EULER'S USE OF i TO REPRESENT AN IMAGINARY.

IN the *Introductio in Analysin Infinitorum*, Lausannæ, 1748, Euler uses i to represent an infinitesimal, an infinite, and a positive integer, but in Tom. II, p. 290, we find "Cum enim numerorum negativorum Logarithmi sint imaginarii, * * erit $l. - n$, quantitas imaginaria, quæ sit $= i$."

In a paper *De formulis differentialibus angularibus maxime irrationalibus, quas tamen per logarithmos et arcus circulares integrare licet, M.S. Academiae* [Petropolitanae] *exhibit. die* 5. *maii* 1777, reproduced in Vol. IV., *Institutiones Calculi Integralis*, Petropoli, 1845, pp. 183–194, there occur such passages as "** formulam $\sqrt{-1}$ littera i in posterum designabo, ita ut sit $ii = -1$, ideoque $\frac{1}{i} = -i$." "* * loco $\cos.\varphi$ has duas partes substituamus

$$\tfrac{1}{2}(\cos.\varphi + i\sin.\varphi) + \tfrac{1}{2}(\cos.\varphi - i\sin.\varphi)."$$

"Constat autem esse

$$(\cos.\varphi + i\sin.\varphi)^n = \cos.n\varphi + i\sin.n\varphi."$$

"* * ubi tam x quam y imaginaria involvit, hanc ob rem ponamus brevitatis gratia $x = r + is$, $y = r - is$."

These extracts would seem to dispose of the claim that "Gauss introduced the use of i to represent $\sqrt{-1}$." (See Baltzer, Fink, Wolf, Holzmüller, Thomae, Suter, Harnack, Durège, Chrystal, etc.)

W. W. BEMAN.

UNIVERSITY OF MICHIGAN.

NOTE ON THE ROOTS OF BESSEL'S FUNCTIONS.

VARIOUS proofs have recently been given of the theorem that between two successive positive (or negative) roots of $J_n(x)$ lies one and only one root of $J_{n+1}(x)$. The following proof (which is contained along with other investigations concerning the roots of Bessel's functions and the hypergeometric series in a paper sent last June to the *American Journal of Mathematics*) is simpler and more elementary than those heretofore given. It depends on the formulæ:

$$\frac{d\,[x^{-n}J_n(x)]}{dx} = -x^{-n}J_{n+1}(x), \quad \frac{d\,[x^{n+1}J_{n+1}(x)]}{dx} = x^{n+1}J_n(x).$$