be regarded not as quadratic equations in x and y but as linear equations in x^2 and y^2; and the use of infinite solutions is of doubtful value.

In the chapter on the theory of equations it would have been well to give a few examples of numerical equations with irrational coefficients, since the equations which come up in practice are frequently of this type. Again, in the chapter on logarithms, the awkward double-headed rule for finding the characteristic which is now in common use might well have been replaced by the older and simpler rule which refers to the units' place instead of to the decimal point.

The later chapters of the book are a veritable store-house of accurate and well-selected information concerning infinite series, infinite products, continued fractions, etc. Special attention should be called to the last chapter of all, which contains a brief but admirable discussion of the continuity of functions of one and two variables, including correct definitions of such terms as upper and lower limit and oscillation of a function, a proof of Weierstrass's theorem that a continuous function actually reaches both its upper and its lower limit in any closed interval, and Argand's proof of the fundamental theorem of algebra. This chapter is of course not intended for a first-year class, but will be valuable for reference in the later years of the student's course.

It may be noted in conclusion that the book has an excellent index, and that answers to the exercises will be supplied by the publishers on the request of a teacher.

E. V. HUNTINGTON.

FREUND'S TRANSLATION OF BALL'S HISTORY OF MATHEMATICS.

Histoire des Mathématiques. Par W. W. ROUSE BALL. Edition française revue et augmentée. Traduite sur la troisième édition anglaise par L. FREUND. Tome premier. Paris, A. Hermann, 1906. viii + 422 pp.

IT is an interesting fact that France, which produced in Paul Tannery one of the greatest, if not the greatest historian of mathematics of our time, has not for a hundred years produced a great general history of the subject. Libri was a scholarly collector of material, and a charming writer, but he did not produce a balanced history. Chasles was an investigator of unusual merit, but he never wrote a general history. Marie, with his twelve volumes based on forty years of study, produced only a mass of inaccuracies. Hoefer's little book, although mentioned as an authority by Ball, is unworthy of any attention from considerations either of style or of scholarship. Boyer's works have no merit, and Rebière's are simply amusing. Indeed France seems to have exhausted her powers in Montucla, whose work even yet is a model in style and is not without value as an authority a century and a half after the first edition appeared.

In view of these facts it is gratifying that the translator can in a measure supply what French scholars fail to produce. Zeuthen has recently been published in Paris, the Encyklopädie (a history in the best sense) is also appearing, and various other contributions to history are finding their way into the French language. It is a matter of congratulation that Ball's "Short Account of the History of Mathematics" should be included in this list. For whatever may be said against the work, the fact remains that it is one of the best arranged and most readable handbooks on the subject ever published. It has its weak features, and these have already been mentioned in various journals as the successive editions of the original have appeared, but it is doubtful if any similar work is more often used. Mr. Ball, for example, gives only about sixteen pages to his chapter on the development of arithmetic in the important period from 1300 to 1637, and less than a dozen pages to the primitive arithmetic, and he rarely mentions the subject elsewhere; he makes no mention of the Nana Ghat inscriptions; he shows familiarity with a relatively small range of historical literature, and he derives very little information from the original sources. We can reasonably pardon him for saying that the Quadrivium of Psellus appeared in 1556, although there was an edition published in Venice in 1532, and several other editions appeared before the given date. We can overlook the fact that he gives V. M. de Kempten's Practique as appearing in 1556 instead of 1550; that (page 142) he leads us to infer that the works of Cassiodorus did not appear until 1729, although the Paris edition of 1598 is well known, not to mention numerous other editions of the Compendium; that he gives 1496 as the date of publication of Sacrobosco's Algorismus, although it appeared in 1488, and that he gives 1502 instead of 1495 for the first publication of Bradwardin's arithmetic; that he asserts so positively that Campanus was a canon at Paris, and that he tells us that Ahmes was a priest, when he was only a scribe. For a not particularly scientific handbook these and similar errors may, perhaps, be overlooked for the pleasure of having at hand such a readable manual. Mr. Ball has a good eye for the selection of important topics and of interesting facts, and this has enabled him to produce a book well worth translating.

