

SHORTER NOTICES.

Correspondance d'Hermite et de Stieltjes, publiée par les soins de B. BAILLAUD, H. BOURGET. Avec une Préface de ÉMILE PICARD. Vol. I, xx + 477 pp.; vol. II, vi + 464 pp. 8vo. Paris, Gauthier-Villars, 1905.

It is all too seldom that the mathematical public gets even a furtive view of the private life of its masters, or is allowed to penetrate into their ateliers and observe how their illustrious works are elaborated. In other fields of science, in literature and art, we are more fortunate. How great would be our loss if we knew nothing of a Darwin, a Goethe, or a Michel Angelo except their finished works. It is true that mathematicians are not altogether destitute of information regarding the inner life of their leaders. Who can read the biographies of Hamilton, Abel, or Jacobi, or the letters of Gauss to Schumacher and Bessel without being touched and edified?

The present collection of letters which passed between Hermite and Stieltjes between the years 1882 to 1894 forms a very precious accession to the mathematician's scanty library of personalia, and will be welcomed by all those who love to associate with the works of a great man the man himself.

A sketch of Hermite's life has already been given in the BULLETIN,* we will therefore devote here only a few lines to the all too brief career of Stieltjes. The son of a distinguished civil engineer, Thomas-Jean Stieltjes was born at Zwolle, Holland, December 29, 1856. He studied at the Ecole Polytechnique of Delft, where his great talents were recognized both by teachers and classmates. In spite of his exceptional abilities he did not graduate, being prevented by an instinctive dislike for competition, which seems to have pursued him all his life. In 1877 he entered as computer the Observatory of Leyden and later took part in the work of observation. But soon celestial mechanics and the higher mathematics began to absorb his attention, and interfere to such an extent with his routine duties, that he entertained the very bizarre idea of giving up his position (although living then in straitened circumstances), and coming to America to study under Sylvester, who was lecturing at the Johns Hopkins University.

* Vol. 13 (1906-7), pp. 182-190.

At this critical moment (1882) he had the good fortune to come under the beneficent influence of Hermite.* This was brought about by a letter, afterwards published in the *Comptes Rendus*, in which Stieltjes communicated to Hermite some of his results on the coefficients of the development of the perturba-tive function when the inclination of the orbits is considerable. Other letters followed, and soon the most friendly relations were established between the veteran Hermite and the youth-ful Stieltjes. His passion for mathematics now became so absorbing that Stieltjes the following year (1883) gave up his post in the observatory and devoted himself entirely to his favorite study. Numerous papers were the results of his efforts, and public recognition came in the form of the doctorate, honoris causa, from the University of Leyden, and an election to the Academy of Amsterdam.

But his hopes to an academic career received a cruel blow by his failure to obtain the vacant chair in mathematics at the University of Groningen, although the favorite candidate of the faculty. Writing to Hermite (March 13, 1884) he modestly states: "Probablement la raison aura été que n'ayant point eu l'occasion de suivre le chemin ordinaire, je n'ai point obtenu un grade à l'Université."

In 1885 Stieltjes left Holland for good, and took up his abode in Paris. He became a French citizen, and took his doctor's degree at the University of Paris the following year (June, 1886) presenting a thesis, "Sur les séries semi-conver-gentes." By virtue of his great talents, and supported by the powerful influence of Hermite, Stieltjes obtained at once a posi-tion as chargé de cours at the University of Toulouse, and three years later he received the professorship there. But his ardu-ous and incessant labors began to undermine his health. In 1892 and 1893 he was obliged to pass the winters in the mild climate of Algiers, but without permanent benefit. He died the last day of the year 1894, with a brain teeming with fertile ideas and projects unfulfilled.

