$B_{ij\lambda}$ represents the substitution which alters only ξ_i, replacing it by $\xi_i + \lambda\xi_j$.

Another symmetric definition employs the generators $B_{ii+1\lambda}$ and $B_{i+1i\lambda}$ ($i = 1, 2, \cdots, n-1$ variables).

As corollaries are deduced symmetric definitions of the abstract groups isomorphic with the linear fractional and linear non-homogeneous groups on $n - 1$ variables.

10. In his second paper Professor Dickson points out that the symmetric group on n letters is simply isomorphic with the abstract group generated by the operators $T_{ij} \equiv T_{ji}$ ($i, j = 1, \cdots, n$; $i \neq j$) subject to the relations $T_{ij}^2 = I$, $T_{rk}^{-1}T_{rs}T_{rk} = T_{sk}$, T_{ij} commutative with T_{kl} when i, j, k, l, are all distinct. The proof is immediate. Another symmetric definition by fewer generators is due to Professor Moore, *Proceedings of the London Mathematical Society*, volume 28, page 357.

11. Professor Haskell has shown before that there is a birational transformation between the coëfficients of a binary cubic and of its cubicovariant. In seeking for a generalization of this relation he finds that the binary n-ic, if $n = 4m - 1$, possesses a covariant of order n whose coefficients are then related to those of the ground form; if $n = 4m$, there is such a relation provided a certain invariant vanishes; while for $n = 4m + 1$ or $n = 4m + 2$, there is no covariant bearing this property.

The next meeting of the Section will be held at Stanford University on February 24, 1906. The newly elected officers will begin their term of office with this meeting.

G. A. MILLER,
Secretary of the Section.

NOTE ON LOXODROMES.

BY PROFESSOR C. A. NOBLE.

(Read before the San Francisco Section of the American Mathematical Society, September 30, 1905.)

A LOXODROME is a curve on a surface of revolution which meets the meridians at a constant angle. If the meridians and the parallels of latitude on such a surface can be so selected as to constitute a network of similar infinitesimal rectangles, the diagonals of these rectangles will give loxodromes.

Let the surface be given by

$$x = p(u) \cos v, \quad y = p(u) \sin v, \quad z = q(u).$$

The meridians are the curves $v = \text{const.}$, the parallels the curves $u = \text{const.}$

Let the network be conditioned by prescribing for u and v the increments

$$du = \alpha(u) \cdot \epsilon, \quad dv = \epsilon,$$

where ϵ is an infinitesimal and $\alpha(u)$ a function to be determined.

Two consecutive rectangles of the network, *e. g.*, those with corresponding vertices at (u, v) and $(u + du, v + dv)$ respectively, will be similar if the angle between the two diagonals of the network which pass through (u, v) is equal to the corresponding angle between the two diagonals of the network which pass through $(u + du, v + dv)$. The condition for this equality, to within second order infinitesimals, is

$$\frac{\alpha'(u)}{\alpha(u)} = \frac{p'(u)}{p(u)} - \frac{\sigma'(u)}{\sigma(u)} \quad \left(\sigma(u) = \sqrt{[p'(u)]^2 + [q'(u)]^2}\right),$$

where accents denote differentiation.

This relation yields

$$\alpha(u) = \mu \cdot \frac{p(u)}{\sigma(u)} \qquad (\mu = \text{const.}).$$

The diagonals passing through (u, v) are given by

$$dv = \frac{du}{\alpha(u)},$$

that is, by

$$v = A \int \frac{\sigma(u)}{p(u)} du + B \qquad (A, B = \text{const.}).$$

This, then, is the finite equation of the ∞^2 loxodromes on a surface of revolution.

