THE APRIL MEETING OF THE SAN FRANCISCO SECTION.

THE fifth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held at Stanford University, on Saturday, April 30, 1904. The following members were present:

Professor R. E. Allardice, Dr. E. M. Blake, Professor H. F. Blichfeldt, Professor R. L. Green, Professor M. W. Haskell, Professor L. M. Hoskins, Dr. D. N. Lehmer, Mr. W. A. Manning, Professor G. A. Miller, Professor H. C. Moreno, Professor C. A. Noble, Professor Irving Stringham.

The attendance also included a number of teachers of mathematics who are not members of the Society. A morning and an afternoon session were held, Professor Allardice acting as chairman at both sessions. It was decided to hold the next meeting at the University of California on October 1, 1904.

The following papers were read at this meeting:

(1) Professor M. W. HASKELL: "The construction of conics satisfying given conditions."

(2) Professor H. C. MORENO: "On a class of ruled loci."

(3) Dr. D. N. LEHMER, "On a cylinder the intersection of which with a sphere will develop into an ellipse."

(4) Professor R. E. ALLARDICE: "On the envelope of the directrices of a system of similar conics through three points."

(5) Mr. A. W. WHITNEY: "The application of actuarial methods to fire insurance."

(6) Professor IRVING STRINGHAM: "Analytical treatment of certain metric relations in the non-euclidean plane."

(7) Professor G. A. MILLER: "Addition to a theorem due to Frobenius."

(8) Professor H. F. BLICHFELDT: "On the primitive collineation groups in four variables."

(9) Professor H. F. BLICHFELDT: "A theorem concerning the invariants of linear homogeneous groups, with some applications to substitution groups."

(10) Professor M. W. HASKELL: "The construction of a twisted cubic from six points."

(11) Professor M. W. HASKELL: "Triangles in perspective and the collineations derived therefrom."

Mr. Whitney's paper was read by Professor Stringham. All the other papers were read by their authors. Abstracts are given below. The abstracts are numbered to correspond to the titles in the list above.

1. The main feature of Professor Haskell's paper is the relation between the various problems in the determination of conics and the involutory quadratic transformation. The author finds that the points of contact of the two conics passing through four given points and tangent to a given line are converted into each other by the involutory quadratic transformation of which the four given points are the self-corresponding points, the diagonal triangle of the complete quadrangle of these four points being the fundamental triangle of the transformation.

There are four conics passing through two points and tangent to three given lines, and they will be fully determined if the poles of their common chord are given. It is shown that these poles are the self-corresponding points of the involutory quadratic transformation interchanging the two given points, the fundamental triangle being the triangle of the three given tangents.

2. Professor Moreno's paper is devoted to an extension of a theorem due to Salmon (Geometry of three dimensions, 1882, page 429). The problem as extended is to find the locus of all the lines that meet n $(n-2)$-spreads in space. The simplest and most interesting case presents itself when the director spreads are flat. In this case the degree of the locus is $n-1$; $(n-1)!$ different generators go through any one point on this locus and these generators belong to different systems. There are, in general, $n(n-1)$ $(n-2)$-flats on this locus. Exceptions occur for $n=3$. The locus when $n=4$ has been studied by Segre. In case the director spreads are of orders $m_1, m_2, \cdots, m_n$ respectively, the orders of the loci are, in general, $(n-1)m_1 m_2 \cdots m_n$, and the spread of order m_1 is generally of multiplicity $m_2 m_3 \cdots m_n$.

3. In his Mathematical Recreations published in 1624 Van Etten (Jean Leurechon) describes a method of drawing an

"oval" with a pair of compasses, leaving the angle of the instrument unaltered. His scheme is to stretch the drawing paper around a circular cylinder. The resulting curve is not an ellipse, the equation being $y^2 + 4b^2 \sin^2 (x/2b) = a^2$, where b is the radius of the cylinder and a the spread of the compasses. The curve is clearly the development of the intersection of a sphere with a cylinder, the center of the sphere being on the cylinder.

The question suggests itself to find the cylinder such that its curve of intersection with a sphere, the center of which is on the cylinder, will develop into an ellipse. Dr. Lehmer shows that a right section of the cylinder is a logarithmic spiral. The solution indicates that if an elliptical hole be made in a sheet of paper and the sheet wrapped about a sphere the radius of which equals the minor axis of the ellipse, the sheet wraps up into a cylinder whose right section is a logarithmic spiral.

4. Mr. Whitney's paper relates to the construction of fire insurance tables showing the premiums for various ratios of insurance to value. The derivation of such tables for several classes of risks has been published in the *Proceedings* of the twenty-eighth annual meeting of the Fire underwriters' association of the Pacific.

