

The author employs the generalized theorem of Cauchy to formulate the generalized problem of Cauchy which, if solved, would effect the integration completely; to study singular solutions which he classifies according as they are simply, doubly, and so on to  $n$ -ply singular; and finally to investigate precisely the transformation which changes a system where the unknown is present into one where it is absent.

The theorem on the reduction to a single equation furnishes immediately a method of integration of linear systems, the method of Mayer, the simplest known. For the integration of non-linear systems the author presents the theory of the complete integral in the ordinary manner, but adds the solution of the problem of Cauchy by means of such an integral without the geometric considerations relative to characteristics.

The exposition of the method of Jacobi and Mayer is made without using any algebraic properties of the bracket expressions and without any reference to the presence or absence of the unknown; moreover Delassus shows that the method leads always to a complete integral. He finds finally the method of Lie as an immediate consequence of the above reduction theorem, and without raising any question relative to the unknown function.

A list of forty examples of simultaneous systems forms the second part of this most valuable contribution to the theory of the linear partial differential equation.

E. O. LOVETT.

---

#### NOTES.

THE Chicago Section of the AMERICAN MATHEMATICAL SOCIETY will meet at the University of Chicago on Thursday and Friday, December 28 and 29 next. Titles and abstracts of papers to be read should be in the hands of the Secretary of the Section not later than December 12.

AMONG the officers of the American association for the advancement of science for the coming year are: Professor R. S. WOODWARD, Columbia University, president; Professor CHARLES BASKERVILLE, University of North Carolina, general secretary; Professor ASAPH HALL, JR., University of Michigan, vice-president of section A, mathematics and astronomy; Dr. W. M. STRONG, Yale University, secretary of section A.

THE meeting of the British association for the advancement of science was held at Dover, September 13–21. Professor J. H. POYNTING was president of the mathematical section, and Mr. E. T. WHITTAKER secretary. The following papers were presented to the section: Report of the committee appointed to calculate tables of certain integrals; Report of the committee appointed to calculate tables of certain mathematical functions; Mr. FRANCIS GALTON, F.R.S., "The median estimate;" Professor A. R. FORSYTH, F.R.S., "A system of invariants for parallel configurations in space;" Professor J. D. EVERETT, F.R.S., "On the notation of the calculus of differences;" Professor A. C. DIXON, "On the partial differential equation of the second order;" Professor A. R. FORSYTH, F.R.S., "On singular solutions of ordinary differential equations;" Professor IRVING STRINGHAM, "On the fundamental differential equations of geometry;" Mr. E. T. WHITTAKER, "Report on recent progress in the problem of three bodies;" Professor E. O. LOVETT, "An application and interpretation of infinitesimal transformations;" Lieut. Col. A. J. C. CUNNINGHAM, "On Fermat's numbers."

THE German mathematical association held its annual meeting at Munich from the 17th to the 23d of September in connection with the association of scientists and physicians. The preliminary programme includes the following papers in the order of announcement:—K. HENSEL, Berlin, "On the analytico-arithmetical theory of algebraic functions of two variables;" K. HEUN, Berlin, "The kinetical problems of scientific technology;" E. STUDY, Greifswald, "Geometry of dynames;" R. MEHMKE, Stuttgart, and J. BAUSCHINGER, Berlin, "Discussion of the decimal division of angles and time;" H. WEBER, Strassburg, and G. HAUCK, Berlin, "On the organization of university mathematical instruction to accord with the new Prussian examination requirements;" K. DÖHLEMANN, Munich, "On hyperboloidal straight lines;" F. ENGEL, Leipzig, "On two remarkable groups in space of five dimensions;" P. GORDAN, Erlangen, "On symmetric functions;" D. HILBERT, Göttingen, "On the number concept;" "On Dirichlet's principle;" J. HORN, Charlottenburg, "Divergent series in the theory of differential equations;" G. KOHN, Vienna, "On the geometry of Klein's group of 360 collineations and 360 correlations;" M. LERCH, Freiburg, Switzerland, "Arithmetical considerations on infinite series;" L. MAURER, Tübingen, "Invariance under special groups of linear substitutions;" F.

