

of singular points first determined, is represented by a finite number of formulas, in each of which one coördinate is expressed as an analytic function of the other two. If the curve does, however, contain multiple factors the introduction of a quadratic transformation of the type

$$x = x'z \quad (2)$$

(preceded by a certain other transformation of the form  $x + py = x_1$ ) secures the same result as in the other case.

Each of the new singular points is then subjected to the same treatment as the original point, and the process is repeated until all the singular points disappear, the parametric representation then becoming at once possible. It is shown that this result will be reached by means of a finite number of transformations of types (1) and (2), together with certain other one to one transformations.

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## THE ITHACA COLLOQUIUM.

THE Third Colloquium of the AMERICAN MATHEMATICAL SOCIETY was held at Cornell University, Ithaca, N. Y., beginning on Wednesday, August 21, 1901, and extending over the following three days. Before describing the proceedings, it may be of interest to recall the work of the previous colloquia.

The first colloquium\* organized by the Society was held in connection with its third summer meeting at Buffalo, N. Y., September 2-5, 1896. Two courses of six lectures each were delivered before an audience of thirteen members. Professor Maxime Bôcher discussed "Linear differential equations and their applications," and Professor James Pierpont, "The Galois theory of equations." The innovation proved so successful that the participants recommended to the Council that the same plan should be adopted the following summer; but the meeting at Toronto, with

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\* See the report, including abstracts of the courses of lectures, by Professor T. S. Fiske in BULLETIN, volume 3, pp. 49-59. Professor Bôcher's lectures were in part reproduced in the *Annals of Mathematics*, 1st series, vol. 12 (1898), pp. 45-53. Professor Pierpont's lectures were published in the same journal, 2d series, vol. 1 (1899), pp. 113-143, and vol. 2 (1900), pp. 22-56.

the British association for the advancement of science, rendered this impracticable. The second colloquium\* was therefore held, after an interval of two years, in connection with the fifth summer meeting, at Cambridge, Mass., August 22-27, 1898. Professor W. F. Osgood delivered six lectures on "Selected topics in the general theory of functions," and Professor A. G. Webster six lectures' on "The partial differential equations connected with wave propagation." Twenty-six members attended the two courses.

On account of the intervening Paris exposition a period of three years elapsed between the second and third colloquia. At the December meeting, 1900, the Council appointed a committee, consisting of Professors F. N. Cole, W. F. Osgood, James Pierpont, J. H. Tanner, and H. S. White, to arrange for a third colloquium in connection with the eighth summer meeting. A preliminary notice containing the plans agreed upon was issued in May, 1901. In accordance with these plans, the following twenty-five members of the Society assembled Wednesday morning, August 21, 1901, in one of the lecture rooms of White Hall, Cornell University, and registered for the colloquium :

Dr. G. E. Bliss, Professor Oskar Bolza, Professor E. W. Brown. Professor F. N. Cole, Professor L. L. Conant, Professor T. S. Fiske, Mr. W. B. Ford, Mr. B. F. Groat, Dr. E. R. Hedrick, Professor T. F. Holgate, Dr. J. I. Hutchinson, Dr. Edward Kasner, Dr. G. H. Ling, Professor J. L. Markley, Professor E. H. Moore, Professor G. D. Olds, Professor W. F. Osgood, Dr. I. E. Rabinowitch, Mr. F. G. Radelfinger, Professor E. R. Skinner, Dr. Virgil Snyder, Professor J. H. Tanner, Professor E. B. Van Vleck, Professor L. A. Wait, Professor Alexander Ziwet.

The Society has always been fortunate in securing for the colloquia the services of lecturers of high standing. These serve purely as a matter of honor, receiving no compensation, except a very moderate allowance for their travelling and other expenses provided for by the collection of a nominal fee from each auditor.

