appears a note concerning an important misprint in Dini's discussion of Fourier's series, referring also to a letter from Dini. Earlier in the text Dini was mentioned as the only one among several writers who had given a rigorous proof of the correctness of the expansion of $f(x)$ in a series of terms each involving a Bessel function. This expansion caused Todhunter to state that many German writers credit Fourier with its authorship, though in fact he did not give it; and into this category it may be inferred that Nielsen has fallen.

Last of all comes a most important part of the book, a very complete bibliography, giving referencces to both theoretical and applied work in cylindrical harmonics. In addition, at the bottom of many pages are references to the original sources of nearly all formulas, in many cases proved by methods different from those in the text. A paper by Glaisher on Riccati's equation appeared in *Philosophical Transactions* in 1881, not in 1882, while to Schläfli's credit may be added an extensive article in *Annali di Matematica*, series 2, volume 6. In 1867 Lommel mentioned nine writers, while in this list appear one hundred and fifty-five.

With all the work which Dr. Nielsen has brought within the compass of a volume of moderate size, and in which he has had so great a share, there remain unexplored fields. Apart from his frank statement that we do not know the necessary and sufficient conditions under which a function is developable in a Fourier's series, there are other topics more closely related to the text, such as the remainder terms in null developments, and the single valuedness of developments in a Schlömilch's series, also many topics not fully treated in the present work.

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Space and Geometry in the Light of Physiological, Psychological and Physical Inquiry. By Dr. ERNST MACH. Translated from the German by THOMAS J. McCORMACK. Chicago, The Open Court Publishing Company, 1906. 148 pp.

To appreciate this work it is necessary to view it in its relation to two complementary movements in modern mathematical thought, namely, the logical movement and another that may be significantly called biological. The aim of the former has been to detect and to enumerate all definite ideas or terms that are indefinable and all definite propositions that are indemonstrable, to combine these primitives into all logically available sets of compatibles, and then by processes of pure inference to render explicit in systematic form the various contents implicit in the various sets. The method has been that of abstraction, postulation and deduction; the concern has been, not with external validity or applicability, but solely with consistence, with logical coherence, with conceptual and propositional harmony; and the effect has been more and more to eliminate intuition, to detach mathematics from experience, from reality, from life, from the sensuous world of things and events. The advance has followed two paths, the path of the well-known mathematical rigorists, and the path of symbolic logic under the leadership of C. S. Peirce, Schröder, and especially Peano; and these paths unexpectedly to most have been found to converge in the remarkable thesis that pure mathematics is symbolic logic and that pure logic is symbolized mathematics. A notable mark of the movement has been a strong tendency to nominalism, as witness, for example, Hilbert's Fundamental principles of geometry, in which the point, the line and the plane are nothing but names of undefined entities (replaceable by other names or entities or both) satisfying a prescribed system of postulates.

Meanwhile another movement has been going on and in recent years rapidly gaining in interest and force. It contains two principal components. These are easily confounded because they agree in seeking to reattach mathematics to experience, in demanding a reunion and a more intimate union than ever before of mathematical science and reality. Nevertheless the two components are entirely distinct. They differ radically in respect to the kind of union they severally contemplate. The one aims to establish or to reestablish a union supposed never to have existed or to have been broken up. The other aims to discover a union supposed to have existed always as in the nature of things. The former is partly due to a reaction against the logical movement, the tendency to nominalism, and partly to the increasing demand of an increasing number of sciences such as engineering, chemistry, anthropology, economics, psychology and statistics, that mathematics shall cease to dwell apart, interested exclusively in its own evolution as a pure science, and adapt itself, its teaching and investigation, to their needs. The latter component is not of the nature of such reaction and is not due to such a demand. Its aim is to unite mathematics and experience, not in the sense of rendering the science applicable in other fields of investigation, but in the sense of showing that mathematical concepts, however tenuous or pure or remote or recondite, have been literally evolved continuously in accordance with the needs of the animal organism out of the elements (feelings) of physiological experience. This is why the second movement may properly be described as biological. The extent to which the movement may be destined to succeed is a question that only time can answer. The enterprise is undoubtedly legitimate and is one that men as rational beings were bound sooner or later to undertake. The significance of it, its bearings on theory and on practice, on teaching in particular, and on the conception that the future mathematician may have of his science and of its relations to other modes and forms of intellectual activity, can scarcely fail to be profound.

The leading contributor to the biological movement in mathematical criticism, at least the most widely known contributor, is Professor Mach, whose Science of mechanics, Popular scientific lectures, and Contributions to the analysis of the sensations are well-known and are highly valued in scientific and critical circles everywhere throughout the western world. The book in hand ought to be read and pondered by every teacher of mathematics and by every educated guardian of the young. Physiological space, the space of vision, the space of touch, the space of audition, metrical space, the space of geometry, the correspondences, likenesses and differences of these, feelings of constance, of motion, of velocity, of acceleration, of locations, the interplay and biological functions of these sensations, their contributions to geometry,— such are some of the themes of the discussion, which, running from simple facts of vision through a critical characterization and comparison of the chief varieties of metageometry, is remarkable alike for the questions it propounds and for the answers it gives or suggests. The Kantian philosopher will find here ample reason to reconsider his master's doctrine of space, the famous saying of Kant that "Thoughts without contents are empty, intuitions without concepts are blind" being happily transformed by Mach into "Concepts without intuitions are blind, intuitions without concepts are lame." The indications are clear that the psychologist of the future will, contrary to the rash predictions of Dr. Hall, find a rich field for psychological research in the concepts of mathematics; and the mathematician in his turn will not fail to find evidence that the purest offspring of his thought may trace a legitimate lineage back and down to the rudiments of physical and physiological experience. The author's discourse carries waters from numerous confluent sciences, and is a living witness to the unity of knowledge.

The translation, as one knowing Mr. McCormack's previous work would expect, is well-nigh perfect. The Open Court Company is again to be congratulated on its excellent judgment and on its generosity in the service of science.

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