two such sextics, and $u_a^2 = 0$ the conic carrying the related involution, then the envelope

$$(clu)^6 = 0$$

is the triply counted conic: $(clu)^6 \equiv \rho \cdot (u_a^2)^3$. So for the similarly related pairs of higher order.

VASSAR COLLEGE.

CANTOR'S HISTORY OF MATHEMATICS.

Vorlesungen über Geschichte der Mathematik. By MORITZ CANTOR. Vierter Band. Von 1759–1799. Leipzig, Teubner, 1908. 8vo. vi + 1113 pp. 32 Marks.

IT is a long period that Moritz Cantor looks back upon to the time when he made his first noteworthy contribution to the history of mathematics. He was little more than a boy when he published his Inaugural-Dissertation "Ueber ein weniger gebräuchliches Coördinaten-System" in 1851,—only twenty-two years old; and it was only five years later that he entered upon his life work as historian of mathematics, by the publication of his paper "Ueber die Einführung unserer gegenwärtigen Ziffern in Europa." He was thirty-four when his first noteworthy treatise on the history of mathematics appeared, the "Mathematische Beiträge zum Kulturleben der Völker" (Halle, 1863). And next August he will be eighty years old, with over a dozen books to his credit, with hundreds of memoirs, with a notable record as editor of the *Zeitschrift für Mathematik und Physik* and the *Abhandlungen zur Geschichte der Mathematik*, and with academic honors and governmental recognition commensurate with the great work that he has accomplished.

For a man of nearly eighty to undertake a work of the magnitude of this fourth volume of the Geschichte would seem foolhardy, were he not such a man as Professor Cantor. When the third volume was completed in 1898 it seemed proper that it should be called the last, as was the case, and under all the circumstances any man might have been well content to say his *nunc dimittis.* That Professor Cantor was not thus content is a cause of gratification to all who are interested in the history of mathematics, and a worthy lesson to all who feel that threescore years mark the bounds of man's working life.

The present volume is not the work of Cantor himself in the same way as the earlier part of the history, although it was planned and edited by him and contains one chapter from his own pen. He has brought to his assistance several of the best men in Germany and in other countries, each an expert in the line upon which he has written. The following are the chapters and authors: Histories of mathematics, by S. Günther; Arithmetic, algebra, and theory of numbers, by F. Cajori; The combination theory, doctrine of probabilities, series, and imaginaries, by E. Netto; Elementary geometry, by V. Bobynin; Trigonometry, polygonometry, and tables, by A. von Braunmühl; Analytic geometry of the plane and of space, by V. Kommerell; Perspective and descriptive geometry, by G. Loria; Infinitesimal calculus, by G. Vivanti; Total and partial differential equations, sums and differences, and the calculus of variations, by C. R. Wallner; Survey of the period from 1758 to 1799, by M. Cantor.

Professor Günther's article relates to the histories of mathematics, including monographs and editions of the classics, published during the period in question. The chapter is particularly valuable for the reason that we have no worthy bibliography of the subject in any period preceding the founding of *Bibliotheca Mathematica* and the *Jahrbuch über die Fortschritte der Mathematik,* and this particular period is practically the one in which serious historical investigation begins. A rather careful examination of this chapter fails to show the omission of any names of importance, and the chapter will stand as a worthy bibliography of the period. One or two minor misprints have been noticed, but they are not worth mention at this time.

