portion than is customary of the clothed (eingekleidet) type, the formal exercises to be put at the end of the treatment of topics, to develop mechanical skill, after a larger amount of practice is had in the translation of verbal into formal language. For a considerable time a necessary part of the solution of every problem would be the setting up of the necessary equations. The former mode seeks technique first, then undertakes to infuse thinking into a formal frame-work. The latter seeks to secure the thinking first, and then to develop the technique *as a means of facilitating*, not result-finding, but *thought*. In the reviewer's opinion this collection errs in the over-stressing of the formal side of algebra at the beginning of developments of topics. Of course, teachers may select problems in whatever order they choose. But these lists have evidently been prepared for the special service of teachers whose practice is "through the art to the science."

In this compilation the formal problems outnumber the clothed, or verbal, about in the ratio of 3 to 1. The verbal problems are taken from physical science, from geometry, from elementary mechanics, and from the customary topics of arithmetic. A very large percentage of them are of real modern interest. Any teacher of high school algebra will find this manual a valuable source from which to select exercises to replace many of the dead ones of the standard texts. In this day of correlated algebra and geometry it will be a great help to learn, as these lists of problems show, how many types and varieties of algebraic equations may be based on geometric relations. The chief value of the book for American high school teaching is its adaptability to this service. Many teachers will be glad to find so practical a means as such problems afford of holding the ground made in algebra during the first high school year while the second-year geometry is being taught.

G. W. MYERS.

An Elementary Treatise on Pure Geometry. By J. W. RUSSELL, M.A. Oxford, The Clarendon Press, 1905. xii + 366 pp.

THIS is the second edition of a work in which "the author has attempted to bring together all the well-known theorems and examples connected with harmonics, anharmonics, involution, projection (including homology), and reciprocation" (preface to first edition, line 1). In other words, it is concerned with the material of the usual courses on projective geometry, including problems of the first and second degree in the plane. It starts with a metric foundation, Menelaus's and Ceva's theorems, etc., and uses metric geometry freely throughout the book.

Some of the fundamental concepts are introduced in a way that is not likely to impress a beginning student (or any one else) with its logical sharpness. For example, at the bottom of page 9, the notion of "point at infinity" is introduced as follows:

"Take any fixed point O and a fixed line l. Then any line x through O cuts l in a point P. Now rotate x about O so that x may become more and more nearly parallel to l. Then P recedes indefinitely along l; and in the limit when x is parallel to l, P is said to be the point at infinity upon l. Hence *two parallel lines intersect in a point at infinity.*" (Author's italics.)

From here on the "point at infinity" is used quite freely. At the beginning of Chapter III occurs the first mention of imaginary points. It is as follows:

"1. EVERY *line meets a circle in two points, real, coincident, or imaginary.*

"For take any line l cutting a circle in the points A and B. Now move l parallel to itself away from the center of the circle. Then A and B approach and ultimately coincide when l touches the circle. But when l moves still further from the center, the points A and B disappear; yet, for the sake of continuity, we say that they still exist, but are *imaginary* (see also XXVII)."

The reference at the end is to Chapter 27, where the aid of algebraic geometry is invoked: "* * * this quadratic equation will have two solutions, real, coincident, or imaginary. Hence we conclude that a line always meets a circle in two points, real, coincident, or imaginary."

It may be true, as many teachers evidently believe, that clear statements on subjects like those to which our quotations apply are too difficult for most elementary students. But on the other hand must be set the distrust inspired in thoughtful students by statements which, taken literally, are untrue, and the flabbiness of mind which comes from arguing with vaguely defined terms.

These criticisms apply only to a limited number of places in Mr. Russell's book and do not in any way impeach its usefulness as a source of theorems and examples. The latter are particularly rich in metric cases of projective theorems. The arrangement of material has a number of elegant features as, for example, the way in which theorems on conic sections are derived by projection and reciprocation from the corresponding theorems on circles.

O. VEBLEN.

Wahrscheinlichkeitsrechnung und Kollektivmasslehre. By Dr. HEINRICH BRUNS. Leipzig, Teubner, 1906. 310 + 18 pp.

ABOUT one third of this book is devoted to the theory of probability, and two thirds to Kollektivmasslehre. The theory of probability is treated as a theory of frequency, and from this point of view the part on probability is presented in excellent form for application to Kollektivmasslehre. The intimate relation between the two parts of the book stands out so clearly as to make it an important feature, especially because in the work of Fechner Kollektivmasslehre appears much more as an independent subject than as one so closely related to the theory of probability.

While Fechner and Pearson have, to a certain extent, treated Gauss's law of distribution as a "scientific dogma," and have presented generalized probability curves which fit well a large class of data, the conclusion that all distributions conform to one of these curves would have the same kind of logical weakness as the dogma of Gauss. Bearing on this point, Bruns makes a distinct advance by obtaining what seems to be a "suitable" analytic representation for an arbitrary frequency distribution. I use the term "suitable" because it is not difficult to get an analytic representation whose algebraic and numerical complications make it of no value for describing populations such as arise in applications.

Starting with a frequency distribution, the author constructs what he calls a "Summentafel" which gives the number of variates below given values. He uses the term "Summenfunktion" $S(x)$ to represent the relative frequency with which a variate lies below x. This function $S(x)$ and its derivative $V(x)$ (Verteilungsfunktion) are the functions for which the author obtains analytic representations. He treats the Summenfunktion as of fundamental importance rather than the distribution function, and in the process of reduction or smoothing which he employs the Summentafel is unchanged, while the frequency distribution may be very much changed.