The translator has arranged to produce the work in two volumes, with a larger and more open page than the original, and the first of these volumes is the one now under review. The opportunity which came to him was an excellent one for a scholar. A real student of the history of mathematics would have made use of all of the good features of the original, would have corrected the manifest errors, and would have added bibliographical notes of value and amplified the text where it was necessary. Unfortunately Lieutenant Freund has done nothing of the sort. He has merely made a mediocre translation, adding a few footnotes of little value, and showing that his taste is really in the line of physics by an appendix devoted almost exclusively to this subject and to logarithms. His translation is not free from errors, it is carelessly made even when not otherwise objectionable, and it preserves all of the blemishes of the original.

That the work is carelessly done is seen in such features as the common lack of uniformity in titles. Ball's Mathematical Recreations is sometimes cited in French, and sometimes in English, and with the dates 1899 (page 21), 1898 (page 228), each of which should be 1897 according to the advertisement on the cover. (See also pages 31, 39, 128). Paul Tannery appears sometimes as P. and sometimes as S. P. Tannery, although always in the latter form in the original. On page 192 the title of Taylor's work is given differently in the two notes. On page 190, "De Karlsruhe" should not be in small capitals, nor should "Zwickau" on page 222. The fact that a list of names occupying seven lines on page 49 is omitted, for no apparent reason, renders line eight meaningless: "Parmi tous les mathématiciens dont nous venons de donner les noms." In the matter of proper names the treatment is even less satisfactory than in the original. Althought it is gratifying to see such Greek forms as Antiphon and Heron instead of Antipho and Hero, the French forms are often used even in cases where the classical ones are the more common. The translator also takes liberties that are quite unwarranted, as when he uses sacro-Bosco for Sacrobosco. If he were to change the form used by Ball it would have been better to adopt a more common one like Sacrobusto or Sacro Bosco.

That the footnotes added by Lieutenant Freund are of little value is seen in the fact that there are only a dozen all told. Of these, four mention articles by Loria, one mentions a work by Sedillot, one devotes seven lines to giving the useless information as to who the Admirable Crichton was (and then misspells his name !), and one says that Charles Kingsley wrote a novel about Hypatia. Of recent literature on the history of mathematics there is nothing save the four references to Loria. Hoefer is retained as an authority, but Braunmühl is unknown; Marie is retained, but there is no mention of Zeuthen or of those who are making the Abhandlungen a storehouse of first-hand material.

That no originality of scholarship is shown by the translator in his work is seen not only in the paucity of his footnotes but in the appended matter. This consists merely of a series of extracts from the works of various writers as follows: 1. A brief note by Chasles on Vieta (page 327); 2. An extended note by Biot on Napier and the invention of logarithms (pages 328–353); 3. A note, chiefly from Bertrand, on Kepler (pages 354–358); 4. A note by Mach on the development of the principles of dynamics (pp. 359–409); 5. A note by Duhem on the origin of statics (pages 410–412).

That Lieutenant Freund has preserved all of the errors and doubtful statements of the original is continually manifest. For example, we do not know, as Ball asserts, that the Mexicans used the abacus (page 132); the figure on page 135 is not a Chinese swan pan but a Japanese saroban; Alkwarizmi was born in Khuwarism, not in Khorassan (page 165); we do not know that Fibonnacci was born in 1175 (page 175), nor that Psellus was born in 1020, nor is there justification for similar positive statements not infrequently made as to dates; Widman did not use the + and − "pour indiquer l'addition et la subtraction," but for a purpose explained later in the book; Recorde did not use the symbol Z to indicate equality of ratios, for, to select one of his examples, he says: "I ſet them one ower the other, and 8 vndermoſt, thus $\frac{3}{8}\!\not\!\!\searrow$, with ſuch a crooked draught of lines. Then doe I ſet the other number which is 16, againſt 3 at the right ſide of the line, thus $^{3}_{8}\!\underline{\angle}^{16}$" In other words, the lines merely indicate a direction of operations, not any equality of ratios.

In general the mere translation seems satisfactory. It is a question however whether Recorde had in mind any such idea as is found in the interpretation of his "Grounde of Artes" as meaning "Jardin des Artes" (page 221).