It is a matter of gratification to Americans that Professor Cajori should appear as one of the collaborators in this work. To him was assigned the subject of arithmetic, theory of equations, and theory of numbers. The period has not been carefully studied before, and it is apparent even on first reading that Professor Cajori's best work has been done here. He has gone to a sufficient number of original sources to trace the development at first hand, and his secondary authorities are standards. Not the least valuable is the brief but comprehensive sketch of early American textbooks. Some points are open to criticism, and naturally so when secondary authorities are taken. For example, Trenchant (1566) commonly used miliar for 10^9, and miliar de miliars for 10^{12}. As to the date of Ruffini's Teoria generale delle equazioni, upon which Professor Cajori raises a question, it is not improbable that the authorities giving 1798 (and the excellent Elogio by Giuseppe Bianchi might well have been included) are quite as correct as those giving 1799. In my own copy the title pages of both volumes originally read MDCCXCVIII, but in both cases an additional I has been added, perhaps by a type held in the hand. It is probable that some questions may arise with respect to a few of the other dates, particularly of memoirs, for so much material has been used that slight errors are to be expected.

Netto's contribution begins with the so-called kombinatorische Schule, founded by Hindenburg whose first publication on the subject was made at the age of thirty-nine, although Æpinus, van Schwinden, and Euler had already made some use of the idea in connection with the binomial series, the last named having suggested the common German symbolism for the coefficients and having stated two well-known problems in the theory. In the doctrine of probabilities the author shows our great indebtedness to D'Alembert, who, building upon the work of Jakob Bernoulli, contributed much more to the theory than has commonly been supposed. The work of Laplace has generally been appreciated, probably because of the extensive use of least squares, and possibly because of the interest immediately aroused by his study of vital statistics, and Professor Netto has given an excellent resumé of this phase of Laplace's work. In the study of series the beginning is made with the work of Landen. This is followed by the contributions of Euler, Daniel Bernoulli, Pfaff, and other continental writers, and also of the too generally neglected English scholars, Vince, Waring, and Hellins. In the subject of imaginaries Professor Netto also gives much more credit to D'Alembert than has generally been the case. He shows that Euler first used i for $\sqrt{-1}$, in his Institutiones calculi integralis appearing the words, "formulam $\sqrt{-1}$ littera i in posteriorem designabo." As to the graphic representation of complex numbers he gives the credit, of course, to Wessel.

Professor Bobynin begins his chapter on elementary geometry by a discussion of textbooks, showing very effectively by table the great advance of Germany and France, in this phase of education, over the other European countries, and especially over England. Here the influence of D'Alembert again stands out prominently, and Bobynin's appreciation following immediately that of Netto suggests that in spite of the praise that

D'Alembert has generally received he is entitled to still more recognition. It is, however, to Legendre that the author assigns the greatest honor, and with good reason, and the essential features of his revision of the Euclid sequence are carefully set forth. The second topic in this chapter is on practical geometry or surveying, and here the noteworthy advance naturally centers around the establishing of the basal unit of the metric system. We are already in possession of an excellent treatise on this subject in the work of Bigourdan, but Professor Bobynin has approached it more definitely from the geodetic standpoint. The third topic relates to specific propositions, and particularly to Lexell's contributions. The fourth topic considers the theory of parallels, necessarily closing before the modern Lobachevsky-Bolyai developments.

The contribution of the late Professor von Braunmühl is along the line of his great life work, the history of trigonometry. It begins with the period of Euler and his contemporaries, the same as is covered, often more fully, in the author's history, chapters 4 and 5. The date of Blake's birth, on the second page, is wrong, by the way; it should be 1708 instead of 1718. The period is one of peculiar interest, covering a good part of Euler's most prolific years, but the treatment is necessarily more condensed than in von Braunmühl's larger work. The second topic relates to efforts to improve the teaching of trigonometry, especially as to its foundation principles. The third part relates to tetragonometry, polygonometry, and polyhedrometry, and the fourth part to tables, cyclometry, and trigonometric series. In several places the latter part of von Braunmühl's contribution is more complete than his history.

Professor Kommerell's chapter on analytic geometry of the plane and of space covers a field that has been less worked than that of the preceding chapter or two. He begins with some general notes on conics and then takes up higher plane curves, proceeding from Waring's Proprietates algebraicarum curvarum, of which he gives a careful resumé. The period was one of great activity in this line of work, and some fifty pages are assigned to the subject. Space curves and surfaces are next discussed, beginning again with Waring, this part of the treatment also covering about fifty pages.

