THE FIFTY-FOURTH ANNUAL MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

THE fifty-fourth annual meeting of the American Association for the Advancement of Science was held in Philadelphia during the holiday week, December 27 to 31, 1904.

Professor W. G. Farlow, of Cambridge, Mass., was president. The address of the retiring president, Honorable Carroll D. Wright, entitled "Science and economics," was given in the Gymnasium of the University of Pennsylvania on the evening of December 28. The address has been published in full in *Science* for December 30. The attendance at the meeting reached a total of nearly a thousand, which may be considered as satisfactory in view of the many other meetings of scientific societies in progress at the same time.

The buildings of the University of Pennsylvania furnished suitable and ample accommodations for the various sections of the association itself and for the thirty affiliated societies; while the cordial welcome extended by the faculty of the university and the citizens of Philadelphia was most gratifying to the visiting scientists. The arrangements of the local committee were perfect in every detail.

The meetings of Section A (mathematics and astronomy) alternated with those of the Astronomical and Astrophysical Society of America. The programmes of both societies were well filled and the meetings were well attended. The officers of Section A were: vice-president, Alexander Ziwet; secretary, L. G. Weld; councilor, J. R. Eastman; member of the general committee, G. B. Halsted; press secretary, J. F. Hayford; sectional committee, O. H. Tittmann, J. A. Brashear, J. R. Eastman, Ormond Stone, E. B. Frost and E. O. Lovett, together with the vice-president and the secretary. Professor E. S. Crawley presided at the closing session of Section A. The following astronomers and mathematicians were elected by the Council to fellowship in the Association: Ellen Hayes, Willis I. Milham, and John J. Quinn. Others will receive fellowship through the adoption of a recommendation of the Committee on the policy of the Association by which all members belonging also to national scientific societies having a qualification

for membership equal to the qualification for fellowship in the Association shall be nominated as fellows.

The next meeting of the Asssociation will be held in New Orleans during the week beginning December 28, 1905, under the presidency of Professor Calvin M. Woodward, of Washington University. Dr. W. S. Eichelberger, of the U. S. Naval Observatory, will be vice-president of Section A. The present secretary will continue in office until after the meeting of 1907. Boston was recommended as the place of meeting in 1906.

The vice-presidential programme, which, in accordance with the recommendation of the committee on policy, was given a broader scope than heretofore, included the address of the retiring vice-president, Superintendent O. H. Tittmann, upon the subject "The present state of geodesy" and a paper by Professor Josiah Royce entitled "Symmetric and unsymmetric relations in the exact sciences." General discussion of each of these subjects was in order. Superintendent Tittmann's address has been published in *Science* for January 13, 1905. The paper of Professor Royce, which presents some further developments along the line followed by his address on "The sciences of the ideal" at the St. Louis Congress (see *Science*, October 7, 1904), will appear in the same journal at an early date.

The following papers were presented at the regular meetings of the section:

(1) Mr. H. W. CLOUGH: "Synchronous variations in solar and meteorological phenomena." To be published in the *Bulletin of the U. S. Weather Bureau.*

(2) Professor C. L. DOOLITTLE: "Temperature corrections of the zenith telescope micrometer, Flower astronomical observatory."

(3) Professor J. R. EASTMAN: "Results from observations of the sun, moon and planets for 26 years."

(4) Mr. PHILIP FOX: "Determination of the solar rotation period from flocculi positions." To be published by the Carnegie institution.

(5) Mr. O. E. GLENN: "Determination of all non-divisible groups of order $p^m q$ which contain an abelian subgroup of order p^m and type [1, 1, 1, $\cdots$ to m units]."

(6) Dr. G. H. HALLETT: "A note on groups of order 2^m which contain self-conjugate subgroups of order 2^{m-2}."

(7) Professor G. B. HALSTED: "Biology and mathematics."

(8) Professor ELLEN HAYES: "The path of the shadow of a plummet bead." To be published in *Popular Astronomy.*

(9) Professor J. F. HAYFORD: "The computation of the deflections of the vertical due to the topography surrounding the station."

(10) Professor G. A. MILLER: "Extension of a theorem due to Sylow." To be published in the BULLETIN.

(11) Mr. J. J. QUINN: "On inversions."

(12) Professor DAVID TODD: "On an optical method of radial adjustment of the axes of the trucks of a large observatory dome."

(13) Mr. FRANK SCHLESINGER: "On systematic errors in determining variations of latitude." To be published in the *Astronomical Journal.*

(14) Mr. FRANK SCHLESINGER: "Some experiments on the distortion of photographic films." (By title.)

(15) Mr. J. D. THOMPSON: "Bibliography and classification of mathematical and astronomical literature at the library of congress." To be published by the library of congress.

(16) Professor C. A. WALDO." "An exhibition of a new form of frame for straight line mathematical models."

(17) Professor L. G. WELD: "The application of Mayer's formula to the determination of the errors of the equatorial."

