

NOTE ON SUBSTITUTION GROUPS OF EIGHT LETTERS.

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I HAVE found a non-primitive group of degree eight and order twenty-four which is not contained in Professor Cayley's list, vol. 25, p. 140, *Quarterly Journal of Mathematics*, nor in Professor Cole's supplementary list published in the BULLETIN, vol. 2, pp. 187 and 189. The group may be written in the form

$$(abcd . efgh) \text{ pos. } (ae . bg . cf . dh).$$

The substitutions are

1	<i>abc . efg</i>	<i>ab . cd . ef . gh</i>	<i>afch . bgde</i>
	<i>abd . efh</i>	<i>ac . bd . eg . fh</i>	<i>ahcf . bedg</i>
	<i>acd . egh</i>	<i>ad . bc . eh . fg</i>	<i>afdg . bhce</i>
	<i>bcd . fgh</i>	<i>af . be . cg . dh</i>	<i>agdf . bech</i>
	<i>bdc . fhg</i>	<i>ag . bf . ce . dh</i>	<i>agbh . cfde</i>
	<i>acb . egf</i>	<i>ah . bf . cg . de</i>	<i>ahbg . cedf</i>
	<i>adb . ehf</i>	<i>ae . bg . cf . dh</i>	
	<i>adc . ehg</i>	<i>ae . bh . cg . df</i>	
		<i>ae . bf . ch . dg</i>	

Two of the six groups of order 16 given by Professor Cole in the BULLETIN, vol. 2, p. 187, are identical, viz.:

1	<i>ac . bd</i>		1	<i>eg . fh</i>
<i>ac</i>	<i>bd</i>		<i>ef . gh</i>	<i>eh . fg</i>
<i>ab . cd</i>	<i>ad . bc</i>		<i>efgh</i>	<i>ehgf</i>
<i>abcd</i>	<i>adcb</i>		<i>eg</i>	<i>fh</i>

and

1	<i>ac . bd</i>		1	<i>eg . fh</i>
<i>ac</i>	<i>bd</i>		<i>efgh</i>	<i>ehgf</i>
<i>ab . cd</i>	<i>ad . bc</i>		<i>eg</i>	<i>fh</i>
<i>abcd</i>	<i>adcb</i>		<i>gh</i>	<i>eh . fg</i>

The substitutions of the group of order 12, given just below these six, should be

1	<i>abc . efg</i>	<i>ab . cd . ef . gh</i>
	<i>acb . egf</i>	<i>ac . bd . eg . fh</i>
	<i>abd . efh</i>	<i>ad . bc . eh . fg</i>
	<i>adb . ehf</i>	
	<i>acd . egh</i>	
	<i>adc . ehg</i>	
	<i>bcd . fgh</i>	
	<i>bdc . fhg</i>	

Since two of the published groups of degree eight are identical and a new one is added, their total number is still 199.

It remains to note a typographical error in Professor Cole's supplementary list. The combining substitution in the third group of order 32, p. 188, should be *aecg . bf . dh* instead of *aecg . bfdh*; the latter substitution gives the same group as *ae . bf . cg . dh*, which is already in the list. There is a considerable number of other typographical errors in each of the lists referred to, but they are of such a nature as to be easily detected by those employing the groups and require therefore no special mention.

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With the above corrections, for which I am under great obligations to Dr. Miller, the question of the possible substitution groups of eight letters is now settled beyond doubt.
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