MEYER, Königsberg, "On the notion of equivalence of geometrical propositions;" M. NOETHER, Erlangen, "Riemann's lectures of 1861-62 on abelian functions;" E. SCHIMPF, Bochum, "Introduction of a measure of convergence and divergence in the theory of infinite processes (series, products, continued fractions, etc.);" L. SCHLESINGER, Klausenburg, "On Riemann's posthumous paper on the theory of linear differential equations;" J. SOMMER, Göttingen, "On quadratic manifolds in five-dimensional space;" A. SOMMERFELD, Clausthal, "On Weierstrass's criteria in the calculus of variations;" E. v. WEBER, Munich, "A fundamental classification of differential problems." Pedagogical papers from RUDEL, of Nürnberg, SCHOTTEN, of Halle, and TREUTLEIN, of Karlsruhe, were also announced.

ON the seventeenth of last June the statue erected to the memory of CARL FRIEDRICH GAUSS and WILHELM WEBER was unveiled at Göttingen with appropriate ceremonies. The monument in bronze, on a granite base, represents Gauss seated in an arm-chair and Weber standing by his side and apparently discussing with him their joint invention, the electric telegraph. It is the work of the Berlin sculptor Hartzer. The principal address of the celebration was delivered by Professor Woldemar Voigt who sketched in brief and pregnant words the scientific career of the two men. A large number of delegates from various scientific institutions and other guests were present at the unveiling and the succeeding banquet. An exhibition of manuscripts by Gauss and Weber, of instruments used by them, and of other relics was arranged in the Aula of the University and attracted much attention. The Festschrift published in commemoration of the day contains two valuable memoirs, one by Professor Hilbert, "On the foundations of euclidean geometry," the other by Professor Wiechert, "On the theory of electrodynamics." The University of Göttingen took this opportunity to confer the honorary degree of doctor of philosophy on Professor G. H. Darwin, of Cambridge, Professor Geitel, of Wolfenbüttel, Professor J. Hadamard, of Paris, Professor Lorentz, of Leiden, Professor E. H. Moore, of Chicago, Professor Righi, of Bologna, and Professor von Sterneck, of Vienna.

THE international congress of philosophy in connection with the Paris exposition of 1900 will be held from the 2d to the 7th of August. The section devoted to the history of the sciences has made liberal provision in its programme for the discussion of the fundamental notions of mathematics.

THE Paris academy of sciences has been authorized to increase the number of its national and foreign correspondents from one hundred to one hundred and sixteen.

A MATHEMATICAL society has been organized at Liverpool, England, to be known as the Liverpool mathematical society. Mr. Walter Stott is honorary secretary.

THE London mathematical society has decided to issue its *Proceedings* in two volumes per annum.

M. GAUTHIER-VILLARS, of Paris, announces the early appearance of the following mathematical works:—A two volume treatise on the theory of forms and higher analytic geometry, by ANDOYER; an elementary treatise on the theory of numbers, by CAHEN; a treatise on geometrical optics, by WALLON; the scientific works of G. ROBIN, including the memoirs on the theory of functions founded exclusively on the idea of number, and those on mathematical physics and general thermodynamics, edited by L. RAFFY; an index to the *Comptes rendus* from 1881 to 1895, including volumes 92 to 121.

THE fall announcements of M. Hermann, of Paris, include *Recherches sur les fonctions de variables réelles*, by R. BAIRE; *Sur les fondements de la théorie des ensembles transfinis*, by G. CANTOR, translation by F. MAROTTE, of the two well known memoirs of the author published in the *Mathematische Annalen*; *Leçons nouvelles sur les applications géométriques du calcul différentiel*, by W. de TANNENBERG.

