be divided into two large classes: "the first embraces the forms for which the exponential law may be regarded as at least a first approximation; the second, on the contrary, embraces the forms for which this law can not be regarded as even a crude approximation. Thus, the exponential formula, which for a long time was conceived as a sort of law of nature, is put in its proper light. For a large group of distributions arising in Kollektivmasslehre it plays a role similar, say, to the representation of the earth by means of a sphere in measurements of the earth."

In the chapter on the mixture of the arguments, it is shown that this process produces a tendency towards the exponential law. This result is of significance as accounting in part for the large number of distributions which approach this law. In the chapter on the mixture of distributions the essentials of correlation theory are treated.

Of the twenty-four chapters contained in the book, the last five deal with numerical applications. These numerical cases show the systematic methods of carrying the theory into practice, and indicate what parts of the work can be done once for all in many applications.

Dealing with a subject which we should like to see treated rigorously, this book takes a high place in point of mathematical elegance, and it should serve to make much better known this important field of applied mathematics.

H. L. RIETZ.

Hermann Grassmanns gesammelte mathematische und physikalische Werke. Herausgegeben von FRIEDRICH ENGEL. Bd. 2: Theil 1, *Die Abhandlungen zur Geometrie und Analysis*, x + 451 pp., 1904; Theil 2, *Die Abhandlungen zur Mechanik und zur mathematischen Physik*, viii + 266 pp., 1902. Leipzig, B. G. Teubner.

THE first volume of Grassmann's works is in two parts, each containing one of the two Ausdehnungslehren; the second volume reprints the miscellaneous papers and Nachlass. It is interesting to note that the earliest paper is a "Programm" on crystals and bears the inscription: Stettin, 1839. This was only five years before the publication of the first Ausdehnungslehre. The last papers are dated 1877 to 1879 and are concerned with various applications of the calculus so intimately associated

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with the name of the author. As a matter of fact practically all the intervening matter deals with some phase of mathematics closely allied to one or the other of the Ausdehnungslehren. The author's whole mathematical activity may therefore be said to be confined to this one somewhat narrow and neglected field. The adjective mathematical is, however, a very necessary qualifier of activity; for during the forty years, from 1839 to 1879, Grassmann had been occupied with other researches. In particular, he had published in the early seventies, an exhaustive dictionary to the Rig-Veda, a work more laborious and in its field no less a standard, even to this day, than the two Ausdehnungslehren. To have accomplished such results in two so different fields is a rarity even in a genius.

Of the twenty-four memoirs in the first part of the volume under review, by far the greater number treat the geometric theories of plane cubics and quartics. It is here that one finds the well known methods developed by Grassmann for the construction of plane curves of degree three and four. Of the other contributions perhaps only two deserve individual notice: those entitled "Sur les différents genres de multiplication" and "Stücke aus dem Lehrbuche der Arithmetik." Grassmann's general definition of multiplication is worth quoting here. He says: To multiply extensive magnitudes, first multiply each by each and in the order indicated the units of which the magnitudes are composed; then multiply these products by the products of their respective coefficients and add. He then goes on to state that other products are merely relative. That was in 1859. The standpoint here outlined was that adopted later by Gibbs. In most of Grassmann's work, whether earlier or subsequent, a more specialized position is maintained relative to products. In no other way can one account for the comparatively meager attention given by Grassmann to the dyadics or polyadics (Lückenausdrücke). The selections from a treatise on arithmetic are in reality nothing but a discussion of the theory of numbers taken broadly. It resembles to a great extent the present method, especially Hilbert's method, of laying down axioms or postulates and building up the analysis therefrom. In form the presentation differs considerably from that now adopted. The words postulate and axiom nowhere occur. In their place stands "Erklärung." In the prefatory remarks the author states that certain parts of the treatment are intended for successive grades in the gymnasium. It is difficult to see

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how students so immature could profit greatly by the study of such material.

The first memoir on mechanics is again a "Programm" and is intended as an introduction to the subject for students in the gymnasium. Here once more it is difficult to see how the students could have derived much profit from the study. To be sure the presentation is logical and explicit. That seems to be the most serious objection to it. The student would get too much mathematics and logic, too little physics and mechanical intuition from his studies. This would be an excellent argument for having mechanics taught by physicists and engineers rather than by mathematicians, were it not a still better argument for having mathematicians more or less forget their mathematics and really learn mechanics as such before trying to teach it. These two treatises on arithmetic and mechanics are very interesting and highly worth studying. In their way, they are quite distinctive. Grassmann's other work on mechanics is largely a presentation of the subject from the standpoint of the Ausdehnungslehre. The contributions to physics are partly original, partly explanatory. They would not greatly interest anybody now actively engaged in research or in teaching in physical science. This is largely true of all but the highest work done on physics thirty or forty years ago. Mediocrity is short lived in physical science; and it is evident that as a contributor to physics and mechanics Grassmann was not of high rank. This is no reproach: the Ausdehnungslehre and the dictionary to the Rig-Veda are there to silence all specious criticism.

E. B. WILSON.

Die Grundlagen der Bewegungslehre, von einem modernen Standpunkte aus. Von DR. G. JAUMANN, Professor der Physik an der Deutschen Technischen Hochschule in Brünn. Mit 124 Abildungen. Leipzig, Johann Ambrosius Barth, 1905. vi + 421 pp. M. 11; M. 12 gebunden.

IN the author's view the transformation which Fresnel wrought in the theory of light, and Faraday in electromagnetic phenomena, furnishes a prototype for all domains of theoretical physics, mechanics not excepted. The latter, although preserving its old form unaltered, proceeds now in part from points of view which lay quite beyond the reach of its founder and, in this respect, it appears to be nearing a decisive change. This rather remote aim gave direction to the treatment of this book.

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