

particular order higher than the fourth still asserts its independence and resists stubbornly every attempt to introduce law and order. If we except [certain special researches upon quintic curves] we may say that up to the present nothing has occurred to illuminate the thick darkness which conceals from us the properties of those curves which should come next after conics, cubics and quartics. Not only so, but there is not even any special curve of fifth order which has received a particular name."

The course here advocated is somewhat modernized from the traditional type of college courses in geometry, but it is not, therefore, chimerical. A survey of the field should convince us that in three semesters a college student can become posted on the essentials of conics, quadric surfaces, and cubic curves. Individual teachers will prefer a different order from that here advocated, either for theoretical reasons or on account of the special qualifications of the students under their charge; but it is believed that the main thesis will command general approval — the most modern methods and the most general propositions of present day geometry must be made available for the college student.

VASSAR COLLEGE,
November 29, 1905.

FOUR BOOKS ON THE CALCULUS.

Die Anfangsgründe der Differentialrechnung und Integralrechnung. Für Schüler von höheren Lehranstalten und Fachschulen sowie zum Selbstunterricht. Dargestellt von DR. RICHARD SCHRÖDER. With numerous exercises and 27 figures in the text. Leipzig, B. G. Teubner, 1905. vii + 131 pp.

Hauptsätze der Differential- und Integralrechnung, als Leitfaden zum Gebrauch bei Vorlesungen. Zusammengestellt von DR. ROBERT FRICKE. Fourth edition, with 74 figures in the text. Braunschweig, Friedrich Vieweg und Sohn. xv + 217 pp.

Repetitorium und Aufgabensammlung. Von DR. FRIEDRICH JUNKER. Second, improved edition, with 46 figures in the text. Leipzig, G. J. Göschen, 1905. 16mo. 127 pp.

Sammlung von Formeln und Sätzen aus dem Gebiete der elliptischen Functionen, nebst Anwendungen. Von J. THOMAE. Leipzig, B. G. Teubner. 43 pp.

THESE four titles are here grouped for the purpose of making certain comparisons both among themselves and with current American texts in the corresponding fields.

The first volume, as its title indicates, is an introduction to the calculus and is intended for use in the upper classes of gymnasia, in accordance with the permission afforded in the Prussian curriculum of 1901. While the text is strictly elementary, it is clear and logical in its development, beginning with limits notions and geometric interpretation of the derivative. The derivatives of all ordinary forms, including inverse and implicit functions, are deduced before any exercises are introduced; then over one hundred exercises, very many of which are complicated combinations of algebraic and transcendental functions, are given in one list, with the results in each case printed in juxtaposition. Eleven pages, or one-twelfth of the book, are devoted to indeterminate forms, with illustrations but no exercises. Thirty-three pages are given to tangents, normals, subtangents, subnormals, etc., the forms of which are all worked out for the circle, parabola, ellipse, hyperbola, semicubical parabola, cissoid, cycloid, lemniscate and cardioid, with no exercises left for the student. Likewise rectifications and quadratures for these curves are worked out in full in the text.

The second volume is the fourth edition of Fricke's well-known text, improved and enlarged. When we consider the clearness and rigor of presentation and contemplate the number of subjects here treated within a little more than two hundred pages, including all the ordinary topics of elementary calculus, together with chapters on the theory of definite integrals, ordinary differential equations, and functions of a complex variable, we wonder how it can be done in so small a space. The explanation is that the book contains very few exercises for the student and the development of principles goes on uninterruptedly from beginning to end. From the standpoint of the purely theoretical science, this presentation of the subject has its advantage in that the attention is not diverted from the steady swing of the doctrine from topic to topic; and the student whose mind quickly and readily grasps the principles of the calculus in the abstract must

surely find great delight in the clear and vigorous presentation, the pleasing arrangement of the page, and the skillful development step by step, from the elementary to the more involved theorems. But the student who must handle the concrete before he grasps the abstract, who must work through many exercises up to the comprehension of the principles, will need to combine with such a text as Fricke's a good collection of exercises and applications in the differential and integral calculus.

Fortunately such exercise books are not wanting in Germany, and the third title in the list above is a book of this kind for the differential calculus. It is a small volume containing nearly five hundred exercises, more than half of which are set forms for differentiation or for the formal application of the rules under the various topics, and in all cases the results are given with the exercises, leaving no chance for independent discovery. Of the remaining exercises which are more in the nature of problems, all are geometric, relating to maxima and minima, asymptotes, tangents, normals and numerous properties of curves and surfaces, and under all the topics large numbers of illustrations are worked out in full. There are no application to mechanics or other subjects.

These two volumes of Fricke and Junker (with a corresponding exercise book on the integral calculus) would seem to constitute a somewhat typical provision for the needs of the German student of the calculus, quite in contrast with the plan of our own text-books — on this and other subjects. In favor of the former plan is the greater compactness of the text and the more closely knit continuity in the development of the principles; while the latter plan secures the juxtaposition of principles and applications in a single volume and seems more certain to secure careful attention to the concrete side of the work on the part of the student.

The last title here under consideration represents still another scheme of arrangement and presentation of formulas and principles of a subject, quite in contrast with the other three volumes and also with publications in this country. Thomae has collected the chief formulas and theorems in the realm of elliptic functions, without proofs or explanations beyond what is necessary for a clear understanding of the notation. These are included compactly but clearly under forty-two divisions

covering only twenty-six pages and the remaining eighteen pages are devoted to brief expositions of twenty typical applications of the elliptic functions. The collection of formulas is based on the Jacobi-Legendre fundamental functions and notation, as being, in the author's judgment, better adapted to applications involving numerical computation than the Weierstrassian forms used by Schwarz in his collection, though these latter are of the highest theoretical importance.

Thomae's collection puts in concise and accessible form the whole range of the elliptic function doctrine, as based on the theta function, the zeta function, the omega function, and the Legendre normal forms, and shows fully how useful and practical this development becomes in application to a wide class of problems.

In conclusion it seems quite clear that collections of principles and formulas are highly appropriate and useful for students in advanced stages of progress, but that for elementary students the form of presentation to be commended is that in which problems and exercises are skilfully used in the text to lead up to the statement and proof of principles, as well as to illustrate and clarify the theory in immediate connection with its formal development.

H. E. SLAUGHT.

THE UNIVERSITY OF CHICAGO,
May 28, 1906.

SHORTER NOTICES.

Einleitung in die Funktionentheorie. Abteilung 2. By OTTO STOLZ and J. ANTON GMEINER. Leipzig, B. G. Teubner, 1905. viii + 355 pp.

WITH the publication of the second part of the Stolz and Gmeiner *Funktionentheorie*, the revision of Stolz's *Allgemeine Arithmetik* is complete. The *Theoretische Arithmetik* and the *Funktionentheorie*, which must still be regarded as parts of the same whole, together present a course in analysis which begins with the integers and includes all the usual operations except differentiation and integration. Taken in connection with Stolz's *Calculus*, they form a kind of German *Cours d'Analyse*.