although this property of the surface curve is very often referred to and used by modern writers upon these subjects.

This last section of the book is very much compressed; but the reader will find there at least the fundamental theorems necessary for a more extensive study of surface curves.

Very few misprints which could mislead the reader have been observed in the volume. One which is, perhaps, worth mentioning is the use of the word "Flächen" for the word "Ebenen" in theorem 7, page 26.

The figures are drawn with care, and are numerous, which is a very helpful feature of the work for a beginner. The references, in footnotes, to other treatises and to original sources, are full and instructive. Tables containing the principal formulas established are appended at the end, as is also an excellent index.

J. M. PAGE.

UNIVERSITY OF VIRGINIA,
April, 1902.

SOME RECENT BOOKS ON MECHANICS.

The Principles of Mechanics: An Elementary Exposition for Students of Physics. By FREDERICK SLATE. Part I. New York, The Macmillan Co., 1900. Crown 8vo., vii + 299 pp.

A Treatise on Elementary Dynamics. By H. A. ROBERTS. London, Macmillan & Co., 1900. 16mo., xi + 258 pp.

Die Dynamik der Systeme starrer Körper von E. J. ROUTH. Autorisierte deutsche Ausgabe von ADOLF SCHEPP mit einem Vorwort von FELIX KLEIN. Zwei Bände. B. G. Teubner, Leipzig, 1898–1900.

DURING the past decade many works which treat mechanics from one standpoint or another have been published. The list of authors includes Appell, Boltzmann, Föppl, Gray, Hertz, Routh, Volkmann, and others perhaps less well known. The different authors differ greatly both in their aims and in their methods of presenting the subject; but on the whole they show a tendency toward something which is to a considerable extent a recent development—to the careful consideration of the underlying principles of mechanics. The amount of attention given to this phase of

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the subject varies from one extreme in the work of Hertz, in which every step from the first to the last is built up with geometric logic by means of more or less artificial devices and points of view, to the other extreme in those works, especially by English authors, in which force and mass are taken as practically innate, *à priori*, ideas and are left with no further refinements of conception than the naïve intuition of the student may supply. In one case the careful examination of the concepts involved usually leads to the result that the fundamental dynamical equation

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

is merely the definition of either force or mass. In the other case, where force and mass are taken for granted, this equation becomes a physical law or mathematical axiom founded, like Euclid's parallel axiom, upon universal experience.

Notwithstanding that this latter point of view still finds able advocates there is undoubtedly a general trend toward a more thorough discussion of the foundations of mechanics. The laws of Newton, as stated by him and as used for two centuries, no longer satisfy us. This lack of satisfaction is but one of the many similar manifestations of the present state of mathematical instruction and mathematical science. We are no longer content to bear with superficially clear statements which seldom if ever lead into actual error—nor does it suffice to start with inaccurate statements and, as we advance, to modify them so as to bring them into accord with our wider vision and our more stringent requirements. No. We must from the beginning bring up ourselves and our pupils on not only the truth but the whole truth. How soon the recent researches of Hilbert and others on the foundations of geometry must take their place in elementary text-books on plane and solid geometry cannot be said. But that is purely a matter of time unless some reactionary tendency sets in—and there are some who think it already beginning to set in.

At present, however, the treatises and text-books on mechanics are filled with a deal of half scientific, half metaphysical, and generally quite crude discussion of the underlying principles of mechanics. The "principles of mechanics" as denoting the "elements of mechanics" and as denoting the "scientific foundations of mechanics" are becoming synonymous. Thus the task of the reviewer is rendered triply hard, inasmuch as not only must he sift out the good from the bad, but must do this in each of the

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various aspects, mathematical, physical, metaphysical, which the author touches upon. In the following series of reviews an attempt will be made to keep each of these aspects separate from the others and still give a connected idea of each work under discussion.

While authors in England, Germany and France have been flooding the market with expositions of all the various branches of mechanics, instructors in our own universities and schools other than technological have not written upon the subject. Works on the mechanics of engineering, graphical statics, strength of materials, and so forth we have in great number; but, with the exception of Professor Ziwet's book, scarcely a work upon theoretical mechanics has been published in this country. What the type of our instruction in theoretical mechanics shall be, whether we shall lean toward the English, the German, or the French, is a question which is not yet settled. It is, however, a question of moment and must inspire at present an exceptional interest in such treatments of mechanics as may come from American authors. But Professor Slate's book needs not rely upon the fact that it is American to awaken interest. It contains numerous novel features and many improvements which will recommend it strongly to teachers and students.

