asymptotes there touch the multiple line ; finally, every one of the $n$ generators is cut by the same asymptotic line in but one point not on the directrix; the whole order is

$$2m(n-1) + 2(m-n) + n - 2\delta = 2mn - n - 2\delta.$$

In particular, if the surface is unicursal, the order reduces to $2m + n - 2$.

For the surface $\overline{[2, 2]}$ without double line (Cayley, IV) the order is 6, with a double line (Cayley, V) the order is 4. For these surfaces the multiple line cannot be a generator.

The surface $\overline{[3, 1]}$ (Cayley, VI) is unicursal ; its asymptotic lines are of order 5. The triple line is double generator.

For the cubic $\overline{[2, 1]}$ (Cayley's cubic scroll) the order is 3 ; the double directrix counts as single generator.

CORNELL UNIVERSITY,
*November*, 1898.

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## WILLSON'S GRAPHICS.

*Theoretical and Practical Graphics.* By F. N. WILLSON, C.E., M.A., Professor of Graphics, Princeton University. New York, The Macmillan Company, 1898. 4to, 300 pp. Price, $4.00.

THIS work, primarily, is a text book on graphics compiled by an experienced teacher to meet the needs of his own classes. Few student have, heretofore, been called upon to make larger expenditures for books than has the embryo engineer and in combining, under the comprehensive title of graphics, much that is essential for such students, for example, chapters upon freehand and mechanical drawing, theory of the helix, link motion, trochoidal and other mechanical curves and the theory of descriptive geometry, a real need has been recognized. This volume is far more than a collection of class room notes. Every page bears evidence of conscientious care and research. The grouping of the chapters, the concise and useful table of contents, the clear cut and often elaborate illustrations and the exceptional typographical excellence cannot be too highly commended.

The book appeals at once to the artistic sense and to the admirers of descriptive methods, but, valuable as are these qualities, a review of them in a strictly mathematical journal would be out of place were it not for the fact that the treatment is mathematical, involving problems of interest to students of geometry.

In the opening paragraphs of Chapter I, much space is devoted to a consideration of the nomenclature, an effort being made to harmonize the "positional" properties discussed in descriptive geometry with those that would be defined by Cayley or Clebsch as projective. This will undoubtedly subject the author to criticism. Descriptive geometry is defined as "that branch of mathematics in which figures are represented and their *descriptive* properties investigated and demonstrated by means of projection." Descriptive properties are defined to be such as remain unaltered by projection, and, as one example, we find stated: "If a line is perpendicular to a plane, any plane containing the line will also be perpendicular to the plane." Concluding, the author says: "The main province of projection is obvious." Perpendicularity is not a projective property as defined by Cayley and Cremona; indeed, in all Euclid it is difficult to find one that is, Proposition 2, Book XI, being a conspicuous exception. Moreover, other methods than those of projection, the invariant theory, for example, are apt to be employed to demonstrate projective properties. The work in fact has little to do with such properties and the word *positional*, used by the author as synonymous, had better be substituted and otherwise defined if the definition of descriptive geometry as given by Church is to be replaced. This misconception is unfortunate, especially as, in general, the author has appreciated the importance of accurate definitions. When projections are made upon two perpendicular planes we have the orthographic projection first logically investigated by Gaspard Monge; and, reserving the general definition, the author calls this special branch to be considered "Monge's descriptive," an appropriate and historically valuable limitation.

It is evident that the author is at one time writing a treatise and at another a text book, and as a consequence the reviewer is sometimes at a loss to determine from what standpoint to criticise his work. For example, the quality of the paper, the elaborate and beautiful illustrations and the large and unusually fine quality of type would be out of place in a treatise, while desirable in a work intended to afford to draftsmen a daily illustration of neatness, precision, and the power of artistic expression.

Again, the numerous historical and explanatory notes accompanying the text are very desirable for the student, but at times are so worded as to leave the reader in doubt as to whether they have been verified or not. For example, we find on page 31, a construction for obtaining approximately a straight line equal in length to any semicircle and the accompanying note reads : "According to Böttcher it is due to a Polish Jesuit, Kochansky, and was first published in the Acta Eruditorum Lipsiae, 1685." Again in article 114, the sections of an "annular torus" made by a plane parallel to but not containing the axis are said to cut the surface in "Cassian ovals." Why Cassian instead of Cassinian, which last is the generally accepted spelling, and is the author quite sure of the truth of his statement?

