will continue to derive most of their working knowledge. Moreover the insistence on the numerical side of the subject, to which we have referred, is only one feature of the book. Long sections of a decidedly theoretical character follow in which affine and perspective transformations of the plane play an important part. Indeed the ellipse, hyperbola, and parabola are introduced as the images of the circle under these transformations, and the whole theory of these curves is made to depend on this point of view. Homogeneous coordinates are also treated at length in the last chapter.

On the whole the book departs less essentially than one would at first suppose from the traditional German text-book in which the subject is presented rather from the point of view of *kennen* than *können*. It is still a body of doctrine which is presented, though the choice of subjects is somewhat unusual, including as it does, besides the subjects already mentioned, a section on the computation of stresses in frameworks of light rods, and an introduction to some of the most elementary aspects of the use of vectors. Even the questions of numerical computation may fairly be regarded as part of this scheme rather than as an attempt to put the student on his own mettle. To a teacher collecting material for a course on numerical computation the section to which reference has just been made will be found most useful. MAXIME BÔCHER.

Gruppen- und Substitutionentheorie. By Dr. EUGEN NETTO. Sammlung Schubert LV. G. J. Göschen, Leipzig, 1908. viii + 175 pp.

IN conformity with the general plan of the Sammlung Schubert Professor Netto aims to give in this book an introduction to the theory of groups of finite order. He has succeeded admirably in his purpose. Those readers who are not already familiar with the details of the theory will find Chapter II particularly valuable in fixing for them the fundamental notions of the subject, if they take the pains to work through the details. Indeed we do not know if there is another place where this particular phase of the subject is treated so happily. But this is by no means the only good chapter. They are all excellent and the book as a whole is a fine example of clear and attractive exposition.

The author introduces some new notation which is of value in the interest of brevity. On page 35 the greatest common subgroup of the groups $G, H, K, \cdots$ is denoted by the symbol $\} G, H, K, \cdots \{$. The largest subgroup of a group G within which a given subgroup J of G is invariant is called the "Zwischengruppe" of J in G (page 54). New notations in the interest of precision of statement are found on page 58.

The book is not free from misprints, some of which we note. In lines 8 and 7 from the bottom of page 45 H and G should be replaced by H and Γ respectively. The inequality sign in line 7 from the bottom of page 88 should be reversed. The c_λ on page 102, line 3, should be replaced by σ_λ. In line 5 from the bottom of page 103 J_1 should be in the place of H_1. The latter part of the first formula on page 169 should be

$$(p^\kappa - 1)\, p^{\kappa-1}\, (p^{\kappa-1} - 1)\, p^{\kappa-2}\, r_{\kappa-2}.$$

Netto's definition of a group (page 1) is exactly Dickson's definition of a semigroup.* If only a finite number of elements are under consideration, as is the case practically throughout the book, the two notions coincide. But some of the illustrations of groups given on page 2 are, according to Dickson, and also according to de Séguier,† not groups at all, but semigroups.

The book contains a few slight inaccuracies that should be noticed. The statement at the beginning of § 116 applies obviously only to groups of composite order. Moreover, the word transitive in this statement is redundant. The summing up of § 54, page 71, should not apply to the factors of composition $r/r_1, r_1/r_2, \cdots, r_{\nu-1}/r_\nu$. The statement near the bottom of page 102 that there is a $(p^\beta, 1)$ isomorphism between G and Γ is not true in general. The $(s, 1)$ isomorphism that is mentioned near the bottom of page 139 in reality exists between G and D, and not between G and S, as the author states it. As is well known, there are five primitive groups of degree 5. In the enumeration of these on page 156, the cyclic group is omitted.

The definition given of the *class* of a substitution group (page 128) implies that this term is applied only to k-fold transitive groups, where $k>2$. This is not the definition given by Jordan.‡

The statement on page 123, in regard to solvable groups of

* *Trans. Amer. Math. Soc.*, vol. 6 (1905), p. 205.

† Éléments de la théorie des groupes abstraits, p. 8.

‡ *Liouville's Journal*, ser. 2, vol. 16 (1871), p. 408. A somewhat different definition is given by Jordan in the *Comptes Rendus* of the Paris Academy, vol. 73 (1871), p. 853.

order $p_1^{a_1}p_2^{a_2}$ could, in view of Burnside's results,* have been made much more general.

This enumeration of a few points in which we think the book might be improved should not be understood as detracting from our statement at the beginning of this review commending the book. In publishing so excellent a treatment of the subject Professor Netto has performed a service of value to the mathematical public.

W. B. FITE.

Einführung in die höhere Mathematik. Von EMANUEL CZUBER. Teubner, 1909. 382 pp.

THIS book is an amplification of the lectures on differential and integral calculus given by Professor Czuber at the technical school in Vienna. The treatment has been extended so as to form a good introduction to higher mathematics, adapted for students other than technical students. The subjects are developed with much care and rigor.

There are essentially three divisions, viz., Functions of real variables, Algebra, and Analytical geometry. The opening chapter is on real and imaginary numbers. The number concept is developed for use in the functions of a real variable. The second chapter is a short, concise, and elegant presentation of infinite series and products. Besides simple demonstrations the subject is made easier for the student by the many well-chosen examples to illustrate the points in question.

Chapters III and IV begin the theory of functions of real variables. The general idea of a function, limit of a function, and continuity are the principal topics discussed. Here again we find many well-chosen examples. The function

$$f(x) = \lim_{n=\infty} \frac{nx + 2}{nx^2 + 1}$$

is given to show the difference between the value of a function given by direct substitution and that obtained by the limiting process. The substitution of $x = 0$ gives $f(0) = 2$, while if we first proceed to the limit and then put $x = 0$ we have $f(0) = \pm \infty$.

The example

$$f(x) = \frac{x}{1 + e^{1/x}}$$

is a good illustration of a function which is continuous for $x = 0$, but whose derivative is discontinuous at this point.

* *Proceedings Lond. Math. Society*, series 2, vol. 1 (1904), p. 388.