At the commencement of the preface, Professor Slate states that the ideas which have guided the selection and presentation of the material are three: first, to select the subject matter with close reference to the needs of college students; second, to bring the instruction into adjustment with the actual state of their training; and third, to aim continually at treating mechanics as a system of organized thought having a clearly recognizable cultural value. Three better aims could not be chosen. Moreover the students for whom the book is intended are to have had a working knowledge of the calculus and a good ground work of experimental physics. This preparation is ideal, but perhaps more that can fairly be expected of students beginning mechanics. Certain it is, however, that to attempt to teach mechanics as mechanics before the student has had calculus is next to useless and equally certain it is that some previous knowledge of experimental or descriptive physics is a great aid though by no means an absolute necessity in acquiring true mechanical intuition. The preparation Professor Slate expects of his students and the aims which he sets himself are alike harbingers of a merited success for his text-book.

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The fundamental idea of a vector is emphasized in connection with velocity and acceleration to the extent of introducing a distinctive notation—a circumflex accent—for vectors. Thus v is the scalar speed ; $\widehat{v}$, the vector velocity. The notation is unfortunately clumsy at best and would become excessively so if used consistently throughout the book. Why could not the established custom of using heavier faced type for vectors be followed? The intrinsic value of the emphasis laid upon the idea of a vector nevertheless more than atones for the awkwardness of the accidental notation. In later chapters moments and work are mentioned as the vector and scalar products of forces and distances, although no subsequent use is made of this fact.

The first two chapters, fifty-eight pages, deal with kinematics. The treatment is by no means conventional and is noteworthy in a number of points. From the start there is a clearly defined distinction between the assumed fixed reference system and the coördinate system which may be moving relative to it. Many theorems are proved by geometric or partially geometric instead of by purely analytic methods. This is particularly true of the discussion of instantaneous centers and of accelerations. Perhaps a treatment slightly more analytic would not be amiss. Possibly too the author would do well to give both treatments as he does in several instances. For grasping the meaning of a theorem a geometric proof is better ; but for specific applications the analysis is generally necessary. With no further aid than that given in the text a student would have difficulty in determining the space and body centrodes for any given case of motion. It is interesting to note that in the attempt to simplify the difficult subject of acceleration the author begins with the definition of acceleration parallel to a line or component acceleration and afterwards combines the components into the resultant. He then points out the fundamental fact that this resultant acceleration is really nothing but rate of change of velocity regarded as a vector. From a careful perusal of the text it does not appear that this method of treatment succeeds in simplifying matters to any great extent.

The third chapter, entitled Mass and Force, is decidedly the poorest in the book. It can offer few apologies for itself on any score. An attempt is made to discuss mass and force from the "modern" standpoint. The discussion for the most part is extremely crude—a jumble of empirical physics and partially accurate logic—neither clear nor satisfying to logician or practical physicist.

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In the first place the author seems to be thoroughly imbued with the idea that kinematics and dynamics are two sciences which differ not in degree alone but in kind; that kinematics is a branch of pure mathematics, but dynamics a branch of physics founded, unlike the former, upon experiment.* Why time and space, equally with inertia, matter, and force, are not derived from universal experience is difficult to see. In fact, speaking from the standpoint of psychology, would it not be true that force, qualitatively at least, earlier than either space or time makes a definite impression upon the individual and does not force always appear more real than either? Perhaps force enters more critically into the struggle for existence. Passing from the qualitative to the quantitative, is it not clear that the so-called parallel axiom of Euclid must be derived in the final test from experience? Experiments by parallactic measurements upon fixed stars have been suggested seriously for making the test. Mass and force may be perceived and defined independently of the truth of this axiom. Non-euclidean mechanics is as possible as non-euclidean geometry. Surely it is not well to lay so great stress on the inherent difference in empiricism between mechanics and geometry. Professor Slate would probably not have done so, had Newton been a contemporary of Euclid.

