greater than one in u. The presence in the differential equations of derivatives of y of order greater than one does not affect the result in the latter case, while in the case of equation (3), as in the calculus of variations, we are led to a resulting differential equation more complicated than (5).

Mr. Coolidge's paper began with a definition of imaginary geometrical elements after von Staudt and August. The usual method of representing imaginary points in a real line by real points in a plane was then mentioned, with an explanation why this proceeding could not be so extended as to include all imaginary points in a plane. It was then shown that every imaginary point in a real plane might be represented by the real base of its projection from a real point on an imaginary plane, and the necessary construction was explained. The assemblages of lines representing points on real and imaginary lines were discussed, as well as the relations of those lines which represented chains of imaginary points. The paper closed with a brief reference to the congruences of lines representing points lying on coinc sections, and a discussion of the advisability of finding a method of representing all the imaginary points in space.

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NOTE ON THE SIMPLY TRANSITIVE PRIMITIVE GROUPS.

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(Read before the American Mathematical Society, October 28, 1899.)

The simply transitive primitive groups present many difficulties which have not yet been solved. In his "Traité des substitutions" Jordan proved some fundamental theorems in regard to these groups.* Several additional theorems appeared in the "*Proceedings of the London Mathematical Society*," 1897, pp. 534–536. The following theorems and corollaries are closely connected with the results of this article.

Every primitive group G of degree n contains a maximal subgroup G_1 of degree $n-1$. When G is simply transitive

* P. 284.

G_1 is intransitive and the order of each of its transitive constituents is divisible by the same prime numbers.†

THEOREM I. *When G_1 is of order p^m, p being any prime number, it cannot contain as a transitive constituent any regular group whose order is less than the order of G_1.*

The subgroup of G_1 which would correspond to identity in this transitive constituent would be self-conjugate and it would have to occur in each of the conjugates of G_1 which do not involve all the elements of the given transitive constituent. In each of these conjugates it would be transformed into itself by substitutions that are not found in G_1, since each subgroup of any group of order p^m is transformed into itself by substitutions that are not contained in it. These conditions are impossible, since G_1 is a maximal subgroup of G.

COROLLARY I. *When G_1 is of order p^m it cannot contain any self-conjugate subgroup of degree less than $n-1$ and of order greater than p^{m-3}, $m > 2$.*

COROLLARY II. *When one of the transitive constituents of G_1 is of order p^2, G_1 must be of this order and G must be of class $n-1$.*

THEOREM II. *If the order of G_1 is p^5 and if G_1 contains a transitive constituent of order p^3, then any two of the p^2 conjugates of G_1 which do not involve all the elements of the given transitive constituent contain a common abelian group of order p^3.*

It follows from theorem I that the given transitive constituent must be of degree p^2. Each of the given p^2 conjugates contains a subgroup of order p whose substitutions are commutative to every substitution of the group and whose degree does not exceed $n-1-p^2$. This subgroup must occur in G_1 since the self-conjugate subgroup S of G_1 which corresponds to identity in the given constituent of order p^3 cannot be transformed into itself by any substitution of G except those that are contained in G_1. It cannot occur in S since each subgroup of S is transformed into itself by at least p^4 substitutions of G_1. The given subgroup of order p and S must therefore generate an abelian group of order p^3 which includes all the substitutions of G_1 that are common to it and the corresponding conjugate to G_1.

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July, 1899.

† Jordan, loc. cit.