancy is often at odds with the receiver's ability to correct possible errors. The fundamental problem of coding theory asks: How does one minimize redundancy while maximizing error-correcting robustness? In the late 1990s so called turbo codes were developed as the first practical binary code that approaches the information theoretic Shannon limit on code efficiency. RAA codes are a specific family of turbo code. In this talk we introduce a possible family of RAA codes based on the algebraic structure of underlying permutation groups. (Received September 11, 2023)

## 1192-94-31266

Mckenzie West*, University of Wisconsin-Eau Claire. Parameters of Fiber Product Codes Constructed using Curves from ManyPoints.org. Preliminary report.
In an attempt to explore the possibilities of parameters produced through the construction of algebraic geometry codes through fiber products of curves, the authors constructs codes from fiber products of curves that appear in the table at lurl\{ManyPoints.org\}. The purpose of this construction is that there are two recovery sets for each coordinate in a codeword, a reasonable requirement of a cloud storage system. In an effort to further understand possible dimensions and rate, we explore more examples of these objects. For these codes, often the dimension relative to their size is beyond the useful complexity. In many cases the dimension and size parameters for the lower genus curves are slightly better than previous examples when comparing across fixed base fields.
(Received September 11, 2023)

## 1192-94-31473

Eduardo Camps Moreno*, Instituto Politécnico Nacional, Hiram H. Lopez, Virginia Tech, Eliseo Sarmiento, Instituto Politécnico Nacional, Ivan Soprunov, Cleveland State University. Affine permutations of some evaluation codes. Let $S_{i} \subseteq \mathbb{F}_{q}$ for $1 \leq i \leq m$ and $S=S_{1} \times \cdots \times S_{m}=\left\{P_{1}, \ldots, P_{n}\right\}$. Let $R=F\left[x_{1}, \ldots, x_{m}\right]$ be a polynomial ring, $I_{S}$ the vanishing ideal of $S$ and let $\mathcal{M}$ be a set of monomials. A monomial Cartesian code is a code

$$
C_{S}(\mathcal{M})=\left\langle\left(u\left(P_{1}\right), \ldots, u\left(P_{n}\right)\right): u \in \mathcal{M}\right\rangle
$$

Several well-known families of codes are monomial evaluation codes, such as Reed-Muller codes or some polar codes. We study the permutation group of monomial cartesian codes. In particular, we focus on those permutation that can be understood as affine transformations, these are linear function $f\left(x_{1}, \ldots, x_{m}\right)$ such that $f(S)=S$ and $f(\mathcal{M}) \subseteq\langle\mathcal{M}\rangle$. We see how different set of points can reduce the size of the affine permutation group and how this can be read from the linear relation graph of the defining monomial set.
(Received September 11, 2023)

1192-94-31483
Shuichi Katsumata, PQShield, Ltd, Yi-Fu Lai, University of Auckland, Jason LeGrow*, Virginia Tech, Ling Qin, University of Auckland. CSI-Otter: Isogeny-Based Blind Signatures from the Class Group Action with a Twist.
We construct the first provably-secure isogeny-based (partially) blind signature scheme. While at a high level the scheme resembles the Schnorr blind signature, our work does not directly follow from that construction, since isogenies do not offer as rich an algebraic structure. Specifically, our protocol does not fit into the linear identification protocol abstraction introduced by Hauck, Kiltz, and Loss (EUROCYRPT'19), which was used to generically construct Schnorr-like blind signatures based on modules such as classical groups and lattices. In more detail, our blind signature exploits the quadratic twist of an elliptic curve in an essential way to endow isogenies with a strictly richer structure than abstract group actions (but still more restrictive than modules). The basic scheme has public key size 128 B and signature size 8 KB under the CSIDH-512
parameter sets-these are the smallest among all provably secure post-quantum secure blind signatures.
(Received September 11, 2023)

1192-94-31506
Giuseppe Cotardo, Virginia Tech, Gretchen Matthews, Virginia Tech, Alberto Ravagnani, Eindhoven University of Technology, Julia Marie Shapiro*, Virginia Tech. Recent Results on the Multishot Capacity of an Adversarial Network. Adversarial network coding studies the transmission of data over networks affected by adversarial noise. In this realm, the noise is modeled by an omniscient adversary who is restricted to corrupting a proper subset of the network edges. In 2018, Ravagnani and Kschischang established a combinatorial framework for adversarial networks. The study was recently furthered by Beemer, Kilic and Ravagnani, with particular focus on the one-shot capacity. The one-shot capacity is a measure of the maximum number of symbols that can be transmitted in a single use of the network without errors. In this talk, we share recent results on the capacity of a network in multiple transmission rounds.
(Received September 11, 2023)

## 1192-94-31520

Welington Santos*, University of Wisconsin Stout. Unmasking Deceitful Servers: Ensuring Integrity in Secure Distributed Matrix Multiplication. Preliminary report.
In this talk, we consider the problem of Secure Distribute Matrix Multiplication (SDMM), where a user can compute the product of two matrices $A$ and $B$ with the help of $N$ servers under the constraint that any information about either $A$ or $B$ is not leaked to any server. We introduce $\mathbb{F}_{q}$ linear codes in $\left(\mathbb{F}_{q}^{a \times b}\right)^{n}$ and establish a duality theory for these codes. Furthermore, our talk unveils a versatile technique for detecting and isolating dishonest servers across a diverse range of SDMM schemes. We will illustrate the efficacy of our approach through practical examples, providing valuable insights into its applicability and impact on secure matrix multiplication protocols.
(Received September 11, 2023)

## 1192-94-31566

Ryann Cartor*, Clemson University. Total Break of a Public Key Cryptosystem Based on a Group of Permutation Polynomials Singh, Sarma, and Saikia introduced the Permutation Polynomial Encryption Scheme in 2020. We simplify the private key and prove the scheme can be completely broken by a direct algebraic attack. Furthermore, we show that the direct attack also completely breaks the $\ell$ IC cryptosystem. Although other attacks on this scheme were known, it was previously incorrectly asserted that Gröbner basis method is not feasible against $\ell$ IC. We also highlight that this attack is effective against any generalization of these schemes that contain specific properties necessary for inversion.
(Received September 11, 2023)

1192-94-31587
Kathryn Haymaker, Villanova University, Beth Malmskog*, Colorado College, Gretchen Matthews, Virginia Tech. Locally Recoverable Codes with Availability and Hierarchy from Fiber Products of Curves.
Curves (or higher dimensional varieties) over finite fields have rich geometric structure that can lead to natural constructions of codes with locality, availability, and hierarchy. This talk will discuss how locally recoverable codes with availability from fiber products of curves are also endowed with a hierarchical recovery structure, and connect these codes with more general geometric constructions.
(Received September 11, 2023)

## 1192-94-31651

Allison Beemer, University of Wisconsin-Eau Claire, Jessalyn Bolkema*, California State University, Dominguez Hills. Community Detection through Error Correction. Preliminary report.
Clustering, or community detection, refers to the task of partitioning the nodes of a network into densely connected groups and can be a powerful tool for data analysis. Continuing in a rich information-theoretic tradition, we propose a novel formulation of the graph clustering problem as an error correction problem. In this talk, we present the associated coding paradigm, discuss the parameters of the resulting code, and consider the implications for clustering performance. (Received September 11, 2023)

Christine Ann Kelley, University of Nebraska - Lincoln, Kirsten Morris*, University of Nebraska-Lincoln, Tefjol Pllaha, University of Nebraska-Lincoln. Analyzing connections between absorbing sets and iterative graph-based decoder performance for QLDPC codes.
Key to understanding the performance of quantum low density parity check codes (QLDPC) is to understand their decoding failures. Using the Tanner Graph representation of QLDPC codes we study the subsets of variable nodes that, when in error, cause a decoding failure under the Gallager B syndrome-based iterative decoder. In this talk we extend results in the classical LDPC setting on the impact of a specific class of variable nodes, known as absorbing sets, to the QLDPC context. (Received September 12, 2023)
has motivated a growing interest in understanding the correlations between emotions and cognitive processes. In particular for STEM-based programs, cognitive processes, such as critical reasoning and problem solving becomes the foundation of students' outcomes and therefore clear strategies for enhancing this type of learning are an active research subject in the educational field. In this context, the scientific literature reports that learning can be improved with an adequate emotional state, however the development of optimal protocols that supports the required emotional state is still a challenge. A general approach for developing these protocols is based on a three-step methodology that starts with an emotional stimulus generated by the teacher's activity, followed by an emotional response in the student, and a final stage in which the teacher detects and classifies the student's emotion. The implementation of each stage varies according to the emotional health policy of each University. In the case of the Universidad del Rosario the emotional health policy has been implemented through a specialized center for emotional education, in which emotional data is acquired and analyzed for the establishment of actions that contributes to academic, personal, and social well-being throughout students' life. Due to the high impact of the actions established by the emotional education center in the university, it is required a continuous improvement in the emotional health protocols. In this regard, from the School of Engineering, Science and Technology it is proposed an automatization strategy for the acquisition and processing of emotional data that aims to improve the protocols inside the emotional education center. Specifically in this study, we present the design of an emotional acquisition system based on the detection, processing, and classification of bio-signals, such as heart rate, and skin conductivity. The proposed design includes the biosensors schematics, the processing of bio-signals to obtain emotions' descriptors, and a classification of emotions based on machine learning algorithms.
(Received September 12, 2023)

1192-94-31951
Alexander Barg*, University of Maryland, Madhura Pathegama, University of Maryland. Smoothing of codes, uniform distributions, and applications.
The action of a noise operator on a code $C \in\{0,1\}^{n}$ transforms it into a distribution on the respective space. Some common examples from information theory include Bernoulli noise acting on a code in the Hamming space and Gaussian noise acting on a lattice in the Euclidean space. We aim to characterize the cases when the output distribution is close to the uniform distribution on the space, as measured by Rényi divergence of order $\alpha \in[1, \infty]$. If this divergence approaches zero as the code length increases, we way that the code family achieves asymptotically perfect smoothing. A version of this question is known as the channel resolvability problem in information theory, and it has implications for security guarantees in wiretap channels, error correction, discrepancy, and other problems. We derive expressions for the minimum rate of codes required to attain asymptotically perfect smoothing. Using these results, we prove that nested Reed-Muller codes achieve strong secrecy on the binary wiretap channel.
(Received September 12, 2023)

1192-94-32057
Cicero Carvalho, Universidade Federal De Uberlandia, Hiram H. Lopez*, Virginia Tech, Gretchen Matthews, Virginia Tech. High-rate norm-trace codes.
We define high-rate norm-trace codes, which depend on the evaluation of certain polynomials on the affine points of the normtrace curve. We see how the Gröbner basis of the vanishing ideal of the affine points and their indicator functions help us to find the dual of the code. We show that the trace function helps recover an erased entry of a codeword using partial information from the rest of the entries. This recovery property has applications for distributed storage systems. This research is joint work with Cicero Carvalho and Gretchen Matthews.
(Received September 12, 2023)

1192-94-32128
Alejandro Cohen*, Technion. Post-Quantum Security for Ultra-Reliable Low-Latency over Heterogeneous Networks. Preliminary report.
One of the important and main challenges in advanced communication systems in the post-quantum age is security and privacy. We consider the problem of post-quantum secure and ultra-reliable communication through a heterogeneous network consisting of multiple connections. Three performance metrics are considered: security, throughput, and in-order delivery delay. In this setting, previous work has looked, individually, at the trade-offs between in-order delivery delay and throughput, and between security and throughput. This is the first work considering the trade-off between all three for heterogeneous communication networks, while taking the computational complexity into account. We present LL-HUNCC, a low latency hybrid universal network coding cryptosystem. LL-HUNCC is an efficient coding scheme which allows for secure communications over a noisy untrusted heterogeneous network by encrypting only a small part of the information being sent. This scheme provides post-quantum security with high throughput and low delivery delay guarantees. (Received September 12, 2023)

1192-94-32140
Alejandro Cohen, Technion, Rafael D'Oliveira*, Clemson University, Ken R. Duffy, Northeastern University, Muriel Medard, MIT, Jongchan Woo, MIT. Cryptosystems as Error Correcting Codes. We characterize a family of computational cryptosystems that can be used as well performing error correcting codes. In particular, we show that a simple padding followed by a cryptosystem with uniform or pseudo-uniform outputs can approach the error correcting performance of random codes. We conduct an empirical comparison between our approach, utilizing AES, to Random Linear Codes and CA-Polar codes to assess their performance in practical scenarios. Our results show that the AESbased approach achieves nearly identical performance to the other well established error correcting codes. (Received September 12, 2023)

We show how information-theoretic security can be combined with any cryptosystem to obtain a hybrid universal networkcoding cryptosystem (HUNCC). To achieve this, we employ a secure network coding scheme to premix the data, followed by encrypting a small portion of the mixed data before transmitting it across an untrusted network. By encrypting only a small fraction of the mixed messages, HUNCC is able to amortize the various costs associated with encryption, including communication throughput, energy consumption, and computational time complexity, while still providing strong and provable security guarantees.
(Received September 12, 2023)

## 1192-94-32160

Christine Kelley, University of Nebraska-Lincoln, Kirsten Morris, University of Nebraska-Lincoln, Tefjol Pllaha*, University of Nebraska-Lincoln. Quantum Absorbing Sets.
The recent discovery of asymptotically good quantum LDPC codes has shifted the focus toward decoding algorithms and related issues. Iterative decoders, while low complexity, can get trapped in detrimental graphical configurations. Absorbing sets are combinatorial structures that have played a significant role classically. In this talk, we discuss the extended notion of quantum absorbing sets that capture degenerate and logical errors arising from quantum error-correction. In particular, we will discuss their role in various classes of quantum LDPC codes. (Received September 12, 2023)

## 1192-94-32386

Rowan David Hess*, Cornell University, Lionel Levine, Cornell University. How to quantify the coherence of a set of beliefs. Given conflicting probability estimates for a set of events, how can we quantify how much they conflict? How can we find a single probability distribution that best encapsulates the given estimates? One approach is to minimize a loss function that quantifies the dissimilarity between the given estimates and the candidate probability distribution. We propose, justify, and compare two specific loss functions and prove some continuity properties of general loss functions. Then, we explore a possible application to eliciting the beliefs of large language models. We characterize the facets of the polytope of coherent beliefs, and apply this to situation in which the estimates come in coherent groups and deductions are allowed to be made within those groups.
(Received September 12, 2023)

## 1192-94-32400

Spencer Kuhn, Adelphi University, Anil Venkatesh*, Adelphi University. Periodicity Detection and Consonance of Empirical Audio Samples.
The mathematical study of auditory consonance dates back at least to Helmholtz's 1863 theory of roughness arising from clashing overtones. Contemporary research has identified the metric of relative periodicity as a leading explanation of pitch processing in humans that was shown to largely match the results of clinical psychoacoustic research. Stolzenburg used combinatorial methods in 2015 to explicitly compute the relative periodicity of pure tones. In this talk, we extend that work to empirical measurements of relative periodicity of audio samples, demonstrating the effects of phase shifts and timbre on relative periodicity.
(Received September 12, 2023)

1192-94-32440
Henry Chimal-Dzul*, University of Notre Dame, Anthony Gomez-Fonseca, University of Notre Dame. Short Quasi-Cyclic LDPC Codes with Girth at Least 6.
Since the publication of Fossorier's work on the necessary and sufficient conditions for a $(k, \ell)$-regular QC-LDPC code to have a desired girth $g$, many researchers have been attracted to the problem of determining the smallest lifting degree $N_{\min }$ for which such a code exists. In this work, we translate the problem of determining $N_{\min }$ into one about the existence of mutually adjacent partial orthomorphisms of $\mathbb{Z}_{N}$ with certain deficit $d$, and determine its value for certain parameters. These particular orthomorphisms can also be used to determine, for a fixed $\ell$ and $N$, the maximum number $k_{\max }$ for which there exists a
( $k_{\max }, \ell$ )-regular QC-LDPC code with girth $g$.
(Received September 12, 2023)

## 1192-94-32455

Sarah Arpin, University of Colorado Boulder, Tyler Raven Billingsley, St. Olaf College of Northfield, MN, Daniel Rayor Hast, Boston University, Jun Bo Lau, Boston University, Ray Perlner, NIST, Angela Robinson*, NIST. A graph-theoretic approach to analyzing decoding failures of BIKE. Preliminary report.
We present experimental findings on the decoding failure rate (DFR) of BIKE, a fourth-round candidate in the NIST Post Quantum Standardization process, at the 20-bit security level using graph-theoretic approaches. We select parameters according to BIKE design principles and conduct a series of experiments using Rust to generate significantly more decoding failure instances than in prior work using SageMath. For each decoding failure, we study the internal state of the decoder at each iteration and find that for $97 \%$ of decoding failures at block size $r=587$, the decoder reaches a fixed point within 7 iterations. We then consider the corresponding Tanner graphs of each decoding failure instance to determine whether the decoding failures are due to absorbing sets. We find that $81 \%$ of decoding failures at $r=587$ were caused by absorbing sets which includes $4401(d, d)$-near codewords. (Received September 12, 2023)

1192-94-32468
Kathryn Haymaker*, Villanova University. Group testing from Reed-Solomon codes using subsets and composition.
Kautz and Singleton introduced a method for constructing binary superimposed codes from nonbinary error-correcting codes. This construction has been used to yield binary group testing matrices, which have applications to disease detection in
pandemics such as COVID-19. The disjunct value of a matrix is the maximum number $D$ such that any Boolean sum of up to $D$ columns of the matrix does not contain the support of any other (distinct) column. We prove results on the disjunct value of subsets of the construction of group testing codes from Reed-Solomon codes. We also apply the idea of Kautz and Singleton to use a composition map to construct group testing matrices, and investigate the disjunct values that result.
(Received September 12, 2023)

1192-94-32475
Narayanan Rengaswamy*, University of Arizona. Algebraic Codes for Quantum Fault-Tolerance.
Quantum error correction must carefully balance two critical tasks: efficient decoding of errors and fault-tolerant operations on encoded qubits. While codes with sparse factor graphs are well-matched to the former task, algebraic codes are often the answers for the latter task. In particular, for performing a universal set of logical gates on the encoded qubits of a quantum stabilizer code, it is necessary to fault-tolerantly realize all Clifford gates and at least one non-Clifford logical gate such as the T gate. For many codes, there are efficient methods to perform the former whereas the latter is achieved through a process called magic state distillation and injection. This process requires codes with very specific properties that guarantee faulttolerance. After introducing the key background and motivation, we will discuss these properties in terms of the mathematical conditions. We will supplement this with a few examples that show why algebraic codes are well-suited for this problem. In contrast, we will highlight how codes with sparse graphs are appropriate for the decoding task. Finally, we will discuss an important open problem in the field that connects sparse graphs with algebraic codes and argue that a positive answer will represent groundbreaking progress in the field.
(Received September 12, 2023)

1192-94-32496
Sivakanth Gopi*, Microsoft Research. Higher Order MDS Codes.
In this talk, we introduce 'higher order MDS codes', a fundamental generalization of MDS (maximum distance separable) codes. An order-m MDS code, denoted by MDS(m), has the property that any m subspaces formed from columns of its generator matrix intersect as minimally as possible. Ordinary MDS codes correspond to $m=2$. We will then show that higher order MDS codes are equivalent to various seemingly unrelated concepts in coding theory: (1) optimally list-decodable codes (2) maximally recoverable tensor codes and (3) MDS codes with support constrained generator matrices (such as in GM-MDS theorem). This along with the GM-MDS theorem for Reed-Solomon codes implies that random Reed-Solomon codes over large enough alphabet are optimally list-decodable!
(Received September 12, 2023)

1192-94-32525
Eduardo Camps, Virginia Tech, Ignacio García-Marco, Universidad de La Laguna, Hiram H. Lopez, Virginia Tech, Edgar Martinez, University of Valladolid, Irene Márquez-Corbella, Universidad de La Laguna, Eliseo Sarmiento*, Instituto Politécnico Nacional. Generalized Hamming Weights of Hyperbolic Codes
Hyperbolic codes are a class of evaluation codes that improve Reed-Muller codes by increasing the dimension while maintaining the same minimum distance. In this talk, we will offer a thorough review of hyperbolic codes, including their definition, construction, properties, and applications. We will also present new insights on their generalized Hamming weights, which are important parameters for evaluating the performance of error-correcting codes. (Received September 12, 2023)

## 1192-94-32527

Susana Jaramillo*, Whittier College, Fernando Luis Piñero, University of Puerto Rico In Ponce, Jeffrey Charles Venable, California State University, Stanislaus. Hermitian LRCs.
Lifted codes are an excellent way to construct Locally recoverable codes from lines. Lifted Reed-Solomon codes use the lines of the affine space to construct the recovery sets. Likewise one can use the lines of the Hermitian curve to make an LRC. The goal of our research is to create Locally Recoverable Codes (LRCs) from fiber products of Hermitian curves and their intersecting lines. Most LRCs from fiber products are made by restricting the degree of each variable. Our construction emulates the LRC construction obtained by restricting to lines on the affine space. Thereby emulating the construction of Lifted Reed-Solomon codes. From all possible lines, we chose specifically lines that intercept the Hermitian fiber product on $q$ +1 points. Through the use of Linear Algebra techniques and incidence techniques we are able to estimate the dimension of the Resulting LRC. We then test the efficiency of the resulting LRCs by passing them through an iterative bit decoder. Our hope is to find other techniques to make LRCs from fiber products of curves.
(Received September 12, 2023)

1192-94-32747
Christine Kelley, University of Nebraska-Lincoln, Daniel Welchons*, University of Nebraska--Lincoln. The Hunt for Capacity Achieving Codes using Automorphism Groups. Preliminary report.
In 2016, it was shown that Reed-Muller codes achieve capacity on the binary erasure channel and more generally any family of codes with doubly transitive automorphism groups achieves capacity on the $q$-ary erasure channel. In the same year, the result was extended to any family of codes with transitive automorphism groups and increasing minimum orbits under the action of the stabilizers. This result opened the possibility identifying families with large automorphism groups that would be capacity achieving on the $q$-ary erasure channel. Promising candidates include codes with known large automorphism groups like cyclic and quasi-cyclic codes, and constructions which extend a family of desirable base codes, such as product and half-product codes. This talk will examine recent developments in the hunt for capacity achieving codes.
(Received September 12, 2023)

Computation with Differential Privacy. Preliminary report.
Group testing has recently regained researchers' attention due to its practical applications in disease prevention and control. The goal of a group testing algorithm is to identify a group of infected individuals by performing a minimum number of pooled tests. In this talk, we will focus on settings where differential privacy is required to prevent the disclosure of an individual's infection status. In such situations, it is not possible to identify the exact group that is infected. Our objective is to provide the best possible approximation of the group while ensuring privacy. We provide both achievable schemes and converse bounds for system performance.
(Received September 12, 2023)

1192-94-32886
Pedro Soto*, Virginia Tech. Coded Distributed Batch Matrix Multiplication via an Additive Combinatorics Lens. Preliminary report.
Fault tolerance is a major concern in distributed computational settings. In the classic master-worker setting, a server (the master) needs to perform some heavy computation which it may distribute to $n$ other machines (workers) in order to speed up the time complexity. In this setting, it is crucial that the computation is made robust to failed workers, in order for the master to be able to retrieve the result of the joint computation despite failures. A prime complexity measure is thus the recovery threshold, which is the number of workers that the master needs to wait for in order to derive the output. In this talk, I address the fundamental and well-studied task of matrix multiplication; i.e., when the master needs to multiply a batch of $n$ pairs of matrices. Several coding techniques have been proven successful in reducing the recovery threshold for this task. One approach that is also very efficient in computation time is called Rook Codes. I also present a recent lower bound proof that says that any Rook Code for batch matrix multiplication must have a recovery threshold of at least $\omega(n)$ which was recently discovered by my collaborators and myself. Notably, we employ techniques from Additive Combinatorics in order to prove this, which may be of further interest. Moreover, we show a Rook Code that achieves a recovery threshold of $n^{1+o(1)}$, establishing a near-optimal answer to the fault tolerance of this coding scheme.
(Received September 12, 2023)

## 1192-94-33024

Kathryn Haymaker, Villanova University, Emily McMillon*, Virginia Tech. Parity Check Codes from Disjunct Matrices. Preliminary report.
Disjunct matrices are used extensively in group testing but have not been well-studied as parity check matrices for binary linear codes. In this talk, we consider building parity check codes from disjunct matrices. We give results on code parameters including rate, distance, and girth for some specific classes of disjunct matrices.
(Received September 13, 2023)

1192-94-33072
Veronika Kuchta*, Florida Atlantic University. Demystifying post-quantum ZK-SNARKs.
Zero-Knowledge Succinct Non-interactive Arguments of Knowledge (ZK-SNARKs) are cryptographic protocols between a prover and a verifier which allow a prover to prove knowledge of a statement to a verifier without revealing any secret information of the statement. Post-quantum constructions of these protocols are relevant for many applications such as blockchain, e-voting, obfuscation whenever quantum computers will be widely available. In this talk I'll focus on lattice-based ZK-SNARKs and summarize the main techniques used in it, such as linear probabilistically checkable proofs and homomorphic encryption schemes.
(Received September 13, 2023)

## 1192-94-33251

Alex Pellegrini*, Eindhoven University of Technology. Analysis of REDOG and Layered-ROLLO-I.
Cryptosystems based on rank metric codes have been seen as a promising alternative to the more traditional cryptosystem based on Hamming metric codes in the post-quantum cryptography scenario. Research in this area led to the proposal of rank metric code-based candidates to post-quantum competitions. In this talk I will present the cryptanalysis of REDOG and
Layered-ROLLO-I, two rank metric code-based cryptosystems submitted to the Korean post-quantum Cryptography Competition. REDOG combines Gabidulin codes with different techniques aiming at reducing decryption failures to 0 . I will show that REDOG's decryption fails with high probability, give an efficient attack on its implementation, using tools from the Hamming metric, and finally provide possible fixes to the found problems. Layered-ROLLO-I is a modification of the ROLLO-I cryptosystem that aims at providing improved performances w.r.t. the latter. I will describe an efficient direct transformation of Layered-ROLLO-I into ROLLO-I, showing that the two cryptosystems are essentially identical.
(Received September 13, 2023)

## 97 Mathematics education

## 1192-97-25374

Sylvester James Gates, Jr*, Clark Leadership Chair in Science, University of Maryland; past president of American Physical Society, National Medal of Science. What Challenges Does Data Science Present to Mathematics Education?. Preliminary report.
The importance of data science is increasing so rapidly that mathematically based education is facing a challenge to effectively engage this issue. The presentation will discuss these community challenges.
(Received May 31, 2023)

Human Endeavor. Preliminary report.
Culturally Relevant Mathematics Pedagogy encompasses a teaching approach that actively involves and empowers students, fostering connections between the subject matter and their personal lives, communities, and the broader world. This presentation offers a vision of how this teaching approach has paved the way for more equitable teaching and learning experiences in K-16 mathematics classrooms. The speaker will emphasize selected instructional strategies from her book, 'Engaging in Culturally Relevant Math Tasks,' illustrating how educators can deliver meaningful mathematical instruction from an asset-based perspective. Participants will have the opportunity to partake in discussions exploring how these strategies relate to their own experiences and to identify how to enhance their current teaching practice.
(Received July 13, 2023)

1192-97-26605
Mary Beisiegel, Oregon State University, Mary E Pilgrim*, San Diego State University, Rebecca A Segal, Virginia Commonwealth University. Mathematics Graduate Teaching Assistant Preparation for Teaching through the ELITE PD Program.
Mathematics graduate teaching assistants (MGTAs) make up part of both the current and future teaching force at the undergraduate level; however they are often not adequately prepared to teach in engaging, inclusive, and equitable ways. Without robust professional development (PD) for teaching that focuses on active, engaged teaching practices, MGTAs often replicate the lecture-based teaching practices that they have experienced and observed as students. Our project aims to implement and study a PD program for MGTAs focused on engaged learning, inclusive teaching, and equity (ELITE PD). Spanning multiple semesters, the ELITE PD program will be implemented with cohorts of MGTAs across three institutions. In this presentation, we will share the ELITE PD program model, research questions the project aims to answer, and contextual adaptations.
(Received August 01, 2023)

1192-97-26649
Zenaida Aguirre Munoz*, University of California, Merced, Melissa Almeida, University of California, Merced, Sarah Frey, University of California, Merced, Changho Kim, University of California, Merced, Yue Lei, University of California, Merced, Lolita Oka, California State University, Fresno, Erica Marie Rutter, University of California, Merced, Bianca Salazar, University of California, Merced, Comlan Souza, California State University, Keith C. Thompson, University of California, Merced, Mayya Tokman, University of California, Merced, Khang Tran, California State University, Fresno, Maribel Viveros, University of California, Merced. The Role of Active Engagement and Mathematics-Related Factors in Calculus Performance.
Performance in Calculus courses is seen as a major barrier to students' progress in STEM majors. The problem is particularly amplified for disadvantaged groups such as underrepresented minoritized students (UMS) and first-generation college students. Extant literature and guidance increasingly point to the need to address the psychosocial factors that can bolster resilience, persistence, and positive self-concepts in science, technology, engineering, and mathematics (STEM) fields. Improving the Calculus experience requires understanding the complex networks of relationships among individual and classroom variables that impact UMS performance in first year calculus courses. Much of the extant research in this area involves universities with small proportions of UMSs. Thus, continued study of the impact of these factors on more diverse student populations is also necessary to better capture the calculus experience of UMS majoring in STEM. The purpose of the study was to examine student and classroom-level factors that influence course performance measured by course grade. This study focused on two STEM-related psychosocial factors : (1) STEM self-efficacy and (2) STEM sense of belonging, and three mathematics-specific psychological factors which we refer to as math motivators, (1) math interest, (2) self-concept, and (3) anxiety. Classroom level factors included active engagement practices, proportion of females, proportion of UMS students and proportion of first-generation students in classes. Hierarchical Linear Model analyses showed individual and class-level factors impact course grades in complex ways. These data were taken from an ongoing study examining the impact of a calculus redesign project on student performance. Both student and classroom level factors were found to predict course performance. Implications will be discussed.
(Received August 02, 2023)

## 1192-97-26772

Yvonne Lai*, University of Nebraska-Lincoln. (Why) To Build Bridges in Mathematics Education.
It can be easy to silo ourselves with those that think "like us". This is the opposite of what we want to do if we want to be better teachers. In this talk, I will use the case of designing courses for prospective high school mathematics teacher to argue for the value of building bridges in mathematics education. Along the way, I will discuss recent work that examines the cost of uncivil discourse in mathematics education. I will conclude with examples and suggestions for bridge building in mathematics teaching, departmental structuring, and writing policy for the mathematical sciences community. (Received August 04, 2023)

Frederick M Butler*, York College of Pennsylvania. Standards-Based Learning in the Calculus Sequence. Preliminary report. In standards-based learning, students receive multiple attempts to fulfill each of the objectives of a course. Student grades are largely based on the level at which each objective is met. This paper details the outcomes of standards-based learning in multiple sections of Calculus II and Differential Equations courses. Practical considerations for implementation of standardsbased learning in both courses are given. Theses include specific course objectives, management of repeated student attempts, scoring rubrics, and grade breakdowns. Preliminary data on student learning will also be shared.
(Received August 07, 2023)

Mohammad K. Azarian*, University of Evansville. Combinatorics of Discrete Functions.
We discuss combinatorics of discrete relations, (total) functions, as well as partial functions. For two sets A and B, we present formulas for the calculations of the number of relations from $A$ to $B$ that are not: (i) functions, (ii) one-to-one functions, or (iii)
onto functions. Next, we provide formulas to calculate the number of functions from A to B that are not: (i) one-to-one or (ii) onto. Also, we determine the number of partial and proper partial functions from A to B. Moreover, we state conjectures and pose questions.
(Received August 12, 2023)

Pavneet Kaur Bharaj, California State University, Bakersfield, Amber Simpson*, Binghamton University. Parent-Child Pairs' Humanistic Approaches to Mathematics through Engagement in Engineering Design Tasks in their Home Environment. Engaging in engineering design processes may serve as an everyday experience to support children's understanding and application of math concepts; yet, little is known how this may unfold between children and parents in out-of-school contexts. This study utilized observational methods to observe how children and parents engaged in authentic and humanistic approaches to mathematics through engineering design tasks in their home environments. Through the analysis of approximately 15 hours of video collected from nine families engaged in three different engineering design tasks (e.g., create a shelter for stray animals), we observed parents and children engaged in four mathematical concepts - geometric reasoning, informal measurement, fractional understanding, and proportional and covariational reasoning. In this special session, the focus will be on the authentic and humanistic ways parent-child pairs utilized geometric reasoning and informal measurement through the discipline of engineering. As an example, in creating a trendy tennis shoe for Olaf, we observed a parent-child pair using an "inch finger" as a way to measure the length and width of the sole of the shoe. The significance of this study lies in the seeds of mathematical thinking being developed in an out-of-school context, as well as the new opportunities for engaging parents and children in authentic and humanistic approaches to mathematics in their home environment - engineering design tasks. This study further elevates parents as unique educators that can support formal schooling opportunities. One question of interest for the special session is whether or not it is important for parent-child pairs to recognize the different ways they engage in mathematics in their home environment or is building a foundational understanding of mathematics as "hidden" within the engineering design process - mathematics that may not resemble mathematical work in school settings - enough. (Received August 13, 2023)

## 1192-97-27509

Javier Gonzalez Anaya*, UC Riverside. Exploring combinatorial aspects of max-pooling layers with undergraduates. Convolutional neural networks (CNN's) are prominent in audio, image, and text processing because of their ability to identify characteristic features of complex sets of data with a relatively low computational cost. Part of this success is due to the robustness provided by the pooling layers within their architectures. In this talk, I report on my experience co-mentoring three undergraduate students on a project about the combinatorial complexity of max-pooling layers (MPL's). MPL's can be thought of as piecewise linear functions. The students tackled this problem by using a novel technique that relates the number of linear regions of a MPL to the problem of counting vertices in some concrete polytopes. This experience resulted in a poster, which the students have presented in several regional conferences. Two of these students will be attending different masters programs this Fall. (Received August 15, 2023)

## 1192-97-28056

Alvaro Alfredo Ortiz Lugo*, University of Cincinnati. Harnessing the Power of Artificial Intelligence in Education: Tools, Challenges, and Innovations in Course and Instructional Design.
Integrating Artificial Intelligence (AI) into educational settings has revolutionized how educators design and deliver courses. This talk explores the multifaceted landscape of utilizing AI in the classroom, focusing on the tools available and the inherent challenges faced in course and instructional design. As the educational paradigm evolves, AI-powered tools have become invaluable aids for educators. These tools encompass a broad spectrum of applications, including intelligent tutoring systems, adaptive learning platforms, and automated grading systems, which have the potential to enhance the learning experience for students and streamline the teaching process for instructors. However, this transformation has its share of challenges. This presentation delves into the complexities of integrating AI into the classroom, addressing issues such as data privacy, algorithmic biases, and the ethical considerations surrounding AI-assisted education. It also explores the need for effective professional development for educators to harness the full potential of AI tools in their instructional design. Moreover, the presentation sheds light on innovative course and instructional design approaches that leverage AI's capabilities. Examining real-world examples and best practices will highlight successful strategies that empower educators to create engaging and personalized learning experiences that cater to diverse student needs.
(Received August 24, 2023)

## 1192-97-28060

Maritza M. Branker*, Niagara University. Surprising benefits to playing the card game SET in a foundations of mathematics classroom.
Many mathematicians develop a fondness for the card game SET in their graduate school days however it's a surprisingly useful classroom exercise in an undergraduate foundations of mathematics course. It's become more valuable post COVID since playing the game in small groups serves as an ice breaker and enables informal peer to peer interactions. I have used it successfully in my class multiple times (pre-pandemic) to ascertain the suitability of pairings for group work and to foster a general spirit of playfulness and collegiality amongst my students.
(Received August 25, 2023)

1192-97-28238
Steven Craig Clontz, University of South Alabama, Drew Lewis*, Unaffiliated. Team-Based Inquiry Learning in Lower Division Mathematics Courses. Preliminary report.
Team-Based Inquiry Learning (TBIL) is a novel form of active learning developed with a goal of facilitating the adoption of inquiry-based learning techniques in lower division mathematics courses such as Calculus and Linear Algebra. In this talk, we report on the progress of the TBIL project in developing curricular resources, training faculty, and documenting the impact of TBIL on students across a variety of instructional contexts.

