As three particular solutions, take

$$\xi = 2U/(u+v) - U', \quad \eta = 2V_1/(u+v) - V_1',$$
$$\zeta = 2V_2/(u+v) - V_2'.$$

Substitution in (6) gives for S_0 in this case,

$$x_0 = 2(V_2 V_1' - V_1 V_2')/(u+v) + \int (V_1' V_2'' - V_2' V_1'')dv,$$
$$y_0 = 2(V_2 U' + V_2' U)/(u+v) - U' V_2',$$
$$z_0 = -2(V_1 U' + V_1' U)/(u+v) + U' V_1'.$$

Substitution in (7) gives at once the equations of the group of associated surfaces.

The method above outlined for determining associated surfaces may be applied to the problem of determining surfaces which admit of continuous deformation with preservation of conjugate lines, since such surfaces are the associates of those whose curvature in terms of the parameters of the asymptotic lines is of the form $K = [\phi(u) + \chi(v)]^{-2}$.* In that case (4) may be written in the form

$$\frac{\partial^2\theta}{\partial u \partial v} = \left(-\frac{1}{4\rho^2}\frac{\partial \rho}{\partial u}\frac{\partial \rho}{\partial v} - f\right)\theta.$$

PURDUE UNIVERSITY,
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NOTE ON THE PRACTICAL APPLICATION OF STURM'S THEOREM.

BY J. E. WRIGHT, M.A.

IT is strange that the following small point is not mentioned in the text-books. The only remark I can find bearing on it is in Encyklopädie der Mathematik I., 1, 4, page 417. Suppose $f, f_1, \cdots, f_n$ are the successive Sturmian functions in *e. g.*, the case when f has only simple roots. If all the real roots of f_r are known, then we can separate the roots by using $f, f_1, \cdots, f_r$ only. For let a, b be two consecutive roots of f_r. Then between a and b f_r does not vanish, and therefore no alteration in the number of changes of sign can arise from the functions $f_r, \cdots, f_n$.

* Bianchi-Lukat, l. c., p. 337.

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Hence the number of roots of f between a and b is determined from $f, \cdots, f_r$ only. Practically to avoid zeros of f_r, we substitute $a + \epsilon$, $b - \epsilon$ in the first $r + 1$ functions, when ϵ is small. To complete the work it is necessary to discover whether or not any of the roots of f_r are roots of f, and to apply the theorem to all the intervals given by $\pm \infty$ and the roots of f_r.

Example :

$$f(x) = x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1,$$

$$f_1(x) = 5x^4 - 20\,x^3 + 27x^2 - 18x + 5, \quad f_2(x) = x^3 - x.$$

Roots of f_2 are $-1, 0, +1$; of these $+1$ is a root of $f(x)$.

	$-\infty$	$-1-\varepsilon$	$-1+\varepsilon$	$-\varepsilon$	$+\varepsilon$	$+1-\varepsilon$	$+1+\varepsilon$	$+\infty$	
f	$-$	$-$	$-$	$-$	$-$	$+$	$-$	$+$	Altogether three real roots.
f_1	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$+$	
f_2	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	
	no roots		no roots		one root		one root		

BRYN MAWR,
January, 1906.

THE MOVEMENT FOR REFORM IN THE TEACHING OF MATHEMATICS IN PRUSSIA.

Ueber eine zeitgemässe Umgestaltung des mathematischen Unterrichts an den höheren Schulen. F. KLEIN. Leipzig, 1904, pp. 82.

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THESE works are recent manifestations of an important movement for the improvement of the teaching of mathematics

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