SHORTER NOTICES.

Calcul Graphique et Nomographie. Par M. d'Ocagne. Paris, Octave Doin, 1908. xxvi + 381 pp.

Le Calcul Simplifié. Par M. d'Ocagne. Paris, 1905. 225 pp.

The first mentioned volume occurs in the division Sciences Appliquées of the Encyclopédie Scientifique, now publishing under the direction of Dr. Toulouse. It forms a part of the section Science du Calcul. The author's object is to develop as an independent discipline the theory of graphical processes of calculation. The work is therefore mainly to give system and revision to previously published material of his own and that of Culmann, Massau, Mehmke, and others. To this end the volume is divided into Livres I and II.

Livre I, Calcul Graphique, is subdivided into two chapters: Arithmétique et Algèbre graphique and Intégration graphique. In reading these two chapters one would perhaps better forget that any but graphical methods are available. Such methods of necessity often involve trying detail and sometimes sag under their own weight. They lead in the last step to the measurement of a segment by a scale. The author begins therefore with a discussion of "Echelles métriques," and easily converts one to the elegance of the "contre-échelle."

The dominating construction of this first book is that for graphic multiplication: "Si, entre deux parallèles, mM et nN à Oy (* * * lignes de rappel) separées par l'intervalle $mm = a$, nous tirons une droite MN de coefficient angulaire b, la différence NH ou c des ordonnées des points M et N est égal à ab. Pour avoir * * * b il suffit * * * ayant pris sur la partie negative de Ox le segment OQ égal à l'unité * * * de porter sur Oy le segment OQ égal à b, pris avec son signe, et de tirer PQ. La droite MN est dès lors parallèle à PQ." This paragraph is typical of the character and detail of presentation of much of the text.

The value of $a_1b_1 + a_2b_2 + \cdots + a_nb_n$ is then constructed by laying off the coefficients a on OX and b as angular coefficients consecutively and then measuring the segment on the nth *ligne de rappel* or ordinate. Such a construction serves also to solve the linear equation

$$a_0 + a_1 z_1 + \cdots + a_{n-1}z_{n-1} + a_nz_n = 0$$

in ∞^{n-1} ways by merely closing the polygon of angular coefficients on OX at a_n.

Again, a system of n linear equations of n unknowns can be solved directly if they form a "système étagé."

$$a_0 + a_1 z_1 = 0,$$
$$b_0 + b_1 z_1 + b_2 z_2 = 0,$$
$$\cdots\cdots\cdots$$
$$l_0 + l_1 z + l_2 z_2 + \cdots + l_n z_n = 0.$$

By a construction due to Van den Berg any linear system may be reduced to the above form by graphic elimination. An alternative method for the general case is also presented.

A graphic solution of the general equation

$$a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n = 0$$

is based on the method of Lill, in which the coefficients are laid off in succession at right angles and the polygon of multiplication closed by trial and error.

A graphic method of multiplying $F(x)$ by x enables one to plot polynomials in x entirely by rule and compass; and graphic interpolation or determination of the coefficients of the parabola Π_n of the n^{o} degree

$$y = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n,$$

so that the graph passes through a large number of plotted points, is skillfully accomplished.

The second chapter on graphic integration aims at the evaluation of

$$\int F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \cdots\right) dx.$$

The reader must be content, however, if the shot is made at very close range and with a firm rest. The graph C with ordinate y is presupposed and is usually a Π_n, also it does not appear advisable to have F much worse than $x^n y$. The integral curve, whose ordinate measures the desired integral, is usually constructed by inscribing or circumscribing approximate polygons. When a Π_n is to be handled, the construction of the simple successive integrals can be made rigorous. One would scarcely resort, however, to graphic methods in such an emergency.

There is included in this chapter a section on the graphic integration of

$$dy/dx = f(x, y)$$

by approximate graph of the integral curves. The construction is dependent upon plotting the "isoclines"

$$f(x, y) = k.$$

Where possible, use may be made of the loci of inflections or cusps and the envelope.

The author has brought order and unity to a large variety of constructions and made available many scattered results. If the reader is disappointed at the calibre of some of the results, it is a solace to remember that they are *strictly* graphical throughout.

Livre II, Nomographie, takes one into a new atmosphere and presents in slightly over two hundred pages the essentials of this recent branch of mathematics. Those who have read the author's "Traité de Nomographie" with a desire for more compact treatment will find it here in excellent form. There is also a clear separation of the subject into its three essential parts: Représentation nomographique par lignes concourantes, * * * par points alignés, and * * * au moyen de points cotés diversement associées."

To give an idea of nomography itself the author's preface is illuminating: "Il est aisé de faire saisir a priori ce que constitue son essence au moyen d'une comparison familière. Toute le monde connait les graphiques au moyen desquels le Bureau central météorologique indique la repartition des hauteurs barométriques à une date donnée. Sur ces cartes figurent trois systèmes de lignes munis chacun d'une certaine chiffraison: les méridiens dont la cote est la longitude, les parallèles dont la cote est la latitude, les lignes, dites isobares, qui unissent tous les points où la hauteur barométrique atteint une même valeur, inscrite comme cote, à côté de chacune d'elles. Voici donc un nombre, la hauteur barométrique, que depend de deux autres, la longitude et la latitude, qui en est fonction, comme disent les mathematiciens, (* * *) *et dont on a la valeur en lisent simplement la cote d'une certaine ligne, l'isobare, passant par le point de rencontre de deux autres lignes*, le méridien et le parallèle, cotées au moyen des valeurs des nombres donnés."