During the four days of the colloquium, two courses of four lectures each were delivered by Professor Oskar Bolza, of the University of Chicago, and Professor E. W. Brown, of Haverford College. The titles and outlines of the courses are as follows :

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\*See the report by Professor H. S. White, BULLETIN, volume 5, pp. 57-58. Professor Osgood's lectures were published in the same volume, pp. 59-87.

PROFESSOR BOLZA :—THE SIMPLEST TYPE OF PROBLEMS IN THE CALCULUS OF VARIATIONS.

The principal object of the course was to give a detailed account of the simplest type of problems in the calculus of variations, in its historical development, with special emphasis upon the contributions of Weierstrass and his followers. After a critical account of the older theory of Euler, Legendre, and Jacobi, the lecturer passed to Weierstrass's theory and to the contributions made since Weierstrass by Schwarz, Kneser, Osgood, and others. The course concluded with an exposition of Hilbert's existence theorem, and with an indication of the lacunæ which must be filled before the theory of even the simplest type of problems can be declared complete.

PROFESSOR BROWN :—MODERN METHODS OF TREATING DYNAMICAL PROBLEMS AND IN PARTICULAR THE PROBLEM OF THREE BODIES.

The object of this course was to set forth some of the later attempts to introduce more rigor into the methods of solving dynamical problems, mainly the researches of Poincaré. The course was chiefly descriptive, in showing the principles of the methods, the mathematical difficulties which arise, and the results which have been obtained. Amongst the subjects treated were the following: The various forms of the differential equations of dynamics; the theorems of Bruns and Poincaré on the non-existence of integrals; absolute and relative invariant integrals; solution by infinite series, including divergent series; periodic and asymptotic solutions; stability and instability. No special knowledge of any one subject was assumed.

Two lectures were delivered daily, one in the morning and one in the afternoon, the hour varying to suit the convenience of the lecturers and auditors. Each lecture extended through two hours, the period being divided by a recess of ten minutes. Professor Bolza's course was given on the mornings of Wednesday and Saturday, and the afternoons of Thursday and Friday; Professor Brown's at the other four sessions.

The librarian of the University kindly offered the facilities of both the reading room and the stack rooms to members of the Society. The literature bearing on the subjects discussed in the colloquium, including autograph notes of Weierstrass's lectures, and a bibliography of the calculus of variations prepared by Dr. J. I. Hutchinson, was collected

and arranged for the convenience of the auditors. Printed syllabi of both courses were distributed, so that every opportunity was offered for intelligent participation. At the close of the courses, the members present, by a rising vote, unanimously expressed their appreciation of the value of the lectures, and of the self-sacrificing labor of the lecturers.

Friday evening the participants of the colloquium dined together at the Glenwood Hotel, and afterwards enjoyed a cruise around Cayuga Lake. During the remaining evenings there were social gatherings at the Town and Gown Club, whose privileges were thrown open to members of the Society. The hospitality of Cornell University, and of the mathematical department in particular, deserves the most grateful acknowledgment.

Detailed reports of the courses, prepared by the lecturers themselves, will appear in later numbers of the BULLETIN.

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## UPON THE NON-ISOMORPHISM OF TWO SIMPLE GROUPS OF ORDER $8!/2$ .

BY MISS IDA M. SCHOTTENFELS.

(Read before the American Mathematical Society, August 20, 1901.)

1. *Introduction.*—The proof offered in this paper of the non-isomorphism of the ternary linear fractional group Galois field  $[2^2]$  and the alternating group of degree eight is shorter, simpler and more direct than that presented by the author in the *Annals of Mathematics*, volume 1, No. 3, April, 1900.

In a paper read December 29, 1900, before the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY, by making use of the present method involving the three conjugate sub-groups (1)  $G_{168}$ , (2)  $G_{168}'$ , (3)  $G_{168}''$ , the identity was established between the ternary group  $G.F. [2^2]$  and the literal substitution group of degree 21,  $G_{81/2}^{21}$ .

2. *The Ternary Group.*—In the above mentioned paper the group was defined as a group of fractional matrices, access being had to the group by means of the following matrices :