THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and thirtieth regular meeting of the Society was held in New York City on Saturday, October 27, 1906, the following thirty-three members being in attendance:

Professor G. A. Bliss, Professor E. W. Brown, Dr. W. H. Bussey, Professor F. N. Cole, Miss E. B. Cowley, Dr. W. S. Dennett, Professor T. S. Fiske, Miss Ida Griffiths, Mr. S. A. Joffe, Professor Edward Kasner, Professor E. O. Lovett, Professor T. E. McKinney, Professor James Maclay, Professor Max Mason, Professor Richard Morris, Professor W. F. Osgood, Professor James Pierpont, Mr. H. W. Reddick, Professor L. W. Reid, Dr. R. G. D. Richardson, Miss S. F. Richardson, Miss I. M. Schottenfels, Professor Charlotte A. Scott, Mr. L. P. Siceloff, Professor D. E. Smith, Professor P. F. Smith, Professor J. H. Tanner, Professor Oswald Veblen, Mr. H. E. Webb, Professor H. S. White, Miss E. C. Williams, Professor J. E. Wright, Professor J. W. Young.

President W. F. Osgood presided at the morning session, Ex-President T. S. Fiske at the afternoon session. The Council announced the election of the following persons to membership in the Society: Professor A. F. Carpenter, Hastings College; Dr. H. M. Dadourian, Yale University; Mr. T. E. Gravatt, Pennsylvania State College; Rev. A. S. Hawkesworth, Allegheny, Pa.; Mr. H. R. Higley, Pennsylvania State College; Dr. Mario Kiseljak, Fiume, Hungary; Dr. Emanuel Lasker, New York, N. Y.; Professor Ernest Lebon, Lycée Charlemagne, Paris; Dr. R. L. Moore, Princeton University; Mr. W. P. Russell, Pomona College; Professor J. H. Scarborough, State Normal School, Warrensburg, Mo.; Mr. L. P. Siceloff, Columbia University; Professor Cyparissos Stephanos, University of Athens. One application for membership in the Society was received.

A list of nominations of officers and other members of the Council was adopted and ordered placed on the official ballot for the annual election at the December meeting. A committee was appointed to audit the Treasurer's accounts for the current year.

Professor W. F. Osgood tendered his resignation from the Editorial Committee of the *Transactions*, finding it impossible to assume the burdens of the office. The vacancy was filled by the appointment of Professor H. S. White.

The following papers were read at this meeting:

(1) Miss S. F. RICHARDSON: "Note on poristic systems of polygons."

(2) Professor R. D. CARMICHAEL: "Multiply perfect numbers of four different primes."

(3) Dr. ARTHUR RANUM: "On Jordan's linear congruence groups."

(4) Professor BEPPO LEVI. "Geometrie projettive di congruenza e geometrie projettive finite."

(5) Professor CHARLOTTE A. SCOTT: "Note on regular polygons."

(6) Professors MAX MASON and G. A. BLISS: "Some problems of the calculus of variations in space with variable end points."

(7) Professor EDWARD KASNER: "Note on the transformations of dynamics."

(8) Professor G. A. MILLER: "Groups of order p^m containing exactly $p + 1$ abelian subgroups of order p^{m-1}."

(9) Professor G. A. MILLER: "The groups in which every subgroup is either abelian or hamiltonian."

Professor Levi's paper was communicated to the Society through Professor Osgood. In the absence of the authors, the papers of Professor Carmichael, Dr. Ranum, Professor Levi, and Professor Miller were read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Miss Richardson's paper appears in full in the present number of the BULLETIN.

2. In this paper Professor Carmichael shows that there are but two multiply perfect numbers of (only) four different primes, and that these are $2^5 \cdot 3^3 \cdot 5 \cdot 7$, of multiplicity 4; and $2^9 . 3 \cdot 11 \cdot 31$, of multiplicity 3.

3. Dr. Ranum proves that if G is any Jordan group, mod p^a, p being prime, and if G_i $(i = 0, 1, \cdots, a)$ is the invariant subgroup whose matrices are congruent to identity, mod p^i, then, except for certain low values of p, every matrix in G of

period $p^i(i = 0, \cdots, a - 1)$ is contained in M_{a-i}, but not in M_{a-i+1}, and conversely; moreover every matrix of period p^i $(i = 0, \cdots, a)$ is commutative with every matrix of period p^{a-i}. These properties of Jordan groups are derived for the sake of their application to the more general linear congruence groups given in a paper entitled "The group of classes of congruent matrices, etc.," presented at the summer meeting of September, 1906. The great majority of the latter groups are thus shown to be not only new linear groups, but abstractly new, *i. e.*, they are not simply isomorphic with any Jordan groups of the same degree, or subgroups thereof.

