It is safe to state that the student who has mastered the contents of both volumes will be in a position to read technical literature with intelligence, and it is equally safe to say that no student can consider himself equipped to work in any corner of the mathematical field without an equivalent body of knowledge ready at hand, or without the spirit of careful inquiry which pervades Professor Czuber's two volumes.

L. WAYLAND DOWLING.

Traité de Mathématiques générales à l'Usage des Chimistes, Physiciens, Ingenieurs, et des Élèves des Facultés des Sciences. Par E. FABRY. Avec une preface de G. DARBOUX. Paris, Hermann, 1909, 8vo. 10 + 440 pages.

DURING the last few years the requirements for the "diplôme de licencié ès sciences" have undergone considerable change. The tendency has been to imitate somewhat the German custom and allow the student greater freedom in the choice of subjects taken for this diploma. Under the present system each faculty issues certificates for the work taken in that faculty and a student who has obtained three of these certificates, whether at the same session or at different sessions, has the right to a diploma. Previously there were only three modes of obtaining this diploma, but under the new regulations Darboux tells us that at Paris alone it can be obtained in 1771 different ways.

Among the certificates most frequently issued those of general mathematics are certainly included. This course in general mathematics forms a sort of transition between the mathematics of the Lycée and the advanced mathematics of the University. Such a course has always been necessary for the student of mechanics and physics and is fast becoming necessary for the student of chemistry. It was to meet this new demand for a course in general mathematics that Professor Fabry undertook the difficult task of writing a book that should meet the needs of the engineer, physicist, and chemist and at the same time serve as a preliminary training for those who expect to pursue the study of mathematics for its own sake. Such a book must combine rigor with simplicity and in this lies the difficulty of the task. It seems that the author has succeeded remarkably well in combining these two things. To be sure at times the subject becomes quite abstract, but the student of applied science can omit these parts without in the least marring the course for him.

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The book is divided into four parts: Algebra, Analytical Geometry, Analysis, Mechanics.

Part I (128 pages) starts with a discussion of incommensurables, limits, and continuity, leading to the idea of the maximum of a function $f(x)$ which remains finite between a and b. In the first 46 pages is given all that the student will need of the binomial theorem, determinants, series, and exponential functions. The subject matter of these introductory chapters is remarkably well chosen and only those things are given which are really needed for further developments. In the chapter on exponential functions and logarithms the number e is defined by the series

$$e = 1 + \frac{1}{1} + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \cdots$$

and no attempt is made to show that $(1 + 1/n)^n$ approaches e as n becomes infinite. The function e^x is then defined and the convergence of the series discussed.

Derivatives are defined and discussed in Chapter VI, page 46, and thus early in the work the student has this tool at his disposal. It was rather curious to note that the symbols dy, dx are used to denote increment instead of the old familiar ones Δy, Δx. The formulas of differentiation and Rolle's theorem are disposed of in 6 pages.

The derivation of Taylor's series in its finite form is rather interesting. Given the function $f(x)$ which possesses the first $n + 1$ derivatives, the following expression is written down

$$F(x) = f(b) - f(x) - (b - x)f'(x) - \frac{(b-x)^2}{\underline{|2}} f''(x) - \cdots - \frac{(b-x)^n}{\underline{|n}} f^{(n)}(x) - A(b-x)^p,$$

where p is a positive integer and

$$A = \frac{1}{(b-a)^p}\left[f(b) - f(a) - (b-a)f'(a) - \frac{(b-a)^2}{\underline{|2}} f''(a) - \cdots - \frac{(b-a)^n}{\underline{|n}} f^{(n)}(a)\right].$$

Now since $F(a) = 0$, $F(b) = 0$, and $F'(x)$ exists for all values

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of x between a and b, we have by Rolle's theorem

$$F'(x) = -\frac{(b-x)^n}{\underline{|n}} f^{(n+1)}(x) + Ap(b-x)^{p-1} = 0$$

for some value x_0 of x between a and b; and as $b - x_0 \neq 0$

$$A = \frac{(b-x_0)^{n-p+1}}{\underline{|n}} f^{(n+1)}(x_0).$$

Equating this to the value of A above and making the substitutions

$$a = x, \quad b = x + h, \quad x_0 = x + \theta h \qquad (h < 1),$$

we have Taylor's series in its finite form. The remainder is of the form

$$\frac{h^{n+1}}{\underline{|n}} \cdot \frac{(1-\theta)^{n-p+1}}{p} f^{(n+1)}(x + \theta h).$$

If $p = n + 1$, we at once have Lagrange's form of the remainder. If $p = 1$, we obtain Cauchy's form of the remainder.

