THE FEBRUARY MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and twenty-seventh regular meeting of the Society was held in New York City on Saturday, February 24. The attendance at the two sessions included the following thirty members of the Society:

Professor G. A. Bliss, Professor C. L. Bouton, Professor Joseph Bowden, Dr. W. H. Bussey, Professor F. N. Cole, Dr. W. S. Dennett, Professor L. P. Eisenhart, Professor H. B. Fine, Mr. A. M. Hiltebeitel, Dr. Edward Kasner, Professor C. J. Keyser, Dr. G. H. Ling, Professor Max Mason, Mr. A. R. Maxson, Professor H. B. Mitchell, Professor Richard Morris, Professor W. F. Osgood, Miss I. M. Schottenfels, Professor Charlotte A. Scott, Professor C. S. Slichter, Dr. Clara E. Smith, Professor D. E. Smith, Professor P. F. Smith, Mr. A. W. Stamper, Dr. C. E. Stromquist, Professor H. D. Thompson, Professor E. J. Townsend, Professor Oswald Veblen, Miss E. C. Williams, Professor J. W. Young.

The President of the Society, Professor W. F. Osgood, occupied the chair. The Council announced the election of the following persons to membership in the Society: Mr. M. J. Babb, University of Pennsylvania; Mr. William Betz, East High School, Rochester, N. Y.; Mr. G. D. Birkhoff, University of Chicago; Mr. W. C. Breuke, Harvard University; Mr. B. E. Carter, Massachusetts Institute of Technology; Dr. H. L. Coar, University of Illinois; Miss Anna Johnson, Harvard University; Mr. W. D. Lambert, U. S. Coast Survey; Mr. W. A. Luby, Central High School, Kansas City, Mo.; President W. J. Milne, New York State Normal College; Professor Richard Morris, Rutgers College; Mr. W. J. Newlin, Harvard University; Miss R. A. Pesta, Wendell Phillips High School, Chicago, Ill.; Dr. H. B. Phillips, University of Cincinnati; Mr. A. R. Schweitzer, University of Chicago; Mr. C. G. Simpson, Michigan College of Mines; Mr. A. W. Stamper, Columbia University; Mr. F. C. Touton, Central High School, Kansas City, Mo.; Mr. M. O. Tripp, College of the City of New York. Ten applications for admission to the Society were received.

The date of the next annual meeting of the Society was fixed as Friday–Saturday, December 28–29. The summer meeting and colloquium will be held at Yale University, extending through the week September 3–8. Courses of colloquium lectures have already been arranged, and a preliminary announcement will be issued in May.

The following papers were read at the February meeting:

(1) Dr. W. H. BUSSEY: "On the tactical problem of Steiner."

(2) Miss I. M. SCHOTTENFELS: "On linear fractional transformations of functions of the complex variable $u + \epsilon v$, where $\epsilon^2 = 0$" (preliminary communication).

(3) Professor C. J. KEYSER: "On the linear complex of circle ranges in a plane."

(4) Professor E. B. WILSON: "Note on integrating factors."

(5) Miss R. L. CARSTENS: "A set of independent postulates for quaternions."

(6) Dr. W. B. FORD: "On the analytic extension of functions defined by double power series."

(7) Professor OSWALD VEBLEN: "Remark on a measure of categoricalness."

(8) Professor VIRGIL SNYDER: "Surfaces generated by conics cutting a twisted quartic curve and a line in the plane of the conic."

(9) Dr. CLARA E. SMITH: "Development of a function in terms of Bessel's functions (second paper)."

(10) Professor L. P. EISENHART: "Surfaces with the same spherical representation of their lines of curvature as spherical surfaces."

(11) Professor PAUL STÄCKEL: "Die kinematische Erzeugung von Minimalflächen (erste Abhandlung)."

(12) Professor OSKAR BOLZA: "A fifth necessary condition for a strong extremum of the integral

$$\int_{x_0}^{x_1} F(x, y, y')dx."$$

Miss Carstens's paper was communicated to the Society by Dr. Epsteen, Professor Stäckel's by Professor E. H. Moore. In the absence of the authors, the papers of Professor Wilson, Miss Carstens, Dr. Ford, Professor Snyder, Professor Stäckel and Professor Bolza were read by title. Abstracts of the papers follow below; the abstracts are numbered to correspond to the titles in the list above.