In the absence of the authors the papers by Mr. Fox (4), Professor Miller (10), Professor Todd (12) and Mr. Thompson (15) were presented either by the vice-president or the secretary. Mr. Frank Schlesinger's first paper (13) was presented by Professor Doolittle. Outlines of such of the papers presented as deal with purely mathematical subjects are given below.

5. Burnside proposes, as the most general problem in the theory of finite groups, the determination and analysis of all distinct types of groups whose order is a given integer. Mr. Glenn, in his paper, suggests, as a still more comprehensive problem, the *generalization* of all types belonging to a given integer, and gives a determination of the sets of defining relations which include as special cases all groups of orders pq and p^2q and a family of the known groups of order p^3q.

Letting G be the group under consideration and H its subgroup of order p^m, then, if p appertains to the index m (modulo q), none of the $p^{m-1} + p^{m-2} + \cdots + p^2 + p + 1$ subgroups of

order p in H is invariant in G. The latter is then defined by the relations

$$P_i^p = Q^q = 1, \quad P_iP_j = P_jP_i, \quad Q^{-1}P_kQ = P_{k+1}$$
$$(i = 1, 2, \cdots, m; \; j = 1, 2, \cdots, m; \; k = 1, 2, \cdots, m-1.)$$
$$Q^{-1}P_mQ = P_1^{(-1)^{m-1}}P_2^{(-1)^{m-2}\Sigma(\lambda\lambda^p \cdots \lambda^{p^{m-2}})} \cdots P_{m-1}^{-\Sigma(\lambda\lambda^p)}P_m^{\Sigma(\lambda)},$$

wherein λ is a mark of the $G \cdot F[p^m]$ and a primitive root in that field of the congruence $\lambda^q \equiv 1 \; [\text{mod. } p]$.

In case $p \equiv 1 \; (\text{mod. } q)$, at least m subgroups of order p are permutable with Q so that the defining relations of G are

$$P_i^p = Q^q = 1, \quad P_iP_j = P_jP_i, \quad Q^{-1}P_iQ = P_i^{\alpha x_i}$$
$$x_1 = 1, \quad \alpha^q = 1 \quad [\text{mod. } p].$$

The number of types comprised in the first set of relations is one. The number in the second set is given by the formula

$$N = \frac{1}{m}\left[\sum_{\sigma=0}^{(m-1)(q-2)} P(0, 1, 2, \cdots, q-2)^{m-1}\sigma - \psi\right],$$

ψ being a determinate function of q and m.

6. In the list of groups of order 2^m which contain self-conjugate subgroups of order 2^{m-2} as given in Burnside's Theory of Groups, there are six types. Dr. Hallett indicates a simple type of group non-isomorphic to any one of these six and establishes its defining relations, viz:

$$P^{2^{m-2}} = 1, \qquad Q^4 = P^{2^{m-3}}, \qquad Q^{-1}PQ = P^{-1}.$$

7. Professor Halsted called attention to certain analogies which have been assumed to exist between the mathematical doctrine of continuity and the evolution of new species through natural selection. He then proceeded to show that the analogy between mathematics and biology is much closer if we emphasize, on the one hand, the idea of discontinuity as it appears in modern mathematics and, on the other, those phases of the process of evolution supposed to be more readily explained by the theory of mutations.

10. Every group G of order p^m, p being any prime number, contains at least p invariant operators. This fundamental

theorem, due to Sylow, is shown by Professor Miller to be included in the following: Every non-abelian group of order p^m contains at least p invariant commutator operators, and its commutator quotient group is always non-cyclic. The paper is devoted to a proof of this theorem and the following closely related theorems: It is possible to construct a non-abelian group having any arbitrary abelian group as a commutator quotient group. Every non-cyclic abelian group of order p^a is the commutator quotient group of some non-abelian group of order p^m.

11. Mr. Quinn exhibited and explained a number of new linkages for describing the right line, in each of which the principle of inversion was applied.

16. A new form of thread model for ruled surfaces was exhibited by Professor Waldo, the frame of the model being conformed to the surface of a sphere; thus permitting the location of the points of attachment of the threads with much greater ease than in the ordinary forms, in which the limiting surface is discontinuous. The method of construction was also explained.

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A CALCULUS FOR GEOMETERS.

Cours D'Analyse. By G. HUMBERT. Paris, Gauthier-Villars. 8vo, 2 vols.: Vol. I, 483 pp., 1903; Vol. II, 493 pp., 1904.

THE rapidity with which French treatises on the calculus follow one another is at times confusing to the American mind. Picard's monumental work is still unfinished; Jordan's is a recent production; Vallée-Poussin is as yet scarcely familiar; Goursat's second volume is only partially complete; and a number of others, more or less well-known, are certainly upon the horizon. Meanwhile, in the interim between the appearance of the separate volumes of some of the works just mentioned, another first class treatise — the subject of this review — has been published in its entirety.

In general, Humbert's work is characterized by a predominance of geometry, and in particular by applications to the