"It may be too bold an attempt to include in the brief space of a small manual the principles of two vast and important theories. Nevertheless it seemed useful, in spite of the greater conciseness required, to unite both theories in order to avoid the repetition necessary in treating them separately, and also to show that the icosahedron theory, which is capable of appearing in the field of analysis as an elegant and independent creation of a special type, is only the first of a series of related constructions very closely bound together. From this point of view it would have been better still to take another step and include also the theory of the automorphic functions; but this would clearly be impossible.

"To save space, and for greater symmetry of treatment, I have assumed that the reader, besides having a certain familiarity with the more elementary parts of mathematics, has also some notions of several more advanced theories: analysis situs, functions of a complex variable and Riemann surfaces, elliptic functions, abelian integrals, linear differential equations, theory of numbers."

The program which the author thus places before himself has been carried out, we think, with much care and good judgment. He has treated his subject with admirable simplicity, directness, and unity. Not only will this work greatly facilitate the efforts of the reader to master these comprehensive theories, but it will be of particular service to those whose main interests lie outside this special field, by enabling them to become familiar with its general topography without an undue expenditure of time.

The book is printed in large, well leaded type. By using a paper as thin as is consistent with opaqueness and by cutting down to a narrow margin, the publisher has produced a small and handy volume without any sacrifice of legibility.

J. I. HUTCHINSON.

Vorlesungen über Zahlentheorie. Einführung in die Theorie der algebraischen Zahlkörper. By J. SOMMER. Leipzig, B. G. Teubner, 1907. iv + 361 pp.

THE generalizations of the ordinary theory of numbers which, following Gauss's introduction of complex integers, have been made by Kummer, Dirichlet, Dedekind, and Kronecker constitute an extensive and exceedingly interesting part of mathematics.* Nevertheless it is true that this subject is not as widely read as is proper in view of its importance; and undoubtedly the reason is to be found in its abstract nature.

The recent books of Bachmann † and König ‡ upon this general subject are excellent, but many will find them very difficult because of the generality of their treatments. Indeed, Professor Dickson in reviewing them § felt it necessary to give a brief exposition of the theory for a special case, and there is no doubt of the desirability of an extensive treatment of the simpler cases which will lead up to the general discussion. The volume under review has been prepared to serve those who have hitherto been hampered by the lack of an introduction to the subject.

The author has successfully undertaken to present the chief points of the theory by means of the elementary cases of (absolute) quadratic and cubic number fields and fields quadratic with respect to a quadratic fundamental field. The book is divided into five chapters, the first of which constitutes an exceedingly brief introduction, so brief in fact that it must be regarded as simply a statement of those topics of the ordinary theory of numbers whose generalizations are to be given later and are to form the foundation of the discussion. And, naturally enough, these topics are the divisibility of integers, the function $\phi(n)$, congruences, and Fermat's theorem.

The second and third chapters constitute two thirds of the book and are the most important ones. In the second the theory of quadratic number fields is worked out in detail in connection with the topics above mentioned, and in this chapter the reader is given an opportunity to become intimately acquainted with the ideas of number field, ideals, and the generalizations of the results obtained by Fermat and Gauss. In chapter three applications of the theory of quadratic fields are made to several important problems, and particularly welcome to many will be the introduction of the famous "last theorem" of Fermat.

To give the reader a closer approach to the generalities of the subject, the author devotes a fourth chapter to such a broadening of methods as will make the treatment adequate for the

* See Hilbert, *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 4.

† Zahlentheorie, Teil 5, 1905.

‡ Einleitung in die allgemeine Theorie der algebraischen Grössen, 1903.

§ BULLETIN, April, 1907.

case of cubic number fields. A fifth chapter deals with fields which are quadratic with respect to another one, itself a quadratic field. Both of these chapters are relatively short and are not intended to be exhaustive.

The value of the volume is distinctly increased by the introduction of a considerable number of simple illustrations of the theory in addition to the more elaborate applications made in the third chapter.

The book is timely, has been put out by the publishers in an attractive form, and should materially increase the number of those who will undertake to familiarize themselves with this branch of general algebraic theory.

GEORGE H. LING.

Les Principes des Mathématiques, avec un appendice sur la philosophie des mathématiques de Kant. Par LOUIS COUTURAT. Paris, Felix Alcan, 1905. viii + 310 pp.

THE present work owes its origin, according to the author, to the appearance of "the masterly work of Bertrand Russell which bears the same title." It seems that originally it was intended to be merely a review of the older work. But it is much more than a review of Russell's treatise. We can readily sympathize with the author when he tells us in the preface that in commenting and illustrating Russell's theories he was led gradually to include in his review abstracts of most of the recent papers dealing with the same questions. The result has been that the author has written a comprehensive and careful report on the present state of the logical foundations of mathematics, which on account of its clear style and admirable arrangement of content is valuable, not merely as a work of reference but also as a book well adapted to the needs of anyone desiring to acquaint himself with the fundamental ideas and methods of the subject with which it deals.

The book is divided into six chapters, to which are added two "Notes" and an appendix. The first chapter deals with the principles of formal logic, as developed by Peano and others. The next four chapters discuss respectively the notions of number, order, the continuum, and magnitude. The last and by far the longest chapter deals with the foundations of geometry. It is divided into four parts treating respectively of the dimensions and topology, projective geometry, descriptive geometry,