For the spheroid formed by the revolution about the z axis of the ellipse,

$$x = a \cos u, \quad z = b \sin u,$$

the above equation becomes

$$v=A\left\{\tfrac{1}{2}e\log\left[e^2\cos^2 u-\frac{1+e^2}{2}+e\sqrt{1-(1+e^2)\cos^2 u+e^2\cos^4 u}\right]\right.$$

$$\left.+\tfrac{1}{2}\log\left[\sec^2 u-\frac{1+e^2}{2}+\sec^2 u\sqrt{1-(1+e^2)\cos^2 u+e^2\cos^4 u}\right]\right\}+B$$

where e denotes the eccentricity of the ellipse. For the unit sphere this reduces to

$$v = A\log\tan\left(\frac{u}{2}+\frac{\pi}{4}\right)+B.$$

It is obvious, and well known, that if the sphere be stereographically projected upon the plane of the equator, the loxodromes will map into logarithmic spirals. The equation of these spirals can be found readily, with the aid of the principle that every conformal mapping of one surface upon another transforms the ∞^2 minimal curves of the one surface into the ∞^2 minimal curves of the other. The minimal lines on the unit sphere are given (Scheffers: Einführung in die Theorie der Flächen, page 64) by

$$\bar{u} = \log\tan\left(\frac{u}{2}+\frac{\pi}{4}\right)+iv = \text{const.},$$

$$\bar{v} = \log\tan\left(\frac{u}{2}+\frac{\pi}{4}\right)-iv = \text{const.}$$

If these lines are selected as parameter lines, the equations of the sphere become

$$x=\frac{\bar{u}+\bar{v}}{\bar{u}\bar{v}+1},\qquad y=\frac{-i(\bar{u}-\bar{v})}{\bar{u}\bar{v}+1},\qquad z=\frac{\bar{u}\bar{v}-1}{\bar{u}\bar{v}+1},$$

and the loxodromes on the sphere become

$$\cos^{-1}\frac{\bar{u}+\bar{v}}{2\sqrt{\bar{u}\bar{v}}} = A\log\sqrt{\bar{u}\bar{v}}+B.$$

Stereographic projection establishes the relations (Scheffers, page 80)

$$\bar{x}=\tfrac{1}{2}(\bar{u}+\bar{v}),\quad \bar{y}=-\tfrac{1}{2}i(\bar{u}-\bar{v})$$

between the coördinates $\bar{x}$, $\bar{y}$ in the plane of projection, and the minimal parameters $\bar{u}$, $\bar{v}$ on the sphere. These relations yield, as the equation of the curves into which the loxodromes project,

$$\cos^{-1}\frac{\bar{x}}{\sqrt{\bar{x}^2+\bar{y}^2}} = A \log \sqrt{\bar{x}^2+\bar{y}^2} + B$$

or, in polar coördinates,

$$\rho = \alpha e^{\beta\theta} \qquad (\alpha, \beta = \text{const.}).$$

The case of the oblate spheroid is interesting, in that the loxodromes on it project stereographically into the same spirals as do the loxodromes on the sphere which is tangent to the spheroid along the equator.

BERKELEY, *October*, 1905.

STOLZ AND GMEINER'S FUNCTION THEORY.

Einleitung in die Funktionentheorie. Abteilung I. By OTTO STOLZ and J. ANTON GMEINER. Leipzig, B. G. Teubner. 1904. vi + 242 pp.

THE Funktionentheorie of Stolz and Gmeiner is a working over of certain parts of Stolz's Allgemeine Arithmetik which do not appear in the new Stolz and Gmeiner's Theoretische Arithmetik. The two new books are evidently to be thought of together as a single course in the elements of analysis. The contents of the Theoretische Arithmetik correspond in a general way to sections 1 to 7, part of 10, and most of 11 of volume I, and of sections 1, 2, 6, and part of 5 of volume II of the Allgemeine Arithmetik. The Theoretische Arithmetik begins with the theory of whole numbers; then, after discussing the system of rational numbers, positive and negative, develops for the real and ordinary complex numbers the theory of addition, subtraction, multiplication, division, exponents and logarithms. In the course of this development appears a short discussion of complex numbers of n units, including quaternions, some geometric applications, and the fundamental theorems on infinite series.

The Funktionentheorie, on the other hand, corresponds to sections 9, 11, and part of 10 of volume I, and to sections 3, 4,