Professor Loria's article is divided into four topics. The first treats of perspective, but unlike the other contributors the author has found it necessary to begin with the middle ages on account of omissions in the earlier volumes of the history. He brings out in a clear if not new light the indebtedness of mathematics to the graphic arts in the fifteenth and sixteenth centuries, and makes further mention of the work of Desargues in the seventeenth century. His second topic is The golden period of theoretic perspective, in which he begins with the work of s'Gravesande and properly pays high tribute to the genius of Lambert. The third topic touches briefly upon the immediate predecessors of Monge, and the fourth devotes some fifteen pages to the great founder of descriptive geometry.

The treatment of the infinitesimal calculus by Professor Vivanti opens with a consideration of the foundations of the subject, carrying the work on from where it was interrupted in volume III. The second topic deals with the textbooks of the period. The third is subdivided, the first part treating of processes of differentiation, in which the work of Pfaff is most prominent, and the second treating of integration, a subject particularly fertile of results in this period. The fourth topic relates to definite integrals, beginning and necessarily chiefly concerning itself with the contributions of Euler. The fifth topic relates to the analytic applications of the infinitesimal calculus, beginning with the English contributors to the theory of maxima and minima, and including the development of indeterminate forms and series. Professor Vivanti's contribution closes with an exhaustive discussion of transcendents and elliptic integrals, some eighty pages in all, beginning with Vandermonde's treatment of the theory of factorials, but chiefly devoted to the theory of elliptic integrals. These functions, while first suggested somewhat earlier, had their greatest development in this period, and chiefly at the hands of Euler. While the subject has already been well treated by Enneper and by Bellocchi, the author has examined at first hand so much source material that his own work must be used to supplement that of his two predecessors.

Professor C. R. Wallner's contribution to the history of total and partial differential equations, differences and sums, and the theory of variations, is one of the most exhaustive in the work. While the theory of differential equations had already been established before 1750, their solution at the outset had been looked upon as means to an end rather than an object in itself. It was during the latter half of the eighteenth century, the period covered by this volume, that the theory came to its complete independence, and chiefly through the influence of Euler, D'Alembert, Danie Bernoulli, Lagrange, Laplace, and Legendre.

Professor Cantor's own contribution is largely bibliographical, consisting of a list of the most important works mentioned by his collaborators. He makes the same assignment of date to Ruffini's first work on equations of the fifth degree as made by Professor Cajori. It is evident that the printing of the work began in 1798 and was completed in 1799.

It is too early to enter into critical details of such a work. That numerous errors will be found is certain, as witness the partial list given by Müller, and the fact that the *Bibliotheca Mathematica* gives every month a list of corrections extending back to the first edition of the first volume, published nearly thirty years ago. On the other hand it will be a long time before any one will attempt to treat so exhaustively this remarkable period in which the genius of Euler, D'Alembert, Lagrange, Laplace, and Legendre showed at its best, and in which Gauss was beginning the labors that placed his name among the leaders of modern times. DAVID EUGENE SMITH.

SHORTER NOTICES.

High School Algebra. By H. E. SLAUGHT and N. J. LENNES. *Elementary Course*, 1907, vii + 297 pp., $1.00; *Advanced Course*, 1908, vii + 194 pp. $0.65. Boston, Allyn and Bacon.

THE common weakness of our college students in elementary algebra shows a great need for improvement in the teaching of this subject. The appearance of these admirable texts, constructed after a new model, marks a distinct advance in the teaching of algebra in our high schools. Recent discussions have shown that it is quite generally agreed that high school algebra should be divided into a first course of elementary algebra during the first year, and a second course of review and advanced algebra during one half of the third or fourth year, preceded by one year of plane geometry. The authors have met this demand of teachers by dividing their text into two parts, and in doing so have succeeded in presenting the subject of high school algebra in the most concrete and teachable form we have yet seen.