

general expression for the order of the group associated with each of these figures is also given. The omission of a phrase from the theorem at the top of page 256 is easily corrected by reference to the formula at the bottom of the preceding page.

The fourth and final section treats "Die runden Polytope." In the work on the hyper-sphere one notices such topics as, the sphere passing through $n + 1$ points, the sphere touching $n + 1$ R_{n-1} 's, the sphere touching $n + 1$ other spheres, the configuration of the centers of similitude of $n + 1$ spheres, the content and surface content of the sphere, and the content of the spherical sector and segment. The hyper-cone and cylinder are similarly treated. Under general rotation figures, one finds the quadric spreads generated by revolving a flat spread about a flat spread as an axis, the torus spreads and the Guldin spreads obtained by revolving hyperspheres and linear polytopes respectively.

Since the book is largely a compilation of previously known results, one regrets that references to the literature of the subject are not more numerous and specific. This second volume is better than the first in this respect, but still leaves much to be desired.

The book will doubtless prove to be a valuable reference work to those who are interested in, and have use for, the metrical formulas of higher-dimensional geometry; but many readers will doubtless share with the present writer a regret that the author's point of view has been so largely metrical, both in his choice of topics and in his method of treatment.

W. B. CARVER.

Theory of the Algebraic Functions of a Complex Variable. By J. C. FIELDS. Berlin, Mayer & Müller, 1906. v + 186 pp.

THE work before us is not intended as a treatise or textbook on the theory of algebraic functions along any of the well-established lines of treatment. It is, on the contrary, a new and distinctive mode of approach to this class of functions, although grounded on principles which in their essence are already familiar. The methods employed are purely algebraic, we might almost say arithmetic, in character, and in this respect the influence of Weierstrass may be said to predominate.

The fundamental idea on which the work is based is the notion of "order of coincidence." A given class of algebraic functions is defined as usual by a rational expression in (z, v)

the dependence of v on z being determined by an algebraic equation $F(v, z) = 0$ of degree n in v . The n branches of the function are assumed in the form $v = P_k(z - a)$ ($k = 1, 2, \dots, n$) in the vicinity of a given point $z = a$, the exponents in the power series being rational numbers only a finite number of which are negative. The order of coincidence of any function $H(z, v)$ with the branch $v - P_k = 0$ is then defined to be the least exponent obtained in developing H in powers of $z - a$, after substituting for v the series $P_k(z - a)$. For each point a there is accordingly a set of n (or fewer) numbers constituting the orders of coincidence of the given function with the several branches of the curve $F(v, z) = 0$. This set of numbers will be zero except for a finite number of points a . These sets of non-vanishing orders of coincidence are studied and the conditions which they must satisfy are determined.

On the other hand, if the orders of coincidence be completely given in advance, the function H is thereby conditioned, and it is then required to determine its form and properties. Chapter VI, for example, is devoted to finding the relation between the degree of a function and its orders of coincidence at infinity.

The methods of the author, growing as they do out of a single fundamental idea, are naturally characterized by a high degree of simplicity, unity, and generality. This is especially observable in Chapters XIII, XIV, and XV where the efficiency of the new treatment and its culminating "complementary theorem" of Chapter XII is tested by deducing for any algebraic relation in (z, v) , reducible as well as irreducible, the Riemann-Roch and related theorems and formulas, and the general forms and properties of the ϕ -functions and abelian integrals. The brief discussion of the abelian integrals contained in the last chapter is merely intended as a suggestion of ways in which the previous theory might be utilized in the study of this class of transcendental functions.

As remarked in the preface, the methods of this work, at least in part and with suitable modifications, are applicable outside the field of algebraic functions and to functions of any number of variables.

The book is printed in a style uniform with the elegant edition of Weierstrass's works issued by the same publishers.

J. I. HUTCHINSON.