Brian P Katz*, California State University, Long Beach. Establishing a Small Axiomatic Foundation in Class. Preliminary report.
For a classroom community to collaborate and communicate about an axiomatic system, they must build a shared understanding of what counts as an acceptable justification, including which unproven statements can be used as axioms. This can be challenging in undergraduate mathematics classrooms because (i) students are often just starting to grapple with the idea of axiomatic methods and (ii) it is unclear what from their diverse prior learning experiences should be taken-as-shared and what will need to be justified within and accepted by the community. In this talk, I will report preliminarily on ways that my students and I negotiate a small, shared set of axioms, mostly in the context of a Geometry course.
(Received September 02, 2023)

1192-97-28552
Jennifer A Czocher*, Texas State University. Scaffolding Undergraduate STEM Majors' Learning of Mathematical Modeling. Mathematical modeling is a key skill for undergraduate STEM majors. In my view, learning to do learning to do mathematical modeling means that students know (i.e., can justify) that their models align with real world conditions and with their mathematical schema. Successfully guiding students to know mathematics in this way draws on awareness of the typical ways students think about mathematical content in real-world contexts and the kinds of explanations that are sensible to students. Generating this knowledge of students' reasoning is an essential step towards equipping instructors with the pedagogical skills needed to contingenly interpret and respond to student reasoning patterns that arise during modeling. The CAREER project Scaffolding Strategies for Undergraduate Mathematics Modeling Skills (SUMMS) studies how STEM majors learn to use mathematics as a representational system for solving real world problems and synthesizes their ways of reasoning into actionable scaffolding strategies. And then it tests those strategies. Ultimately, the project's goal is to draw conclusions about what scaffolding (support) moves a teacher might make that will progress the student's reasoning, without "giving it all away" - a problem that transcends content area in mathematics education. Using qualitative, quantitative, and mixed methods, the project elaborates on the interactions among features of modeling problems, students' mathematical reasoning, and teachers' attempts to draw out or change student reasoning patterns. In this talk, I'll share an overview of the activities the SUMMS team has carried out to get us closer to this goal and some of the great successes the doctoral research assistants on the project have enjoyed. I'll also point towards avenues that have opened for future inquiry.
(Received August 31, 2023)

1192-97-28557
Jennifer A Czocher, Texas State University, Eric Deeds, UCLA, Alan Garfinkel*, UCLA, Brendan A Kelly, Harvard.
Dynamical Systems Modeling for Biologists \& Doctors - without Prerequisites!.
Nearly 20 years after Steen (2005) observed that biology is the "dominant scientific partner for mathematics" and argued that "science has moved on" from the 19th century analytic approaches that supported physics, biology majors still suffer through outdated and out-of-touch mathematics requirements. Beyond providing students the wrong content, leaving them ill-prepared for science research, traditional math pathways too often act as a filter in the STEM pipeline - and they're not even filtering in the modeling and computational skills that biologists need. In this presentation, we will share an overview of UCLA's highly successful Life Sciences 30 course which teaches the art of modeling positive and negative feedback loops in ecology, physiology, and molecular biology, all without a calculus prerequisite. Courses like this have been called for by the National Academy of Sciences, the American Association of Medical Colleges and the NSF, with very little positive response. In our course, students encounter sophisticated 21st century perspectives on derivatives, state spaces, vector fields, and the simulation methods that are necessary to approach feedback models. The course succeeds on multiple measures: it changed student attitudes towards math, narrowed the gender gap, and reduced the achievement gap for minoritized students. (Received August 31, 2023)

1192-97-28565
Jennifer A Czocher*, Texas State University, Eric Deeds, UCLA, Alan Garfinkel, UCLA, Brendan A Kelly, Harvard. A Master Class in Modeling the Life Sciences, for Mathematics Faculty.
In Summer 2023, we ran a professional development workshop at Harvard University for 24 faculty to support them in adopting a modeling-first approach to teaching mathematics to biology and pre-med majors. The 35-hour workshop ran concurrently with an adaptation of UCLA's Life Sciences 30 course for secondary students attending Harvard Summer School. The workshop addressed three major threads: content, pedagogy, and leadership. Attendees learned scientific and mathematical skills for understanding the modern use of dynamical systems as models in biology. Since systems featuring feedback loops (e.g., in ecology, hormone physiology, and molecular biology) are typically nonlinear, faculty learned computational approaches and geometric methods in nonlinear dynamics. We drew on educational research to host sessions dedicated to developing faculty's modeling-specific pedagogical knowledge, knowledge of equitable teaching practices and knowledge of how students reason during modeling problems (as opposed to pure math problems). Finally, some sessions focused on developing leadership skills for effectively sustaining change at attendees' home institutions. We observed a statistically significant gain in faculty's knowledge of the relevant biology content, knowledge of the relevant mathematical content from a modeling perspective, their confidence for teaching the content from a modeling perspective, and their level of comfort with student-centered teaching practices. Our mission is to support faculty who are enthusiastic about a modelingbased approach to introductory mathematics courses in adopting and implementing this (or a similar) curriculum at their home institutions. Our first-run results are encouraging and we are eager to welcome new faculty to the workshop in Summer 2024. (Received August 31, 2023)

## 1192-97-28630

Nicholas E Long*, Stephen F. Austin State University. The CalcVR Project.
The CalcVR app was developed at SFASU in 2017 as a cost-effective virtual reality resource for multivariable calculus. As part of the NSF grant DUE-1820724, the CalcVR team developed both in-app and supplementary materials. We will discuss the
three different student interview structures we have used as assessment (including results of each interview type), the other types of formative assessment we have used in the past 5 years, and the current state of the VR and supplemental materials as a result of this feedback. The CalcVR app is available at no cost on the Google Play Store, the iOS App Store, and the Meta Quest Labs Store and the CalcVR learning materials are available on the project website, calcvr.org.
(Received September 01, 2023)

## 1192-97-28632

Silviana Amethyst*, University of Wisconsin - Eau Claire, Warren Shull, University of Wisconsin-Eau Claire. Purpose-driven Calculus content in Webwork. Preliminary report.
Teaching Calculus is hard: classes have students with many interests and career paths; for many students the class is terminal, while others will go much further; and for some, the class is non-consensual. In this talk, we want to share practice and results from a Summer 2023 project to create purpose-driven Calculus content in the free system Webwork, aiming to move from "here's a problem" to "this problem teaches something specific". In particular, we created distinct problem sets: training, speedruns, and challenges. Training sets aim to teach something specific with each problem, and each comes with a statement of purpose and direct instruction; many training problems incorporate interactive Desmos graphs, and we make use of scaffolded problems to guide through stages of understanding. These lead to speedruns, focusing on core computational skills necessary for later coursework, and then challenge sets which synthesize on trainings and speedruns. We used Python heavily to augment the Webwork system, and the project is ever a work-in-progress. Our ultimate goal is to free classtime for impactful activities focusing on concepts and principles, especially in the face of rapidly evolving AI systems which trivialize the computational aspects of Calculus. We wish to create time to spend a few class periods at the river flowing through our campus, as it so richly embodies change - the stuff of Calculus. (Received September 01, 2023)

## 1192-97-28854

Perla Myers*, University of San Diego. Teaching Visuospatial Skills through Paper Play.
Research shows that visuospatial skills are an important predictor of long-term achievement and attainment in Science, Technology, Engineering, and Mathematics (STEM). However, there is a lack of ecologically valid learning design that can be easily integrated into existing STEM curricula and teaching practices. Grounded in the social constructivist framework, collaborative inquiry-based learning, and multiple modalities of learning, VisMO (Visuospatial Skills, Mathematics, Origami) Lessons leverage the visuospatial tools in paper folding and designed protocols and resources to help engage students in visuospatial reasoning and use of spatial language. This session will introduce the VisMO curriculum and share findings from our research based on this curriculum.
(Received September 04, 2023)

## 1192-97-28926

Henry Segerman*, Oklahoma State University. Learning by doing with 3D printing.
I'll describe a course I have been teaching since 2016. It is a capstone, project-based undergraduate course in which students learn how to use how to use CAD software and 3D printers. Student projects delve into advanced topics, requiring them to investigate a mathematical form, understand the geometry, and model it.
(Received September 05, 2023)

## 1192-97-28955

Sean S Warner*, Clark Atlanta University. Leveraging Community Assets to Increase the African American and Latinx STEM Teacher Workforce.
African American and Latinx professionals are severely underrepresented in the STEM disciplines. Several researchers have purported that the scarcity of these racial groups is tied to an amalgamation of sociocultural variables in these communities that have made it challenging to sufficiently recruit black and brown high school and college students to the STEM teaching field. Moreover, the research suggests that retention is an even greater hurdle to surmount. Challenges with African American and Latinx STEM teacher recruitment and retention has been widely documented and the reasons why cut across several areas, however much of evidence points to their early teaching and learning experiences and the kind of impact it had on their career aspirations and self-efficacy. This presentation describes how one project (NSF \#215104), currently underway, seeks to mitigate the aforementioned dilemma by attracting, preparing, supporting and retaining 19 black and brown math teachers for two (2) high need school districts, by leveraging the community assets. Specifically, the project is the culmination of a partnership cultivated over several years between the two (2) school districts and Clark Atlanta University to co-create an accelerated math teacher preparation program that embeds, mathematical literacy coaching, mentorship, culturally responsive teaching and wellness training to sufficiently equip the novice teachers increasing their chances of success and persistence in the profession. Outside of what is already known about recruiting and retaining BIPOC math teachers, there are no findings to draw on at this time as the assessment of project goals and targets won't begin until 2025.
(Received September 07, 2023)

## 1192-97-28999

Philip B Yasskin*, Texas A\&M University. Spatial Visualization in MY Math Apps Calculus.
Many students have trouble with spatial reasoning. MY Math Apps Calculus is an online text for a 3 semester calculus course for STEM students which emphasizes 3D visualization. You can see a sample of about half the chapters at https://mymathapps.com/mymacalc-sample/. I will show the structure of the text with emphasis on the interactivity and graphics, both 2D and 3D, static and animated, visual and manipulatable. Some of the 3D plots can be manipulated with sliders or rotated with a mouse. Course surveys show the students actually read the book, like the graphics, find the interactivity engaging and like having solutions to many exercises. Nearly all theorems have proofs which are either in the text or accessible for interested students by clicking a button. The order of material is slightly modified to enhance the learning process.
(Received September 05, 2023)

## 1192-97-29015

George Marufu Marufu Chirume*, Leva Foundation, South Africa. Using Offline Tangible Coding Games as Tools for a Mathematics Class. Preliminary report.
Offline low-cost tangible coding games, for example: RANGERS problem-solving game [1] focuses on the key 4IR Skills, namely: Computational Thinking, Critical Thinking, Collaboration and Problem solving. Offline coding games[2] and Unplugged math and coding activities[3] are inspired by computer science but can be easily implemented without the use of a computer and used to support the learning and teaching of mathematics concepts to learners who come from any learning environment, like the African underserved communities such as the rural areas, farms and informal townships. During the presentation, participants will be introduced to the fundamentals of coding concepts and how we can apply the concepts and results to improve learners' problem solving skills through a series of engaging interactive activities which have a direct application to the teaching and learning of school mathematics. Following this, they will interact with the Rangers problemsolving offline coding game. Afterwards, a time of reflection will help to identify the connections between coding activities and mathematics concepts.
(Received September 05, 2023)

## 1192-97-29283

David Weisbart*, University of California Riverside. Principles of Reasoning for structuring a Calculus Curriculum. We will discuss what it means for a course to be principle-based and describe an underlying philosophy of teaching and learning that supports principle-based instruction. Using concrete examples, we will discuss some advantages that a principlebased calculus course offers, as well as some challenges in constructing and implementing such a course. Principle-based instruction improves access by reducing artificial barriers to participation while simultaneously increasing the learning standards.
(Received September 06, 2023)

## 1192-97-29334

Gregory D Foley*, Ohio University. Creating a Statewide Network to Support High-Quality Teaching in Gateway Quantitative Reasoning. Preliminary report.
With NSF support, from 2022 through 2025, a team of researchers and practitioners is developing a statewide network to support instructors who teach a freshman-level course in Quantitative Reasoning public college and universities in Ohio. This talk will address the premises, plans, progress, and prospects of this project while answering several questions: What is quantitative reasoning? Why is it important? How can it be developed within undergraduates in nonscientific majors? What does high-quality teaching of Quantitative Reasoning entail? Who teaches Quantitative Reasoning at colleges and universities across Ohio? What type of professional development is needed to support this QR teaching workforce? What are the implications of this project for higher education across the United States? How can this network be "replimented" in other states?
(Received September 06, 2023)

## 1192-97-29337

Ornella Mattei*, San Francisco State University. Alternative forms of assessments in Math: video exams, class reports, and peer reviewing.
This talk is intended to showcase some alternative forms of assessments of students' understanding, and the related pros and cons. It is not meant to be a Math Education talk. I will share some successful techniques I have been implementing in my courses with the goals of (i) actively engaging my students, (ii) favoring retention of the course material, and (iii) helping my students learn the language of Mathematics by improving their communication skills. All the classes I teach are flipped classes in which students watch short videos before coming to class, and then they work on group activities on the board during class. Much time is spent to ensure that the language the students use and the notation they adopt is rigorous. Indeed, being able to communicate complicated Math concepts in a clear and professional way is of paramount importance, especially in sight of the future career paths for Math majors. With this in mind, in Fall 2020, together with Dr. Kim Seashore (San Francisco State University), I introduced activity reports in my calculus class. Students had to submit a report of the in-class group activities with the goal of favoring retention and improving students' communicating skills. Students used reports as a tool to study the material and this lead to incredibly detailed and professional reports. Given this initial success, I implemented reports in other Math courses I teach. This time, to facilitate social and cognitive presence, as well as to empower my students, the activity reports were graded by the students themselves through a peer review process. Again, both the reports and the peer reviews were detailed and professional. In the attempt to export the benefits of class reports to other assignments, I implemented video exams in all my courses. For each exam, students have to submit videos, no longer than 12 minutes, in which they explain in their own words how they solved the problem and the reasoning behind any technique and formula they used. The video format leaves space for creativity, and the time limit forces students to record themselves multiple times before being able not to exceed it, thus gaining the benefits of repetition. In the talk I will be sharing samples of each alternative form of assignment.
(Received September 06, 2023)

## 1192-97-29340

Hortensia Soto*, Colorado State University/MAA President. Mathematics, Education, \& Cognition.
Embodied cognition is a philosophy that hypothesizes that learning is body-based. In other words, learning is not only a brain activity - the brain and body work in tandem. In this presentation, I will discuss my research related to embodied cognition in the teaching and learning of undergraduate mathematics. I will provide examples from classroom and research settings specifically from complex analysis, abstract algebra, and linear algebra. The audience will have an opportunity to become cognizant of how they already integrate embodiment in their teaching subconsciously and learn how to begin to integrate embodiment intentionally in their teaching. Bring your fun-meters.
(Received September 06, 2023)

Christina Edholm*, Scripps College, Maryann Hohn, IDA/CCS, Ami Radunskaya, Pomona College. Teaching Just-In-Time Modeling with Differential Equations.
Mathematical modeling took a front line during the COVID epidemic, and this provided fertile ground for teaching the impact of mathematical models on decision-making. In May 2020, administrators of residential colleges struggled with the decision of whether or not to open their campuses in the Fall semester of 2020. To help guide this decision, we formulated an ODE model capturing the dynamics of the spread of COVID-19 on a residential campus. The model accounts for the different behaviors, susceptibility, and risks in the various sub-populations that make up the campus community. In particular, we start with a traditional SEIR model and add compartments representing relevant variables, such as quarantine compartments and a hospitalized compartment. We then duplicated the model for ten interacting sub-populations, resulting in a large system of differential equations. The model predicts possible outcomes based on hypothetical administrative policies such as masking, social distancing, and quarantining. With a basic model in hand, we involved students in updating the model as the pandemic developed to account for new policies, such as testing and vaccination. Students also calibrated the model to data gathered from local sources, exposing them to the full modeling process. The findings of our model were then extrapolated for a general audience, which, along with plots and an interactive app, makes model conclusions accessible to all, democratizing the policymaking process. We will share some reflections on how modeling efforts were used to engage students.
(Received September 07, 2023)

## 1192-97-29511

Yanping Ma*, Loyola Marymount University, Los Angeles CA USA. The modeling first approach in an ODE classroom: utilizing fictional dragons as a practical modeling illustration. Preliminary report.
This work introduces an innovative example of teaching ordinary differential equations (ODEs) by harnessing the captivating world of fictional dragons. Drawing inspiration from popular culture, specifically Game of Thrones and A Song of Ice and Fire we present a structured methodology for employing ODEs to model the growth, ecology, and energy dynamics of mythical dragons. We will demonstrate how to utilize these mythical creatures as illustrative examples within ODE instruction, facilitating students' comprehension of ODEs. Our aim is to provide educators with a novel teaching tool that sparks student engagement and deepens their understanding of ODEs while enhancing critical thinking and problem-solving skills. We hope this demonstration inspires educators to explore new modeling topics in ODE classes, fostering students' interest and encouraging their active participation.
(Received September 07, 2023)

## 1192-97-29563

Amrit Bahadur Thapa*, Ohio University. Why do Undergraduates Dislike Mathematics?. Preliminary report. This presentation unveils preliminary findings from my PhD dissertation, "Perspectives of University Undergraduates Enrolled in Quantitative Reasoning on Their Dislike of Mathematics." The study investigates college students' perspectives on their dislike of mathematics, scrutinizing its origins and contributing factors. Additionally, it assesses the role of a college quantitative reasoning course in mitigating this dislike. Qualitative research methods are being employed to delve into students' subjective experiences in mathematics education, spanning from early years to college. Semi-structured, in-depth interviews were conducted with 15 students enrolled in a quantitative reasoning course at an Ohio-based university, with interviews that took place at the beginning and end of the Spring 2023. Initial findings indicate that students' dislike of mathematics often emerges as early as the third grade. Key contributing factors include unengaging learning materials, insufficient teacher encouragement, perceived irrelevance to real-world contexts, learning disabilities, and an emphasis on standardized testing. Furthermore, this presentation offers preliminary insights into the ways the quantitative reasoning course may address the dislike of mathematics. Notably, the inclusion of real-world contexts in teaching appears promising. However, it's essential to note that none of the students reported a complete transformation of their dislike of mathematics through this course. Finally, considering that a substantial number of students and individuals openly express their dislike of mathematics, this presentation aims to assist educators in comprehending the origins and contributing factors behind this dislike. While this research remains ongoing, this presentation seeks to inspire educators with the potential valuable insights it can offer.
(Received September 07, 2023)

## 1192-97-29646

Ryan Joseph Rogers*, University of Kentucky. Real Analysis and Undergraduate Students' Understanding of Function Continuity.
Undergraduate students' understanding of function continuity has not been explored broadly in previous research. The relevant findings in the literature are predominantly concerned with calculus students' understanding and misconceptions of continuity. Many of these misunderstandings are tied to the relationships which continuity has with limits and differentiability. We have conducted a multiple-case study to explore how, if at all, an introductory real analysis course impacts undergraduate students' understanding of function continuity and its connections to the notions of limits and differentiability. We embed our findings within the theoretical framework of Tall's three worlds of mathematics, namely, the embodied, symbolic, and formal worlds. The cases in this study are seven undergraduate students who enrolled in an introductory real analysis course during the Fall 2022 semester. Each participant took a pre-test to assess their comprehension of function continuity and its relations with limits and differentiability before the topic was covered in the course, as well as a post-test that measured their understanding of the subject after learning about it in real analysis. Finally, six of the seven cases engaged in an individual, audio-recorded oral interview with questions designed to triangulate, enrich, and clarify the data collected from the written instruments. The pre- and post-tests were scored numerically, with each subject receiving pre- and post-embodied, symbolic, and formal scores, while the qualitative data collected from the interviews were coded with respect to the three worlds of mathematics and analyzed via constant comparison. In all, we found that the students' perspectives on function continuity generally shifted from an embodied viewpoint to a more symbolic and formal outlook, though several of the misconceptions held by calculus students, per the existing literature, were possessed by the real analysis students as well, even after the material was covered in class. Of high interest was the strength of the cases' concept images regarding continuity and the ways in which inadequate concept images permeated their understanding of the topic throughout the study.
(Received September 07, 2023)

Qingxia Li*, Fisk University. African American Students' Perceptions Towards STEM in a Flipped Classroom Setting. Preliminary report.
Black students persist in STEM field at a lower rate than Asian and White students, and this same trend is seen with entering research careers in biology. One way to possibly increase engagement and retention in STEM research is through authentic research experiences linked to statistics courses. Additionally, flipped classroom approaches can provide in-class time where authentic analyses and interpretation of data can be directly guided by an instructor. The purpose of this session is to provide a preliminary overview of student performance and perceptions of a flipped classroom biostatistics course at a Historically Black University (HBCU). Fifty-eight students (90
(Received September 07, 2023)

1192-97-29907
Adam Rumpf*, Florida Polytechnic University. Getting to the Good Part: An Applications-First Approach to Calculus II through Differential Equations Modeling.
We present a plan to re-envision the Calculus sequence in a modern, engaging, and applications-driven way by making differential equations modeling an immediate and central part of the Calculus II experience, using differential equations models as the course's introduction and regularly referring back to them as a motivating application for the later material. Differential equations are exceptionally applied and provide the opportunity to discuss many real-world uses of calculus and its intersections with other STEM fields, they give the opportunity to show interesting phenomena unlike anything the students will have seen in Calculus I, and they can be studied and understood well before the students possess robust tools for evaluating antiderivatives. We will discuss our efforts to develop and implement this plan at Florida Polytechnic University during Fall 2023, including what went well, what we would change, and how you might be able to update your own approach to teaching Calculus.
(Received September 08, 2023)

## 1192-97-29942

Jessica Deshler, West Virginia University, Danielle Maldonado, West Virginia University, Tim McEldowney*, West Virginia University, Lynnette Michaluk, West Virginia University, Edwin "Ted" Townsend, West Virginia University. KnowledgeGAP: Barriers in Applying to Graduate School.
The Undergraduate Knowledge of the Mathematics Graduate School Application Process (Knowledge-GAP) project seeks to understand how undergraduate student knowledge about the graduate school application and admissions processes may act as a barrier for students seeking advanced degrees in mathematics. The project is especially interested in experiences of students historically marginalized in STEM disciplines as they attempt to pursue graduate education in mathematics. This presentation on this NSF funded project (Award \# 2126018) will discuss what undergraduate students view as barriers to them applying to graduate school in mathematics and how perception of these barriers varies by demographics.
(Received September 08, 2023)

1192-97-30013
Ali Bhai*, LAUSD, Yvonne Lai, University of Nebraska-Lincoln. What data do teachers find useful, helpful, and salient?. Preliminary report.
K12 teachers will discuss in a panel how they view the use of data, the narratives that data are used to tell about teaching and learning, and the kind of data they find useful, helpful, and salient.
(Received September 08, 2023)

1192-97-30162
Younhee Lee*, Southern Connecticut State University. Panel on Looking Forward and Back: Common Core State Standards in Mathematics (CCSSM), 12 Years Later I.
This session will be a panel discussion on Looking Forward and Back: Common Core State Standards in Mathematics
(CCSSM), 12 Years Later I.
(Received September 09, 2023)

## 1192-97-30449

Sudhir Murthy*, University of California, Riverside. The Potential of Lean Theorem Prover in AI and Education.
Lean is an interactive theorem prover that serves as a powerful tool for digitally organizing mathematical knowledge and rigorously verifying proofs. It has established robust foundations across diverse mathematical disciplines such as number theory, analysis, and algebra. In this context, we present a student's account of learning Lean and highlight its inherent differences from set-theoretic foundations. We emphasize the advantages of interactive theorem provers in undergraduate mathematics and interdisciplinary curricula. Additionally, we discuss its relationship with proof automation, AI-generated mathematics, and ongoing efforts to expand the necessary training data for integrating large language models with it. (Received September 10, 2023)
to ideas and concepts in and beyond school mathematics, expose them to mathematics applications and careers, and foster positive mathematics attitudes. We have conducted screening sessions and focus group interviews with K-8 students and teachers and found that mathematicians' stories engage K-8 students in several ways, including by expanding their mathematical knowledge and imaginations, piquing their interest in mathematical applications, establishing personal connections to mathematicians' identity and their work. While this digital database was developed with K-8 students in mind, these videos have the potential to inform college mathematics policy and practice and be impactful for college mathematics students as well.
(Received September 10, 2023)

## 1192-97-30863

Sepideh Stewart*, University of Oklahoma. Teaching and Learning Linear Algebra Proofs: Students' Perspectives. Preliminary report.
Proof plays a vital role in mathematics. In this talk, I will share a case study on students' perspectives on teaching and learning proofs in a second linear algebra course.
(Received September 11, 2023)

## 1192-97-30957

Mary Beisiegel, Oregon State University, Patrick Kimani, Maricopa Community Colleges, Vilma Mesa*, University of Michigan, Vmqi Research Team, University of Minnesota. Mathematics Education Research in Post-Secondary Settings. What does it take to assess high quality mathematical instruction? If you have already designed a measure that captures that quality, what does it take to include a diversity, equity, and inclusion (DEI) lens? In this talk I will present ongoing work done in collaboration with community college faculty and researchers from several institutions that seeks to assess high quality of mathematical instruction at community colleges. Despite having a validated instrument that captures key dimensions of highquality instruction at community colleges, modifying it to incorporate a DEI lens required a shift in our approach to the work, to investigate instead the various ways in which community college students experience with equity, diversity, and inclusion in the community college mathematics classroom. I will describe the approach we are using to collect the data and some preliminary themes about students' understanding of equity.
(Received September 11, 2023)

1192-97-30967
Dusty Grundmeier*, The Ohio State University, Deborah Moore-Russo, University of Oklahoma, Stepan Paul, North Carolina State University. Three-Dimensional Manipulatives in Integral Calculus: Student Achievement and Confidence in Solids-of-Revolution Tasks.
We present the results from a randomized controlled study to evaluate the impact of 3D manipulatives on student learning of volumes of revolution in calculus
(Received September 11, 2023)

## 1192-97-30979

Shashini Sanjana*, University of Colombo. Gem Symmetry Game.
Gem Symmetry is a puzzle game for young students that asks them to use their knowledge of rotational symmetry to create specified shapes and patterns. Players must slot gems in the right positions, using a "drag-and-drop" mechanic, to connect angles of the right measure to create corresponding regular shapes. To form a square, for example, the player must use 90degree gems or the gems will shatter. Set in a mine, this game provides a themed environment in which students can practice skills that will eventually lead to the development of geometry knowledge such as the formulae for internal angles of a polygon. The complexity of the game also increases gradually, with the introduction of higher-order regular polygons and limitations on the player's resources. In the future, we hope to expand the game to include direct connections to other geometric concepts, including isosceles triangles, as well as accessibility options such as a color palette for users with visual impairments, and a time-based mode for users who desire an additional challenge.
(Received September 11, 2023)

## 1192-97-31045

Sukhithi Chamali*, University of Colombo, Sri Lanka. Roman Army Battle Matrix Game. Preliminary report. Roman Army Battle Matrix Game S.A. Sukhithi Chamali University of Colombo Abstract Matrix operations, especially multiplication, are somewhat counter-intuitive for novices of linear algebra. The Roman Army Battle Matrix game is a 2dimensional strategic game that aims at developing this intuition of basic matrix operations, such as subtraction and multiplication. It is aimed at high school students, undergraduates, and anybody else who will be learning linear algebra for the first time. This game integrates the mathematical concepts of matrices into the structure of rectangular Roman Armies (Legions). The goal of the game is for the player to choose from a variety of power-up matrices to build the best possible army to fight a given enemy army. Applying the power-ups to the Roman Army follows the structure of matrix multiplication. The final fight with the enemy follows matrix subtraction; the player wins if their army has more soldiers alive at the end of a round. This game progresses gradually, starting from single row/column power-ups coming from one side, to those coming from both sides, and finally to power-ups with different orders coming from either side. The visualization of the powering up, which is displayed as an animation demonstrating each step, builds the player's intuition on how and why matrix multiplication works the way it does. The game also has a healing potion alongside the power-ups which can be applied to the entire army at once which would help the player understand scalar multiplication. The final fight at the end of each round would facilitate the player's understanding of matrix subtraction. Overall players will be able to gain an intuitive understanding of matrix operations and will be able to practice performing such operations. The game could be expanded to include further learning goals in linear algebra (e.g.: transpose - army could be made to re-arrange accordingly). A tutorial round could be used at the beginning of the game to familiarize the player with the nature of the game. Further, an even more guided tutorial version could be introduced for players who have not learnt linear algebra at all. Moreover, timed rounds could be introduced for more advanced players who already understand the operations and would just want to practice doing them. Keywords - linear algebra, matrices, matrix operations

Sam Macdonald*, University of Nebraska - Lincoln. Carceral Mathematics: The Parallels of Collegiate and Prison
Mathematics Education.
Prisoners participating in prison education programs are 43\% less likely to be re-incarcerated following their release, and estimates suggest that every dollar spent on correctional education saves five in re-incarceration costs. With the United States spending $\$ 80$ billion each year on incarceration, the statistical benefits of prison education cannot be understated. Yet, on a smaller scale, these figures fail to communicate the monumental impact on instructors and how we choose to conduct ourselves within the classroom. In this talk, I discuss my experiences teaching mathematics behind bars through the lenses of equity, ethics, and empathy, highlighting the parallels that exist between carceral and collegiate classrooms.
(Received September 11, 2023)

## 1192-97-31103

Shawn Wiggins*, City College of San Francisco. Mathematics Education in Transition: Embracing Tech and Refocusing. Preliminary report.
As a community college mathematics instructor, I've grappled with changes in undergraduate Probability and Statistics courses due to adapting to Common Core standards and implementing various technologies. Time constraints and fixed course objectives limit exploration, leading to overly guided technology use and rushed conclusions dependent on template language. I've worked with several people to introduce an introductory data science course modeled after UC Berkeley's DATA 8 to our students. The pedagogy of this course prioritizes utilizing a scripting language like Python, forming plans for statistical analysis, and utilizing the computer for making predictions in a way that accomplishes similar goals in a less contrived manner. The subtle change in course objectives could also set a precedent for similar shifts in undergraduate courses in Algebra and Calculus, easing prerequisites and enhancing accessibility in math education. Join me in exploring how this adaptation can empower students and improve math instruction in a post-Common Core era.
(Received September 11, 2023)

## 1192-97-31152

Jie Zhong*, California State University Los Angeles. Guiding the Data-Driven Generation: Mentoring Students in Transformative Undergraduate Research.
In an era where data drives innovation and decision-making across industries, the realm of undergraduate research is no exception. This presentation explores the transformation of undergraduate research through a data-driven lens, highlighting the profound benefits for both students and mentors. From cultivating essential skills tailored for a competitive job market to uncovering novel mathematical insights rooted in real-world challenges, data-driven research offers a unique avenue for academic and professional growth. However, this transformative journey is not without its hurdles. Challenges such as data access, aligning academic and industry objectives, and managing diverse student skillsets require thoughtful navigation. The presentation offers practical strategies to overcome these obstacles and thrive as a faculty mentor in the data-driven research sphere. Drawing from case studies and personal experiences, attendees will gain valuable insights and actionable advice for embarking on or enhancing their own data-driven mentoring journeys. (Received September 11, 2023)

## 1192-97-31196

Sandra Laursen*, U. Colorado Boulder. Is the needle moving? Measuring change in college mathematics teaching in response to professional development.
High-quality professional development is a powerful lever for change in the national movement toward evidence-based undergraduate mathematics teaching. From education research, we know much about teaching approaches that college instructors can use to offer engaging and inclusive learning experiences of the type that strengthen student success and persistence toward their goals, and that support all students to achieve these good outcomes. More difficult to come by is highquality evidence about how well we teach these teaching approaches to teachers. That is, it is challenging to determine whether professional development programs move the needle-to establish from evidence if professional development makes enough of a positive difference in enough classrooms to benefit substantial numbers of students. I will outline some conceptual and methodological issues in studying change in teaching that make these studies difficult. Then I will discuss recent work in our research group to develop and test measures of teaching that seek to capture instructors' movement toward evidencebased teaching, using examples from recent studies of national-scale professional development programs. Finally, I will outline some worthy challenges for practitioners and researchers alike. (Received September 11, 2023)

## 1192-97-31479

Mili Shah*, The Cooper Union. Using the Mathematics of Art and BodyTracking to Encourage Research at PUI. Preliminary report.
The Cooper Union is a Primarily Undergraduate Institution (PUI) in New York City consisting of just three schools Architecture, Art, and Engineering. This talk will showcase interdisciplinary projects that use the mathematics of art and bodytracking to encourage research between the three schools. Live interactive demonstrations will also be presented. (Received September 11, 2023)

John Paul Cook*, Oklahoma State University, Elise Nicole Lockwood, Oregon State University, Zackery K Reed, Embry Riddle Aeronautical University Worldwide, April Richardson, Oklahoma State University. Examining key ideas across mathematical domains: How do mathematicians disentangle equivalence and equality?.
Equivalence - and the specific example of equality - are among the most foundational topics in all of mathematics. We note,
however, that these core ideas have not been treated consistently in the educational literature. Characterizations of the relationship between equivalence and equality are either context-specific (and thus do not generalize across domains) or are contradictory. While an incoherent literature base is problematic in its own right, there are also implications for students' learning. A prevailing explanation for some of the difficulties students experience with equivalence and equality is that they struggle to identify and characterize the various forms of equivalence at play in a given situation. Indeed, if we, as researchers and educators, are to support students in reasoning coherently about equivalence and equality, a key precursor to these efforts is to clearly characterize these key ideas in a way that is both coherent and generalizable. To this end, in this talk we report on a series of interviews we conducted with mathematicians to see how they characterize and differentiate between these terms. We report on themes that emerged across these interviews, which include (a) coherent and accessible characterizations of equivalence and equality at both a local (element-to-element) and a global (equivalence class-based) level, along with (b) a multitude of illustrations and examples from the K-16 curriculum. We conclude with implications for both research and instruction.
(Received September 11, 2023)

## 1192-97-31698

Sepideh Stewart*, University of Oklahoma. Teaching Proofs in a Second Linear Algebra course. Preliminary report. In this talk, I share some insights from the mathematics education literature on teaching and learning proofs in first and second linear algebra courses.
(Received September 11, 2023)

## 1192-97-31755

Emily Atieh, Stevens Institute of Technology, Jan Cannizzo*, Stevens Institute of Technology, Andrey Nikolaev, Stevens Institute of Technology. Integrating community engagement into the curriculum: A report on undergraduate-led math circles for elementary school students. Preliminary report.
We report initial findings from the implementation and study of a community engagement program that brings together three parties-math faculty, STEM undergraduates, and community partners (public schools, public library, and Boys and Girls Club) -to organize math circles for elementary school students. As part of the program, undergraduate students enroll in a creditbearing course in which they learn about community engagement and extracurricular mathematics, lead math circles at community partner sites, and engage in weekly reflections about their experiences. Data collection primarily consisted of observations, interviews, and the collection of course artifacts. Through qualitative analysis, we sought to understand the manner and extent to which the undergraduates' experiences as a math circle facilitator-including their training-shaped their perceptions of mathematics and its role in their education, future careers, and within society as a whole. These preliminary results are part of a larger investigation that seeks to evaluate the success of this community engagement program and describe the critical components and best practices for its integration into the broader undergraduate mathematics curriculum. This work was conducted with Emily Atieh, Gail Baxter, and Andrey Nikolaev and supported by NSFIUSE Grant No. 2236229.
(Received September 12, 2023)

