SOME RECENT ELEMENTARY WORKS ON MECHANICS. I.

The Laws of Motion, an elementary treatise on dynamics. By W. H. LAVERTY, late fellow of Queen's College, Oxford. London, Rivingtons. 1889. 8vo, pp. 212.

IN a recent number of the *Bulletin* (No. 2, pp. 48–50) Professor T. W. Wright complains of the confusion existing in the nomenclature of elementary mechanics. It would be easy to answer his questions from a purely theoretical point of view; indeed, in theoretical mechanics no difficulty is encountered in this respect. But it must be admitted that in elementary works, particularly in those of a more "applied" character, the confusion is great, both as to the use of terms and the way of presenting the fundamental laws.

By reviewing somewhat at length a few of the better recent works on elementary mechanics it may perhaps be possible to "fix the ideas" and arrive at some conclusions, at least as to what is the best modern usage in treating the subject.

Mr. Laverty's little work is rather different from the ordinary English text-book. There is no reference in the preface to the "examinations of the Science and Art Department for the elementary stage," nor any gentle hint to the reader that "most of the examples are taken from actual recent examination papers."

"The object of this treatise," says the author (p. v.), "is to put the subject of dynamics on a thoroughly sound basis, avoiding unsatisfactory illustrations and definitions which do nothing towards defining, and to endeavour to give the student such an accurate idea of the subject that he may be able *e.g.* to give explanations and illustrations of the laws without just merely copying these from the book."

The author's objections to definitions that do not define, to inadequate illustrations of the fundamental laws, and to the loose and confused ways of stating these laws found so often in elementary works are certainly well taken. The book is evidently the result of careful independent thinking and treats a well-worn subject in a fresh and original way. Newton's laws are given in good English and in modern scientific language; the discussion of their meaning and interdependence is noteworthy in many respects.

The outward appearance of the book is pleasing; the little volume is neatly printed and furnished with an alphabetical index in addition to an ample table of contents. The matter is well arranged and distributed into sections of convenient size; every subject is illustrated by a few "worked" examples followed by a large number of exercises for which the answers are given at the end of the volume.

Before discussing the points of principal importance a few minor matters might be mentioned which could readily be corrected in a second edition.

In art. 9, the terms "standard" and "unit" are used as if they meant the same thing. It is preferable to make a distinction. Thus, the standard of mass in the C. G. S. system is the kilogramme, that is a certain bar of platinum preserved in Paris, while the unit of mass is a gramme, that is any mass equal to a one thousandth part of that kilogramme.—The statement of art. 267 that "the laws of friction between bodies, as found by experiment, are surprisingly simple," gives a surprisingly optimistic view of the case.—The factor 2 in the first expression for $n-n'$ on p. 170 is a misprint; it should be dropped.—In example E, pp. 110–111, the factor g should be inserted in the expression for the work, or rather in the problem itself "42400000 ergs" should read "42400000g ergs."—

The numerical data in the exercises are usually so selected as to lead to answers expressible in round numbers. This method has obvious advantages for class work and examinations; it saves time and allows a certain display of ingenuity in arranging the numerical work conveniently for cancelling. But it accustoms the student to methods that are far from being the best in examples as they occur in actual practice. If the working of numerical examples is to be of any value it should lead the student to understand the bearing that every quantity involved in the formula has on the final result. The beginner should in particular learn to select for any constant the proper number of decimal places necessary in order to obtain the required accuracy of the result; he should also determine from the data the accuracy obtainable with the data of the problem. Thus, on p. 7 we find the problem: "How many metres are there in a mile, if there is .305 of a metre in a foot?" The answer is correctly given as 1610.4. But actually there are 1609.3 metres in a mile; the given constant .305 is not sufficiently exact to give the result correct within a decimetre. Would it not be better to refer the student to the more exact value of the constant given on a previous page (p. 5) and require him to select the proper number of decimal places?

It must be said, in general, that the author has an excessive fondness for such merely speculative problems as the following: "If the unit of area and time be 10 acres and 10 seconds; what is the unit of velocity expressed in miles per hour?" (p. 27.) Such meaningless problems occur in great number throughout the book. With this exception the exercises are very well selected and constitute a valuable feature of the book.

