analysis by which such product is represented. It is even independent of the kind of product. Thus if $\mathbf{a}$ and $\mathbf{b}$ are vectors representing length, the scalar product $\mathbf{a}\ \mathbf{b}$, the inner product $[\mathbf{a}|\mathbf{b}]$, the vector $\mathbf{a} \times \mathbf{b}$, the bivector $[\mathbf{ab}]$, the dyad $\mathbf{ab}$, the quaternion $\mathbf{ab}$, all have the dimensions $[L]^2$. It will not do for the physicist to lose sight of the fact for a moment. The followers of Grassmann, however, use the term dimension in a more geometric sense. If $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ be vectors representing lengths, $\mathbf{ab}$ is a bivector or vector of the second class (zweiter Stufe) and $\mathbf{abc}$ is a trivector, a scalar, a quantity of the third class. Again, by the law of regressive multiplication, $(\mathbf{abc})\mathbf{a}$ becomes a vector of the first class. As a matter of fact the physical dimensions of $(\mathbf{abc})\mathbf{a}$ are $[L]^4$. As a geometer one may take great delight in the theory of geometric dimensions in multiple algebra, but as a physicist he must discard it as confusing. This is precisely what is done by the physicists who use Grassmann's notation.

The whole matter may, however, be let pass with good grace inasmuch as the book is otherwise singularly free from blemishes and well fulfils the requirements for an elementary text on geometric or multiple algebras. Jahnke's lectures supplemented first with the elementary geometric theory of imaginary numbers and of quaternions and second with Gibbs's theory of dyadics, where the ideas of products are considerably more extended than anywhere else, would form an excellent introductory course on higher algebra and would furnish, in addition, a very respectable knowledge of some parts of projective geometry, mechanics, and physics.

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YALE UNIVERSITY,
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CELESTIAL MECHANICS.

An Introduction to Celestial Mechanics. By F. R. MOULTON, Ph.D., Instructor in Astronomy in the University of Chicago. New York, The Macmillan Company, 1902. xv + 384 pp.

THE need of a work written in the English language which would properly introduce astronomical students into the subject of celestial mechanics has been keenly felt for some time. In many of our universities the teaching of astronomy is confined to practical and to what is commonly called theoretical astronomy, the latter involving chiefly the derivation of suitable formulas for the numerical solution of parabolic, elliptic, and hyperbolic orbits and of special perturbations. As a result of this, the young astronomer often enters upon his work as an observer without a proper understanding of the general problems of astronomy to the solution of which his observations are to contribute, or as a mechanical computer without a real insight into the methods which he is applying, or both. It is probably not far from true that the sum total of the knowledge in celestial mechanics of many American students at the close of their university career is confined to the subject matter of Watson's Theoretical astronomy, supplemented sometimes only by a mechanical working familiarity with the excellent collections of formulas in von Oppolzer's Lehrbuch zur Bahnbestimmung. The importance of this subject is considered so great that two works along these lines have appeared in Germany in rather close succession: a new edition by Dr. Buchholz, Göttingen, of Klinkerfuss's Theoretische Astronomie, and Bauschinger's Bahnbestimmung, just off the press. All of these serve an excellent, though very special purpose, and are priceless in the hands of the teacher, but if properly absorbed, they perhaps take more than their just proportion of the student's time. They are, unfortunately, not directly designed to develop a taste nor the necessary preparation for the study of the methods of investigation and the results attained in celestial mechanics, such as would inspire to the study of Tisserand's Mécanique céleste, or Poincaré's Les nouvelle méthodes de la mécanique céleste, or Brown's Lunar theory, or other standard works along the same lines.

In the opinion of the writer, Dr. Moulton has succeeded in filling this gap and has exhibited rare skill in the selection and treatment of the necessary material. But, to quote from the preface, the work will also be welcomed by mathematical students who "desire to get an idea of the processes by means of which astronomers interpret and predict celestial phenomena." Moulton's work will develop in the astronomical student a desire for the study of the advanced mathematics necessary for further pursuit of celestial mechanics, and in the mathematical student a greater love for his subject through the practical applications to which he sees it put. It will serve, no doubt, to draw the astronomer and mathematician into closer relation. The book contains for its size an abundance of material, including the results of some of the author's own investigations; the treatment is original, clear and concise. Mention should be made in this connection of the method adopted in Chapter VII in developing Lagrange's particular solutions of the problem of three bodies; of his investigations regarding the validity of the expressions for the elements α_i and β_i in power series in m_1 and m_2 (pages 266 and 257); of his general method of determining the elements of an orbit of a comet or planet after the solution of the heliocentric and geocentric distances is accomplished (article 236), etc.

