criteria (given in § 498) appears at once from the example $u = (y^2 - 2px)(y^2 - 2qx)$ [Peano]. The origin is a point satisfying the preliminary conditions; taking then for $x, y,$ small quantities $h, k,$ the terms of the second degree are positive for all values except $h = 0$; when $h = 0$, the terms of the third degree vanish, and the terms of the fourth degree are positive; nevertheless the point does not give a minimum, which it should do by the test of § 498. For we can travel away from O in between the two parabolas, so coming to an adjacent point at which u has a small negative value, while for points inside or outside both parabolas the value of u is positive. The truth is, the nature of the value a of the function u at a point (x_0, y_0) at which $\frac{\delta\varphi}{\delta x}$ and $\frac{\delta\varphi}{\delta y}$ vanish, depends on the nature of the singularity of the curve $u = a$ at this point. If this curve has at (x_0, y_0) an isolated point of any degree of multiplicity, we have a true maximum or minimum of u; but if through (x_0, y_0) pass any number of real non-repeated branches of the curve, we have not a maximum or minimum; in Peano's example the branches coincide in the immediate neighbourhood of the origin, but then they separate, and therefore we have not a minimum value for u.

We object, then, to Mr. Edwards' treatise on the Differential Calculus because in it, notwithstanding a specious show of rigour, he repeats old errors and faulty methods of proof, and introduces new errors; and because its tendency is to encourage the practice of cramming "short proofs" and detached propositions for examination purposes.

CHARLOTTE ANGAS SCOTT.

BRYN MAWR, PA., *May* 18, 1892.

NOTE ON RESULTANTS.

BY PROF. M. W. HASKELL.

ON page 151 of Prof. Gordan's lectures on determinants* is to be found the theorem

$$R_{f,\phi} = R_{f+\phi.\psi,\phi}$$

where $R_{f,\phi}$ denotes the resultant of two functions f and ϕ of a single variable x of degree m and n respectively. This

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* *Vorlesungen über Invariantentheorie,* herausgegeben von KERSCHENSTEINER. Erster Band. Leipzig, 1885.

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theorem is proved under the restriction that n is not greater than m and that the degree of the arbitrary function ψ shall not be greater that $m - n$. The author goes on to say: "es wäre eine schätzenswerthe Arbeit, auch den Fall zu untersuchen, in welchem $m < n$ ist, d. h. allgemein die Frage zu behandeln: Wie hängen die Resultanten $R_{f+\phi.\psi,\phi}$ und $R_{f,\phi}$ zusammen, wenn wir über den Grad der diesbezüglichen Functionen keinerlei Voraussetzung machen?"

This statement is somewhat remarkable on account of the ease with which it may be shown that the theorem is true in general in exactly the form given above.

I. Let $F = f + \phi . \psi$. Then, no matter what the degree of the functions involved may be, if the degrees of F and ϕ be m and n respectively, n is certainly not greater than m, and the degree of ψ cannot be greater than $m - n$. Hence, by Gordan's result as quoted,

$$R_{F,\phi} = R_{F-\phi.\psi,\phi} = R_{f,\phi}.$$

That is, the theorem is true without restriction.

II. Suppose the resultant $R_{f,\phi}$ to be found in the usual way by the method of greatest common divisor. Two functions A and B of x, of degree $n - 1$ and $m - 1$ respectively can be found to satisfy the relation:

$$1 = A . f + B . \phi.$$

The coefficients of A and B are rational, but not integral, functions of the coefficients of f and ϕ, whose least common denominator is the resultant $R_{f,\phi}$.

It follows that

$$1 = A(f + \phi . \psi) + (B - A\psi)\phi$$

and the resultant $R_{f+\phi.\psi,\phi}$ is the least common denominator of the coefficients of A and $(B - A\psi)$. But the coefficients of ψ will evidently not occur in the denominators at all, and the least common denominator is therefore identical with that of the coefficients of A and B, viz. $R_{f,\phi}$.

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