## 1192-97-31783

Megan Chang-Lee*, Brown University. Teaching with Team-Based Inquiry Learning: Joyful Teaching.
Attrition of college students from underrepresented backgrounds in STEM fields often occurs during their initial exposure to large introductory STEM courses. At Brown University, with the leadership of Jordan Kostiuk, a significant overhaul of the lower-division linear algebra and multivariable calculus courses has recently taken place. Both classes, each enrolling several hundred students every semester, have been transitioned to a team-based inquiry learning format. In this talk, I will discuss some of the logistics of implementing the team-based inquiry learning structure, share my experience as a graduate student facilitator within the framework of team-based inquiry learning methods in mathematics education, and discuss how this instructional approach not only impacted students but also was a profoundly joyful and restorative experience for myself as an educator.
(Received September 12, 2023)

1192-97-32031
Ann Edwards*, WestEd. Policy as a Lever for Change: Looking Back at the Common Core.
This session will reflect on the role of policy in shaping mathematics education. We will discuss policy implementation, focusing primarily on the Common Core State Standards for mathematics, as a lever for change, examining the challenges and promise of policy efforts to improve equity and outcomes in mathematics.
(Received September 12, 2023)

## 1192-97-32064

Widodo Samyono*, Jarvis Christian University. Integrating Ethical Conversations in Biocalculus. Preliminary report.
Mathematicians, with some exceptions, have not had opportunities to describe what it means to practice and use mathematics ethically. This talk describes the integration of ethics into Biocalculus (Calculus I) classroom and ethical issues surrounding the teaching of mathematics at Jarvis Christian University, a small liberal arts HBCU, Hawkins, East Texas. This paper addresses Calculus I (Biocalculus) for students in biology, chemistry, and mathematics majors as a redesigned course in the mathematical sciences. The course addresses ethics in mathematics course Biocalculus by integrating ethical conversations into the course. The paper discusses how ethics in Biocalculus were addressed, include reflection or assessment on success, describes challenges and barriers how to implement the integration of ethical conversations in Biocalculus, and share strategies to address those barriers.
(Received September 12, 2023)

Aditya P Adiredja*, The University of Arizona, Marta Civil, University of Arizona, Becca Jarnutowski, University of Arizona. Project ADeLanTe: An Anti-deficit Design for a Professional Development Program of University Instructors. Preliminary report.
In this report, we share the design of a year-long professional development (PD) program for university instructors that we developed and refined as part of the Anti-deficit Learning and Teaching Project (ADeLanTe). We approach instructors' professional development as a community learning project, wherein minoritized students, peer mentors, and instructors (graduate students and instructional faculty) share their knowledge and experiences with one another. Adiredja's anti-deficit perspective is embedded in all aspects of the learning experience for everyone involved. We work with the assumption that minoritized students have cultural and intellectual assets for learning. But we also recognize that many of these students are implicated in deficit narratives related to their capacity to learn, which we need to challenge. Instructors worked from this assumption and explicitly challenged deficit narratives as they engaged in the core activities on the program. The core activities for the PD include: 1) Four PD meetings on anti-deficit teaching and Inquiry Based Learning (IBL) teaching method; 2) Leading a five-day math summer bridge workshop in Pre-Calculus, Calculus I, II, Vector Calculus, or Linear Algebra with live coaching from the PI; 3) Participating in Critical Conversations about Race and Gender in STEM at the university cultural centers; 4) Conducting a Funds of Knowledge inspired interview with a student from the summer workshop; and 5) Group reflection meetings throughout the year. As the research team, we also take an anti-deficit approach to instructor development. We focus on instructors' individual growth and agency, their joy in teaching, and maintaining their mental health. Preliminary analyses of two of the three cohort of participants found that most instructors developed a more humanizing approach to their teaching and students after hearing students' stories. IBL helped instructors to explicitly challenge deficit narratives about minoritized students in the classroom. Interviewing their student also shifted deficit narratives that developed in the classroom. The workshop became a space for instructors to try out previously learned teaching ideas. Doing so reinvigorated their teaching. We interpret these findings vis-a-vis Ladson-Billing's idea of Culturally Relevant Pedagogy.
(Received September 12, 2023)

## 1192-97-32177

Gail F Burrill*, Michigan State University. Preparing Students for Tomorrow: Rethinking What is Important to Teach. Technology today can retrieve, manage, and analyze vast amounts of data; create complex interactive visualizations; and manipulate mathematical symbols. This power, increasing exponentially since the CCSSM, can allow the field to focus on understanding and interpreting mathematical results rather than on the traditional content concentrated on developing procedures. Statements from those considering the skills and competencies students will need in the future such as, "We are preparing students for a world that no longer exists;" "America's math curriculum doesn't add up;" and "Assessment of trustworthiness is moving away from mathematical methods and towards AI and decision-making methods" are becoming more frequent. While it is clear that a sound mathematical foundation is necessary for these competencies and that understanding new topics such as risk and working with data should be part of the picture, it is not at all clear what that foundation should look like nor how it should be structured from secondary to post-secondary courses nor is it clear for whom mathematics as has been traditionally perceived is necessary. To prepare students for the world in which they will live, we need to rethink what we are teaching, why it is important, to whom, and when it should be taught. (Received September 12, 2023)

## 1192-97-32204

Yvonne Lai*, University of Nebraska-Lincoln. Panel On Looking Forward And Back: Common Core State Standards In Mathematics (CCSSM), 12 Years Later II.
This session will be a panel discussion on Looking Forward and Back: Common Core State Standards in Mathematics (CCSSM), 12 Years Later II.
(Received September 12, 2023)

## 1192-97-32214

Tyler Kloefkorn*, American Mathematical Society. Panel On Looking Forward And Back: Common Core State Standards In Mathematics (CCSSM), 12 Years Later III.
This session will be a panel discussion on Looking Forward and Back: Common Core State Standards in Mathematics (CCSSM), 12 Years Later III.
(Received September 12, 2023)

## 1192-97-32219

Luis E Saldivia*, ETS. Analyzing NAEP Trends in the Common Core Era. Preliminary report.
The National Assessment for Education Program (NAEP) is the largest nationally representative, continuing evaluation of the what the students in the United States know and can do. The assessments have been administered by the National Center for Educational Statistics since 1969, and provides the basis for the Nation's Report Card. Since the introduction of the CCSS-M back in 2010, NAEP has conducted six different administrations of its mathematics assessments at grades 4 and 8 , and three at grade 12. In this presentation, we will discuss the results of these administrations and provide comparisons of their results to those from previous administrations. State results, based on the adoptions of the CCSS-M will also be compared. Additionally, the presentation will discuss the NAEP transition from paper and pencil to digital delivery in 2017. At the end of the presentation, the shifts introduced by the new NAEP Mathematics Framework will be discussed. These shifts include an explicit emphasis on mathematical practices. A comparison between the NAEP mathematical practices and the practices described by the CCSS-M will also be discussed.
(Received September 12, 2023)

1192-97-32293
Matthew Beckman, Penn State University, Neil Hatfield, Penn State University, Alyssa Hu*, Penn State University. Exploring Computational Thinking in a Data-Driven Context.
Modern statistics education requires that we support students in building powerful and productive ways of computational
thinking. We begin with an overview of key statistical education developments. Following this background, we describe our research goal to understand the ways of computational thinking that individuals employ as they engage with data. Through task-based interviews with participants using the R programming language, we discuss several problem-solving approaches that emerge. We consider these ways of thinking in context of existing literature and work towards developing a framework highlighting beneficial ways of thinking when working with data.
(Received September 12, 2023)

## 1192-97-32356

Missy D Cosby*, University of Tennessee, Maisie L Gholson, University of Michigan. Curricular Interventions for the Development of Positive Mathematics Identities: Tensions and Possibilities. Preliminary report.
The proposed presentation relies on a five-year NSF study mathematics identity development of successful Black youth. An aspect of this study examines an out-of-school mathematics curriculum that supports positive math identity construction. In particular, we explore: What are the affordances and tensions in facilitating and promoting positive mathematics identity development through curricular interventions? The development of the mathematics identity curriculum occurred over a multiyear cycle (spanning the COVID-19 pandemic), using design-based research methods, and implemented with approximately 150 students each year. Due to the need for remote learning opportunities, the curriculum shifted from (1) an in-person whole class, social justice focused mathematics content to (2) asynchronous, individual-focused psycho-social mathematics content. The initial curricular intervention designed for in-person instruction included six social justice activities, three mathematics identity activities, and four problem-solving activities. The current curricular intervention designed for asynchronous, individual learning included six mathematics-identity focused lessons around topics such as, the mathematics gene myth, the utility of math, metacognition, and racial stereotypes. The preliminary analysis suggests two competing findings. First, curricular interventions that change "the face of math" to promote engagement, such as teaching mathematics for social justice lessons and activities, can productively shift social interaction around mathematics; however, children do not recognize their engagement as doing math, so this positive engagement does not succeed as an identity intervention. Second, curricular interventions that focus on internalizations and intra-psychic constructions of mathematics seem to provide new resources for sense-making and narrating of self, but fail to shift practice or expand conventional (often erroneous) views of mathematics as a discipline. Through the presentation, we share sample lessons and materials along with student artifacts to describe the tensions that arose from these efforts in curricular intervention in an effort to deepen our understanding of ways to impact the development of positive mathematics identities for Black youth.
(Received September 12, 2023)

## 1192-97-32362

Brittany Oletti*, United States Military Academy. Fostering Mathematical Curiosity: Using Real-Life Modeling to Teach Differential Equations.
Imagine a paradigm shift in higher education: where instead of tossing mathematically apt and advanced students into Calculus II or III as their first college class, suppose that we put them into Differential Equations instead. Furthermore, what if we made that Differential Equations class a modeling class as well, where students would have to simultaneously learn how to model as they learned various techniques including variation of parameters, integrating factor, Euler's Method and more? What if I told you that we are doing just that? The framework we've developed for teaching our advanced population of students involves a parallel learning schematic of mathematical modeling and differential equations. As we scaffold the techniques of solving first-order, second-order, and systems of ODEs; we also introduce modeling at its basic levels of transforming the variables, through the solve step and ultimately interpreting meaningful solutions. We do this as a three-part process: by engaging in in-class lectures and associated practice problems; with smaller, engaging modeling application days where students focus on parameters in ODEs; and finally with a large scale project and presentation at the end of the course on a differential equation modeling problem of their choice. During this talk, we will discuss how we facilitate this innovative educational journey, where students don't just learn mathematics; they become mathematical explorers, discovering the boundless possibilities that await when modeling meets differential equations.
(Received September 12, 2023)

1192-97-32458
Skylyn Irby*, The University of Alabama. Investigating STEM Retention Program Participants' Sense of Belonging in Mathematics. Preliminary report.
Studies show that a sense of belonging in mathematics may impact students' desires to persist in STEM fields. Summer bridge programs and Louis Stokes Alliance for Minority Participation (LSAMP) programs are resources used by institutions to improve retention in STEM fields by helping students become acclimated to college STEM environments. However, little is known about how students' sense of belonging in mathematics evolves while participating in these programs. This study seeks to fill this gap by exploring the mathematics sense of belonging and mathematics identity of students pursuing STEM degrees and participating in these programs. In particular, this study explores the change or growth in students' sense of belonging in mathematics and mathematics identity over the course of their participation in an LSAMP program. Student surveys, interviews with students and staff, and site observations within the LSAMP program at a single institution will provide insight into students' sense of belonging in mathematics, mathematics identity, mathematics confidence, and experiences in these programs. This study may help us understand STEM retention programs' roles in diversifying STEM via a lens of mathematics sense of belonging and mathematics identity.
(Received September 13, 2023)

1192-97-32467
Leo Livshits*, Colby College. Bases pronto in "Not Your Grandpa's First Introduction To Linear Algebra". Preliminary report. In the 2022 article "A Gauche Perspective on Row Reduced Echelon Form and Its Uniqueness" published in The American Mathematical Monthly, E.L. Grinberg proposed an approach for producing the RREF of a matrix through a construction of "Gauche" bases. Encouraged by Grinberg's lively article, we will report on a related approach for introducing bases and dimension for subspaces of $\mathbb{F}^{n}$ which we have been using in Linear Algebra at Colby College, and which is included in our forthcoming book "Not Your Grandpa’s First Introduction To Linear Algebra." Our approach is based on demonstrating that every non-trivial subspace of $\mathbb{F}^{n}$ has two canonical bases, "initial" and "terminal" bases, which can be described explicitly and quickly upon defining the notion of a subspace, and which, for instance, immediately demonstrate that dimensions of a
subspace and its ortho-complement add up to $n$ (when $\mathbb{F}=\mathbb{R}, \mathbb{C}$ ). Time permitting, we will put this approach into the context of our course, in which orthogonality in $\mathbb{R}^{n}, \mathbb{C}^{n}$ is introduced from the get-go and is then used as a foundation for further developments and applications, as dictated by the modern needs.
(Received September 12, 2023)

## 1192-97-32482

Marlen Vasquez*, City College of San Francisco. Making sense of the data.
In this session, I will share my experiences as a community college English faculty member and coordinator during the rollout of California state assembly bill AB705. In particular, I will share how data was presented to the English faculty regarding the persistence of equity gaps in transfer level English. As part of this discussion, I will discuss what quantitative data shows us and what it does not show us. Furthermore, I will share how qualitative data can complement quantitative data by addressing the gaps in our understanding. In this concluding section, I will highlight some parts of my current dissertation study, which explores the impact of Latina community college students' relationships with their parents on their abilities to navigate their college education.
(Received September 12, 2023)

1192-97-32522
Jill R. Faudree*, University of Alaska, Fairbanks. The Results of a Quasi-Experimental Study of Embedded Precalculus Instruction in Calculus I.
The purpose of this study was to examine the impact of embedded precalculus instruction in Calculus I. Common measures of mathematical content knowledge and early engagement were collected in the first two weeks of the semester for all students enrolled in Calculus I for two years. Based on this data, instructional support was designed and embedded in Calculus I for all students who scored low on these early measures. Note that the historical data set was obtained prior to the COVID pandemic and the intervention was implemented after the institution returned to in-person instruction. In addition, all sections of Calculus I were tightly coordinated. Statistical comparisons of the course grades, final exam scores, and success in Calculus II between two populations (before and after the intervention) were made controlling for mathematical content knowledge and engagement at the beginning of the semester. The findings suggest that very modest additional academic support can result in significant gains for students with weak mathematical content knowledge. However, there were several crucial limitations. Evidence of grade inflation was detected. Improvement in precalculus content knowledge was marginal. For students with very low scores on mathematical content knowledge, few gains were observed. Success in Calculus II appears to be unchanged. (Received September 12, 2023)

1192-97-32553
Alessandra Pantano, University of California, Irvine, Roberto C Pelayo*, University of California, Irvine. Teacher Noticing in an Introduction to Mathematical Proof Course.
Introduction to mathematical proof courses are typically important transitional experiences for mathematics majors. As such, low pass rates often serve as barriers to timely progress towards a mathematics degree, and do so in a disproportionate way for minoritized students. To address this, we utilize teacher noticing, an instructional technique more frequently found in K-12 spaces, in an introductory proof writing course at UC Irvine. In particular, our project investigates the effectiveness of a videoannotation curriculum designed to support responsive and equitable mathematics instruction in undergraduate prooftransition courses. This training curriculum was presented to entire instructional teams (instructor, graduate teaching assistants, and undergraduate learning assistants), in an effort to improve student engagement and learning outcomes in this critical mathematics transition course. We discuss the structure and content of the teacher noticing curriculum and efficacy results from our initial analysis.
(Received September 12, 2023)

## 1192-97-32557

Patti Frazer Lock*, St. Lawrence University. Using Intro Stats, and the GAISE report, as a Vehicle to Recruit a Diverse Population of Mathematics Students. Preliminary report.
The College GAISE report (Guidelines for Assessment and Instruction in Statistics Education) has been a widely cited and influential tool to help instructors assess and improve their introductory statistics courses. The speaker is co-chairing the committee to revise this report. One of the specific goals of this revision is to enhance equity and inclusion in this course. A strong and inclusive Intro Stats course can be a very effective alternate pathway into mathematics. We will present ideas and invite input from the audience on ways to enhance the effectiveness and inclusivity of Intro Stats as a way to recruit a diverse population to mathematics.
(Received September 12, 2023)

## 1192-97-32584

Jim Fowler*, The Ohio State University, Duane Q. Nykamp, University of Minnesota, Matt Thomas, Cornell University. Distributed Open Education Network (Doenet).
The Distributed Open Education Network (Doenet) is a project to improve the educational technology ecosystem for the teaching of mathematics. As a collaborative effort involving the University of Minnesota, The Ohio State University, and Cornell University, Doenet is designed to enable authors to produce dynamic, interactive worksheets and then to evaluate student performance. One tool in the Doenet ecosystem is DoenetML, which simplifies the creation of interactive content. We will dive into the vision behind Doenet, its potential integration with other platforms, and the future of the Learning Management System (LMS).
(Received September 12, 2023)

Alternate Narratives Creating the Mathematics Ethos that May Have Been.
What would the history of mathematics told through the accomplishments and identities of women and people of color look like? What ethos and semblance would the discipline take in a world that naturally centered cultures, experiences, and ways of knowing that have lied historically at the margins? And how might that ethos and semblance suggest alternate narratives, permeate classroom discourse and reshape the fabric of the mathematics learning experience? Working from an asset-based, culturally sustaining lens (Ladson-Billings, 1995; Paris \& Alim, 2014), we propose ways to center alternate mathematical narratives in the mathematics classroom, leveraging two recent resources, AWM's EvenQuads: Notable Women in Math Playing Cards, and AMS-MAA's Testimonios, Stories of Latinx and Hispanic Mathematicians. Our discussion is based on our forthcoming chapter, baring the same title as this presentation.
(Received September 12, 2023)

## 1192-97-32667

Fred Bonner, Prairie View A\&M University, Orion Ciftja, Prairie View A\&M University, Alphonso Keaton, Prairie View A\&M University, E. Gloria C. Regisford, Prairie View A\&M University, James R. Valles*, Prairie View A\&M University. Mentoring Activies and Effectiveness in an S-STEM Program. Preliminary report.
This is a preliminary report on the effectiveness of various mentoring activities on the mathematical anxiety of students participating in a multi-disciplinary NSF S-STEM program. The PVAMU SUMS program is a multi-disciplinary NSF S-STEM program that recruited students in the following majors: Biology, Chemistry, Physics, and Mathematics. As part of their participation in the SUMS program, these students (SUMS Scholars) participated in various activities meant to enhance their academic experience as well as their professional development. Part of their experience is taking Calculus I as annual cohorts, and in doing so the Scholars were provided tailored mentoring and tutoring. This presentation will evaluate the participants' mathematics anxiety and mathematics self-efficacy and how this has changed since they took Calculus I. Discussion will also focus on effective measures as well as measures that can be improved upon.
(Received September 12, 2023)

## 1192-97-32699

Stephen Wang*, Rice University. Inquiry-based Activities for Exploring Orthogonality.
Orthogonality is an important concept in linear algebra, from its "super-independence" properties to its role in facilitating faster computation. How might its significance be communicated to students? In this talk, I will present some guided inquiry activities that try to do just that in an experiential and intuitive manner. These activities have been used and refined in flipped linear algebra courses in multiple semesters.
(Received September 12, 2023)

## 1192-97-32732

Kelly Buch*, Austin Peay State University. A First Attempt at Teaching Mathematical Modeling: Sharing Successes and Lessons Learned.
Teaching any course for the first time is a challenge. Course design must be delineated early on, but it's hard to anticipate how students will react to course content without prior experience teaching the course. Inevitably, you end the course with some successes, a few pleasant surprises, and a long list of things you wish you had done differently. This is especially true if the course involves open-ended learning activities like major projects or undergraduate research. In this talk, I'll share my first experience designing and teaching an upper-level mathematical modeling course with an undergraduate research component. I'll share overarching goals which shaped my course design, changes I made to that design mid-semester, and (hopefully) a few success stories involving the students' research projects. I hope that through sharing my experience, the audience can prepare themselves for incorporating undergraduate research within their own courses.
(Received September 12, 2023)

1192-97-32770
Emily J Evans*, Brigham Young University. Linear Algebra Revisited: An intermediate course for students interested in data science.
With the increase in popularity in data science, more students are interested in a second course in linear algebra that focuses on linear algebra topics that are especially relevant to data science. In this talk, I will discuss two classes developed at Brigham Young University for students interested in data science that dig deeper into the relevant linear algebra. I will also discuss how to balance the need for rigor, computational projects, and real-world applications while maintaining student interest.
(Received September 12, 2023)

## 1192-97-32820

Ryan H Allaire*, United States Military Academy, Andrew C Lee, United States Military Academy, Margaret Ann Reynolds, United States Military Academy. Designing Problems with Nice Solutions for Improved Instruction and Learning. One of the grand challenges of Mathematics instruction is to provide the students with problems that are both accessible and have a reasonably nice solution. To determine such problems instructors often utilize resources such as online homework platforms or other software that generate problems automatically. These resources give the instructor little control over the parameters of the problem and may lead to complicated answers that detract from teaching the solution approach. In this talk, we focus on the process of reverse engineering nice problems in Linear Algebra and Calculus and show that this leads to mathematically challenging inverse problems. We present algorithms, along with Mathematica code, that can be utilized to (i) create an integer matrix with integer inverse, (ii) easily design a system of equations with a desired number of solutions, (iii) create arc length problems that are easily integrable, and (iv) create nice surface area problems. We show that these techniques can be utilized in many subsequent problems with the overarching goal of enhancing instruction and student confidence.
(Received September 12, 2023)

Tyler J. Jarvis*, Brigham Young University. BYU's Applied and Computational Math Emphasis (ACME) program after 10 years.
Ten years ago the DUE funded a project to build a new undergraduate curriculum in applied and computational mathematics (DUE-1323785) at Brigham Young University. That funding allowed us to develop a new degree program, including several textbooks and lab manuals for undergraduates to learn powerful tools in applied and computational mathematics. We've been using and sharing those resources ever since then, and we've had over 350 students graduate from the program and go on to great careers and to top graduate programs in applied math, pure math, computer science, and many other subjects that rely on applied math. The grown of the ACME program has led to new resources coming to our department, many more students choosing to be math majors, and much more alumni engagement (and donations). In this presentation I'll review some of the program's successes and challenges, and some of the lessons we learned along the way that might be useful to others looking to start similar programs at their institutions.
(Received September 12, 2023)

1192-97-32887
Sarah K Bleiler-Baxter, Middle Tennessee State University, Amanda Lake Heath*, Middle Tennessee State University. Reasons Students Leave (and Stay) in Mathematics: Identifying Ways to Promote Retention in the Mathematics Major. Preliminary report.
In our quickly evolving world, there is a pressing need for more students with degrees in science, technology, engineering, and mathematics. However, nationwide, an overwhelming $52 \%$ of declared mathematics majors change their major in the course of their college career (Leu, 2017). In an effort to promote retention in the mathematics major, we pose the research question, what reasons do students report for choosing to stay in and leave the mathematics major? This project, commissioned by Middle Tennessee State University's College of Basic and Applied Sciences to be carried out in Fall 2023, is currently in the survey development phase. This survey, to be administered in October 2023, will be administered nationwide to students enrolled in a mathematics major over the last five years and solicit responses to open-ended questions regarding their decisions either to leave the mathematics major, reluctantly stay in the mathematics major, or enthusiastically complete their degree in mathematics. Qualitative analysis of survey responses will provide insight beyond quantitative measures and predictors of persistence in mathematics. In our presentation, we will describe the qualitative methodologies used to analyze responses to our national survey and present our findings. The results of this project will provide implications for K-12 mathematics education, as well as tertiary mathematics instruction and administration to promote the retention of a diverse population of mathematics students. Reference: Leu, K. (2017). Beginning college students who change their majors within 3 years of enrollment. National Center for Education Statistics. https://files.eric.ed.gov/fulltext/ED578434.pdf (Received September 12, 2023)

## 1192-97-32895

Mine Cekin, Columbia University, Baldwin Mei*, Columbia University. Impact of Chavrusa-Style Learning in Mathematics Courses on International and Non-International Students.
The increasing diversity of mathematics classrooms in the United States requires educators to continuously ensure students' needs are being met. One way this can be done is to use pedagogies that are inclusive and culturally responsive.
MathChavrusa is one such example, a novel implementation of the Talmudic chavrusa learning model in the context of mathematics education. MathChavrusa-style learning holds significant promise in fostering student interaction between learners from diverse cultural, social and linguistic backgrounds in mathematics classrooms. This study investigated how differently international and non-international students perceived the impact of MathChavrusa-style learning. Data collection was performed in Fall 2021 and consisted of semi-structured interviews with both international and non-international students in graduate mathematics courses using MathChavrusa-style learning and classroom observations of participants interacting with their MathChavrusa partners. Our findings were divided into three sections based on the research questions: how MathChavrusa impacted international and non-international students' academic performance and confidence in doing mathematics, social integration in the classroom, and social integration outside the classroom. The data suggested that both types of students valued MathChavrusa's ability to improve their learning experience through increased contact with their classmates. However, non-international students had fewer mentions of incompatible scheduling and personalities being roadblocks for learning. International students cited these concerns as well as cultural differences between partners as roadblocks to learning. Our findings also implied that the mode of classroom interaction (i.e., in-person, hybrid, and online) had an impact on how learners experienced MathChavrusa-style learning, with hybrid learning reported as being the worst because of difficulties communicating between students during class.
(Received September 12, 2023)

1192-97-32911
Adam Castillo, University of Texas at Arlington, Pablo Duran Oliva, Virginia Commonwealth Universtiy, Eddie Fuller*, Florida International University, Laird Kramer, Florida International University, Geoff Potvin, Florida International University, Charity Watson, Florida International University. Improving Student Learning Using Active Learning Approaches in Calculus: the Modeling Practices in Calculus Curriculum.
Calculus is a gateway course that occupies an important role many STEM disciplines. Students at universities and colleges often struggle with the traditional format of the course, and this struggle can limit their access to STEM degree programs. We conducted a pragmatic randomized trial in order to compare an active learning focused classroom approach, the Modeling Practices in Calculus curriculum, to the existing instructional style, a predominantly lecture-based format. From an initially assigned population of 1019 students who sought to enroll in the course, a sample of 811 university students were allocated to 16 treatment and 16 control sections taught by 19 instructors over three semesters at a large, R1 Hispanic-serving institution. In this talk we will present the results of a mixed-effects model used to analyze the outcome data from the study. Learning outcome measures were derived from embedded final exam items that were graded anonymously by independent teams using a calibrated rubric. Effect sizes for student learning outcome gains in the treatment condition were found to be medium-large, establishing strong evidence for the classroom practices implemented as a part of the curriculum. Subsequently all students in Calculus 1 now experience the new curriculum and success rate increases have been sustained for over two years so far. (Received September 13, 2023)

Amanda Lake Heath*, Middle Tennessee State University. Implications for the Teaching and Learning of Proof: Student Perceptions of Individual and Group Creativity. Preliminary report.
Feeling creative and viewing mathematics as a creative subject have been seen to promote high self-efficacy and positive identity formation in mathematics. Moreover, students often engage in collaboration with their peers outside of the classroom and in inquiry-oriented courses. To gain insight into how students view themselves and their peers as creative during collaboration, student-written narratives describing their experiences during a collaborative proving exercise in an introduction-to-proof course were qualitatively analyzed to reveal themes among student perceptions of how they (and their group) functioned creatively during a proving exercise. In this presentation, I will describe the major themes of studentreported actions and moments within collaboration that made them feel creative as well as the implications these student perceptions of creativity and collaboration may have on both informal learning and inquiry-based mathematics instruction. (Received September 13, 2023)

## 1192-97-32939

James A M Alvarez*, National Science Foundation. Interpreting Data in STEM Settings. Preliminary report.
The National Science Foundation (NSF) plays a pivotal role in advancing STEM (Science, Technology, Engineering, and Mathematics) education in the United States. The Directorate for STEM Education at NSF supports research, development, and implementation of data-centric strategies that empower educators, engage students, and improve STEM outcomes. Relevant ways that making sense of data plays a role in NSF STEM education proposals and projects will be discussed as well as important aspects of using data outcomes to re-imagine connections among stakeholders in the Pre-K-20 mathematics community.
(Received September 13, 2023)

## 1192-97-32955

Mary Ann Barbato*, Fitchburg State University. OER for Algebra, Patterns and Functions course for Elementary Educators. With the support of the ROTEL (Remixing Open Textbooks through an Equity Lens) grant, Massachusetts state institutions of higher education have been developing OER accessible to students with various learning types and backgrounds. This presentation summarizes the creation of an OER titled Algebra, Patterns and Functions for Elementary School Teachers: A Workbook Approach, created by adapting a combination of relevant existing OER materials. The text aims to use a universal language with clarity of expression that is reader friendly for all. It also includes word problems and activities that strive to be relevant to a variety of cultures and genders. Mathematical concepts are defined in multiple ways and extra explanations are included to help English Language Learners. A variety of examples and activities (including scaffolding) are incorporated to accommodate different learning types. The text includes corresponding mathematics standards from both the National Council of Teachers of Mathematics (NCTM) and the Massachusetts Mathematics Curriculum Framework which is based on the Common Core State Standards (CCSS).
(Received September 13, 2023)

## 1192-97-32992

Frank Uhlig*, Auburn University. Overcoming the challenges of modernizing the first Linear Algebra course. Preliminary report.
This talk studies plans of the ILAS LINEAR ALGEBRA Education Study group.
(Received September 13, 2023)

## 1192-97-33007

Ronald Page*, ELITE Public Schools (Board). Reflection: A Transition from Pre-Requisite to Co-Requisite Instruction and its Impact on Expectations in a College Math Classroom.
While CCSSM have significantly impacted pedagogy in the K-12 system over the past decade, AB705 and AB1705 will have similar implications at the Community College level in California as time moves forward. This presentation is a reflection on the transitional steps taken to change curricula from a pre-requisite to a co-requisite model and a glance at student experiences and expectations within community college math classrooms. This exploration of how policy changes should include varied levels of support/implementation will highlight prospective new expectations and outcomes for many faculty and students to come.
(Received September 13, 2023)

## 1192-97-33058

Tuto LopezGonzalez*, San Francisco State University. The intersection of Arts and Mathematics cognition.
This work examines the potential of using the Pirie-Kieren ( $\mathrm{P}-\mathrm{K}$ ) framework to explore the development of mathematics understanding and art appreciation when engaging in an action-based embodied cognition task involving the Fibonacci spiral and the wood-block print, The Great Wave off Kanagawa. Task-based interviews were conducted with four pairs of pre-calculus students. Qualitative inductive analysis revealed how the P-K framework is useful in unpacking the math and visual-rich aspects of cognition. The implications of this work include designing mathematical learning experiences that integrate artwork storytelling, and action-based embodied cognition tasks to support students' entering mathematical formalizing when abstracting mathematical concepts. Future work may investigate the electrical brain waves when engaging in such mathematics learning experiences to develop new interventions and instructional strategies to support the potential integration of arts and mathematics.
(Received September 13, 2023)

From the parents' perspective, mathematics should play a crucial role in preparing students for future life. Integrating parental perspectives into mathematics education can contribute significantly to advancing social justice within the field. However, many teachers and preservice teachers (PSTs) may hesitate to invest the necessary time and effort in involving parents in mathematics education, often due to a deficit mindset regarding parents' perceived lack of knowledge or approach to mathematics. This hesitation can lead to a gap in mathematical understanding between parents and their children Sometimes, children may not fully appreciate their parents' mathematical knowledge, while some parents may be unfamiliar with their children's experiences with mathematics at school. This study is built upon a project that establishes sustainable connections among institutions, schools, and communities by incorporating Family Math Night into a mathematics methodology course. The project has two primary goals: (1) to foster reciprocal understanding among PSTs, families, and children regarding mathematics learning, and (2) to transform fieldwork within teacher preparation programs by integrating community knowledge and linking it with practical and methodology courses. This study explores the impact of participating in Family Math Night (FMN) on the awareness of culturally responsive mathematics teaching among preservice teachers (PSTs). The study involved sixteen PSTs who were enrolled in an elementary methods course at a Hispanic Serving University on the West Coast. In collaborative groups, they designed and implemented activities during a Family Math Night event hosted at a local elementary school, primarily serving a Hispanic/Latino population. Data were collected through class discussions, reflections, and debrief sessions. Thematic analysis was used to interpret the PSTs' reflections and debriefings, resulting in emerging themes from the data analysis. The findings demonstrate that PSTs benefited from this community-based field experience, resulting in personal transformations in their perception of mathematics and a recognition of the potential for parental involvement in mathematics.
(Received September 13, 2023)

## 1192-97-33182

Lily Lum*, City College of San Francisco. Advising the post-Common Core Student - trends and observations.. Preliminary report
I will share my experience as the Math Advisor at City College of San Francisco. I will start by describing the population of students that I work with in my role - enrollment data as well as completion data from San Francisco Unified School District broken down by equity populations. We will discuss the SLAM (Statistics and Liberal Arts Math) and STEM (Science Technology Engineering and Mathematics) tracks at City College, and examine the populations of students in the two tracks, as well as their success rates. My day to day experience involves meeting with a lot of students who are interested in taking calculus, citing a desire to study Data Science. But are these students representative of the whole student body? We will look at representation of the various equity student populations in the STEM track, and discuss ways that educators can be instrumental in addressing persistent equity gaps.
(Received September 13, 2023)

## 1192-97-33209

Cathy B Kessel, Consultant, Liping Ma*, HCC. Looking back: Two ancient approaches illustrated in the past century of elementary school mathematics. Preliminary report.
Euclid's Elements (Greece, c. 300 BCE) and The Nine Chapters on the Art of Mathematics (China, c. 200 BCE) are two famous repositories of mathematical knowledge which were used as textbooks for hundreds of years. Although their aims were the same-to organize and present a body of mathematical knowledge-their pedagogical approaches are different. The Elements is organized deductively, presenting theorems derived from "first principles": postulates, definitions, and "common notions." In contrast, each chapter of the Nine Chapters deals with a type of practical problem. For example, Chapter 1, Field Measurement, shows how to find the areas of fields with various shapes: rectangle, triangle, trapezoid, circle, bowl, portion of circle, and annulus. Within each section of the chapter are "problem groups" consisting of a prototype problem and more complicated variants that lead learners to deeper understanding. Early twentieth-century US arithmetic textbooks followed a Euclidean approach, but this was later abandoned. We suspect that the Nine Chapters tradition was a major reason why elementary mathematics with an underlying Euclidean system survived in China during the twentieth century. Aspects of both approaches appear in the Common Core State Standards. These can be emphasized or obscured in textbooks and other instructional materials. Consciousness of the two approaches and their different pedagogical features may be an aid for designers of these materials.
(Received September 13, 2023)

## 1192-97-33210

Jean Nanjo*, City College of San Francisco. Before, after, and behind the scenes: What high school teachers, community college instructors, and parents could learn by walking in each others' shoes. Preliminary report As a former public high school teacher, current community college instructor, and parent of students in both systems, I have a unique insight into where students have been, where they are going, and what it's really like to go through it all. I'll be sharing my experiences, and how they have affected the way I teach and lead, both as an instructor of co-requisite support courses, and as the leader of my department's community of practice, where we engage in models of collaboration that continually result in truly transformative changes to our teaching, including peer reciprocal observations, consultancies, and more. (Received September 13, 2023)

## 1192-97-33215

Ekaterina Fuchs*, City College of San Francisco. AB705 and AB1705-policy shaping community college mathematics education.. Preliminary report.
Assembly Bills 705 and 1705 have greatly impacted mathematics education at California Community Colleges. In this session, I will give an overview of the two bills, and discuss some of the consequences, both intended and otherwise, of their implementation. Speaking from my experience as Mathematics department chair, as well as President of the Northern California affiliate of the American Mathematical Association of Two Year Colleges (AMATYC), I will engage participants in a conversation and reflection around the impacts of legislative solutions to nuanced problems facing mathematics educators. (Received September 13, 2023)

