n into itself contains α substitutions which are commutative with every substitution of this transitive group when its subgroup composed of all the substitutions which omit a given letter is of degree $n - \alpha$. In the case of the holomorph of G, $\alpha = n$.

UNIVERSITY OF ILLINOIS,
July, 1907.

SHORTER NOTICES.

Leçons d'Algèbre et d'Analyse à l'Usage des Elèves des Classes de Mathématiques spéciales. Par JULES TANNERY. Tome Premier. Paris, Gauthier-Villars, 1906. 423 + vii pp.

THIS text is prepared for students of the classe de mathématiques spéciales of the French lycées, in which boys of 18 or 19 prepare themselves for admission to the Ecoles normale and polytechnique. In so doing, they go over an amount of mathematics which seems overwhelming.* If the contents of the volume under review are even approximately an indication of what is taught successfully in one year, in any one of several subjects pursued by classes of boys of ordinary ability, then we have indeed much to learn from the teachers in the French lycées. But the contrast is less pronounced when we consider the difference in aim. The French adapt their courses to the abilities of the 2 per cent. or 5 per cent. who are the most gifted; Americans adapt their work to the capacity of the average boy. In the French lycées the 95 per cent. or 98 per cent. who fail in the first trial repeat the course during a second year, whereupon about 25 per cent. succeed. Less than half are said to succeed even after the third trial. Much can be said for and much against such a system of highly competitive examinations.

The first chapter of Tannery's book, covering 58 pages, contains a detailed exposition of irrational numbers. Adopting an easy conversational style, the author makes the subject very clear, except that, in a few instances, it is not stated from the start what the underlying assumptions are. Whenever such doubt lurks in the mind of the reader, he will find that the

* BULLETIN, vol. 6 (1900), p. 233.

matter is cleared up later on, when a restatement of the argument is made. Thus, on page 6, the author states in a footnote the assumption, now usually called the assumption of Archimedes, which was tacitly made on page 4. Tannery calls irrational numbers (page 2), "une façon de parler plutôt qu'une réalité." His theory of the irrational, although originally developed independently, resembles that of Dedekind in starting out with the notion of a partition (coupure). In the elegant development of this theory in the volume under review, the author makes ingenious use of a color system. Of the two classes into which a segment incommensurable to the unit segment on a line divides the points standing for rational numbers, he imagines the inferior class to be represented by blue points and the superior class by red points, while the irrational number in question is pictured by a white point. When the partition is effected by a commensurable segment, the points representing rational numbers are divided into three classes, namely the inferior (blue), the superior (red), and the single intermediate (white). This color imagery greatly facilitates the discussion.

Thus, two irrational numbers A and B are said to be equal when the blue and red points of the one are the very same, respectively, as those of the other. The sum (product) of A and B is a number greater than the sum (product) of any two blue numbers belonging, respectively, to A and B; this sum (product) is a number less than the sum (product) of any two red numbers.

Chapters II and III are given to polynomials. The real object of algebra is asserted to be the study of polynomials from different points of view. The author explains the graphic representation and the differentiation of polynomials, derives Taylor's theorem by the method of undetermined coefficients, and touches upon the singular points of graphs.

In chapters IV–VII are treated rational fractions, the greatest common divisor, and imaginaries. The resolution into partial fractions is effected by a process unusual in our text-books. Suppose $f(x)$ is a polynomial of lower degree than $\phi(x) \equiv (x-a)^\alpha (x-b)^\beta \cdots (x-l)^\lambda$. Writing $x = a + h$, $\phi_1(x) \equiv (x-b)^\beta \cdots (x-l)^\lambda$, and

$$P_\alpha\left(\frac{1}{x-a}\right) \equiv \frac{A_0}{(x-a)^\alpha} + \frac{A_1}{(x-a)^{\alpha-1}} + \cdots + \frac{A_{\alpha-1}}{x-a},$$

we obtain

$$\frac{f(a+h)}{\phi_1(a+h)} = A_0 + A_1 h + \cdots + A_{\alpha-1}h^{\alpha-1}\frac{h^\alpha R(h)}{\phi_1(a+h)}.$$