MESSRS. Carré and Naud, of Paris, have a history of mathematics by BOYER in the press, and an annuaire of mathematicians in preparation.

PROFESSOR R. S. BALL is engaged on a treatise on the theory of screws which is to be considerably larger than his former book on that subject. It will be published by the Cambridge university press.

THE second volume of Professor A. R. FORSYTH's theory of differential equations is in the press.

The following courses of lectures have been announced for the winter semester 1899–1900:—

UNIVERSITY OF BERLIN.—By Professor H. A. SCHWARZ: Analytical geometry, four hours; Maxima and minima, two hours; Elliptic functions, four hours; Colloquia, two hours; Mathematical seminar, two hours.—By Professor E. R. HOPPE: Analytical geometry, four hours; Differential cal-

culus and theory of series, four hours.—By Professor J. KNOBLAUCH: Differential calculus, four hours; Applications of elliptic functions, four hours; Exercises in differential calculus, one hour.—By Professor LEHMANN-FILHÉS: Integral calculus, four hours; Method of least squares, one hour.—By Professor G. FROBENIUS: Theory of determinants, four hours; Theory of numbers, four hours; Mathematical seminar, two hours.—By Professor K. HENSEL: Synthetic geometry, four hours; Geometry of numbers, three hours; Exercises in the geometry of numbers, one hour.—By Professor G. HETTNER: Theory of probability and of errors, four hours.

UNIVERSITY OF Breslau.—By Professor J. ROSANES: Exercises in mathematical seminar, one hour; Differential calculus and elements of integral calculus, four hours; Elements of the theory of determinants, two hours.—By Professor R. STURM: History of mathematics, one hour; Mathematical seminar, two hours; Theory of geometrical correspondences second part, three hours; Non-euclidean geometry, two hours.—By Professor F. LONDON: Introduction to the theory of functions, three hours; The technology of insurance, two hours.

UNIVERSITY OF Erlangen.—By Professor P. GORDAN: Differential and integral calculus, four hours; Algebra, four hours; Seminar, three hours.—By Professor M. NOETHER: Analytical geometry, four hours; Differential geometry, four hours; Mathematical exercises.

UNIVERSITY OF Göttingen.—By Professor F. KLEIN: Mechanics of deformable bodies, four hours; Mathematical seminar, two hours.—By Professor D. HILBERT: Integral calculus, four hours; Curvature of surfaces, two hours; Number-concept and quadrature of circle, two hours; Exercises in mathematical seminar, one hour.—By Dr. F. SCHILLING: Descriptive and projective geometry, four hours; Exercises in descriptive geometry, four hours; Exercises in differential calculus, one hour.—By Dr. G. BOHLMANN: Chapters in the mathematics of insurance, two hours; The mathematical foundations of insurance, three hours; Mathematical exercises in seminar, two hours.—By Dr. ZERMELLO: Theory of functions, four hours.—By Dr. SOMMER: Theory of numbers, four hours; Exercises in the theory of numbers, one hour.

UNIVERSITY OF Greifswald.—By Professor W. THOMÉ: Analytical geometry, two hours; Theory and applications

of determinants, two hours ; Exercises in mathematical seminar, two hours.—By Professor E. STUDY : Mechanics, four hours ; Applications of the infinitesimal calculus to geometry, four hours ; Exercises in seminar, one hour.

UNIVERSITY OF KIEL.—By Professor L. POCHHAMMER : Plane analytical geometry, three hours ; Theory of definite integrals, three hours ; Mathematical seminar, one hour.—By Professor P. STÄCKEL : Integral calculus, three hours ; Higher geometry, three hours ; Seminar, one hour.

