ness as a source of theorems and examples. The latter are particularly rich in metric cases of projective theorems. The arrangement of material has a number of elegant features as, for example, the way in which theorems on conic sections are derived by projection and reciprocation from the corresponding theorems on circles.

O. VEBLEN.

Wahrscheinlichkeitsrechnung und Kollektivmasslehre. By Dr. HEINRICH BRUNS. Leipzig, Teubner, 1906. 310 + 18 pp.

ABOUT one third of this book is devoted to the theory of probability, and two thirds to Kollektivmasslehre. The theory of probability is treated as a theory of frequency, and from this point of view the part on probability is presented in excellent form for application to Kollektivmasslehre. The intimate relation between the two parts of the book stands out so clearly as to make it an important feature, especially because in the work of Fechner Kollektivmasslehre appears much more as an independent subject than as one so closely related to the theory of probability.

While Fechner and Pearson have, to a certain extent, treated Gauss's law of distribution as a "scientific dogma," and have presented generalized probability curves which fit well a large class of data, the conclusion that all distributions conform to one of these curves would have the same kind of logical weakness as the dogma of Gauss. Bearing on this point, Bruns makes a distinct advance by obtaining what seems to be a "suitable" analytic representation for an arbitrary frequency distribution. I use the term "suitable" because it is not difficult to get an analytic representation whose algebraic and numerical complications make it of no value for describing populations such as arise in applications.

Starting with a frequency distribution, the author constructs what he calls a "Summentafel" which gives the number of variates below given values. He uses the term "Summenfunktion" $S(x)$ to represent the relative frequency with which a variate lies below x. This function $S(x)$ and its derivative $V(x)$ (Verteilungsfunktion) are the functions for which the author obtains analytic representations. He treats the Summenfunktion as of fundamental importance rather than the distribution function, and in the process of reduction or smoothing which he employs the Summentafel is unchanged, while the frequency distribution may be very much changed.

The analytic representation of the arbitrary Summenfunktion is primarily in terms of two functions, ϕ and R. The function $\phi(x)$ is defined by

$$\sqrt{\pi}\phi(x) = 2\int_0^x \exp.\ (-t^2)dt,$$

and the function $R(x)_q$ is given by the recursion formula

$$\exp.\ (-2xv - v^2) = \sum_{q=0}^{q=\infty} R(x)_q(2v)^q.$$

If the qth derivative of $\phi(x)$ is denoted by $\phi(x)_q$ the function $S(y)$ is capable of representation in the form

$$2S(y) - 1 = \sum_{q=0}^{q=\infty} \mathfrak{D}\,[R(u)_q]\,\phi(v)_q,$$

where

$$u = h(x - c), \quad v = h(y - c)$$

and the operation $\mathfrak{D}$ is defined, in general, by

$$\mathfrak{D}\,[T(x)] = \int T(x)\,V(x)dx,$$

where $V(x)$ is the distribution function.

The c and h are parameters so chosen that

$$c = \mathfrak{D}(x), \quad h^2 = \frac{1}{2\mathfrak{D}\,[x - c]^2}.$$

When thus selected, c is the arithmetic mean and h is the measure of precision.

It is next shown that the above analytic representation gives

$$2S(x) - 1 = \sum_{q=0}^{q=\infty} D(c,\ h)_q\phi(u)_q.$$

On choosing c and h as just explained,

$$D(c,\ h)_1 = 0, \quad D(c,\ h)_2 = 0.$$

Then

$$2S(x) - 1 = \phi(u) + D_3\phi(u)_3 + D_4\phi(u) + \cdots.$$

If the D_3, D_4, $\cdots$ are zero or so small that they can be neglected, the Verteilungsfunktion gives the simple exponential law. According to this, the author says that populations can be divided into two large classes: "the first embraces the forms for which the exponential law may be regarded as at least a first approximation; the second, on the contrary, embraces the forms for which this law can not be regarded as even a crude approximation. Thus, the exponential formula, which for a long time was conceived as a sort of law of nature, is put in its proper light. For a large group of distributions arising in Kollektivmasslehre it plays a role similar, say, to the representation of the earth by means of a sphere in measurements of the earth."

In the chapter on the mixture of the arguments, it is shown that this process produces a tendency towards the exponential law. This result is of significance as accounting in part for the large number of distributions which approach this law. In the chapter on the mixture of distributions the essentials of correlation theory are treated.

Of the twenty-four chapters contained in the book, the last five deal with numerical applications. These numerical cases show the systematic methods of carrying the theory into practice, and indicate what parts of the work can be done once for all in many applications.

Dealing with a subject which we should like to see treated rigorously, this book takes a high place in point of mathematical elegance, and it should serve to make much better known this important field of applied mathematics.

H. L. RIETZ.

Hermann Grassmanns gesammelte mathematische und physikalische Werke. Herausgegeben von FRIEDRICH ENGEL. Bd. 2: Theil 1, *Die Abhandlungen zur Geometrie und Analysis*, x + 451 pp., 1904; Theil 2, *Die Abhandlungen zur Mechanik und zur mathematischen Physik*, viii + 266 pp., 1902. Leipzig, B. G. Teubner.

THE first volume of Grassmann's works is in two parts, each containing one of the two Ausdehnungslehren; the second volume reprints the miscellaneous papers and Nachlass. It is interesting to note that the earliest paper is a "Programm" on crystals and bears the inscription: Stettin, 1839. This was only five years before the publication of the first Ausdehnungslehre. The last papers are dated 1877 to 1879 and are concerned with various applications of the calculus so intimately associated