That the book abounds in typographical errors is shown by the list given below. This is merely the result of a hurried reading, and is doubtless very incomplete. It shows, however, such an utter lack of care on the part of the translator as to assure us that an opportunity has been missed. Such a list makes it seem almost hopeless to expect a worthier effort in the second volume. Page 21, for 1899 read 1897; page 33, for Ænopides read Œnopides; page 39, for Ritz Patrick read Fitz Patrick; page 49, for Ménœchme read Ménæchme, and so on page 414, and on pages 81, 82, change Menechme; page 53, for Grichen read Griechen; and for modern read modernen; page 55, for Euclides read Eucleides (De Morgan's article), and for studien read Studien (the German capitalization being elsewhere followed); page 56, for Strobacus read Strobaeus; page 81, for Grow read Gow; page 86, for F. Hiller read E. Hiller; page 87, for Chrichton read Crichton; page 90, for phœnomena read phænomena; page 93, for J. S. Greenwood read J. G. Greenwood; page 95, for Αύτοματα read Αὐτόματα; page 101, for Hache read Hoche; page 102, for Ptolemœus read Ptolemæus, and for 3,1416 read 3,1416; page 103, for Συναγωγη read Συναγωγή; page 112, for μ^{δ} read $\mu^{\hat{o}}$ four times; page 112 for ψ read ᛦ twice; page 136, for ἑτεως read ετεος, or ἕτεος if the accents are taken, the inscription referring to the seventh year, not to seven years; page 143, for C. Werner read K. Werner (cf. page 145); page 147, for Malmesburg read Malmesbury; page 154, for Appolonius read Apollonius; page 155, for Batha read Bhata; page 156, for M. Kern read H. Kern; page 161, for Arhya read Arya; page 179, for algebriste read algebrista; page 183, for Medita read Inedita; page 186, for Radolt read Ratdolt; page 190, for Encyclopœdia read Encyclopædia, and drop "Supplement" from note 1; page 263, for Breitscwert read Breitschwert; page 287, for $(n - 1^2)^2)$ read $(n - 1)^2$; page 296, for $(x - x^3)^2$ read $(x - x^2)^1$; page 298, for Schoeten read Schooten (as on page 313, 315);

page 307, for $p\frac{1}{3}a^{\frac{5}{3}}e$ read $p^{\frac{1}{3}}a^{\frac{5}{3}}e$; page 313, for Dictionnary read Dictionary ; page 316, for d read ∂ four times, and three times on page 324 ; page 321, insert *di* after *Bulletino*, as elsewhere.

DAVID EUGENE SMITH.

SHORTER NOTICES.

Abhandlungen zur Geschichte der mathematischen Wissenschaften. 18. Heft. Inhalt ; J. L. HEIBERG, Mathematisches zu Aristotles ; C. H. MÜLLER, Studien zur Geschichte der Mathematik insbesondere des mathematischen Unterrichts an der Universität Göttingen im 18. Jahrhundert ; R. LINDT, Das Prinzip der virtuellen Geschwindigkeiten. Leipzig, B. G. Teubner, 1904. 196 pp. Price, 6 marks.

FROM the standpoint of the history of pure mathematics the first two parts of this volume of the *Abhandlungen* are of great interest and value. The death of Paul Tannery left no one so well prepared to speak with authority upon a question involving both Greek mathematics and literature as Professor Heiberg. Although primarily a student of the classics, this prolific scholar has so long devoted his attention to the ancient mathematicians that he has become one of the great authorities upon their contributions.

The various histories of mathematics have always recognized the impetus given to mathematics by both Plato and Aristotle, by the former in fixing the foundations, and by the latter with reference to the history and the applications of the science. There is, however, a lack of definite information regarding the mathematical contributions of both of these leaders of philosophic thought. If we try to find exactly what Plato contributed to the advance of mathematics, Cantor, Gow, Zeuthen, and even Tannery give answers that are far from satisfactory. For Aristotle this has also been the case, and hence this concontribution of Professor Heiberg is timely and welcome, especially as it throws much light on the work of Plato as well.

The essay opens with a discussion of the sources of information and then by detailed references to the works of Aristotle it shows his influence upon the subsequent work of the Greeks. In particular the influence of this writer upon the Greek mathematical terminology is shown to be much greater than would be