Having set forth the notion of "echelles fonctionelles," viz., scales graduated with lengths proportional to $f(z)$ and inscribed with corresponding z, we are introduced to the important nomogram for

$$(1) \qquad f_1(z_1) g_3(z_3) + f_2(z_2) h_3(z_3) + f_3(z_3) = 0,$$

which is solved once for all as follows:

Set $$x = f_1(z_1), \quad y = f_2(z_2).$$

Then (1) becomes

$$g_3 x + h_3 y + f_3 = 0,$$

a system of straight lines z_3. The solution for, say z_2, when z_1 and z_3 are known, requires us to read the z_2 on the line passing through the point of intersection of the lines marked z_1 and z_3. The idea of the function scales on the axes is due to Lalanne and was given the name *anamorphose.*

From the fact that the vanishing of a third order determinant determines the concurrence of three straight lines developes the most general equation in three variables

$$(2) \qquad \begin{vmatrix} f_1 & g_1 & h_1 \\ f_2 & g_2 & h_2 \\ f_3 & g_3 & h_3 \end{vmatrix} = 0,$$

whose nomogram has three systems of straight lines. This nomographic representation gives rise to a beautiful application of the projective transformation in the plane to obtain any desirable disposition of the straight lines involved.

It may be well to state here that the general problem, when and only when any given equation is representable by any given type or group of types of nomograms, is still in the main unsolved. Fortunately however nomography finds perhaps its greatest use in engineering where a formula can frequently be identified, by inspection, with some standard type. The question is of mathematical interest however and has been successfully treated in part by E. Duporcq and Goursat.

The most general nomogram of *lignes concourantes* which handles an equation of three variables will be one having three systems of inscribed curves. Furthermore if, instead of graduating the X and Y axes with function scales of one variable, we use the equations

$$x = f_{12}(z_1, z_2), \quad y = f_{34}(z_3, z_4),$$

we are in a fair way to realizing a nomogram for

$$F(f_{12}, f_{34}, z_5) = 0.$$

It remains only to adopt a wing nomogram on each axis to segregate their respective pairs of variables. The axes are then said to have each a *binary scale*.

It is to D'Ocagne himself, in 1884, that we owe "Représentation nomographique par points alignés." He points out clearly the origin in the idea of duality and still continues the constructions with line coordinates. The nomogram of three systems of straight lines gives place to one of three *curved scales*, the key to which is to read the third value where the line joining the given two cuts its scale. The line is of course a moveable straight edge. The advantage gained is appreciable with very little practice. For American engineers the idea of duality and use of parallel coordinates will be unattractive, but the results may be obtained entirely without the use of either as the author points out in Remarque II, page 229.

From this point of view the equation (2) is regarded as determining the alignment of the points defined by

$$x = \frac{f_i}{h_i}, \quad y = \frac{g_i}{h_i} \qquad (i = 1, 2, 3).$$

Such equations are classified by introducing the notion of *nomographic order* on which depends the number of curved and straight function scales. This idea of order is not in complete accord with that used by Soreau * and neither definition appears to determine the order of a given equation absolutely. There are other slight but unfortunate discrepancies in the notations of the two authors, for example, in the meaning of the term "*double alignement*."

A more complete discussion would require us to reproduce figures and extend this review unduly. The book is thoroughly readable and emphasizes fewer unessential features than the larger treatise. A larger number of examples would improve it; for instance in connection with the nomograms of two straight scales and a web of curves. The volume closes with a note on the general theory.

The second volume appearing in the title is a compact treat-

* R. Soreau, "Contribution à la théorie et aux applications de la Nomographie," *Mém. et Comptes rendus de la Soc. des Ing. civils*, 1901 (tiré à part).

ment with many cuts and pictures of the following six topics: 1) Les instruments arithmétiques; 2) Les machines arithmétiques; 3) Les instruments et machines logarithmiques: 4) Les tables numériques (barèmes); 5) Les tracés graphiques; 6) Les tables graphiques (nomogrammes ou abaques). The volume also includes an interesting introduction and many historical notes.

L. I. HEWES.

Theorie der elliptischen Funktionen. Von H. DURÈGE. In fünfter Auflage neu bearbeitet von LUDWIG MAURER. Leipzig, B. G. Teubner, 1908. 436 + viii pp.

IT is difficult to understand on what ground this work can appropriately be called a new edition of Durège's well-known and admired book, since scarcely a trace of the original seems to have been left.

The first edition appeared in 1861, and was followed by successive editions at intervals of ten years or less, until the fourth in 1887, which was hardly more than a reprint of the third. There can be little doubt as to the inadvisability of further revising this work, now over twenty years old in its latest form, and which represents the state of the theory substantially as it was at the time of Jacobi. Since then this field has been transformed by the new theories of Weierstrass and Riemann, and has been more or less modified or influenced by other lines of mathematical activity such as the theory of groups and the Galois theory of equations. Moreover, the recently developed elliptic modular functions and the still more general class of the automorphic functions afford an extension or generalization which has not only placed the elliptic functions themselves in a new light, but has laid stress on their properties when the periods are regarded as additional independent variables.

It would evidently be quite out of the question to engraft all of these new methods and ideas on to the older theory as expounded by Durège, and Professor Maurer has started *de novo* in his treatment without attempting, as far as we can observe, to incorporate any of the material of the older work except that of course the jacobian functions are given their proper share of attention. The Weierstrassian functions and methods, however, predominate, and the influence of the great work of Klein and Fricke on the elliptic modular functions is observable throughout.