

## THE APRIL MEETING OF THE CHICAGO SECTION.

THE third regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of Chicago on Saturday, April 9th, the first session opening at 10 o'clock a. m. The following members were present:

Dr. E. M. Blake, Dr. Harris Hancock, Professor Thomas F. Holgate, Mr. Herbert G. Keppel, Dr. Kurt Laves, Professor Heinrich Maschke, Professor E. H. Moore, Professor J. B. Shaw, Dr. H. F. Stecker, Professor H. S. White, Professor Alexander Ziwet.

Professor E. H. Moore, Vice-President of the Society, occupied the chair.

The following papers were read:

(1) Professor JAMES BYRNIE SHAW: "Dual algebras."

(2) Dr. E. M. BLAKE: "The ellipsograph of Proclus and its inverse."

This paper, which embodied an earlier one\* upon the same mechanism, described, first, the surfaces generated by any straight line carried by either of two circles, of radius  $r$  and  $2r$  respectively, lying in the same plane and rolling the one upon the other, these surfaces being of the fourth order; and second, the envelopes of carried planes, these being of the sixth order and fourth class for the ellipsograph, and cones for the inverse mechanism. The paper was illustrated by thread models showing the principal types of these surfaces.

(3) Professor ROBERT J. ALEY: "A triangle related to Nagel's triangle."

(4) Dr. L. E. DICKSON: "The structure of the hypo-abelian groups."

The paper announced marked simplifications in the general conception and in the detailed developments of the theory of the hypo-abelian groups of Jordan, and of the author's generalization of the first hypo-abelian group just appearing in the *Quarterly Journal of Mathematics*.

(5) Dr. KURT LAVES: "On the most general form of the inner potential consistent with the complete integration of the differential equations of motion of a free system of two bodies" (preliminary communication).

---

\*"On a ruled surface, etc.," abstract in BULLETIN, vol. IV., pp. 186. This paper will not be published separately, owing to its incorporation in the present one.

(6) Dr. SHUNKICHI KIMURA (presented by Professor J. B. Shaw) :

I. "Quaternion notes."

II. "Introduction to the theory of functions of a quaternion or a vector variable."

(7) Professor W. H. METZLER : "On the roots of a determinantal equation."

In the *American Journal of Mathematics*, vol. 19, Dr. Thomas Muir showed that the roots of the equation

$$\begin{vmatrix} 11 - x, & 21 & , & 13 & , & \dots \\ 21 & , & 22 - x, & 23 & , & \dots \\ 31 & , & 32 & , & 33 - x, & \dots \\ \dots & & \dots & & \dots & \\ \dots & & \dots & & \dots & \end{vmatrix} = 0$$

are all real in case  $1\mu \cdot \mu\nu \cdot \nu 1 = \mu 1 \cdot \nu\mu \cdot 1\nu$  and  $1\mu \cdot \mu 1$  is positive. In the present paper the writer shows that the roots of this determinantal equation are pure imaginaries when  $1\mu \cdot \mu\nu \cdot \nu 1 = -\mu 1 \cdot \nu\mu \cdot 1\nu$ ,  $1\mu \cdot \mu 1$  is negative, and  $\lambda\lambda = 0$ .

(8) Professor H. MASCHKE : "Concerning the case where a linear substitution-group of finite order in  $n$  variables breaks up into groups in a lower number of variables."

If a linear substitution-group  $G$  in  $n$  variables is of finite order and contains at least one substitution, the characteristic equation of which has all its roots distinct, then the following theorem holds true : If at least one coefficient occupying a certain place (not in the main diagonal) in the matrix of the substitutions of  $G$  vanishes for *all* the substitutions, then the group breaks up into two groups in  $n - p$  and in  $p$  variables, respectively. The paper containing the proof of this theorem will be published in the *Mathematische Annalen*, vol. 50.

(9) Professor E. H. MOORE : "A two-parameter class of solvable quintics, in which the rational relations amongst the roots, by threes, do not contain the parameters" (preliminary communication).

At the opening of the afternoon session Professor Michelson, of the University of Chicago, favored the section with an exhibition of the workings of his new harmonic analyser, a description of which was published in the *American Journal of Science* and in the *Philosophical Magazine* for January, 1898.

THOMAS F. HOLGATE,  
*Secretary of the Section.*