cated the general character of the laws of resistance of continents, mountains, and oceans to the motion of the atmosphere as also the probable general characteristics of the desired solutions of the whole system of equations. His fundamental departures from all previous analytical studies consisted in the recognition of a so-called convectional friction or resistance due to the interference with each other of ascending and descending currents of air, and in the recognition of the division of the globe into land and water hemispheres having their poles at London and anti-London respectively.

Mr. Pawling's first paper gave a brief history of the action of Congress in regard to weights and measures. The author also spoke of the "English decimal association" and the good that a similar association might do in this country. In his second paper he gave an account of the new National bureau of standards which is to be located in the suburbs of Washington. It has an appropriation of $300,000 and a laboratory costing about $200,000 is to be erected. The Bureau will employ a number of young men just graduated from universities giving them opportunities to develop along the lines which they wish to follow. It will also employ specialists to do work in their line. It standardizes three grades of weights and measures: 1° Those for commercial use, 2° those for manufacturing and technical processes and professions, 3° those for extreme accuracy for scientific purposes. Professor Nichols, of Cornell, added that the prospects seemed very good that the United States was about to take a place among the leading nations in scientific work along this line.

G. A. MILLER.

STANFORD UNIVERSITY.

RIEMANN-WEBER: PARTIAL DIFFERENTIAL EQUATIONS OF MATHEMATICAL PHYSICS.

Die Partiellen Differentialgleichungen der Mathematischen Physik. Nach Riemann's Vorlesungen in vierter Auflage neu bearbeitet von HEINRICH WEBER, Professor der Mathematik an der Universität Strassburg. Braunschweig, Friedrich Vieweg und Sohn. Erster Band, 1900, xvii + 506 pp. Zweiter Band, 1901, xi + 527 pp.

RIEMANN'S lectures on the partial differential equations of mathematical physics and their application to heat conduc-

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use

tion, elasticity, and hydrodynamics were published after his death by his former student. Hattendorff. Three editions appeared, the last in 1882; and few books have proved so useful to the student of theoretical physics. The object of Riemann's lectures was twofold: first, to formulate the differential equations which are based on the results of physical experiments or hypotheses; second, to integrate these equations and explain their limitations and their application to special cases. However, since Riemann's death, the development and extension of physics has been so great, especially in our knowledge of the properties of the luminiferous ether and of the properties of moving electrical charges—ions, and the advances in mathematical analysis have been so marked, that simply a revision of Hattendorff's treatise has become impossible. It has been felt for some years that what was most needed by students of physics was not a new edition of Riemann's lectures, but rather an entirely new book with the same purposes in view and with possibly the same general title. This has been given us at last by Professor Heinrich Weber, of Strassburg. The fact that Professor Weber still retains on the title-page of his two volumes the words, "nach Riemann's Vorlesungen" is certainly a tribute of the highest respect of one great scholar for another; for to Weber, and to him alone, belongs all the credit for this most important work.

These two volumes include all the subjects—and more—treated by Riemann in the two books which were prepared for publication by Hattendorff: the one already referred to on "Partielle Differentialgleichungen," and the other on "Schwere, Electricität, und Magnetismus." In the first volume of Weber are given: (1) a discussion of those analytical theorems and methods which are useful in a study of mathematical physics, (2) a brief presentation of the principles of geometry and mechanics, (3) a treatment of the mathematical theory of electricity and magnetism. The second volume contains: (1) selections from the theory of linear differential equations, (2) heat conduction, (3) theory of elasticity, (4) electrical vibrations, (5) hydrodynamics.

Some idea of the scope and usefulness of the work may be formed from a more detailed statement of the contents. In the section devoted to analytical theorems and methods there are eight chapters, viz: Definite integrals, Fourier's theorem, Infinite series, Fourier's series. Multiple integrals, Functions of a complex variable, Differential equations, and Bessel's functions. In the second section, that entitled

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use

"Principles of geometry and mechanics," there are six most interesting chapters: Linear infinitesimal deformations, Vectors, Potentials, Illustrations of potentials, Spherical harmonics, and a brief discussion (Ueberblick) of the Principles of mechanics. The third and last section of volume I—Electricity and magnetism—has nine chapters: Electrostatics, Problems of electrostatics, Magnetism, Electro-kinetics, Electrolytic conduction, Steady electric currents, Flow of electricity in plates, Flow of electricity in space, and Electrolytic displacements.

The first section of volume II., that dealing with linear differential equations, includes four chapters: Integration by means of hypergeometrical series, Integration by means of definite integrals, Riemann's p-functions, Oscillation theorems. The portion on heat conduction has three chapters: The differential equation of heat conduction, Problems of heat conduction in one dimension, Heat conduction in a sphere. There are seven chapters given up to the theory of elasticity: General theory of elasticity, Statical problems of the theory of elasticity, Pressure on an elastic support, Motion of stretched strings, Riemann's method of integration, Vibrations of a membrane, General theory of the differential equation of a vibrating membrane. The section on electrical vibrations has three chapters: Electrical waves, Linear electrical currents, Reflection of electrical vibrations. In hydrodynamics there are six chapters: General principles, Motion of a rigid body in a fluid—hydrodynamic and mechanical portions, Discontinuous motion of fluids, Propagation of pulses in a gas, Air vibrations of finite amplitude.