Russell Campisi*, Pleasanton Unified School District. California Core State Standards - impacts on special education and other programming in public school settings.. Preliminary report.
Russell Campisi is secondary school principal with extensive experience in K-12 education. He will be presenting on the updated California Core State Standards and how the changes overlap with special education and other academic programming in public school settings. Please come for a fun and engaging presentation that will be interactive and timely. (Received September 13, 2023)

## 1192-97-33236

Brandy S. Wiegers*, College of Idaho. Bringing Math Circle Problem Solving into the Central Convergence REU. Preliminary report.
For all of you out there who run Math Circles or REUs, I am making an argument that the problem-solving and communitybuilding that we do in both programs are so similar that we should be talking a lot more. The Central Convergence REU was a 3-year NSF-funded REU program at Central Washington University (DMS-2050692). Over the three-year program, we brought together more than thirty undergraduates to have their first research experience. By using Math Circle problems during the first week of the program we created an environment that encouraged open-ended exploration rather than worksheet final answers. This presentation will share the problems used, the pedagogical approaches that we used with these problems, and the many ways our students then took the introductory problems and turned them into full summer research work. The presentation will also discuss how we brought this work back into our Math Circle, we some of our local summer undergraduates creating and presenting in the after-school program during the following school year. (Received September 13, 2023)

1192-97-33262
Jeffrey S Meyer*, California State University, San Bernardino. A Student-Centered Curriculum For Teaching Linear Algebraic Proofs.
How do you teach students formal linear algebraic reasoning? Your answer to that question informs your choice of textbook and pedagogy. My answer to that question inspired me to create a student-centric curriculum for a proof-based second course in linear algebra in which students construct all the proofs themselves. In this talk, I will go over important elements of this curriculum, including the textbook I wrote and pedagogy I implement to support student-driven learning. I will share some of my experiences teaching with this curriculum, some student feedback, and some observed learning outcomes. This is part of ongoing research to better understand the teaching and learning of proof in a second course in linear algebra. (Received September 13, 2023)

## 101 Teaching and learning

## 1192-101-25731

Michael Olinick*, Middlebury College. Analyzing the Richardson Arms Race Model at the Beginning, Middle and End of a Differential Equations Course.
The Richardson Arms Race Model describes the dynamics of arms expenditures of two nations, each of which is driven to increase its spending by the spending level of its rival, but faces internal resistance as its own expenditures grow and experiences external drivers of good or ill will. The original formulation of the model is $x^{\prime}=a y-m x+r, y^{\prime}=b x-n y+s$ where $x$ and $y$ are arms expenditures as functions of time, $a, b, m, n$ are positive constants and $r, s$ are constants of arbitrary sign; differentiation is with respect to time. Phase plane analysis of the model early in a differential equations course reveals the possibilities of sinks, sources, and saddle points. Students can find exact solutions using eigenvalues and eigenvectors in the middle of the term. They can explore more realistic models by replacing the constant parameters with their own choices of continuous functions and apply numerical solution methods to simulate the behavior later in the semester. In an advanced DE course, delay differential equations can replace the ODEs as neither nation may be able to change its arms budget instantaneously in response to its opponent. .
(Received July 06, 2023)

## 1192-101-26638

Kinnari Atit, University of California Riverside, Robert C Wilbur*, University of California Riverside. The Role of Spatial Cognition in Postsecondary Math Learning. Preliminary report.
Spatial cognition entails reasoning, visualizing, and mentally manipulating objects in both real and imagined spaces. Research has shown that spatial processing factors are intimately linked with student understanding across all educational levels and branches of mathematics. Moreover, postsecondary mathematics instructors and researchers assert the importance of processing and thinking about spatial information to understanding the calculus domain. In postsecondary gateway mathematics courses such as precalculus where attrition from STEM pathways is highest, especially for women and underrepresented minority (URM) students, efforts to improve performance and retention can benefit from a consideration of these factors. In this presentation, the role of spatial cognition in mathematics and data describing introductory postsecondary students' spatial skills and their relations to student demographics will be discussed.
(Received August 02, 2023)

1192-101-26875
Alec M. Bodzin, Lehigh University, Grace I.L. Caskie, Lehigh University, Lisa A. Grossbauer*, Lehigh University. $A$ Measurement Tool for the Impact of Self-Beliefs upon Performance on College Math Placement Exams. Although science and engineering (S \& E) fields continue to grow, certain groups including first-generation students remain underrepresented among S\&E degree recipients. Mathematics, specifically calculus, is often the gatekeeper to entering STEM majors that open the pathway to financially lucrative careers in S\&E. Although incoming college students typically take math placement exams to determine their first college mathematics course, the use of noncognitive measures, such as math selfbeliefs, in addition to cognitive skill assessments has been recommended due to high rates ( $25 \%-33 \%$ ) of misplacement with
skill assessments. A self-beliefs survey about math performance was collected for 355 incoming first-year college students ( $49.9 \%$ women; $25.2 \%$ first-generation). This study compared a hypothesized four-factor model (self-efficacy, expectancy, value, and cost) and a three-factor model (ability beliefs, value, and cost) for the self-beliefs data. The theoretical framework for the three-factor model was expectancy value theory (EVT) while the four-factor model was a blend of EVT theory and Bandura's social cognitive theory. Confirmatory factor analysis indicated the four-factor model was a better fit for the data (CFI $=.949, \mathrm{TLI}=.929$, RMSEA=.078). Factorial invariance was also tested across first-generation status to ensure measurement equivalence. Analyses indicated that the four-factor model for self-beliefs achieved strict invariance (CFI=.93, TLI=.92, RMSEA=.06) indicating that the measure's structure, factor loadings, item intercepts, and measurement errors were invariant across first-generation and non-first-generation students. Thus, the self-beliefs measure is a valid way to assess noncognitive aspects of math performance that may help to promote entry into STEM majors and careers. (Received August 07, 2023)

1192-101-26921
Christopher Oehrlein*, Oklahoma City Community College. First-Order Linear Models with Non-Smooth Forcing: Learning by Exploring. Preliminary report.
Introducing the Laplace Transform early in an ODE course, right after First-Order Linear Models, has several advantages. Students get to see more first-order linear equations, reinforcing the concepts about the form of solutions to linear equations Students do not have basic concepts about the Laplace Transform and its uses crammed and rushed at the end of the term. Students see forcing functions that are not continuous or differentiable early in the term and have time to become comfortable with notation and application of unit step functions. From a teaching and learning perspective, the early introduction of unit step functions and the Laplace transform also has some advantages. There is more time for exploration of similarities and differences among different "types" of first order linear equations. Laplace Transforms can be used for nonhomogeneous linear systems without having to introduce more linear algebra theory. There may be more time at the end of the course to spend exploring concepts about convolution and its applications. I will share a couple of models that I use for exploration and concept development when teaching the Laplace Transform.
(Received August 07, 2023)

## 192-101-26959

Gizem Karaali*, Pomona College. Oblique Strategies for Classroom Poetry. Preliminary report.
Oblique Strategies was originally "a card-based method for promoting creativity jointly created by musician/artist Brian Eno and multimedia artist Peter Schmidt, first published in 1975" (Wikipedia). In some of my classes integrating mathematics and the arts, I have begun to ask students to use prompts from an Oblique Strategies app to write form-constrained mathematical poetry. The resulting combination of randomness and constraint leads to much fun in the classroom, and sometimes, sublime poetry. In this talk I describe the current version of my Oblique Strategies Poetry assignment, and I share some resulting work
(Received August 08, 2023)

1192-101-26967
Gizem Karaali*, Pomona College. ChatGPT and New Ethical Considerations for the Mathematics Classroom. Preliminary report.
ChatGPT, a large language model (LLM) artificial intelligence (AI) engine, burst onto the educational scene in November 2022. Since then, educators around the world have been clamoring to figure out its full implications for the college classroom. As mathematics educators, we are somewhat more experienced in the disruptive effects of new technology (e.g., the electronic calculator, WolframAlpha, etc). However, LLM technology, and more broadly, AI, offers some new challenges (as well as some new opportunities) for us as well. In this talk, I open up and analyze some of the standard concerns educators have about AI in the classroom that have ethical implications (such as access and academic integrity) with a view towards constructive solutions.
(Received August 08, 2023)

## 1192-101-27178

Brendan W. Sullivan*, Emmanuel College. Suggestions for teaching voting methods in a general education mathematics course.
In their Curriculum Guide from 2004, the MAA's Committee on the Undergraduate Program in Mathematics expressed concern that "students and faculty outside mathematics perceive mathematics departments as uninterested in adapting instruction to new circumstances, especially to the needs of non-majors and those in entry-level courses. Indeed, many view the formal study of mathematics as irrelevant or tangential to the needs of today's society." Sadly, almost two decades later, these perceptions of undergraduate mathematics education persist. However, I believe we have an opportunity to combat these perceptions by capitalizing on the growing conversation in the United States around electoral reform and voting systems. In this talk, I will share some insights gleaned from my experience in recent years teaching a "math for liberal arts" course and writing a book about voting methods designed for a non-technical audience (i.e. the students in that course). I will give suggestions about in-class activities, assignments, and course design that will help students approach the topic of voting methods with an open mind and leave your course with improved skills in problem-solving and quantitative literacy, along with a newfound appreciation for the relevancy of mathematics to our modern world.
(Received August 11, 2023)

1192-101-27248
Maritza M. Branker*, Niagara University. Viewing our students as ambassadors of the discipline: a new approach to a mathematics senior seminar.
The capstone course is traditionally an opportunity for students to delve deeper into the mathematical discipline. This talk outlines the rationale and details of designing a senior seminar course on the theme of the relevance of mathematics to society. Viewing students as ambassadors of the discipline provides the flexibility to accommodate math majors, preservice teachers with a math concentration and actuarial science majors within a single section of the course. More importantly it
provides a venue for students to articulate their personal philosophy on the significance of mathematics and grapple with ethical ramifications of the discipline on our society. By the end they are cognizant of the need to consider not only if they are capable of crafting an effective solution to any given problem but also the implicit duty to avoid causing harm with their mathematical knowledge.
(Received August 12, 2023)

1192-101-27292
Steven Joel Miller*, Williams College. AMS Expects that every Mathematician will do their ODEs: From the Battle of Trafalgar to Calculus (or Nelson to Newton).
Differential equations successfully model numerous theoretical and real world problems, with examples often drawn from physics and, especially in the covid era, mathematical biology. We discuss a beautiful problem in a very different area that can be done easily in a first calculus class: modeling the famous Battle of Trafalgar, where Nelson's strategy led to one of, if not the, most decisive and lopsided naval victories ever. This example illustrates the power of creating good mathematical models that capture enough of the key features, and having techniques to solve the resulting differential equations. (Received August 13, 2023)

1192-101-27315
Prashansa Agrawal, University of California, Riverside, Catherine Lussier*, University of California, Riverside. Broadening Engagement in Postsecondary Math by Examining Instructor Active Learning Strategy Usage: Preliminary Report. Preliminary report.
Much research notes the benefit of embedding active learning strategies into curriculum to enhance student motivation and cognitive capability in STEM content fields, particularly in mathematics. Research on the efficacy of a new tool developed to aid this process as part of a California Learning Lab (CLL) grant developed while revising precalculus/calculus curriculum at the community college and university level will be reviewed. Specifically, the Mathematics-Fidelity of Implementation Tool (MFIT) was designed as a checklist to evaluate STEM-based pedagogical strategies contained within different curriculum resources (e.g., lecture, tutorial videos, handouts) and are being revised. Initial outcomes appear to indicate that student performance on Calculus resource videos was positively influenced by those videos with greater active learning strategies and scaffolding as measured by the M-FIT checklist. Implications for applying the M-FIT to assist mathematics educators who seek to make their content more inclusive and comprehensible to a wider audience is discusse (Received August 14, 2023)

## 1192-101-27331

Bryan Carrillo, Saddleback College, Catherine Lussier*, University of California, Riverside, Mark Lydon, Yuba College,
Dylan Noack, Yuba College, Mayya Tokman, University of California, Merced. Community Development: The joy and challenge when adopting innovative curriculum tools.
This panel seeks to support a lively discussion on community development methods for generating collaboration for adopting innovative mathematical curriculum tools or other learning strategies to raise the number, quality, and diversity of participating students in your classes. Questions to be addressed by invited panelists will include: revising curriculum with new evidence-based pedagogical strategies, building partnerships across two-year and four-year institutions, addressing realworld concerns (e.g., will new teaching methods water down content or reduce critical problem-solving rigor), engaging student interest, and the surprising benefits of forming interdisciplinary communities of practice. Audience members are encouraged to share their own experiences and challenges as part of the conversation. (Received August 14, 2023)

## 1192-101-27681

Li Zhang*, The Citadel. Promoting Technology in a Mathematical Modeling Class. Preliminary report.
In this talk, we present the use of technology in an undergraduate mathematical modeling class to obtain solutions. In particular, for mathematical models using differential equations where exact solutions are difficult for the students to obtain, students are taught to use computer software to obtain solutions numerically and graphically. This practice not only helps students who struggle with the techniques of obtaining solutions by hand, but it also provides our students an opportunity to learn and expand their technical skills.
(Received August 18, 2023)

## 1192-101-27787

Christoph Borgers*, Tufts University. Some thoughts on what we should teach in an undergraduate course on differential equations. Preliminary report.
A traditional course on differential equations is primarily about mathematical ideas and techniques of paper-and-pencil computation. Modeling and applications may well appear, but as a teaching vehicle that may help engage students, and that could probably be replaced by some other effective way of engaging students. I would like to build a course instead in which the applications are central, and the mathematical techniques are introduced only when and insofar as they help gain insight into the applications. I don't think it is a good idea to do this completely radically, but I have been trying to move in that direction. Such a course would be about planetary motion, epidemics, population growth, the extinction of populations, fluid flow, the law of mass action, mass-spring systems and the pendulum, vibrating strings, heat loss through walls, the synchronization of nerve cells, the flocking of birds, perhaps even the flocking of people towards one or the other opinion about some issue, or the flow of wealth from the poor to the rich or vice versa, with mathematical ideas introduced as needed The course would have to include a significant computational component (on the computer). I will report on my efforts and what motivates them, give examples, and speak about obstacles. (Received August 19, 2023)

Violeta Vasilevska*, Utah Valley University. Projects that Enrich Students' Technical and Communication Skills. Preliminary report.
In this presentation, we describe the structure of projects used in upper-level mathematics classes that enrich students' technical and communication skills. In addition to the structure, the expectations of the assignments and the criteria for grading will be shared. The objectives of these projects are to enrich students' knowledge with additional topics from the course material, research a real-life problem where these concepts are applied, deepen their collaborative skills, and improve their technical writing and presentation/communication skills. Namely, these projects require students to do research about their chosen topic, produce a written math paper, present in class about their research topic, and to do peer evaluations on another project and the in-class peer presentations. In addition, survey feedback collected about some of those projects will be discussed. These projects provide students with opportunities to improve their thinking and problem-solving skills, as well as enhance their math technical and communication skills.
(Received August 25, 2023)

## 1192-101-27800

Biyong Luo*, University of Massachusetts Dartmouth. Differential Equations for a Changing World: How to Engage Students in Learning and Applying Differential Equations.
Building upon foundations taught in calculus I and II, the Differential Equations course potentially has tremendous impact on students' field of study and future careers. During the last ten years, I have taught a condensed-format Differential Equations summer course over five weeks with in-person sessions as well as an asynchronous (fully online) option. Our course objectives were expansive: to understand the fundamental methods of ordinary differential equations (ODEs), to master qualitative analysis and numerical methods in solving nonlinear differential equations, and to gain substantive understanding of mathematical modeling with ODEs or other advanced methods. This abstract will highlight my strategies to optimize students' engagement in class and how I empower students in learning through a dynamic rigorous curriculum including the use of multimedia, interactive discussion forums emphasizing real-life application, and daily quizzes to emphasize key problemsolving skills and to decrease test anxiety. These opportunities are designed to be low-stake and the goal is to augment mathematical fluency as students communicate mathematical concepts and explain their problem-solving approach. The outcomes and student feedback to this dynamic curriculum have been impressive. It is the emphasis on how differential equations can applied to real-world situations, engaging students outside the classrooms through online discussion forums, empowering them and building confidence through leading and participating in group exercises and presentations in a lowstake environment that I believe will be able to show the 21st-century learner that mastery of differential equations can be impactful across a variety of careers.
(Received August 20, 2023)

1192-101-27815
Blain Patterson*, Virginia Military Institute, Sarah Elizabeth Ritchey Patterson, Virginia Military Institute. The Use of Chat GPT in Mathematics and Physics Departments. Preliminary report.
ChatGPT is an Artificial Intelligence (AI) large language model released by the company OpenAI in November 2022. Its use by students has recently become a controversial topic due to concerns over academic integrity and hinderance to critical thinking. This study looks to determine how ChatGPT can be used by undergraduate students in physics and math while addressing these concerns. In particular, it aims to answer three research questions: 1 . How can ChatGPT enhance learning without hindering critical thinking and problem-solving skills? 2. Are there certain types of classwork for which ChatGPT is appropriate to use? 3. What kinds of citations are appropriate for assignments when using ChatGPT? These questions are answered using both survey and focus group data. Both sets of data are gathered from professors from Virginia Military Institute's (VMI) Department of Physics and Astronomy and Department of Applied Mathematics. The survey provides quantitative statistical data, while the focus group provides qualitative data that is analyzed using a three-step qualitative coding process. Results show some inconclusive answers, as professors believe the technology is not in danger of hindering students at the moment because of its inaccuracy, but may become more of an issue as the technology advances. (Received August 21, 2023)

## 1192-101-27836

Behzad Djafari-Rouhani*, UTEP. A Nonlinear Oscillator with Damping in Continuous and discrete time.
We consider a second order differential equation modeling a nonlinear oscillator with damping. With suitable assumptions on the damping coefficient, the forcing term and the nonlinear operator, we give necessary and sufficient conditions for the equation to have a bounded solution. In this case, we prove convergence theorems for the solutions, and show that the limit is a zero of the nonlinear operator. We also consider the second order difference equation corresponding to the discrete version of the above equation and show similar results for the solutions to this discrete version. Finally, we provide some examples of applications and numerical experiments for our results.
(Received August 21, 2023)

## 1192-101-27920

Li Zhang*, The Citadel. The Challenging Task of Teaching Undergraduate Differential Equations. Preliminary report.
In this talk, we present an overview of an undergraduate ordinary differential equations course offered at our institute, and the challenges that we are facing in teaching the course. Also, we present several ways implemented by our instructors for improving teaching and students' learning experience in this course.
(Received August 22, 2023)

Hawaiian fishpond, coastal region conservation, and more. Immersing the participants outside and planting seeds of Kupuna (ancestor) consciousness, to paddle and do their own exploring away from what is seen as the typical classroom, allows them to gather their own experience and make sense of mathematical phenomena firsthand. We will be presenting a lesson plan for a real-life example of this execution, and share the reflections from the teacher participants as evidence of impact. (Received August 22, 2023)

## 1192-101-27969

Girija Nair-Hart*, University of Cincinnati, Clermont. Concept Equilibration, Aligning Formal Concept Definition with Student's Concept Images - the Focus of a Redesigned Calculus 1 Course. Preliminary report.
During the Fall Semester of 2023, I am teaching a university-level Calculus 1 course that I redesigned. In my class, in a constructivist classroom setting, students generate knowledge by themselves and in groups. The interactive classroom activities are geared towards attaining "equilibration" between students' "concept images" and the formal "concept definition" of the mathematics concept discussed. The core calculus concepts of limits and derivatives are first introduced in the contexts of real-life applications and later facts, theorems, and problems are discussed. Labs for hands-on learning and several onepage papers detailing students' understanding of various concepts are collected. To maintain student's focus, instant responses are collected via a point-solutions response feedback system. Some of these assessment methods help lessen student fatigue that is caused by traditional assessment methods. A mathematics project in collaboration with a local high school that was aimed at STEM outreach could provide students with additional off-campus learning experiences. In addition to these assessment activities, there will be regular homework, in-class exams, and a final examination. During my JMM presentation, some of the classroom activities and assessments will be highlighted along with a discussion of how these activities positively impacted student motivation and their learning of calculus in the re-designed calculus course. (Received August 23, 2023)

## 1192-101-28050

Feryal Alayont, Grand Valley State University, Korana Burke, University of California, Davis, Erin Leigh Griesenauer, Eckerd College, Jeremy Shaw*, Oregon State University-Cascades, Rohit Thomas, University of California, Davis. A General Framework for Incorporating Ethical Reasoning into Mathematical Modeling.
Ethical reasoning is an essential component of applying mathematical modeling in solving real-life problems, both in research and business settings. Our mathematical answers exist in a context of a larger system and have implications on the lives of others, the planet, and future generations. However, mathematics instruction often treats the mathematical work as if it exists in a vacuum devoid of context and omits careful consideration of stakeholders, validity of data, assumptions made, and limitations of the analysis. In this talk, we present a general framework that can be used to modify any mathematical modeling problem or project in a way to help students focus on the missing ethical reasoning perspective in the problem/project. Our framework is generalized in the sense that it can be applied to any course at any level, including the K-12 instruction. The framework provides flexibility to instructors in terms of the types and level of questions that can be asked, as well as the quantity. In addition to describing the general template, we will demonstrate the use of the framework on a specific example to clarify the application of the framework. (Received August 24, 2023)

## 1192-101-28088

Mihhail Berezovski*, Embry-Riddle Aeronautical University. Incorporating AI-Powered ChatGPT in Undergraduate Research: Enhancing Learning, Collaboration, and Ethical Awareness.
As the landscape of education evolves, the integration of Artificial Intelligence (AI) presents transformative opportunities. This talk delves into the experience of leveraging ChatGPT in the Embry-Riddle Aeronautical University (ERAU) Research Experiences for Undergraduates (REU) program, Research Projects in Data-Enabled Industrial Mathematics, over the past two years, along with its application in project-based classes. This presentation emphasizes the advantages of using ChatGPT as a cutting-edge tool for enhancing learning outcomes, encouraging collaborative learning experiences, helping with coding tasks, report writing, and even assisting in the structuring of research publications. The talk additionally discusses the moral issues surrounding the inclusion of AI in mathematics instruction. It covers important issues including data privacy, transparency, and any biases that could surface when using ChatGPT and other AI models. Examining these ethical issues can help educators and institutions approach AI integration with a greater understanding of the duties and factors required to maintain an equitable and inclusive learning environment.
(Received August 25, 2023)

## 1192-101-28094

Viktoria Savatorova*, Central Connecticut State University. Parameter Estimation and Sensitivity Analysis in Mathematical Modeling with ODEs
This presentation provides insights into the implementation of parameter estimation and sensitivity analysis within the context of mathematical modeling using ordinary differential equations. Illustrative examples include models of infectious disease spread and Lotka-Volterra dynamics. Parameter estimation is executed through the utilization of experimental data. Numerical examples are provided. The application of parameter sensitivity analysis deepens the understanding of system's behavior. Both local and global sensitivity analyses are undertaken, with their results visually depicted.
(Received August 26, 2023)

## 1192-101-28190

Donna A Dietz*, American University. Parallel Projects in Basic Statistics.
Curating data sets for student use can be a daunting task. In order for course objectives to be met, data sets must be consistent between students, yet data should also be somewhat different in order to stimulate student interest and to encourage independent work. This talk will show how to design engaging "Parallel Projects" by utilizing public data sets which are then split along one or more categorical variables to create smaller data sets which are structurally identical yet yield very different analytical results. The talk will also cover tricks and tips on how to do this efficiently, to reduce grading strain while optimizing student learning and excitement.

1192-101-28568
Sami S Kanderian*, WikiModel LLC. WikiModel: A Web-based Software Application to Rapidly Create, Simulate, Fit and Share Mathematical Models. Preliminary report.
WikiModel (https://www.wikimodel.com) was created to enable teachers, students, scientists, and engineers to create custom mathematical models consisting of any number of closed form closed form and/or Ordinary Differential Equations (ODEs) with no programming. The cloud-based application requires no installation and is run via a web-browser to facilitate rapid implementation. Equations are typed in as they appear in a text book. ODEs are automatically integrated via Runge-Kutta methods. Once created, these models can be simulated with parameters whose values are user defined or fit to experimental data whereby optimal parameters are identified. The software implements non-linear or linear least squares methods as required. Desired state variables and datasets are selected from a drop down to be output in one or more plots. Once created, models and datasets can be shared to the public, a defined group of users, or remain private. The library of public models already contains pre-existing model examples in various scientific disciplines. WikiModel was built to rapidly simulate, and fit models in academia or industry. It is a tool that will hopefully motivate students and scientists to explore and benefit from the value mathematical models bring in solving real world problems without being bogged down in their implementation. (Received August 31, 2023)

## 1192-101-28734

Dylan Noack, Yuba College, Roberto C Pelayo, University of California, Irvine, Yat Sun Poon*, University of California, Riverside, Christina White, Saddleback College. The Challenges and Options of Embedding Sciences in a Calculus Course. Preliminary report.
Calculus is known to be a fundamental tool for modern sciences. While some institutions may afford specialty courses or methods of calculus courses such as calculus for engineers and calculus for biologists, many institutions could only afford a general calculus course. Other institutions may also choose to address some of the general principles of calculus rather than the method of calculus. In the latter context, we invite a panel of scientists and mathematicians from different sectors of the California institutions to discuss the challenge and options of embedding sciences effectively to enable STEM students' success beyond their calculus course.
(Received September 02, 2023)

## 1192-101-28804

John A. Rock*, Cal Poly Pomona. Parsing analysis with arbitrarily close. Preliminary report.
The definitions of convergence and continuity are notoriously difficult to teach and to understand. In this talk, we present a parsing of these concepts through a technical definition for arbitrarily close: A point is arbitrarily close to a set if every neighborhood of the point contains an element of the set. The transition to convergence then follows from changing a few words: A point is the limit of a sequence if every neighborhood contains a tail. Continuity is the preservation of closeness: A function is continuous when points and sets arbitrarily close in the domain have images arbitrarily close in the range. The talk focuses on how the scratch work and proofs for arbitrarily close lead to that for convergence and continuity.
(Received September 03, 2023)

## 1192-101-28948

Jerome Zegaigbe Amedu*, University of New Hampshire, Ruby Ellis, North Carolina State University. An In-Depth Analysis of Informal Learning Effects on Mathematics Teacher Knowledge and Practices..
While there is a growing interest in researching teacher informal learning and an increasing acknowledgment of the critical role it plays in teacher continuous professional development, fewer studies focus on its effects or outcomes. This study, part of a larger mixed-methods project, explored math teachers' perceptions of informal learning's impact on their knowledge and practices based on in-depth interviews and analysis of closed and open-ended survey items administered to a sample of 258 math teachers across the United States. The study found positive correlations between math teachers' participation in informal learning activities (media interaction, colleague interaction, stakeholder interaction, individual reflection) and their selfreported efficacy in select TPACK domains (content knowledge, pedagogical knowledge, technological knowledge, and technological pedagogical content knowledge domains). Specifically, participants' frequencies of media interaction, colleague interaction, and individual reflection significantly predicted their self-reported efficacy in at least one TPACK domain. No significant differences in frequencies of participation in informal learning activities were found based on teacher career stage or gender. Additionally, the interviews and analysis of open-ended survey responses revealed informal learning substantially contributed to math teachers' content, pedagogical, and technological knowledge, alongside enhancing their motivation and emotional well-being. Predominantly, informal learning benefited teachers' pedagogical knowledge; teachers also sought informal learning opportunities mostly to meet immediate practice needs. Besides primarily aiding immediate problem-solving, informal learning further contributed to enduring changes in equity considerations and student engagement approaches in the classroom. The results of this study corroborate similar studies that challenge the notion of teachers undertaking formal learning activities as their primary means of developing professionally.
(Received September 04, 2023)

## 1192-101-28966

Bryan Carrillo*, Saddleback College. Implementing a Principle-based Calculus Curriculum.
A variety of active learning pedagogies support the implementation of a principle-based Calculus curriculum in programs that require both high standards for student learning and an entry point with minimal prerequisites. This talk will focus on concrete examples, classroom observations, and lessons learned. We will also discuss some challenges to implementation at both the classroom and institutional-level. Finally, we will discuss how our curriculum and implementation can achieve the goals of California Assembly Bills 705 and 1705.
(Received September 05, 2023)

Amanda J. Mangum*, Converse University, Nicole Marie Panza, Francis Marion University. Modern Pen Pals in ODEs. One concern we have as educators is whether our students can effectively communicate mathematics. The Modern Pen Pals project seeks to address this concern through written, oral, and interpersonal communication. ODE classes at two different universities created video lessons and supporting materials, including homework, quizzes, and solutions, to send to the other university, "pen pal" style. Making our students the teacher of their topics gave them an opportunity to take an in-depth look at the material and encouraged them to think deeply about the details they needed to convey to an audience who has never seen this material before. (Received September 05, 2023)

## 1192-101-29058

Amanda J. Mangum*, Converse University. Benefits and Challenges of Finding Data Aligned with Students' Interests.
Data intersects almost every field now and provides a natural connection to students' other interests. This talk will discuss several data-based projects ranging from modeling a network in $X$, formerly known as Twitter, to predicting the amount of aid a local nonprofit will give to an incoming client. Outcomes and challenges will be discussed along with ideas for projects in various settings.
(Received September 05, 2023)

1192-101-29103
Dywayne A Nicely*, Ohio University Chillicothe. An Online Synchronous Delivery of Discrete Mathematics.
Discrete Mathematics is the highest-level mathematics courses that we regularly offer at the regional campuses of Ohio University and the course is mainly populated with students who are pursuing a Middle Childhood Education degree with a mathematics concentration. Pre-COVID, the course has been delivered on each regional campus through traditional face-toface lectures with justifiable enrollments and, during semesters with enrollment challenges, to multiple regional campuses via the Ohio University Learning Network (OULN) using a rotating group of faculty from different regional campuses. During and post-COVID, video conferencing technology has been utilized to deliver the Discrete Mathematics online synchronously using Zoom or Teams. In Fall 2022, it was my turn in the faculty rotation to teach Discrete Mathematics and the course was delivered online synchronously using Teams. This presentation will focus on the Fall 2022 online synchronous course and discuss pros (e.g. recorded lectures and chat features) and cons (e.g. maintaining exam integrity and combating A.I.) from the instructor's perspective. Additionally, student comments and grade analysis from the Fall 2022 course and past Discrete Mathematics courses from the instructor will be shared and compared.
(Received September 05, 2023)

## 1192-101-29153

Michael Hess Ernst*, United States Air Force Academy. 3D Printing and its applications towards learning and student comprehension in Calculus 3 Classrooms..
Both 3D Printing and Calculus 3 have something in common, 3 dimensions. Students often struggle with Calculus 3 due to this extra, foreign dimension to wrestle with. This creates the need for a different approach to making student activities and reviews that have the same or greater impact to comprehension as worksheets do for Calculus 1 or 2 . Here lies the entry point for 3D Printing in math classrooms. Newly printed objects and activities are presented to revamp the existing Calculus 3 toolbox for teachers so that learning, understanding, and even student enjoyment can be improved drastically across the board.
(Received September 05, 2023)

## 1192-101-29162

Peyam Ryan Tabrizian*, Brown University. If Eigendoit, then so can you!.
In this talk I use diagonalization to do some fun operations with matrices. Did you know that you can calculate a matrix to the power of another matrix? Or the matrix-th root of a matrix? Or matrix choose a matrix? The motivation of this comes from my 10 years of teaching linear algebra, as well as from my YouTube channel Dr Peyam which currently has over 156,000 subscribers and over 100 linear algebra videos.
(Received September 05, 2023)

## 1192-101-29165

Peyam Ryan Tabrizian*, Brown University. The Factoring Method that will change your life!.
In this talk, I will present a very elegant method for solving constant coefficient 2 nd order ODE such $y^{\prime \prime}-5 y^{\prime}+6 y=0$ by factoring out operators instead of using auxiliary equations. The advantage of this technique that it is much more direct and does not require any guesswork whatsoever. Moreover, it generalizes easily to inhomogeneous ODE and can even be used to solve some PDE such as the wave equation.
(Received September 05, 2023)

1192-101-29259
John A. Kerrigan*, Rutgers University. Syllabus 2.0: Using Video and Choice to Make the Math Syllabus Active.
The first iteration of our math syllabus study examined the implementation of two multimedia syllabi developed for large undergraduate lecture math courses: a graphic syllabus and a video version. Findings indicated that students preferred choosing which modality of syllabus they experienced and more than creating a video was needed to create an active experience with the syllabus. The current study allowed math students to select the type of syllabus (graphic or video) and required them to either complete annotations in the graphic syllabus or questions embedded throughout the video syllabus to engage students in an active learning experience with the syllabus. While no significant differences were found between groups in the perception survey and syllabus quiz, there was a statistically significant difference between the results of the two studies. Students in the current study performed better on the syllabus quiz and had better perceptions of the course than
those from the first iteration. This indicates that incorporating students' ability to choose the best syllabus mode for them and/or incorporating active learning into the syllabus review is essential for learning syllabus content and initial positive perceptions of the course.
(Received September 06, 2023)

1192-101-29277
Shanna Dobson*, University of California, Riverside, Julian Scaff, ArtCenter College of Design. Qurio: QBit Learning, Quantum Pedagogy, and Agentive AI Tutors. Preliminary report.
We propose Qurio, which is our new model of pedagogy incorporating the principles of quantum mechanics with a curiosity AI called Curio AI equipped with a meta-curiosity algorithm. Curio is an agentive AI tutor that has a curiosity profile that is in a quantum superposition of every possible curiosity type. By adapting to different learning types, Curio is more inclusive of a wide range of students, including those who are neurodivergent and underserved by more standardized educational models. We describe the ethos and tenets of Qurio, which we claim can create an environment supporting neuroplasticity that cultivates curiosity powered by tools that exhibit their own curiosity. By harnessing the principles of quantum theory and the neuroscience of curiosity, learning becomes inherently adaptable and dynamic, enabling students to explore multiple facets of the same topic simultaneously. The quantum curiosity algorithm, inspired by the multi-dimensional curiosity types of humans, encourages students to delve deeper into their interests, fostering a naturally engaging learning experience. We give examples of how to incorporate the principles of non-locality, complementarity, and quantum lateral thinking into pedagogy. We then futurecast the scholarship of curiosity and quantum pedagogy by hypothesizing and examining curiosity's event horizon. (Received September 06, 2023)

1192-101-29422
Shelby Stanhope*, U.S. Air Force Academy. Enhancing Multivariable Calculus Instruction with 3D-Printed Models. To support students' understanding of spatial concepts in multivariable calculus, I have incorporated several in-class activities which leverage 3D-printed models. During this talk, I will present two such activities: one centered on contour plots and the other on the topic of Lagrange multipliers. Students work in small groups, using models which they can write on using dryerase markers. I will delve into the pedagogical rationale behind these activities and share my insights concerning practical implementation.
(Received September 06, 2023)

1192-101-29513
Amy Futoma, King's College (PA), Karen B McCready*, King's College (PA). Reflective Learning in Mathematics. We all reflect on events, ideas, and interactions on a regular basis. Why not encourage students to do so in math class? We will consider how we can use reflection to help students to be more intentional about their learning progress and process. We will also look at reflective thinking in general and share a variety of reflective learning strategies that can be used in the classroom.
(Received September 07, 2023)

## 1192-101-29702

Timothy P Chartier*, Davidson College. Math-nificently Creative.
Math is a creative venture. Like a poet or artist, a mathematician often begins with a blank page awaiting the scribblings of written thought to unfold. Integrating creativity into the mathematical classroom folds this important part of mathematics into the classroom and more importantly demonstrates to students that their creativity is welcome and an asset in the field. This talk will demonstrate ways creativity has been used to teach class and been integrated into assignments in classes such as calculus, differential equations, numerical analysis and mathematical modeling
(Received September 07, 2023)

