ON A REGULAR CONFIGURATION OF TEN LINE PAIRS CONJUGATE AS TO A QUADRIC.

BY PROFESSOR F. MORLEY.

(Read before the American Mathematical Society at the Meeting of October 29, 1898.)

AT the meeting of the London Mathematical Society, June, 1898, I showed a model of ten lines in space, each intersecting three others perpendicularly.* Subsequently Professor Study, seeing the model, remarked that there should be an analogous configuration of ten pairs of lines conjugate as to a quadric. This new configuration can be established as follows:

(1) Consider a ruled quadric H, and denote by Q an inscribed quadrilateral formed by two right generators and two left generators. The diagonals of Q are a pair of conjugate lines, say P. Thus Q determines P.

(2) Call two pair of generators harmonic when they cut another generator in harmonic pairs of points. Then two harmonic pairs of right generators will cut any conic of the surface H at the ends of conjugate chords, and the left generators through these ends will also be harmonic pairs. Hence calling two inscribed quadrilaterals Q and $\overline{Q}$ harmonic when their right generators and also their left generators are harmonic pairs, we can say that *harmonic quadrilaterals determine meeting conjugate pairs*, where we mean that each of the one pair meets each of the other.

(3) Now two quadrilaterals Q and Q' have a common harmonic quadrilateral; therefore two conjugate pairs are met by a third conjugate pair; that is, the two lines which met the four are themselves conjugate.†

(4) Take now three conjugate pairs P_1, P_2, P_3. Let the pair meeting both P_2 and P_3 be P_1'; thus we have three new pairs P_1', P_2', P_3'. Let the pair meeting P_1 and P_1' be P_1''; thus we have three new pairs P_1'', P_2'', P_3''. It is to be proved that these are met by one conjugate pair.

Replace the pairs by inscribed quadrilaterals, and the matter being merely one of harmonic constructions consider

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* See *Proc. Lond. Math. Soc.* vol. 29.

† We set aside the special case in which the two pairs are met by an infinity of lines, which arises when the two quadrilaterals have the same right (or left) generators.

separately the right and left generators. Refer the pairs of right generators to the pairs of points where they meet a conic of the surface.

Then we are to prove that if p_1, p_2, p_3 are pairs of points on a conic, and the pair harmonic with p_2 and p_3 be p_1', and the pair harmonic with p_1 and p_1' be p_1'', then p_1'', p_2'', p_3'' are in involution. In other words, replacing the pairs of points by the lines through them, we are to prove that a triangle and its polar triangle as to a conic are in perspective, a known elementary proposition. The ten pairs of points on the conic are in fact the points in which the lines of the configuration 10_3, the intersection of a plane with a complete five point cut the conic to which it is self polar.

Thus as three pairs of points on a conic determine the configuration of ten pairs of points, so three quadrilaterals in the surface determine a configuration of ten quadrilaterals, and replacing each quadrilateral by its diagonal pair we have the configuration of Professor Study.

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BESSEL'S FUNCTIONS.

Einleitung in die Theorie der Bessel'schen Funktionen, von Professor J. H. GRAF und Dr. E. GUBLER. Erstes Heft: Funktionen erster Art. Bern, 1898, Wyss & Co.

IN the last few years the study of Bessel's functions has been enriched by many contributions, the subject generalized for the complex variable, and the premises much more carefully defined.

Among English speaking investigators the interest seems to lie chiefly in two directions, theoretical consideration of the differential equation, and practical applications, numerical tabulation, etc.

The book under review does not favor any particular side but may be said to put the most emphasis upon the definite integral. Numerical tabulation is not touched upon in the part which has appeared; the remaining part, soon to appear, may have a chapter devoted to this purpose.

The first feature is a carefully arranged historic introduction, which contains an extensive list of books and memoirs