student of mechanics proper may be interested in this matter is a question. The advisability of including such a treatment here cannot, however, be well open to doubt. Mathematical astronomy as one conclusion of mechanics is quite in the spirit of the present time.

Personally we should have been happy to see introduced, before these pages crammed with necessarily complicated analysis, a treatment of two problems which are of prime importance in physics and without which no student of mechanics for its applications in physics rather than in astronomy can feel himself well equipped. The first is the study of the application of Lagrange's equations and the kinetic potential to problems in physics and chemistry. This was early introduced by Maxwell into electricity and magnetism, and is treated at length in J. J. Thomson's little book on the subject. It goes a long way toward giving the lagrangian function real physical interest. Such a chapter might properly find its place just before the tenth. The second problem is that of statistical mechanics which both in theory and its applications to the kinetic theory of gases and to thermodynamics is well available, though in too great detail for the débutant, in the works of Gibbs and Jeans. A chapter on this subject, placed perhaps right after the tenth, would go far on the way toward completing a treatise on dynamics in a direction likely soon to be of greater general interest than the special methods of astronomy. May we not hope that the author's interest in astronomy (he was recently appointed astronomer royal of Ireland, a position illustrious with great names) will not prevent him from giving adequate treatment of these two physical problems in later editions of his book?

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YALE UNIVERSITY,
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SOME RECENT FOREIGN TEXT BOOKS.

A Course in Practical Mathematics. By F. M. SAXELBY. London, Longmans, Green, and Co., 1905. 8vo. viii + 2 unnumb. + 430 pp. Price, $2.25.

Die Planimetrie für das Gymnasium. By GUSTAV HOLZMÜLLER. Leipzig and Berlin, B. G. Teubner, 1905. 8vo. viii + 240 pp. Price, M. 2.40.

Methodisches Lehrbuch der Elementar-Mathematik. By GUSTAV HOLZMÜLLER. Leipzig and Berlin, B. G. Teubner, 1903. 8vo. xiv+370 pp. Price, M. 4.40.

Vorbereitende Einführung in die Raumlehre. By GUSTAV HOLZMÜLLER. Leipzig and Berlin, B. G. Teubner, 1904. 8vo. x+123 pp. Price, M. 1.60.

IT is probably a fact that in America we look upon the educational system of England as rather unprogressive. It is also a fact that we are accustomed to look upon its mathematics as less interesting and fruitful than that of the other leading countries of Europe. In elementary work we think of the English school as so bound to Euclid, so given to algebras of the abstract, uninteresting Todhunter type, so repelled by the practical in its adherence to what it deems the cultural, that we feel we shall find nothing suggestive in this line on British soil. And so our teachers today hear more of what is done to improve mathematics and education in France, Germany, and Italy, than in the mother country to which we are so largely indebted for our early text books and curricula. At the present time we indulge in much talk and we write numerous essays upon making our elementary mathematics more practical, upon early setting forth such of the uses of the science as are within the range of the beginner's understanding, and upon relating mathematics to the other sciences and to the general life about us, and if we give the matter a passing thought we pity England because its educators have not awakened to this vital issue.

In view of this feeling on our part, it is interesting to see what England is really doing in the way of improving elementary education in general, and mathematics in particular. Ten years have seen a very marked change of view with respect to Euclid, and although the standard classical schools still use what seem to us very uninteresting algebras and geometries, it is to England that we have to go for some of the best elementary text books we have today on the applications of mathematics. We have talked a great deal, but English teachers have acted, and neither France nor Germany is facing the issue more earnestly.

Of the several elementary text books on applied mathematics that have appeared in the last few years, Mr. Saxelby's is by far the best. It is written for institutions of a class at present more highly developed in England than here, although our technical schools are beginning to make themselves felt in a very serious way. It recognizes directly the utilitarian needs of these institutions, and in twenty-nine chapters, with less than four hundred pages, it presents the essential parts of plane trigonometry, graphs and analytic geometry, the differential and integral calculus, vector analysis, solid analytic geometry, and differential equations. This seems like a large domain, and the natural prejudgment would be that the work must be scrappy, unscientific, and incomplete. But after all, this depends upon the point of view. All elementary mathematics is scrappy, for the beginner cannot be expected to exhaust each subject before he proceeds to the next one; all is more or less unscientific, as when a pupil uses the binomial theorem before he knows anything about differentiation and convergency; and the treatment of no topic is complete. Thoroughness is a matter of degree.

Mr. Saxelby has condensed a great deal of theory into a small compass; he has eliminated what is not essential from the practical standpoint, and has arranged his material on a psychological rather than a logical plan. For example, he proves the addition formula for sin $(A+B)$ in ten lines, but only for the case of A and B acute. The general case he postpones until he takes up vector analysis. This may not seem logical, but it is very psychological, and from the standpoint of teaching it is commendable. He does little with conics, which probably brings down upon his head much criticism from the traditional schoolmaster, but he investigates a great many curves such as the engineer and the statistician need. He does not study subnormals and subtangents, but he plots $i=50 \sin 600t+20 \sin 1800\,t$ for the purpose of studying an alternating electric current, and he considers an interesting set of curves representing compound periodic oscillations, and the educational results of such work are not to be discounted by the usual remark about utilitarianism.