## 1192-101-29839

Leann Ferguson*, United States Air Force Academy. I can't read your mind ... what's on the paper must refect what you know and understand about a particular mathematical concept. Preliminary report. When we (the "experts") write-up a solution to a workout or word problem, we write something that is coherent, logical, wellexecuted, well-communicated, and correct. In other words, we build an argument that our answer is correct and answers the question(s) asked. We then demonstrate this ideal for our students (the "apprentices") in class - often with lots of commentary - and encourage them to shoot for this ideal when they write-up their own solutions. We then grade their solutions, and we must do so without the luxury of the students sitting next to us providing commentary on their work; their solutions must stand alone. We not only encourage them to write coherent, logical, well-executed, well-communicated, and correct solutions, but to also write something that accurately reflects what the individual student knows and understands about a given mathematical concept(s). Building to this expectation takes practice, specifically practice communicating in this manner, and to a level that can be understood and followed by someone else. I present an assignment, creatively called the "Communication Practice," I developed to help the students practice and achieve this standard before the higher-stakes events, and to become familiar with the grading rubric we use to assess their work. Proverbial lightbulbs come on when the students exchange their Communication Practices in class and have to read/interpret/figure out someone else's solution in order to award it a grade using the provided rubric. These lightbulbs illuminate the need and value of the communication ideal and more often than not, the students carry this insight into their studies (often helping them uncover misconceptions and/or procedural issues in time to fix them) and performance on the various graded events. Their solutions are the coherent, logical, well-executed, wellcommunicated, and correct solutions that reflect good knowledge and understanding about a given mathematical concept(s). (Received September 12, 2023)

Classroom Approach in an Introduction to Functions Course. Preliminary report.
After teaching multiple semesters of a college Introduction to Functions course using conventional methods, it became evident that student engagement during class sessions was lacking, and both student attitudes and success rates fell short of expectations. In a pursuit to elevate engagement, promote equity, and improve overall outcomes, I introduced a flipped classroom approach. In this talk, we discuss strategies, obstacles encountered, and outcomes of the initial attempt at implementing a flipped classroom model in the context of an Introduction to Functions course.
(Received September 08, 2023)

## 1192-101-29995

Katharine A. Ott*, Bates College. Limits and Bounds: Local changes and global ideas for teaching Real Analysis. In this talk, I will present on ideas for inclusive teaching practices in undergraduate real analysis.
(Received September 08, 2023)

## 1192-101-30048

Mckenzie West*, University of Wisconsin-Eau Claire. An Active Approach to Linear Algebra. Preliminary report. As most students' first step into proof-based mathematics, Linear Algebra is ripe for exploration in small groups. While teaching the course in a physical space designed for student collaboration, I was able to flip the course and rarely lecture. This format allowed me to speak with each student every day of class, track their understanding of the content, and apply adjustments as needed. Since development in 2019 the content I created has been successfully adopted by several colleagues in and out of my department. In this talk, I will share my philosophy for the course, the nuts and bolts of how our meetings are structured, some student feedback, and where I intend to go next with the course.
(Received September 08, 2023)

## 1192-101-30197

Denise Hum*, Skyline College. Turning Introduction to Statistics into an Onramp to STEM. Preliminary report.
STEM classes, particularly calculus and introductory computer science classes, traditionally have few women and
underrepresented minorities enrolled. Skyline College, a Hispanic Serving Institution in the San Francisco Bay Area, is no exception. A 2019 state-mandated shift in placement practices has resulted in more students who are decidedly non-STEM majors being placed directly into Introduction to Statistics. This traditionally terminal course for non-STEM majors was redesigned to include coding and project-based learning, serving as an on-ramp to data science and computer science. A 2020 NSF IUSE grant supported these efforts, including intentionally recruiting women and underrepresented minorities in Introduction to Statistics classes, sustained faculty development, and curriculum redesign. Three years into the project, the redesigned statistics course has succeeded in engaging students and sparking an interest in STEM. The past few semesters, over $2 / 3$ of Introduction to Statistics students in project-based classrooms have express interest in learning more coding and statistics. Also, almost all faculty who have been involved with the project still to use core parts of the curriculum, meet regularly, and continue to innovate. However, despite the high demand for data science careers, especially in the local area, many Skyline students remain unaware of it. While there has been enrollment growth in the Introduction to Data Science course, enrollments from core Skyline students, even other STEM majors have been relatively low. So, we have had to pivot and adapt our plans to meet students where they are at. In this talk, how we are overcoming these challenges, connecting more decidedly non-STEM students to STEM and data science, and supporting the newly formed Data Science Club and their efforts to reach a broader student audience will be discussed.
(Received September 09, 2023)

1192-101-30202
Vinodh Kumar Chellamuthu*, Utah Tech University. Unlocking Authentic Learning through Data-Driven Interdisciplinary Collaboration. Preliminary report.
This presentation delves into the fusion of data-driven research and interdisciplinary teamwork between Utah Tech University students and Southern Utah's key sectors, including business, industry, and government. By participating in these projects, students not only apply classroom theory to tangible problems but also assume greater responsibility for their learning. Highlighting specific case studies, the session will showcase the innovative solutions crafted by student teams. It will also offer insights into the unique mentoring landscape-capturing both the highs and lows-that accompanies this kind of hands-on, real-world educational experience. The talk will address how these endeavors stand apart from traditional academic research, prepare students for a demanding job market, and pose fresh challenges, especially in the realm of mathematics. (Received September 09, 2023)

1192-101-30352
Molly Sutter*, Washington State University. Mathematics Teacher Identity Formation During the First Years of Teaching: The Use of Autoethnography and Reflective Practices in Identity Formation. Preliminary report.
Identity is something that is hard to define, but has a huge impact on someone's life and sense of belonging to the communities in which they live, work, interact, and learn. For this study, I have decided to look into the formation of identity in preservice to in-service mathematics teachers as both mathematicians and teachers. Semi-structured individual interviews were conducted with six participants who wrote an autoethnography as an assignment for a math methods course in their teacher prep program and are now continuing their journey in math and/or teaching. A focus group interview was also conducted with four of these same six participants to discuss the impact of reflection on identity formation. In addition, I prompted further reflection on other experiences that may be more dormant in their lives, but still impact identity formation. Preliminary themes and results will be discussed to show the range of experiences participants shared that shape their math and teacher identities. The use of autoethnography will also be highlighted as a way to provide opportunities for math teachers to understand their own lived experiences' impact on their identity and in turn, better understand their students' identities in the math classroom.
(Received September 09, 2023)

Patricio Gallardo*, UC Riverside. Mentoring Undergraduates in Neural Networks for Mathematical Data Exploration. I report on my experience mentoring first- and second-year undergraduate students in learning the fundamentals of neural networks for exploring mathematical data. Specifically, the students worked with a labeled database of polytopes and their volumes, focusing on using a one-layer RELU neural network. After understanding how to theoretically set up the problem and train the corresponding neural network, we explored the concept of transfer learning for new databases constructed through a change of coordinates. We will discuss our experiences with students grappling with concepts like equivalence classes and group actions and the role of coding in these discussions. (Received September 09, 2023)

## 1192-101-30380

Feryal Alayont*, Grand Valley State University. Incorporating Programming into a Differential Equations Course.
Most mathematics majors have long had a programming course requirement for graduation, and programming is a skill that will be helpful to students in industry or research careers. However, not all mathematics majors at an institution may be required to take a programming course and not all students may complete this course before enrolling in a differential equations course. On the other hand, with the availability of free to use programs like Python that do not even require installation on the computer, it is impossible to miss the opportunity to help students both to practice programming and to strengthen their understanding of differential equations concepts through using Python activities. In this talk, I will describe the setup for using Python-based group lab activities in a differential equations course with no programming prerequisite that is open to all math majors and minors, along with examples of the labs and student feedback.
(Received September 09, 2023)

1192-101-30572
Stepan Paul*, North Carolina State University. Exploring geometry with non-Euclidean paper.
The author presents a novel teaching tool for concepts in differential and axiomatic geometry-paper-like plastic sheets endowed with non-Euclidean geometry through thermoforming technology. We report on demonstrations and activities using this non-Euclidean "paper" both in the classroom and in outreach events. We will show how these physical models can make accessible concepts such as hyperbolic geometry, surface isometry, and Gaussian curvature.
(Received September 10, 2023)

## 1192-101-30578

Stephen Featherstonhaugh, BMCC-CUNY, Jorge Florez, BMCC-CUNY, Yi Annie Han*, BMCC-CUNY, Elisabeth Jaffe, BMCC-CUNY, Glenn Miller, BMCC-CUNY, Oleg Muzician, BMCC-CUNY. Creating Data Science Pathways for STEM Student Success at BMCC-CUNY. Preliminary report.
The BMCC Data Science Pathways model contextualizes mathematics course content, reduces attrition in lower-division gateway courses, and facilitates senior college transfer. Our project promotes structural, systemic reforms through the integration of support services within a sustainable, institutional framework and addresses the educational achievement gap among groups traditionally underrepresented in STEM. The project phase one focuses on curricular reforms through the development of data science content. These efforts aim to develop students' data literacy by enhancing their conceptual understanding, integrating real-life data, and fostering active learning. The curriculum development has involved the infusion of data science concepts into the existing mathematics courses and the creation of a new Data Science Associate Degree program. The project developed a Data Science Research component in the second phase to reinforce data literacy competencies and encourage student confidence. We adopted and modified the PIC Math model (Preparation for Industrial Careers in Mathematical Sciences) developed by the Mathematical Association of America, which involves students working collaboratively and individually on undergraduate Data Science research problems in business, industry, or government. During the Summer of 2023, twenty BMCC students in STEM majors and ten faculty mentors participated in the first Summer Data Science Research project. In this presentation, we will share the successes and challenges we experienced with the project.
(Received September 10, 2023)

## 1192-101-30615

Timothy P Chartier*, Davidson College. Elevator Speeches to Consulting Presentations.
Communication is an essential part of mathematics. Effective communication disseminates work to the mathematics community and allows those outside the mathematics community to understand and use results. This talk will present different techniques utilized within a math class to communicate mathematical content which includes infographics prepared and presented to community partners, assignments enabling students to answer the "when would I use this" question, and consulting projects developed with campus or community partners that lead to a poster session and formal paper of findings and recommendations. Techniques for STEM and non-STEM majors will be discussed.
(Received September 10, 2023)

1192-101-30716
Steven Sofos DiSilvio*, Columbia University, Anthony Ozerov, University of California, Berkeley, Leon Zhou, Columbia University. Fungi, Trash, Bikes, and Wordle: Remembrance of Models Past.
Every year, thousands of undergraduate students compete in the Mathematical Contest in Modeling (MCM), gaining experience with real-world data and problems. We present our Outstanding Winner submission from the 2023 MCM and share advice from our experiences that may be useful for academic mentors and other undergraduates. Problem C of the 2023 MCM asked teams to use past Twitter-reported Wordle results to predict future ones for a target word on a given date. In our submission, we developed a Bayesian model consisting of three submodels and used Markov Chain Monte Carlo to obtain posterior predictive distributions. Along the way, we developed novel entropy-based word difficulty metrics and a twoparameter representation of observed word difficulty. We discuss these methods and how we arrived at them in the short contest time-frame, from brainstorming to putting the final components together. We also present productivity and sleep data for one competitor in the past three MCMs, and discuss advice we did or did not follow when competing. As 3 - and 2 -time
competitors who are now in or approaching graduate school, we also reflect more broadly on the competition, what it has given us, and how it compares to our other research experiences. Finally, we discuss how Columbia-which has had 3 Outstanding Winner teams in the past 2 years-has used MCMs to generate interest in mathematical modeling among undergraduates, both creating training workshops and a summer research program. We hope these experiences will inspire others to engage with and reap the benefits of MCMs and mathematical modeling.
(Received September 10, 2023)

## 1192-101-30754

Anamika Megwalu*, San Jose State University. A Meaningful Intersection: Mathematics, Computer Programming, and Art. Computer programming is used to design, construct, and implement new and improved technology. Mathematics is the foundation and language of computer programming. Algorithms, data structures, and computational complexities use mathematical principles, operations, and logic. The dependence of computer programming on math is obvious and inevitable. Does art have a place in what is created using mathematics and computer programming? Can educators create a space in their classrooms where students are given the opportunity to be an artist in a Computer Engineering class? In this session, the presenter will discuss the role of art in the creation, use, and interpretation of technology, and her pedagogical strategies to foster a creative space for computer engineering students. While the building of a technology requires mathematics and coding, the concept and value of a technology is creatively expressed using art. Thus, for inventors of technology, whether students or seasoned researchers, it is imperative that they think beyond the disciplinary boundaries and operate in a space where disciplines intersect.
(Received September 11, 2023)

1192-101-30822
Samuel M Graff*, Department of Mathematiics and Computer Science, John jay College of Criminal Justice, CUNY, Yi Li, n/a. Using a Model to Give a Grand Tour of a First Course in Differential Equations.
Formulating a versatile population growth model affords an opportunity to survey some of the important concepts that are presented during a first course in ordinary differential equations. Initially, the classical Malthusian law allows for a discussion of first order linear differential equations including the notion of an integrating factor. While the full model is nonlinear with respect to the dependent variable, it can be solved explicitly, yielding an implicit representation of the solution using the separation of variables method. The implicit representation of the solution suggests that an analysis of the long term behavior of all solutions might be challenging. Fortunately, the fact that there are three equilibrium solutions offers a gateway to the geometrical theory of differential equations. In this context, the phase line, the stability of the equilibria, and the long term behavior of the entire family of solutions may be explored. Students are often spellbound by the geometric analysis and the reality that so much information can be obtained just from the differential equation. For many students, this experience offers a compelling answer to their query concerning the utility of calculus itself.
(Received September 11, 2023)

## 1192-101-30911

Carmen Caiseda, Inter American University of Puerto Rico, Michael A. Hill, UCLA, Padmanabhan Seshaiyer, George Mason University, Jianzhong Su, University of Texas at Arlington, Edouard Tchertchian*, Los Angeles Pierce College. Scaling Up Research Experiences for Community College Students. Preliminary report.
Please, join us in this share-out session as we share progress on our mentoring/research program for community college students. With support from the Improving Undergraduate STEM Education: Hispanic-Serving Institutions (HSI Program), this Track 1 project aims to engage underrepresented minority (URM) community college students and faculty in meaningful research experiences led by faculty at four-year local university partner institutions. Our goal is to increase the participation rate of URM CC students into great REU programs while aware of their socio-economic status and life hardships. (Many of them work full-time jobs while attending school, support children or other family members, and cannot give up employment or drop other responsibilities for a prolonged period ( $6-8$ weeks) to solely participate in a traditional REU.) Additionally, CC faculty's primary focus and responsibilities semester-to-semester are on duties related directly to providing great instruction in the classroom. Our goal is to fix this by relying on a proven model as a backbone - the work done in RE-C^\{2\}: Research Experiences in Community Colleges - and expanding our reach nation-wide by utilizing a virtual community of practice and technology.
(Received September 11, 2023)

## 1192-101-30939

Robert M Riehemann*, AMS, MAA, APS. Small Projects at Small Institutions.
At "teaching institutions" graduating mathematics majors are required to complete a research project. Faculty have heavy teaching loads of at least 12 hours/semester. To maintain student interest, departments often try to tailor projects to individual students. In such cases, a wide variety of projects over similarly diverse areas are required. Thus the interpretation of a research project is modified. Research projects typically involve working a graduate level problem in an area that is tangentially related to the training of the faculty member. Consequently, this becomes a mathematical learning experience for both student and faculty. We illustrate this situation with a brief review of projects at Thomas More University in Kentucky. (Received September 11, 2023)

1192-101-30977
Timothy D Comar*, Benedictine University. Writing, Research, and Presentation in a Modern Geometry Course.
The highlight of the modern geometry course at Benedictine University is the independent course project. This project gives the students the opportunity to learn about a new topic, create a significant pedagogical activity, or engage in research and then present their work in both written in oral forms. The students learn new mathematics and often prove significant results themselves. They learn how to write a mathematics paper as well as to prepare the paper in LaTeX. The project also requires a public presentation at the end of the course. These projects often incorporate geometry software; attention is paid to how to incorporate effective figures and demonstrations. The writing process is supported by shorter writing assignments throughout the course and through a revision process involving feedback from the instructor and from peers in the course.

1192-101-31018
Qiang Shi*, Emporia State University. Using 3D Modeling Projects and 3D-Printed Objects to Improve College Mathematics Course Learning Outcomes.
A MakerGear M2 3D printer has been used in our mathematics program since 2017. This presentation will showcase some 3D modeling projects and 3D-printed objects we have created for a variety of undergraduate mathematics courses. Examples include hyperbolic functions and Gateway Arch, estimating volumes of symmetrical fruits, and a few others. Additionally, we will discuss the design programs for these projects, such as CalcPlot3D, Maple, Tinkercad, and Selva3D. Students' feedback on these projects and the use of 3D-printed objects will also be presented.
(Received September 11, 2023)

## 1192-101-31021

Christopher A Patterson, U.S. Air Force Academy, Kevin Treat*, U.S. Air Force Academy. Using an Air Force Scenario to Make Graph Theory Relevant to Undergraduate Students.
We describe how we used contextual teaching and learning principles to build a graph theory project for math courses at the U.S. Air Force Academy. In the project we leveraged an Air Force targeting scenario to make the mathematics relevant to our students. In this talk we will pay special attention to how the relevancy of the scenario helped students describe the mathematical concepts involved. We explain the details of the project, the math behind it, and how we incorporated contextual teaching and learning to improve the learning experience of our students. Although our project focused on an Air Force application, due to the ubiquitous nature of graph matchings, we believe it could be easily modified for other audiences. (Received September 11, 2023)

## 1192-101-31027

Kirsten Hogenson*, Skidmore College. Supporting Student Understanding Of 3D-Coordinate Systems in Multivariable Calculus.
Multivariable calculus courses typically include instruction on the 3-dimensional Cartesian, cylindrical, and spherical coordinate systems. This knowledge can be useful when modeling real-life problems involving structures like pipes or domes. However, sometimes students have difficulty developing a conceptual understanding of these coordinate systems from written descriptions and 2-dimensional sketches. In this talk, I will share three 3D-printed models which I have designed to help students visualize the relevant quantities corresponding to each of these coordinate systems. I will also share a lab assignment from my Calculus 3 course in which students design, describe, and 3D-print a 3-dimensional object that is best suited to a particular coordinate system.
(Received September 11, 2023)

## 1192-101-31086

Christian Gilde, The University of Montana Western, Tyler Seacrest*, The University of Montana Western. Teaching Modeling with Case Studies in Undergraduate Mathematics Courses. Preliminary report.
There are few case-based undergraduate courses outside of business schools and nursing schools. This becomes even more sporadic when considering STEM disciplines, such as engineering and mathematics. The case method in the context of modeling places students in the role of decision-makers, confronting real-world challenges marred with the constraints and incomplete information found in real life. In this respect, cases involving mathematical modeling create a student-centered learning environment-an environment that is central to successful experiential learning and at the heart of most case-method teaching. This work showcases the presenters' experience adapting the case study method combined with models for undergraduate mathematics courses. The presentation describes the structure of the course, including the process of choosing a case, approaching the case discussion in a successful manner, assessing student contributions, and reflecting on the results of the process. The session provides a brief background about the case teaching method using models. The student feedback of this exploratory work suggests that such a case course enhances the learning experience through a pedagogy that is steeped in learning-by-doing, provides a platform that allows for the application and practice of mathematical modeling concepts learned in earlier classes, exercises analytical skills, and addresses the gap between theory and practice through simulated real-world challenges.
(Received September 11, 2023)

1192-101-31181
Matthew Krauel*, California State University, Sacramento. Analyzing the effectiveness of an opt-in peer-led learning class. The Peer Assisted Learning Program at California State University, Sacramento, offers student-led classes to supplement certain mathematics and science courses. Prior analysis of data showed these classes boost students' GPA in the course. However, the possibility this success stemmed from voluntary enrollment remained. In this talk, we discuss the use of propensity score matching to account for this opt-in factor of the program when analyzing its effectiveness.
(Received September 11, 2023)

## 1192-101-31194

Daniel C Garvey, Phillips Exeter Academy, John E Mosley*, Phillips Exeter Academy. An Experiment in Transcript Mastery Assessment in High School Algebra.
At Phillips Exeter Academy, we have been experimenting with Transcript Mastery Assessment in our Math 1 classes (high school algebra). After a brief introduction to standards-based assessment in general and TMA specifically, we report on the current state of TMA in mathematics at Phillips Exeter Academy, and detail our ongoing efforts to study the efficacy of this assessment strategy with our student population.
(Received September 12, 2023)

Melissa A Stoner*, Salisbury University. Using our Medical Simulation Center to Bring Modeling of Lung Capacity to Life in The Differential Equations Classroom. Preliminary report.
How can we utilize the medical simulation center on campus to enhance learning when teaching differential equations? We expand on a textbook-based project requiring students to model the volume of lungs by solving a linear differential equation for the pressure balance of the respiratory system. By varying parameters and then simulating those patient scenarios in our medical simulation center using an ASL 5000 breathing simulator, students explore the dynamics of the lungs and compare key characteristics across scenarios. Students also analyze the quality of their model and potential improvements. We share how the project was developed, implementation and student reactions. (Received September 11, 2023)

## 1192-101-31257

Mark C McClure*, UNC at Asheville. Integrating Observable and Discourse for classroom discourse. Preliminary report. Discourse is open source forum software. You might think of it like a Math StackExchange that you can build and host yourself. Observable is an online notebook platform for computational data exploration and visualization. You might think of it like an instantaneously reactive version of Jupyter that uses Javascript to make your demonstrations easily deployable to the web. Discourse and Observable are both easy to for students to use and can be integrated to powerful effect. Students can learn quite a few basic skills in the process including: - ize
- Use of Markdown to set up a Discourse post or Observable notebook that's nicely laid out with sections, images, and/or tables.
- Use of LateX snippets to incorporate mathematical notation. \enditemize They both have their own strengths. Discourse provides a gentle introduction to online communities where you (the instructor) have some control. Observable is more of a programming tool that you can use to liven up your forum. In this presentation, I'll go over some of the ways I use these tools together in my classroom, using this live discourse instance: https://discourse.marksmath.org/t/42
(Received September 11, 2023)


1192-101-31346
Jingyue Gao*, Borough of Manhattan Community College, Yi Annie Han, BMCC-CUNY, Jun Ha KIm*, Borough of Manhattan Community College, Miguel Silva*, BMCC-CUNY, Matthew Sunderland, STASH Tech. Early BMCC "Connect2Success" System Alert Enhance Students' Performance. Preliminary report. Our project used data collected from the BMCC Connect2Success system to assess the effectiveness and efficiency of the three different time period alerts from the Connect2Success academic advising system. A multitude of data mining techniques were implemented, including classification, regression, clustering or k-nearest neighbor algorithm (KNN), linear regression, and association analysis in this study. The preliminary finding shows that the KNN for predictive modeling analysis yielded a result of $\mathrm{R}^{\wedge}\{2\}=0.8001$ for Connect2Success alerts sent before the $3 \mathrm{rd} / 4$ th week of the semester. Alerts during the middle or end of the semester minimally impact student performance.
(Received September 11, 2023)

## 1192-101-31466

Sarah Wolff*, Denison University. Teaching and Learning in South Africa. Preliminary report.
The South African university and high school system and mathematics curriculum has many similarities to that in the United States but also some major differences. From small differences of wording and notation to differences of classroom environment to a completely different, spiraled, high school mathematics curriculum, there are many interesting conversations that open up when comparing with the United States. After spending six months teaching at both the university and high school level in South Africa as a US Fulbright Scholar, I will reflect on what we can learn from these differences in teaching and learning.
(Received September 11, 2023)

## 1192-101-31476

Sudhan Chitgopkar*, HARVARD UNIVERSITY, Gerasim Kiril Iliev, UNIVERSITY OF GEORGIA. The Pond: An Analog and Digital Model for Mixing-Type ODE Problems.
"The Pond" is a discretized model of the mixing-type problems used in early ordinary differential equations courses. It can be executed in two parts - an 'analog' in-class activity using manipulatibles to introduce the model and language associated with the problem, as well as a 'digital' web application that allows for modular changes to the initial conditions, transient solution, and the long-time equilibrium. The analog component allows students to simulate the composition of a pond over time using a deck of cards. Students "fish" from the pond and "replenish" it at discrete time steps, using a standard deck of cards and are able to see how the fish concentration behaves over time. The digital component allows students to scale this process up once they understand it, using a now-virtual deck of cards. Using an interactive parameter mode, students may modify aspects of 'The Pond' such as pond size, fish concentration, and number/speed of simulation iterations. By tinkering with such parameters, students are exposed to features of the solution to such problems, including dependence on initial conditions, rise of a transient solution as well as the approach to equilibrium when the number of time steps gets very large. The collected information is represented both as tabular and graphical data, and can be compared and contrasted with the in-class activity for a more complete understanding of mixing-type problems and ODEs more broadly. (Received September 11, 2023)

## 1192-101-31492

Linda C. Burks*, Santa Clara University, Kathy Liu Sun, Santa Clara University. The influence of an experiential learning social justice class on undergraduate students' beliefs about mathematics.
In this presentation we will describe the influence of an experiential learning social justice math class on undergraduate students' beliefs about mathematics. During a ten-week long quarter, class sessions consisted of small group problem-solving tasks and student-led class discussions of readings. The experiential learning component involved 16 hours of tutoring in local
high schools whose student population was traditionally underrepresented in mathematics. Throughout the quarter, the undergraduate students reflected on their learning by responding to weekly journal prompts; at the end of the quarter, they designed an equitable learning product for a high school math topic. Undergraduate students' journal entries and final projects were analyzed to answer the research question: How does an experiential learning social justice math class influence students' beliefs about mathematics? Students' views of mathematics expanded from a focus on application and problemsolving to a broader view which included intrinsic values of mathematics, such as beauty and exploration.
(Received September 11, 2023)

1192-101-31531
Vikram Duvvuri, NYU Tandon School of Engineering, Amakoe Gbedemah, NYU Tandon School of Engineering, Miguel Modestino, NYU Tandon School of Engineering, Ingrid Paredes, NYU Tandon School of Engineering, Abby Rabinowitz NYU Tandon School of Engineering, Andrea Silverman, NYU Tandon School of Engineering, Lindsey G Van Wagenen*, NYU Tandon School of Engineering. Sustainability Modeling. Preliminary report.
As a part of NYU Tandon's Sustainable Engineering Initiative we have implemented a set of sustainability-themed differential equations worksheets and projects to engage and empower STEM students to understand and solve problems related to climate change and sustainability. By applying the skills and concepts they have learned in their calculus, physics, engineering and differential equations courses to issues of climate change, the goals are to: - ize
- 1. Increase student engagement. litem2. Deepen student understanding of course material.
- 3. Prepare students to craft the climate solutions of tomorrow.
- 4. To create and support a community of practice among faculty involved in these courses. \enditemize In differential equations, we have piloted worksheets and projects aligned with the above goals. These worksheets and projects are both original and based on materials developed by SIMODE and others. The preliminary results are encouraging. This talk will present an overview of our interdisciplinary initiative's work, course materials and report on current progress.
(Received September 11, 2023)


## 1192-101-31591

Michelle L. Ghrist*, Gonzaga University. Utilizing Personalized Application Assignments in Writing-Enriched Math Courses. Preliminary report.
There are many ways for instructors to incorporate technical writing into mathematics courses, ranging from informal "writing to learn" exercises to extensive projects; in this talk, I discuss several assignments that are somewhere in the middle of that scale. In recent integral calculus and ordinary differential equations courses, I gave two kinds of "Application Assignments", which I termed formal and informal, both of which focused on real-world applications. The formal assignments were essentially mini-projects, while the informal assignments were more personalized and focused on mathematical ideas and communication. One of my main goals for the informal assignments was to encourage students to find ways to express mathematical ideas in understandable language without relying on symbols or jargon; for example, one assignment had students write about a scenario at their home which could be modeled with a system of differential equations. I present four informal assignments, two from each course. I discuss the all-important instructions given to help set students up for success as well as how students perceived these assignments. I also examine several benefits for the instructor, including ease of grading and less opportunities for students to copy others' work.
(Received September 11, 2023)

## 1192-101-31796

Ron Buckmire, Occidental College, Maila B. Hallare, US Air Force Academy, USAFA CO USA, Geraldine Maskelony, Arlington Public Schools, Trinidad Morales, Framingham State University, Alonso Ogueda Oliva, George Mason University, Christopher Adam Perez, Loyola University New Orleans, Padmanabhan Seshaiyer, George Mason University, RN Uma*, NC Central University. An Instructional Framework for Educating at the Intersection of Data Science and Social Justice. Preliminary report.
We present a new pedagogical framework, particularly for educating at the intersection of data science and social justice, based on the constructivist learning theory that integrates elements of the 5E Instructional Design Model, the Project-Based Learning (PBL) approach, and the data science life cycle, seamlessly. The proposed framework also embeds active learning and inquiry-based learning pedagogical approaches to engage student learning. The framework incorporates the different stages of the data science life cycle and appropriate PBL elements into the 5E instructional design components - Engage, Explore, Explain, Elaborate, and Evaluate. What makes this framework unique is the 6th E that augments the 5E Framework which aims to motivate the students to Expand their education into research and activism through a social justice topic explored. We exemplify this framework through the use of two social justice topics, namely, mass shootings and environmental justice. This new teaching framework is designed to make data science accessible across age groups (K-16), across disciplines (STEM vs. non-STEM), and across demographics.
(Received September 12, 2023)

1192-101-31844
Kristin A Camenga*, Juniata College. Filling in Gaps for Calculus I Students: Building in supplemental instruction at a small liberal arts college. Preliminary report.
How do we help Calculus students who missed mathematics instruction during the pandemic succeed? To help students be successful in Calculus, we have added a 5th hour of instruction for our 4-credit Calculus I course and added a Precalculus assessment early enough that students can switch to Precalculus. Students prepare for the 5th hour by watching videos to work through prerequisite topics and then work in groups on problems that use the prerequisite topics in the context of Calculus. We share details of the implementation and reflect on its success in the first semester. (Received September 12, 2023)

1192-101-31913
Eric Hogle*, Gonzaga University. Peer editing vs. ChatGPT.
In an introductory proofs course, the skill of producing proofs can eclipse the skill of evaluating them. However as procedurally generated content becomes more prolific, close and critical reading will only become more important. Can AI-
generated proofs be useful in teaching students to tease apart style and content?
(Received September 12, 2023)

## 192-101-31990

Martha Asare, University of Texas Rio Grande Valley, Luis Miguel Fernandez, The University of Texas Rio Grande Valley, Mayra Ortiz, University of Texas Rio Grande Valley, Cristina Villalobos*, University of Texas Rio Grande Valley. Student Attitudes in Specifications Grading Calculus 1 classes. Preliminary report.
For approximately two years, specifications grading combined with collaborative problem-solving sessions has been implemented in a subset of Calculus I classes. In this session, we aim to engage in a comprehensive examination of the practical execution of this assessment methodology. Furthermore, we present some exploratory outcomes derived from presurvey and post-survey evaluations, with a particular emphasis on the application of Cribb's framework for evaluating student mathematical identity. This analysis compares the experiences of students participating in specifications grading-based classes against those enrolled in conventional lecture-based courses featuring traditional grading systems. Our overarching objective is to facilitate a nuanced exploration of the efficacy and impact of the specifications grading paradigm in enhancing student engagement, attitudes, and self-concept in the context of Calculus I education.
(Received September 12, 2023)

## 1192-101-32011

Maila B. Hallare, US Air Force Academy, USAFA CO USA, Alonso Ogueda Oliva, George Mason University, Padmanabhan Seshaiyer*, George Mason University. Creating a dynamic instructional environment through innovative pedagogical and technological tools for teaching and learning of ODEs. Preliminary report.
Effective instructional methods and active learning strategies can help students develop a strong foundation in ordinary differential equations (ODEs). In this talk, we will expose the audience to learning ODEs through a 5E instructional design for organizing and structuring learning experiences of ODEs. Specifically, we will present examples on how to use this framework to have the students Engage, Explore, Explain, Elaborate, and Evaluate as they learn first order, second order and systems of ODEs. The 5E instructional design model is a learner-centered approach that promotes active engagement throughout the learning process. This pedagogical approach will be integrated with no-code, low-code and high-code technology tools to a) help students construct and reinforce knowledge through their experiences and interactions and, b) encourage educators to create a dynamic learning environment that fosters inquiry and critical thinking.
(Received September 12, 2023)

1192-101-32030
Padmanabhan Seshaiyer*, George Mason University. Novel pedagogical approaches to enhance technical communication for students in mathematics. Preliminary report.
Effective technical communication skills through mathematics courses is an important part of professional and scientific development of all students. In this talk, we will share some of the best practices learnt from a mathematics capstone course offered to both undergraduate and graduate students which incorporated novel pedagogical approaches, problem solving frameworks, brainstorming strategies and technology tools that helped to enhance the technical communication skills of all students.
(Received September 12, 2023)

## 1192-101-32153

Cara Sulyok*, Lewis University. Making the MBRG: Engaging Undergraduate Students in Mathematical Biology Research. In my first two years as a pre-tenured faculty member, I have been fortunate to advise fourteen undergraduates across eight research projects. In this talk, I will discuss how the MathBio Research Group (MBRG) was established at Lewis University and how we have involved undergraduate researchers in extensions of my dissertation at a small, primarily undergraduate institution. I will discuss how we identify and recruit promising undergraduate researchers, prepare them to begin research in mathematical biology, construct interesting and accessible research projects, and mentor students alongside our heavy teaching loads. I will also highlight the essentials needed to build a successful research group with limited funding. Drawing upon experiences, I will showcase how to create an inclusive and supportive research environment, ensuring that the collaborations between faculty and students are meaningful. Ultimately, this talk aims to inspire and equip early-career faculty members to develop and engage undergraduate students in mathematical biology research by sharing experiences, best practices, and lessons learned.
(Received September 12, 2023)

## 1192-101-32258

Gabriel Martins*, CSU Sacramento. Incorporating Programming in the Instruction of Linear Algebra.
We will describe a few useful strategies that have been implemented to incorporate programming tools in a linear algebra class. These tools allow students to engage more deeply with linear algebraic concepts without getting bogged down by unnecessarily repetitive computation, as well as develop relevant programming skills. We discuss the usage of symbolic libraries, such as SymPy, in order to perform row reductions, compute characteristic polynomials, inverses, etc. We also discuss a number of visualization techniques for concepts like the Rank-Nullity theorem and the least-squares method. (Received September 12, 2023)

1192-101-32355
Andrés Forero, University of California, Irvine, Daniel Morrison, University of California, Irvine, Alessandra Pantano*, University of California, Irvine, Sandra Simpkins, University of California, Irvine, Taylor Michelle Wycoff, University of California, Irvine. Empowering mathematical identities of Latinx youth through culturally responsive informal pedagogical practices. Preliminary report.
Afterschool programs can support youth in seeing their future selves in STEM, by promoting STEM knowledge and skills and
by enhancing youth's interests, attitudes and motivation related to STEM, particularly for youth from historically marginalized groups who attend lower quality schools and do not feel supported there. To be most effective, afterschool programs need to meet high-quality standards. A necessary and defining aspect of program quality is the implementation of culturally responsive practices. A culturally responsive program leverages the cultural knowledge, experiences, and perspectives of participating youth to make the learning more relevant and more effective to them. As a result, students feel safer, more valued and better prepared for their futures. Research also shows that youth from ethnic minorities experience higher levels of retention, engagement, and positive social interactions in afterschool programs where they feel that their culture is respected. Although many scholars agree on the importance of promoting cultural responsiveness as an inseparable and integral part of afterschool program quality, there is limited knowledge on what cultural responsiveness looks like in practice and how to measure it. This talk describes a framework for implementing culturally responsive teaching within the context of an afterschool math educational outreach program engaging Latinx youth. (Received September 12, 2023)

## 1192-101-32358

Therese Shelton*, Southwestern University. Safe Contact with Disease: Student Models using Differential Equations. We present models using differential equations from student projects. We discuss student backgrounds, resources used, parameter values, and resulting models. Diseases include mononucleosis and polio. (Received September 12, 2023)