4. In a note published in the *Transactions* (April, 1906) Drs. Veblen and Bussey have characterized the spaces consisting of a finite number of points in which a projective geometry exists. Professor Levi recalls that he has already occupied himself with spaces which include these as a particular case, in a paper on the "Fondamenti della metrica projettiva" (*Memorie* of the Turin Academy, series 2, volume 54, 1904). The coördinates of the points of these spaces are classes of numbers and rational expressions with rational (or entire) coefficients of a certain system (finite or not) of symbols, corresidual according to a determined irreducible modulus M. In these spaces a projective geometry exists which the author calls one of congruence.

Professor Levi indicates further how for such geometries the theorem of von Staudt is changed. After showing that, in any projective geometry, a transformation ϕ which converts harmonic groups into harmonic groups and leaves fixed the points $0, 1/0, 1$ is characterized by the equality

$$\phi[f(\xi, \eta, \zeta, \cdots)] = f[\phi(\xi), \phi(\eta), \phi(\zeta), \cdots],$$

where f is a rational function, he demonstrates that, if the number of the points of a right line is finite and the modulus M does not contain numbers, the study of the transformation ϕ corresponds to the study of the birational transformations with rational coefficients of the variety which represents the modulus M in itself. If, on the contrary, to the modulus M belongs a prime number p and the question is precisely about finite geometries in which the abscissas of the points of a right line belong to a Galois field $G(p^n)$, instead of the theorem of Staudt the following can be enunciated: Determine any succession whatever $n', n'', n''', \cdots, n$ of divisors of n, with $n' > 1$, and let each of them be equal to the preceding multiplied by a prime. To this succession of numbers $n^{(k)}$ one can get to correspond a succession of Galois fields $G(p^{n^{(k)}})$ contained in $G(p^n)$ and each containing the preceding, and a series of transformations ϕ characterized by the fact that each of them leaves fixed the points whose abscissas belong to one of these fields $G(p^{n^{(k)}})$, but removes the points of the next. The points belonging to one of these fields remain fixed as soon as one of them, not belonging to the preceding, remains fixed. To make sure that a ϕ comes to the identity it is necessary to verify at the most that it has as many fixed points, each rationally independent of those formerly considered, as there are prime divisors of the number n.

5. Miss Scott gave constructions for the regular pentagon, heptagon and nonagon by means of rectangular hyperbolas cutting a given circle at four vertices of the desired polygons; and from these deduced constructions for four vertices of the polygon on a given rectangular hyperbola.

6. The problem of finding a curve $y = y(x)$, $z = z(x)$, joining two given fixed points and minimizing an integral

$$I = \int f\left(x, y, z, \frac{dy}{dx}, \frac{dz}{dx}\right) dx$$

has been discussed according to recent methods by several writers, but comparatively little seems to have been done on the problem in which the end points are allowed to vary on surfaces or curves. In the paper of Professors Mason and Bliss the necessary conditions for a minimum when the end points are variable, analogous to those in the plane, and also sufficient conditions, have been derived. The problem in space has peculiarities essentially different from those of the corresponding problem in the plane on account of the fact that the end points may vary not only on curves but also on surfaces. The theory of congruences of curves plays an important part because the family of extremals transversal to a curve or surface is itself a congruence. A feature of the paper is the use of the parameter representation.

7. Appell has shown that the system of trajectories of a particle moving in a (positional) field of force is converted by any projective transformation into the system of trajectories corresponding to some other field of force. Professor Kasner showed, synthetically and analytically, that no other contact transformations possess this property.

8. Professor Miller's first paper appears in full in the present number of the BULLETIN.

9. Professor Miller's second paper is devoted to an extension of the results in the paper on "Non-abelian groups in which every subgroup is abelian" published in volume 4 of the *Transactions*. The principal theorems may be stated as follows: There is one and only one group of order 2^m which involves operators whose orders exceed four and satisfies the additional conditions that every subgroup is either abelian or hamiltonian and that at least one subgroup is hamiltonian. If every subgroup of a group of order 2^m, $m > 4$, is either abelian or hamiltonian and if it contains at least one hamiltonian subgroup, the entire group is hamiltonian. If a group contains at least one hamiltonian subgroup and if all its subgroups are either abelian or hamiltonian, it is the direct product of the hamiltonian group of order 2^m and an abelian group of odd order, unless it is the group of order 24 which does not contain a subgroup of order 12. There are only two non-hamiltonian groups which contain at least one hamiltonian subgroup and whose other subgroups are either abelian or hamiltonian.

F. N. COLE,
Secretary.

THE STUTTGART MEETING OF THE DEUTSCHE MATHEMATIKER-VEREINIGUNG.

THE annual meeting of the Deutsche Mathematiker-Vereinigung was held at Stuttgart September 17–20, 1906, forming a section of the seventy-eighth convention of the Deutsche Naturforscher und Aerzte. The following papers were presented:

(1) O. BLUMENTHAL, Aachen: "Integral transcendental functions and Picard's theorem" (report).

(2) A. PRINGSHEIM, Munich: "Fourier's integral theorem."