

integrable expressions when the modulus is near unity; this is misleading, as a transformation enables us in such a case to approximate as close as we wish to the numerical value of the integral. The writer of this notice has not tested any of the formulæ but they appear to be carefully worked out and the author seems to be familiar with the best methods of computation.

ERNEST W. BROWN.

*Vorlesungen über Geschichte der Trigonometrie.* VON DR. A. VON BRAUNMÜHL. Erster Theil. Leipzig, B. G. Teubner, 1900. 260 pp.

VON BRAUNMÜHL's history of trigonometry is to be in two parts. The volume before us is the first part, and carries the narrative down to the invention of logarithms. If the second half of the work rises to the level of the first, the book will at once take its place as the most complete and authoritative work on the history of trigonometry.

In the perusal of this volume we realize the rapid progress which has been made in the study of mathematical history. The "restitution of decayed intelligence" has been carried on with great diligence. Formerly the origin of trigonometry was ascribed to Hipparchus and Ptolemaeus; now the earliest attempts to establish relations between the sides and the angles of a right triangle can be traced back to a period as long before the time of Hipparchus, as Hipparchus was before our time. The Ahmes papyrus of about 2000 B. C. contains five problems involving such relations.

In Whewell's History of the inductive sciences we read of the "sterility of Arabian genius," but in recent years a closer acquaintance with Arabic research in mathematics forces us to the admission that the Arabs displayed considerable originality. Von Braunmühl makes it plain that in trigonometry much of what has been ascribed to Europeans of the fifteenth century was worked out at an earlier period by the Arabs. The author suggests that even our present knowledge may require considerable revision after Arabic manuscripts have been read, which at present, covered with the dust of centuries, lie untouched on the shelves of Spanish libraries.

The author has embodied in this volume a great deal of original research. He has brought to light the fact that the Greeks possessed a graphic method of solving spherical triangles which was extended by the Hindus and Arabs so as to become a fruitful aid in trigonometric computation. This

remained in use as late as the seventeenth century. Von Braunmühl traces the beginning of the graphic treatment of spherical triangles back to Anaximander, but suspects that these graphic processes are much older, that they were known to the Egyptian and possibly also to the Chaldean astronomers.

The graphic method of solving spherical triangles is the oldest trigonometric *method* known to us. Ptolemaeus gives graphic processes in which *sine* is used, but curiously enough, in all trigonometric computation he employs instead *the chord of double the arc*. This anomaly finds its explanation in the fact that the Greeks treated the graphic solutions and the numerical solutions apart from each other; it remained for the Hindus and Arabs to unite the two methods and to recognize the advantage, in all cases, of using half the chord in place of the whole chord.

We have noticed no errors of importance. On page 88 the date of Alcuin's birth is given as 736. The same date occurs in Felix Müller's *Zeittafeln*, but the correct date is uncertain; it is probably 735. Snellius's baptismal name is spelled on page 70 *Willebrod*, in other places it is given correctly as *Willebrord*.

FLORIAN CAJORI.

*Histoire des Mathématiques.* Par JACQUES BOYER. Illustrée de fac-similés de manuscrits et de portraits. Georges Carré et C. Naud, Éditeurs. Paris, 1900. 250 pp.

ON opening this book the reader is attracted by several facsimile reproductions from old mathematical books or documents and by a number of portraits of mathematicians. Thus he has before him a facsimile of part of the Egyptian Akhmim papyrus, of the title page of the *Acta Eruditorum*, of a page of Euclid's *Elements*, from a manuscript preserved in the Bibliothèque Nationale of Paris. There are nineteen portraits of mathematicians. The list comprises eleven Frenchmen, three Englishmen, two Germans, two Russians, and one Swiss. Two likenesses are of women, namely, of Mme. Du Chatelet and Mme. Kovalevski.

To write a general history of mathematics and confine it, as M. Boyer does, to the small compass of 247 pages is no easy task. Anything like completeness cannot be looked for. Perhaps all one can expect is that the information offered be accurate, that the broad movements in mathematical thought be brought before the reader and that the narrative be made attractive so as to invite more thorough study in larger treatises. In this last respect we think that