the relatively prime pairs α, β and a, b. We shall prove this fact in the following paragraph.

Since G is abelian $(s_1s_2)^\alpha = (s_2s_1)^\beta = (s_1s_2)^\beta$, and hence $(s_1s_2)^{\alpha-\beta} = s_1^{\alpha-\beta}\, s_2^{\alpha-\beta} = 1$, or $s_1^{\alpha-\beta} = s_2^{\beta-\alpha}$. Combining this equation with $s_1^a = s_2^b$, there results $s_1^{a(\alpha-\beta)} = s_2^{a(\beta-\alpha)} = s_2^{b(\alpha-\beta)}$, and hence

$$s_2^{(a+b)(\beta-\alpha)} = 1 = s_1^{(a+b)(\alpha-\beta)}.$$

As the orders of s_1, s_2 are limited and these operators must be commutative, this proves that *only a finite number of groups can be generated by two operators which satisfy both of the equations*

$$(s_1s_2)^\alpha = (s_2s_1)^\beta \quad \text{and} \quad s_1^a = s_2^b,$$

where α, β and a, b represent two pairs of relatively prime numbers. For instance, when these numbers are 4, 5 and 2, 3 G is the group of order 5. That is, if s_1, s_2 satisfy both of the equations

$$(s_1s_2)^4 = (s_2s_1)^5, \; s_1^2 = s_2^3,$$

they must generate the group of order 5. This result establishes close contact between the present note and the paper "On groups which may be defined by two operators satisfying two conditions," *American Journal of Mathematics*, volume 31 (1909), page 167.

A NOTE ON IMAGINARY INTERSECTIONS.

BY PROFESSOR ELLERY W. DAVIS.

In the plane let there be a conic C and a line L. Set up a system of coordinates such that L is the line infinity, its pole O with regard to C is the origin, the axes OX and OY are conjugate with regard to C, while X and Y are their intersections with L. Furthermore let $x = \pm 1$, $y = \pm 1$ be tangents to C through Y and X respectively. Then $x =$ a constant passes through Y, while $y =$ a constant passes through x. All these lines are to be determined by the fact that any four convergents form a harmonic set when the constants in the right member are a harmonic set of numbers. In brief, C, L, and the coordinates are projectively transformed from a circle $x^2 + y^2 = 1$, the line infinity, and a rectangular system whose origin is the center of the circle. The equation of any line in the transformed coordinates is precisely the same as that of which it is the projection in the rectangular coordinates.

In the transformed coordinates, let us represent the point with complex coordinates $(x' + ix'', y' + iy'')$ by a vector from (x', y') to $(x' + x'', y' + y'')$. If we change to another pair of conjugate axes OX_1, OY_1 the coordinates are changed to $x_1' + ix_1''$, $y_1' + iy_1''$, but the vector is left unchanged since its end points are.

Consider, however, the intersections of C with L. These are given by $x' = \infty$, $x'' = 0$, $y' = 0$, $y'' = \pm ix'$. That is, the points x', $\pm y''$ are the intersections of $y = \pm x$ with L.

But the lines $y = \pm x$ change when we change the axes as above. Thus the vectors taken to represent the intersections of C with L also change.

If with Cauchy we represent $(x' + ix'', y' + iy'')$ by a vector joining the real points on the circular rays through it, we meet the same difficulty. For these real points are $(x' + y'', y' - x'')$ and $(x' - y'', y' + x'')$; that is, in the case under consideration, they are $(\infty, 0)$ and $(0, 0)$ so that the direction changes as OX does.

Von Staudt's representation is, however, unimpaired; since his imaginary intersection is the involution of all the point pairs cut out on L by the involution of all the line pairs $y = \pm x$. The vector representation should then be regarded as a symbol, adapted to the particular coordinates used, for the more complete Von Staudt representation.

UNIVERSITY OF NEBRASKA,
July 16, 1909.

MAUROLYCUS, THE FIRST DISCOVERER OF THE PRINCIPLE OF MATHEMATICAL INDUCTION.

BY DR. G. VACCA.

Introductory Note. — Soon after the publication of my review of Voss's address (BULLETIN, volume 15, page 405), wherein I considered at some length the history of mathematical induction, I received a note from Professor Moritz Cantor of Heidelberg, in which he called my attention to Dr. Vacca's research on this same historical topic. As Dr. Vacca's research was not accessible to me, I wrote to him for information and received, in reply, the following notes, which will doubtless be of general interest to American readers. — FLORIAN CAJORI.

Many years ago I published in the *Formulaire de Mathé-*