with pupils of the ages named. Such an example is furnished in Borel's texts. As Stäckel remarks, this publication is intended only for teachers. Since the reform movement in France and Germany is essentially the same as in the United States, the book under review, coming from authors of distinction, cannot fail to be of interest to American readers.

FLORIAN CAJORI.

A Treatise on the Mathematical Theory of Elasticity. By A. E. H. LOVE, M.A., D.Sc., F.R.S. Second edition. Cambridge University Press, 1906. xvii + 551 pp.

THE first edition of this important work was published in two volumes in 1892 and 1893. The present edition is a new treatise which contains some extracts from the old one. The object of the book is threefold, namely, to be useful to engineers, to set forth the physical notions and analytical processes which are also used in other branches of physics, and to afford a complete picture of the present state of the science of elasticity. The book commences with an excellent historical introduction which explains the parts taken by various eminent mathematicians in establishing the theory. The first four chapters are concerned with the analysis of strain and stress, the equations of equilibrium and small motion, the expression of the stresses as functions of the strains, and the connection between the mathematical theory and technical mechanics. Chapter V opens with a useful recapitulation of the essential parts of the preceding chapters and the author proceeds to illustrate them by a number of simple examples which are needed for the subsequent development of the subject. In Chapter VI there is a discussion of the elastic constants. The 6 components of stress are linear functions of the 6 components of strain and hence depend on 36 constants. The law of conservation of energy reduces the number of constants to 21. The hypotheses of Navier and Cauchy concerning the constitution of matter (according to which bodies are regarded as made up of material points which are supposed to act on each other so that the mutual action between each pair of points is along the line joining them and is a function of the length of the line) leads to 6 relations which are called Cauchy's relations, so that the number of constants is reduced to 15. As the author remarks in the historical introduction, our views concerning the constitution of matter have changed so that the

argument as to whether the number of constants is 21 or 15 holds a subordinate position. The development of the atomic theory in chemistry, of statistical molecular theory in physics, and the discovery of electrical radiation, have shaken our confidence in the hypothesis of central forces between material points. In the case of crystals possessing certain kinds of symmetry the number of constants may have various values from 21 down to 2 for an isotropic body. Chapter VII discusses the uniqueness of a solution and also Betti's reciprocal theorem, which subsequently proves to be of great importance in developing a general method of solution. The various singular solutions analogous to $1/r$ in ordinary potential theory are next obtained, and an interesting application follows, namely, Hertz's theory of the distribution of pressure between two bodies in contact. Chapter X opens with an excellent resumé of the theory of potential which explains the two lines of attack, namely, the method of a series of harmonic functions and the method of singularities or the use of Green's functions. The author here states that little progress has been made with the existence theorem of elasticity. Shortly after the present work appeared however, Tedone published in the Encyklopädie der mathematischen Wissenschaften a valuable account of the memoirs dealing with the existence theorem and remarked on the probability of using Fredholm's integral equations. Lauricella and others have since successfully applied integral equations to the problem and the new book on the Equations of Fredholm by d'Adhémard contains an account of these researches. Professor Love proceeds to give a full account of Betti's method of integration and of Cerruti's application to the problem in which the boundary is a plane. In the next chapter the problem of the sphere is discussed by means of harmonic functions. Tedone has solved the same problem by the use of Green's functions. After discussing the vibrations of solids, the author takes up the important practical problems of the torsion and flexure of beams and regrets the slowness of engineers in adopting the exact methods of solution. The sections of a beam do not usually remain plane, with the curious result that the greatest stress is not at the point furthest from the center. The remainder of the book is devoted to the theory of deformable bodies and is substantially identical with the second volume of the first edition. In the case of thin rods the theory is fairly simple, although a

long analysis is required to justify the approximation that the bending couple is proportional to the curvature. In the cases of plates and shells the necessary analysis is still more lengthy. The problem is complicated by the fact that the bending is usually accompanied by stretching particularly near the edges. The book concludes with an account of the important practical problem of the stability of cylindrical shells. It is perhaps needless to say that the treatise can be heartily commended both as a text-book and a book of reference. A German edition was published by Teubner in 1907.

F. R. SHARPE.

Les Découvertes modernes en Physique. Par O. MANVILLE. 2ème édition. Paris, A. Hermann et Fils, 1909. 463 pp.

ONLY a year after the first edition of Manville's short book of 182 pages on Les découvertes modernes en physique a second edition was needed. The author evidently did not have to contend with costly electrotype plates in which the publisher would allow few changes, for he has practically written a new book about three times the size of the first — the term second edition is really a misnomer. The new work is divided into two parts, entitled Electricité et matière and Les ions et les électrons dans la théorie des phénomènes physiques — La matière et l'éther. This entire rewriting and expansion of the original is very fortunate. The state of fundamental electrical theory is to a considerable extent still speculative, and experiments which reveal new and sometimes nearly crucial results are still of frequent occurrence. To write at all on this subject brings with it the liability and desirability of rewriting after the lapse of a very short period.

From the title of the work we might be inclined to fear that the author had written a popular and unreliable essay on the wonders of recent discoveries. Fortunately this is by no means the case; many chapters contain considerable hard physics and more or less hard mathematics, which require and repay close application on the part of the reader. The presentation, however, let it be stated, deals with a vast variety of interrelated physical data after the manner of the experimental physicists rather than with the broad mathematical groundwork of electrical theories as treated by such theorists as Larmor, Lorentz, or Minkowski. Sooner or later, theory and experiment in regard to atomic electricity will probably be well knit together;