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The letters number somewhat more than 400, and are alto-gether charming both on account of the frankness and freedom of their style, and on account of the great variety of the topics

* In 1892 he writes Hermite: "J'ai perdu mon père bien avant, en 1878; cette date est, pour moi, la fin d'une jeunesse heureuse; la période qui l'a suivie a été bien plus tourmentée, et pas sans difficultés."

discussed. A large share of the earlier letters treat of the theory of numbers, particularly those parts connected with the elliptic functions. Both Hermite and Stieltjes had a strong arithmetical bent. "Je vois aussi, Monsieur, que vous êtes un ami de l'arithmétique, et que vous partagez mon admiration pour Gauss et Eisenstein" writes Hermite in one of his first letters. Another time he writes to Stieltjes "Je confie maintenant à votre cœur arithmétique . . ." Stieltjes was much interested in Riemann's work on prime numbers and devoted considerable labor to prepare a doctor's thesis on this subject.* We therefore find an occasional discussion of Riemann's ζ function, supplemented in the appendix by four noteworthy letters to Mittag-Leffler. As is well known, Hermite first showed the transcendence of e , and a number of letters treat of this and π . In reply to Hermite's praise, Stieltjes writes with characteristic modesty "Quant à ma démonstration de la transcendence du nombre e , elle est évidemment fondée entièrement sur vos principes, et je n'ai fait que généraliser votre démonstration." Stieltjes efforts relative to π were without result and in a later letter he is "heureux d'avoir échappé au nombre π qui m'a causé un vrai cauchemar."

Although both Hermite and Stieltjes were deeply interested in the higher arithmetic, by far the larger part of their correspondence relates to analysis. Naturally the theory of continued fractions, to which Stieltjes contributed so much that is fundamental, is often treated. From a letter to Hermite we know that Stieltjes began their study as early as 1877 or 1878; while for Hermite such algorithms had formed a familiar instrument of research for two score years. We can well understand the benevolent interest that the aged Hermite took in the brilliant discoveries of his young friend in this field, and it is pleasing to hear Hermite break out in such heartfelt praise as this: "Vous êtes un merveilleux géomètre, les recherches nouvelles sur les fractions continues algébriques que vous me communiquez sont un modèle d'invention et d'élégance; ni Gauss, ni Jacobi ne m'ont jamais causé plus de plaisir."

A great deal of space is occupied in discussing Legendre's functions of the first and second kind, questions of development, asymptotic expressions, roots of $Q_n(x)$, etc.

Klein's article of 1890 on the roots of the hypergeometric function at once attracted Stieltjes' attention and he writes

* He afterward selected another subject, as stated above.

Hermite: "La démonstration de M. Klein est extrêmement intéressante, elle repose entièrement sur la conception des feuilles multiples de Riemann et des considérations géométriques. Cela prouve bien l'utilité de ce mode de représentation, mais je vous dirai que la géométrie y intervient trop, à mon gout. . . . Dès que j'ai vu l'énoncé du théorème je n'ai pu m'empêcher de penser qu'on pourrait y arriver par la méthode de Sturm en s'appuyant sur les relations entre les fonctions F contiguës, etc." Hermite replies "Mais la communication extrêmement intéressante que vous m'avez faite du beau théorème de M. Klein . . . m'arrache à la torpeur qui m'a envahi depuis plusieurs jours. Je vous chicanerai au sujet de votre prétention de le démontrer par la méthode de Sturm . . . Il me semble que cette méthode n'a jamais été appliquée et n'est applicable. . ." Whereupon Stieltjes in his next letter playfully begins to develop his method with: "Maintenant, permettez moi de défendre mon idée de démontrer le théorème de M. Klein à l'aide de la méthode de Sturm." A subject of unfailing interest especially in the later letters is the Γ function in all its astonishing and intricate relations. Mechanical quadrature and asymptotic developments are also favorite topics.

Thus we might go on describing these delightful mathematical causeries which follow one another in rapid and unaffected succession, but we hope that the above will give an idea of the rare treat these letters will afford.

JAMES PIERPONT.

Cartesian Plane Geometry. Part I.: *Analytical Conics.* By C. A. SCOTT, D.Sc. London, J. M. Dent and Company, 1907.

IF this most excellent book is really meant to be a text-book for schools, we are afraid that some one will now have to make a school for the text-book. If it is meant for the libraries of teachers and prospective teachers, it is as near perfection as one could wish; full of the best ways of doing things, of excellent examples and of inspiration for every real teacher. So, whatever faults we may see in the book are those which arise when we consider it as a function of the things it is meant to be as a text-book.

The book, rather unprepossessing in its outward appearance, contains 428 pages, a great number of illustrative examples, and 700 problems. Its thirteen chapters, which of course are