In the second place Professor Slate, after drawing such a distinct line of demarcation, proceeds with a strange ironical perverseness to obliterate it. One would think this line of demarcation but the natural precursor of the old Newtonian treatment, a justification for assuming mass and force as innate ideas and for stating the postulate that force equals mass times acceleration. But, no!, the following definitions are laid down.†

> 1° Bodies are said to manifest *inertia* in proportion as it is more difficult to set them in motion.
>
> 2° *Force* is said to be acting whenever the physical conditions are such that velocity is changed in magnitude or direction.
>
> 3° Two sets of physical conditions are considered as calling into play *forces of equal magnitude* if they are capable of producing in a given body accelerations that are numerically equal. The direction of the force is that of the acceleration which it produces.

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* See pp. 2–3 and p. 59.

† For convenience the definitions are arranged more formally than in the text.

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4° The term *mass* is in general use to denote the measured inertia of bodies. The *mass ratio* of two bodies is defined as the inverse ratio of the accelerations producible in them by equal forces.

5° Since, when accelerations are equal, the greater force must be applied to the greater mass and since "it is also accepted in our conception of force" (How? By physics or metaphysics?) "that it increases with the acceleration exhibited in the same body," then "in conformity with these two aspects of the relation—(1) when masses are different and accelerations equal—(2) when accelerations are different and masses equal—force is in fact measured as *proportional* to mass and acceleration conjointly; *i. e.*, to the product of mass and acceleration."

Just what this fifth statement is intended to mean or what standpoint it is intended to represent is nearly a matter for pure conjecture. Professor Slate's none too lucid style becomes unfortunately obscure at this critical point. Then, too, apart from the fact that the first definition naïvely implies an innate knowledge of force in the word *difficult*, it is clear that empiricism plays no rôle here. These definitions are as purely mathematical and free from experiment as if geometric. There is no necessity for previously laying stress on the physical side. When it comes to applying theoretical results to practical experience, a few words concerning the assumed applicability may be in place but not before. A careful examination of Boltzmann's Vorlesungen über die Principe der Mechanik, Leipzig, 1897, will show what may be done in the line of accurate definition in mechanics. That which appears in the book under review is so unfinished and inaccurate as to be valuable in no other way than to call attention to the difficulties involved and give the student a few well chosen illustrations to think over if he finds leisure.

Finally, even if the treatment which Professor Slate has attempted had been carried out in a manner beyond adverse criticism, would his attempting it be unquestionable? Does any such treatment fall in with those three aims he set himself at the start? Not with the first. Not with the second. Not necessarily with the third. Few of the students for whom the book is intended would understand such definitions as given, fewer could appreciate them, fewer still could use them. It would probably have been better, all things considered far better, to have stated the laws of motion in a more customary form; to have explained, discussed, and

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illustrated them with that aptness of which Professor Slate is evidently a master; possibly to have pointed out at the end those lines along which modern mathematicians are trying to perfect geometry and mechanics. The students would then have a more distinct conception of the ideas involved in mechanics a greater ability to make practical use of them, and no less appreciation of "improvements" recently introduced or yet to be introduced. Fortunately, however, there is little need of worrying about the students. They will survive this chapter and disregard it. They invariably feel quite sure that they know what mass and force really are.

Passing on to the remaining chapters of the book, there is little save unconditioned praise to be said. Professor Slate has done that which has long needed to be done. He has shown that the principles of elementary analytical mechanics are few and simple, the applications many and various. Students who use his book will be under the constant necessity of thinking for themselves. It will be useless for them to look for a formula by which each assigned problem may be solved. The unthinking, formula hunting undergraduate will doubtless be perplexed and at a loss at first; but elementary mechanics affords the best of opportunities for learning to think.

