While many teachers may not see their way to using the book as a text during the early part of the course, it will nevertheless commend itself for collateral reading from the very beginning, and its ultimate introduction as the sole or chief text-book of the latter part of the first course and of parts of the second course is a matter that teachers of calculus will do well carefully to consider.

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HARVARD UNIVERSITY, CAMBRIDGE, MASS.,
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SHORTER NOTICES.

Éléments de la Théorie des Nombres. Par E. CAHEN, Professeur de Mathématiques spéciales au Collège Rollin. Paris, Gauthier-Villars, 1900. 8vo, viii + 403 pp. Price, 12 fr.

THE purpose of the author, as stated in his preface, is to supply the lack of a modern treatise in French on the theory of numbers. The works of Bachmann, Dirichlet-Dedekind, and Tchébyscheff are mentioned; but no reference is made to French books. But the second volume of Serret's Algèbre Supérieure treats of congruences, quadratic residues, Galois imaginaries, and the number of primes in a given interval. Of these topics, the latter two are not treated by Cahen. Again, chapters IV, V and XIV of Tannery's Leçons d'Arithmétique form a most excellent introduction to the ordinary theory of numbers, aside from quadratic forms.

Cahen treats not merely of the properties of integers, congruences and quadratic forms, but also of irrational numbers, particularly of algebraic numbers of the second degree. Fractions are introduced (on page 20) from the standpoint of pure analysis as symbols each defined by a system of two integers, called numerator and denominator. By definition

$$\frac{a}{b} \gtreqless \frac{c}{d} \quad \text{according as} \quad ad \gtreqless bc.$$

The usual results for the sum and product of two fractions are taken as the definitions of sum and product.

Irrational numbers and operators upon them are defined by means of Dedekind's *cut*.

In treating congruences of general degree with respect to a prime modulus (pages 69–77), the author leaves to the reader the proof of eight theorems on the divisibility of polynomials (mod p), analogous to theorems previously established for integers. In this matter the German texts are obviously simpler.

Two proofs are given of the law of reciprocity of quadratic residues, that by Pastor Zeller, and one of those due to Kronecker. The latter proof is said to be much more artificial than the former, at least at first view. This is not apparent, as both depend immediately upon the celebrated lemma of Gauss. This lemma is given on page 118 without mention of Gauss. In general, references are given sparingly and usually several pages after the theorems are stated.

The results on pages 131–133 are stated carelessly, the limitation "premiers" being omitted in four places. The results stated hold only for prime numbers.

Cahen employs (page 136) Tannery's modification of the Schering-Kronecker method of defining Jacobi's symbol $\left(\frac{n}{P}\right)$, P being any odd number, to equal $(-1)^\mu$, where μ is the number of negative minimum residues (mod P) of the products $n, 2n, \cdots, \frac{1}{2}(P-1)n$.

Chapter VI (pages 201–315) is devoted to binary quadratic forms and their application in the solution of an indeterminate equation of the second degree in two variables. For the form

$$f \equiv ax^2 + 2bxy + cy^2,$$

the quantity $b^2 - ac$ was called by Gauss the *determinant* of the form f, and designated by the letter D. This notation has since been employed by Bachmann, Dirichlet, Dedekind, etc. But Cahen makes its negative, the *discriminant* $ac - b^2$, play the fundamental rôle, employing for it the same designation D.

In adopting (page 208) the notation ABC for the product $(AB)C$, where A, B, C are binary linear substitutions, Cahen fails to state that $(AB)C = A(BC)$, a result very simply proved.

With Gauss and Dirichlet the form f of negative determinant is called *reduced* when $c \gtreqless a \gtreqless 2\,|b|$. Cahen adds the conditions $2b \neq -a$; $b \gtreqless 0$ if $a = c$, thereby ruling out one of the forms in each of two pairs of equivalent Gauss-reduced forms. In Klein's geometric phraseology, these added conditions indicate which boundaries are to form part of the fundamental region [Klein, Ausgewählte Kapitel der Zahlentheorie I, p. 216].

A simplification is made by Cahen by employing as roots of a form the reciprocals of the values used by Dirichlet. Then, in Cahen's notation, a substitution which transforms a form into a second transforms the roots of the second into the roots of the first form.

On page 301, sixth line from bottom, and on page 306, first line of §376, the word impropre should read propre.

The concluding pages (316–400) are devoted to notes and tables, the latter being borrowed from Tchébyscheff. There is a note on prime numbers in which are proved special cases of Dirichlet's theorem on an arithmetical progression. A note on the decomposition of numbers into prime factors shows how the problem can be solved by finding a quadratic form $x^2 + Dy^2$ which represents the given number, using the tables on pages 391–400. In the headlines to pages 397–400, $x^2 + \Delta y^2$ should read $x^2 - \Delta y^2$. There is a note on the calculation of, primitive roots of prime numbers and tables (pages 375–390) giving the primitive roots and indices for all prime numbers < 200. The final note gives Gauss's theory of complex integers $a + bi$, their geometrical representation being emphasized.

Cahen's book will prove of special interest to those students who desire numerous illustrative examples and numerical applications of the general theorems. The amount of detail, which has added considerably to the size of the book, can not fail to allure the reader to the fascinations of number theory.

L. E. DICKSON.

Essays on the Theory of Numbers: I. *Continuity and Irrational Numbers.* II. *The Nature and Meaning of Numbers.* By RICHARD DEDEKIND. Authorized translation by W. W. Beman. Chicago, The Open Court Publishing Company, 1901. 115 pages.

THE essays of Dedekind, Stetigkeit und irrationale Zahlen (Braunschweig, 1872), and Was sind und was sollen die Zahlen? (Braunschweig, 1888) have already become classics in the literature of mathematics. In giving a fairly literal translation of them, Professor Beman performs a service for which one must feel grateful, especially as one needs whatever advantage one's own language gives in attempting to master the abstruse second essay.

The word Abbildung is translated transformation (page