## 1192-101-32378

Susan A Ruff*, Massachusetts Institute of Technology. Teaching undergraduates to communicate as mathematicians: proofs and papers; research presentations and learning seminars; and informal collaborative communication.
The M.I.T. Department of Mathematics teaches communication in 14 different undergraduate subjects: 3 focus on writing proofs (in classes on discrete mathematics and analysis), 1 on writing and presenting research, and the remaining 10 are seminars in which students teach each other the mathematics; each seminar is in a different subdiscipline, from logic to information theory. We teach both formal writing and presenting and informal communication for research collaboration. We teach not only genres (e.g., what typically goes in the introduction of a research paper), but also processes and tools (e.g., strategies for preparing a talk, and using LaTeX), and strategies for communicating complicated mathematical logic effectively (e.g., ordering information, guiding readers through the logic, and writing economically). The teaching of communication is often a collaboration between the course lead and a lecturer from M.I.T.'s Writing, Rhetoric, and Professional Communication (WRAP). This talk will be by a WRAP lecturer who has been collaborating with the math faculty for two decades: it will include examples from a range of the classes, as well as measures of effectiveness, and links to resources. (Received September 12, 2023)

1192-101-32393
Geoffrey Moon, Santa Fe Public Schools, James C Taylor*, MathAmigos. Pairing Math Competitions with Math Wrangles throughout a School District.
Working with most of a New Mexico district's schools (primarily Title I), we have been able help almost 200 upper elementary and middle school students develop positive math identities through district-sponsored math competitions and wrangles. Our approach is to engage and prepare these students yearlong in problem solving techniques and math circle activities, have them participate in five individual competition tests throughout the year, and to debrief them with post-test problem analysis. Students have engaged in practice math wrangles during the year, on teams within a school and between schools, culminating in a district-wide wrangle at the end of the school year. While this was a considerable success, we believe that formal study (beyond our informal surveys) of student resilience and joyful tenacity in the face of hard problems, and more broadly the examination of changing student attitudes and tolerance for frustration should and will be added to our project. (Received September 12, 2023)

1192-101-32401
Jennifer Austin*, The University of Texas at Austin. Developing Ethical Reasoning Skills as a Mathematics Major. I will share three mini-lessons for guiding mathematics majors to develop ethical reasoning skills. I am piloting these activities in a one-hour seminar for entry-level mathematics major in the fall of 2023 as part of my participation in the three-year NSFfunded project "Normalizing Ethical Reasoning in Mathematics as a Foundation for Ethical STEM." In the first activity students collaboratively develop a set of Ethical Guidelines for Mathematics Majors. Then in the second activity students apply these guidelines to analyze vignettes posing an ethical challenge linked to be an undergraduate math major. Finally, in the third activity students reflect on their own experiences through the lens of the collaborativly developed Ethical Guidelines for Mathematics Majors. I will share my lesson plans, reflect on my implementation, and offer suggestions as to how you might incorporate these into your institution's undergraduate program as I challenge you to offer lesson activities that support future mathematician's ethical reasoning development.
(Received September 12, 2023)

## 1192-101-32419

Peter Oden Kagey*, Harvey Mudd College. Illustrating multivariable calculus concepts in the Makerspace.
Makerspaces are facilities that provide access to tools and technology such as 3D printers, laser cutters, sewing machines, electronic equipment, and more. On college campuses, these spaces can provide an opportunity for students inside and outside of the classroom to find applications of multivariable calculus concepts. In particular, laser cutting, water jet cutting, and pen plotting provide a context for understanding parametric curves in the plane, and 3D printing with filament printers can be used to motivate concepts like tangent planes and angles between vectors. We will discuss ways in which makerspaces can enrich undergraduate learning experiences, while building skills which allow students to express themselves creatively. (Received September 12, 2023)

Jeffrey Musyt*, Slippery Rock University. Hashiwokakero Puzzles: Trying to Bridge the Gap between Student and Researcher. Hashiwokakero are a form of Japanese logic puzzle that are often called "bridge puzzles" because their solutions look similar to a road network between a collection of islands. After introducing these puzzles, we'll discuss how their use both inside and outside of the classroom leads to increasing students' interest in doing mathematical research. Additionally, we'll share some other ways that math circle type of activities can help students internalize the belief that they are capable of doing research. (Received September 12, 2023)

## 192-101-32523

Kun Wang*, Texas A\&M University. From Play to Proof: Exploring Red Ball Puzzles and Beyond. Preliminary report. I'd like to present this intriguing game to my math circle students. Imagine we have an equal-arm balance scale, but it lacks balance weights. In this challenge, we start with five identical red balls, among which one is uniquely weighted (either lighter or heavier) compared to the others. The task is to devise a strategy to determine the distinct ball and to ascertain how many times we need to use the balance scale for this purpose. Now, consider a variation of the problem: what if we have six identical red balls, and one of them is the special one? Again, we aim to figure out the unique ball's identity and determine the minimum number of times we must use the balance scale. Furthermore, we can extend this problem to a more general case: when presented with $n$ identical red balls, how can we determine the special one and find a formula that tells us the minimum number of weighings required? Taking it a step further, we can generalize this problem: when confronted with $n$ identical red balls, how can we identify not one but two special ones? Can we formulate a mathematical expression to determine the minimum number of weighings necessary? This opens up exciting research possibilities for college students, and I believe there are practical applications for these types of problems. I look forward to hearing your innovative strategies and solutions. (Received September 12, 2023)

## 1192-101-32609

Rebecca I Swanson*, Colorado School of Mines. Mastery Based Testing in Linear Algebra. Preliminary report.
In this talk, I discuss a first foray into implementing Mastery Based Testing in Linear Algebra. In collaboration with members of our Teaching and Learning Center, we collected pre- and post-course survey data from students as well as comparative data on student performance. In addition to providing an overview of this implementation, we will discuss the results of this study. While our results uphold previous work demonstrating a decrease in student anxiety, students also reported on both their changes in study habits as well as their views on the representativeness of their grades.
(Received September 12, 2023)

## 1192-101-32610

Paul E. Seeburger*, Monroe Community College, Shelby Stanhope, U.S. Air Force Academy. Learning Activities using 3DPrinted Models to Explore Volumes of Revolution \& Partial Derivatives. Preliminary report.
Come discover two hands-on, active learning activities that use 3D-printed surfaces to help students develop a better understanding of two visually interesting geometric calculus topics: Shells and Washers used to approximate a Volume of Revolution in single variable calculus and the first and second partial derivatives of a function of two variables in multivariable calculus. The PDFs and STL files are provided so you can use these small group activities with your own classes! This project is funded by NSF-IUSE \#2121152. See: https://sites.monroecc.edu/multivariablecalculus/3d-learning-activities/ (Received September 12, 2023)

## 1192-101-32663

Jing Xie*, Northeastern Illinois University. A Path to Improving Learning Attitudes through the Analysis of the College-Level Mathematical Teaching and Learning Model. Preliminary report.
The significance of students' attitude toward learning in relation to their mathematics performance cannot be overstated. However, the connection between students' underperformance in mathematics and their attitude towards learning remains a subject of uncertainty. We are currently confronted with the challenge of assisting and understanding students' evolving attitude as they progress in their learning journey. To tackle the challenges, we employ statistical tools such as the t-test and ANOVA to enable us to delve into error analysis within the college-level mathematics teaching and learning model. This comprehensive evaluation considers not only academic aspects but also factors related to health and social influences within the context of the mathematical educational framework. Moreover, we extend our approach by integrating machine learning algorithms into the evaluation process, enhancing our error analysis and predictive capabilities. This augmentation also provides a valuable tool for gaining insights through rigorous analysis. In essence, our research findings hold the potential to offer meaningful explanations for the disparities that become evident. By dissecting these disparities, we acknowledge the possibility of errors stemming from flawed assumptions, particularly when considering social factors. Through a comprehensive understanding of these dynamic interactions, our findings become a pivotal steppingstone towards more informed and inclusive strategies aimed at enhancing students' attitudes towards learning mathematics. This approach addresses the multifaceted challenges arising from the convergence of various factors within the learning process. (Received September 12, 2023)

## 1192-101-32697

Kiran R Bhutani, The Catholic University of America, Kathryn E Bojczyk*, Educational consultant, Guoyang Liu, The Catholic University of America. Enhancing and Understanding Help-Seeking Behaviors in an Online Precalculus Review Course for Incoming College Students: A Follow-up Report.
In the past few years, as educators we have witnessed the impact of the pandemic on students' academic performance. Changes in instructional modalities and lack of in-person interactions during their high school years led to challenges in students' mathematics readiness and level of rigor in their preparation. These factors made it difficult for incoming college freshmen to have a smooth transition and success in their programs of study. At the same time, faculty have been facing barriers due to the widespread disparities in the level of mathematics preparation of Science, Technology, Engineering, and Mathematics (STEM) majors. As a result, research is needed on ways to prepare high school students for college level work in mathematics. Calculus is a gateway course for college students majoring in engineering and other STEM fields. The goal of our
research is to investigate the help-seeking behaviors of students enrolled in an online precalculus review course (OPRC). Our intervention methodologies have focused on helping underprepared students, including women, first generation college students, and non-native speakers of English to be successful in their college study. At the 2023 Joint Mathematics Meeting in Boston, we shared our preliminary research findings on the first-year implementation of a 3-year study funded by the Nationa Science Foundation. This year we will report on the results of the study conducted between the OPRC intervention group and the comparison group. We will present quantitative and qualitative evidence of the impact of the intervention program and differences between the two groups. We will share updates on the changes we made this summer to improve our training of research assistant, teaching assistants, our collaboration with community partners, outreach and recruitment strategies for both the course and the research study.
(Received September 12, 2023)

## 1192-101-32703

Scot Appleton Gould*, Claremont McKenna, Pitzer, Scripps colleges, Karishma Punwani, Maplesoft. Learning Introductory Undergraduate Physics Through Maple Immersion.
This talk reports on the outcomes of incorporating Maple into an experimental first-semester undergraduate introductory physics course and several upper-division physics courses. The talk covers what works and fails and attempts to improve integration. Some reasons why Maple was chosen: * Maple is capable of solving numeric and symbolic mathematics problems. * Using the clickable math components and the easily readable "2d-math" input, students spend less time learning how to code than many popular programming systems. * Maple is more readable than Python and Mathematica. Maple math tends to look like the math they have learned in their math courses. * Maple allows one to incorporate subject-appropriate mathematics, such as solving sets of linked differential equations, into a course that rarely contains it. * Maple allows students to generate animations, create simple "what if" applications and practice presenting results through publishable graphs. For the initial attempts to include Maple in a course, it was taught primarily at the beginning of the semester and during class time. Later courses were modified in the following ways: * Students learned the basic skillsets of Maple through a collection of ten-minute videos and associated problem sets. * Each skillset was introduced at the time it was useful for solving the problems related to the physics principle being studied. * All presentations and all homework submissions were in the form of a Maple worksheet using the 2D input style. By the end of every course, the protocol by virtually every student was to solve the physics problem using Maple. In addition, most students stated that in comparison with traditional science courses, Maple allowed them a greater opportunity to concentrate on understanding the physics principles while spending less time performing the mathematical minutia that typically exists within such courses. (Received September 12, 2023)

1192-101-32721
Douglas B Meade, University of South Carolina, Paul E. Seeburger*, Monroe Community College. Gaining Insight in Differential Equations using Interactive GeoGebra Figures. Preliminary report.
Visual exploration can add significant meaning and promote a more general and intuitive understanding of many topics in differential equations. This presentation will demonstrate interactive figures that illustrate and explore the motion of a spring (undamped or damped, and unforced or forced), bifurcation, and the phase portrait of a system of differential equations. These interactive figures were created in GeoGebra for integration into an online differential equations textbook. They can be used as classroom demonstrations or for independent student learning. The presenter will also show an OER textbook section on Bifurcations that he created on the LibreTexts platform. This section includes a series of interactive figures to help make these concepts clear. See: https://sites.monroecc.edu/multivariablecalculus/visualizing-differential-equations/ (Received September 12, 2023)

1192-101-32740
Warren Shull*, University of Wisconsin-Eau Claire. Taking the leap: How I transitioned to Grading for Growth in a semester. Preliminary report.
In this talk, I will discuss my transition into a Grading for Growth in two courses: Calculus 1 and College Algebra. I had taught both courses with more traditional grade structures before, though not at the same institution. While I was fortunate to have very supportive colleagues throughout my department, my implementation of this grade structure was largely independent (not coordinated across the entire department or the entire course). Also, while I had tried out a smaller version of a Grading for Growth structure in a different course in the past, this was the first time it applied to more than half (60\%) of the students' course grade.
(Received September 12, 2023)

## 1192-101-32751

Meghan Maureen De Witt*, St. Thomas Aquinas College. Class Schedules to Serial Killers, and Everything in Between. Preliminary report.
We explore the experience of mentoring data-driven undergraduate research projects and how the mentorship differs from a more theoretical project, focusing especially on the benefits and challenges associated with allowing a student to choose a data topic outside your field of experience. In the process, we will discuss how you might find yourself learning about theme park construction, the best scheduling method for your own classes, or even the numerical patterns of serial killers across multiple decades.
(Received September 12, 2023)

1192-101-33151
Lawrence C Udeigwe*, Manhattan College \& MIT. Using Student Projects for General Public Education. Preliminary report. Recent events and technological advancements have led to many innovations on how students are instructed and assessed in mathematics courses. Teaching a traditional differential equation course during the COVID-19 pandemic forced me to think of innovative and interesting ways of assessing the students while making sure I cover all the required topics. Inspired by SIMIODE materials, I helped the students design public education projects. The goal of this project was to use a SIMIODE article to introduce the general public to an application of differential equations. An important punch line of the project
description is: "Your target audience is a high school student. Your job is to teach this student without intimidating him." The projects were disseminated in long form via YouTube, and in short form via instagram and tiktok. In this talk I will discuss my takeaways including (1) feedback from students; (2) areas that need improvement; (3) scaling beyond differential equations. (Received September 13, 2023)

1192-101-33156
Svetlana Roudenko*, Florida International University. Innovations in Undergraduate Education via REU and STEM programs. Preliminary report.
We discuss outcomes from the specific undergraduate programs that we could run at the Dept of Math \& Stats of Florida International University such as Summer REU together with S-STEM programs, both supported by NSF. This talk is geared toward a conversation about some of the approaches used as well as discussion on various other avenues in the improvement of undergraduate education.
(Received September 13, 2023)

## 1192-101-33162

Jane Holsapple Long*, Stephen F. Austin State University, Clint Richardson, Stephen F. Austin State University. A "Math Without Words" Puzzle Leading to Research Questions.
A visual puzzle by James Tanton forms the basis for a session that has been successfully implemented with various audiences. Designed to be presented with no directions or description, the puzzle requires participants to discover the goals themselves and to generate their own questions for investigation. Undergraduate student work connected to this problem will be discussed as well as many deep questions suitable for long-term investigation.
(Received September 13, 2023)

## 192-101-33169

Lawrence C Udeigwe*, Manhattan College \& MIT. Framing Ethics through General Public Education. Preliminary report. Recent events and technological advancements have led to many innovations on how students are instructed and assessed in mathematics courses. Teaching a traditional differential equation course during the COVID-19 pandemic forced me to think of innovative and interesting ways of assessing the students while making sure I cover all the required topics. Inspired by SIMIODE materials, I helped the students design public education projects. The goal of this project was to use a SIMIODE article to introduce the general public to an application of differential equations. An important punch line of the project description is: "Your target audience is a high school student. Your job is to teach this student without intimidating him." The projects were disseminated in long form via YouTube, and in short form via Instagram and Tiktok. In this talk I will be exploring and discussing (1) the ethical issues that may arise from this mode of assessment; and (2) how this mode of assessment may be an avenue for framing ethics in mathematics.
(Received September 13, 2023)

1192-101-33318
Tifin Marie Calcagni*, Global Math Circle, Taylor Yeracaris, Global Math Circle. The Sum of Two Squares as a Math Circle Activity.
"Which numbers are the sum of two squares?" is a deceptively simple question that ignites mathematical curiosity and problem-solving within math circles. As younger math circle participants grasp its connection with visual proofs of the Pythagorean theorem, this query spirals into much more complex topics, delving deep into modular arithmetic, number theory, and Gaussian integers. We have conducted this math circle with diverse groups: 8-10-year-old students in the US, 9-12-yearold students from around the world, pre-service teachers, math circle leaders, and experienced educators. As many good math circle questions do, the sum of two squares is accessible by novice learners and challenges seasoned mathematicians. (Received September 13, 2023)

1192-101-33319
Benjamin R S McLaughlin*, Asbury University. Preparing Students and Building a Stronger Department Through the COMAP Competition in Mathematical Modeling. Preliminary report.
For more than three decades, the Department of Mathematics at Asbury University has fielded teams to compete in the COMAP (Consortium for Mathematics and its Applications) Mathematical Contest in Modeling (MCM) and Interdisciplinary Contest in Modeling (ICM). Modeling contests are an excellent tool to help students bridge the gap between mathematics achievement in the undergraduate classroom and mathematics achievement in graduate research and the workplace. In the Asbury University Department of Mathematics' experience, student participation in the contest has become a key component of student engagement within the department and also a key tool for the student's development and preparation for future academic and vocational achievement. In this talk, we will present some of the themes we have consistently heard from students and alumni who have participated in the contest. These themes include ways in which participation in the competition made them more motivated and engaged in their university studies and ways in which they found themselves more prepared to pursue their goals after graduation as a result of the competition. We will also describe the experience of our faculty advisors, and present some additional practices of faculty advisors leading up to to the modeling competition and following on after the modeling competition that enrich the student experience and their formation as mathematics practitioners. (Received September 13, 2023)

1192-101-33914
Kayla Blyman*, Saint Martin's University, Keith Erickson, Georgia Gwinnett College. 100 Hours of MCM/ICM: The Abridged Version.
COMAP's Mathematical Contest in Modeling (MCM) and Interdisciplinary Contest in Modeling (ICM) provide amazing opportunities and experiences for students from across the disciplines to demonstrate their modeling, problem solving, teamwork, and communication skills by solving a real-world problem. In 2023, over 24,000 teams of up to 3 students each
solved one of six problems. What type of problems do they solve? In the past, students have addressed dozens of topics including, but not limited to, designing the longest lasting sandcastle, analyzing the influence of music, examining our food systems, exploring migration patterns of fish, investigating the spread of giant hornets, evaluating the health of higher education, and considering the relocations of environmentally displaced persons. Come to this session ready to dive into the modeling process and consider how your students might model some of the MCM and ICM problems.
(Received October 06, 2023)

## 102 Recreational mathematics


#### Abstract

1192-102-26353 Michael Raul Carrion, Alvernia University, Nathan Hurtig, Rose-Hulman Institute of Technology, Maggie X. Lai*, Tulane University, Sarah Lohrey, Bryn Mawr College, Brittany Ohlinger, Albright College. Multi-hop Duotaire. Peg duotaire is an impartial two-player combinatorial game on simple graphs, where players alternate making hops to remove pegs. We study the multi-hop variant proposed by Eppstein and Moore in One dimensional peg solitaire, and duotaire, where a player is forced to continue removing pegs through successive hops if there are unique pegs and holes available for them to do so. We determine the $N$ - and $P$-positions for games played on complete, cycle, and path graphs in normal and misère play and on complete bipartite graphs in normal play. (Received July 26, 2023)


## 1192-102-27655

Barry Cipra*, Freelance. The Bricklayer's Challenge.
A famous (albeit partly aprocryphal) story has Gauss as a schoolboy speedily adding the numbers 1 to 100 . When the speaker pondered how he might have approached the same assignment, he stumbled into an unsolved problem that might have tested Gauss himself.
(Received August 17, 2023)

## 1192-102-28247

John Urschel*, MIT. Fundamental Checkmates on an Extended Chess Board.
In the game of chess, and the goal is to checkmate ones opponent, e.g., obtain a position in which the opposing king is unable to avoid capture. Chess contains a variety of pieces, each with their own unique geometric movement. On a standard chess board, there are five fundamental checkmates - a checkmate performed against a lone king using the minimum material needed. In this talk, we consider checkmate on an extended chessboard and discuss how (1) the piece configurations capable of checkmate, (2) the techniques required to deliver mate, and (3) the number of moves required to do so vary as the size of the chessboard increases.
(Received August 28, 2023)

1192-102-28298
Persi W Diaconis*, Stanford University. The Mathematics of Solitaire.
It's one of the embarrassments of my field that we can't analyze the usual game of solitaire (Klondike). What's the chance of winning, how to play well, how do various changes of rules change the answers? In Vegas, you can 'buy a deck' for 52 dollars and get 5 dollars for each card on the ace piles. How unfavorable is this? In work with David Aldous, we studied a toy solitaire, patience sorting, which looks a bit like the real thing and can be analyzed. This linked to the deepest corners of modern probability (Baik-Deift-Johansson theorem). Surely you say, the computer can do this. Not at present, not even close. I will review all this, including recent results and lots of new games to try.
(Received August 29, 2023)

1192-102-28720
Istvan G Lauko, University of Wisconsin-Milwaukee, Gabriella A. Pinter*, University of Wisconsin, Milwaukee. When diagonals cannot meet - twists and turns.
Math Circles provide an opportunity for students to engage in open-ended problem solving. Sometimes seemingly simple problems open up new avenues of investigations. In this talk we present one such problem that provided some surprises and stimulated original research at the UWM Math Circle a few years ago. New questions connected to the problem will also be shown.
(Received September 02, 2023)

## 1192-102-28925

Henry Segerman*, Oklahoma State University. Variants of the 15-puzzle and the effects of holonomy.
I'll discuss some variants of the classic sliding tile "15 puzzle" that involve holonomy - the phenomenon of traveling around a loop in a curved surface and coming back rotated. I'll demonstrate physical puzzle designs with positive and with negative curvature, and discuss design considerations and consequences.
(Received September 04, 2023)

Peyam Ryan Tabrizian*, Brown University. Teaching with YouTube.
In this talk, I will present my YouTube channel Dr Peyam, which currently has over 156,000 subscribers and close to 1,000 videos. On this channel, I present a variety of math videos, ranging from Calculus and Linear Algebra, all the way to Partial Differential Equations. I also give some tips on how to start your own YouTube channel, as well as some advice in how you can use YouTube in your own teaching.

1192-102-29520
Jonathan Herbert Schaeffer*, University of Alberta. The Joy of Computational Recreational Mathematics.
Games and puzzles play a special and important role in many research areas. For some of these applications, or subsets of these applications, human and/or mathematical analysis can be used to either "solve" the application or provide useful insights. Unfortunately, some games and puzzles are too complex for meaningful analysis. In such cases, the computer can be an invaluable tool. While brute-force computational approaches may not be mathematically satisfying, modern machine learning and search technologies may be able to glean new insights from game/puzzle data. This can lead to deeper human insights and, potentially, new mathematics. This presentation will discuss the past, present, and future of computational approaches to unlocking the secrets of popular games and puzzles.
(Received September 07, 2023)

## 1192-102-29926

Tomas Rokicki*, California. Twenty Moves Suffice for Rubik's Cube.
This talk will present the mathematics behind the proof that every position of the Rubik's Cube can be solved in twenty moves or less. We start with some history of the mathematical and computational exploration of the cube, highlighting Kociemba's amazing subgroup-based algorithm that calculates near-optimal solutions. We turn this algorithm inside-out, to solve an entire coset of positions at the same time, enabling resolution of the diameter of the cube group with a reasonable amount of computation. We give some details of the effort and finally close with some recent results and open questions.
(Received September 08, 2023)

## 1192-102-29994

Richard E. Korf*, Computer Science Department, University of California, Los Angeles. Rubik's Cube, the Jeep Problem, and an Open Rectangle-Packing Problem.
This talk will consist of three parts. First I will discuss two pieces of work related to Rubik's Cube: my Ph.D. thesis from 1983 on a program that learned to solve the Cube, and my work from 1997 that found the first optimal solutions to random instances of the Cube. Next I will discuss recent work on the Jeep Problem. The problem is to cross a desert in a jeep which doesn't have sufficient range to make it in one trip, by making multiple trips and carrying and caching fuel along the way. The problem is to find the most efficient strategy. In the classical version of the problem, solved in 1947, the distance across the desert is known, but I considered the case where we don't know the distance in advance. The optimal strategy turns out to be rather surprising. Finally, I will discuss an open rectangle-packing problem. The sum of the infinite series
$1 \cdot \frac{1}{2}+\frac{1}{2} \cdot \frac{1}{3}+\frac{1}{3} \cdot \frac{1}{4}+\cdots$ is one. Consider the infinite set of rectangles of these dimensions. The question is whether they can all fit into a unit square without overlap.
(Received September 08, 2023)

1192-102-30072
Vigneswaran Madappan Chinnasami*, University of South Carolina. On the Rotation and Reflection of Magic Square Type Sliding Games. Preliminary report.
A magic square is a square grid in which the numbers of each row, each column, and each of the two diagonals sum up to the same number. By subtracting the smallest number in the square from each number and using the space of 0 as an empty block, we then create a magic square type sliding game. To play the game, the player needs to arrange all the numbers, by moving one number at a time, to an increasing order form from left to right, and from top to bottom. Even though any reflection or rotation of a magic square is considered the same, each resulting sliding game after reflection or rotation is a new game. Since it is known that not all sliding games are solvable, in our project we analyze how the rotation and reflection affect the solvability of some magic square type sliding game.
(Received September 08, 2023)

1192-102-30127
Jessalyn Bolkema, California State University, Dominguez Hills, Sharon Lanaghan, California State University, Dominguez Hills, Carolyn Yarnall*, California State University Dominguez Hills. Cauldrons and Hexes: The alchemy of turning math circles into authentic research experiences for undergraduates. Preliminary report.
With the goal of providing an authentic research experience in mathematics to undergraduate students, we mentored students in summer research in 2022 and 2023 based on problems discovered through our participation in math circles. Students were introduced to a few math circle sessions and opted to work on the Bubbling Cauldrons and Hexagonal Hidato problems. The student researchers came from our home institution, a public Hispanic-serving institution with a diverse, largely firstgeneration student population. The math circle problems provided multiple entry points which allowed for students of different levels of preparation and mathematical experiences to explore and pose their own questions, ensuring that they were active participants in all phases of the research process. In this session, we will discuss how problems were chosen, how students engaged in the research process, and our future plans. We will also share student perspectives on participating in the summer research experience.
(Received September 08, 2023)

## 1192-102-30486

Klara Mundilova*, CSAIL, MIT (subject to change by January). Art-Inspired Curved-Crease Origami Analysis and Design. Preliminary report.
Curved geometries that can be obtained from flat sheets of material have numerous applications in design and engineering. Inspired by artistic sculptures, we explore shapes formed by joining planar patches of material along their curved boundaries, with a particular focus on curved-crease origami. We discuss the theory behind the computation of shapes made of developable patches and highlight a simplified computational approach for cases where the connected patches are either cylinders or
cones. Moreover, we illustrate how to compute a crease connecting a patch to a generalized cylinder or cone. Drawing inspiration from artistic origami, we discuss examples and show parametric design tools for curved crease origami design (Received September 10, 2023)

## 1192-102-30500

Michael Raul Carrion, Alvernia University, Nathan Hurtig, Rose-Hulman Institute of Technology, Maggie X. Lai, Tulane University, Sarah Lohrey, Bryn Mawr College, Brittany Ohlinger*, Albright College. Peg Duotaire on Graphs. Preliminary report.
Peg solitaire is a game played on a board consisting of a pattern of holes. Play begins with all of the holes containing a peg. This pattern of holes can be represented by a simple graph where the vertices indicate holes. Peg duotaire is a two-player, impartial variation of peg solitaire in which players alternate taking turns. The first turn consists of a player choosing one peg to remove. Subsequent turns consist of one peg being moved from a vertex to an empty vertex so long as they share a neighbor containing a peg. The neighboring peg is then removed. Games in which the first player has a winning strategy in normal play (last person to make a valid move wins) have been determined for several classes of graphs, including complete, complete bipartite, path, and cycle graphs. Notably, this does not include the class of tree graphs. In this talk, I will discuss conditions on tree graphs of diameter four in which the first player has a winning strategy. I will also provide analogous results for each of these graph classes under misere play (last person to make a valid move loses).
(Received September 10, 2023)

## 1192-102-30581

Angelina Chirichella, Marist College, Matthew Glomski*, Marist College, K. Peter Krog, Marist College, Elizabeth Reid, Marist College. Lonely distancing dance graphs. Preliminary report.
We speak of lonely distancing as the concept that people can crave company at the same time they seek to avoid too much of it. We construct a scenario in which $k$ players on an $n \times n$ grid jockey to occupy space next to a single companion while maintaining at least a minimum distance from everyone else. The tension behind these conflicting needs-companionship versus isolation-induces a dynamical system on the grid we model with directed graphs. Depending on $n$ and $k$, players are likely to redistribute themselves into one or more equilibrium states. In very special cases no such resolution is possible, and the players enter a cycle of perpetual redistribution in what we call a never-ending lonely distancing dance. (Received September 10, 2023)

## 1192-102-30656

Erik Demaine*, Massachusetts Institute of Technology. A conversation with Ernő Rubik.
In conversation with its inventor, we will explore the Rubik's Cube as arguably the most popular embodiment of a mathematical concept and the ultimate metaphor of problem solving. The Cube conveys its meaning without numbers or language, readily available to anyone regardless of prior education, social status, cultural background, age, or gender. It is tactile and playful, evoking a wide range of strong emotions. It is directly educational and widely inspirational, creating a new genre of mathematical ("twisty") puzzles; stimulating research from robotics to cryptography; and lending itself to art. Hollywood blockbusters, documentaries, podcasts, news circuits, theatrical plays, and novels keep it at the forefront of public attention. And of course ever new generations immerse themselves in scrambling it fast or slow. Joined by Ernő Rubik, we will discuss the process of the Cube's invention; the reasons for its universal relevance half a century after its birth and Ernő's wishes for it for the future. Most importantly, we will exchange ideas on how best to put this unparalleled global appeal to use in maths-popularization. Forging international partnerships among researchers, educators, corporations, and philanthropies, Rubik's Cube could finally bring together the best traditions of serious recreational mathematics and amplify them at every twist and turn.
(Received September 10, 2023)

## 1192-102-30657

Martin Demaine*, Massachusetts Institute of Technology. Fun with Fonts: Algorithmic Typography.
Over the past decade, my son and I have designed dozens of typefaces based on mathematical theorems and open problems, specifically computational geometry. These typefaces expose the general public in a unique way to intriguing results and hard problems in computational origami design, hinged dissections, pencil-and-paper puzzles, and video games. Most of our typefaces include puzzle fonts, where reading the intended message requires engaging in the mathematics itself, inviting mathematicians and nonmathematicians alike to solve a series of puzzles which illustrate the challenge of the underlying mathematical problem. To play with the fonts, visit \url\{https://erikdemaine.org/fonts/\}
(Received September 10, 2023)

## 1192-102-30661

Erik Demaine*, Massachusetts Institute of Technology. Puzzles and Games Meet Algorithms and Complexity. One hypothesis for why humans like to play games and solve puzzles is that they enjoy the challenge. We can formalize the notion of challenge using computational complexity theory: if it's challenging for even a computer to play perfectly, then it's going to be challenging for a human as well. I will describe several such results, for example, that Tetris and Rubik's Cubes are NP-complete, while Super Mario Bros., Mario Kart, and Rush Hour are PSPACE-complete, which essentially means that you can build typical computers inside these games. Proving these hardness results is itself a fun kind of game/puzzle, where we need to construct pieces of game levels to implement pieces of a computer, like memory and wires. On the other hand, some puzzles and games can be solved by efficient computer algorithms, yet are still challenging for humans. In both cases, we can use the improved mathematical understanding to design new puzzles or levels. (Received September 10, 2023)

Clovis North High School. Games with Special Moves. Preliminary report.
In our Math Circle, we often play two-person strategy games, such as Nim and its variations. The participants try to find winning strategies for these games and learn various ideas: symmetry, working backwards, modular arithmetic, bases, etc. At the local Math Field Day, however, games are often modified by allowing each player to make one special move per round. For example, a player may move counter(s) in a way not allowed by the rules of the regular game. We wonder and explore how such special moves change the distribution of winning positions for each player.
(Received September 10, 2023)

## 1192-102-30965

Peter M Winkler*, Dartmouth College. Probabiility and Intuition. Preliminary report.
Puzzles are a wonderful medium for exposing holes in one's probabilistic intuition. Several puzzles will be presented, solved, and examined for what they say about intuition.
(Received September 11, 2023)

## 1192-102-32054

Bram Cohen*, none. Frameless N-ary Puzzles.
A common type of puzzle is ' N -ary', meaning a puzzle which includes a repeated pattern of pieces where the number of steps necessary to solve it is exponential on the number of pieces. Usually such puzzles involve two large pieces whose relative positions are an important part of the puzzle. Past work has involved reducing that to a single large piece. In this work we demonstrate how to go even further, to have a completely 'frameless' N-ary puzzle, where any number of pieces can be assembled but the number to end with must be chosen at the start of the assembly process and the number of steps is exponential on the number of pieces used. We also have the further requirement that all pieces be identical. We will show several different constructions which meet these requirements and their mathematical analyses.
(Received September 12, 2023)

## 1192-102-32170

Chaim Goodman-Strauss*, National Museum of Mathematics. The Hat Tile and The Rosenthal Prize.
The recently discovered aperiodic "Hat" monotile has been a remarkably rich source of new perspectives on aperiodic tiling, showing many phenomena not seen before. In the first part of the presentation, we will catch up on evolving developments, and discuss the Hat in the context of the computational complexity of tiling problems and self-assembly algorithms. Would you like to win $\$ 25,000$ ? Submit an innovative and inspiring activity for the elementary or middle school classroom to the Rosenthal Prize! In the second half of the presentation we'll share information about applying to the contest, sponsored by the National Museum of Mathematics.
(Received September 12, 2023)

## 1192-102-32171

Bodie Tanner Cole*, Pi Mu Epsilon. Tiling Deficient Squares With L-and T-Tetrominoes.
We examine a subset of the tetrominoes that includes all rotational and reflective symmetries of L - and T - tetrominoes, and square regions with one square missing, called deficient square regions. We show that all deficient square regions of size $n \times n$ are tileable when $n \geq 7$ and $n$ is odd. We also show that some cases of deficient $3 \times 3$ and $5 \times 5$ regions are untileable with the chosen tile set.
(Received September 12, 2023)

## 1192-102-32411

Denise A Rangel Tracy*, Francis Marion University. Understanding EvenQuads.
EvenQuads is a double-sided card deck that can be used to play a variety of mathematical games. One side of each card features a profile of a woman mathematician who has made significant contributions to the field of mathematics, and the other shows a number of symbols that appear in varying colors and quantities. Originally known as the Notable Women in Math Playing Cards Project, EvenQuads was created in 2019 to celebrate the 50th anniversary of AWM (Association for Women in Mathematics). We will explore the variety of games that can be played with the EvenQuads decks, such as Quad Collector and EvenBetter, and the mathematics underlying them. We will also give an overview of the project including a brief history, origins of the symbols used, and a description of the five pillars created to categorize significance of individuals within the mathematics community which provided the framework for the selection of honorees in EvenQuads. Finally, we will highlight the ongoing research scholarship related to the mathematics of EvenQuads.
(Received September 12, 2023)

1192-102-32738
Susan Goldstine*, St. Mary's College of Maryland. Counting Stitches: Enumerative Problems in Knitting. Preliminary report. With the recent explosion of interest in mathematical fiber arts, enthusiasts have discovered an abundance of mathematics embedded in knitting, crochet, embroidery, and other fabric-oriented art forms. Here, we will survey some of the ways that knitting and knitwear design, filtered through combinatorics, graph theory, and geometry, lead to complex and fascinating numerical results.
(Received September 12, 2023)