Chapter VII is devoted to applications of differentiation; but as no geometry has yet been developed, about the only applications made are to maxima and minima and the evaluation of indeterminate forms. Chapter VIII treats of development in series. Uniform convergence is defined and criteria are given for the differentiation of series term by term. The expansion of a function by means of Taylor's series is discussed and the expansion is applied to a few particular functions. In Chapter IX functions of several variables are taken up. It is remarkable that the subject should be developed thus far (80 pages) before the first figure appears. It would seem extremely difficult for the beginner to grasp the full meaning of much of the abstract mathematics without some sort of a geometrical picture. It is rather unfortunate that the whole subject of maxima and minima should be developed without the use of a single graph.

Chapter X is a short introduction to the study of imaginaries, and the remainder of Part I is devoted to algebraic equations, rational fractions, elimination, etc.

Part II, Analytical geometry (145 pages) contains the elements of both plane and solid analytical geometry besides most of the applications of differential calculus to geometry contained in elementary treatises on the calculus. Multiple points,

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asymptotes, unicursal curves, curvature, torsion, osculating plane, ruled and developable surfaces are among the topics discussed. The chapters on the conic and quadric are short and yet contain the important properties of these curves and surfaces. The conics are discussed purely from the equation and not as a locus problem. One chapter however is devoted to geometric loci, in which the general idea of a locus is developed and such curves as the strophoid, cissoid, and cycloid are discussed as examples in loci. The idea of a multiple point is introduced after 28 pages. It seems as if it might have been better to delay this until the student had a little more familiarity with curves in general.

Part III, Analysis (85 pages), begins with a discussion of infinitesimals and differentials. Here the notation is changed, Δy now being used to represent increment and dy to represent differential. In the first chapter the handling of differentials is pretty thoroughly drilled into the student. The expression for radius of curvature which has already been derived in terms of derivatives is now expressed in terms of differentials in both rectangular and polar coordinates. The definite integral is then defined and it is shown that the limit of $\Sigma m_i\delta_i$ is the same no matter how the interval be divided. It is then shown that the limits of $\Sigma m_i\delta_i$ and $\Sigma M_i\delta_i$ (M_i and m_i being the maximum and minimum of $f(x)$ in the interval δ) are the same, if $f(x)$ is a continuous function. This is rather abstract mathematics for students of applied science, but the author has a scientific conscience to satisfy, and besides it will be useful to those who expect to continue the study of pure mathematics. In a few pages the ordinary rules of integration are developed. The evaluation of the definite integral is then taken up and the criteria that the integrals

$$\int_a^b f(x)dx,$$

where $f(x)$ becomes infinite between a and b, and

$$\int_a^\infty f(x)dx$$

should exist, are given. Evaluation of a definite integral by expanding the integrand and integrating term by term is shown by means of a single illustrative example. Multiple integrals

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are defined and the method of evaluating them is given without a rigorous demonstration, and applications are made to area, volume and surfaces of solids. In Chapters VI and VII line and surface integrals are discussed and Stokes's and Green's theorems are derived.

The last twenty pages of Part III are devoted to differential equations. Here again the author shows his ability to present the essentials of a subject clearly and in a small space. The linear equation, singular solution, systems of equations and partial differential equations, all come in for their share in these few pages. The student of applied science will find here about all the differential equations ordinarily necessary for his purpose.

Part IV, Mechanics (60 pages). In these few pages the student will find an introduction to theoretical mechanics. The subject is opened by a discussion of velocity and acceleration and the representation of the acceleration by means of the hodograph. The components of the acceleration along the tangent and principal normal of the curve of motion are then formed, which leads to the equation

$$\gamma^2 = \left(\frac{dv}{dt}\right)^2 + \frac{v^4}{\rho^2} \quad [\rho \text{ being the radius of curvature}].$$

By means of this equation various kinds of motion are discussed. Motion expressed in terms of polar coordinates is then taken up and the expression for the acceleration is derived. The instantaneous center of curvature is introduced both for plane and solid figures and the parametric equations of its path are obtained. This first chapter then closes with a good discussion of relative motion. The chapter contains only eight pages and from the amount of material which it contains some idea can be formed of the amount of mechanics which the author compressed into sixty pages. The following chapters are entitled respectively, Dynamics of a free point; Dynamics of a point not free to move; Center of gravity; Dynamics of systems. The whole treatment is concise and to the point and mathematical in all its details.

At the end of each chapter there are a few well chosen examples. The book closes with a table of formulas which contains, besides the principal results derived in the text, also all the most important trigonometric and geometric formulas useful to the student.

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