

The square with its scales, and the moveable line, constitute then the abacus of the equation (1). Naturally, the square is marked with vertical and horizontal lines to assist the reading.

The questions raised in the book should appeal not only to the technical man, but also to the teacher of elementary analytic geometry, at least to those teachers who care to heed that class of students whose cry is "of what use is this?"

FRANK MORLEY.

*An Elementary Treatise on the Theory of Equations.* By S. M. BARTON, Ph.D. D. C. Heath & Co., Boston, 1899. 8vo, xii + 198 pp.

THE title of Professor Barton's book suggests at once to the English-speaking student of mathematics the well-known treatises of Todhunter and of Burnside and Panton, and calls to mind how consistent English practice has been in assigning the theory of equations to distinctive works on the subject. Among the comparatively few books thus styled may be mentioned Chapman's "An Elementary Course in Theory of Equations," published in this country a few years ago.

The present treatise is much more limited in scope than the English works above mentioned. There is no attempt to deal with the formal side of the higher algebra. The book is intended expressly for undergraduate instruction in our colleges and technical institutions, and the contents and treatment are accordingly quite narrowly prescribed by the requirements of the usual college course.

The work falls into two parts: I., an elementary exposition of determinants; II., the theory of equations proper.

*Part I.*—The first two chapters give the principal theorems of determinants, and the third consists of applications to linear equations and a consideration of special determinant forms. The subject is introduced by considering the permutations of a group of elements, after which the development follows the usual course. The author here undoubtedly exposes himself to criticism for devoting so much space, some seventy-five pages in all, to a theory of which he is able to make very little use in the remaining chapters.

*Part II.*—In this part Burnside and Panton have been laid under heavy contribution. An inspection of the table of contents and a review of the pages show, as the author states, that the development has followed very closely in the lines of the first ten chapters of the treatise cited. It

will perhaps be fair to characterize this part as an attempt to adapt a portion of the Burnside and Panton to the needs of the general student, and at the same time to provide an introduction to the more extensive treatises on the subject. To secure the requisite condensation, an occasional change in arrangement has been made, statements and proofs abridged, and certain parts omitted. In all essential respects, however, the proofs, method, and arrangement of the English book have been retained.

In so far as the excellent features of the large treatise are found reproduced in the smaller work, to that extent is this grade of text-book enriched. A decided gain is the retention of the purely algebraical methods of proof. There is undoubtedly a tendency, quite marked in some of our college text-books on algebra, to replace the algebraical proofs of certain theorems by the methods of the calculus. The gain in an elementary text-book by a substitution of this character is neither apparent nor real, and does not warrant so radical a departure from algebraical methods; on the contrary, the change is attended by a loss in simplicity and uniformity of development.

It is to be regretted that Professor Barton has not availed himself of the opportunity to improve upon Burnside and Panton in the matter of drawing a sharp distinction between the variable  $x$  and its absolute value. The need of a careful discrimination in this direction is particularly felt in the chapter on the properties of entire polynomials.

The condensation and abridgements previously referred to have not always been effected without loss. Occasionally indeed there are statements which, unqualified, are not true. Again, there is a failure at times to express with sufficient directness and simplicity the nature and purposes of the propositions. Attention may be here directed to several cases in which changes seem desirable. In the introduction to Part II., page 77, the double assumption that  $\alpha$  and  $\beta$  are distinct roots and that  $\alpha$  is not zero, is a rather unhappy conjunction of hypotheses. So in chapter VII., page 125, following the statement of the general theorem, the corollary: "Corollary I. Every root of an equation is a divisor, *whole or fractional*, of the absolute term of the equation" should be eliminated. As it stands, it is incorrect; and if restored to the form in which it appears in Burnside and Panton, it is void of significance. On page 162, the reference to the sign of the intermediary Sturm function  $f_r(x)$  is superfluous, and, moreover, the function does not necessarily change sign when  $x$  passes through  $\alpha$ .

For completeness of statement, we note that the introduction to the second part is followed by a chapter of sixteen pages on complex numbers, containing De Moivre's theorem and the trigonometric solution of the binomial equation. There is also a chapter of six pages on elimination, with illustrations of Euler's and Sylvester's methods. The volume concludes with the methods of approximating to the real roots of numerical equations. Each theorem of the text, as far as possible, is immediately supplemented by problems showing its applications and place in the theory.

In conclusion, there are many admirable features in the work which would make it in some respects a valuable adjunct to instruction, and it is to be hoped that in a second edition a careful revision and the elimination of the rather numerous typographical errors will bring these qualities more clearly to light and place it in a position of considerable usefulness.

JAMES MACLAY.

*The Theory and Practice of Interpolation*: Including mechanical quadrature and other important problems concerned with the tabular values of functions. With the requisite tables. By HERBERT L. RICE, M.S. The Nichols Press, Lynn, Mass. 1899. Cr. 8vo., 234 + ix pp.

THE theory and practice of interpolation is, in its essentials, based on the fact that in nearly all functions which arise in physical problems, a small change of the variable produces a small change of the function. The simplest case is the one in which we may consider the ratio of the two small changes as constant: in the language of the subject, the first differences are constant. When this assumption will not give sufficiently accurate results, we have to consider the difference of the first differences or the *second* differences, and even differences of higher order. Ultimately we neglect differences of some definite order and thus implicitly reduce the problem to the consideration of the values of a function which, between certain limits and to a given degree of accuracy, may be considered rational, integral and algebraic. Round this problem a mass of literature has grown up. The values of a function are required for certain values of the variable. To obtain these every time from a formula may be troublesome, or even impossible if no such formula is known; tables are therefore made giving the values of the function for certain values of the variable, and the subject teaches how we can thence