## 1192-102-32839

Noam D Elkies*, Harvard University. Life Update.
We report on some old and new work and remaining open questions in and near Conway's Game of Life.
(Received September 12, 2023)

## 1192-102-32863

Robert A. Hearn*, Gathering 4 Gardner. The Puzzling Origins of Compound Symmetry Groups.
Symmetry is at the heart of much of mathematics, physics, and art. Traditional geometric symmetry groups are defined in terms of isometries of the ambient space of a shape or pattern. If we slightly generalize this notion to allow the isometries to operate on overlapping but non-identical metric spaces, we obtain what we call compound symmetry groups. A natural example - derived from the study of two-dimensional "twisty puzzles" - is that of the groups generated by discrete rotations of overlapping disks in the plane. Investigation of these groups reveals a new family of fractals, as well as a rich structure that is intriguing both mathematically and artistically. We also argue that through the lens of compound symmetry groups, twisty puzzles such as the Rubik's cube can be seen as pure mathematical objects on a par with, for example, the 17 wallpaper groups, and not simply as arbitrary permutation puzzles.
(Received September 12, 2023)

## 1192-102-32932

Lauren L Rose*, Bard College. Avoiding Triples in the Card Game Spot It!. Preliminary report.
Spot It! is a card game where each card has 8 symbols and shares exactly one symbol with every other card. A standard Spot It! deck can be viewed as the projective plane of order 7. My undergraduate research group began by creating Spot It! decks for projective planes of order $\mathrm{n} \leq 16$. For each deck, we studied sets of cards where each symbol appears at most twice. We will describe the process of finding research problems related to the game, followed by results and work in progress. (Received September 13, 2023)

## 192-102-32977

Lauren L Rose*, Bard College. EvenQuads, Finite Geometry, and Sidon Sets.
Quads is a SET-like card game, published by the AWM as EvenQuads. The cards can be viewed as points in the affine geometry $A G(6,2)$, which easily generalizes to $A G(n, 2)$ for any $n$. A quad in the card game corresponds to a plane in $A G(n, 2)$. We are most interested in collections of cards that don't contain a quad, which turn out to be Sidon Sets. We describe an analog of the "Cap Set problem" for the game SET, providing results and work in progress.
(Received September 13, 2023)

## 1192-102-33046

Lily Chung*, MIT. How Not To Get Around In Video Games.
Many video games involve exploration by the player character, who must interact with and manipulate the game's environment to reach their goal. From a computational perspective, many of these games can be shown to be intractable. We give an exposition of the motion-planning-through-gadgets framework, a general method for analyzing problems involving a moving agent. We show applications of this framework and its extensions in proving hardness of various puzzles and games. In particular, we show how recent work on checkable gadgets was used to prove the PSPACE-completeness of Push-1F, a theoretical abstraction of many "Sokoban-like" puzzle games.
(Received September 13, 2023)

1192-102-33439
Matvey Borodin, Brookline High School, Eric Chen, Acton Boxborough Regional High School, Aidan Duncan, MIT PRIMES STEP, Boyan Litchev, Lexington High School, Jiahe Liu, MIT PRIMES STEP, Veronika Moroz*, Lexington High School,
Matthew Qian, MIT PRIMES STEP, Rohith Raghavan, MIT PRIMES STEP, Garima Rastogi*, Virtual Learning Academy Charter School, Michael Voigt, MIT PRIMES STEP. The Stable Matching Problem and Sudoku.
Are you having trouble getting married? These days, there are lots of products on the market for dating, from apps to websites and matchmakers, but we know a simpler way! That's right - your path to coupled life isn't through Tinder: it's through Sudoku! Read our fabulous paper where we explore the Stable Marriage Problem to help you find happiness and stability in marriage through math. As a bonus, you get two Sudoku puzzles with a new flavor.
(Received September 16, 2023)

1192-102-33481
Ian Nicholas Bridges*, Florida State University. Tilings in the 3 dimensional lattice with L-tetrominoes. Preliminary report. We consider three dimensional L-tetrominoes. We show that there exists at least one way to tile every three dimensional rectangle whose side lengths are at least three and area is congruent to $1(\bmod 4)$ such that one square goes untiled. In addition, we show that every three dimensional rectangle is tileable provided one side has length at least 2 and the other is a multiple of 4.
(Received September 18, 2023)

## 103 Professional development and professional concerns

## 1192-103-27940

Ron Buckmire, Occidental College, Carrie Diaz Eaton, Bates College, Joseph Edward Hibdon, Northeastern Illinois University, Jakini Kauba, Clemson University, Drew Lewis*, Unaffiliated, Omayra Ortega, Sonoma State University, Jose Luis Pabon, New Jersey Institute of Technology, Rachel Roca, Michigan State University, Andrés R. Vindas-Meléndez, UC Berkeley. Quantifying Inequities and Documenting Elitism in Mathematics.
The myth that mathematics is a meritocracy is pernicious but prevalent in our discipline. This talk reports on ongoing work to quantify inequities in mathematics whose existence is inconsistent with the notion that mathematics is a meritocracy. In particular, we examine gender representation and NSF funding among Ph.D.-granting departments, finding that "prestigious"
departments unsurprisingly amass the vast majority of grant funding while having fewer women faculty than less "prestigious" departments.
(Received August 22, 2023)

## 1192-103-29168

Peyam Ryan Tabrizian*, Brown University. Making the real analysis experience an epsilon-bit more bearable.
I will present some fun analogies that will help the students better grasp the concepts taught in real analysis. What do Cauchy sequences have to do with concerts? What does sequential compactness have to do with scissors? Come (converge) to this talk and find out! I will also highlight a project that I undertook at the start of the pandemic where I taught a whole real analysis course through my YouTube channel Dr Peyam (over 156,000 subscribers) by splitting it up into shorter 10-20 mins long videos.
(Received September 05, 2023)

1192-103-29574
Sarah van Ingen Lauer, University of South Florida, Eugenia Vomvoridi-Ivanovic, University of South Florida, Jennifer
Ward*, Kennesaw State University. Advancing Professional Development Through Informal Learning: Cultivating a Community of Practice of Mathematics Educators Who Are Parents.
We report findings from a collaborative self-study on knowledge gained from our experiences as parents and how this expertise serves as informal professional development for our work as university-based mathematics educators. Unearthing and sharing tacit knowledge requires interaction and informal processes such as storytelling, conversation, and coaching that communities of practice provide. Through a communities of practice approach we were able to explore the knowledge gained from our experiences that lie in the intersection of our participation in both the parenthood community and the academic community - two communities that have been traditionally viewed as distinct and at odds with one another. Our goal is to humanize mathematics educator expertise by unearthing and giving credence to the unique and often invisible knowledge of mathematics educators who are parents and by attending to how mathematics educators leverage expertise from informal parenting experiences in their academic teaching. We use the TEACH Math collaboration's four lenses of classroom practice; teaching, learning, mathematics task, and power and participation, to examine how these informal experiences contribute to our work as mathematics educators. We will also engage in discussions around ways in which informal learning experiences of mathematics educators can inform their practice and serve as counter-narratives to how parenthood has traditionally been viewed in academia.
(Received September 07, 2023)

1192-103-30509
Tuncay Aktosun, University of Texas at Arlington, Jianzhong Su*, University of Texas at Arlington. MENTORING PROGRAMS IN MATHEMATICS AT THE UNIVERSITY OF TEXAS AT ARLINGTON AND THEIR LOCAL AND BROADER IMPACTS.
We describe three mentoring programs in the Department of Mathematics at the University of Texas at Arlington, and we provide some data to present their local and broader impacts. The first of these three programs is the S-STEM program funded by the NSF aiming to provide undergraduate mathematics majors with scholarships and mentoring, and it has been running in our department since 2008. The second is the Bridge-to-Math-Doctorate program supported by the NSF aiming to provide scholarship and mentoring for students to help them transition to doctoral studies, and it has been running in our department since 2016. The third one is an NSF conference grant program to support the Gulf States Math Alliance conference in 2023 for faculty development and student mentoring for encouraging students to pursue mathematics career pathways. We elaborate on our vision and strategy in running these programs and on the improvements we have implemented. We also provide some data to demonstrate how the Covid-19 pandemic has been affecting our undergraduate mathematics majors in our NSF SSTEM program.
(Received September 10, 2023)

1192-103-31709
Shandy Hauk*, San Francisco State University, Sean P Yee, University of South Carolina. Improving the Preparation of Graduate Students to Implement Student-centered, Inclusive Teaching. Preliminary report.
Entry-level college mathematics classes are often taught by graduate students (about 35\%). In fact, 94\% of graduate students will teach at some point. Research indicates that college mathematics instructors, particularly novices, benefit from wellstructured teaching-focused professional development (TPD). Decades of work in undergraduate mathematics teaching, learning, and curriculum development have created an evidence-based foundation of resources for equitable teaching and effective learning. Yet, getting implications from research into the preparation of graduate students for teaching remains a challenge. The project Improving the Preparation of College Mathematics Instructors to Implement Student-centered, Inclusive Teaching (DUE-2020952; 2021139) is addressing the challenge. The project has completed a national survey of departments about their TPD for graduate students and done interviews with expert providers of TPD. These survey and interview results, along with what is being learned from other post-secondary projects, are informing the development of a web-based tool to guide departments in designing TPD programs for graduate students. Slated for piloting in Spring 2024, the tool has users describe where the department is (point A), where they want to go (point B) with the local TPD program, and give data on local resources and constraints. The tool responds with recommendations for program design next steps based on paths others have taken for a similar journey (from A to B).
(Received September 12, 2023)

Camilo Mejía, DevSavant, Rafael Alberto Méndez-Romero*, Universidad del Rosario, Daniel Peña-Ronderos, DevSavant,
Yofer Quintanilla-Gómez, Universidad del Rosario. Joining Forces for Success: Rescuing Synergies between Industry \& Academia to Strengthen Technological Startups—DevSavant \& the School of Engineering, Science \& Technology at Universidad del Rosario.
Traditional approaches in higher education often overlook the crucial relationship between academia and industry. Generally,
they focus mainly on strengthening disciplinary competencies, neglecting other skills that are equally essential for improving students' employability. However, in recent times, the emergence of artificial intelligence has sparked unprecedented interest from both academia and industry, signalling a remarkable convergence of interests that has been absent for quite some time. This convergence underscores the growing need to establish strong links between education and industry, making it fundamental and highly relevant. In this context, this presentation highlights the importance of revitalizing these connections and considering industry as an essential ally in the formation of competent technological citizens. A paradigmatic success case is presented: the collaboration between Universidad del Rosario, through its School of Engineering, Science and Technology, and DevSavant, a services consultancy company dedicated to boosting, transforming and expanding the SaaS products of Savant Growth portfolio companies. This partnership, aligned with the shared goals of both parties, not only fosters gender equity, but also serves as a key driver for training new talent in the software development field. The most recent milestone of this collaboration has been the creation of CodeSavant, a Hackathon in which teams, supported by generative artificial intelligence, can propose innovative and creative processes to optimize software development. This communication aims to inspire other organizations to follow this collaborative model, which guarantees the emergence of exceptional young talents dedicated to technology. In short, our presentation seeks to be an inspirational catalyst for other academic institutions and companies that wish to forge similar relationships, in order to nurture and empower the next generation of technology professionals.
(Received September 12, 2023)

## 1192-103-31942

Brandon Armstrong, Valencia College, Deb Howard, Valencia College, Ryan Kasha, Valencia College, Keri Siler, Valencia College, Sidra Van De Car*, Valencia College. Cultivating Confidence: Professional Development that Encourages Active Learning Experimentation. Preliminary report.
How do you transform mathematics education at your institution into a culture of support empowering faculty to experiment with novel active learning and inclusive practices in the classroom? Through an NSF grant (Award Number: 2111262), Valencia College created a tiered cohort-model professional development series for mathematics instructors. Mentors, who redesign one class to be based in inclusive active learning techniques, work with mentees in the program to design and implement two new inclusive active learning lessons. Mentees in the first cohort reported feeling very supported and less hesitant to experiment in the classroom. The study design and professional development will be described, and an overview of the first cohort's preliminary results will be discussed. Access to the lessons developed under the grant, designed for general education mathematics courses, will be provided.
(Received September 12, 2023)

## 1192-103-32608

Natalie E. Dean*, Emory University. Tales From the Front Lines of Pandemic Communications.
All of our lives changed in 2020, and I'm no exception. As a biostatistician specializing in emerging infectious disease research, I was eager to contribute to the cause through my work. But a surprise to me was the level of impact I was able to make via public engagement. My communications took many forms, including television, radio and podcast appearances, opinion pieces and editorials, and a lot of talking to reporters. It also included direct engagement with the public via Twitter. I digested new studies and synthesized these with existing findings. I taught others how to interpret results, particularly surrounding vaccine effectiveness. And I advocated for higher quality data. I will talk about my experiences diving into the deep end of communications during the pandemic. I will recall times that were alternately funny, challenging, humbling, unnerving, and incredibly rewarding. I will share some of the most important lessons I've learned along the way, and talk about what we stand to gain by fostering communication skills among mathematicians and statisticians. (Received September 12, 2023)

## 1192-103-33109

Kristin E. Lauter*, Meta AI Research (FAIR). Women In Numbers (WIN) and AWM: creating Research Networks for Women in Mathematics.
I like to say that serving as AWM President (2015-2017) and starting WIN (Women in Numbers) with Rachel Pries and Renate Scheidler in 2006 is the proudest achievement of my career. Because I view them as two parts of the same 15-year long, ongoing effort: to build and sustain Research Networks for Women in mathematics. We now have 25 different Research Networks for Women in many areas of math, from the early networks, WIN, WiSh, WIT, WIMB, ACxx, to many newer and interdisciplinary networks. We have published 35 volumes of research papers in the AWM Springer Series, and many articles in the newly launched La Mathematica AWM journal. This talk will highlight aspects of the history of these efforts and reflect on next steps.
(Received September 13, 2023)

1192-103-33184
Patricia Klein*, Texas A\&M University, Allechar Serrano Lopez, Montana State University. Beyond Roots of Unity: Mentor and Organizers Panel.
In this panel, mentors and organizers of the Roots of Unity workshop will provide advice and share their experiences. (Received September 13, 2023)

1192-103-33913
Catherine Roberts*, COMAP. About the Consortium for Mathematics \& Its Applications.
For over four decades, the Consortium for Mathematics and Its Applications (COMAP) has produced classroom modules, journals, newsletters, textbooks, videos, and national reports in support of applied mathematics teaching and learning, particularly related to mathematical modeling. Since 1985, COMAP has annually held its international Mathematical Contest in Modeling (MCM) for undergraduate students, and in 1999 added its Interdisciplinary Contest in Modeling (ICM). COMAP also offers contests at the school level (MidMCM, HiMCM, and IM2C). During this introductory portion of this Special Session, we will overview COMAP's curricular materials and resources, and its many contest opportunities. (Received October 06, 2023)

## 104 Wider issues

## 1192-104-25446

Kamuela E. Yong*, University of Hawaii West Oahu. When Mathematicians Don't Count. Preliminary report.
A systemic issue of Indigenous invisibility within the mathematical community persists, rooted in practices that obscure Indigenous individuals in demographic data. Whether through aggregation with broader groups, categorization as "other," or complete omission due to identifiability concerns, they remain statistically invisible. This not only impedes accurate representation but also perpetuates the false narrative that mathematics is devoid of Indigenous presence. Simultaneously, Indigenous voices remain critically absent within educational spaces. In this talk, I will not only address these challenges but also share our ongoing efforts to build a thriving community of Indigenous mathematicians. Furthermore, I will discuss my personal journey in transforming my curriculum, infusing it with examples of ancestral knowledge and Indigenous perspectives integrated into mathematical concepts. By shedding light on these issues and offering actionable strategies for change, this presentation seeks to inspire hope and promote a more inclusive and welcoming environment for Indigenous individuals within the mathematical community.
(Received June 09, 2023)

## 1192-104-25870

Ranthony A Clark*, The Ohio State University. Quantitative Justice: Intersections of Mathematics and Society. Preliminary report.
In this talk, we use a historical lens to consider intersections of mathematics and society. We look internally and externally by considering two perspectives-(1) how society impacts equity within the mathematical community, and (2) how the mathematical community impacts equity within society. With respect to the latter, we focus on one particular external effortan emergent new field of interdisciplinary research called Quantitative Justice. Quantitative Justice comprises the mathematical, computational, and statistical analysis of real world problems related to social inequity. In this context, mathematical tools are used to quantify notions of 'fairness' in a given domain, generating both new mathematics and impacting society at large. We will give current examples of how math is being used to shift societal systems, and discuss how this research complements historical efforts to improve equity in mathematics by bridging the divide between scholarship and activism.
(Received July 13, 2023)

1192-104-25982
Arezoo Islami*, San Francisco State University. The Unreasonable Effectiveness of Mathematics: Dissolving Wigner's Applicability Problem.
To be announced
(Received July 17, 2023)

1192-104-27058
Victor Piercey*, Ferris State University. Using Data Science to Promote Genocide Reliance to Policymakers. Preliminary report.
Social scientists use data science algorithms to forecast and predict genocide. For example, the "Early Warning Project" uses a modified logistic regression to identify at-risk countries that are put on a watch-list. We propose to use data science to invite policymakers to strengthen their resilience to genocide. We proceed from the assumption that policymakers act in their selfinterest. To that end, given an at-risk country or region, we seek to find previous genocide events that data suggests have similar profiles (which we call "nearest neighbors") and use data-storytelling to tell a story about the impact of the event on those nearest neighbors. In this talk, we will share our progress toward such a program.
(Received August 09, 2023)

## 1192-104-27665

Amanda Harsy Ramsay*, Lewis University. Benefits and Challenges of Mentoring Students in Data-Driven Sports Analytics Research.
Sports analytics is an exciting and assessable research area which draws student interest and allows students the opportunity to explore various data-driven projects. In this talk, we will share some examples of data-driven undergraduate research projects within the realm of sports analytics, discuss the benefits and challenges of mentoring and conducting these projects, and offer practical advice and lessons learned from these experiences.
(Received August 17, 2023)

1192-104-27935
David A Reimann*, Albion College. One Hundred Quotes for One Hundred Numbers.
A curated list of literary quotes was compiled, one for each of the integers from one to one hundred. Each quote was chosen for its content and range from humorous to inspirational. The quotes provide a sampling of the broad categories writers cover when using numbers. Example quotes will be shared to illustrate broad categories such as counting items, measurements of space and time, ages of individuals, and labels. The process for finding interesting quotes will be discussed along with challenges in authenticating the original sources.
(Received August 24, 2023)

## Romantic Era Music using Diophantine Approximation.

The Romantic period of music is known for its intricate and emotionally expressive harmonic style. Nevertheless, recreating authentic melodies of the Romantic period in contemporary compositions poses a considerable challenge for composers and musicians. In this paper, I present a method for simulating chromatic harmony in Romantic Era music through the use of Diophantine Approximation. Inspired by the works of renowned composers of that period, such as Richard Wagner, Franz Liszt, and Frederic Chopin, my transitional model was constructed to preserve the essence of the preceding melody as the harmonic progression unfolds. The model helps with the creation of harmonic progressions that contain the nuances of the preceding harmonic structure and musical style, providing composers with a novel way to explore a wide range of musical possibilities. Experimental validation involving case testing and human hearing shows that the model is successful in imitating authentic Romantic-era harmonic structures. This finding suggests that the model is a promising tool that could inspire contemporary composers to create almost authentic chromatic harmonic progressions. (Received August 26, 2023)

## 1192-104-28222

Vladimir Bulatov*, Shapeways. Creating Symmetry Using Dynamics on Orbifolds. Preliminary report.
The motivation of this work is the development of algorithms to create animated seamless patterns with discrete symmetry in various geometries. It is relatively easy to create a seamless pattern with symmetry generated by pure reflections. The basic kaleidoscope is constructed in such a way. Reflections are continuous functions and the resulting patterns are continuous. Animation of such kaleidoscope keeps the pattern continuous but discontinuity of derivatives along reflection lines generates obvious visible artifacts. In case of more general symmetries it is rather difficult to make the pattern even continuous. A way to make symmetric patterns with arbitrary symmetry is the rubber stamp approach used by M.C.Escher in his tessellation work. This requires very difficult manual fitting of the tiles. No general way to animate such patterns exists. Another approach is to construct a function with fine tuned symmetric properties and use the function as a tool for domain coloring. It is difficult to construct such functions. We are offering a general approach to make seamless patterns using dynamic (ODE or PDE) on an orbifold. An orbifold can be cut and flattened. The flattened orbifold will tile the whole space. If the pattern is seamless on the orbifold the tiled pattern will be seamless. So the problem of making a seamless symmetric pattern is reduced to creating a seamless pattern on the orbifold. This problem can be successfully solved using appropriate boundary conditions. Such patterns can be animated in real time in web browsers using javascript and WebGL. Animated results of such an approach are presented in youtube videos. Dynamics of several interacting points https://tinyurl.com/4ywt3z8y Gray-Scott reaction diffusion https://tinyurl.com/4zffnxbm
(Received August 28, 2023)

## 1192-104-28269

John Leo*, Halfaya Research. Chinese Remainder Theorem
Mathematics is an art, whose best theorems and proofs have a deep beauty unlike any other art form. Different from other arts, however, appreciation of this beauty requires hard work and years of specialized study. In some cases, such as fractals, it may be possible by visualization to get a glimpse into the fundamental mathematics, but this is rare. Music is the art form closest to mathematics, and math's foundational use in harmony, counterpoint and rhythm is well-known. Math has also inspired mathematical composition, although to this point the level of math used has been fairly simple. There has also been work done on sonification of math, including by the author, but again there is room to go much further. It seems clear that there is an opportunity for considerable advances in two complementary areas: the use of advanced mathematics to structure musical composition, and the use of music to provide a means to assist in understanding the deeper beauty of math. The musical work "Chinese Remainder Theorem", based on the author's favorite theorem and composed for a small ensemble of Chinese instruments, is an attempt at a small step toward these goals. The composition is simultaneously a musical reflection of the proof of the theorem and an application of the theorem to the music itself. The presentation will include a discussion of both of these aspects, as well as a short demonstration of part of the work.
(Received August 29, 2023)

## 1192-104-28346

Jennifer M. Wilson*, Eugene Lang College, The New School. Jennifer Bartlett: Working with Grids.
The artist, Jennifer Bartlett, organized all her works on a grid. Whether painting landscapes and interior scenes or experimenting with abstract patterns, the grid served as both structure and subject. In this talk, we will focus on Bartlett's earlier work, looking at several of her works involving counting patterns and simple algorithms. We will discuss how these works give rise to interesting mathematical questions that can be used in the classroom and/or as the starting point for students' creative coding projects.
(Received August 29, 2023)

## 1192-104-28408

Annalisa Crannell*, Franklin \& Marshall College. The moon tilt illusion and perspective geometry.
The Moon Tilt illusion confuses the viewer about the direction of illumination of a waxing or waning moon. We give several examples of this phenomenon and explain how the illusion arises from standard (but surprising) aspects of perspective projections. Familiar perspective drawings and photographs of objects such as clocks and cubes help us further analyze the unfamiliar explanations of pictures of illuminated portions of spheres.
(Received August 30, 2023)

## 1192-104-28752

Jaya Bishnu Pradhan*, Tribhuvan University. Ethnomathematics and Indigenous Epistemology. Preliminary report. Indigenous peoples have their own mathematical knowledge, which they have used for survival and transcendence for a long time. The mathematical knowledge they generate is a result of different cultural activities of making patterns, designing artifacts, playing games, measuring, calculating, and applying to solve problems in everyday life. Ethnomathematics recognizes mathematical knowledge, ideas, and thinking present in various practices and experiences across diverse societies. It is a research program that investigates how cultural groups understand, articulate, and use concepts and practices. Thus,
ethnomathematical research aims to reveal how indigenous peoples perceive, understand and practice mathematical phenomena developed by different groups of people to interact with their immediate environment. In the AMS special session, I explore indigenous epistemology on how members of the Nepalese indigenous people like Chundara and Bhujel think, how they perceive, explain things, and what it means for them. This involves acknowledging the mathematical significance in indigenous artifacts, languages, and cultural practices as well as celebrating the diverse interpretations and applications of mathematics within indigenous cultures through the lens of indigenous epistemology.
(Received September 02, 2023)

1192-104-28799
Xiaoye S Li*, Lawrence Berkeley National Laboratory. Interplay of linear algebra, machine learning, and high performance computing.
In recent years, we have seen a large body of research using hierarchical matrix algebra to construct low complexity linear solvers and preconditioners. Not only can these fast solvers significantly accelerate the speed of large scale PDE based simulations, but also they can speed up many AI and machine learning algorithms which are often matrix-computation-bound. On the other hand, statistical and machine learning methods can be used to help select best solvers or solvers' configurations for specific problems and computer platforms. In both of these fields, high performance computing becomes an indispensable cross-cutting tool for achieving real-time solutions for big data problems. In this talk, we will show our recent developments in the intersection of these areas.
(Received September 03, 2023)

## 1192-104-28834

Michelle R. DeDeo*, University of North Florida. Data-Driven Research Projects: Benefits and Challenges for Faculty. Preliminary report.
Drawing on extensive experience with students from diverse fields such as math, statistics, biology, psychology, and computer science, this talk explores the tangible advantages of mentorship in data science projects from skill development and research opportunities to portfolio creation. The talk provides practical strategies for successful faculty mentorship, inspired by data science experiences in collaboration with clinicians, community partners, and scientists, both within and outside of the university. Mentoring also presents its share of complexities. The talk candidly addresses the challenges including fostering a collaborative research environment, managing project goals, and best practices for dealing with various complications that arise in data science projects.
(Received September 03, 2023)

## 192-104-28939

Tong Chen, Santa Clara University, Frank A Farris*, Santa Clara University, Jingxuan Hou, Santa Clara University, Reza Shariatmadari, Santa Clara University, Yanni Zhou, Santa Clara University. The Artistic Potential of Bessel Functions. Preliminary report.
The humble sine and cosine functions have led to quite a large body of mathematical artwork. And yet, the Bessel functions, which are formally similar to the trigonometric functions in so many ways, are relatively unexplored. Like the trig functions, Bessel functions are solutions to second order differential equations that arise naturally in physics, their simplest formulas require power series, and, apart from some special values, we must turn to computers to know their values to some desired accuracy. We explain how Bessel functions (of the first type) occur in modeling a circular vibrating membrane with a fixed edge, then show a variety of artworks we created based on those vibrations. This illustrates a belief that beautiful shapes naturally arise from solutions to the classical wave equation.
(Received September 04, 2023)

1192-104-29042
Thomas L Drucker*, University of Wisconsin--Whitewater. Lewis Carroll and Mr. $B^{*} r t r^{*} n d R^{*} s s^{*} l l:$ P.E.B. Jourdain on the State of Mathematical Philosophy in 1918. Preliminary report.
Philip E.B. Jourdain's 'The Philosophy of Mr. B*rtr*nd R*ss*ll' (1918) has generally been taken as a humorous assault on the writings of philosophers having to do with mathematics. He is working in the tradition of Lewis Carroll with entertaining results. There is also, however, a serious thread running through the book, as illustrated by Jourdain's ongoing correspondence with Bertrand Russell. This talk looks at the philosophical points made by Dodgson, even in his humorous writings, and Jourdain's demonstration that some of those points had not been satisfactorily addressed in the half-century since Dodgson wrote.
(Received September 05, 2023)

1192-104-29424
Donald G Palmer*, Numbers Unlimited Ltd. Bridging the Boundary of Applied and Pure Mathematics: A Philosophical Argument for Expanding Mathematics and Scientific Disciplines via a New Numeric System.
This presentation is about new directions of study and discovery in both Mathematics and Science. Over the last several centuries, science has discovered objects in the world along a continuum of scale. In one direction, we have found planets and stars, galaxies and galaxy clusters. In the other direction we have found cells and proteins, atoms and neutrinos. In order to locate and model this world, we use the 3 traditional directions of length, width and height. However inherent in all our measurements is the scale of the objects we are measuring - a continuum we do not directly see with our eyes. A key reason we do not include this direction as part of our scientific models is that we do not have the appropriate mathematical tools to take measurements along this continuum. New mathematical tools may require a numeric representational system with more power than our traditional decimal or positional based numerals. Such a more powerful system may provide a single value for complex numbers able to measure across scale and adds to its structure the reversing operations of integration and differentiation. The ability to calculate integration and differentiation results would be a powerful mechanism for applied and for pure mathematics. The author presents some opening remarks on what is anticipated to be a much larger discussion, looking at a model of reality where objects at all levels of scale can be located and considers directions for generating more powerful mathematical tools than we have today.

Rosemberg Toala-Enriquez*, George Mason University. The Outreach program at GMU's Mason Experimental Geometry Lab.
The MEGL team is a group of students and faculty at George Mason University with a passion for sharing the fun side of mathematics with K-12 students. We have been offering free mathematics enrichment activities to local schools and other organizations since 2015. In this talk, I will share with you our outreach activities, through some examples. I will also give you insights into how we handle the logistics, preparation, and organization behind the scenes. Finally, I will present some challenges and areas of improvement of the program.
(Received September 06, 2023)

## 1192-104-29575

Beyza Caliskan Aslan*, University of North Florida. Crocheting Islamic Geometric Art. Preliminary report.
A common feature of Islamic art is the covering of surfaces with geometric patterns. This use of geometry is thought to reflect the language of the universe and help the believer to reflect on life and the greatness of creation. In Islamic art, the circle represents unity and the ultimate source of all diversity in creation. The division of the circle and use of symmetries is the starting point for many traditional Islamic patterns. Such symmetries have now been classified as belonging to distinct mathematical groups. In my work, I crocheted a design after Paul Marchant, which is based on elements of medieval designs typically found in Cairo. The pattern consists of underlying polygons such as hexagons, triangles, squares, dodecagons. As it is the case in most Islamic geometric art, the underling tiling pattern is hidden beneath the final design but the viewer is given an intimation of the hidden order and this is part of what gives the design its meditative power. In crocheting the different polygons involved, specific stitch numbers are assigned for the degrees of the polygons, for example 2 stiches for 60 degrees, around the circle. Details of crocheting each polygon is explained, as is the patterns involved in the entire design. (Received September 07, 2023)

## 1192-104-29615

Mark Alan Branson*, Stevenson University, Whitney George, University of Wisconsin, LaCrosse. Math for the People: A Collaborative Quantitative Justice Textbook for Non-STEM Students.
While many of the methods used in quantitative justice are more advanced, there are numerous topics and approaches which can be made accessible to a non-STEM audience. Math for the People is an OER textbook designed to provide a framework for instructors to engage non-STEM majors in quantitative justice applications of mathematics. Each module covers a social justice topic, like predatory lending or racially biased policing, and introduces elementary quantitative justice methods which can be used to explore the issue and work for change. We will provide an update on the current status of the project, opportunities to participate, and future work.
(Received September 07, 2023)

## 1192-104-29790

Raegan J Higgins*, Texas Tech University, Alison M. Marr, Southwestern University, Amy Oden, Pomona College. From Absence to Excellence: The Impact of the EDGE Summer Program on Women in Math. Preliminary report. The EDGE (Enhancing Diversity in Graduate Education) Summer Program has been an instrumental force in transforming graduate education in the mathematical sciences and fostering diversity in academia for the past quarter-century. This engaging talk provides a comprehensive overview of the program's evolution, impact, and ongoing commitment to empowering women mathematicians. The talk highlights the pivotal role the EDGE Summer Program has played in nurturing talent, fostering leadership, and promoting inclusivity within the mathematical sciences. It explores the stories of its alumnae, showcasing their significant contributions to education, research, and the corporate and government sectors. As the EDGE Summer Program looks forward to the next 25 years, this presentation offers a vision for the future, highlighting its commitment to advancing diversity and shaping the next generation of leaders. Join us in celebrating two and a half decades of achievements and envisioning the continued transformation of graduate education through diversity and inclusion in the mathematical sciences.
(Received September 07, 2023)

## 1192-104-29864

Kylie Loftis*, Virginia Commonwealth University, Citlali Rocha, Kansas State University. Essential Contributing and Predictive Factors of Bravery Through Advanced Data Analytics.
What makes a person brave? Brave Enterprises is a company that designs and implements dynamic training that enables people to recognize fear as a cue to take brave action. Part of the training sessions, requires participants to complete a presurvey prior to the 2-hour session and a post-survey following the session. Embry-Riddle REU students were given the opportunity to research and analyze various datasets provided by Brave. This analysis used more than a thousand matched surveys from various sessions done by Brave Enterprises since their founding in 2016. Our goal was to identify what factors lead to a person's sense of bravery. The advanced analysis performed included: feature importance, cross correlations that lead to high bravery scores, and efficiency of the program. It was expected that leadership style, choice of role model, and sense of purpose were strong indictors of one's bravery score. By understanding which factors are the most predictive, regarding bravery score, we can contribute and support Brave's mission of helping people tackle their obstacles, grow confidence, and be more brave.
(Received September 08, 2023)

## 1192-104-30085

Treena Basu*, Occidental College, Ron Buckmire, Occidental College, Osei Kofi Tweneboah, Ramapo College of New Jersey. An Unusual Application of Machine Learning: The Educational Data Mining Context.