1. The problem considered in Dr. Bussey's paper was proposed by Steiner in the *Journal für die reine und angewandte Mathematik*, volume 45, page 181. It has to do with the arrangement of N elements in triads, tetrads, pentads, etc. That part of the problem which relates to triads has been solved (see Encyclopédie des Sciences mathématiques, volume 1, page 80). The other parts are more difficult and have been made the object of but little study. By means of the properties of linear homogeneous equations in the Galois field of order 2, the author has completed the solution for the linear triple systems in $2^k - 1$ elements.

2. Miss Schottenfels treated the developments essential to the study of the linear fractional transformations of functions of the complex number $u + \epsilon v$, where $\epsilon^2 = 0$, including certain transformations such as $z' = z + \epsilon$, $z' = \mu z$.

3. Professor Keyser's paper, like that presented by him at the January meeting, deals with the circle range geometry of the plane. The range enjoys four degrees of indetermination. The ranges satisfying a single condition constitute a complex. A pencil of ranges is the ensemble of ranges having a common circle and lying in a circle congruence (totality of circles orthogonal to a given circle). The degree of a complex of ranges is the number of ranges common to it and an arbitrary pencil. When this number is 1, the complex is linear. The ranges of such a complex that have a given circle in common constitute a pencil, polar to the given circle. The ranges of the complex that lie in the congruence of a given pencil constitute a pencil, pole of the congruence. Hence a linear complex is a means of reciprocal transformation pairing the circles with the congruences of the plane and the ranges with the ranges. The ranges of a pair are conjugates. The circles of a range correspond to the congruences of the conjugate range, and reciprocally. The anharmonic ratio of any four circles of a range is equal to that of the corresponding congruences of the conjugate range, and reciprocally. To any configuration of circles, ranges and congruences corresponds a (reciprocal) configuration of (the polar) congruences, ranges and circles. Every range having a circle in common with each of two con-

jugate ranges belongs to the complex. If a range r of the complex has a circle in common with a given range r_1, r contains a circle of the conjugate range r'_1 of r_1. According as two conjugate ranges contain or do not contain a common circle (or congruence) they do or do not belong to the complex. If they do, they coincide. Three independent ranges r_1, r_2, r_3 determine an infinity of ranges each intersecting each of the given r's in a circle. The ranges so determined are readily constructible. They constitute a system S of generating ranges of a quadric configuration of ranges, analogous to the simple hyperboloid of space. The second system S' of generating ranges is determined by any three ranges of S. If the ranges of S belong to a complex C, they are each self-conjugate as to C. Then no range of S is in C but the conjugate of every range of S is in S'. The radical axes of the ranges in S envelope a conic; similarly for S', and the conics coincide. In case of the analogue of the hyperbolic paraboloid, the conic degenerates into a pair of pencils of lines.

4. Professor Wilson gave an elementary proof of the relation $M\sqrt{X_1^2 + X_2^2 + \cdots + X_n^2} = dF/dn$ between the coefficients X_i, the integrating factor M, and the solution F of an integrable total differential equation $\sum X_i dx_i = 0$. This relation was then used to discuss, geometrically and somewhat more in detail than is usual, the matter of singular solutions, limiting solutions and the factors of $M^{-1} = 0$. The paper is to appear in the *Annals of Mathematics*.

5. In this paper Miss Carstens defines Hamilton's quaternions by a set of independent postulates based on Professor Dickson postulates for hypercomplex number systems (*Transactions*, volume 6, No. 3, pages 344–348; 1905).

6. Dr. Ford's paper considers the functions $f(x, y)$ defined by the double power series

$$\sum_0^\infty \sum_0^\infty a(m, n) x^m y^n. \tag{1}$$

It is shown that under certain conditions for the function $a(m, n)$ the function $f(x, y)$ may be extended analytically outside the circles of convergence of the series (1) and throughout the entire x, y planes with the exception of the cuts 0 to $+\infty$ along the real x and y axes. The paper will appear in the *Transactions*.

7. Professor Veblen's note appeared in the March BULLETIN as the last part of his review of Huntington's Types of Serial Order.

8. If a line and a twisted quartic be given, and a correlation between the planes and points of the line be established, then a conic is uniquely fixed by the four points on the curve and the point associated with the plane. The surfaces studied by Professor Snyder are described by the conic when the plane turns about the line, which was chosen in various ways with regard to the curve.

