

atic residues ; the works of Stieltjes appertain here also. Certain differences between the various orders of residues explain themselves by difference in properties of spaces of different dimensions. The geometric reasoning of the author presupposes familiarity with transcendental geometry on the part of the reader ; where the theorems admit of immediate extension to space of any number of dimensions the exposition is made for five dimensional spaces. It is beyond the scope of this short notice to recapitulate the results, which, in the nature of the subject, resist compression.

E. O. LOVETT.

Leçons sur la théorie analytique des équations aux dérivées partielles du premier ordre. Par ÉTIENNE DELASSUS, chargé de conférences à l'Université de Lille. Paris, Hermann, 1897. 8vo, 88 pp.

THE theory of partial differential equations has reached a high degree of perfection, but the theory in its present form lacks unity, its various parts resulting from particular properties and remaining almost independent of one another.

Thus the integration of linear systems rests on a partial extension of the theorem of Cauchy ; in non-linear systems it is necessary to distinguish two cases according as the unknown function does or does not enter, since the method of Jacobi and Mayer experiences modifications in passing from one to the other ; further the method of Lie is demonstrated only for systems in which the unknown does not figure, and is the consequence of a theorem which results from the theory of characteristics or from a method of Jacobi.

The object of the memoir of Delassus is to unify this system of details by determining a canonical form altogether general and establishing a general existence theorem. In the case of systems in which the unknown does not enter, this canonical system coincides with the form in involution, but it is no longer the same when the unknown enters explicitly. Upon this general canonical form Delassus establishes a fundamental existence theorem which he calls for short the generalized theorem of Cauchy ; the demonstration of it yields the following theorem which plays a capital rôle throughout the theory : The integration of a system of partial differential equations of the first order having but one unknown can always be referred to that of a single equation of the first order. This theorem was already known for the case where the unknown function is absent, having been demonstrated by Lie by means of the general theory of characteristics and then by Mayer who employed a method of integration of Jacobi.

The author employs the generalized theorem of Cauchy to formulate the generalized problem of Cauchy which, if solved, would effect the integration completely; to study singular solutions which he classifies according as they are simply, doubly, and so on to n -ply singular; and finally to investigate precisely the transformation which changes a system where the unknown is present into one where it is absent.

The theorem on the reduction to a single equation furnishes immediately a method of integration of linear systems, the method of Mayer, the simplest known. For the integration of non-linear systems the author presents the theory of the complete integral in the ordinary manner, but adds the solution of the problem of Cauchy by means of such an integral without the geometric considerations relative to characteristics.

The exposition of the method of Jacobi and Mayer is made without using any algebraic properties of the bracket expressions and without any reference to the presence or absence of the unknown; moreover Delassus shows that the method leads always to a complete integral. He finds finally the method of Lie as an immediate consequence of the above reduction theorem, and without raising any question relative to the unknown function.

A list of forty examples of simultaneous systems forms the second part of this most valuable contribution to the theory of the linear partial differential equation.

E. O. LOVETT.

NOTES.

THE Chicago Section of the AMERICAN MATHEMATICAL SOCIETY will meet at the University of Chicago on Thursday and Friday, December 28 and 29 next. Titles and abstracts of papers to be read should be in the hands of the Secretary of the Section not later than December 12.

AMONG the officers of the American association for the advancement of science for the coming year are: Professor R. S. WOODWARD, Columbia University, president; Professor CHARLES BASKERVILLE, University of North Carolina, general secretary; Professor ASAPH HALL, JR., University of Michigan, vice-president of section A, mathematics and astronomy; Dr. W. M. STRONG, Yale University, secretary of section A.