A feature of the work to be particularly commended on account of the breadth which it gives to the subjects treated is the inclusion of applications to astrophysical problems, and of principles useful in geodesy. The importance of the study of celestial mechanics is thus brought home to the student specializing in astrophysics or geodesy. There are articles on the velocity from infinity applied to the escape of atmospheres (Chapter II, Article 36); the heat of the sun, the temperature of meteors, the meteoric theory of the sun's heat, Helmholtz's contraction theory (Chapter II, articles 39–43); Gegenschein (Chapter VII, Article 124).

At the end of each of the eleven chapters is given a carefully selected set of problems, the solution of.which will add materially to a clear understanding of the text, and an historical sketch and bibliography which goes to prove a thorough familiarity with the literature of the subject, although in places omissions may be noted of references of at least equal importance to those given. The work is strictly up to date, touching as it does on the most recent researches of Hill, Poincaré, Darwin and others.

In the last two chapters of the book the author attempts to derive the working formulas necessary for the computation of a parabolic or general orbit of a comet or planet. This attempt is a radical departure from the otherwise excellent plan of the book. Although a mere outline of these methods might have been in place, the last two chapters could have been entirely omitted without in the least impairing the excellency of the work. Why should not Hansen's method of general perturbations be given a chapter, if it is considered worth while to consider in detail such very special topics as the adaptation of the solution of the problem of two bodies to the numerical computation of an orbit? In the first place, in the brief space which the author has devoted to the subject, it is not possible to do this subject justice so as to be of actual practical use to the student. The inclusion also of formulas for the reduction from apparent to mean place, etc., without proof, encourages mechanical computing. Such matters really have no place in an introduction to celestial mechanics. The presentation is patterned somewhat after Watson's Theoretical astronomy, excellent certainly in its time but now improved upon by other works. Take, for instance, the discussion on page 344 of the various fundamental formulas: why not follow Oppolzer's plan of merely stating that six conditions being available for the determination of five elements, one observation is made incomplete by substituting for it the condition that the middle place shall lie in an arbitrary auxiliary circle passed through it; that the formulas become simplest when the great circle is passed through the solar place at the middle date, and that in the so-called "Ausnahmefall" the most accurate result is obtained by adopting an auxiliary great circle through the middle place perpendicular to the geocentric motion. This latter is Oppolzer's method, and does not fail for parabolic orbits. Similarly, Oppolzer's method for determining elliptic orbits is certainly to be preferred to the original method of Gauss owing to its higher degree of approximation. But I consider these matters of really too special a nature to have a place in Moulton's work. As said before, they do not give the astronomical student a real insight into the nature of the numerical problem, nor can they be of great interest to the mathematical student.

Chapter I is designed to leave no doubt in the student's mind regarding the foundations of the subject. There might be a doubt as to whether or not the reader ought to be familiar with most of the elementary principles considered, but a review of them on the basis of Moulton's logical presentation can do no harm.

Chapter II deals with rectilinear motion and applications referred to above. It includes the consideration of forces varying directly as the distance, inversely as the square of the distance, proportional to the velocity, and proportional to the square of the velocity, the general method of solution being given the place it deserves.

Chapter III is on central forces. After a consideration of simultaneous differential equations, the orbits corresponding to a given law of force are discussed, and vice versa. The proper foundation is then laid for a consideration of the universality of

Newton's law on the basis of observed double star orbits, which leads to the conclusion that if the attraction does not depend upon the direction of the bodies from each other, Newton's law of gravitation operates in binary systems.

Chapter IV contains a judicious selection of topics relating to the potential attraction of bodies, and should particularly appeal to the geodesist. Chapter V is on the problem of two bodies. Chapter VI gives the fundamental principles of the problem of n bodies.

Chapter VII is on the problem of three bodies. Its subject matter is particularly well chosen, leaving the reader with a clear conception of all that has been accomplished in attempting its solution. The particular solutions are clearly presented, the treatment of the surfaces of zero velocities, the stability of particular solutions, etc., is concise and to the point.

Chapter VIII contains the geometric interpretation of perturbations and is valuable as a means of assisting the student to a proper understanding of the effects of perturbation on the elements of an orbit.

Chapter IX treats of analytic methods of determining perturbations. The object of the chapter is stated by the author in the following words: "The chief difficulties which arise in getting an understanding of the theories of perturbation come from the large number of variables which it is necessary to use, and the very long transformations which must be made in order to put the equations in a form suitable for actual computations. It is not possible, because of the lack of space, to develop in detail the explicit expressions adapted to computation; and indeed it is not desired to emphasize this part, for it is much more important to get an accurate understanding of the nature of the problem, the mathematical features of the methods employed, the limitations which are necessary, the exact places where approximations are introduced, if at all, and their character, the origin of the various sorts of terms, and the foundations upon which the celebrated theorems regarding the stability of the solar system rest" (page 256). The author has performed his task in an exceptionally satisfactory manner.

Chapters X and XI contain the derivation of formulas for the computation of the orbits of comets and asteroids, and have been referred to above.

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