In the matter of symbols and names for the units Mr. Laverty is unusually radical. He manufactures them without the slightest compunction. The British unit of velocity (foot a second) is called *fas*, the C. G. S. unit (centimetre a second) *cas*; similarly the unit of acceleration are *sfas*, *scas*; those of momentum (a *fas* in a pound): *fasp* and *casgram*; of kinetic energy: *faspen* and *casgrammen*; of impulse: *bim* and *cim*; of force: *sfasp* and *scasgram*. This new notation is as ingenious as it is simple; *bim* for impulse strikes one as particularly happy. But will it be possible to bring this brilliant new coinage into circulation? And before this is accomplished, what is the poor student to do as soon as he leaves Mr. Laverty's class-room? Nobody will understand him when he begins to talk of *casgrammen* and *sfasp*, and he will have difficulty in understanding the old-fashioned rest of the world.

A new notation of this kind is entirely out of place in an elementary text-book. Originality is no doubt a good thing; but in a work for beginners it is to be used with moderation; an over-dose may become fatal. It is another question whether the notation is in itself good and its acceptation desirable.

It may be seriously questioned whether there is any actual need for special names and symbols for all these units. The British Association Committee on Units suggests the name *kine* for "a speed of 1 cm. per sec.;" J. B. Lock uses *vel* and *cel* for the units of velocity and acceleration; the term "quickening" has been proposed for unit acceleration. Mr. Laverty's scheme has the advantage over these separate efforts of being methodical and comprehensive; it also lends itself readily to farther extension. A "mile an hour" might be called a *mah*, a "yard a minute" a *yam*, etc. But the fact of the matter is that these numerous symbols and names can be of use almost exclusively in the elementary text-book. Later on we can get along without them. In most cases the unit can be understood from the context, as when the physicist says that the acceleration of gravity at a certain place is 981, meaning "centimetres per second," or when the engineer gives the angular velocity of his fly-wheel as 25, meaning "revolutions per minute." It is mere pedantry to require the unit to be stated explicitly under such circumstances. In other cases it is best to state the unit completely.

Mr. Laverty says, in regard to his notation (preface, p. x.): "These words should be looked upon simply as abbreviations (perhaps in some cases as aids to the memory); I have no desire to add new words to the language." But if they are not to become new words of the language, what is their use?

Are they to be learned only to be forgotten as soon as possible? And does not Mr. Laverty himself use them throughout as if they were new words of the language? Let us give the student in the elementary text-book nothing but the most approved notation of the science and the less perfect equivalents used in the applications; he will have enough to do in mastering these.

All these slight strictures, however, do not detract materially from the value of Mr. Laverty's work, which gives an admirable presentation of Newton's three laws of motion. After explaining the ideas of velocity and acceleration in the simplest cases, the idea of mass is introduced; the fundamental equations $v = \alpha t$, $\frac{1}{2}v^2 = \alpha x$ are multiplied by m and the quantities mv, $\frac{1}{2}mv^2$, $m\alpha$ are given the names *momentum, kinetic energy,* and *mass-acceleration,* respectively. There is no good reason why the term *force* should not be used here instead of mass-acceleration. If force were thus defined, the fundamental relations

$$mv = m\alpha \cdot t, \qquad \tfrac{1}{2}mv = m\alpha \cdot x$$

would at once show that force *is* the rate of change of momentum with the time, or the rate of change of kinetic energy with the distance.

The author prefers to call force that which *produces* change of momentum. At the same time he objects to calling this a definition of force. If this be not what the logicians call a *definitio realis,* it certainly is a *definitio nominalis:* we observe in nature a change of momentum, and to the cause of this change we give the name force.

Newton's first law is stated very clearly in the following terms (p. 46): "The momentum in a mass (or system of masses) cannot be increased or diminished except by the action of external force." This becomes a self-evident truth with the above definition of force as the cause of change of momentum; for when there is no cause there can be no effect. But unfortunately Mr. Laverty neglects to give a definition of force. And yet what concept needs definition more than force? In ordinary language the term is used in a variety of meanings; and on the other hand, force itself cannot be directly observed in nature (excepting the case of muscular force with which we are not concerned here), it is only its effects, *i.e.* changes of momentum, that can be directly measured. In all other respects Mr. Laverty's explanations and illustrations of the first law can only be commended.

