$$\frac{hg' - gh'}{f^2 + g^2 + h^2} = s_{11}f + s_{12}g + s_{13}h,$$

$$\frac{fh' - hf'}{f^2 + g^2 + h^2} = s_{21}f + s_{22}g + s_{23}h,$$

$$\frac{gf' - fg'}{f^2 + g^2 + h^2} = s_{31}f + s_{32}g + s_{33}h,$$

in which f, g, h are functions of u and f', g', h' their derivatives, while s_{11}, $\cdots$, s_{33} are constants. These equations also give the Scherk's surfaces, which admit an infinite number of generations in the way described; moreover the ordinary helicoidal surfaces appear as special cases of this result.

12. It is well known that the conditions of Euler, Legendre, Jacobi, and Weierstrass are not sufficient for a strong extremum of the integral

$$J = \int_{x_0}^{x_1} F(x, y, y')dx.$$

In Professor Bolza's paper a fifth necessary condition is established. The paper will be published in the current volume of the *Transactions*.

F. N. Cole,
Secretary.

THE FIFTY-FIFTH ANNUAL MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

The American Association for the Advancement of Science held its fifty-fifth annual meeting in New Orleans, the sessions continuing from December 28, 1905, to January 3, 1906.

The president of the meeting was Professor Calvin M. Woodward of St. Louis. Dr. L. O. Howard, Washington, D. C., is the permanent secretary of the Association. The enrollment was small, reaching a total of only 233, and the programmes of many of the sections were unusually brief, but the meeting as a whole can by no means be considered unsuccessful. It is believed by many that, though the attendance may always be small, one of the most important purposes of the organization — the stimulation of popular interest in science — will be better achieved by meeting more frequently in the remoter cities of the country. It is to be noted, however, that few of the affiliated societies find it advisable to meet at a great distance from the more populous educational centers. Thus, another important function of the Association — the coördination of scientific interests — is certain to be suspended as often as meetings at distant points are undertaken. It should in any case be said, with reference to the New Orleans meeting, that the welcome accorded the Association was remarkably cordial.

The address of the retiring president, Professor W. G. Farlow, of Cambridge, Mass., was given at Sophie Newcomb College on the evening of December 29, the subject being 'The popular conception of the scientific man at the present day.' It has been published in full in *Science* for January 5 of the current year.

The meetings of Section A (mathematics and astronomy), as well as those of the other sections, were held in the buildings of Tulane University. The officers of the section were: vice-president, W. S. Eichelberger; secretary, L. G. Weld; councilor, C. S. Howe; member of the general committee, G. B. Halsted; press secretary, the secretary of the section; sectional committee, Alexander Ziwet, J. R. Eastman, Ormond Stone, E. B. Frost, E. O. Lovett, Harris Hancock, together with the vice-president and the secretary of the section. In the absence of the vice-president the retiring vice-president, Professor Ziwet, acted as chairman. The following mathematicians and astronomers were, upon nomination by the sectional committee, elected by the council to fellowship in the Association: E. W. Brown, W. A. Granville, Harris Hancock, M. W. Haskell, E. J. Townsend, Irving Stringham, Paul Wernicke.

The next annual meeting of the Association will be convened in New York City, on Thursday, December 27, 1906. In addition to this meeting it was decided by the general committee to hold a special summer meeting at Ithaca, New York, to close on or before July 3. There will be no presidential or vice-presidential addresses at the summer meeting. All officers elected at the New Orleans meeting will hold over to the close of the New York meeting. Dr. W. H. Welch of Baltimore, Md., was elected president of the New York and Ithaca meetings, Dr. Edward Kasner of New York City will be vice-president of section A. The present secretary of the section is to continue in office. Chicago was recommended as the place of meeting in 1907.

The address of the retiring vice-president of section A, Professor Alexander Ziwet, on 'The relation of mechanics to physics,' was presented on the afternoon of December 29. It has already been published in full in *Science* for January 12. At the regular programme meeting of the section, held on December 30, eleven papers were presented. The titles of these are given below, with abstracts of such as deal with purely mathematical subjects.

(1) Dr. O. E. GLENN: "On the groups of order $p^m q^n$ having abelian subgroups $H_{p^{mn}}$ of type $[n, n, \cdots, n]$."

(2) Professor G. B. HALSTED: "A new straight in non-euclidean geometry."

(3) Professor HARRIS HANCOCK: "A chapter in the present state of development of the elliptic functions."

(4) Mr. H. B. HEDRICK: "A catalogue of 1607 zodiacal stars for the epochs 1900 and 1920, reduced to an absolute system."

(5) Dr. EDWARD KASNER: "A class of central forces."

(6) Professor F. H. LOUD: "Solar photographs."

(7) Professor G. A. MILLER: "The groups of order p^m which contain exactly p cyclic subgroups of order p^α."

(8) Mr. J. J. QUINN: "Inversion and inversors."

(9) Professor DAVID TODD: "Observations of the total solar eclipse of 1905, August 30, at Tripoli, Barbary."

(10) Professor DAVID TODD and Mr. R. A. BAKER: "Computed traces and totality durations of the total eclipses of the twentieth century."

(11) Professor L. G. WELD: "A possible extension of the theory of envelopes."

1. This paper is supplementary to one presented by the author at the Philadelphia meeting of the Association, in which the case $n = 1$ was discussed. The defining relations of all groups described in the title are tabulated, and their properties discussed in relation to the properties of the Galois field $GF[p^{mn}]$ determined by the automorph of the subgroup H. It is found that all of the groups in question are members of a general family and have one general set of defining relations.

