and Paciuolo (pages 127, 259). The form "Mohammed ben Musa Al Hovarezmi" is probably the least satisfactory of any for the name of the great Arab mathematician, particularly as it is followed by the statement that he was a native of Chwarizm, and as the form "Alhowarizmi" appears on page 207. It is unfortunate that we have as yet no generally accepted norm for such transliterations, but there is no good authority for such a mixture of languages as this. A similar criticism might justly be passed upon most of the other oriental names in the work, particularly Al Fahri (page 194), Al Karhi and Alkarhi (pages 194, 195), and Alhayyami (page 199).

DAVID EUGENE SMITH.

Leçons de Géométrie Supérieure. Professées en 1905–1906 par M. E. VESSIOT. Lyon, Delaroche et Schneider, 1906. 4to., 326 pp. (autographed).

THESE lectures delivered by Vessiot during the year 1905–1906 were published in the present form at the demand of his students. The author remarks in the preface that he is hopeful that they may be of service to those who are beginning the study of higher geometry and may serve them as a good preparation for the reading of original memoirs and such works as Darboux's Théorie des surfaces. It is the opinion of the reviewer that the lectures serve these purposes admirably. The attack is direct and the end to be reached is kept clearly before the reader, in fact the whole presentation is such as to lead the beginner to an appreciation of the subject. A glance at the table of contents will convince one that the book will serve as a good introduction to the study of Darboux.

The principal object of the lessons is the study of systems of straight lines but owing to the close relation between lines and spheres it is quite natural that systems of spheres should be studied also. It is assumed at the outset that the student is familiar with the elementary notions of twisted curves and surfaces (tangent planes, tangent lines, etc.), and that he has some acquaintance with the elements of the theory of contact.

In Chapter I, Frenet's formulas for twisted curves are derived and the simple properties of developable surfaces obtained. The rectifying and polar surfaces are discussed as examples of developables. Chapters II, III, and IV are devoted to the general surface theory. Throughout these chapters the importance of the two differential forms of Gauss

$$\Phi(du, dv) = Edu^2 + 2Fdu\,dv + Gdv^2,$$
$$\Psi(du, dv) = E'du^2 + 2F'du\,dv + G'dv^2$$

is insisted upon quite emphatically. These chapters are only introductory to the discussion of systems of lines and spheres, so quite naturally conjugate directions, asymptotic lines, geodesic lines and lines of curvature are the principal subjects treated. In these chapters good geometric interpretations are given to nearly all the analytic results. In Chapter V the foregoing theory is applied to scrolls and developable surfaces and the general properties of these surfaces are derived. Chapters VI, VII, and VIII are devoted to the study of congruences of lines and the correspondence set up by such congruences. Chapter VII is given up entirely to normal congruences and their applications. In this chapter the close relation between lines and spheres is first pointed out and discussed in an elementary manner. The analogy between asymptotic lines and lines of curvature is also pointed out. In Chapter VIII homogeneous and tangential coordinates are introduced. Here the lines of the congruence are defined by corresponding points on two surfaces between which a correspondence has been established. The dual of any line of the congruence is the line of intersection of the tangent planes at the points which define the line of the congruence. Chapters IX and X treat the general and linear line complex; curves and surfaces belonging to a congruence are the principal subjects discussed. Chapter IX is devoted to dualistic transformations and the transformation of Lie. A short discussion of contact transformation is given and then applied to dualistic transformations and the line-sphere transformation of Lie. In this chapter the correspondence between lines and spheres is again taken up and discussed more in detail by means of the Lie transformation.

Chapter XII deals with triply orthogonal systems. After demonstrating Dupin's theorem: On each surface of a triply orthogonal system the intersections with the other surfaces of this system are lines of curvature, — it is applied to the discussion of triply orthogonal systems which contain given surfaces as part of the system. Chapter XIII discusses congruences of spheres and cyclical systems. In this the concluding chapter spheres are discussed without reference to Lie's transformation. At the beginning of the discussion of focal points of a congruence of spheres another theorem of Dupin is proved: A normal congruence of lines is reflected or refracted on any surface into a normal congruence. It is shown that each sphere of the congruence touches the focal surface in two points. The congruence of lines formed by joining these points is discussed and some very pretty relations between the surface of centers of the spheres and their envelope are derived therefrom. The chapter closes with a short account of the cyclical systems of Ribaucour and Weingarten surfaces.

An excellent set of exercises is given to accompany each chapter. C. L. E. MOORE.

Nichteuklidische Geometrie. Von HEINRICH LIEBMANN. Leipzig, G. J. Göschen (Sammlung Schubert, XLIX), 1905. 12mo. viii + 248 pp.

IN this volume of the Schubert collection, Professor Liebmann has succeeded in presenting an introduction to non-euclidean geometry that is brief, readable, and well-balanced. Its brevity will recommend it to the student whose interest in the subject has been aroused by the numerous references in literature, but whose time and maturity are scarcely sufficient for a study of the many longer and more difficult works. It might well appeal also to a teacher of elementary geometry. The recent literature on non-euclidean geometry naturally falls into two classes: the one dealing with the lives and writings of Lobachevsky, Bolyai, and Gauss; and the other consisting of systematic developments of particular phases of the subject. In this book there is a happy combination of the two methods, giving a broad outlook, and yet not sacrificing the unity.

The first chapter contains an interesting account of the parallel axiom and of the attempts at its proof, considered from an historical point of view. The next five chapters, comprising three fourths of the book by pages, are devoted to hyperbolic geometry, beginning with a very simple account of its picturing by means of circles in the euclidean plane. One regrets that references are not given here to some, at least, of the articles that have appeared during the last twenty years on this picturing. With this one exception, the many exact references to a comparatively wide range of literature form one of the most attractive features of the book. There are other chapters on hyperbolic geometry, dealing with the synthetic and the analytic geometry and the trigonometry in the hyperbolic plane. After