§ 14. *Isomorphisms between Various Linear Groups.*

The conception of "the compounds of a given linear homogeneous group," introduced in recent papers by the writer,* has proven to be a powerful means of setting up isomorphisms. The group of quaternary linear homogeneous substitutions of determinant unity in the $GF[p^n]$ is isomorphic with a subgroup of the senary linear group leaving invariant $\xi_1\eta_1 + \xi_2\eta_2 + \xi_3\eta_3$, and hence, according as $p^n = 2^n$, $4l + 1$, or $4l - 1$, to the senary first hypoabelian group, the senary orthogonal group, or the group of § 10.

The simple group of order $\frac{1}{2}(p^{4n} - 1)(p^{2n} - 1)p^{4n}$ derived from the quaternary abelian group and that from the quinary orthogonal group, each in the $GF[p^n], p > 2$, are simply isomorphic.

The following four simple groups of order 25920 are simply isomorphic: †

1°. The abelian group on four indices modulo 3.

2°. The second hypoabelian group on six indices modulo 2.

3°. The orthogonal group on five indices modulo 3.

4°. The hyperabelian group on four indices in the $GF[2^2]$.

In the paper cited an abstract group, a substitution-group on 36 letters, and one on 27 letters are given, each isomorphic to the above groups of order 25920.

SHORTER NOTICES.

Leçons sur la détermination des orbites, professées à la Faculté des Sciences de Paris. Par F. TISSERAND; redigées et developpées pour les calculs numériques, par J. PERCHOT; avec une Préface de H. POINCARÉ. Paris, Gauthier-Villars et Fils. 1899. 4to, xiv + 124 pp.

TISSERAND is chiefly known outside of France for his Traité de mécanique céleste—a monument of labor undertaken and carried out amid the pressure of many duties. He had in fact an enormous capacity for work and in his numerous memoirs left few parts of theoretical astronomy untouched. The volume of 120 quarto pages before us, edited by M. Perchot, shows him working for the practical

* "Concerning a linear homogeneous group in $C_{m,q}$ variables isomorphic to the general linear homogeneous group in m variables," BULLETIN, Dec., 1898. "The structure of certain linear groups with quadratic invariants," *Proc. Lond. Math. Soc.*, vol. 30, pp. 70–98. "A new definition of the general abelian linear group," *Transactions* (read Feb. 1899).

† *Proc. Lond. Math. Soc.*, vol. 31. Abstract in BULLETIN, May, 1899, p. 384.

astronomer. The problem of the determination of an orbit from three observations is not exactly an enticing one. It is a tiresome piece of analysis which appears to offer little scope at the present time for the aspiring mathematician. In the case of a parabolic orbit the solution can even be made without successive approximations as M. Poincaré points out in the preface. In the case of an elliptic orbit, almost the only point of mathematical interest is the presence of a transcendental equation.

The treatises on this subject are well known, if not numerous. Gauss, in his Theoria motus, laid the foundations of the method at present in practical use, while Laplace a short time before had given a different form of solution. In the preface to the 'Leçons,' M. Poincaré has devoted some space to showing that Gauss's method is really equivalent to that of Laplace in spite of their apparent dissimilarity. The former has, however, been fully worked up with tables and formulæ for practical use in the great treatises of Oppolzer (of which there is a French translation) and Watson. In spite of the fact that a computer who has at his finger ends the notation and formulæ given in either of the last named volumes, will probably not care to change to another, M. Tisserand's work may nevertheless be found of use. The equations are developed quickly and easily, and moreover are all put together in a form which admits of immediate application. The young astronomer, unless he has acquired a firm grasp of his mathematics, may perhaps find it somewhat difficult to see the bearing of all the formulæ owing to the brevity of the explanations.

There are two chapters. The first on the method of Olbers for the determination of the orbit of a comet from three observations; the second on Gauss's method for the similar determination of the orbit of a planet. A numerical example of the latter, fully worked out, is set forth, showing the form in which the calculations would be actually made. Tables VIII and IX of Oppolzer's treatise are reproduced.

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Vorlesungen über mathematische Physik, Band I, Mechanik. Von GUSTAV KIRCHHOFF. 4te Auflage von W. WIEN. Teubner, Leipzig.

KIRCHHOFF's four volumes on mathematical physics which have appeared at various times since 1876 are so well known that little need be said about them here. The first volume