SHORTER NOTICES.

Théorie élémentaire des Séries. Par MAURICE GODEFROY, Bibliothécaire de la Faculté des Sciences de Marseille, avec une préface de L. SAUVAGE. Paris, Gauthier-Villars, 1903.

La Fonction Gamma. Par M. GODEFROY. Paris, Gauthier-Villars, 1902.

THE text-book literature of European mathematics has, since Euler's day, been blessed with numerous and most excellent books treating the elementary convergent processes from the algebraic standpoint. In such books we find a systematic application of the algorithm of inequalities to such questions as the convergence of infinite series, products and continued fractions and, after Cauchy's day, uniform convergence and termwise differentiation and integration of series.

These books which used to go by the generic title of books on algebraic analysis * afford an admirable introduction to a *theoretic* course in the calculus, in that they present its basal ideas in a form at once most concrete and comprehensible to the beginner.

It has been the custom of continental text-book writers on the calculus in recent years to incorporate such of this matter as relates to the more essential notions concerning infinite series in the initial chapters of the differential calculus. This course, while it has no doubt been dictated by necessity, seems from some points of view unfortunate.

It has thus come about that since Cauchy's Analyse algébrique few books of similar purpose have appeared in French and of these the only one that need here be mentioned, Tannery's Théorie des fonctions d'une variable, while admirable in every way and destined not soon to be supplanted, is perhaps too extensive for the beginner.

M. Godefroy has given us a book on the elementary theory of series that has much to recommend it. The exposition has the traditional French clearness and evinces a pedagogic insight that one would only expect in a writer accustomed to actual instruction.

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* This name, apparently coined by Cauchy, is still used in Italy.

The book begins with a short (too short) account of irrational numbers defined by a "section" and establishes certain fundamental theorems due to Cauchy concerning the equality of the limits

$$\lim_{n=\infty}(x_{n+1}-x_n)=\lim_{n=\infty}(x_n/n), \quad \lim_{n=\infty}(x_{n+1}/x_n)=\lim_{n=\infty}\sqrt[n]{x_n}, \text{ etc.}$$

After a section on continuous functions of a single variable, series with positive terms are taken up and the powerful and elegant test of Kummer is established.

This remarkable criterion, which contains as special cases the geometric test, Raabe's test and the test of DeMorgan-Bertrand, admits of a very simple proof; * it necessitates, however, the construction of divergent scales of positive numbers. Our author, however, gives an independent deduction of the geometric test and after establishing the familiar theorems concerning conditionally convergent series and sundry theorems of Cauchy and Mertens concerning double series proceeds to the consideration of series whose terms involve a variable parameter.

The regional property of uniform convergence is defined and illustrated, unfortunately without any graphs, and Weierstrass's m-test is established. The rest of the book is devoted to power series and the more important functions defined by them.

The statement of Abel's theorem concerning the interval of convergence of a power series is open to the objection that our author fails to distinguish — although the distinction is made sharply enough in an earlier part of the book — between a suite of numbers that are always finite and one that has an upper and lower boundary.

The demonstration of the term by term differentiability of a power series, apparently due to Biehler † is very simple and has the merit that it goes back to first principles; this result might however have been more readily deduced from the continuity of such series.

Noteworthy features of the book are the proof of the existence theorem for homogeneous linear differential equations with analytic coefficients and the proof of the transcendence of the natural base e. The collection of excellent examples that fol-

* Cf. Cesàro: *Nouvelles Annales*, vol. 7, 3d series, p. 406, the proof in the text is however sufficiently simple.

† *Nouv. Annal.*, vol. 7, 3d series, p. 200.

lows each chapter will be found useful and the bibliography appended seems adequate.

The last chapter of the book is devoted to the gamma function and the auxiliary functions of Prym. It is condensed from La function gamma: Theorie, Histoire, Bibliographie of our author, and although the available analytic processes are necessarily restricted, the proofs are elegant and compact.

M. B. PORTER.

Höhere Analysis für Ingenieure. Von Dr. JOHN PERRY. Autorisierte deutsche Bearbeitung von Dr. ROBERT FRICKE und FRITZ SÜCHTING. Leipzig, B. G. Teubner, 1903. 8vo., viii + 423 pp.

IT is interesting to note in what numbers the last few years have produced treatises on the differential and integral calculus which have for their aim the fulfilment of the needs of some special class of students. For instance, there are for students of physics the elaborate treatise of Boussinesq and the smaller book by H. A. Lorentz, for chemists the work of Nernst and Schoenfliess recently translated into English by Young and Linebarger, for students of political economy a small primer by Irving Fisher, and for engineers the work of Perry. The question naturally arises whether so much subdivision in the study of calculus is necessary. It will prove a matter of serious inconvenience if each specialist must have a special course and a special book to suit his needs. There can be little doubt that the present tendency to this subdivision is due partly to the fact that mathematicians are apt to wish too selfishly to make their elementary courses strictly mathematical instead of practical, and in this respect we hope they will mend. A great part of the difficulty, however, is due to the inertness of the students of special branches, who wish to learn so much and only so much of calculus as appears to them necessary for their immediate needs. The short-sightedness of this attitude renders it dangerous. To-day many a chemist or economist, whose elementary education was finished a decade ago, complains of experiencing inconveniences because he did not study calculus. Perhaps before twenty-five years are past those who now are trying to learn as little of it as possible will be wishing that they had not tried to economize so much. In the earlier years of instruction time is not so precious as later, symbolic processes fix themselves more readily upon the mind which