The equation of the torus (Salmon, Geometry of Three Dimensions, 2d edition, p. 348) is

$$(1) \qquad (x^2 + y^2 + z^2 + a^2 - r^2)^2 = 4a^2 (x^2 + y^2).$$

If $y = c$, we may write

$$(2) \qquad (x^2 + z^2 + c^2 + a^2 - r^2 - 2ax)(x^2 + z^2 + c^2 + a^2 - r^2 + 2ax) = 4a^2c^2,$$

or

$$ss' = \text{constant},$$

$s$ and $s'$ denoting circles. The sections, it is true, are declared, in the article quoted above, to be "*lemniscates of various kinds.*" The ovals of Cassini might, therefore, by inference be included. But, in the judgment of the reviewer, Willson has been misled by Salmon, who errs in this statement, for the accepted equation of the Cassinian is

$$(3) \qquad (x^2 + z^2 + a^2)^2 - 4a^2x^2 = m^4$$

(Salmon, Higher Plane Curves, p. 44, ex. 3).

Equation (2) has three constants and (3) has but two and if these definitions are to hold we must have $y = r$, giving (3), which is then a Cassinian for every value of $a$, reducing to the lemniscate of Bernouilli only when $a = m$.

In Chapter V, keeping always in view the double object of the book, namely, to afford examples for the draftsman and to develop mathematical principles of importance to the engineer, the author discusses in turn the helix, the conic sections, link motion curves, centroids, trochoids, limaçon, cardioid, trisectrix, spirals, conchoid, quadratrix, witch, Cartesian and Cassinian ovals and the catenary. Well known as these curves have become, Professor Willson with new and beautiful figures, interesting descriptions, and notes has made out of this material a delightful chapter that a mathematician must read with pleasure. The trochoids are treated with unusual elaboration. Here, again, the author is writing a treatise, for, after developing the subject extensively for ten pages, an appendix, primarily devoted to a reprint of his classification of the trochoids, is added to a volume already too large to be easily handled by the student.

One hundred pages are devoted to a discussion of descriptive geometry and its applications. Compared with the treatise of Mannheim it might suffer, but as an English text-book it is a decided improvement on its predecessors. Only a glance is necessary to convince the student familiar with Church, for example, that most, if not all, of the old difficulties and special cases are here explained with clearness. Indeed the one criticism may be that too little is left for the student's ingenuity and for the cultivation of his imagination, the faculty deriving the most benefit from this study.

As interesting examples of the surfaces considered the following are noted: Of the third order, the conoid of Plücker; of the fourth, the cyclide of Dupin, the conocuneus of Wallis, the cylindroid of Frézies, the corne de vache, the conchoidal hyperbola of Catalan and the torus; in addition, many that are transcendental.

Chapter X is devoted to practical applications of the principles developed, treating in turn of projections and intersections by the third angle method, the development of of surfaces, projections, intersections, and tangencies of developable, warped, and double curved surfaces.

It is by far the most interesting chapter and contains all that is needed in this connection by the undergraduate student of engineering. The treatment of perspective is elaborate but the absence, in a work otherwise well supplied in this respect, of notes connecting the subject with covariants is worthy of comment. We have indeed an illustration of the fact that, whereas the pure mathematician has but slight knowledge of the draftsman's methods, the engineer on the other hand is rarely well informed concerning the algebra of his subject.

The five pages of Chapter XI are devoted to the graphical solution of spherical triangles, constituting a departure from usual methods that is theoretically interesting but likely to be omitted by most teachers using the book.

Orthographic, stereographic, and allied projections of the spherical surface form the subjects of Chapter XII, and the next three deal with the principles of shade, shadow, and perspective.

While it is not written expressly for mathematicians, no student of geometry, especially of surfaces, can review this book without profit. In the main it is firmly grounded on mathematical principles accompanied with illustrations of the best practice of modern draftsmen.

J. B. CHITTENDEN.

COLUMBIA UNIVERSITY.

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# PASCAL'S REPERTORIUM OF HIGHER MATHEMATICS.

*Repertorio di matematiche superiori. I. Analisi.* Per ERNESTO PASCAL, Professor ordinario nella R. Università di Pavia. Milan, U. Hoepli, 1898. 16mo, xv + 643 pp.

THAT the increasing need of encyclopædic literature in mathematics is being promptly met is indicated by the appearance during the last decade of Carr's synopsis, Láska's compendium, the first two volumes of Hagen's synopsis, and most recently the first volume of Pascal's repertorium and the first part of Burkhardt and Meyer's encyclopædia. The body of doctrine of mathematics is expanding at such a rapid rate that the difficulty of securing a liberal education in the science is becoming wellnigh insurmountable, a difficulty that is obvious on contemplating the marvelous development centering about the notions function and group, to cite particular examples. In no subject is special specialization growing more imperative than in mathematics ; in the midst of difficulty and demand the student should hail with delight the valuable services of a work so admirably adapted to purposes of orientation as Professor Pascal's repertorium promises to be.

The author's plan, adhered to without deviation, is to present with regard to each theory of modern mathematics the fundamental definitions and notions, the characteristic necessary theorems and formulæ, and citations to the principal works of its bibliography. The definitions are clear and unequivocal; the statements of the theorems, always given