THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and thirty-fifth regular meeting of the Society was held in New York City on Saturday, October 26, 1907, a single morning session sufficing for the unusually brief programme. The attendance included the following twenty-eight members of the Society:

Professor G. A. Bliss, Professor Maxime Bôcher, Professor Joseph Bowden, Professor E. W. Brown, Dr. J. E. Clarke, Professor F. N. Cole, Miss L. D. Cummings, Mr. G. W. Hartwell, Professor E. V. Huntington, Mr. S. A Joffe, Mr. E. H. Koch, Dr. G. H. Ling, Professor E. O. Lovett, Mr. E. B. Lytle, Professor Max Mason, Professor Mansfield Merriman, Dr. R. L. Moore, Mr. H. W. Reddick, Professor L. W. Reid, Mr. F. H. Smith, Professor P. F. Smith, Professor H. D. Thompson, Mr. C. A. Toussaint, Professor E. B. Van Vleck, Professor Oswald Veblen, Mr. H. E. Webb, Professor H. S. White, Professor J. W. Young.

Vice-President Professor P. F. Smith occupied the chair. The Council announced the election of the following persons to membership in the Society: Mr. V. R. Aiyar, Gooty, India; Professor P. P. Boyd, Hanover College, Ind.; Dr. Charles Haseman, Indiana University; Professor C. A. Proctor, Dartmouth College; Mr. J. M. Rysgaard, University of North Dakota; Mr. C. A. Toussaint, College of the City of New York. Thirteen applications for admission to membership in the Society were received.

A list of nominations of officers and of the members of the Council was adopted and ordered placed on the official ballot for the annual election at the December meeting.

The following papers were read at this meeting:

(1) Professor R. D. CARMICHAEL: "A certain class of quartic curves."

(2) Professor R. D. CARMICHAEL: "Geometric properties of quartic curves possessing fourfold symmetry with respect to a point."

(3) Professor OSWALD VEBLEN: "On magic squares."

(4) Professor L. E. DICKSON: "On triple algebras and ternary cubic forms."

(5) Sir G. H. DARWIN: "Further note on Maclaurin's spheroid."

(6) Dr. J. L. COOLIDGE: "The equilong transformations of space."

(7) Professor EDWARD KASNER: "Note on isothermal systems."

(8) Dr. R. L. MOORE: "A note concerning Veblen's axioms for geometry."

(9) Professor JOSEPH BOWDEN: "Proof of a formula in combinations."

Professor Darwin's paper was communicated to the Society through Professor E. B. Van Vleck. In the absence of the authors, the papers of Professor Carmichael, Professor Dickson, Professor Darwin, and Dr. Coolidge were read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. In this paper Professor Carmichael studies the general nature of a class of quartic curves whose equation may be represented thus: Set

$$m_i = c_i \sqrt{[(x-a_i)^2 + (y-b_i)^2]} \quad (i = 1, 2, 3),$$

in which $c_i \neq 0$ and the radical is to be taken with the positive sign. Then the equation of the curve studied takes the form

$$(m_1 + m_2 + m_3)(m_1 + m_2 - m_3)(m_1 - m_2 + m_3)(m_1 - m_2 - m_3) = 0,$$

which clears of radicals when the parentheses are removed. A method is also given for the construction of the curve by continuous motion when the c's are commensurable.

2. In a previous communication Professor Carmichael has shown that plane algebraic curves possessing fourfold symmetry with respect to a point divide into two groups. In the present paper he studies the geometric properties of each of these two classes for the case of quartic curves. Special attention is given to the forms of the loci, and the number of singularities in each case is completely determined.

3. Professor Veblen's note shows how it is possible to derive magic squares with p^n elements in a line from the corresponding finite euclidean geometries. The method is a generalization of that of G. Arnoux: Arithmétique graphique. Les espaces arithmétiques hypermagiques, Paris, 1894.

4. Professor Dickson's paper appears in full in the present number of the BULLETIN.

5. In volume 4 of the *Transactions*, Sir George Darwin showed that the form of Maclaurin's ellipsoid could be determined by spherical harmonics to a higher order of approximation than had usually been supposed possible. In the paper referred to he had been unable to determine the angular velocity corresponding to a given ellipticity as far as the cube of that ellipticity. In the present paper he shows that if certain terms previously omitted were retained the defect can be remedied.

6. Scheffers has given the name "equilong" (German, aequilong) to those transformations of the plane which carry a line into a line, keeping invariant the distance between the points of contact of a line with any two envelopes which it touches. These transformations are, in a sense, dual to the conformal ones, and Scheffers has shown that, whereas the latter depend on an arbitrary function of the usual complex variable, the former may be expressed by an arbitrary function of a complex variable of different type. Dr. Coolidge's paper dealt with the corresponding transformations in three dimensions, namely, those which carry a plane into a plane, keeping invariant the distance of the points of contact with any two envelopes which the plane may touch. In this case the duality between conformal and equilong transformations exists in the statement only, for whereas the former group depends on ten arbitrary parameters, the latter involves two arbitrary functions.

7. Professor Kasner's paper appears in full in the present number of the BULLETIN.

8. In volume 5, number 3, pages 343–384 of the *Transactions*, Professor Veblen has exhibited a set of axioms for geometry in terms of point and order. Dr. Moore shows that Axiom II becomes redundant if Axiom V is strengthened by assuming that C is different from B.

9. Professor Bowden gave an elementary proof by mathematical induction of the formula

$$C_r^{m+n} = \sum_{k=1}^{k=r+1} C_{r-k+1}^m C_{k-1}^n.$$

F. N. Cole,
Secretary.

ON TRIPLE ALGEBRAS AND TERNARY CUBIC FORMS.

BY PROFESSOR L. E. DICKSON.

(Read before the American Mathematical Society, October 26, 1907.)

1. For any field F in which there is an irreducible cubic equation $f(\rho) = 0$, the norm of $x + y\rho + z\rho^2$ is a ternary cubic form C which vanishes for no set of values x, y, z in F, other than $x = y = z = 0$. The conditions under which the general ternary form has the last property are here determined for the case of finite fields. One formulation of the result is as follows:

Theorem. *The necessary and sufficient conditions that a ternary cubic form C shall vanish for no set of values x, y, z in the $GF[p^n]$, $p > 2$, other than $x = y = z = 0$, are that its Hessian shall equal mC, where m is a constant different from zero, and that the binary form obtained from C by setting $z = 0$ shall be irreducible in the field.*

Although I have not hitherto published a proof of this theorem, I have applied it to effect a determination * of all finite triple linear algebras in which multiplication is commutative and distributive, but not necessarily associative, while division is always uniquely possible. I shall here (§ 11) determine these algebras by applying directly the more fundamental conditions from which the preceding theorem is derived.

These ternary cubic forms arise in various other problems; for instance, in the normalization of families of ternary quadratic forms containing three linearly independent forms.

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* *Amer. Math. Monthly*, vol. 13 (1906), pp. 201–205. References are there given to my earlier papers on the subject.