In Chapter IV the analytic discussion of the motion of the center of gravity of a system or of a particle, which is the same thing, is reduced to the three fundamental equations—the force equation, the momentum equation, the work equation. In Chapter V these results are extended to rotation about a fixed axis and to uniplanar motion in the most natural and simple manner, merely by treating a rigid body as made up of an infinite number of elements dm to each of which the foregoing equations apply. Later in the chapter, motion and systems of forces in three dimensions are touched upon. Chapter VI deals with harmonic motions and pendulums. The discussion is elaborate, including damped and undamped vibrations, simple, compound, torsion, and bifilar pendulums. An excellent collection of thought existing problems concludes the chapter. Chapter VII treats the law of the inverse square, potential, and orbits described under the attraction due to gravitation. Chapter VIII exhibits the application of the three fundamental equations to a vast number of dissimilar problems. Chapter IX takes up the theory of dimensions together with the C.G.S. and weight system of units. The concluding Chapter X discusses moments of inertia and

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centers of gravity. One hundred and sixty-three exercises which are well chosen if not a trifle too difficult and an extensive index close the volume.

In conclusion we would say that with the exception of the third chapter the book carries out the three aims of its author to our entire satisfaction. We would recommend it most heartily to all who wish to think themselves into the heart of mechanics with as little purely formal analysis as possible. Teachers will probably find that as they teach the book they will have to assign numerous problems easier than those inserted at the end of the chapters. This, however, is a small matter. We are glad to notice that Professor Slate is planning a second part to his work which will treat mechanics in three dimensions. We wish him the best of success, and hope that he will not delay the publication of this sequel.

Mr. Roberts's Treatise on Elementary Dynamics is one of the multitudinous English text-books which attempt to teach mechanics without the differential and integral calculus, generally without analytical geometry, often even without trigonometry. In this case a knowledge of trigonometry and the elements of analytical geometry is assumed. This book is therefore as advanced as any of its kind can be. In numerous places the notation of the calculus is also given for the benefit of those students who may be beginning that subject. The treatment is excellent from first to last. Addition and subtraction of vectors, (vector) velocities, (vector) accelerations, angular velocities, the laws of motion including a short statement of the "modern" point of view, motion under gravity and other simple accelerations such as that afforded by elastic strings, work and energy, potential energy and conservative systems, dimensions, direct and oblique impact, projectiles, cycloidal and simple pendulums are among the subjects discussed.

The book may be of great use in England; but to what class of pupils it may be of service in this country does not appear. We seldom teach mechanics between analytical geometry and calculus. Moreover this treatise looks too difficult, too much extended, for those who have not had calculus. It would, however, be very serviceable to such as were studying elementary mechanics by means of the calculus, but found that the none too familiar analytic methods created so much confusion and distraction as to render the ideas involved in mechanics unreal and vague.

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Even students of considerable advancement in mechanics might do well to read this little book for the sake of the perspective which they might thus acquire.

Dr. Routh's treatises upon the various branches of mechanics, statics, dynamics of a particle, stability of motion, and in particular rigid dynamics, are so well known as to need no notice. It is therefore not surprising that Teubner, who is ever ready to publish a German translation of the best scientific literature of all nations, should now print this edition of Dr. Routh's Rigid Dynamics. As Professor Klein points out in his preface, this work is wholly different from any which has previously been available in German. To teach mechanics even in the most advanced portions from the standpoint of solving problems is furthest from the German method. We remember one instance in which a German reviewer recently said of a work under review that it was remarkable for its numerous examples. A count showed not more than forty in about five hundred pages of text. How Dr. Routh's work impresses this reviewer may be difficult to imagine. These English books are, however, an extreme. Placing such emphasis on the solution of problems is a result of the system of examinations at the colleges. The student is too apt to lose his perspective and to forget what the *theory* of mechanics is. In this the Germans are far ahead. We daresay that our English publishers would render no less service to our own scientific literature by translating the best German presentations of mechanics into English than Teubner has now rendered to German scientific literature by his translation of Dr. Routh's classic treatise on rigid dynamics.

EDWIN BIDWELL WILSON.

YALE UNIVERSITY,
March 30, 1902.

THE GALOIS THEORY IN BURNSIDE AND PANTON'S THEORY OF EQUATIONS.

ONE of the most welcome additions to Burnside and Panton's "Theory of Equations" is the appearance in the new edition (the fourth) of a chapter devoted to the theory of substitutions and the theory of equations from the Galois standpoint. The British interest in the methods of Galois never has been very deep and about all the national literature is comprised in the last two pages of Cayley's article "Equation" in the Encyclopedia Britannica and four or

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