respects it can be commended highly to those who are attracted by marvellous relations among natural numbers. The author is looking forward to a second edition in which a number of slight errors will be corrected, and he has had the courtesy to send the reviewer a marked copy in which the following changes are suggested : The term "perfect square" as used on page 2 is replaced by "regular square." In the second and third lines from the bottom of page 5 "twenty-eight" and "sixteen" are replaced by twenty and eight respectively, and in the first line of page 6 "twelve" is replaced by sixteen. The term "prime number" as used on page 14 and in many other places in the book is replaced by primary number. In the last line on page 65 the expression "first and last" should read last and first. Near the middle of page 179 the statement marked I. should be followed by "with four exceptions." These errors are, however, not sufficiently serious to detract much from the value of the volume.

G. A. MILLER.

Exercices et Leçons d'Analyse. By R. D'ADHÉMAR. Paris, Gauthier Villars, 1908. 208 pp.

THE subtitle of this volume is "Quadratures, équations différentielles, équations intégrales de M. Fredhoim et de M. Volterra, équations aux dérivées partielles du second ordre." It will be seen that the topics treated are thoroughly up to date. The book is meant, as the author says in his preface, to supplement the larger Traités and Cours. An introduction of 22 pages presents a brief statement of some of the principal theorems on differential geometry and analysis, together with references for their proofs and for further developments. Then follow chapters on quadratures; the functions of Legendre, Bessel, Euler, etc.; partial differential equations of the elliptic type, including a brief treatment of Fredholm's integral equation ; equations of the hyperbolic and parabolic types ; and two chapters on miscellaneous problems. The book, in spite of its decidedly fragmentary character, will prove useful both by furnishing a source of interesting problems and by giving the reader at least a superficial idea of many recent developments in analysis. The indications given as to the scope and purport of theorems mentioned (to say nothing of their proofs) are, however, frequently so meagre that the reader seeking to gain information will often be in doubt as to how they should be interpreted. Occasionally an actually misleading statement occurs, as when on page 116 in discussing the essential difference between the Cauchy-Kowalewski existence theorem for Laplace's equation and the "problem of Dirichlet" it is at least strongly implied that the former theorem does not apply to closed curves, whereas it applies exactly as well to closed curves as to open ones, but in both cases, and this is the essential point, to only a small neighborhood of the curve.

Judiciously used by a person who is able to perceive that he does not understand a thing when that is the actual case, the book will prove a source of inspiration.

MAXIME BÔCHER.

Analytische Geometrie auf der Kugel. Von Dr. RICHARD HEGER. Sammlung Schubert, LIV. Leipzig, G. T. Göschen, 1908. 12 mo. vii + 152 pp.

THERE is a spherical trigonometry; why not also a spherical analytical geometry? This question interested mathematicians towards the end of the eighteenth century and the beginning of the nineteenth, and there resulted numerous papers published in the periodicals of the time. Certain problems had been solved at an earlier date and others have appeared up to within a decade ago. It is the object of the author of the little volume before us to bring this material together in a convenient form and to arrange it along the lines of the usual text upon plane analytic geometry. The book begins by explaining several coordinate systems upon the sphere. Dr. Heger adopts that one in which the homogeneous coordinates of a point are the sines of the angles whose arcs are drawn perpendicularly from the point to the sides of the spherical triangle of reference. This triangle of reference is assumed to be trirectangular. The homogeneous coordinates of a great circle are taken to be the point coordinates of one of its poles. Many of the formulas are exactly the same as the analogous formulas in plane geometry. For instance, the necessary and sufficient condition that a point (x, y, z) lie upon a great circle (u, v, w) is

$$ux + vy + wz = 0.$$

Small circles and conics in general are represented by quadratic equations. There is a theory of poles and polars and of tangents, all of which is analogous to plane analytic geometry.