Each of these chapters is brief and contains only the theorems and discussions which are necessary for the matter in hand; yet, until one reads each chapter and each paragraph, it is impossible to form an idea of the amount of information which is contained in these pages and of the enormous difficulty it must have been to condense the matter into such a small space and to select suitable illustrations. These last are, without exception, well chosen; and the references to recent literature are particularly noteworthy. Several of the chapters on electricity are specially interesting, as they deal with problems not ordinarily discussed in text-books. If it is possible to select from the many most excellent chapters of Professor Weber's book that one which seems the most noteworthy, that chosen would be the one entitled a "Brief discussion of the principles of mechanics." In this we have, in the short space of thirty-three pages,

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use

the most satisfactory deduction and discussion of Hamilton's and Lagrange's equations that is known to the reviewer. Weber refers in this chapter several times to the important memoir by Hölder in the *Göttingen Nachrichten*, 1896, on the "Principles of Hamilton and Maupertius," which is by no means as well known as it should be. It would have been, however, so simple a matter to have added a few sentences in regard to dissipative forces and concealed motions that one regrets that Professor Weber exercised such self-restraint.

There is need of only a few words of what might be called criticism; for one reads the whole book with feelings of the keenest satisfaction. There are several cases, however, where the author has not used the simplest or most direct proof of certain theorems, *e. g.*, Stokes's theorem; yet, in defense, it must be acknowledged that the proof given in each case is one which was chosen in order to illustrate some important mathematical method. Again, in the discussion of differentiation under the integral sign, it would have been better to have included the case where the limits are functions of the parameter; and many other times the reader wishes for slight extensions of the treatment given. All English speaking students are so thoroughly accustomed to calling by the name "Green's theorem" the well known volume and surface integral transformation, or the identity that follows from it, that to have the name given to any other theorem of Green's, however important, seems wrong to them. In any case, it would have been well to have alluded to the Green's theorem just referred to, even if only as a special case of Gauss's theorem. As has been stated before, the excellence of the references to current literature is most marked; and one regrets the more the absence of some brief statement of the history of certain theorems; for one feels sure that Professor Weber's remarks on the subject would be so valuable. For instance, few mathematical theorems are more useful in physics than the statement of the possibility of the expression of any vector as the sum of a potential (or lamellar) vector and a solenoidal one; and a brief sketch of the history and application of this theorem would have been most interesting. A discussion of polarized distribution might have been added with advantage, as well as a more complete treatment of the energy relations of various distributions. Again, Fourier's method of approaching the problem of heat conduction might have been referred to, and the absence of Stokes's name in speaking of hydrodynamics seems unaccountable.

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use

These remarks on what seem desirable additions to the volume of Professor Weber's are not to be regarded in the least as criticisms of the work; for there is no other book in any language which provides the student of physics with so much necessary imformation, so well selected and prepared. The only book which even attempts in part to do this is Professor Webster's treatise on "Electricity and magnetism," the first part of which is devoted to the general theorems of mathematics and mechanics. But while Webster's treatment on the whole is fuller and more satisfactory in some respects than the corresponding chapters in the volumes before us, it is by no means so extended. In the former, the physical facts and ideas are brought to the front; while in the latter emphasis is laid upon the limitations imposed by mathematical analysis.

In conclusion, the utmost satisfaction must be expressed with the paper, type, and cuts of the book. It is an unusual pleasure to have in one's hands a volume which has been rendered so attractive and intelligible by the efforts of the printer and publisher.

J. S. AMES.

NOTES.

THE closing (October) number of Volume II. of the *Transactions* of the AMERICAN MATHEMATICAL SOCIETY contains the following papers: "Geometry of a simultaneous system of two linear homogeneous differential equations of the second order," by E. J. WILCZYNSKI; "Theory of linear groups in an arbitrary field," by L. E. DICKSON; "On certain aggregates of determinant minors," by W. H. METZLER; "Ueber die Anwendung der Cauchy'schen Multiplicationsregel auf bedingt convergente oder divergente Reihen," by A. PRINGSHEIM; "Ueber den Goursat'schen Beweis des Cauchy'schen Integralsatzes," by A. PRINGSHEIM; "New proof of a theorem of Osgood's in the calculus of variations," by O. BOLZA; "On certain pairs of transcendental functions whose roots separate each other," by M. BÔCHER; "On the system of a binary cubic and a quadratic and the reduction of hyperelliptic integrals of genus two to elliptic integrals by a transformation of the fourth order," by J. H. MCDONALD; "On the theory of improper definite integrals," by E. H. MOORE; "On the convergence and character of a certain form of continued fraction," by E. B. VAN VLECK; Notes and errata, volumes 1 and 2.

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use