The second law is given in this form (p. 68): "When momentum is produced, it is by the action of force; and the amount of momentum produced in a given time is proportional to and in the direction of the force."

It will be noticed that the first clause is but a re-statement of the first law ; and the author very justly remarks (preface, p. VII., foot-note) that the first law might be dropped and the science of mechanics be based on only two laws, "the law of force (or momentum), and the law of work (or energy)."

While the first law merely states that we shall give the name force to the cause of any observed change of momentum, the second law defines force more accurately by saying that this cause is proportional to the effect produced and that the direction of the force shall mean the direction of the momentum produced. It also implies the independence of the action of two or more forces applied at the same point. Thus it follows that the parallelogram law applies to forces just as it applies to velocities and accelerations.

The third law is expressed as follows (p. 86) : "The work done by a force (or any agent) on any mass (or system of masses) has its equivalent in the kinetic energy exhibited, and in the work done against molecular forces, gravity, and friction." The usual short form "action and reaction are equal and opposite" is rejected as meaningless as long as action and reaction are not carefully defined. "The fact is," says the author, p. VIII., "that, if by 'action' and 'reaction' are meant force and resistance, the third law is but an easy deduction from the second ; while if d'Alembert's principle is really to be ultimately deduced from the law, it is better to enunciate it at once in proper form, and not in the usual indefinite and undefined terms." Thus, the third law in the simplest case is expressed by the equation

$$m\alpha \cdot x = \tfrac{1}{2}mv^2,$$

while in the most general case it leads to d'Alembert's principle (p. 92) : "The internal pressures of any system of rigid bodies are in equilibrium amongst themselves."

After discussing each law for itself the author devotes several sections to illustrations and applications of the laws ; these embrace the theory of the pendulum, Atwood's machine, the inclined plane, collision, projectiles, and circular motion. Only the most elementary mathematics are used throughout the book.

As a point not usually touched upon in elementary textbooks it may be mentioned that Mr. Laverty calls special attention to the fact that the parallelogram law would not hold for forces if they were not defined as they are by the second law, *viz.* as the time-rate of *momentum,* but *e.g.* as the timerate of *kinetic energy.* It is well known that on this point turned the long controversy on the nature of force and energy between Descartes, Leibnitz, and their followers.*

The closing section contains some interesting general remarks on the nature of the three laws and the ways of testing their truth.

ALEXANDER ZIWET.

ANN ARBOR, MICHIGAN, *January* 1, 1892.

WEIERSTRASS AND DEDEKIND ON GENERAL COMPLEX NUMBERS.

WEIERSTRASS †—*Zur Theorie der aus n Haupteinheiten gebildeten complexen Grössen. Göttingen Nachrichten,* 1884.

DEDEKIND—*Zur Theorie der aus n Haupteinheiten gebildeten complexen Grössen. Göttingen Nachrichten,* 1885.

DEDEKIND—*Erläuterungen zur Theorie der sogenannten allgemeinen complexen Grössen. Göttingen Nachrichten,* 1887.

IN closing his second memoir on biquadratic residues ‡ Gauss makes this remark: "Our general arithmetic, which goes so far beyond the limits of the geometry of the ancients, is entirely the creation of recent times. Starting with the notion of whole numbers its field has widened little by little. To whole numbers came fractions, to rational numbers the irrational ones; to the positive came the negative and to the real came the imaginary."

Once convinced that $\sqrt{-1}$ was properly an algebraical quantity and that it had a meaning, mathematicians began to look for other quantities of a similar nature. "Why," they asked themselves, "should algebra yield an imaginary unit which makes it possible to represent two dimensions of space analytically; and fail to yield a second imaginary unit which can be used to represent the third dimension?" The thing needed only to be sought for apparently, and at first they looked amongst the functions of $\sqrt{-1}$. Unfortunately it turned out that even the most promisingly irrational of these could all be broken up into a real part and $\sqrt{-1}$ times a second real quantity; algebra had done her best; if mathematicians wanted more imaginaries they must invent them. From the time of Gauss, then, until the present day the architects and the masterbuilders have turned occasionally

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* See for instance E. MACH, *Die Mechanik in ihrer Entwickelung,* Leipzig, Brockhaus, 1889, pp. 254–259.

† Extract from a letter to Schwarz.

‡ *Werke,* II., p. 175.