2. The paper sets forth the discovery that, in Riemann non-euclidean geometry, the six mid-points of the parts of the

six rays from the vertices of any triangle obtained by prolonging the sides are costraight. This is a new and noteworthy straight associated with every triangle. The theorem is demonstrated, and then interpreted in ordinary euclidean space.

3. This paper is an attempt to show that practically all (American and European) writers on the elliptic functions have been giving too much emphasis to certain parts of Weierstrass's theory, while they have neglected many of the lines of thought which Weierstrass himself considered fundamental.

It is shown that the so-called Weierstrass normal form is not due to Weierstrass. The introduction of new functions gives a different aspect to the presentation of the elliptic functions although little that is new has been added thereby to the theory itself. Weierstrass's great work lies in a somewhat different direction. With him the problem of determining all analytic functions which have algebraic addition theorems is the leading idea.

The paper also cites several fundamental theorems of Hermite and indicates some of the characteristics of Riemann's theory.

4. This paper will be published in the Astronomical Papers of the American Ephemeris, volume 8, part 3.

5. There exists no field of force in which a particle started from an arbitrary position with arbitrary velocity will describe a circular path. In the case of a central force the only possible circular trajectories are, in general, those whose centers are at the origin of force. If, however, the force varies according to a function of the form $br(r^2 - a)^{-3}$, then a quadruple infinity of the trajectories are circular. In the simplest case, arising when a vanishes, the force varies inversely as the fifth power of the distance, and the circles all pass through the origin. In the general case they are orthogonal or diametral to a fixed sphere.

7. The main theorems proved in this paper may be stated as follows: If a group of order p^m, p being any odd prime, contains exactly p cyclic subgroups of order p^α, $\alpha > 2$, it contains exactly p cyclic subgroups of every order which exceeds p and divides p^{m-1}. Hence it is one of the two non-cyclic groups of order p^m which contain operators of order p^{m-1}. When $\alpha = 2$

and $p > 3$ the theorem is still true. In fact, the only possible exception occurs when $\alpha = 2$, $p = 3$ and $m = 4$. In this special case there are three groups which contain exactly p cyclic subgroups of order p^α.

When $p = 2$ the preceding theorem is replaced by the following: If a group of order 2^m contains exactly two cyclic subgroups of order 2^α, $\alpha > 2$, it cannot contain more than two cyclic subgroups of any higher order. If a group of order 2^m contains exactly two cyclic subgroups of order 2^β but does not contain any cyclic subgroup of order $2^{\beta+1}$, then m cannot exceed $2^{\beta+2}$. These theorems involve the fundamental properties of all possible groups whose order is a power of any prime p and which involve exactly p cyclic subgroups of any given order p^α. From a well known theorem it follows that α is not unity, but it can have every other possible value less than m.

8. In this paper are presented two new theorems relating to inversion, besides an explanation of the construction of certain linkages exhibiting the operation of inversion.

11. (a) In the equation $f(\alpha, x, y) = 0$, representing a family of loci, by giving to α, first an increment and then a corresponding decrement, each of magnitude $\Delta\alpha$, solving the resulting equations for the coördinates of the point of intersection and, finally, letting $\Delta\alpha \doteq 0$, there will be obtained $x' = \phi(\alpha)$, $y' = \psi(\alpha)$. These equations define a point of the envelope of the given family of loci and eliminating α between them gives $F(x', y') = 0$, the equation of the envelope.

The point (x', y'), determined as above, may be called the tracing point of the locus; that is, the point which, for the moment, is tracing the envelope. It was shown in the paper, by way of illustration, that the tracing point for the envelope of the family of ellipses

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1, \quad \alpha + \beta = c$$

is the Fagnani point.

(b) The inverse of the above notion was next developed with reference to the right line, viz: A point on the line

$$\frac{x}{\alpha} + \frac{y}{\beta} = 1$$

being assigned at will, to find the functional relation between the intercepts, $\Phi(\alpha, \beta) = 0$ (*i. e.*, the law governing the motion of the line), in order that the given point may trace an envelope and, finally, to obtain the equation of the envelope. The required relation is given by either of the differential equations

$$x' = \phi(\alpha, \beta) = \frac{\alpha^2}{(\alpha - \beta)d\alpha/d\beta}, \quad y' = \psi(\alpha, \beta) = \frac{\beta}{(\beta - \alpha)d\beta/d\alpha}$$

In general both equations will be needed in order to determine the constants of integration. Having thus obtained the function Φ, which is, in effect, the tangential equation of the envelope, the equation in rectangular coordinates readily follows.

Several examples applying the principles were presented and its application to other families of loci was suggested as a promising field of investigation for the amateur mathematician.

LAENAS GIFFORD WELD,
Secretary.

THE STATE UNIVERSITY OF IOWA,
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A PROOF OF THE FUNDAMENTAL THEOREM OF ANALYSIS SITUS.

BY PROFESSOR G. A. BLISS.

(Read before the American Mathematical Society, December 28, 1905.)

THE theorem that a Jordan curve divides the plane into two regions, an interior and an exterior, has in recent years received much attention. The proofs which have been given may be roughly divided into two classes, those in which the object has been to prove the theorem with the fewest possible hypotheses on the curve,* and those in which generality has to a certain extent been sacrificed for simplicity.† The following proof belongs to the second class. In § 1 it is assumed that the curve considered is continuous and has a continuously turning tangent at every point; but in § 3, by extending the proof of one of the auxiliary theorems, curves with a finite number

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* Veblen, *Transactions Amer. Math. Society*, vol. 6 (1905), p. 83.

† Ames, *Amer. Jour. of Math.*, vol. 27 (1905), p. 353. Bliss, BULLETIN, vol. 10 (1904), p. 398. For further references, see the paper by Ames.