THE DECEMBER MEETING OF THE SAN FRANCISCO SECTION.

THE fourth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held on Saturday, December 19, 1903, at the University of California. The following fourteen members were present:

Professor R. E. Allardice, Professor H. F. Blichfeldt, Professor G. C. Edwards, Professor R. L. Green, Professor M. W. Haskell, Dr. D. N. Lehmer, Professor A. O. Leuschner, Professor G. A. Miller, Dr. H. C. Moreno, Professor C. A. Noble, Dr. T. M. Putnam, Professor Irving Stringham, Dr. S. D. Townley, Mr. A. W. Whitney.

A morning and an afternoon session were held, Professor Stringham acting as chairman at both sessions. During the morning session the following officers were elected for the ensuing year: Professor R. E. Allardice, chairman; Professor G. A. Miller, secretary; Professors M. W. Haskell, Irving Stringham and G. A. Miller, programme committee. In the by-law relating to the time of meetings the dates of the regular meetings were changed from May and December to February and September. This change is to go into effect after the next regular meeting, which will be held in May at Stanford University.

The following papers were read at this meeting:

(1) Dr. E. M. BLAKE: "Exhibition of models of polyhedra bounded by regular polygons."

(2) Professor M. W. HASKELL: "Brianchon hexagons in space."

(3) Professor R. E. ALLARDICE: "On the locus of the foci of a system of similar conics through three points."

(4) Professor IRVING STRINGHAM: "On curvature in absolute space."

(5) Professor H. F. BLICHFELDT: "On the order of linear homogeneous groups, II."

(6) Professor E. J. WILCZYNSKI: "Studies in the general theory of surfaces."

(7) Professor E. J. WILCZYNSKI: "A fundamental theorem in the theory of ruled surfaces."

(8) Professor G. A. Miller: "On the roots of group operators."

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(9) Dr. D. N. LEHMER: "On the Jacobian curve of three quadric surfaces and a certain ruled surface connected with it."

(10) Dr. D. N. LEHMER: "On a new method of finding factors of numbers."

(11) Mr. W. A. MANNING: "On the primitive groups of classes six and eight."

(12) Professor M. W. HASKELL: "Approximations to the square root of a positive number."

Professor Haskell exhibited Dr. Blake's models, Professor Wilczynski's papers were read by Dr. Lehmer, and Mr. Manning's paper was read by the secretary. The other papers were presented by their authors. Abstracts are given below. The abstracts are numbered to correspond with the titles in the list above.

1. Dr. Blake has prepared a number of models of convex irreducible polyhedra in which all the faces are regular polygons, and developed some general theorems relating to the construction of such solids. He has found all the convex irreducible polyhedra bounded by regular triangles and also all of those having only trihedral angles (cf. Brüchner : Vielecke und Vielflache, page 83).

2. Professor Haskell developed a number of theorems on the tetrahedra determined by a Brianchon hexagon in space, and the three-fold infinity of conicoids through the vertices of such a hexagon, together with the following simple construction for the polar plane of any point P with reference to a given quadric :

Through P draw any three non-coplanar lines meeting the quadric in A_1 and A_2, B_1 and B_2, C_1 and C_2 respectively. The pairs of planes $A_1B_1C_1$ and $A_2B_2C_2$, $A_1B_2C_2$ and $A_2B_1C_1$, $A_2B_1C_2$ and $A_1B_2C_1$, $A_2B_2C_1$ and $A_1B_1C_2$ meet in lines lying in the polar plane of P, which is therefore determined by any two of these lines.

3. Steiner proposed the problem of finding the loci and the envelopes of certain points and lines associated with a system of conics passing through three fixed points. A solution of these problems is given by Schoute in Darboux's *Bulletin* by means of Chasles's method of characteristics ; but only the

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degree of each locus and the class of each envelope are stated, together with the nature of the singularities; and in each case the degree or class is given to be twice as great as it ought to be, while the statements with respect to the singularities are correspondingly inaccurate. Professor Allardice had previously shown, in papers in the *Annals of Mathematics*, volume 3, page 154, and in the *Transactions*, volume 4, page 103, that each axis and each asymptote envelopes a three-cusped hypocycloid; and in the present paper he shows that the locus of the foci is a bicircular sextic, having the three fixed points as double foci. The singularities are determined, and some typical forms of the curve are traced. For a parabola, the sextic degenerates into the straight line at infinity, and a circular quintic.

4. Professor Stringham applied the well known geometric method of determining the radius of curvature of curves in non-euclidean space. The homogeneous coördinates of points in space being represented as functions of arc in the form $w(s)$, $x(s)$, $y(s)$, $z(s)$, the expression for curvature is developed in the form

$$1/\sin_\kappa^2 \rho = w''^2 + x''^2 + y''^2 + z''^2,$$

where w'', x'', y'', z'' are the second derivatives with respect to s and $\sin_\kappa \rho$ is defined by

$$\sin_\kappa \rho \equiv \tfrac{1}{2}\kappa\,(e^{\rho/\kappa} - e^{-\rho/\kappa}).$$

5. This paper is a continuation of the article published in the *Transactions*, volume 4, page 387, in which Professor Blichfeldt proved that if the order of a linear homogenous group G of substitutions of determinant 1, primitive and on n variables, is divisible by the prime p, then is $p \leqq (n-1)(2n+1)$. In the present paper, certain theorems are given from which it appears that, if p^N is a factor of the order of G, then is N certainly not greater than

$$\frac{n}{p-1} + (n-1)\left(1 + \frac{\log n^2k}{\log p}\right),$$

where k is the largest term in the expansion of $(1+1)^n$.

