

Fite has translated another article by Lebon in which the computation and use of a larger table, with base  $30,030 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$  instead of 2310, are described.\*

In the factor tables of Glaisher, the numbers are listed and opposite to each is given the smallest prime divisor of the number. Lebon's idea is to divide the numbers prime to a given base  $B$ , e. g.,  $B = 30,030$ , into classes of the form

$$KB + I \quad (K = 0, 1, 2, \dots)$$

according to the values of their residues  $I$  with respect to  $B$ . The possible prime divisors  $D$  less than  $B$  are listed, and opposite to each and under each  $I$  is placed the characteristic  $k$ , if there is any, which specifies the smallest number divisible by  $D$  of the class belonging to  $I$ . It can then be readily discovered whether or not any other given number in the  $I$ -class is divisible by  $D$ , since for such a number  $K - k$  must be divisible by  $D$ .

In the article which is the subject of this review, the characteristics with respect to the base 2310, for prime divisors from 13 to  $\sqrt{30,030}$  have been listed, so that the table can be applied to the factorization of numbers between 1 and 30,030. If the characteristics for prime divisors up to 2310 had been given, the table would have been applicable to numbers as large as  $(2310)^2 = 5,336,100$ . The table described in the article translated by Professor Fite could be used to factor numbers between 1 and  $(30,030)^2 = 901,899,900$ . The factor tables already published by others, according to Lebon's statement in his introduction, give the factors of numbers from 1 to 10,000,000. All except those by Glaisher for the fourth, fifth, and sixth millions are out of print.

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*Serret's Lehrbuch der Differential- und Integralrechnung.* Dritte Auflage, neu bearbeitet von GEORG SCHEFFERS. Zwei Bände: I, xvi + 624 pp.; II, xiv + 585 pp. Leipzig, B. G. Teubner, 1907.

SERRET's name, which is in the title of this well known work, bears about the same relation to the edition under review that Webster's name bears to the latest edition of Webster's dictionary.

\* "Theory and construction of tables for the rapid determination of the prime factors of a number," BULLETIN, vol. 13 (1906-7), p. 74. The original article appeared in the *Comptes Rendus*, vol. 151 (1905), p. 78.

The original "Cours de calcul différentiel et intégral," appeared in the seventies and Harnack's translation into German was published in 1884. In 1897-99 a revised edition of this translation by Bohlmann appeared. This second edition, while perhaps the best-known text-book on the calculus in Germany, suffered greatly from the many typographical and mathematical errors. The figures were poorly drawn, many of them being altogether wrong. An incomplete list of over two hundred of these errors was published in connection with the volume on differential equations which was the third volume of the work.

This third edition, by Georg Scheffers, is not only revised, but entirely rewritten and in part rearranged. All the figures have been drawn anew and many new figures added. The theorems have been stated more clearly and more use has been made of italics to make them stand out from the body of the text. Many more references to previous paragraphs have been inserted.

Some of the more important changes which have been made in this edition are the following. The articles on number have been remodeled according to Dedekind's theory and the proofs of the theorems on continuous functions and on the convergence of series have thus been given a real backbone. A new chapter on implicit functions, with a thorough discussion of the functional determinant and of the independence of functions and of equations has been inserted. The difficulties of maxima and minima of functions of several variables receive considerable detailed discussion and the limitations of the parameter method of discussing an extreme value are clearly shown. Many improvements and corrections have been made in the chapters dealing with the application of the calculus to geometry. For instance, the plus or minus sign in the expression for the curvature of a space curve has been removed and in all cases where the square root occurs, a discussion of its sign has been given. The chapter on the complex variable has been entirely rewritten and shortened, the subject matter being limited to just what is necessary for the beginner to read understandingly the remainder of the work and the volume on differential equations which is now in preparation. The rather weak proof of the fundamental theorem of integral calculus in the second edition has been revised and completed. A proof of the theorem, "If  $f(x, a)$  is a continuous function of  $x$  and  $a$ , then the integral

$$F(z, a) = \int_{x_0}^z f(x, a) dx$$

is a continuous function of  $z$  and  $a$ ," is added to the chapter on definite integrals.

Harnack's appendix on Fourier's series and Fourier's integral has been kept unchanged, but several pages of explanatory notes have been inserted in the form of an introduction to the appendix. The valuable notes and references to other works, given at the end of the volumes in the second edition, have been omitted in the third edition for the rather insufficient reason that they might discourage the student.

This calculus is a geometer's calculus. Over three hundred of the twelve hundred pages are devoted to applications to geometry. With the exception of the paragraphs on center of gravity, there are practically no references to mechanics or physics. All the problems given are worked out in detail. In fact, detail is one of the features of the work. The reviser rejoices in saying that as far as possible he has eliminated from the text such phrases as "the reader will easily see," "the proof is left to the student."

Profiting by the example of the second edition, this edition is quite free from typographical errors. The few found by the reviewer are unimportant. But there is one which may be misleading. The symbol  $\Delta$  in the second volume is not from the same font of type as that in the first volume, and the difference in the two is sufficient to cause confusion to a beginner.

A detailed table of contents and a copious index make the work very valuable as a reference book, and Serret's *Lehrbuch* will no doubt continue to be one of the most used books on the shelves in the mathematical reading room at Göttingen.

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*Vorlesungen über mathematische Statistik (Die Lehre von den statistischen Masszahlen)*. Von ERNST BLASCHKE, Professor an der Technischen Hochschule in Wien. Leipzig, Teubner, 1906. 263 pp.

THE present work has to do with the theory and application of statistical constants (*statistische Masszahlen*). It opens with views of different writers as to the field to be included under mathematical statistics. While it contains but little that is new in the line of theory, nearly every point is accompanied by an