UNIVERSITY OF KOENIGSBERG.—By Professor F. MEYER : Integral calculus, three hours ; Exercises to preceding, one hour ; Elliptic functions, four hours ; Seminar, one hour.—By Professor A. SCHOENFLIES : Determinants, one hour ; Projective geometry, four hours ; Exercises to preceding, one hour ; General introduction to higher mathematics, two hours.—By Professor L. SAALSCHÜTZ : Differential equations, three hours ; Exercises in differential equations, one hour.—By Dr. T. VAHLEN ; Calculus of variations, three hours ; Exercises, one hour.

UNIVERSITY OF HALLE-WITTENBERG.—By Professor G. CANTOR : Analytical mechanics, four hours ; Chapters of the infinitesimal calculus, two hours ; Exercises in seminar, two hours.—By Professor A. WANGERIN : The most important partial differential equations of mathematical physics, one hour ; Theory of surfaces and space curves, four hours ; Synthetic geometry, three hours ; Mathematical seminar, two hours.—By Professor V. EBERHARD : Chapters of analytical geometry, one hour ; Theory of numbers, four hours.

UNIVERSITY OF HEIDELBERG.—By Professor L. KOENIGSBERGER : Analytical mechanics, four hours ; Calculus of variations, differential equations, two hours ; Elliptic functions, two hours ; Seminar, two hours.—By Professor M. CANTOR : Infinitesimal calculus, four hours ; Exercises in the calculus, one hour ; Political arithmetic, two hours.—By Professor F. EISENLOHR : Theoretical optics, four hours ; Infinitesimal calculus, five hours ; Potential, two hours.—By Professor K. KOEHLER : Analytical geometry of space, three hours.—By Professor G. LANDSBERG : Descriptive geometry, three hours ; Introduction to the theory of algebraic equations, two hours.

UNIVERSITY OF JENA.—By Professor J. THOMAE : Algebraic analysis, four hours ; Mathematical geography, three

hours; Seminary, two hours.—By Professor G. FREGE: Analytical mechanics, four hours; Symbolic algebra, two hours.

UNIVERSITY OF LEIPZIG.—By Professor A. MAYER: Higher analytical mechanics, four hours; Exercises to the preceding, one hour.—By Professor C. NEUMANN: Infinitesimal calculus, four hours; Seminar, one hour.—By Professor O. HÖLDER: Introduction to the theory of functions of a complex variable, four hours; Chapters in the theory of substitution groups and equations, two hours; Seminar, one hour.—By Professor F. ENGEL: Analytical mechanics, four hours; Theory of continuous transformation groups, two hours; Seminar, one hour.—By Dr. F. HAUSDORFF: Chapters of higher geometry, four hours.—By Dr. G. KOWALEWSKI: Algebraic invariant theory.

UNIVERSITY OF MUNICH.—By Professor G. BAUER: Plane analytical geometry; Conduction of heat in fixed bodies; Seminar.—By Professor F. LINDEMANN: Differential calculus; Abelian functions; Mathematical seminar on theory of automorphic functions and application of elliptic functions.—By Professor A. PRINGSHEIM: Introduction to the theory of analytic functions; Elementary theory of infinite algorithms (series, products, continued fractions).—By Dr. H. BRUNN: Introduction to the mathematical foundations of natural science.—By Dr. K. DÖHLEMANN: Descriptive geometry and exercises; Synthetic geometry and exercises.—By Dr. E. v. WEBER: Introduction to analysis; Introduction to the Cauchy-Riemann theory of functions.

THE London mathematical society has conferred its De Morgan medal on Professor W. BURNSIDE for his mathematical researches in the theory of groups of a finite order.

THE Adams prize offered by the University of Cambridge for an essay on the theory of the aberration of light has been divided between Mr. J. LARMOR, fellow of St. John's college, senior wrangler 1880, and Mr. G. T. WALKER, fellow of Trinity college, senior wrangler in 1889.

THE Madrid academy of sciences recently announced the results of the competition for its mathematical prize set in 1897. There were three competitors. Professors GINO LORIA, of the University of Genoa, and GOMES TEIXEIRA, of the Polytechnic of Oporto, each received a prize, and JOAQUIN DE VARGAS Y AGUIRRE, of Salamanca, an honorable mention.