5. In this paper Professor Allardice continues the discussion of a series of problems, suggested by Steiner. An explicit equation for the envelope of the directrices is obtained in a comparatively simple form. The envelope is a curve of the fourth class with one double tangent, the straight line at infinity, the circular points being the points of contact. The centres of the inscribed circle and the three escribed circles of the triangle formed by the fixed points are double points on the envelope, for every value of the eccentricity. In a paper in the *Bulletin des sciences mathématiques* the envelope has been stated by Schoute to be a curve of the eighth class.

6. Professor Stringham's paper sets up the purely analytical criteria for determining the characteristics of the fundamental types of the non-euclidean geometries. The coördinates of points and of straight lines being w, x, y and ω, ξ, η respectively, the assumed fundamental relations are $w^2 + x^2 + y^2 = 1$, $\omega^2 + \xi^2 + \eta^2 = 1$, and the distance functions are the following :

Between points and points $f = ww' + xx' + yy'$, between lines and lines (angle) $c = \omega\omega' + \xi\xi' + \eta\eta'$, between points and lines $g = k(\omega' w + \xi' x + \eta' y)$, where k is the fundamental space constant.

If the points (w), (w') lie upon the lines (ω), (ω') respectively, it is shown that maxima and minima of f exist and that the conditions for these maxima and minima make the points (w), (w') lie on the absolute polar of the intersection of the lines (ω), (ω'). This result makes it possible to replace the distance function f by g and to express g as a function of w alone, and the criteria for the maxima and minima of g are then easily determined. They show that g has maxima when k^2 is positive, minima when k^2 is negative, provided $\xi y - \eta x$ and $\xi' y' - \eta' x'$ be not infinite. The well known result follows that any two distinct straight lines in the elliptic plane (k^2 positive) have a maximum distance apart measured along the absolute polar of their real point of intersection, and that any two non-intersectors in the hyperbolic plane (k^2 negative) have a minimum distance apart measured along the absolute polar of their imaginary point of intersection.

7. In a paper recently presented before the London mathematical society, Professor Miller proved that the number of cyclic groups of order $p^\alpha (p > 2, \alpha > 1)$ in any group G whose Sylow subgroups of order p^m are non-cyclic is always a multiple of p. From this it follows directly that a non-cyclic group of order p^m contains $p^\alpha k$ operators whose order is p^α and hence the number of its operators which satisfy the equation $s^{p^\alpha} = 1$ is a multiple of $p^{\alpha+1}$. The theorem of Frobenius states that this number is a multiple of p^α. In any group G whose Sylow subgroups of order p^m are non-cyclic the number of operators of order $p^\alpha m$, m being prime to p, is a multiple of p^α.

8. Professor Blichfeldt's paper is devoted to an enumeration of the primitive collineation groups in four variables.

9. The results given in the second paper by Professor Blichfeldt are as follows: Let $\theta_1, \theta_2, \cdots, \theta_n$ be the multipliers of a substitution S_i of a linear homogeneous group G of order N in n variables. If the sum of the homogeneous products of degree m be formed from $\theta_1, \theta_2, \cdots, \theta_n$ and be designated by W_i, then

$$N^{-1}\sum_{i=1}^{N} W_i$$

is equal to the number of linearly independent absolute invariants of degree m of the group G. Also, let w_i be any symmetric function of the multipliers $\theta_1, \cdots, \theta_n$ then the same sum is an integer (positive or negative) or zero. Applying this principle to substitution groups, a number of known theorems concerning such groups are derived.

10. Professor Haskell's second paper is in abstract as follows: Let $A_1, A_2, A_3, A_4, A_5, A_6$ be the given points, and let it be required to find the third point A_7 which lies on any plane through A_5 and A_6. The lines joining the first four points in pairs meet this plane in the six vertices of a complete quadrilateral. Denoting by A_{ik} the intersection of this plane with the line joining A_i and A_k, then A_{12} and A_{34}, A_{13} and A_{24}, A_{14} and A_{23} are interchanged by the involutory quadratic transformation of which $A_5\,A_6\,A_7$ is the fundamental triangle.

11. After determining the condition that two triangles in perspective should be interchanged by a collineation of period 2, Professor Haskell shows that any collineation which leaves a conic invariant can be reduced to the product of two perspective collineations in an infinity of ways and that any collineation whatever is equivalent to the product of four perspective collineations. Then follow the deduction and geometric interpretations of the well-known groups of collineations in the plane.

G. A. MILLER,
Secretary of the Section.