Educational data mining is an emerging discipline concerned with the analysis of data that comes from educational settings. The focus of this talk is to present some examples of educational data mining research, specifically the application of machine learning to college admissions. Meeting targets for the number of students admitted and enrolled is crucial for many institutions of higher education since tuition-based income often serves as a major component of the operating revenue budget. Enrollment targets and diversity goals can be hampered by summer melt: the phenomenon in which students who, after being admitted and having committed to attend a college or university in the spring, do not actually enroll in the fall. Using six years of data from 2014 through 2019 of students admitted to a small liberal arts college in California, we investigate the application of supervised machine learning models to predict and identify those admitted students who will decline their admission offers, those that will accept their admission offers and those students who are in danger of "melting away" over the summer.
(Received September 08, 2023)

## 1192-104-30212

Chamberlain Fong*, San Francisco, CA. Visualizing Squircular Implicit Surfaces.
The squircle is an intermediate shape between the square and the circle. In this paper, we examine and discuss equations for different types of squircles. We then build upon these 2D shapes to come-up with various 3D surfaces based on squircles. The visualization of these surfaces can be considered as an artistic endeavor. There is a preprint document available in
https://arxiv.org/abs/2210.15232
(Received September 09, 2023)

## 1192-104-30334

Margaret Kepner*, Independent Artist. Shoofly Shapes, Stamps, Stencils, and Symmetry. Preliminary report.
Traditional quilt patterns often contain, or suggest, interesting mathematical concepts. For example, the quilt block called
"Shoofly" can be a starting point for exploration and further visual interpretation. I have created a design of mutated Shoofly shapes with various symmetry properties. Further analysis suggests ways to add information and enhance the design through color. I will consider the minimal set of mutated Shoofly blocks under rotation (stamps), and under both rotation and reflection (stencils). I will also discuss the MacMahon 3-color squares.
(Received September 09, 2023)

## 1192-104-30364

Kelly Buch, Austin Peay State University, Alexis Hardesty*, Texas Woman's University, Sofia Rose Rose Martinez
Alberga, Purdue University, Quiyana Murphy, Virginia Tech. EDGE Networking Session.
This session will feature structured time for interaction and networking between early career mathematicians and more senior women in math. A group photo for EDGE individuals will be taken at the start of this session. Anyone is welcome to attend. (Received September 09, 2023)

## 1192-104-30515

Victor Piercey*, Ferris State University. Communicating Ethical Reasoning in Actuarial Science. Preliminary report. Professional mathematicians in business, industry, and government are likely to run into difficult ethical dilemmas. In these situations, they will have to make difficult decisions, and then explain and justify those decisions to others. In this talk, I will share how I have introduced ethical reasoning and communication into the financial mathematics course for actuarial science students. I will describe examples of how the lessons are designed, what I do to support students, and some of their work. (Received September 10, 2023)

## 1192-104-30595

Andrew James Simoson*, King University. Do-it-yourself trammel constructions for the ellipse, the conchoid, and the quadratrix. Preliminary report.
With a bit of wood, a few dowels, pvc pipe connectors, and patience, we show how to construct easy-to-make functional renditions of the trammels used by Archimedes, Nicomedes, and Hippias of Ellis so as to create the ellipse, the conchoid, and the quadratrix that were instrumental in "resolving" the three great impossible Euclidean geometry constructions. (Received September 10, 2023)

## 1192-104-30606

María Del Carmen Bonilla Tumialán*, National University of Education Enrique Guzman y Valle. Quipu and Yupana in Inca mathematics.
The present research investigates the Inca mathematics tools, Quipu and Yupana. Quipu is a Quechua word which means knot. This Inca writing system contains statistical, demographic data, and abstract topics information. The Incas were not just an oral culture, they knew how to preserve historical memory through an annotation system with knots, using three-dimensional tactile signals which contain meaning, combining knots, colors, shapes and rope twisting. Ropes were made with camelid wool or cotton fibers. The Mathematical Quipu was based on the decimal and positional numbering system. It is formed by a thicker main rope, from which hang several thinner ropes, each one represents a number. At the base of the rope there is a knot that represents the units; at the next higher level another knot represents the tens, and so on; the knots go up, each knot in a higher position or level represents a higher power of 10 . The absence of a knot represents zero. On the other hand, Yupana is a type of abacus which was used to perform mathematical operations. Yupana in Quechua means to count. They were made of clay, stone or wood. The numbers were represented with corn kernels, seeds, or pebbles. We can see a Yupana drawing in a book written by an indigenous chronicler in 1615. The illustration suggests that the Quipucamayoc, experts on the issue, calculated mathematical operations with the Yupana and then recorded the data on the Quipu.
(Received September 10, 2023)

Hélène Zapolsky*, University of Rouen, France. Phase-field models at atomistic and mesoscales in materials science.
In recent years, the phase field (or diffuse interface) method has become a powerful tool and has been applied in general methodology for moving interface problems arising from phase transition and solidification of materials, astrophysics, biology, differential geometry or fluid mechanics. This method describes a multiphase system using a set of conserved and nonconserved field variables that are continuous across the interfacial regions. The temporal and spatial evolution of the field variables at mesoscale is governed by the Cahn-Hilliard nonlinear diffusion equation and the Allen-Cahn relaxation equation. Including fundamental thermodynamic and kinetic input data, the phase field method is capable produce a quantitative description of arbitrary morphologies and complex microstructures without explicitly tracking the positions of interfaces. Recently this method has been extended at atomic scale. In this lecture, I will go back to the basics of this approach, introducing the principal equations and main assumptions. Then, I will showcase examples of applications of this method to study the phase transformations in different types of materials at atomistic and meso-scales.
(Received September 10, 2023)

1192-104-30714
Richard Anthony Cisneros*, Bachillerato Bivalente Fray Bartolome de las Casas. Didactic use of the Mayan Numeral System. Preliminary report.
In Southern Mexico, Guatemala, Belize and Honduras, the Mayan prehispanic culture created and used a vigesimal numeral system based on twenty and used only three symbols that represent the values of 0,1 and 5 . My participation as a mathematics and science educator will offer a review of the mathematical nature of aforementioned numeral system and focus on how, through workshop activities for teachers and students, the Mayan numeral system is recuperated within the educational levels of elementary, middle and high school as well as teacher's colleges. The vigesimal numeral system is the basis for architectural planning of ceremonial centers throughout the Mayan areas of influence. The vigesimal numeral system was a determinant factor in the creation of their solar and lunar calendars which could predict the appearance and conjunction of planets and Mayan constellations, lunar and solar eclipses. There will be a description of several didactic activities in the vigesimal numeral system (Mayan), some in the form of games, that permit a comprehension and comparison of the mathematical properties of decimal and vigesimal numeral systems. (Received September 10, 2023)

## 1192-104-30727

Chanwoo Lee*, University of California, Davis. Category Theory as an Explanatory Foundation.
Can category theory be a foundation of mathematics? I argue that category theory can serve as a foundation of mathematics in an explanatory sense. The explanatory sense of foundation is both historically situated and philosophically motivated. Based on the recent scholarship on mathematical explanation, I explain how CT can be explanatory and how it ties in with the more traditional debates on category theory's foundational status.
(Received September 10, 2023)

## 1192-104-30749

John A Christian, Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, Michela Mancini*, Guggenheim School of Aerospace Engineering, Georgia Institute of Technology. Spacecraft state estimation from crater projection in a pushbroom camera image. Preliminary report.
Craters on the surface of planetary bodies are often approximated as ellipses for spacecraft optical navigation purposes. It is well-known that the image of an ellipse with a standard central perspective camera produces a conic on the image plane. However, several spaceflight missions are equipped with pushbroom cameras, whose projection geometry works differently. The images are captured following perspective projection along the direction of the 1D sensor array, and orthographic projection in the direction of motion. By exploiting the canonical parametric representation of a conic, we have developed an analytical expression for the curve produced by a pushbroom camera imaging a conic. We will show how the polynomial description of this curve allows to apply parameter homotopy continuation to solve for the position and velocity of a spacecraft, given the puhsbroom image of a known crater.
(Received September 11, 2023)

## 1192-104-30869

Dan Bach*, dansmathart. Space-Filling Circles: A New Coordinate System. Preliminary report.
By considering concentric spherical shells, we can fill up all of 3D space with disjoint circles of positive radius. In the spirit of rectangular, cylindrical, and spherical coordinate systems, each point in space is specified by a triple of numbers. In our case the coordinates are: $\mathrm{R}=$ radius of spherical shell, $\mathrm{m}=$ slope of intersecting plane giving a circle, and $\mathrm{t}=$ angle along that circle of our point. The recipe: Start with an infinite row of spaced vertical unit rings, and at each spherical shell make a family of circles that avoid the two points at the intersection of a shell and its ring. An interactive 3D model is at https://shorturl.at/xyLV9 (Inspired by Andrzej Szulkin's American Mathematical Monthly article, Nov 1983, pp 640-41). (Received September 11, 2023)

## 1192-104-31002

Henry Segerman*, Oklahoma State University. Mathematical dice design.
Robert Fathauer and I started making mathematically interesting injection molded dice as "the dice lab" in 2014, and have since produced over 30 different dice designs. I'll talk about the mathematical, functional, and aesthetic aspects of our work. (Received September 11, 2023)

## 1192-104-31067

Victor A Piercey*, Fitchburg State University, Victor Piercey, Ferris State University. Introducing Ethics in the Mathematics Classroom. Preliminary report.

Mathematicians have not had the opportunity to explore what it means to practice mathematics ethically, and this has left blind spot in the field. Over the last several years, there have been several challenges and circumstances to illuminate these blind spots. One potential leverage point to build a foundation to address future ethical challenges in the field is in the undergraduate mathematics classroom. We have brought together 16 mathematics instructors to introduce the concept of ethical mathematical practice into their courses, focusing on courses that are not only for mathematics majors but also for other STEM students. We will describe our introductory workshop to integrating ethics and share examples of learning outcomes, integration techniques, resources, and classroom activities.
(Received September 11, 2023)

1192-104-31105
Paul Dancstep, Topos Institute, Daniel Filonik, National Institute of Standards and Technology, Priyaa Varshinee Srinivasan, Topos Institute, Theodore V Theodosopoulos*, Nueva School, Niels Voorneveld, Tallinn University of Technology. Making and using a mathematical artictionary. Preliminary report.
Over the last two decades, Applied Category Theory (ACT) has emerged as a uniquely versatile language for expressing the metaphors that weave mathematical thinking across the sciences. Born at the interface of logic and language, ACT embodies a confluence of insights, from computer science to quantum physics, that are reshaping the questions we ask of our models, and what form we expect the answers to take. Inspired by our work in ACT, we present a curated compilation of artistic renderings that seek to engender the ineffable meanings that populate mathematics. Our hope is that such an artictionary can serve as an inviting, inclusive pathway into the mathematical experience. As such, we envision a catalytic role for it in teaching and learning mathematics.
(Received September 11, 2023)

1192-104-31108
Ron Buckmire*, Occidental College, Joseph Edward Hibdon, Northeastern Illinois University, Drew Lewis, Unaffiliated, Omayra Ortega, Sonoma State University, Jose Luis Pabon, New Jersey Institute of Technology, Rachel Roca, Michigan State University, Andrés R. Vindas Meléndez, MSRI/SLMath \& Harvey Mudd College. Metamath: Applications of Mathematics and Data Science to Analyze the Mathematics Community.
\#MetaMath, also known as "the mathematics of Mathematics" is an example of a quantitative justice project. Similar to "the science of Science" disciplinary area which uses scientific techniques to analyze scientific practices and the scientific community itself, \#MetaMath uses tools, topics, and techniques from the mathematical sciences to analyze the mathematics community. This talk will present several examples of projects that use mathematics and data science to analyze different aspects of the mathematics community itself, with the goal of highlighting issues of equity, diversity, and inclusion. (Received September 11, 2023)

## 1192-104-31638

Carlos Enrique Bustamante, Arizona State University, Jordan Lyerla, University Of Kansas, Aaron Martin, Arizona State University, Fabio Milner, Arizona State University, Elisha Marie Smith*, Kean University, Josean Velazquez, Arizona State University. The Relationship Between Sexually Transmitted Infections and Dating App Use: A Mathematical Inquiry. Incidence of sexually transmitted infections (STIs) is sharply on the rise in the United States. Between 2014 and 2019, male and female incidence has increased $62.8 \%$ and $21.4 \%$, respectively, with an estimated 68 million Americans contracting an STI in 2018 (1). Some human behaviors impacting the rising STI epidemic are unprotected sex and multiple sexual partners (2). Increasing dating app usage has been postulated as a driver for increases in these behaviors. This study attempts to quantify the impact of dating apps on the incidence and prevalence of STIs utilizing a two-sex SIS model of STI transmission. The model is also used to assess the possible benefit of in-app prevention campaigns. Additionally, the mathematical model is used to estimate the percentage of people who seek treatment after contracting an STI. \endabstract 1. Centers for Disease Control and Prevention. "Incidence, Prevalence, and Cost of Sexually Transmitted Infections in the United States". In: Fact Sheets (2022). url: https://www.cdc.gov/nchhstp/newsroom/fact-sheets/std/STI-Incidence-Prevalence-Cost-Factsheet.htm 2. Ashlee N. Sawyer, Erin R. Smith, and Eric G. Benotsch. "Dating application use and sexual risk behavior among young adults". In: Sexuality Research and Social Policy 15 (2018), pp. 183-191
(Received September 12, 2023)

## 1192-104-31743

Ariana Mendible*, Seattle University. Small Town Police Accountability: Case Studies and Opportunities in Data Science. Preliminary report.
Policing data is publicized and analyzed in many large American cities. However, the same transparency is seldom available for small towns where accountability is equally vital. In this talk we will share the work done to obtain and analyze policing data, that was developed by mathematicians, data scientists, community activists, and social scientists of the Small Town Police Accountability (SToPA) Research Lab. We will showcase the SToPA toolkit which uses Python tools for geospatial mapping, machine learning and statistical sampling to better understand the policing landscape. The goal of this work is to present an easily replicable framework for analyzing police and community interactions with accessible on-ramps for activists, developers and researchers.
(Received September 12, 2023)

## 1192-104-31818

Douglas J Dunham*, University of Minnesota Duluth, Lisa Shier, University of Maryland Global Campus. An Embroidered Hyperbolic Butterfly Pattern in the Poincaré Disk. Preliminary report.
Our goal was to create a hyperbolic pattern of butterflies in the Poincaré disk model of the hyperbolic plane using a computercontrolled embroidery machine. We were inspired by M. C. Escher's Euclidean plane Regular Division Drawing Number 70 of butterflies with symmetry group p6 (or [6,3] ${ }^{+}$in Coxeter notation and 632 in orbifold notation). Following Escher, we wanted our butterfly pattern to fill the hyperbolic plane without gaps or overlaps. Our pattern in the Poicaré disk has (uncolored) symmetry group $[7,3]^{+}$in Coxeter notation (or 732 in orbifold notation). In this pattern, seven butterflies meet at left front wingtips and three meet at right rear wings. In order to maintain Escher's perfect coloring in which the circles on the left front
wings are the same for all butterflies meeting at those wingtips and different from any of those butterfly colors, we need eight colors. Our resulting pattern has perfect 8-color symmetry, and the color group is the simple group of order 168. We will also discus several technical details that we had to overcome when programming the embroidery machine.
(Received September 12, 2023)

1192-104-31949
Oscar Vega*, California State University, Fresno. BAMM! A Bridge Program for Master's Students in The Californa State University System.
BAMM! (Bolstering the Advancement of Master's in Mathematics) is an NSF-funded program that leads talented low-income students matriculated at Master's programs at three California State University (CSU) campuses: Fresno, Pomona, and San Francisco. Its main objective is to support students interested in earning a Ph.D. in the mathematical sciences, who for various reasons want/need to go through a Master's program before successfully applying to Ph.D. programs. BAMM! has a system of Benchmarks in place so students navigate the process to apply to Ph.D. programs in a timely fashion while taking courses and engaging in research. In this presentation, we will delineate the BAMM! model and share data obtained this far (the program is in its fourth year).
(Received September 12, 2023)

## 1192-104-32045

Wilfredo Vidal Alangui*, College of Science, University of the Philippines Baguio. Stone walling practice in northern Philippines: Locating 'mathematics' within Indigenous epistemology. Preliminary report.
In the mountainous region of the Cordillera in northern Philippines, stone walling is a well-developed indigenous practice done to hold rice fields that are built in mountain slopes as well as to increase the area for improved rice production. My talk will present the practice of stone walling among the northern Kankana-ey people in Mountain Province. I will show that in the construction of stone walls, quantitative, relational and spatial (QRS) concepts are employed by the practitioners, suggesting a practice that uses a complex system of knowledge that may be considered mathematical. At the same time, such a practice highlights the Kankana-ey people's worldview. I will argue that a holistic understanding of the 'mathematical' knowledge of Indigenous peoples has to be viewed within the context of their worldviews and epistemologies. Not doing so decontextualizes indigenous knowledge and perpetuates epistemic violence.
(Received September 12, 2023)

## 1192-104-32261

Morane Almeida de Oliveira*, Instituto Federal de Educação, Ciência e Tecnologia do Acre - IFAC. The construction of number in the Pano and Aruák linguistic families in the southwestern region of the Amazon. Preliminary report.
Quantifying, measuring, classifying and ordering objects and things is a common activity in indigenous villages. This learning passed on orally by elders for a long time characterizes the richness present in the ways of mathematizing present in indigenous peoples. Among these ways we can mention the process of counting and recording trees on rubber roads. In the indigenous area of the Jordao River (Acre/Brazil), the Indian Augustinho uses a stick with marks on its three sides, each face corresponding to a rubber road, and each mark to a rubber tree, composing, in this artifact (stick), approximately 180 scratches. This work aims to investigate the variability of constructions used to quantify things and objects by the Pano and Aruák linguistic families from the southwestern Amazon region. We carried out a qualitative research and the methodology used in this study was exploratory documentary research. We examined primary and secondary sources contained in mathematics textbooks written in the indigenous mother tongue, doctoral theses in the area of linguistics defended by indigenous and non-indigenous people and documents originating from technical reports on continuing education of indigenous teachers carried out in villages in the state of Acre.
(Received September 12, 2023)

## 1192-104-32286

Sydney Wilson*, Converse University. Predicting the Number of Times Aid is Requested from a Non-Profit.
This project is focused on creating data-driven solutions for the Upstate Family Resource Center (UFRC), a nonprofit in Spartanburg County, SC. The UFRC serves as a central location for those in need to receive multiple types of aid with the goal of helping people regain their self-sufficiency. I will be discussing the results of using various machine learning algorithms to place clients into four unique groups based on projecting the amount of aid they will need to reach self-sufficiency. The accuracy of these models and future extensions will also be discussed.
(Received September 12, 2023)

1192-104-32324
Bailey Flanigan*, Carnegie Mellon University. Fairness and beyond in citizens' assemblies selection.
Citizens' assemblies are a method of participatory policymaking in which randomly-selected citizens, rather than elected representatives, make policy decisions. Citizens' assemblies are gaining traction worldwide, now being administered by more than 40 organizations in over 25 countries. Our work focuses on designing algorithms for randomly sampling the participants of these assemblies. These algorithms aim to satisfy desiderata like fairness, representation, transparency, and robustness to manipulation, and they use diverse techniques from, e.g., convex programming, integer programming, discrepancy theory, and randomized rounding. The algorithms produced by our work have been published in Nature, and they now are now in use by assembly organizers around the world. In this talk, I will discuss the application domain of citizens' assemblies, highlight key aspects of our existing work within it, and identify some of the technical and non-technical challenges that remain for future work.
(Received September 12, 2023)

Wang, University of Texas Rio Grande Valley, Aaron T Wilson*, University of Texas Rio Grande Valley. Promoting high school students' math identity alignment through informal learning and near-peer mentoring. Preliminary report.
With STEM job growth increasing worldwide while numbers of college students selecting STEM degrees declines, there is the need to promote interest in STEM studies among high school students. This paper reports a design-based research (DBR) study wherein college math and science majors engaged high school students in informal mathematical learning activities to promote mathematics identity alignment. Drawing on mathematical identity, informal learning and near-peer mentoring theories, the study explored the high schoolers' engagement and identity work during their participation in 1) MathShows presented by college near-peers, 2) math social media, and 3) a math summer internship. Employing mixed methods, results of the study which was completed in two large school districts in the U.S.-Mexico borderlands region, indicated that high schoolers were overwhelmingly engaged in the MathShows, that some of them followed and interacted with the math social media - especially posts featuring themselves or their peers - and that internship attendees expressed benefits in terms of math self-confidence, value, motivation and more interest in math. In addition to illustrating the design and impacts of the informal learning experiences, the study used an iterative redesign process typical of DBR to develop tools for interrogating participants' mathematician identity alignment. Results from $\mathrm{N}=555$ participants showed that participants explained their own level of mathematician alignment in predictable ways differentially mainly in relation to either their self-confidence or their personal interest in mathematics. Methods and findings reported are relevant for researchers and designers of informal mathematics learning programs.
(Received September 12, 2023)

Irfan Alam*, Department of Mathematics, University of Pennsylvania. What do the infinitesimals tell us about mathematics as an artistic endeavor in the modern society?. Preliminary report.
In 2010, Robert Ely published a paper titled "Nonstandard student conceptions about infinitesimals". Sarah, the student that this paper was a case study on, had developed her own intuitions for the number line which was based on a consistent appeal to infinitesimals. She said that she learned "not from any of her classrooms but that it was her own way of making sense of things." She knew that she was "wrong" and maintained that she did not "know the concepts", primarily because her classroom instruction never vindicated her natural intuitions, which could have been done easily if she was exposed to a course in calculus based in nonstandard foundations, something that is theoretically available but practically unavailable in most schools and colleges because of the standardization of the real line in modern mathematics education. Sarah's mathematical intuitions about infinitesimals should have empowered her to overcome her mathematical difficulties if our mathematics education framework was built in a more intellectually inclusive way - yet she struggled in mathematics because our framework is actually not built in such an inclusive way. There could be lots of Sarahs around the world who might have developed similar conceptions while never recognizing that they do not have to be "wrong" if one is able to have the right (nonstandard) perspective. Abstract art can appear meaningless without an appropriate perspective or interpretation. There is an argument to be made about the "number line" as an abstract object which the mathematicians explicate through their work by how they interpret it. The development of nonstandard analysis in the 1960s shows that alternative interpretations such as those developed by artistic students like Sarah are equally valid even though they get suppressed in our modern educational frameworks. This talk will explore these themes of creative suppression in the practical practice of mathematics from the perspective of the speaker's lived experiences as an autistic mathematician working in nonstandard analysis, who discovered "abilities" in more traditional arts only after his autism diagnosis later in life. Our discussions will lead to difficult questions about what we mean when we say that mathematics is an art, and what "mathematical ability" means. (Received September 12, 2023)

## 1192-104-32463

Jared M. Ifland*, University of California, Davis. Metaontology in Light of the Frege-Hilbert Controversy.
Insights from the philosophy of mathematics have recently been brought to bear upon metaphilosophical disputes pertaining to a priori knowledge, especially by both Justin Clarke-Doane and Jared Warren. My aim is to demonstrate how the Frege-Hilbert controversy over the role of axioms in mathematical theories sheds light upon contemporary debates in metaontology that involve the philosophy of mathematics to any significant degree. Clarke-Doane argues for a pluralistic construal of mathematical realism to guarantee that we could not have easily had false mathematical beliefs. Purportedly, this vindicates a metaphysically realist construal of Rudolf Carnap-inspired philosophical pragmatism. Contrary to Clarke-Doane, Warren argues that mathematical objects are merely byproducts of our use of mathematical language and nothing more. Consequently, Warren's account bears more affinity to Carnap's own pragmatism. Like Frege and Hilbert, Clarke-Doane and Warren adopt different stances toward the relationship between method and metaphysics. For Frege and Clarke-Doane, the reliability of method must be certified by a complementary metaphysical account, whereas for Hilbert and Warren, no appeal to more fundamental principles is needed to justify method. I argue in favor of the stance adopted by Hilbert and Warren, and contra Clarke-Doane, submit that Carnapian pragmatism is in no need of a realist construal and that the more traditional construal suggested by Warren's approach stands on its own.
(Received September 12, 2023)

## 1192-104-32508

Juliana Bukoski*, Georgetown College, Catherine Erbes, Hiram College. Ethical Reasoning In Calculus I. Preliminary report.
As a fundamental course for STEM careers, Calculus I is an ideal place to introduce students to the ethical practice of mathematics. However, it can feel impossible to add extra material to an already-packed syllabus. This summer, as part of the Framing Mathematics as a Foundation for Ethical STEM NSF grant, we developed a series of problems which integrate ethical reasoning and Calculus I topics. We then piloted the materials in our respective Calculus I classes in the fall. In this talk, I will share a sample of the problems as well as our challenges and successes in implementing them. (Received September 12, 2023)

Barry Cipra, Freelance, Paul Zorn*, St Olaf College. Oh What a Complex Rug We Weave When First We Color Then Perceive. This talk reports on some striking patterns that emerge when we obey a simple mathematical rule, borrowed from knot theory, for tricoloring a square weave, or carpet, of "ribbon" or "thread," starting from specified "fringe" conditions across the top and
left edges. Surprisingly to us, Sierpinski carpet-like figures often arise even from the simplest fringe conditions. We have almost no proofs as yet; indeed, it's a challenge just to describe what we see in full mathematical detail.
(Received September 12, 2023)

## 1192-104-32679

Theodore V Theodosopoulos*, Nueva School. Some remarks on collective sense-making. Preliminary report.
What is sense-making? How do we recognize it? How might it become collective? We propose a view of sense as a story, populated by characters and actions, arising from consistently imputable motives. Analogies play a catalytic role in what feels like a cascade of simplifications. What are obstacles to this sense-making cascade and when are we willing to accept something manifestly true as reasonless? When do we entertain the possibility that a claim is genuinely neither true nor false at least not in a forced way, where it feels like we have the freedom to choose? We explore the discontinuous nature of "aha" moments and the sense of agency this engenders. We embrace the irreducibly dialectic nature of sense-making and demonstrate how collaboration facilitates keeping ambiguity from collapsing. It feels like delaying as much as possible the answer to the question "what is being said," while simultaneously saying as much as possible. The resulting contingency across the facets of the emerging story encodes a great variety of logics, which render the whole incommensurate with the parts.
(Received September 12, 2023)

## 1192-104-32735

Patricia Klein*, Texas A\&M University. Panel: Is the Roots of Unity program for me?.
The next iteration of the Roots of Unity workshop will take place June 2024. The purpose of this panel is to give prospective applicants an understanding of the program from the perspective of those who have previously been affiliated with it. Roots of Unity is centered around women and nonbinary graduate students of color. From the Roots of Unity website: "The transition to independent research is a crucial and often jarring point in every graduate student's career. This transition is even more difficult for students from marginalized groups, who often have smaller support systems and may face an actively unsupportive environment at their institution. The goal of this workshop is to support, mentor, and guide students at this crucial stage in their career."
(Received September 12, 2023)

## 1192-104-32742

Felicia Yeung Tabing*, University of Southern California. Creativity in Writing Calculus Exams. Preliminary report.
I will describe the different ways I incorporate creativity and art in the calculus exams I write, with specific examples. I incorporate creativity not only visually in the aesthetics of the exam and layout, but in the problems and theme of the exam. (Received September 12, 2023)

## 1192-104-32759

Stephanie Somersille*, Somersille Math Consulting Services. Algorithms in Society. Preliminary report.
The use and misuse of algorithms can have large societal costs. One area where this is the case is in the redistricting system used in the US democracy. The enormity of the space of all possible districting maps for a state or region creates many challenges. Nevertheless a surprising amount can be determined about districting plans. We discuss Markov Chain Monte Carlo Methods, new advances and new metrics. Additional bonus topics may include school choice models and risk assessment models.
(Received September 12, 2023)

## 1192-104-32766

Susan Goldstine*, St. Mary's College of Maryland. Further Adventures in Branched Brioche Knitting. Preliminary report. At last year's Joint Mathematics Meetings, I spoke about my ongoing experiments with two-color brioche, a distinctive form of nearly-reversible color work in knitting. Brioche is particularly suited to branching, tree-like structures that often sit in negatively-curved fabric. In this talk, we will explore the hidden patterns that emerge when you render the hyperbolic plane in two-color brioche, and see what happens when you take a highly compressible stitch pattern and stretch all the recoil out of it. (Received September 12, 2023)

## 1192-104-32779

Suzanne Sindi*, University of California, Merced. Nonlinear Trajectories: Hard Won Lessons from Meandering One of my favorite things about mathematics is its ability to bring certainty to an uncertain situation. But sometimes the certainty in mathematics had lead some to think that the mathematical community itself need not ever change. In this talk, I'll discuss how my research career in mathematical biology unfolded alongside my own work in advocacy and activism. With each career phase, I've learned new things both about mathematics and how to work towards change.
(Received September 12, 2023)

1192-104-32832
Ha T. Lam*, Oracle Cloud Infrastructure. Monitoring Cryptographic Usage: Some Insights and Challenges. Cryptographic inventory is not only essential for ensuring compliance with security standards and regulations, it is increasingly valuable in enabling crypto agility, especially in the evolving landscape of cryptographic standards. In this talk, I will describe our team's experience in developing a tool that instruments our crypto library and share insights drawn from the data we've gathered. I'll also describe the challenges that we faced, particularly as a team of cryptographers took on the responsibilities of data scientists.
(Received September 12, 2023)

Karl Schaffer*, De Anza College. Dancing with Dienes and Thie.
Zoltán Pál Dienes (1916-2014) was a mathematics educator and theorist and a pioneer in the use of manipulatives and embodied mathematics instruction. Dienes incorporated music, arts, and dance into math instruction materials, but more broadly constantly involved whole body movement and playful explorations in mathematics learning. Joseph Thie (1928-2023) was an internationally known nuclear physicist who combined training in tap dance with time series analysis and other mathematical techniques in his 1964 book Rhythm and Dance Mathematics. Both Dienes and Thie deserve recognition for their early efforts to develop connections between dance and mathematics. We will see how both Dienes's educational activities and Thie's analytical work suggest further possibilities for making new and deeper connections between mathematics and the movement arts.
(Received September 13, 2023)

## 1192-104-32974

Rachel Roca*, Michigan State University. A Friendly Introduction to Topological Data Analysis (TDA) and Applications with the Help of Squirrels. Preliminary report.
Topological Data Analysis (TDA) allows us to examine the shape of data and has been a rapidly growing field for the past few decades. In this talk, I will introduce core concepts used in TDA, such as simplicial complexes and persistent homology. While we explore these concepts, we will ground ourselves in an example of geospatial topological data analysis of the squirrels living in Central Park. We'll then explore non-squirrel related applications of TDA, with a particular focus on social justice. This talk will be accessible for all and aims to demonstrate a minuscule amount of the exciting prospects of using TDA on data in an intuitive and fun way.
(Received September 13, 2023)

1192-104-32997
Ronald Feng*, Western Connecticut State University, Frederick Li*, Western Connecticut State University, Xiaodi Wang, Western Connecticut State University. Wavelet Based Machine Learning Methods for Wildfire Prediction.
It is estimated that there are 70,000 wildfires every year in the United States alone. These wildfires pose a significant danger to humans, both from property damage and health implications; air quality can drastically decrease from the resulting smoke, leading to over 33000 premature deaths each year. As a result, it is imperative that improvements to wildfire prediction models be made. Previous research has demonstrated the effectiveness of Deep Learning (DL) and Machine Learning (ML) methods such as XGBoost and Convolutional Long-Short Term Memory (ConvLSTM). In this paper, we utilise wavelets to preprocess our data by applying a discrete wavelet transform (DWT) in order to increase the accuracies of our models in predicting the next day's wildfire danger in certain counties in California. We input this data into an XGBoost model and compare the model performance to a traditional model (one trained on non-DWT data). We apply a DWT on aerial images from Canada and input the results into a convolutional neural network (CNN) to investigate the effects of DWTs on image data and to determine if such a CNN can accurately distinguish between days with and days without a wildfire. We also investigate the potential use of quantum computing to further improve these models. We find that the use of a DWT significantly improves model performance by denoising the data as well as adding more information in the form of hidden features for the model to train on.
(Received September 13, 2023)

## 1192-104-33150

Mara Alagic*, Professor @ Wichita State University. Artfulness in STEAM: Creativity, Innovation and Change. Preliminary report.
This presentation will provide a reflective overview of art-based practices and approaches to nurture creativity and innovation within a holistic perspective of STEM education, explicating changes that Artfulness brings to STEM: hence STEAM. The overview will be supported with examples explicating mathematical perspectives from the author's educational and professional practices, including (i) Digital MathArt action research project: Preservice elementary teachers were required to create artful representations of mathematical concepts from a different culture. This presentation will include a sample of students' artifacts and reflections regarding their own learning processes and changing (or not) perceptions about ways in which they intend to teach while nurturing creative ways of understanding/doing mathematics. (ii) A case study that investigated what STEAM concepts are underlying the process of designing artifacts in GeoGebra, 3D printing them, and designing an animation to visually capture the process. A reflective follow-up explored students' perspectives on novel ways of introducing STEAM concepts to their own classrooms; integrating artfulness. (iii) DALL•E inspired: An exploratory short study about the use of Artificial Intelligence to investigate its potential for creating images from texts for mathematical visualization in creative and innovative ways and the likelihood of using such visualizations in mathematics classrooms.
(Received September 13, 2023)

1192-104-33188
Ami Radunskaya*, Pomona College. Coffee, wine and tears: the bitter, sweet and salty bits of being a mathematician.. Preliminary report.
Mathematical research for me typically involves an energetic push towards the results, a reveling in the bonds of collaborative creativity, and the joy, exhaustion and frustration in working out the final kinks and getting it all down on paper. Trying to change the ecosystem of academic mathematics similarly involves three stages: planning, taking action, and celebrating having made it through. The rhythm of energy, community and frustration is also felt in both of these efforts: hence the coffee, wine and tears. In this talk I will reflect on particularly memorable research episodes and their interplay with our struggle to create a mathematics community where everyone belongs. Looking forward: where will derive our energy? How can we work together in community? And how can we overcome the tears of frustration and burnout?
(Received September 13, 2023)

## 1192-104-33198

Kristin T. Fesmire, Illinois Institute of Technology, Aakash V. Madabhushi, Illinois Institute of Technology, Kaylee J. Rosendahl, Illinois Institute of Technology, Jacob H. Thomas*, Illinois Institute of Technology. Statistical Analysis of Water Testing Practices: Revealing a Chicago Water Crisis.
Ingesting lead found in drinking water is known to damage cognitive and cardiovascular systems. Lead comes from absorption from lead service lines (LSLs). New LSLs were banned in 1986, and the EPA regulates LSLs through the Lead and Copper Rule (LCR) of 1991. The LCR requires action when $10 \%$ of tap water samples exceed 15 parts per billion (ppb), measured from the 1st liter drawn. There is evidence of higher lead levels in later draws. Chicago is in compliance under the LCR and has until 2077 to replace all LSLs. However, the LCR Revision (LCRR), to take effect in 2024, would require measurements from the 5th liter draws. This brings into question whether Chicago will be in compliance under the new LCRR, and whether the reported data is an accurate representation of lead levels. We approach this by: examining the Chicago sequential draw dataset to determine how well the first and initial draws represents lead levels; using a variety of statistical methods to determine how much more effective, if at all, the LCRR will be at representing lead than the LCR; and performing data analysis to discover if there is a relationship between lead levels and income. With these, it is found that: there is need for further revision to the LCR and LCRR to accurately represent and identify lead levels; there are ethical and practical concerns regarding LCR and LCRR sampling procedures; there is a clear sampling bias, with the households of the lowest income having much less sampling rates compared to expected rates; there is both oversampling and disproportionately high lead levels of low-mid income households, corresponding with the relationship found between lead levels and income.
(Received September 13, 2023)

1192-104-33419
Martin E Flashman*, Humboldt State University. Do We Need a Separate Philosophy of Geometry?.
The philosophy of mathematics has long been focused primarily on topics such as the ontology of numbers and sets and the epistemology of results in the theory of numbers (arithmetic) and sets through issues of axioms and proofs for these theories. Though much of mathematics today seems to involve geometry in one form or another, the philosophical issues of geometry seem to receive little attention, treated as subservient to the general philosophy of mathematics or considered a part of the philosophy of physics. The author will consider why the issues of geometry could use a distinct discussion of its philosophical issues for both intrinsic and pedagogical reasons.
(Received September 15, 2023)

1192-104-33814
Eric A. Autry, Grinnell College, Tanmaie Kailash*, Grinnell College, Anthony Schwindt*, Grinnell College. Race and Traffic Citations in Des Moines, Iowa.
The recent spotlighting of the BLM movement in public consciousness has brought into question not only the increased instances of police brutality against African Americans, but the basic assumptions on which policing rests. Meanwhile, the increase in technological interventions in every sector of life has led to the generation of policing data that may now be interrogated. Our research aims to clean and use traffic citations and bookings data for the Des Moines Police Department from 2016 until 2022 to evaluate racial bias in the same. In particular, we analyze evidence for racial disparities within traffic arrests and citations over this period to determine whether the racial makeup of a census block correlates with its level of policing, and whether the type or severity of charges/citations correlates with the race of the individual charged/cited. (Received September 26, 2023)

## 1192-104-33815

Ethan Collins*, United States Military Academy. Using Natural Language Processing to Advise Policymakers on New Drug Study Regulations. Preliminary report.
Background Brexpiprazole (Rexulti) is an antipsychotic drug used to treat schizophrenia and major depression. In many drug studies, including the study involving Rexulti being observed, patients with a body mass index (BMI) greater than 35 are excluded from trials due to the unique effects a drug can have on them, compared to those with a lower BMI. Furthermore, as many as 60Methods One dataset for the study includes the 9 patient transcripts, ranging from 2000-4000 words each. By using NLP methods such as keyword summarization, n-grams, sentiment analysis, and topic modelling, we can gain a better understanding of the patients' cases, strive for patient advocacy, and create a repeatable method for analyzing post-drug study interview transcripts. A second dataset includes survey results from the International Weight Control Registry (IWCR). Using these survey results, we can determine if people with obesity respond differently to general questions. NLTK will be utilized as the main package for NLP analysis. This includes analysis of both the transcripts and the free text from the survey answers. Results Voice recognition software used in the nine interviews generated nearly illegible transcripts that could not effectively be used by machine learning algorithms. Therefore, preliminary results included pre-processing of the transcripts. With the help of ChatGPT, we were able to fix grammar, formatting, and transcription errors on the documents. This allows for them to be useful in NLTK, our primary package for language processing models. The transcripts were then compared to the originals to ensure consistency of content during the process. Initial analysis, to include keyword summarization and sentiment analysis, showed negative sentiment overall.
(Received September 26, 2023)