The fact is that a student who fairly masters a book of this kind will like mathematics better than one who puts the same amount of time on our traditional courses; he will have a far better working knowledge of the applied science; he will have cultivated more originality; he will be more desirous of studying higher mathematics; and he will not have suffered materially in the pure field. Old school mathematicians may not like the idea of making applications the basis of selection, and they may be content with that most archaic of all our sets of problems, the one generally found in our books on the calculus, but a reaction is bound to come in our college mathematics just as it has come in our school mathematics. The traditional boundaries between subjects must give way more and more, just as Mr. Saxelby has made them give way in his book; the founding of every step upon bed rock before the next step is taken must be recognized as the mission of a mature mathematician, not of a beginner who will probably not become a scientist; and the genuine applications of mathematics must replace the time-worn fictions of the 18th century writers.

This does not mean that Mr. Saxelby's work is a satisfactory text-book for American colleges. Some of his applications would not appeal to us; some of his symbolism is not ours; his text-book has not the appearance which our tastes demand; we have a different system of measures, in part, and we would probably be more traditional than he is in some respects, as in the study of certain quadratic curves. But books of this class are healthy ones for our college and preparatary teachers to have at hand; they breathe of the future; they stand for mathematics as a whole instead of its minor provinces, and for mathematics interrelated and extrarelated instead of isolated in all its parts. It is not improbable that the present agitation in the development of preparatory mathematics in this country will penetrate even into our colleges and stir up some new life. When this is done America will begin to develop for all of its schools what England is developing for its technical institutions, books of the spirit of Mr. Saxelby's, suited to the peculiar conditions and needs of our country.

In Germany of late no one has been more active in the revitalization of elementary mathematics than Professor Holzmüller. He does not represent the movement for which Mr. Saxelby stands, but his work is none the less suggestive and valuable. Germany has never been a slave to Euclid, nor have even the most conservative teachers in that country ever agreed to the interpretation of the dogma of thoroughness which has made much of English and American mathematics so repulsive. It is well known that the German schools begin their algebra and geometry much earlier than we, and extend them over a longer period, adopting a parallel instead of a tandem arrangement. It is in this elementary field, and with these principles to guide him, that Professor Holzmüller has worked.

His Planimetrie is designed for the Quarta to the Untersekunda, say the sixth to the ninth school years. As usual with German elementary text-books, it pays relatively little attention to formal definitions, but devotes much care to developing clear concepts; it is not particularly concerned with what propositions are discussed, but rather as to how they are treated; and it is not so much moved by the desire for a logical sequence as by the hope that interest, originality, and correct ideas may result from the study. As Professor Holzmüller says in his Einführung in die Raumlehre, "Erst methodische Auswahl, dann System! Erst Pestalozzi, dann Euklid!"

Brought up as we are on some revised version of Euclid, and accustomed to the traditional nomenclature, it seems strange to us to see a statement like this for a sixth-grade child: "Eine gerade Punktreihe ist eine solche, die dem Auge durch einen einzigen Punkt verdeckt werden kann." We would be apt to say either that it was too advanced for a child of eleven or twelve, or that it was not sufficiently scientific, depending upon our point of view. And yet this is a type of the statements developed for these beginners. The work is at first largely constructive, with proofs as the capabilities of the child permit. Following the advanced tendencies set forth by Méray and De Paolis there is no distinction drawn between plane and solid geometry. In the sixth school year the tetrahedron is briefly studied before the concept of angle is introduced, and most of the essential propositions of Euclid I are proved along with some treatment of elementary solids. In the Unter- and Obertertia, the seventh and eighth school years if we allow three years before the Gymnasium, plane geometry is fairly well covered, with the exception of the doctrine of similarity. Some of the work laid down would be considered too difficult for our average high schools, particularly the propositions involving cross quadrilaterals and the development of Heron's formula for the triangle. The work of Untersekunda, say the first year in our high school in point of time, relates to similar figures, the radical axis, inversion, elementary projections for map drawing, the regular solids, and the evaluation of π.

The Methodisches Lehrbuch der Elementar-Mathematik, Dritter Teil, now in its second edition, is intended for the Real and Fachschulen, and hence it involves a line of work not found in the Planimetrie. It is an excellent preparation for the study of modern mathematics, combining as it does the essentials of the recent geometry of the triangle, the theory of duality, a little projective geometry, propositions like Feuerbach's and Malfatti's on the circle and Pascal's on the hexagram in a conic, the theory of geometric involution, the theory of conics with its application to physics, stereometry, axonometry, spherics, and the application of higher algebra to geometry. Compared with Mr. Saxelby's work it involves much more mathematics and it is less practical; but on the other hand it is practical in certain directions which are not found in the English work, particularly in the theory of electricity, in higher mensuration, and in cartography. The book is one to be recommended to teachers who wish a brief general introduction to modern mathematics.

Professor Holzmüller's Vorbereitende Einführing in die Raumlehre has not so much to commend it as his other works just mentioned. It is a good book for teachers who wish to supplement the regular work in geometry by interesting side lights on the subject. Such topics as the pythagorean triangle numbers, the geometric proof concerning the sum of n odd numbers, the solids inscriptible in the regular solids, and the various types of semi-regular and stellar polyhedra are here discussed with a simplicity that renders them usable in the school room.

It should be said that none of Professor Holzmüller's works are text-books in the American sense. This is, of course, understood by all who have visited German schools or have read Professor Young's excellent work on the subject. Our type of text-book is practically unknown in the country where the pupils study and recite in the same class period and always under the master's guidance. They are, however, none the less suggestive to the American teacher, for they show what can be accomplished with pupils in certain grades, what can be omitted in the preparation of students from whom some of the world's best mathematicians are being made, what modern mathematics can replace those portions of the ancient science that are eliminated, what attention can be given to applications without interfering with the vague idea of culture which so often possesses us, and what correlation is possible among the various traditionally separate fields of the science of mathematics.

DAVID EUGENE SMITH.