9. Schlömilch determined the coefficients of his development of an arbitrary function $f(x)$ in terms of $J_0(x)$ by applying Abel's relation

$$\frac{2u}{\pi}\int_0^1 dt \int_0^1 \frac{f'(\lambda ut)\lambda d\lambda}{\sqrt{1-t^2}\sqrt{1-\lambda^2}} = f(u) - f(0),$$

to an auxiliary function in terms of which $f(x)$ could be expressed. The justification of this method involves certain conditions on the second derivatives of $f(x)$. Miss Smith showed that this unnecessary restriction can be removed by applying Abel's relation directly to $f(x)$.

Another development in terms of $J_0(x)$, whose coefficients differ slightly in form from those of Schlömilch, can be justified under slightly more general conditions. In simple cases the two developments are identical. Analogous to this second development is one in terms of $J_1(x)$, which is much more easily justified than that which Schlömilch obtained by termwise differentiation of the series in $J_0(x)$. These also are in many cases identical.

10. In several previous papers Professor Eisenhart has considered the surfaces with the same spherical representation of their lines of curvature as pseudospherical surfaces, called for convenience A-surfaces. Now he discusses the case where the representation is that of spherical surfaces, that is, surfaces with constant positive curvature, pointing out his previous results which have a significance in the present case and estab-

lishing theorems which of necessity have no analogues in the other theory. There exists an imaginary transformation of one of the surfaces into another of the same kind, which is similar to the generalized Bäcklund transformations of A-surfaces. Pairs of these transformations can be found which when applied successively to a real surface yield a new real surface of the same kind. Bonnet showed that these surfaces go in pairs — the members of a pair being applicable with correspondence of the lines of curvature; and, moreover, these surfaces are the only ones applicable in this manner. On this account we call them surfaces of Bonnet. The knowledge of a transformation of such a surface enables one to find by algebraic processes the surface of Bonnet applicable to the given one. By means of the above-mentioned transformations a pair of real applicable surfaces of Bonnet can be transformed into a new pair. There exist a large number of surfaces whose coördinates are expressed in forms similar to those for surfaces of Bonnet and the surfaces analogous to the latter considered elsewhere by the author.

11. A curved surface is said to have a kinematic generation if it is generated by a rigid curve moving according to a given law. To surfaces generated kinematically belong the minimal surfaces whose generators are imaginary curves with vanishing line element. Professor Stäckel proposes to publish in a series of articles the results of investigations which he is making on those exceptional minimal surfaces that have more than one kinematic generation. The first paper, which will appear in the *Transactions,* is concerned with the minimal surfaces that can be determined in more than one way as translation surfaces. A new and direct proof is given of Lie's theorem that the Scherk's surfaces are the only minimal surfaces which can be generated by the translation of curves with non-vanishing line element. For, if it be required that a surface of translation be at the same time a minimal surface, then the gaussian parameter representation gives a functional equation for three functions of u and three functions of v. A complete solution for this equation is determined. If the generating curve is plane, the Scherk's surfaces are obtained immediately; but if it is a twisted curve in space, then the solution of the given functional equation reduces to the integration of the system of ordinary differential equations

$$\frac{hg' - gh'}{f^2 + g^2 + h^2} = s_{11} f + s_{12} g + s_{13} h,$$

$$\frac{fh' - hf'}{f^2 + g^2 + h^2} = s_{21} f + s_{22} g + s_{23} h,$$

$$\frac{gf' - fg'}{f^2 + g^2 + h^2} = s_{31} f + s_{32} g + s_{33} h,$$

in which f, g, h are functions of u and f', g', h' their derivatives, while $s_{11}, \cdots, s_{33}$ are constants. These equations also give the Scherk's surfaces, which admit an infinite number of generations in the way described; moreover the ordinary helicoidal surfaces appear as special cases of this result.

12. It is well known that the conditions of Euler, Legendre, Jacobi, and Weierstrass are not sufficient for a strong extremum of the integral

$$J = \int_{x_0}^{x_1} F(x, y, y')dx.$$

In Professor Bolza's paper a fifth necessary condition is established. The paper will be published in the current volume of the *Transactions.*

F. N. COLE,
Secretary.

THE FIFTY-FIFTH ANNUAL MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

THE American Association for the Advancement of Science held its fifty-fifth annual meeting in New Orleans, the sessions continuing from December 28, 1905, to January 3, 1906.

The president of the meeting was Professor Calvin M. Woodward of St. Louis. Dr. L. O. Howard, Washington, D. C., is the permanent secretary of the Association. The enrollment was small, reaching a total of only 233, and the programmes of many of the sections were unusually brief, but the meeting as a whole can by no means be considered unsuccessful. It is believed by many that, though the attendance may always be small, one of the most important purposes of the organization — the stimu-