THE honorary degree of doctor of laws has been recently conferred on Professor A. R. FORSYTH by Dublin University, and that of doctor of civil law on Professor SIMON NEWCOMB, by Oxford University.

PROFESSOR L. FUCHS has been made rector of the University of Berlin for the current year, and Professor E. NETTO has received the same appointment at the University of Giessen.

MR. H. M. MACDONALD, of Clare College, Cambridge, fourth wrangler in 1889, has been appointed to the University lectureship in mathematics made vacant by Professor A. E. H. LOVE's recent resignation to accept the Sedleian professorship in mathematics at Oxford University.

LORD KELVIN has resigned the chair of natural philosophy in Glasgow University after a tenure of fifty-three years.

DR. MILTON UPDEGRAFF, professor of astronomy in the University of Missouri, has been appointed to a professorship in mathematics in the United States naval observatory.

DR. L. E. DICKSON has resigned an assistant professorship of mathematics in the University of California to accept an associate professorship in the University of Texas.

MR. PETER FIELD, fellow in Cornell University, has been appointed professor of mathematics in Carthage College.

THE following have recently qualified as privatdozenten : Dr. GRASSMANN, at the University of Halle ; Dr. KOWALEWSKI, at the University of Leipzig ; Dr. J. GOETTLER, at the University of Munich.

MESSRS. T. BIRTWHISTLE, of Pembroke College, and R. P. PARANJPYE, of St. John's College, were bracketed for the senior wranglership of this year at Cambridge University. The Smith's prizes for 1899 were awarded to Messrs. W. H. AUSTIN and G. W. WALKER, of Trinity College, senior and fourth wranglers in 1897. Mr. PARANJPYE is the first native of India to attain the honor and Mr. BIRTWHISTLE is the first member of Pembroke College to achieve it since Sir GEORGE STOKES who was senior wrangler in 1841.

AT Emmanuel College, Cambridge, Mr. G. T. BENNETT, senior wrangler, 1890, has been elected to a senior fellow-

ship, and Mr. H. S. CARSLAW, fourth wrangler 1894, to a junior fellowship.

MR. A. H. WILSON, instructor in mathematics at Princeton University, has been granted a year's leave of absence, and Dr. L. W. REID has been appointed to an instructorship in the same institution.

THE following mathematicians have been elected to fellowships in the Reale Accademia dei Lincei : Professor P. TARDY and G. VERONESE, as ordinary fellows ; Professor G. RICCI, as corresponding fellow ; Professor G. MITTAG-LEFFLER and J. WEINGARTEN, as foreign fellows.

PROFESSORS F. KLEIN and W. NERNST, of Göttingen, have been elected to foreign membership in the academy of sciences of Budapest.

AT its recent decennial celebration Clark University conferred the honorary degree of doctor of laws on five scientists among whom were Professors PICARD, of Paris, and BOLTZMANN, of Vienna.

MR. J. D. THOMPSON, of Trinity College, Cambridge, has been elected to an assistant professorship of mathematics in the University of West Virginia.

DR. F. H. SAFFORD, of Harvard University, has been made assistant professor of mathematics at the University of Cincinnati.

MR. HOWARD OPDIKE has been promoted to an assistant professorship of mathematics at Union College.

DR. H. Y. BENEDICT and MR. I. N. PUTNAM have been appointed instructors in mathematics at the University of Texas.

THE Right Rev. CHARLES GRAVES, Lord Bishop of Limerick, professor of mathematics at Trinity College since 1843, died recently at the age of eighty-seven years.

THE deaths are announced of Professor ROBERT WILHELM BUNSEN, August 16th, 1899, at the age of eighty-eight years; of M. GRIESS, one of the editors of *L'éducation mathématique*, at the age of forty-two years.