Dividing by h^α and then writing $x - a$ for h, we have

$$\frac{f(x)}{\phi(x)} = P_\alpha\left(\frac{1}{x-a}\right) + \frac{f_1(x)}{\phi_1(x)}.$$

This process is repeated on $f_1(x)/\phi_1(x)$ with respect to $(x-b)^\beta$, and so on. It is thus made evident in a natural and convincing way that, for any factor $(x-a)^\alpha$ of the denominator, all powers of $x - a$, from 1 to α, must in general appear as denominators of partial fractions. The numerators A_0, $A_1 \cdots$ are determined by the process of division.

In the treatment of imaginaries Tannery is influenced by Cauchy, who looked upon i as a real variable. The "fundamental theorem of algebra" is expressed by Tannery on page 223 in this form: Given a polynomial with real coefficients in two variables x and i, then there is a binomial $\alpha + \beta i$ (α and β real) such that, on writing $\alpha + i\beta$ for x, the result is divisible by $i^2 + 1$. The letter i plays only a very special role; for, the remainder resulting from a division of a polynomial by $i^2 + 1$ is the sole object of consideration. Two polynomials involving i are said to be equal when their remainders are equal, or when their difference is exactly divisible by $i^2 + 1$. Tannery remarks that this definition satisfies the three essential conditions of equality: an object is equal to itself; if a first object is equal to a second, then the second is equal to the first; if two objects are equal to a third, they are equal to each other. It is then evident that a polynomial in i is equal to its remainder, and in particular that the polynomial i^2 is equal to -1, which is the remainder resulting from the division of i^2 by $i^2 + 1$. When no confusion arises, one may regard i either as a real variable or as defined by $i^2 = -1$. From either point of view, two complex numbers $a + a'i$ and $b + b'i$ are equal if $a = b$ and $a' = b'$. The definitions of the fundamental operations on imaginaries are then set forth.

In the graphic representation of imaginaries the author starts out by marking off real and pure imaginary numbers along rectangular axes, without attempting to assign any reason for avoiding oblique axes. Later a justification for this procedure appears as the result of successive multiplication of the vector 1 by i. The chapter ends with a trigonometric treatment of the roots of unity.

The remaining three chapters are on combinations and permutations, the binomial formula, equations of the first degree and determinants. Everywhere we recognize the resolute endeavor to present the subject with perfect sincerity. The author says in the preface : " J' ai horreur d'un enseignement qui n'est pas toujours sincère : le respect de la vérité est la première leçon morale, sinon la seule, qu'on puisse tirer de l'étude des sciences."

F. CAJORI.

Leçons d'Algèbre et d'Analyse. Par JULES TANNERY. Tome Second. Paris, Gauthier-Villars, 1906. 638 pp.

THIS volume, which is the second of a comprehensive treatise on this field of algebra and analysis, like the other volumes, is prepared for the use of special students of mathematics of the Sorbonne, Paris. The chapter headings are as follows : series, functions of a real variable, series of functions, applications in the study of a function in the separation and calculation of the roots of an equation, algebraic equations, differential notation and plane curves, and notions of integral calculus.

The chapter on series deduces, with a simplicity and clearness that are ample for beginners of collegiate grade, the fundamental notions of the subject, establishes the ordinary tests for convergence and divergence, examines for convergence many series of frequent occurrence, and closes with seven pages of interesting exercises on convergence, divergence, and equivalence of series. All this is done in the compass of 53 pages and that, too, without material omission of essentials.

The meanings of variable, of function, of the phrases "appertaining to an interval" and "lying within an interval," and of bounds are given and exemplified in the next chapter on functions of a real variable. Common geometric notions are first given, then shown to be too indefinite for the purposes of analysis; then the analytic definitions are given for the following: curve, continuity, functionality, and increasing, decreasing, and discontinuous functions. The meaning and domains of validity of inverse, logarithmic, circular, and exponential functions are pointed out. A development of the properties of