Professor Blichfeldt stated also the following theorem: The number of independent parameters in a primitive continuous group in n variables is less than a fixed number λ which depends only upon n.

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6. In a previous paper, Professor Wilczynski has already considered the congruence made up of all of the generators of the first kind on the osculating hyperboloids of a ruled surface S. In the present paper these considerations are continued. If any other ruled surface S' of this congruence Γ be considered, its osculating hyperboloid H' and that of S have a straight line in common, namely the generator g' of S'. The two hyperboloids must therefore have besides a twisted cubic in common. A large part of the paper is devoted to this cubic, to the null system which it defines, and to the surface made up of these cubics, one cubic corresponding to each generator of S. The osculating linear complex of a ruled surface is also defined and studied, leading to generalizations of some well known theorems of Lie and Cremona. The relations of this complex to the twisted cubic lead to some important results.

The paper further contains the interpretation of certain invariants which present themselves during the discussion. Especially important is the notion of the principal surface of the congruence Γ, which permits the interpretation of the seminvariant condition $\theta_4 =$ const., a result which the author had long looked for in vain, and which is necessary for a complete understanding of the whole theory. The general case only, is completely treated in this paper, *i. e.*, the case when the flecnode curve intersects every generator in two distinct points. The case of coincidence offers some further difficulties not yet overcome, which the author hopes to treat in a future paper. The paper will be published in the *Transactions*.

7. In the general theory of surfaces the theorem is fundamental that a surface is determined uniquely when the six coefficients of the two fundamental quadratic forms are given arbitrarily, subject, of course, to the relations which must exist between them. The following theorem plays the same part in the general projective theory of ruled surfaces. If the four invariants θ_4, θ_6, θ_9, θ_{10} are given as arbitrary functions of x, they determine a ruled surface uniquely, except for projective transformations. If in the second place one determines a second surface by means of the invariants $\bar{\theta}_4$, $\bar{\theta}_6$, $\bar{\theta}_9$, $\bar{\theta}_{10}$, where

$$\bar{\theta}_4 = \theta_4, \quad \bar{\theta}_6 = \theta_6, \quad \bar{\theta}_9 = -\theta_9, \quad \bar{\theta}_{10} = \theta_{10},$$

this second surface is dualistic to the first. The ruled surface is not determined uniquely when θ_4 or θ_{10} vanishes.

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The following theorem on self-dual surfaces follows from this. If a ruled surface is self-dual, it must belong to a linear complex. If however this complex is special, the surface must belong to still another linear complex, *i. e.*, it must have two straight line directrices, which may or may not coincide. All such surfaces are self-dual.

This paper has been published in the *Mathematische Annalen*, volume 58, page 249.

8. Let G represent any group of finite order g. By raising each operator of G to a power prime to g we obtain each operator once and only once. That is, if s represents any operator of G and if m is prime to g, there is one and only one operator s_1 of G such that $s_1^m = s$. In accord with the language of algebra we may say that s_1 is an mth root of s. Hence every operator of G has one and only one mth root under G whenever m is prime to g.

When m is not prime to g there is at least one operator in G (the identity) which has more than one mth root. Since the number of mth roots is always equal to g it follows that G must contain at least one operator which is not an mth root whenever the highest common factor d of m and g exceeds unity. In this case, each of the operators of G whose order is prime to m will have at least one mth root and the mth roots of any operator are identical with its dth roots. Professor Miller developed the theory of roots with respect to abelian groups and those whose order is a power of a prime. The paper will be offered to the *Quarterly Journal* for publication.

9. Steiner has indicated (*Crelle*, volume 53, page 133–141) a simple transformation by which a plane goes into a cubic surface. With a given point he associates the intersection of its polar planes with respect to three quadrics. This furnishes three projective point systems — the usual apparatus for the synthetic theory of the cubic surface. The transformation is in itself interesting in other directions. It leads naturally and simply to the jacobian curve of three quadrics (Salmon, Geometry of three dimensions, page 185), together with a ruled surface of degree eight formed of all lines which meet this curve three times. In the present paper Dr. Lehmer gives a brief development of the theory of this curve and this surface.

10. Lagrange has shown that if the equation

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$$x^2 - Ny^2 = \pm Q$$

be resoluble in integers, Q being less than $\sqrt{N}$, then Q will be found among the denominators of the complete quotients in the expansion of $\sqrt{N}$ in a continued fraction. Dr. Lehmer makes this theorem the basis of a method of finding the factors of numbers. The spirit of the method will be seen from the following theorem: If the number N is the product of two factors p and q, which differ by less than $2\sqrt[4]{N}$, then $[(p - q)/2]^3$ will occur as a denominator in the complete quotients obtained in expanding $\sqrt{N}$ in a continued fraction. Moreover, this denominator will appear before the numerator of the convergent to $\sqrt{N}$ becomes equal to N.

11. In the *Comptes Rendus* of 1872 Jordan reported his determination of the primitive groups of class less than 14. He has never published his proofs and the lists given are not exact. Mr. Manning has studied classes 6 and 8 and reports 14 primitive groups of class 6 and 18 of class 8. Use is made of the theorem that a doubly transitive group cannot have an imprimitive invariant subgroup unless its degree is a power of a prime number.

12. Professor Haskell's paper is mainly historical, analyzing by modern methods some ancient modes of approximation to square roots, as described by Theon of Smyrna, Luca Paciuoli and others, and pointing out their usefulness in introducing the theory of irrational numbers and the idea of a limit.

G. A. MILLER,
Secretary.

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