covering only twenty-six pages and the remaining eighteen pages are devoted to brief expositions of twenty typical applications of the elliptic functions. The collection of formulas is based on the Jacobi-Legendre fundamental functions and notation, as being, in the author's judgment, better adapted to applications involving numerical computation than the Weierstrassian forms used by Schwarz in his collection, though these latter are of the highest theoretical importance.

Thomae's collection puts in concise and accessible form the whole range of the elliptic function doctrine, as based on the theta function, the zeta function, the omega function, and the Legendre normal forms, and shows fully how useful and practical this development becomes in application to a wide class of problems.

In conclusion it seems quite clear that collections of principles and formulas are highly appropriate and useful for students in advanced stages of progress, but that for elementary students the form of presentation to be commended is that in which problems and exercises are skilfully used in the text to lead up to the statement and proof of principles, as well as to illustrate and clarify the theory in immediate connection with its formal development.

H. E. SLAUGHT.

THE UNIVERSITY OF CHICAGO,
May 28, 1906.

SHORTER NOTICES.

Einleitung in die Funktionentheorie. Abteilung 2. By OTTO STOLZ and J. ANTON GMEINER. Leipzig, B. G. Teubner, 1905. viii + 355 pp.

WITH the publication of the second part of the Stolz and Gmeiner Funktionentheorie, the revision of Stolz's Allgemeine Arithmetik is complete. The Theoretische Arithmetik and the Funktionentheorie, which must still be regarded as parts of the same whole, together present a course in analysis which begins with the integers and includes all the usual operations except differentiation and integration. Taken in connection with Stolz's Calculus, they form a kind of German Cours d'Analyse.

The volume before us begins with Chapter VI, containing a thorough discussion of convergence and divergence criteria for infinite series. The chapter is new and, according to the preface, due to Gmeiner, who has based it principally on the article of Pringsheim in the thirty-fifth volume of the *Mathematische Annalen.* The subject is an extremely interesting one, connecting with several of the most difficult problems in analysis. Its connection with the related problem of determining scales to represent the orders of infinity of functions is fairly evident though not emphasized; but its equivalence with the problem of the convergence of

$$\int_a^\infty f(x)dx = \underset{b=\infty}{\mathrm{L}} \int_a^b f(x)dx$$

we did not find referred to, doubtless because the book does not take up the theory of definite integrals.

Chapter VI is somewhat in the nature of a digression since it comes between a chapter on power series and Chapter VII, the theory of monogenic analytic functions. The last mentioned chapter is based chiefly on lectures of Weierstrass and was not contained in the Allgemeine Arithmetik. It is followed by a chapter on the circular functions which contains in more extended and detailed form the contents of chapter 6, volume II of the older work, *i. e.*, the theory of the functions a^x, $\log x$, $\sin x$, $\sin^{-1}x$, etc., for complex values of x.

Chapters IX, X, and XI have all been revised and enlarged by Gmeiner. The chapter on infinite products (IX) differs from the corresponding chapter of the Allgemeine Arithmetik in that the series formed by taking the logarithms of the factors is used as little as possible. There have been added a number of theorems due to Arzelà about products whose factors are functions of an independent variable, thus involving the notion of uniform convergence. The treatment of finite and infinite continued fractions (X, XI) has also been much extended by Gmeiner. Among other changes, such expressions as

$$3 + \frac{2}{7} + \frac{5}{2} - \frac{1}{1} - \frac{1}{2},$$

which in the Allgemeine Arithmetik were regarded as "sinnlos "* are here designated as "improper" continued fractions and their value determined by the following definition: The expression

$$(1) \qquad b_0 + \frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n}$$

determines the quantities Z_k by the equations

$$(2) \qquad \begin{aligned} Z_n &= b_n, \quad Z_{n+1} = 1, \\ Z_m &= b_m Z_{m+1} + a_{m+1} Z_{m+2} \\ & \qquad (m = n-1, n-2, \cdots, 1, 0). \end{aligned}$$

The value of (1) is Z_0/Z_1. The only fractions which now fail to have values are those for which Z_1 is zero. If $Z_\kappa = 0$ for any value of κ other than 1, the fraction is improper. This definition leads to no new extension of the notion of an infinite continued fraction because improper convergents had been used previously even where the corresponding continued fractions were regarded as meaningless. Among the modern developments added to the chapter on infinite continued fractions, it is interesting to note the theorems of Van Vleck published in the second volume of the *Transactions*.

To sum up this notice and also the review of the first volume in last December's BULLETIN — the work as a whole gives an effect of conservatism, maturity and poise. It is not as likely as the modern French books to stimulate research, but it has a permanent value as a repository of accurate information about the conventional functions and processes of analysis.

OSWALD VEBLEN.

Lezioni sul Calcolo degli Infinitesimi, date nella R. Università di Modena da ETTORE BORTOLOTTI. Raccolte dal Dr. ARMANDO BARBIERI. Modena, 1905. vii + 61 pp.

BORTOLOTTI'S Calcolo degli infinitesimi is a short course of lectures on various questions that arise in connection with the determination of the relative orders of two infinitesimals (infinites)

*Because $2 - \frac{1}{1} - \frac{1}{2} = 0$. The significance of this definition of "improper" continued fractions may perhaps be suggested by the equation

$$3 + \frac{2}{7} + \frac{5}{0} = 3 + \frac{2 \cdot 0}{0 \cdot 7 + 5} = 3.$$